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ACCUMULATION, INNOVATION AND CATCHING-UP:
AN EXTENDED CUMULATIVE GROWTH MODEL

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Abstract
This paper presents an extended model of cumulative growth in which the effects of innovation and catching-up are considered. The effect of innovation adds another source of cumulative growth to that of the traditional models and allows for the consideration of the importance of non-price factors as determinants of international competitiveness. Catching-up, on the other hand, is the major force leading to convergence in productivity due to the effect of the diffusion of technology. The model allows for analysing whether cumulative forces may lead to stable growth and whether this solution generates convergence in productivity levels. The structural model is then tested for a set of OECD countries over the period 1965 to 1994 and the results are used for carrying out the comparative dynamics. The results support the view that both the Verdoorn-Kaldor mechanism, and the induced effect of innovation on export performance, are important cumulative forces that interact with the effect of catching-up towards a convergent pattern of growth.

JEL Classification: O3, O4, F43.

Keywords: Cumulative Growth, Innovation, Catching-up

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1. Introduction

The idea of cumulative causation as a process in which the subsequent occurrences reinforce the initial conditions was initially developed by Veblen (1915), although it was not until Myrdal (1957) that it was applied to explain the different performance of countries and regions in terms of growth and development. Kaldor (1966, 1970), based on the ideas of Myrdal, developed the idea of the existence of a cumulative growth mechanism due to the existence of dynamic increasing returns as represented by the Verdoorn-Kaldor mechanism. The basic idea of Kaldor was that growth is demand-led and, particularly, export-led. The growth of output due to the growth of exports would induce a higher increase in the growth of productivity that would feed through into a lower rate of growth of prices. This would improve price competitiveness allowing for a higher growth of exports and, thus, re-starting the process. Kaldor’s arguments were verbally expressed and it was not clear whether his conclusions implied that it was the level of per capita income or the growth rates that would diverge over time. Ever increasing differences in growth rates are most unlikely and not compatible with the observation of the real world. It was not until Dixon and Thirlwall (1975) that the model was formalised. It was shown that under some plausible conditions it would lead to stable differences in rates of growth that may imply diverging per capita incomes. The cumulative mechanism implied by the Verdoorn-Kaldor relationship plays the role of maintaining the growth process and exaggerating growth differences if initially two regions have different growth rates due to different structural characteristics. The Kaldor-Dixon-Thirlwall (KDT) model has been extended and tested in several works of Amable (1993),

Some of the mentioned works already point out that the basic model can be improved on the basis of two arguments. First, it is a model of growth in which the growth rates differences are constant and, hence, it does not allow for the existence of declining (or increasing) growth rates over time. In other words, no reference is made to the relationship between the rate of growth and the level of per capita income (or productivity) and, thus, the analysis of whether there is convergence or divergence in productivity levels is not possible. Secondly, there is no explicit reference to the important role of non-price factors that determine competitiveness, as already pointed out by Kaldor himself (see Kaldor, 1978). In this regard, the role of innovation and the diffusion of technologies seems of crucial importance (see Fagerberg, 1988 and Amable, 1992). Recent developments in growth theory have emphasised the possible beneficial effects of innovation activities and the role of catching-up as major determinants of the growth performance of countries and regions.

In the model in this paper we will allow for the introduction of most of the variables that recent growth theory and empirics emphasise as fundamental in explaining the development and the degree of competitiveness of nations. The effects of learning-by-doing, innovation, education and catching-up will be integrated into the KDT model. This will generate a richer set of dynamics than the traditional cumulative growth models and allow for determining whether there is a stable growth pattern and if this leads to convergence (partial or total) or divergence in productivity levels. It will be shown that cumulative growth models, far from being old-fashioned, allow for the introduction of explicit technological progress variables and give a plausible explanation of the recent growth performance in developed countries. The paper is organised as follows: section 2 presents and discusses the structure of
2. An Extended Cumulative Growth Model

This model is an extended version of Dixon and Thirlwall (1975) that introduces technology variables along the lines of Amable (1993) and De Benedictis (1998). These variables are similar to those emphasised in the “new growth theory” analysis, but a different interpretation will be given to them. As will be shown below, there are several cumulative forces that may lead to divergent growth that interact with the effect that the catching-up - due to the adoption of foreign technologies – has on leading to convergence. Five equations can describe the relations at work. The first relation states that the growth of output \( (y) \) depends on the growth of exports \( (x) \):

\[
y = \theta x, \quad \theta > 0
\]  

which is the dynamic version of Harrod’s foreign trade multiplier and Hick’s super-multiplier (see McCombie, 1985). Implicitly it is assumed that exports are the most important autonomous component of aggregate demand and that it allows for the expansion of all other components of demand without incurring balance of payments problems (Thirlwall, 1979).\(^1\)

The growth of exports, in turn, depends negatively on the growth of relative prices \( (p - pf) \), and positively on world income growth \( (z) \), the investment-output ratio \( (I/O) \) and a technology variable to account for non-price factors \( (K) \) reflecting the flow of innovations that affect export performance.

