Gerlagh, Reyer

Working Paper

Generous Sustainability

CESifo Working Paper, No. 5092

Provided in Cooperation with:
Ifo Institute – Leibniz Institute for Economic Research at the University of Munich

Suggested Citation: Gerlagh, Reyer (2014) : Generous Sustainability, CESifo Working Paper, No. 5092, Center for Economic Studies and Ifo Institute (CESifo), Munich

This Version is available at:
http://hdl.handle.net/10419/105120

Standard-Nutzungsbedingungen:
Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.
Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.
Sollten die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

Terms of use:
Documents in EconStor may be saved and copied for your personal and scholarly purposes.
You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.
If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.
Generous Sustainability

Reyer Gerlagh

CESifo Working Paper No. 5092
Category 2: Public Choice
November 2014

Presented at CESifo Area Conference on Energy and Climate Economics, October 2014

An electronic version of the paper may be downloaded
• from the SSRN website: www.SSRN.com
• from the RePEc website: www.RePEc.org
• from the CESifo website: www.CESifo-group.org/wp
Generous Sustainability

Abstract

I define “generous sustainability” as a combination of two conditions: neither instantaneous maximin income nor attainable maximin income should decrease over time. I provide a formal definition and study applications to an AK economy, a Ramsey economy, and a Climate Economy. Generosity is shown to impose substantially stronger conditions on current actions compared to existing sustainability concepts. As a rule of thumb, generosity requires that GHG emissions are limited to levels that do not cause irreversible system damages if some group of people systematically value these systems.

JEL-Code: D630, D900, Q010.

Keywords: intertemporal choice and growth, intergenerational distribution, sustainable development.

Reyer Gerlagh
Economics Department and
Tilburg Sustainability Center
Tilburg University
Warandelaan 2
The Netherlands – 5037AB Tilburg
r.gerlagh@uvt.nl

This draft: 14 November, 2014
A webversion is available through
https://surfdrive.surf.nl/.les/public.php?service=les&t=9f6047aaf96c76d0baea386d3ca8a345.
I want to thank Geir Asheim, Vincent Martinet, Marc Fleurbay, Paolo Piacquadio, Thomas Sterner, and Jack Pezzey for comments. The usual disclaimer applies.
1 Introduction

"We deserve to do more than just survive; we deserve to thrive." (Kathy Jetnil-Kijiner\textsuperscript{1})

"progress without destruction is possible" (Chico Mendes\textsuperscript{2})

Avoiding regress or even disaster is very important and the core concept of “sustainability” in the literature, but it is not enough. This paper offers a broader concept of sustainability that is based on the potential of a bright future, and that evaluates changes we make to that potential future. In economic theory, sustainability is defined and measured in very different ways (Fleurbay 2013). One approach formalizes sustainability (or more precisely "sustainedness") as an ex-post condition on the utility sequence, for example as in the requirement that generations’ utility should be non-decreasing with time (Pezzey 1997). A second line of analysis frames sustainability in terms of the (inter-generational) welfare function that society should maximize when allocating its resources over time. Chichilnisky (1996, 1997) interpreted sustainability as a non-zero weight given to the interests of the very-far future generations. Zuber and Asheim (2012) present a utilitarian perspective on sustainability, requiring the weights given to generations in intergenerational allocation choices to decrease with increasing generation’s utility levels. A third approach to sustainability formalizes the concept as 'something that must be conserved for the very long run' (Solow 1993). Martinet (2011) and Cairns and Martinet (2014) define sustainability as non-decreasing maximin income (defined in detail below). The advantage of the last approach is that sustainability defined this way can be ascertained without making a precise prediction about the future generations’ decisions, since only their possibility set matters (Fleurbay 2013).

Though the approaches differ fundamentally by use of their method, they share their narrative and focus on the past as the benchmark, and their concern for lower bounds; they aim to protect the weak and poor from further deprivation, both in the present and the future. This focus is explicit in the well-known definition of sustainability given by the World Commission on Environment and Development, as development “that meets the needs of the present, without compromising the ability of future generations to meet their

\textsuperscript{1}A poem to my Daughter, Kathy Jetnil-Kijiner addressing the United Nations Climate Summit Opening Ceremony, 24 September 2014, New York. The poem speaks of the future of the people living on small islands.

\textsuperscript{2}Chico Mendes was a rubber tapper in the Amazon and a campaigner for the sustainable exploitation of the rain forests. The quote is part of the closing lines of a speech at 6th December 1988, in Sao Pablo. Chico Mendes was born 1944 and died 22nd December 1988, shot by the son of a local rancher.
own needs” (WCED 1987). In contrast, Jetnil-Kijiner calls the United Nations Summit to provide her child with the possibility to thrive; such a request marks an important deviation from the minimalistic sustainability concept. Society can and should be more demanding about its contribution to the development of the future prospect (Gerlagh and Sterner 2013). So while it is essential to protect the future against poverty and to ensure that the future can meet its basic needs, and avoiding small risks of total disaster is very important, it is not enough. Narrowing the concept of “sustainability” to the prevention of disaster is too limited. Most integrated assessment models and many observers believe we have the potential to achieve a bright future for society with many decades, possibly centuries, of growth. This harbours the potential for eradication of poverty and of a future where people on average enjoy a better life than today. This is the bright future we stand to (partly) lose with climate change. In such a context, we cannot limit the content of the term “sustainability” to meaning “no worse than today”.

The last century has shown a world with a robust and steady per capita income growth of about 2 per cent per year. We see developing countries rapidly catching up, and the high-income countries continuing their progress. Whereas the developing countries gain from institutional changes, the frontier economies gain from continued progress of technology and knowledge; there is no end in sight to human ingenuity. Compared to the optimist prospect of continued growth, aiming for the conservation of present wealth – as many sustainability paradigms do – reads as an aim for stagnation. We can do better in the future as compared to the past: to eradicate poverty, improve education worldwide, bring more equal chances for all world citizen including closing the gender gap, and make a better place. In this essay I translate the call for contributing to a better future into a formal framework. I will define a perspective labeled ‘contribution’, which requires that living in the future provides equal or better opportunities as living in the past. I also define a more moderate perspective, labeled generous sustainability or generosity, which requires that we at least preserve the best achievable world, that is, we protect the maximal potential of future generations to thrive.

