Ebert, Udo; Moyes, Patrick

Working Paper

Talents, Preferences and Inequality of Well-Being

Wirtschaftswissenschaftliche Diskussionspapiere, No. V-327-10

Provided in Cooperation with:
University of Oldenburg, Department of Economics

Suggested Citation: Ebert, Udo; Moyes, Patrick (2010) : Talents, Preferences and Inequality of Well-Being, Wirtschaftswissenschaftliche Diskussionspapiere, No. V-327-10, University of Oldenburg, Department of Economics, Oldenburg

This Version is available at:
https://hdl.handle.net/10419/105015

CARL VON OSSIETZKY universität OLDENBURG

Wirtschaftswissenschaftliche Diskussionspapiere

Talents, Preferences and Inequality of Well-Being

Udo Ebert and Patrick Moyes

V – 327 – 10

Juni 2010

Institut für Volkswirtschaftslehre
Universität Oldenburg, D-26111 Oldenburg

Talents, Preferences and Inequality of Well-Being*

Udo Ebert † Patrick Moyes ‡

May 27, 2010

Abstract

The distribution of gross income net of taxes and transfers – or equivalently consumption – is generally considered a reasonable approximation of the distribution of well-being in the society. One typically observes differing trends in the distribution of gross incomes across countries or within the same country over time. Where do these inequalities originate from? Considering a simple model with no taxation and where individuals belonging to the same society have identical preferences but different productivities, we investigate the impact on the distribution of gross income of changes in the way productivities are distributed. We also look for those changes in the common preference ordering that result in more equally distributed incomes when the allocation of productivities is fixed. Finally, we want to know how preferences have to be adjusted for less dispersed talents to always imply more evenly distributed incomes.

Journal of Economic Literature Classification Number: D31, D63.

Keywords: Well-Being, Inequality, Talents, Preferences, Lorenz Dominance.

1. Introductory Remarks

1.1. Motivation

Income, well-being, ability and preferences Most economic decisions ultimately involve comparisons of distributions of well-being among the members of the same society or across different societies. In practice such comparisons are currently made by reference to the incomes possessed by the agents in the economy. For instance the assessment of alternative policies – e.g. tax reforms and development programs – is typically based on the comparisons of the distributions of income they generate. Similarly, international studies aiming at evaluating the economic impact of development programs involve comparisons of income distributions across countries or over time. In this paper we are interested in the personal traits that shape the distribution of labour income – or equivalently gross income – in the economy in the absence of government interventions.

* This paper forms part of the research project **Heterogeneity and Well-Being Inequality** (Contract No. HEWI/ANR-07-FRAL-020) of the ANR-DFG programme whose financial support is gratefully acknowledged. We are in particular indebted to Stephen Bazen, Francis Bloch, Alain Trannoy and John Weymark for helpful conversations and suggestions.

† Institut für Volkswirtschaftslehre, Carl von Ossietzky Universität Oldenburg, D-26111 Oldenburg, Germany. Email: ebert@uni-oldenburg.de.

‡ GREThA (UMR CNRS 5113), Université Montesquieu Bordeaux 4, CNRS, Avenue Léon Duguit, F-33608 Pessac, France. Email. patrick.moyes@u-bordeaux4.fr.

Two examples It is convenient at this stage to introduce two examples which provide a good illustration of the questions that may arise.

EXAMPLE 1.1. THE PREFERENCE FOR LEISURE: US VERSUS EU. There is evidence that the US does better than the EU countries if economic performance is measured by GNP per head (see for instance OECD (2006)). Obviously, the ranking of countries on the basis of a summary statistic like mean income pays no attention to the way income is distributed among the society's members. The extent to which the ranking of countries is affected by the recourse to indices that are more concerned with distributional justice is certainly worth exploring. Actually, OECD (2006) reports figures on the *equally distributed equivalent incomes* (EDEI) *à la* Atkinson-Kolm-Sen that incorporate information about inequality. Surprisingly, the implied ranking of the countries proves to be quite similar to that based on GNP per head. The recourse to criteria like first and second degree stochastic dominance enables one to check how robust the above conclusions are with respect to the chosen indices. Again there is evidence that the US still ranks above European countries (see McCaig and Yatchew (2007)). If we assimilate individual well-being with labour income and assume that the impact of taxation is negligible, then we come to the conclusion that the US performs better than the EU countries in terms of well-being. The first explanation that comes to mind is that on average individuals in the US work more than those in Europe (see e.g. Osberg (2002)), which means that, other things equal, they prefer consumption to leisure.

EXAMPLE 1.2. INNATE ABILITIES, EDUCATION AND PRODUCTIVITIES. A major concern in the context of globalisation is the role played by education as a means of improving the productivity and competitiveness of the economy (the efficiency dimension). But education is also generally considered to be a key factor contributing to the attenuation of the inequalities of opportunities arising from the fact that talents are not evenly distributed and that individuals face different social circumstances (the equity dimension). For more on the latter point, see Heckman (2008)'s distinction between the role played by cognitive skills (education) and non-cognitive skills (IQ, family circumstances). To what extent do the distribution of talents (innate abilities) and differences in tastes affect the returns to educational policies? Would the redistribution of labour income through progressive taxation be the most effective means of reducing income differences? Or are the latter more likely to be decreased by the implementation of more progressive educational policies?