\[
x = \eta(p - pf) + \varepsilon z + \zeta K + \delta(I/O), \quad \eta < 0, \varepsilon > 0, \zeta > 0, \delta > 0.
\]  

---

\(^1\) It is possible to introduce a balance of payments equilibrium condition into the model without substantially changing its conclusions.
The first two variables on the right hand side of the export equation correspond to the usual specification of an export function expressed in rates of growth. The introduction of the investment-output ratio as a proxy for capital accumulation is due to the fact that the capacity of an economy to deliver in the international markets depends on the growth of the physical equipment and infrastructures (Fagerberg, 1988). This variable may also capture the effect of embodied technical progress on export performance. Innovation is a key factor affecting the non-price competitiveness of economies. Product differentiation and quality competition characterise the modern international trade. These factors determine the national-specific competitiveness and are different from those dependent on the product composition of exports (Amable, 1992). The former will be reflected in the innovation variable, while the latter are captured by the income elasticity of demand for exports (ε). A country’s ability to differentiate and compete in quality will crucially depend on the degree of innovation of its productive structure, which is reflected in the innovation variable introduced in the export equation (K).

Third, it is assumed that prices are set in imperfectly competitive markets, where the pricing rule is a mark-up on unit labour costs. That is, the level of prices \( P \) is determined by the usual Kaleckian mark-up relationship, i.e. \( P_t = (W_t / R_t) \cdot T_t \), where \( W \) is the level of money wages, \( R \) is the average product of labour and \( T \) is one plus a percentage mark-up over unit labour costs. Assuming, for simplicity, a constant mark-up, we obtain that prices grow as the difference between money wage growth and productivity growth:

\[
p = w - r
\]

The fourth relation of this version of the KDT model determines the rate of growth of labour productivity \( r \). One major determinant of productivity growth stressed in Kaldorian growth models is the induced effect of output growth, that is, the Verdoorn-Kaldor mechanism. This mechanism is responsible for the circular nature of the growth process in the
KDT model. The Verdoorn-Kaldor mechanism reflects the existence of dynamic economies of scale due to increased specialisation (Young, 1928) and embodied technical progress (Kaldor, 1957) and also the existence of static increasing returns. This implies that it is output growth that directly determines productivity growth which, in turn, will increase output growth due to improved price competitiveness. Embodied technical progress is explicitly captured in this model by the introduction of the investment-output ratio \( I/O \) as a second determinant of productivity growth. The third determinant of productivity growth is innovative activity \( K \). Innovation not only leads to a higher degree of product differentiation and quality but also to process innovation leading to increased productivity. The final determinant of productivity growth is the productivity gap \( GAP \). The existence of productivity differences between the frontier economy and the followers opens up the opportunity for imitation and diffusion of more advanced technologies generated by the leader. Authors such as Gerschenkron (1962), Abramovitz (1986), Baumol (1986) and Gomulka (1990) have all stressed this idea. In a simplified version, it implies a positive effect of the productivity gap on the productivity growth of the follower economies, leading to a potential catch-up in productivity levels. Thus, the productivity growth equation stands as follows:

\[
r = \phi y + \lambda (I/O) + \alpha K + \sigma GAP, \quad \phi > 0, \lambda > 0, \alpha > 0, \sigma > 0.
\]

The final set of relationships defining our model is that determining innovative activity or the flow of new national innovations. This will depend on four factors. First, on the rate of growth of output \( y \), reflecting the demand-led innovation hypothesis of Schmookler (1966).

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2 Recent empirical developments in the literature on Verdoorn’s Law have stressed its importance at the regional level. See, for instance, Fingleton and McCombie (1998), Harris and Law (1998) and León-Ledesma (2000). In an international context, however, it may well be the case that a single equation estimation of the Law suffers from a high degree of simultaneity bias.

3 For further qualifications of the concept of catch-up in cumulative growth models see Amable (1993) and Targetti and Foti (1997).
Secondly, on the rate of growth of the cumulative sum of real output \((q)\) as in Amable (1992) and de Benedictis (1998). This variable is a proxy for the effect of learning-by-doing originally formulated by Arrow (1962). Both the new products developed and the new production processes depend crucially on the effect of learning acquired through the accumulated experience of the workers. Thus, the higher the growth of accumulated experience – proxied by cumulative output – the more innovations will be incorporated in the production activities.\(^4\) The third major determinant of the success of an economy to generate innovations is the level of education of its working population \((edu)\).\(^5\) The level of education affects not only directly the capacity to innovate but also indirectly because it raises the ability of the economic system to assimilate and understand the new techniques of production. Finally, the productivity gap affects negatively the innovation activity of an economy. With a low level of development few resources are directed to research and development and patenting activities. In other words, the ability to innovate depends on the technological level of the country. Countries with a lower technological level are more likely to rely on the benefits of knowledge created in the leader economies.

\[\begin{align*}
K &= \gamma y + \beta q + \omega (edu) + \psi GAP, \quad \gamma > 0, \beta > 0, \omega > 0, \psi < 0,.
\end{align*}\]

The model is closed with the formal definition of both the cumulative output growth rate \((q)\) and the gap \((GAP)\) variables. If \(Y(t)\) is the level of output at time \(t\), we have:

\[
q = \frac{d\log \int_{t=0}^{T} Y(t) dt}{dt}.
\]

\(^4\) Note that, in this context, innovations do not necessarily mean the creation of new products or production techniques but also the marginal improvements in the existing ones.