Yet, we also need to confront the optimist future view with history, which shows the side effects of worldwide economic progress (Victor, Gerlagh and Baiocchi 2014). The economic successes of the rapidly emerging countries, the new world middle class, are accompanied by an unprecedented rise in resource use and greenhouse gas emissions. It is thus time to develop a concept of sustainability that supports economic progress -

---

3Neumayer (1999) and Pezzey and Burke (2014) point out that such beliefs remain beliefs, in that they cannot be usefully falsified.
the extra mile - and does not accept economic stagnation as the ultimately sustainable outcome, while at the same time, sustainability has to take serious the protection of the scarce environmental resources. Chico Mendes, cited above, was killed for his vision, though his above quote shows a remarkably modest statement. While generosity is more demanding compared to existing concepts that require the maintenance of opportunity (Martinet 2011, Fleurbay 2013, Cairns and Martinet 2014), it is not too demanding. It does not require huge savings to increase wealth of a future generation that is richer than the present, as zero-discounting does (Nordhaus 1997). It only requires the preservation of resources that are essential to future utility, in a way that we will formalize below.

Before going into the formal analysis, an illustrative example may clarify the core of the concept and conclusions of this paper. Assume that a country’s income can grow by a factor five in hundred years time. Furthermore, assume that if the country cuts it forests, and irreversibly destroys all supporting ecosystems, it can reach the same income level one year earlier, that is, in 99 instead of 100 years. What principles govern the social (il)legitimacy for cutting the forests? The traditional sustainability concepts offer no guidance, unless the citizen love their forests so much that loosing the forests makes them, even with a five-fold income increase, consider themselves worse off. But in such cases, conservation is relatively easy to achieve, and there is no need for some sustainability criterion to support conservation. Azar and Schneider (2002) sketch a similar dilemma for climate change, showing that in most models the cost of climate protection is equivalent to one year of economic growth. But they do not provide principles that can be used to convert the observation into an argument for climate conservation (Gerlagh and Papyrakis 2003). Generosity sets out such guidelines; it asks whether there is a future where citizen, whose income in the very long run has continued to increase, consider themselves irreversibly worse off without the forests and climate conservation as compared to the situation with conservation.\footnote{The term irreversible should not be taken literally as in mathematics; it’s meaning is constrained by our imagination of a meaningful period. Similarly, in the formal analysis, we let the index for time and generations run to infinity. It is clear that, over billion of years, all changes that the current generation makes will wash out.}

If an action irreversibly deteriorates the prospects of a stream of future citizens without bounds, while there is no other group of future citizen whose prospects are permanently improved by that action, then that action conflicts with generosity. Restated, if progress is possible without the irreversible destruction of some resources, and if these resources are fundamentally valued by some group of people in each period (defined precisely in the subsequent sections), generosity stipulates that progress
with conservation is always preferred over progress complemented by destruction, also if the latter leads to faster income growth.

In the next section I shortly define generosity formally and as succinctly as possible. Yet the main aim of this manuscript is not to lay down a strict formal analysis, but to broaden our conceptual perception of a sustainable future and its practical conditions in, for example, the climate change debate. Therefore, subsequent section apply the concept to typical economies of increasing complexity: the AK model of perpetual economic growth, the Ramsey model / logistic renewable resource model, and finally a simple climate-economy model without and with unbounded growth. For these four models I will compare the generous concept of sustainability with four alternatives from the literature: non-decreasing utility (NDU, e.g. Pezzey 1997), zero discounting utilitarianism (ZDU, e.g. Broome, 1992), non-dictatorship of present and future (NDPF, e.g. Radner 1967, Chichilnisky 1996, 1997), and Ranked Discounted Utilitarianism (RDU, e.g. Zuber and Asheim 2012). I then briefly discuss a natural extension of generosity to the context of uncertainty and intra-generational inequality, and conclude.

2 Generosity

To set the stage, we use the most simple set up. We abstract from capital, consumption, state and flow variables, and instead directly consider feasible utility sequences. That is, we do not differentiate between actions and utilities levels, leaving out of sight that different actions can lead to the same utility level but different future opportunities. The subsequent sections discuss more specific economies and translate the concepts developed here to their more specific model context.

Time is discrete, starting at $t = 1$. Let $u = (u_1, u_2, \ldots)$ be the sequence of instantaneous utility levels starting at $t = 1$, and $U$ the set of history-dependent feasible sequences $u$. The set is history-dependent because $U$, what is feasible at $t = 2$, depends on $u_1$, the choices at $t = 1$. The set $U$ completely defines the economy, and $u$ defines all actions (choices). Generosity is defined as a constraint on the actions $u_1, u_2, \ldots$. We do not consider welfare optimization here, generosity has a different basis vis-a-vis zero-discounted utility, non-dictatorship of the present, and rank-discounted utility. It is also incomparable to concepts such as non-decreasing utility, as it does not evaluate ex-post outcomes, but current actions.

At time $t$, the generation inherits the economy $U$ and it has to decide on its action
The economy that it passes on to the next generation is defined by

\[ t+1U(tU, u_t) = \{ t+1u | (u_t, t+1u) \in tU \} \]  

(1)

Note that the set \( tU \) does not denote the set of all sub-sequences from period \( t \) on that are feasible when considered in period 1, but it denotes the set of all feasible paths given the actions \((u_1, u_2, ..., u_{t-1})\) until period \( t \). The simple set up suffices to define a generation’s contribution in a neat way, as an action that increases the set of feasible paths:

**Definition 1** (Contribution) An action \( u_t \) contributes to the economy if and only if

\[ tU \subseteq t+1U \]  

(2)

and it strictly contributes when the set strictly increases. A sequence \( _1u \) is 'contributing' if all actions \( u_t \) are contributing.

