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SUPERVISION AND EFFORT IN AN INTERTEMPORAL EFFICIENCY WAGE MODEL: THE ROLE OF THE SOLOW CONDITION

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Abstract

The Solow condition is examined in an intertemporal model that blends the shirking and the turnover models of efficiency wages with managerial supervision. It is shown that the Solow condition does not hold when shirking and turnover costs are considered. The Solow condition can be a possible outcome when managerial productivity offsets shirking and turnover costs.

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1. Introduction

Efficiency wage models have been developed to explain involuntary unemployment. Firms may benefit from paying their workers more than the market clearing wage, thus generating involuntary unemployment. Among the efficiency wage models, the shirking and the turnover cost models are the most well known and cited. According to the shirking model (e.g. Shapiro and Stiglitz, 1984), firms pay efficiency wages in order to reduce workers' shirking. High wages make the workers fear losing their jobs if they are caught shirking. In the turnover cost model (e.g. Salop, 1979), workers who quit have to be replaced, which makes the firms incur search, recruitment and training costs. Firms have an incentive to minimize these costs and one mechanism for so doing is to set the wage at a level which discourages turnover.

One of the standard results of the efficiency wage models is due to Solow (1979). The Solow condition, as it is known, states that an optimizing firm sets its wage at the level at which the elasticity of work effort with respect to the real wage is unity. However, this result has come under some criticism. As Akerlof and Yellen (1986) point out, an effort-wage elasticity of unity is quite high. If the elasticity is never that high,

> "then there cannot be an equilibrium with unemployment in an efficiency wage model." Akerlof and Yellen, 1986, p.14

As the Solow model does not contemplate shirking, workers who do not work hard do not waste the firm's output since additional workers can be hired. If additional costs related to

low-effort labor are taken into consideration, this will result in an equilibrium effort-wage elasticity lower than unity.

On the other hand, efficiency wages have often been used to explain the pattern of interindustry wages (e.g. Krueger and Summers, 1988). One empirical regularity that they sought to explain is that industries with greater productivity per employee pay their workers more (e.g. Katz and Summers, 1989). Mehta (1998) proposed a model in which supervisors monitor and coordinate their subordinates. However managers are constrained to make tradeoffs in these activities. Therefore,

> "if the productivity of supervisors increases, then other things being equal, they will want to coordinate more and monitor less. Consequently, to maintain effort, they will pay more. Similarly, an increase in the productivity of workers makes supervisors want to employ more of them. Other things being equal, they will now want to monitor each worker less and pay more." Mehta, 1998, p.153

This paper presents a model that combines the shirking and turnover costs models with managerial supervision in an intertemporal optimizing framework. The model encompasses the analyses of Marti (1997) and Mehta (1998) into an intertemporal optimization framework developed along the lines of Lin and Lai (1994). The results reveal that when managerial productivity is not considered, and shirking and turnover costs are taken into account, the Solow condition is not valid. Furthermore, when managerial productivity is considered and offsets shirking and turnover costs, then the Solow condition can be a possible outcome.

2. The Model

Following Mehta (1998), the representative firm has a production function that depends on the product of managers and workers:

$$y = S\big(1 - Mg(s)\big)sN + \Omega\frac{\delta(s)}{s}e(w)sN \tag{1}$$

The first term on the right hand side (RHS) of equation (1) represents the net contribution that supervisors make to directly increasing output, where *S* is the productivity of supervisors, *M* is the difficulty of monitoring, $g(s)$ is the extensive monitoring (assumed to be a convex function, $g_s > 0,\ g_{ss} > 0$), and *s* is the span of control of each supervisor. There are *N* identical teams with *s* workers in each, so the total number of workers is $n = Ns$. The second term on the RHS is the net output of workers and depends on the productivity of workers, Ω. The function $\delta(s)$ indicates how output of the *s* workers in the productive unit, who work for a single supervisor, varies with *s* ($\delta_s > 0,\ \delta_{ss} < 0$). Workers' effort, *e*, is assumed to be an increasing function of the relative wage, *w*, such that $e_w > 0$. The relative wage is defined as the difference between the wage paid by the firm, w_F, and the reservation wage, w_R, so that $w = w_F - w_R$.

It is implicitly assumed in the production function (1) that the probability of detecting shirking is equal to one. It is also assumed that the representative firm is a monopolistic competitor in the goods market. The firm chooses three variables to maximize the discounted profit over an infinite horizon. It uses the span of control (*s*), the number of hired workers (*h*) and the relative wage (*w*) as control variables in the problem below:

$$\underset{h,w,s}{\text{Max}} \int_0^{\infty} [\, p(y)y - wn - c(N) - \tau(h) - \sigma x] e^{-rt}\, dt \tag{2}$$

$$\text{subject to } \dot{n} = h - q(w)n \tag{3}$$

The term in the brackets in equation (2) captures the training costs, $\tau(h)$, $\tau_h > 0$, when $h > 0$, and the search costs, σx, where x is the search effort to hire new workers and σ is the unit cost of search. In order to hire, the firm must expend search effort, where $\theta = h / x$ is the number of hires per unit of search. θ is therefore a measure of search effectiveness of the firm and is assumed to be an increasing function of the relative wage ($\theta = \theta(w)$, $\theta_w > 0$). The function $c(N)$ is a convex cost of coordination function.

Equation (3) describes the time variation of employed workers which depends on the difference between the number of hired workers, h, and the number of workers who decide to quit from the firm, where $q(w)$ is the quit rate which is assumed to be a decreasing function of the relative wage ($q_w < 0$).