\(^5\) The level of education and not its rate of growth has been introduced in the model due to the fact that the role played by education is wider than a simple human capital variable in a production function. This implies that a constant level of education ensures a constant flow of innovations due to the technical competence and skills of the working population.
The productivity gap is one minus the ratio of productivity between the follower \((P)\) and the leader economy \((P^*)\). The gap will be zero if there is no difference in productivity level and approach unity if productivity in the follower country is very low.

\[ \text{GAP} = 1 - \frac{P}{P^*} = 1 - G. \quad (7) \]

We can, thus, identify several forces in the model, some leading to divergence and others to convergence in productivity. On the one hand, the Verdoorn-Kaldor effect is a cumulative force that reinforces initial growth advantages (and disadvantages). This is also the same for the effect of demand-led innovation that affects both non-price and price competitiveness, and that has a similar effect to that of the Verdoorn-Kaldor mechanism. Learning-by-doing is another force that may make growth cumulative due to the positive effect of cumulative experience on non-price competitiveness and output growth. The final force acting towards a diverging growth pattern is the negative effect of the productivity gap on innovation that tends to perpetuate low levels of technological innovation. On the other hand, the catching-up effect arising from the flow of technologies from the leader to the follower economies is the main convergent force of the model. The final outcome will depend on the combination of these multiple effects and their relative power. It will be shown in the next section that possible outcomes of the model include total catch-up, partial catch-up and divergence from the leader. \(^6\) But, given the cumulative forces at work, can the model generate a stable solution for the rate of growth of output? The relevance of this question lies in the empirical fact that explosive behaviour of the growth of output is not observed in the real world.

\(^6\) The model even allows for the case of the follower economies forging ahead from the “old” leader once the catch-up has taken place.
3. Dynamics and Stability

In order to describe the dynamics generated by the relations put forward in the last section, we solve the model represented by equations (1)-(5) for the growth of output \( y \) and the growth of productivity \( r \). We obtain the following two equations:

\[
y = D - EG + Fq - Hr, \tag{8}
\]

\[
r = J - LG + Mq + Ny, \tag{9}
\]

where,

\[
D = \frac{\theta \gamma + \theta \eta (w - pf)}{1 - \theta \gamma} + \frac{\theta \varepsilon}{1 - \theta \gamma} z + \frac{\theta \omega}{1 - \theta \gamma} edu + \frac{\theta \delta}{1 - \theta \gamma} (I / O),
\]

\[
E = \frac{\theta \gamma}{1 - \theta \gamma} < 0, \quad F = \frac{\theta \beta}{1 - \theta \gamma} > 0, \quad H = \frac{\theta \eta}{1 - \theta \gamma} < 0,
\]

\[
J = \sigma + \alpha \psi + \lambda (I / O) + \alpha \omega (edu),
\]

\[
L = \sigma + \alpha \psi > 0, \quad M = \alpha \beta > 0, \quad N = \phi + \alpha \gamma > 0
\]

Differentiating (6) and re-arranging we obtain the following expression for the rate of growth of output:\(^7\)

\[
y = q + \frac{q}{q}. \tag{10}
\]

From the definition of the gap variable (7), we know that the rate of growth of the productivity ratio between the follower and the leader \( G \) is:

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\(^7\) Variables with a dot represent the time derivative, i.e. \( x = dx / dt \).
with \( r^* \) being the rate of growth of labour productivity in the leader economy.\(^8\) Substituting (10) and (11) into (8) and (9), we have the following system of first order non-linear differential equations:

\[
\begin{align*}
\dot{q} &= q[D - EG + (F - 1)q - Hr], \\
\dot{G} &= G[P - LG + M(q - q^*) + N(y - y^*)],
\end{align*}
\]  

(12) and (13)

where,

\[ P = \sigma + \alpha y + \lambda [\frac{1}{I/O} - (\frac{1}{I/O})^*] + \cos [edu - edu^*]. \]

The stability of the system (12) and (13) can be analysed through the stability of the system in brackets, ruling out the possibility that \( q \) and \( G \) are equal to zero (D-stability). In the steady state, \( \dot{q} = \dot{G} = 0 \), and given (10) and (11) then \( r = r^* \), \( y = q^* \) and \( y^* = q^* \). With the steady state solutions for \( y^* \) and \( r^* \) given in Appendix I we can obtain the following system representing the equilibrium paths of both \( G \) and \( q \):

\[
\begin{align*}
- LG + (M + N)q &= -T \bigg|_{G=0}, \\
- EG + (F - 1)q &= -S \bigg|_{q=0},
\end{align*}
\]  