Contribution is a strong condition.\(^5\) We define generosity as a weaker requirement that preserves some of the qualities of the economy, rather than strictly improving the set of opportunities. Generosity preserves two opportunities defined in terms of utilities. The first opportunity, defined by the utility possibility set, measures 'maximin' income.

**Definition 2** Given a feasible utility set, maximin income is the supremum of the infimum utility level

\[ H(tU) = \sup_{u \in tU} \{ \inf_{\tau} \{ u_\tau \} \} \]  

(3)

The second opportunity measures utility in the best achievable stationary world. It is defined recursively, on the basis of maximin income. We could loosely call it 'maximaxmin' or 'maximax' utility:

**Definition 3** Given a feasible utility set, Attainable income is the supremum of maximin income that can be reached

\[ A(tU) = \sup_{\tau > t} \{ H(\tau U) | \tau U \text{ attainable from } tU \} \]  

(4)

where \( \tau U \text{ attainable from } tU \) means that we consider all tails of \( tU \) starting at \( \tau \).

---

\(^5\)Note that the definition of contribution and generosity (below) are most easily understood for autonomous or time-invariant economies, where the current state can be fully characterized by a set of state variables, and time plays no explicit role in technology and utility. Also, note that the definitions require time to have no bound. For 'contribution', both the left and right-hand side present sets of countable infinite and ordered (thus comparable) sequences.
We define an action \( u_t \) to be generous if it keeps the two opportunities intact. First, it must not decrease maximin income, as in Cairns and Martinet (2014), who call the increase in maximin income sustainable savings (different from genuine savings). The second condition distinguishes generosity from the previous definitions of sustainability, and it is much more demanding, as we will see in our applications below.

**Definition 4** An action \( u_t \) is Generous if and only if

\[
\mathcal{H}(tU) \leq \mathcal{H}(t+1U) \quad \text{and} \quad \mathcal{A}(tU) \leq \mathcal{A}(t+1U) \tag{5}
\]

Notice that attainable income cannot increase over time; it can only remain constant or decrease. There cannot be positive attainable 'savings' (an increase in attainable income), though we can loosely speak of the 'attainable deficit' when attainable income decreases. For future reference, we refer to these constraints as the first and second generosity constraint.

The attainable deficit somewhat resembles an idea explored in Pezzey and Burke (2014), who calculate an artificial measure of genuine (adjusted net) savings when temperature change is not allowed to exceed a specific threshold. But where Pezzey and Burke need to assume very pessimistic climate and damage dynamics, the attainable deficit is measures the gap between a bright and a brighter future. Moreover, the concept of attainable deficit is based on rigorous analytics broadly applicable.

Generosity requires non-negative sustainable savings, and the absence of an attainable deficit. We are now in a position to illustrate generosity by use of illustrative economies. We will move from utility sets to state and flow variables in the obvious way, and assume that both Maximin and Attainable income are continuous functions of the state variables.

### 3 Four examples

#### 3.1 An AK economy with perpetual growth

Consider a time-discounted utilitarian welfare function

\[
w_1 = \max_{t=1}^{\infty} \sum_{t=1}^\infty \beta^t u(c_t) \tag{6}
\]

in an AK-economy

\[
c_t + k_{t+1} = Ak_t \tag{7}
\]
We follow the standard assumptions so that \( u(.) \) is strictly increasing and concave. First order conditions are
\[
\frac{d}{dt} = \beta A \frac{d}{dt+1}.
\]
(8)

For \( A = 1 \), the economy describes an exhaustible resource and all consumption paths converge to zero. For \( A < 1 \), all consumption paths decrease to zero faster. For the remainder, we will assume \( A > 1 \) so that constant consumption and perpetual growth are both feasible. For \( \beta > 1 \), when these first order conditions are applied to the Ramsey economy (discussed below), the path becomes dynamically inefficient and thus we assume \( \beta < 1 \) and consider \( \lim \beta \to 1 \) as the special 'zero-discounted utility' case.

The discounted utility economy shows rising consumption if and only if \( \beta A > 1 \). Zero-discounting always leads to consumption growth, but may lead to such large savings and investments in man-made capital stocks that these are considered unrealistic (Nordhaus, 1997), or even an undesirable sacrifice of present consumption for the future benefits (Beckerman 1992).

Non-decreasing utility, when added as a constraint to the welfare maximization program, will be binding but feasible if \( \beta A < 1 \). The constraint leads to a constant consumption path and can be interpreted as implementing an effective discount rate equal to \( \beta' = 1/A > \beta \).

Now consider Chichilnisky’s non-dictatorship condition. Chichilnisky (1996, 1997), elaborating on Radner (1967)), explicitly includes future interests in the welfare objective through appending to the standard welfare function a separate term representing the utility of the generation living at infinity. The ‘sustainable welfare function’ treats present and future interests as two disjoint objectives that are to be treated symmetrically. By this symmetric treatment, she argues, a dictatorship of the present interests over the future, and vice versa, is avoided. We adapt the welfare function through a positive weight for \( \lim_{t \to \infty} u_t \):
\[
w_1 = \max \alpha \sum_{t=1}^{\infty} \beta^t u(c_t) + (1 - \alpha) \lim_{t \to \infty} u(c_t)
\]
(9)

If \( \beta A > 1 \) (perpetual growth) and \( \lim_{c \to \infty} u(c) = \infty \), Chichilnisky’s welfare function is not defined. If \( \beta A > 1 \) and \( \lim_{c \to \infty} u(c) < \infty \), Chichilnisky’s welfare function is well-defined but the infinite-generation term has no effect on current decisions. If \( \beta A < 1 \), Chichilnisky’s welfare function is well defined but has no maximal solution. For any path there exists an alternative path with a higher welfare level (see also Heal 1998, Section 7.1 and 10.3). Non-dictatorship of the present is not an effective criterion for the perpetual...
Consider then rank-discounted utilitarianism:

\[ w_1 = \max \sum_{t=1}^{\infty} \beta^{k(t)} u(c_t) \]  

(10)

where \( \beta \) is the discount factor between ranked generations, and \( k(t) \) is the rank of generation \( t \), such that \( \{k(t) | t = 1, 2, \ldots\} = \{1, 2, \ldots\} \), \( k(t) < k(t') \) iff \( c(t) < c(t') \). Rank-discounted utilitarianism leads to rising consumption if \( A > 1 \), and loosely following the logic of rank-discounted utilitarianism, one can see that the spirit of rank-discounted utility implies constant consumption for \( A < 1 \), though rank-discounted utility is not fully defined for such an economy. The concept of rank-discounted utility performs well for the perpetual growth economy, but the details are complex.