If managers are not productive ($S = 0$) and there are no coordination costs ($c(N) = 0$), and the quit rate, q, and search effectiveness, θ, are independent of the relative wage, we have the shirking model. Similarly, if $S = 0$ and $c(N) = 0$ and the effort of workers is independent of the relative wage, we have a version of the turnover cost model (Marti, 1997). When q, θ, and e are independent of the relative wage and S and $c(N)$ are different from zero, we have a dynamic version of Mehta's (1998) model. Therefore, the above specification combines the shirking and the turnover cost models with the managerial supervision model in an intertemporal optimization framework.

The current value Hamiltonian function, *H*, is written as:

$$H = p(y)y - wn - c\left(\frac{n}{s}\right) - \tau(h) - \frac{\sigma h}{\theta(w)} + \lambda[h - q(w)n]$$

Introducing equation (1) into the Hamiltonian, the first order conditions of this problem are:

$$\lambda = \tau_h + \frac{\sigma}{\theta} \tag{4}$$

$$y_s\, p(y)(1+\psi) + c_n \frac{n}{s^2} = 0 \tag{5}$$

$$(1+\psi)\, p(y) y_w = n(1+\lambda q_w) - \frac{\sigma h \theta_w}{n\theta^2} \tag{6}$$

$$\dot{\lambda} = \lambda(r+q) - [(1+\psi)\, p(y) y_n - w - \frac{c_n}{s}] \tag{7}$$

where $\psi = \frac{dp}{dy}\frac{y}{p}$. Consider the steady state:

$$\dot{n} = 0 \;\Rightarrow\; q(w)n = h$$

and by using (4) and (6) in (7) when $\dot{\lambda} = 0$, we have:

$$e_w \frac{w}{e} = \left[\frac{1 + (\tau_h + \sigma/\theta)q_w - \sigma q \theta_w / \theta^2}{1 + c_n / ws + (\tau_h + \sigma/\theta)(r+q)/w}\right]\left[1 + \frac{S\left(1 - Mg(s)\right)}{\Omega(\delta(s)/s)e}\right] \tag{8}$$

From equation (8), we can examine the Solow condition.

3. The Solow Condition

Consider first a model without managerial supervision (i.e. $S = 0$ and $c(N) = c_n = 0$). Equation (8) then reduces to:

$$e_w \frac{w}{e} = \frac{1 + q_w(\tau_h + \sigma/\theta) - [(\sigma q \theta_w)/\theta^2]}{1 + (\tau_h + \sigma/\theta)[(r+q)/w]} \tag{9}$$

As it is assumed that the number of hired workers, *h*, is positive, then $\tau_h > 0$ and given that $\theta_w > 0$ and $q_w < 0$, it follows that the numerator is less than unity and the denominator is greater than unity in equation (9). Therefore the wage-elasticity of effort is less than unity:

$$e_w \frac{w}{e} < 1$$

From (9), three special situations can be analysed. Firstly, if the voluntary quitting rate does not vary with the relative wage ($q_w = 0$), then equation (9) reduces to:

$$e_w \frac{w}{e} = \frac{1 - [(\sigma q \theta_w)/\theta^2]}{1 + (\tau_h + \sigma/\theta)[(r+q)/w]} \tag{10}$$

which is still less than unity.

The second case is related to the situation in which the search effectiveness of the firm does not depend on the relative wage ($\theta_w = 0$):

$$e_w \frac{w}{e} = \frac{1 + q_w(\tau_h + \sigma/\theta)}{1 + (\tau_h + \sigma/\theta)[(r+q)/w]} \tag{11}$$

which is also less than unity.

As seen above, our optimization problem reduces to a shirking model when $\theta_w = 0$ and $q_w = 0$. Then, combining equations (10) and (11) yields:

$$e_w \frac{w}{e} = \frac{1}{1 + (\tau_h + \sigma/\theta)[(r+q)/w]} \tag{12}$$

Thus, as in Akerlof and Yellen (1986), when shirking is taken into consideration, the effort-wage elasticity is less than unity, and the Solow conditions does not hold.

The third situation occurs when the training costs are absent ($\tau(h) = \tau_h = 0$). From equation (9) we have:

$$e_w \frac{w}{e} = \frac{1 + q_w \sigma / \theta - [(\sigma q \theta_w) / \theta^2]}{1 + \sigma / \theta [(r + q) / w]} \tag{13}$$

which is less than unity.

The three situations above show that the Solow condition is invalid when shirking and turnover costs are taken into account.

Returning to equation (8), the Solow condition can be seen to hold if and only if:

$$e_w \frac{w}{e} = 1 \Leftrightarrow \left[\frac{S(1 - Mg(s))}{\Omega(\delta(s)/s)e} \right] = \left[\frac{1 + c_n / ws + (\tau_h + \sigma/\theta)(r+q)/w}{1 + (\tau_h + \sigma/\theta) q_w - \sigma q \theta_w / \theta^2} \right] - 1 > 0 \tag{14}$$

Condition (14) asserts that the Solow condition depends on the rate of productivity of managers and workers. The weak condition only requires that this rate must be positive. This result is quite appealing since managers are employed, among other things, to reduce shirking and turnover costs. Therefore, when managerial productivity is considered, it can offset the former effect of shirking and turnovers costs in order to guarantee that the Solow condition holds.

4. Concluding remarks

This paper blends the shirking and the turnover models of efficiency wages with managerial supervision in an intertemporal optimization framework. It is shown that the Solow condition does not hold when shirking and turnover costs are considered. However, the Solow condition can be a possible outcome when managerial productivity offsets shirking and turnover costs.

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