(14) and (15)

where \( T \) and \( S \) depend on a set of exogenous variables and the parameters of the model \((\Theta)\):

\[ T = f(I/O, edu, pf, z, I/O^*, edu^*, w^*, \Theta), \]

\[^8\text{Variables with superscript }^*\text{ represent the original variable for the leader economy.}\]
\[ S = g(I/O, edu, pf, w, z, I/O^*, edu^*, w^*, \Theta) \]

Since all the elements in the off-diagonal of the Jacobian of the system (14) and (15) are positive, the stability conditions of this model require:

(i) \(-L < 0\), thus, \(L > 0\) or \(\sigma + \alpha \psi > 0\);

(ii) \(|f| > 0\), or \(\frac{L}{M + N} < \frac{E}{F-1}\), which implies that the slope of the phase path line for \(\dot{G} = 0\) has to be less than that for \(\dot{q} = 0\);

(iii) For condition (ii) to hold, it is required that \((F-1) < 0\).

The steady state equilibrium point is one where both productivity level differences remain the same and output growth is stable and equal in the leader and the follower economy. If catch-up is strong enough, during the transition output grows faster in the follower than in the leader. Once we approach the equilibrium, both rates of growth equal one another and productivity level differences remain stable. The parameters of the model determine where the follower stops catching-up and, thus, whether this process is absolute or just partial. The existence of per capita income convergence and a tendency for the rates of growth to be equal in the long run for the advanced countries is one of the growth facts reported in Evans (1996) and Temple (1999). Two possible stable cases of equilibrium can arise. These are depicted in figures 1 and 2, where the combinations of \(G\) and \(q\) that make \(\dot{q} = 0\) and \(\dot{G} = 0\) are represented. The first one is a stable focus (Figure 1), where the path taken towards the equilibrium gap and rate of growth of the cumulative output (and output
growth) generates cyclical behaviour. The economy oscillates around the equilibrium point until it is reached. The second case is a stable node (Figure 2). In this case, regardless of the initial point, the economy will follow a direct path towards its equilibrium solution, this adjustment being faster than in the former case. In Figure 1 we have depicted an equilibrium point where the laggard country catches-up with the leader and even forges ahead of it ($G > 1$). Contrariwise, Figure 2 shows a situation where the equilibrium only allows for a partial catch-up and, thus, differences in levels of productivity would be maintained through time. Of course, it would be possible to find cases of total falling behind ($G = 0$), if both lines do not cross before the $q$ axis. However, this is an implausible case especially for developed and emerging economies. It is also important to note that a positive value of the parameter of the GAP variable - or negative value of the parameter of $G$ - does not necessarily imply convergence in levels of productivity. The convergence in productivity levels will also depend on the endogenous cumulative mechanism linking the growth of output, learning-by-doing and innovation with productivity growth and price and non-price competitiveness. Despite the fact that the parameter $\sigma$ is positive, convergence may not be the outcome if the cumulative forces in the leader economy are stronger than in the follower. Note also that from the system of equations (1)-(5) we can obtain a reduced form for productivity growth:

$$r = a_{1} + a_{2}G + a_{3}(edu) + a_{4}(I/O) + a_{5}q + a_{6}(p - pf) + a_{7}z$$

(16)

This is similar to those convergence equations used when attempting to test the neo-classical hypothesis of convergence, controlling for the variables that determine the steady state level of productivity (see Barro and Sala-i-Martin, 1995). From the perspective of the model here

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9 This would be the case if the trace of the Jacobian of the system and its determinant are such that $(tr J)^2 > 4|J|$.

10 If $(tr J)^2 < 4|J|$. 
presented, a convergence equation that does not include the rate of growth of the determinants of exports would be mispecified. Thirlwall and Sanna (1996) have shown that one of the most robust variables influencing per-capita income growth in a convergence equation is the rate of growth of exports. This model gives a plausible theoretical explanation of these results and, on the other hand, shows that the Barro-type convergence equations are not necessarily a test of the neo-classical growth model. A test of the structural model underlying the reduced form convergence equation is necessary in order to compare the relevance of competing explanations of the growth and convergence phenomena.

Turning finally to the conditions under which the model presented is stable, it is possible to extract a set of economically meaningful conclusions. The condition (i) that \( L > 0 \) means that the net effect of the productivity gap on productivity growth must be positive, i.e. that the positive effect of technological catch-up is not offset by the negative impact of the gap on innovation and, in turn, productivity. Condition (ii) states that the effect of the productivity gap relative to that of the cumulative mechanisms has to be smaller for productivity growth than for output growth. Keeping the effect of the productivity gap constant, the greater the impact of cumulative forces on output growth \((F)\), the less stable the model will tend to be. This conclusion, together with condition (iii), calls for a limited impact of the cumulative forces on output growth. These conditions are a generalisation, in a continuous time extended system, of the stability conditions stated by Boyer and Petit (1991) in a Kaldorian cumulative growth model, i.e. that the sensitivity of output growth to productivity growth must be smaller than that of productivity growth to output growth. Otherwise, the model would be unstable, giving as a result an explosive behaviour, and this is, in the words of Gordon (1991), “too much cumulation”.