For the AK economy, generosity demands that future options do not deteriorate, thus requiring \( k_{t+1} \geq k_t \). By slight abuse of notation, we can write Maximin and Attainable income as

\[ H(k_t) = (A - 1)k_t \]  

(11)

\[ A(k_t) = \infty \]  

(12)

It is immediately clear that only the first generosity constraint is binding. In a perfect foresight framework generosity effectively implements the same condition as non-decreasing utility. Generosity is, however, more general, as it defines a constraint on current actions only, without invoking the full future consumption path. To see this, consider the case that future preferences are uncertain. Each generation may consider its own preferences for the future, but there is some uncertainty as to the preference of future generations, especially the weight given by future generations to their descendants. The parameter \( \beta \) may come from a distribution, rather than being a fixed number, so that the investment decision is based on expected future strategies. In those circumstances, non-decreasing utility is not well-defined, as the current generation may not know for certain whether a future generation is not willing to drop its own utility in favor of future utility. Generosity does not suffer from this problem as it defines the condition period by period.
3.2 A Ramsey economy: capital build up and renewables conservation

Next we consider the most simple Ramsey economy:

$$c_t + k_{t+1} = f(k_t)$$  \hspace{1cm} (13)

with stationary, continuous and concave $f(.)$, zero-output for zero-inputs, $f(0) = 0$, and feasibility of a strictly positive steady state, $f'(0) > 1$. As Asheim and Ekeland (2014) note, in the Ramsey model the function $f(.)$ can be interpreted as gross economic output, that is, the production of consumption goods and man-made capital, in which case the initial capital stock is typically assumed to be small and historically rising over time. Another possibility is to interpret the function $f(.)$ as a natural growth function for a renewable resource and $k$ as the renewable resource. Then the initial resource stock is often assumed to be large, and historically decreasing.

We write

$$k^* = \arg \max_k \{f(k) - k\}$$  \hspace{1cm} (14)

$$c^* = f(k^*) - k^*$$  \hspace{1cm} (15)

for the maximum constant consumption level and supporting stock.

The discounted utility Ramsey economy is appreciated for its feature that it can be empirically calibrated and then used to assess efficiency of possible future paths. Given a history of world-wide capital accumulation, the calibration of the model results in $\beta f'(k_0) > 1$, and capital accumulation is interpreted as the efficient (optimal) policy. But, as Asheim and Ekeland (2014) discuss, the model is less favorable viewed when applied to renewable resource management. The reason is that in such a context, $\beta f'(0) < 1$ is an empirically realistic assumption, in which case full exhaustion is suggested to be the efficient (and thus optimal) strategy (Clark 1973).

Zero-discounted utilitarianism restores the resource conservation property of the model, but leads to unrealistic and undesirable high savings rate when applied to the macro economy as argued above for the AK economy. Non-decreasing utility and rank-discounted utility have similar implications for the Ramsey economy as for the AK economy.

We can write Maximin and Attainable income for $k > 0$ as

$$\mathcal{H}(k_t) = f(k_t) - k_t \text{ for } k < k^*$$  \hspace{1cm} (16)

$$\mathcal{A}(k_t) = c^* \text{ for } k \geq k^*$$  \hspace{1cm} (17)
Generosity requires $k_{t+1} \geq k_t$ but only for $k_t \leq k^*$. For $k_t > k^*$, generosity allows the capital or resource stock to drop to the green golden rule level $k^*$. We find that generosity implements the same outcome as the 'Chichilnisky game' presented in Asheim and Ekeland (2014).

When the Ramsey economy starts with a capital stock above the maximum sustainable level, a one-time extraction of the stock to this level satisfies the generosity constraints, but it is not 'contributing' as defined above, showing some limitations of the concept of 'contribution'. If the resource stock is above the maximum sustainable yield level, the only possibility to sustain all opportunities is by keeping the stock constant, even as such action is dynamically inefficient. The problem does not arise, though, in a more comprehensive economy that combines the economic growth features of the AK model and the regeneration features of the Ramsey model. We provide the analysis in the appendix.

3.3 A climate-economy model, finite earth

Then we consider a stylized climate-economy model, with economic growth.

\[
\begin{align*}
c_t + k_{t+1} &= A_t \sigma(a_t) \Omega(m_t) f(k_t/A_t) \\
s_{t+1} &= B s_t + (1 - a_t) b k_t \\
m_{t+1} &= h(m_t, s_t)
\end{align*}
\]

where $A_t$ is a measure of labour productivity, $a_t$ is abatement effort in relative terms ($a_t = 1$ means zero emissions), $\sigma(a_t)$ is a measure for the cost of abatement in terms of reduced output, $\Omega(m_t)$ is the relative costs of climate change, dependent on $m_t$, which is the state of the climate affecting output, e.g. a measure of global temperature change, and $s_t$ is a vector of CO2 reservoirs in excess of the natural level, such as the atmosphere and ocean layers, $B$ describes the diffusion between reservoirs, and $b$ is a vector which elements sum to one describing the immediate distribution of emissions over the reservoirs. We normalize the functions such that $\sigma(0) = \Omega(0) = 1$, $\sigma' < 0$, $\Omega' < 0$, $\sigma(1) > 0$, $\Omega > 0$, $B \geq 0$, $b \geq 0$, $h(0,0) = 0$, $0 < h_m < 1$, $0 < h_s$. The capital stock and CO2 stocks are normalized such that, if there is no abatement, one unit of capital emits one unit of CO2. We assume that part of climate change is irreversible: CO2 cannot leave the system, in
technical terms, the columns of $B$ sum to the unit vector $Bu = u$, where $u = (1, 1, ..., 1)$.\(^6\) We assume that the climate system is dynamically stable with persistent effects, and $\lim_{t \to \infty} B^t \gg 0$ is well-defined with strictly positive elements, equal rows up to scaling so that for the long-run consequences only cumulative emissions matter:

$$s^* = \lim_{t \to \infty} B's_1 = s'u's_1$$  \quad (21)

where $s > 0$ with elements that sum to one ($u's = 1$) is the long-term equilibrium relative distribution of CO2 over the reservoirs, and $u$ sums CO2 stocks over the reservoirs in $s_1$, so that $r_1 = u's_1$ measures cumulative historic emissions, and future cumulative emissions are given by

$$r_{t+1} = r_t + (1 - a_t)k_t.$$  \quad (22)

Cumulative emissions are non-decreasing, so that the limit is well-defined:

$$r_\infty = \lim_{t \to \infty} r_t$$  \quad (23)

We can then define the implicit function of long-term consequences of current cumulative emissions, $m^*(r_t)$, by

$$m^*(r_t) = h(m^*(\bar{s}r_t), \bar{s}r_t)$$  \quad (24)

and assume that this function is well defined, and $m^*''(.) \equiv \bar{s}'\partial m^*(\bar{s}r_t)/\partial \bar{s}r_t > 0$. Long-term consequences of current emissions may become small but will never completely vanish.

Zero-discounted utilitarianism ensures the resource conservation property of the model, but also leads to unrealistic and undesirable high savings rate. Non-decreasing utility and rank-discounted utility do not alter the outcome for this economy, unless climate damages are sufficiently strong (a decrease in $\Omega(m_t)$) so that they more than offsets the increase in technology $A_t$. Though such damages are not ruled out by the model set up, most numerically applied studies on climate change do not find such damages (Gerrlagh and Papyrakis 2003). Chichilnisky’s welfare function provides some justification for reduced emissions, but its policy is time inconsistent.

The economy has too many parts to derive a general analytical solution for maximin income. We can derive an expression for attainable income under suitable assumptions, though. We first consider the perspective of 'spaceship earth’, where output is bound by finite resources. Formally, we assume that $A_t$ is increasing and converging to some level,

\(^6\)Our modelling of the carbon reservoirs-climate change dynamics relates to the "trillionth-ton" literature on a limit to cumulative CO2 emissions, see e.g. Allen et al. 2009.
\( A_t \rightarrow A^* \). We can then calculate the bliss steady state, dependent on the current history of CO2 emitted:

\[
\begin{align*}
k^*(r_t) &= \arg \max_k \{ A^* \sigma(0) \Omega(m^*(r_t)) f(k) - k \} \\
c^*(r_t) &= A^* \sigma(0) \Omega(m^*(r_t)) f(k^*) - k^*
\end{align*}
\]

(25) (26)

Attainable income, for \( k_t > 0 \), is now immediately determined as

\[
A(k_t, r_t, m_t) = c^*(r_t)
\]

(27)

We immediately see that, as \( c''(.) < 0 \), generosity implies a stop to the build up of emissions. We state this result as proposition:

**Proposition 1** Under non-negative emissions, \( a_t \leq 1 \), and the 'finite earth' assumption of \( A_t \rightarrow A^* \) and a closed CO2 system (21), generosity requires full abatement, \( a_t = 1 \).

Note that the DICE model (Nordhaus 2008) satisfies the above conditions, so that the proposition implies a zero-emissions policy in DICE. The above discussion naturally leads to the question of future carbon capture and sequestration, allowing \( a_t > 1 \), as such may reverse part of past emissions. If negative emissions are feasible, attainable income is independent of historic emissions:

\[
\begin{align*}
k^* &= \arg \max_k \{ A^* \sigma(0) \Omega(0) f(k) - k \} \\
c^* &= A^* \sigma(0) \Omega(0) f(k^*) - k^*
\end{align*}
\]

(28) (29)

Indeed, under this condition, generosity does not necessarily constrain current emissions, if all consequences are reversible, e.g. through carbon capture and sequestration. But, generosity has a substantial effect on optimal climate policy, nonetheless, as it indirectly changes the conditions for optimal climate policy. Whereas a standard cost-benefit analysis compares the marginal costs of abatement to the net present value of marginal damages, an efficient generous policy compares the marginal costs of abatement with the net present value of future marginal costs to capture and storage, assuming that no irreversible damages occur in between the release and capture of atmospheric CO2. Having written \( A_t \) as labour productivity, we can use it as a proxy for income and emissions without abatement and derive a rule of thumb for generosity:

**Remark 1** Under the 'finite earth' assumption of \( A_t \rightarrow A^* \) and (21), when negative emissions, \( a_t > 1 \) are possible, a rule of thumb necessary condition for generosity is that
any below-full abatement, $a_t < 1$, is matched by a future above-full abatement $a_r > 1$ that leaves future income above current income:

$$A_t \Omega_t \sigma(a_t) < -A_r \Omega_r \sigma(a_r) \text{ and } (1 - a_t)A_t \Omega_t = (a_r - 1)A_r \Omega_r$$

(30)

Effectively, fossil fuel combustion is seen as a borrowing of the future, admissible if it is part of a development process and if it can be ‘repaired’ in the future and if future income is sufficient to pay for the repair costs. The rule of thumb combines the requirement for maximin income and Attainable income. Note that the repair does not need to take place, but it must be feasible. The future generations themselves can decide whether they engage in negative emissions.

### 3.4 A climate-economy model without bounds

We now consider the optimist perspective of perpetual growth, $A_t \to \infty$, $f'(0) = \infty$. The immediate consequence is that output can grow without bound, irrespective of climate change: $\mathcal{A}(k_t, s_t, m_t) = u(\infty)$, as in the AK model. The interpretation of unbounded growth is that adding value is not restricted to physical consumption, combined with the insight that modern services and industrial production can create its own artificial environment if needed and thus is not very dependent on climate conditions. Many applied climate-economy models foresee no need for a drop in future consumption as a consequence of current emissions, so that maximin income is also not decreasing, and thus under such optimistic assumptions, generosity may not require tough climate policies. The above model, where all damages occur in terms of consumption goods, puts only a very weak bound on emissions.