These two examples stress two different – but not mutually exclusive – origins of the differences one observes between income distributions. The first example emphasizes the role played by individuals' *tastes* and the fact that these tastes may vary across societies in the determination of the distribution of labour income. The second example is concerned with individuals' *productivities* and more specifically with the way the distribution of talents among individuals affects the returns to education. Certainly, if individuals were identical in all respects, then there would be little interest in questions of income inequality.

1.2. The Approach Developed in the Paper

Stylized economy In this paper we take the view that: (i) a society's welfare depends on the distribution of its members' well-being, (ii) an individual's well-being is positively associated with her labour income, and (iii) an individual's income is determined by her talents and tastes. Throughout the paper we consider a stylised economy where each individual is completely identified by her preference ordering and her productivity. There are two commodities in the economy: consumption, which is equal to labour income in the absence of taxation, and leisure. The individual's labour – or gross – income is equal to her labour time multiplied

by her productivity. The individual's preference ordering captures her tastes and values, and her productivity indicates her contribution to total output in efficiency units. In this paper we assume that all members of the same society have identical preferences. This assumption reflects the fact that they have the same cultural, social and historical background so that their tastes and values are sufficiently close to justify the assumption of identical preferences. On the other hand, the members of a given society differ in terms of productivity, which reflects the fact that they have different skills. [1] A society can then be completely characterised by a preference ordering and the distribution of productivities among its members. We are interested in the comparisons of the distributions of individual well-being for a given society or across societies with different cultures from the inequality of view. What attribute is to be used when we seek to approximate the distribution of individual well-being: consumption, utility or consumption and leisure? How do we compare these distributions in order to reach conclusions about the way inequality has increased or decreased?

Appraising individual well-being It is common practice to measure an individual's well-being by means of her *labour income* preferably after taxation. The traditional justification for this way of proceeding is the view that the income – or expenditure – of an individual constitutes a good proxy for her well-being because it measures her opportunities. According to this approach the distribution of consumption vectors reflects the tastes of the agents and there is no reason to go against their choices when evaluating their well-being. If an individual prefers to spend more time at work in order to consume more, then it is her choice and we have to respect it. This way of arguing is valid as long as the individuals have the same opportunities, in which case inequalities in consumption result from the fact that individuals have different preferences over consumption and leisure. The difficulty is precisely that – other such things as prices and exogenous incomes equal – the opportunities faced by agents are determined by their talents and the latter may be unequally distributed within the population. Then the observation that two individuals have different consumption levels may arise from the fact that they have different preferences and/or different talents. This heterogeneity in tastes and productivities may be at the origin of the unequal nature of the distribution of consumption one observes.

Another possibility is to use the *utility derived from consumption and leisure* as a measure of individual well-being. This way of proceeding may be considered a more comprehensive approach since in a market economy the level of utility attained by an individual reflects *both* her preferences and talents. The immediate difficulty is that this requires that we choose a particular – admittedly up to some transformation – representation of the individual's preference ordering. This raises a number of difficulties, the most important of which is the fact that these utility levels are not observable. Furthermore, even if they were observable, the question of whether comparisons of utility levels and/or differences between individuals are meaningful would arise. Assuming that a consensus prevails concerning the measurability and comparability issues, one still has to choose for every individual a particular representation of her preferences. For instance, in the case of Cobb-Douglas preferences one has to decide which is the relevant representation: is it the Cobb-Douglas utility function or the square of the Cobb-Douglas utility function or its logarithm? The choice of particular representations of the individuals' preferences has important implications for comparisons of distributions of well-being in different situations. Indeed, there is no guarantee that the results carry over when one selects alternative admissible representations of individuals' preference orderings.

[1] On some occasions one might be interested in the way productivity is determined (see e.g. Example 1.2, where productivity is the result of innate talent and education), but most of the time we assume that the distribution of productivities is given exogenously.

The same problem arises in the optimal taxation model *à la* Mirrlees (1971), where the choice of a cardinal representation of the preference ordering has implications for the shape of the tax schedule. While it may be considered an appropriate approach from a theoretical point of view, it raises difficulties from an empirical point of view because the agent's *actual* utility function – assuming it exists – cannot be observed. This practical difficulty may be seen as an additional argument in favour of the choice of consumption as an approximation of individual well-being.

A third approach, which avoids the difficulties discussed above, would be to adopt a *paternalistic approach* and appeal to bidimensional stochastic dominance criteria in order to compare the joint distributions of consumption and leisure arising in different circumstances (see for instance McCaig and Yatchew (2007)). For a utilitarian ethical observer, a joint distribution of consumption and leisure is no worse than another joint distribution if the sum of the utilities generated by the first distribution is not smaller than the sum of the utilities generated by the second distribution for all utility functions in a given class. By choosing a sufficiently large class of utility functions, one would ensure that the true unknown utility function of each individual – and by a way of consequence her preferences – will be reflected in the evaluation process. Admittedly many more utility functions than the correct one will be used which will render the criterion extremely demanding. Furthermore, the stochastic dominance approach builds on the utilitarian model which has been attacked on various fronts. In fact it can be shown that the rankings of situations implied by the utilitarian rule are identical to those resulting from the application of ethically more acceptable principles provided that the utility functions retained are sufficiently concave (see Gravel and Moyes (2010)).