As an empirical illustration, in the next section we test the model represented by equations (1) to (5) for a set of OECD countries. This will allow for a measure of the impact
of the different variables considered and the relevance of this kind of model for developed economies. The parameters so estimated are used to analyse whether the stability conditions hold. If so, given the estimated values of the coefficients, we can undertake a comparative dynamics exercise in order to analyse the impact of changes in the exogenous variables on the equilibrium levels of output growth and the productivity gap.

4. An Empirical Test for the OECD Countries

As an empirical illustration of the workings of the model we have tested its structural form for a pool of observations of OECD countries. The equations estimated are (1) to (5). In the price equation (3) we have allowed for the parameters of $w$ and $r$ to differ from unity and in all the equations a constant term has been introduced. All the data used are from the same source, i.e. *OECD Statistical Compendium* (1997), except the level of education ($edu$) for which the average years of schooling of the population above 25 years were obtained from Barro and Lee (1993 and 1996). This ensures some homogeneity in the data in order to make cross-country comparisons. Unfortunately, not all the data were available for the complete set of OECD countries. For this reason, we have selected the following 17 countries: Australia, Austria, Belgium, Canada, Denmark, Finland, France, West Germany, Italy, Japan, Netherlands, Norway, Portugal, Spain, Sweden, UK, USA. The periods selected for the pooling are 1965-1973, 1974-1979, 1980-1988, 1989-1994. The main peak years of the business cycle and the starting and end years of the data available determined the periods. Five observations were missing due to the lack of reliable information for the innovation variable.

Details on the definition of the variables can be found in Appendix II. The innovation variable is not devoid of measurement problems. The problems arising when measuring the innovation activity of an economy are well known (see Cohen and Levi, 1989). When an output measure of innovation was used, such as the number of patent applications or grants,
problems of availability and homogeneity of the series were found\textsuperscript{11}. More homogeneous data were found for input measures such as R&D expenditure and personnel. In order to avoid the influence of military expenditure in R&D carried out by the government, we used as a proxy for innovation the ratio of R&D expenditure in the business sector to private investment. Nonetheless, input variables do not reflect the efficiency with which innovative activity is carried out, and value the same any financial expenditure in R&D regardless of its degree of importance in the innovative process. For this reason, the results reported should be interpreted with caution and may not capture adequately the forces at work.

In the model (1)-(5) all the equations are overidentified. The method used for estimation purposes was iterative 3SLS\textsuperscript{12}. The instrumental variables used were the rate of growth of foreign prices ($pf$), the rate of growth of world income ($z$), the investment-output ratio ($I/O$), the rate of growth of the total nominal labour cost ($w$), the productivity gap ($GAP$), the growth rate of cumulative output lagged one period ($q_{t-1}$) and the average years of education ($edu$). The estimations converged after 10 iterations. Given that an important part of the period of estimation covers the two oil crises\textsuperscript{13}, the model presented in Table 1 performs relatively well. All equations have an $R^2$ above 0.4, especially the price equation and the productivity equation. All the parameters take the expected sign. Only the intercept terms in equations (2), (4) and (5) and the $I/O$ and the innovation variable in equation (4) do not seem to be significant\textsuperscript{14}. Thus, the direct impact of innovation on productivity seems to be limited. The insignificance of the $I/O$ variable may be due to the multicollinearity with the

\textsuperscript{11} The data available from the UN, obtained from WIPO, only start in 1970. Furthermore, for many countries there exist years in which the number of patents double or treble due to changes in definitions and legislation about patents.

\textsuperscript{12} All estimation was carried out using LIMDEP 7.0.

\textsuperscript{13} See Boyer and Petit (1991) for an assessment of the breakdown of the cumulative model after the first oil crisis.

\textsuperscript{14} The cumulative output growth variable is not significant at conventional significance levels although it is significant at the 15\% level.
growth of output variable ($y$). Another link that seems not to be significant is the effect of output growth on the innovation variable, that is, the demand-led innovation. It is worthy of note the fact that the growth of labour costs is passed through into increased prices growth in a relation close to one-to-one. However, the productivity gains do not feed back into reduced prices growth to the same extent. These results recommend the relaxation of the assumption of a constant mark-up over unit labour costs in the empirical model.

The system estimation method used does not allow for a direct test of the validity of the instruments used and for undertaking diagnosis tests on the individual equations. For this reason we also carried out the 2SLS estimation method using the same instrumental variables. These results are reported in Table 2. The parameters values and their significance do not change as compared with the results reported in Table 1. The validity of the instruments is confirmed by the fact that the Sargan test rejects the hypothesis of mispecification in all cases except in equation (5). The effect of this equation may have ‘spill-over’ effects on the rest of the system when using system estimation methods. This may be the most plausible explanation of the convergence in 3SLS only after 10 iterations. From the rest of the tests reported we can only find some problems of heteroscedasticity in equations (3) and (5). In general, however, the model seems to be robust to the estimation method used.