**Proposition 2** *Under non-negative emissions, $a_t \leq 1$, and ‘unbounded growth’, $A_t \to \infty$, $f'(0) = \infty$ and a closed CO2 system (21), generosity, by maintaining Attainable income, requires prevention of a full catastrophe. Cumulative emissions must remain below a threshold

$$r_t < r^*$$

that satisfies

$$\Omega(m^*(r^*)) = 0$$

(31)*

If emissions can become negative, the same condition as for finite earth apply, but only after the climate deteriorates beyond the threshold of cumulative emissions $r^*$. When
searching for numbers, let us assume that climate change becomes catastrophic for ecosystems and life-support systems at a 4 degrees Celsius global average surface temperature increase. Abstract from uncertainties, and assume that we know that such global warming will be reached if cumulative emissions between 2010 and 2100 exceed 5 Teraton CO2 (Edenhofer et al. 2014, Table 6.3). Under unbounded growth, generosity sets this threshold to cumulative emissions.

Unbounded growth leads to the weak condition that only a full catastrophe needs to be prevented. The analysis changes fundamentally, however, if climate change indicators enter utility directly,

\[ u_t = u(c_t, m_t), \]  

as proposed by Gerlagh and van der Zwaan (2002) and Sterner and Persson (2008). It is important to realize that potential growth without bounds indeed will lead to actual growth without bounds if the return on capital (interest rate) is bounded from above. This enables us to express long-run utility in terms of cumulative emissions:

**Remark 2** Under 'unbounded growth', \( A_t \to \infty, f'(0) = \infty, \) and a bounded return to capital, \( \sup_t f'(k_t/A) < \infty, \) and climate directly entering utility (32), long-run utility is fully determined by cumulative emissions:

\[ \lim_{t \to \infty} u_t = v(r_\infty) = u(\infty, m_\infty) \]  

where  

\[ m_\infty = m^*(r_\infty) \]  

Given non-negative emissions, attainable utility is determined as \( A(k_t, s_t, m_t) = v(r_t) = u(\infty, m^*(r_t)) \). Gerlagh and van der Zwaan (2002) show that for a two-good economy with one ever-growing good and one good that is bounded from above, long-run utility depends on current actions (cumulative emissions, in our case) that determine the long-run level of the bounded good and the specific features of the utility function. They introduce the terms perfect and poor long-term substitutability to characterize different types of utility functions.\(^7\) If man-made consumption goods are a perfect long-term substitute for the environment, \( v'(r_t) = u_m(\infty, m^*(r_t)) = 0 \), attainable income remains independent of climatic change and generosity does not require additional policies. If, however, man-made consumption goods are a poor long-term substitute for the environment, \( v'(r_t) > 0, u_m(\infty, m^*(r_t)) > 0 \), then attainable utility directly changes with

\(^7\)Gerlagh and van der Zwaan (2002) show that in the long run, there is no intermediate case between "perfect" and "poor" long-run substitutability.
current emissions and Proposition 1 and Remark 1 apply: generosity requires maximal abatement, \( a = 1 \), or security that future generations can negate current emissions.

Besides the two options, perfect and poor long-term substitutability between man-made goods and the environment, it is also possible that there exists a threshold \( \overline{m} \) such that man-made goods are a perfect long-term substitute for \( m_\infty > \overline{m} \), and a poor long-term substitute for \( m_\infty < \overline{m} \) (Gerlagh and van der Zwaan 2002). The third possibility is the most interesting, as it implies a threshold: emissions are acceptable as long as the long-term consequences do not irreversibly decrease attainable utility.

**Proposition 3** Under non-negative emissions, \( a_t \leq 1 \), ‘unbounded growth’, \( A_t \to \infty \), \( f'(0) = \infty \) and a closed CO2 system (21), and climate directly entering utility (32), generosity requires cumulative emissions to be constrained below a threshold

\[
r_t < \overline{r}
\]

where

\[
m^*(\overline{r}) = \overline{m}
\]

Generosity potentially imposes a strong condition, but it will never violate Pareto-efficiency. A corollary of Remark 2, based on Gerlagh and Keyzer (2003, Prop 3), is:

**Corollary 1** Under ‘unbounded growth’, \( A_t \to \infty \), \( f'(0) = \infty \) and climate directly entering utility (32), a non-generous path never Pareto-dominates a generous path.

To assess the practical consequences of the above generosity condition, assume that a restriction of cumulative emissions to 1 TtCO2 will keep global warming below 2 degrees Celsius, and assume that at such levels no irreversible damages will occur to eco- and life-support systems, but that any increase above that level will induce irreversible damages. Furthermore, let us assume that these damages are essential, in the sense that they restrict the utility levels that future citizen can reach. Somewhere in between 1 TtCO2 and 5 TtCO2, there is a state of the world where future increased consumption can compensate for the losses associated with climate change, in the sense that it keeps future generations on the same utility level as current generations. Let us say that 4 TtCO2 cumulative emissions allow us to maintain current utility levels at much higher consumption levels but losing much of nature’s beauty. The essential outcome of the classic sustainability criterion is that it imposes the 4 TtCO2 threshold as a constraint on society’s choices. Generosity, on the other hand, imposes the more stringent 1TtCO2
threshold. Generosity does not admit irreversible and essential damages, but requires that future generations can benefit from unrestricted economic growth and see their utility increase beyond current levels, and not restricted by current actions.