It is important to note that the extensions made to the canonical Kaldorian growth model seem to be of relevance. The innovation variable affects exports positively, accounting for the importance of the innovation processes in determining non-price competitiveness. The catching-up effect is a significant variable in the determination of productivity and, by extension, output growth. The net effect of the gap variable on productivity growth is positive,

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15 Due to collinearity problems, the insignificance, and even incorrect sign, of the capital accumulation proxy is a common finding in the Verdoorn Law literature. See McCombie and de Ridder (1984).
which is a force leading to productivity convergence. Finally, the positive effect of education on innovation and physical investment on exports seems to be confirmed by the model. The interpretation of the effect of these variables, however, differs from that given in the “new growth theories”, where they are introduced in a production function framework assuming a resource constrained economy. In our model it is demand growth that leads output growth and, thus, the economy is not resource constrained. The impact of these variables on output growth, in our model, is due to their effect on demand growth. It is important to note also the strong and significant impact of output growth on productivity growth (with a value of $\phi$ of 0.642 in Table 1). The Verdoorn Law effect, estimated here in an integrated growth model, is highly significant, which highlights the importance of this mechanism for explaining productivity and employment trends in developed countries.\footnote{16 It has been suggested in the literature that, in an international context, due to simultaneity problems, the Verdoorn coefficient should be correctly estimated using a structural model capable of capturing the different interdependencies between the variables. See Bairam (1987) and Boyer and Petit (1991).} Taking the estimated values of the parameters, and substituting in the stability conditions given in section 2, it is confirmed, in both sets of results, that the model leads to a stable solution, i.e. $L > 0$, $L/(M + N) < E/(F - 1)$ and $(F - 1) < 0$.

With the stability assured, we have carried out the comparative dynamics with the values of the parameters obtained using 3SLS.\footnote{17 The results using the values of the 2SLS estimations do not change.} The results of changes of the exogenous variables on both $G$ and $q$ are summarised in Table 3, where the effect on the independent terms $T$ and $S$ of equations (14) and (15) are reported, together with the effect on $G$ and $q$. The results of the exercise are as expected. There are positive effects of the $I/O$ and education variables, and a negative effect of the growth of labour costs on both $G$ and $q$.\footnote{18 Note that a positive effect of any variable on $G$ means a reduction of the productivity gap between leader and follower economies.} However, the
effect of the growth of world income needs further explanation. An increase of the rate of growth of world income leads to a reduction of the productivity gap but also to a lower steady state level of cumulative output (and output growth). This is due to the fact that the effect of the reduction of the gap on productivity and, thus, on output growth is stronger than the combined effect of this change on exports and output growth of the follower. When the economies achieve the steady state position, the reduced impact of the gap on productivity is only compatible with a lower rate of growth of output. This is not to say, of course, that in short and medium run fluctuations a higher rate of growth of world income will affect negatively the growth of the follower economies. The positive impact of $z$ on the productivity gap may explain the fact that the convergence phenomenon is stronger in expansive phases of the cycle. The same arguments as with $z$ may be applied to explain the negative effect of the growth of foreign prices on $G$ and $q$. Nevertheless, some of the net effects have a very low value and, more important, depend crucially on the values of the parameters of the innovation variable. For this reason, the results of this exercise should be taken only as an illustration of the possible outcomes of the model presented.

5. Conclusions

In this paper we have presented an extended version of the canonical Kaldorian cumulative growth model. The model allows for the introduction, among other factors, of technology variables such as innovation and technology gaps that have been stressed as important factors determining the growth performance of modern economies. It allows for the analysis of productivity convergence generating a richer set of dynamics than the traditional cumulative growth models. It has been shown that the model, under some non-restrictive conditions, can generate a stable pattern of growth. Contrary to the popular idea of cumulative growth generating ever increasing differences in per capita output and productivity levels, a
growth process generated by this kind of dynamics is compatible with the existence of catch-up from the followers to the leader economy.

A growth equation similar to those used in recent growth empirical exercises can be derived from the structural form of the model. This fact recommends the estimation of the structural form, in order to avoid second order identification problems that impede discriminating among competing theories of growth. In doing so, the model seems to perform well for the set of industrialised countries analysed, giving a plausible explanation of differences in growth performance. Cumulative growth arises from the effect of the Verdoorn-Kaldor relationship and also from the induced effect that growth itself has on learning and non-price competitiveness. The diffusion of technologies arising from the productivity gap, however, is a significant force that counteracts against these forces favouring a catch-up process.
REFERENCES


Figure 1: Stable focus

Figure 2: Stable node
### TABLE 1

**3SLS Estimation of the Model for the OECD Countries, 1965-1994**

<table>
<thead>
<tr>
<th>Equation</th>
<th>Expression</th>
<th>$R^2$</th>
<th>SSR</th>
<th>n</th>
<th>DW</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>$y = -0.817 + 0.650 \cdot x$</td>
<td>0.40</td>
<td>0.008</td>
<td>63</td>
<td>2.42</td>
</tr>
<tr>
<td></td>
<td>$(1.677)** (7.914)*</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2)</td>
<td>$x = -0.023 - 0.227 \cdot (p-pf) + 1.501 \cdot z + 0.001 \cdot (I/O) + 0.845 \cdot K$</td>
<td>0.49</td>
<td>0.020</td>
<td>63</td>
<td>1.96</td>
</tr>
<tr>
<td></td>
<td>$(1.079) (2.864)* (6.467)* (2.451)* (2.309)*</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\text{Wald rst } \chi^2(1) = 0.186$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(3)</td>
<td>$p = 0.011 + 0.932 \cdot w - 0.374 \cdot r$</td>
<td>0.95</td>
<td>0.004</td>
<td>63</td>
<td>2.27</td>
</tr>
<tr>
<td></td>
<td>$(4.012)* (36.84)* (4.293)*</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(4)</td>
<td>$r = -0.015 + 0.642 \cdot y + 0.0002 \cdot (I/O) + 0.617 \cdot K + 0.021 \cdot GAP$</td>
<td>0.51</td>
<td>0.006</td>
<td>63</td>
<td>1.76</td>
</tr>
<tr>
<td></td>
<td>$(1.213) (6.019)* (0.649) (0.404) (2.113)*</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(5)</td>
<td>$K = 0.019 + 0.499 \cdot y + 0.044 \cdot q + 0.0033 \cdot edu - 0.022 \cdot GAP$</td>
<td>0.42</td>
<td>0.012</td>
<td>63</td>
<td>0.77</td>
</tr>
<tr>
<td></td>
<td>$(1.235) (0.583) (1.619)*** (3.077)* (1.670)**</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Notes:**
1. Method of estimation 3SLS; convergence achieved after 10 iterations.
2. Absolute t-statistics in parentheses; *, **, *** denotes significant at the 5%, 10%, 15% significance level respectively.
3. SSR is the sum of squares of the residuals.
4. Wald rst is the Wald test of the common parameter restriction on $p$ and $pf$.
**TABLE 2**

2SLS Estimation of the Model for the OECD Countries, 1965-1994

<table>
<thead>
<tr>
<th>Equation</th>
<th>Equation Formula</th>
<th>Sargan $\chi^2$</th>
<th>LM test autocorrelation $\chi^2(1)$</th>
<th>LM test normality $\chi^2(2)$</th>
<th>LM test heteroscedasticity $\chi^2(1)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>$y = -0.010 + 0.636 \cdot x$</td>
<td>$6.45 [0.27]$</td>
<td>$1.17 [0.27]$</td>
<td>$0.16 [0.93]$</td>
<td>$2.65 [0.11]$</td>
</tr>
<tr>
<td></td>
<td>$(1.729)** (6.961)*$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2)</td>
<td>$x = -0.027 - 0.231 \cdot (p-pf) + 1.434 \cdot z + 0.001 \cdot (I/O) + 0.820 \cdot K$</td>
<td>$0.131 [0.72]$</td>
<td>$0.00 [0.98]$</td>
<td>$1.29 [0.53]$</td>
<td>$0.64 [0.43]$</td>
</tr>
<tr>
<td></td>
<td>$(1.147) (2.198)* (5.880)* (2.475)* (2.411)*$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(3)</td>
<td>$p = 0.011 + 0.930 \cdot w - 0.354 \cdot r$</td>
<td>$6.43 [0.17]$</td>
<td>$0.65 [0.42]$</td>
<td>$1.02 [0.60]$</td>
<td>$3.33 [0.07]**$</td>
</tr>
<tr>
<td></td>
<td>$(3.824)* (34.89)* (3.977)*$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(4)</td>
<td>$r = -0.015 + 0.672 \cdot y + 0.0002 \cdot (I/O) + 0.672 \cdot K + 0.021 \cdot GAP$</td>
<td>$2.03 [0.38]$</td>
<td>$0.84 [0.36]$</td>
<td>$0.75 [0.68]$</td>
<td>$1.85 [0.17]$</td>
</tr>
<tr>
<td></td>
<td>$(1.486) (5.056)* (0.781) (0.717) (2.134)*$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(5)</td>
<td>$K = 0.015 + 0.502 \cdot y + 0.039 \cdot q + 0.0029 \cdot edu - 0.019 \cdot GAP$</td>
<td>$4.61 [0.10]**$</td>
<td>$0.66 [0.42]$</td>
<td>$2.01 [0.37]$</td>
<td>$3.08 [0.08]**$</td>
</tr>
<tr>
<td></td>
<td>$(1.690)** (0.493) (1.675)** (2.851)* (1.668)**$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Notes:**

1. Method of estimation 2SLS.
2. Absolute t-statistics in parentheses; *, **, *** denotes significant at the 5%, 10%, 15% significance level respectively.
3. p-values in brackets for the specification tests.
4. Wald rst is the Wald test of the common parameter restriction on $p$ and $pf$.
5. Countries: Australia, Austria, Belgium, Canada, Denmark, Finland, France, West Germany, Italy, Japan, Netherlands, Norway, Portugal, Spain, Sweden, UK, USA.
### TABLE 3