4 Uncertainty and intra-generational inequality

The future is uncertain, and the global distribution of wealth is unequal. Sustainability needs to address both the intergenerational inequity as well as the uncertainty and intra-generational inequality. As in the certainty case above, generosity is more demanding compared to the existing sustainability conditions for a stochastic world (e.g. Asheim and Brekke 2002, Dietz and Asheim 2012), given that we do not focus on the possibility of full catastrophes with fat tails.\(^8\)

Under the axiom of within-period anonymity, the utility allocation at time \(t\) is fully captured by the cumulative distribution function for utility levels over all possible states of the world as well as over all individuals within a state of the world. That is, we describe the future in period \(t\) through the cumulative distribution function \(F_t(u)\), which is the share of people with at least utility level \(u\). One can compare the distribution between scenarios or between periods, e.g. period \(\tau\) dominates another period \(t\), which we write as \(F_\tau \succeq F_t\), if the distribution stochastically dominates

\[
F_\tau \succeq F_t \text{ iff } \forall u : F_\tau(u) \leq F_t(u)
\]

For our exposition here, we follow the convenient approach to aggregate utility within a period, defining a weighted average utility as

\[
w_t = \int_0^\infty \alpha(u) udF_t(u) \tag{36}
\]

where we assume that utility levels are defined on the positive domain, and \(\alpha(u)\) is the relative weight given to individuals with utility level \(u\). Propositions 2 and 3 naturally extend to this economy with uncertainty and intra-generational inequality. For Proposition 4, however, we need to consider that the environmental changes can affect people’s utility differently. Equation (32) becomes

\[
\underbar{u}_{t,i} = u_i(c_{t,i}, m_t) \tag{37}
\]

\(^8\)There is also another literature on uncertainty and risks aversion that approaches the question of sustainable development through the adaption of welfare functions to the stochastic environment (e.g. Traeger 2012, Piacquadio 2014).
where label $i$ represents a consumer at a certain state of the world. The best achievable long run allocation is given by

$$v_i(r_{\infty}) = u_i(\infty, m_{\infty})$$

(38)

The corresponding Proposition 4 becomes stronger, in the sense that the maximum cumulative resource use is determined by the consumer type that is most sensitive to climate change damages:

$$r = \min_i \{r_i\}$$

(39)

An action does not satisfy the generosity conditions if there is a state of the world, and a group of consumers with poor long-term substitutability, for whom the maximum attainable utility level is irreversibly reduced. The descendants of the current inhabitants of small islands could be such a group of individuals for the state of the world characterized by sensitive sea-level rise.

5 Discussion

Confronted with the success of worldwide economic growth over the past century, the expected rise of the world middle class in what is still labeled the developing countries, and the threat of the destruction of many of the earth rich ecosystems, including the loss of many small islands due to changing global climate, there is the need for sustainability concepts that provide constructive guidelines which parts of nature we need to conserve, and which parts we can sacrifice in return for higher economic growth. A fundamental but also pragmatic question is by how much we should constrain economic expansion to save nature’s richness. The context of the question is remarkably optimistic: most applied assessments (IPCC 2014) find that we can achieve both economic growth and nature’s conservation, though conservation may still be costly as it delays income progress.

In this paper I propose generosity, keeping or increasing two specific opportunities for the future, as a paradigm for sustainability. The overall message coming from the concept of generosity is simple. If one believes in ’spaceship earth’, that is, a finite space where humankind has to make its living for a very long time to come, then we should keep the fundamental opportunities of this limited space intact, as much as possible. Irreversible damages to the system, when reducing the future utility that can be derived from the system, are not generous. If there is a threshold beyond which climate change threatens to destroy ecosystems beyond recovery, then generosity stipulates that we do not
cross these thresholds. This specific implication of generosity is much more demanding compared to existing concepts of sustainability, which only require that we compensate future generations by sufficient man-made capital for the loss of environmental capital to make them not worse off compared to us. Generosity requires, rather differently, that environmental resources are protected if their contribution to future utility is bounded away from zero, even if future generations enjoy a higher overall level of utility.

If, on the other hand, one believes in infinite ingenuity, where future generations can uncover opportunities that we cannot think of, possibly reaching beyond spaceship earth, then one has to ask which parts of spaceship earth we consider so fundamental to utility that man-made goods coming out of our ingenuity cannot substitute for these, however rich our future descendants may become. The main duty imposed by generosity is then to conserve these fundamental parts, while accepting exhaustion of other parts in return for economic growth.

Though generosity is a strong condition, it does not impose dictatorship of the future (Chichilnisky’s 1996). Present decisions can be directed to the maximization of present generation’s welfare and these interests are given full weight in current decisions. Only when present actions are in direct conflict with utility of an infinite stream of future generations, and only when the costs to future generations are bounded away from zero, does generosity limit today’s actions.

6 Appendix

6.1 An AK+renewable resource model

Here we complement the Ramsey model, interpreting it as a renewable resource, with an AK model. Consider the following economy

\[
\begin{align*}
    u_t &= c_t + r_t \\
    c_t + k_{t+1} &= A k_t \\
    r_t + m_{t+1} &= f(m_t)
\end{align*}
\]  

(40)

where we assume full substitution between man-made goods \( c_t \) and the renewable resource harvesting \( r_t \), we assume that the resource regeneration \( f(\cdot) \) is concave with a maximum sustained level \( f(m^{\text{max}}) = m^{\text{max}} \), and sufficiently strong regeneration for small stocks to make conservation of interest, \( f'(0) > A \). We denote the set of feasible utility paths that can be reached by \( U = U(k_t, m_t) \).
This economy combines perpetual growth of the AK model with possible resource collapse of the renewable resource model. We use the economy to illustrate that for non-trivial economies strategies exist that are strictly contributing, improving the set of feasible utility paths. Let \( m^* \) be the resource level with equal return between the assets, \( f'(m^*) = A \). It is clear that a dynamically efficient allocation must allocate \( m_t = m^* \) to the renewable resource, and if it increases future output such happens by investing in man-made capital \( k_t \).

An action contributes to the economy if it at least maintains the capital and resource stocks, unless the resource stock is above the efficient level \( m^* \), in which case it is a contribution to substitute the man-made capital stock for the resource. The following lemma formalizes this.