Comparative Dynamics

<table>
<thead>
<tr>
<th>Exogenous variable</th>
<th>Effect on the independent term in equation (14) ((T))</th>
<th>Effect on the independent term in equation (15) ((S))</th>
<th>Effect on (G)</th>
<th>Effect on (q)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(I / O)</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>(edu)</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>(w)</td>
<td>=</td>
<td>=</td>
<td>=</td>
<td>=</td>
</tr>
<tr>
<td>(pf)</td>
<td>_</td>
<td>+</td>
<td>_</td>
<td>_</td>
</tr>
<tr>
<td>(z)</td>
<td>_</td>
<td>+</td>
<td>+</td>
<td>_</td>
</tr>
<tr>
<td>((I / O)^*)</td>
<td>_</td>
<td>_</td>
<td>_</td>
<td>_</td>
</tr>
<tr>
<td>(edu^*)</td>
<td>_</td>
<td>+</td>
<td>_</td>
<td>_</td>
</tr>
<tr>
<td>(w^*)</td>
<td>+</td>
<td>_</td>
<td>+</td>
<td>+</td>
</tr>
</tbody>
</table>
APPENDIX I

The Dynamics of Output Growth in the Leader Economy

For the leader economy $GAP = 0$, for $G = 1$. Thus, re-writing equations (4) and (5), we obtain,

$$r^* = \phi y + \lambda (I/O)^* + \alpha K^* \quad (4')$$

$$K^* = \gamma y^* + \beta q^* + \omega edu^* \quad (5')$$

Solving the new system for the rate of growth of output we obtain:

$$y^* = B + Cq^*, \quad (A.1)$$

where,

$$B = \frac{\theta \eta}{1 - \theta [\zeta \gamma - \eta (\phi + \alpha \gamma)]} (w^* - pf) + \frac{\theta \varepsilon}{1 - \theta [\zeta \gamma - \eta (\phi + \alpha \gamma)]} z - \frac{\theta \eta \lambda}{1 - \theta [\zeta \gamma - \eta (\phi + \alpha \gamma)]} (I/O)^*$$

$$+ \frac{\theta \omega (\zeta - \eta \alpha)}{1 - \theta [\zeta \gamma - \eta (\phi + \alpha \gamma)]} edu^*$$

$$C = \frac{\theta \beta (\zeta - \eta)}{1 - \theta [\zeta \gamma - \eta (\phi + \alpha \gamma)]}.$$

From expression (10) we have (Amable, 1993 and De Benedictis, 1998),

$$\dot{q}^* = Bq^* + (C - 1)q^{*2} \quad (A.2)$$

Only in the case that $B > 0$ and $(C - 1) < 0$ do we have a stable and positive solution for the rate of growth of cumulative output (see Amable, 1993). Equation (A.2) is a first order non-linear differential equation of the Bernoulli form, whose solution path is given by:

$$q(t)^* = \frac{1}{\left[\frac{1}{q_0} + \frac{C - 1}{B}\right] e^{-Bt} - \frac{C - 1}{B}} \quad (A.3)$$
where \( q_0^* \) is the initial rate of growth of cumulative output. Given the relationship (17) between \( y \) and \( q \), we obtain the dynamic solution for the rate of growth of output:

\[
y(t)^* = B + \frac{C}{\left(\frac{1}{q_0^*} + \frac{C - 1}{B}\right)e^{-Bt} - \frac{C - 1}{B}}
\]  

(A.4)

When \( t \to \infty \), \( q^* = y^* = -B/(C - 1) \), and \( q = 0 \). Since the value of \( (C - 1) < 0 \), we have a positive and stable solution for the rate of growth of output, despite the fact that cumulative forces are at work. This solution will depend positively on \( z, pf, (1/\lambda)^* \) and \( edu^* \), and negatively on \( w^* \).
APPENDIX II

Variable Definitions

All the variables were obtained from OECD statistics except where indicated. The variables used in the empirical model are the following:

\( y \): average rate of growth of real GDP;

\( x \): average rate of growth of real exports of goods and services;

\( p \): rate of growth of the export price deflator;

\( pf \): rate of growth of the import price deflator;

\( z \): weighted rate of growth of the real GDP of the pre-1994 OECD countries;

\( I/O \): ratio of real investment to real GDP at the beginning of the period considered;

\( K \): ratio of the business sector expenditure on research and development over private investment (interpolated values for the years not available);

\( w \): rate of growth of the total nominal labour costs;

\( r \): rate of growth of real labour productivity;

\( GAP \): one minus the ratio of the level of labour productivity to that of the USA in PPPs;

\( q \): average rate of growth of the cumulative sum of the level of real output \( (Q_t) \). Thus,

\[
q = \log Q_t - \log Q_{t-1}, \quad \text{and} \quad Q_t \text{ is calculated as:}
\]

\[
Q_t = \sum_{i=0}^{t} Y(t),
\]

using 1960 as the starting date \( t_0 \), to allow for the existence of a previous level of learning.

\( edu \): average number of years of schooling of the population over 25 years, obtained from Barro and Lee (1993 and 1996), for the years 1965, 1975, 1980 and 1990.