**Lemma 1** If \( \overline{m}_t \leq m_t \leq m^* \) and \( \overline{k}_t \leq k_t \), or if \( m^* \leq m_t \leq \overline{m}_t \) and \( \overline{k}_t + m_t \leq k_t + m_t \), then \( U(\overline{k}_t, \overline{m}_t) \leq U(k_t, m_t) \).

**Proof.** By induction. We show that if the condition is satisfied at \( t \), then for any feasible sequence \(( \overline{k}_t, \overline{m}_t, \overline{r}_t, \overline{c}_t) \) starting at \( t \) we can construct a next-period state \(( k_{t+1}, m_{t+1}, r_t, c_t) \) such that the condition is met at \( t + 1 \) and \( r_t + c_t \geq \overline{r}_t + \overline{c}_t \).

Consider the case that \( \overline{m}_t \leq m_t \leq m^* \) and \( \overline{k}_t \leq k_t \). It follows immediately that we can replicate \( m_{t+1} = \overline{m}_{t+1} \) and \( k_{t+1} = \overline{k}_{t+1} \), and have \( r_t \geq \overline{r}_t \) and \( c_t \geq \overline{c}_t \).

Now consider the second case, in which \( m^* \leq m_t \leq \overline{m}_t \) and \( \overline{k}_t + m_t \leq k_t + m_t \). If \( \overline{m}_{t+1} \leq m^* \), we can replicate \( m_{t+1} = \overline{m}_{t+1} \) and \( k_{t+1} = \overline{k}_{t+1} \), and have

\[
\begin{align*}
r_t + c_t &= f(m_t) - m_{t+1} + Ak_t - k_{t+1} \\
&= f(m_t) - \overline{m}_{t+1} + Ak_t - \overline{k}_{t+1} \\
&\geq f(\overline{m}_t) + A(m_t - \overline{m}_t) - \overline{m}_{t+1} + A(\overline{k}_t + m_t - m_t) - \overline{k}_{t+1} \\
&= f(\overline{m}_t) - \overline{m}_{t+1} + A\overline{k}_t - \overline{k}_{t+1} \\
&= \overline{r}_t + \overline{c}_t
\end{align*}
\]

On the other hand, if \( \overline{m}_{t+1} > m^* \), we choose \( m_{t+1} = \max\{m^*, m_t + \overline{m}_{t+1} - m_t\} \) and \( k_{t+1} = \overline{k}_{t+1} + m_{t+1} - m_t \), and also find

\[
\begin{align*}
r_t + c_t &= f(m_t) - m_{t+1} + Ak_t - k_{t+1} \\
&\geq f(\overline{m}_t) + A(m_t - \overline{m}_t) - m_{t+1} + A(\overline{k}_t + m_t - m_t) - \overline{k}_{t+1} - m_{t+1} + m_t \\
&= f(\overline{m}_t) - m_{t+1} + A\overline{k}_t - \overline{k}_{t+1} \\
&= \overline{r}_t + \overline{c}_t
\end{align*}
\]
The condition on $m_{t+1}$ ensures that if $m_{t+1} = m^*$, then $r_t \geq f(m_t) - m_t \geq 0$, while for $m_{t+1} = m_t + \overline{m}_{t+1} - \overline{m}_t$, we have

$$r_t = f(m_t) - m_{t+1}$$
$$= f(m_t) - m_t - \overline{m}_{t+1} + \overline{m}_t$$
$$\geq f(\overline{m}_t) - \overline{m}_{t+1} + (A - 1)(m_t - \overline{m}_t)$$
$$\geq \tau_t$$

The inequalities become strict if one of the inequalities in the conditions is strict. ■

The lemma result in the following proposition with a straightforward interpretation.

The actions at time $t$ contribute to the economy if they increase the capital and resource stock. But if the resource stock is above its efficient level, then a decrease is admissible as long as the aggregate capital plus resource stock increases.

**Proposition 4** For $(k_t, m_t)$, when the resource stock is below the efficient level, $m_t < m^*$, then an action at time $t$ is strictly contributing if both the resource stock and capital increase ($m_t \leq m_{t+1} \leq m^*$ and $k_t \leq k_{t+1}$). When the resource stock is initially above the efficient level, $m_t \geq m^*$, then an action at time $t$ is strictly contributing if the combined capital-resource stock increases, but not the resource stock itself ($m_t + k_t \leq m_{t+1} + k_{t+1}$ and $m^* < m_{t+1} \leq m_t$).

For this economy, the long-run utility opportunities are determined by the perpetual increase of capital of the AK model. Discounted utility returns perpetual growth if $\beta A > 1$, convergence to a constant level of $\beta A < 1 < \beta f'(0)$, and collapse to zero utility for $\beta f'(0) < 1$. The effects and applicability of zero discounting, rank-ordered discounting, and Chichilnisky’s welfare function follow directly from their effects in the AK and the Ramsey model. Contribution to the future set of opportunities, maximin, and generosity are closely related, as each of them requires a qualitative maintenance of the broad capital stock. Attainable income is infinite as long as the capital stock is strictly positive, so that generosity essentially comes down to maintaining maximin income.

The conditions for generosity change substantially, however, and a contribution to the economy becomes more difficult to study if we consider an economy with imperfect substitution between the consumption of man-made goods and environmental amenities. This issue of substitution is at is the core of the climate-economy model.
References


the Fifth Assessment Report of the Intergovernmental Panel on Climate Change.
Cambridge University Press, Cambridge, United Kingdom and New York, NY, USA.


Environmental and Man-Made Goods", *J. of Environmental Economics and Man-
agement* 44: 329-345.


[21] Nordhaus (1997), Discounting In Economics and Climate Change; An Editorial

Warming Policies*. (Yale University Press, New Haven, CT).

concern, and axioms versus data", *Land Economics* 73: 448-466.


over time, and discounting", mimeo, presented at CESifo Area Conference on Energy
and Climate Economics, 17-18 Oct 2014, Munchen, Germany.


[29] Traeger C.P. (2012), "Once upon a time preference - how rationality and risk aversion change the rationale for discounting", CESifo working paper 3793.