The preceding discussion suggests that there is no definite argument in favour of one particular approach for measuring individual well-being. However, in the absence of convincing arguments for assimilating an individual's well-being with her utility – were it the utility she actually experiences or that she is assumed to derive – we will follow the first route and assume that her consumption provides a reasonable approximation of an individual's well-being. We note that in our simple model, where all individuals belonging to the same society have identical preferences and where there is no taxation, the optimal consumption level chosen by the agent is totally informative about her tastes and values. In addition, in contrast to utility – whatever its meaning – consumption is an observable variable, something that has an obvious advantage from a practical point of view.

Comparisons of distributions of individual well-being Rather than building on particular inequality indices in order to compare distributions of well-being within and across societies we prefer the dominance approach. We are indeed searching for robust results and the dominance approach avoids much of the arbitrariness due to the choice of a specific inequality index. But this has a cost: it is not always possible to decide whether one distribution is more or less unequal than another. We adopt the standard approach which consists in appealing to the relative and absolute Lorenz quasi-orderings for comparing distributions of individual well-being (see Kolm (1969), Atkinson (1970), Sen (1997), Shorrocks (1983) or Moyes (1999) among others).

Appraising modifications in the distributions of productivities and preferences In our stylised world, welfare improvements in the distribution of individual well-being as well as inequality reductions can only originate in changes in the distribution of productivities and/or preferences. We have therefore to decide: (i) *what the modifications* of the distributions of productivities and preferences are when we investigate the effects on well-being inequality, and (ii) *how we measure* these changes. Other things being equal, the conventional wisdom suggests

that a lower dispersion of productivities in a society leads to a reduction of the inequality in the distribution of individual well-being. As we will show in this paper, these expectations are not always verified and particular restrictions have to be placed on preferences – or equivalently on the representations of these preferences – in order for the inequality of well-being to decrease as a result of more concentrated productivities. Actually, these restrictions can be relaxed to some extent if the reduction in the dispersion of talents goes hand in hand with *either* a more *or* a less efficient allocation of these talents among the population.

1.3. The Questions Addressed in the Paper

The preceding discussion emphasises the fact that the information available in order to make comparisons of well-being is limited. The distributions of talents and utilities are unobservable and the way consumption and labour time are distributed among the population is the only thing that one can observe. In this research, we take a theoretical approach and we do not constrain ourselves with the informational requirements the practitioner typically faces. This does not mean that we are not aware of these constraints but rather that we want to uncover the origins of the differences in individuals' consumption in a stylised world. To this aim we consider a very simple representation of the economy where productivities and preferences are the only factors that shape the distributions of consumption and utility. We adopt a comparative static framework and investigate the implications of *particular changes* in both individuals' tastes and productivities for the distribution of consumption among individuals. This means that we do not consider all possible modifications but rather focus on those that we feel are important and at the same time allow us to derive unambiguous conclusions regarding the direction of the inequality changes.More precisely, we address the following series of questions:

- Is it always the case that less dispersed talents among the population give rise to lower consumption inequality?
- If it were not the case, then is it possible to identify those restrictions that have to be placed on the utility function that would guarantee that consumption inequality decreases when the talents are more concentrated in the population?
- Assuming that individuals' talents are given, which modifications of preferences would lead to more equally distributed consumption levels between individuals?
- How do the distribution of talents and changes in the preferences interact when determining the distribution of consumption?

The above questions focus on the relationship between the dispersion of talents and the inequality in consumption. Similar questions can be formulated concerning the way the distribution of consumption improves according to welfare criteria such as the utilitarian principle or more general welfarist rules when the allocation of talents changes: this is the purpose of another paper (see Ebert and Moyes (2010)). It is expected that the responses to these questions will depend to a large extent on the criteria one appeals to for comparing the distributions of talents and the distributions of consumption. While we certainly do not claim to provide definite answers to the above questions, we nevertheless hope that our approach will identify unambiguous distributional effects in some particular cases of interest.

1.4. Related Literature

At first sight the topic addressed here is not new and an examination of the literature suggests that there are a number of papers investigating the distributional properties of alternative institutions for allocating resources among a society's members. A first strand of literature is concerned with the implication for social welfare of the distribution of the market equilibrium incomes in an economy without production. Chipman and Moore (1973) look for the conditions under which one can conclude that *potential welfare has improved as the result of an increase in real national income*, where the latter is measured in terms of the prices prevailing in each of the two years under comparison. Here a potential welfare improvement is taken to mean that those who are better off in the period when real national income is higher can compensate those who are made worse off. It is shown that these conditions would be realized if and only if all agents have the same preferences and these preferences are homothetic. Imposing the further restriction that the increase in real national income is accompanied by an unchanged distribution of income – in the sense that the income shares of the agents are not modified – Chipman and Moore (1980) found that *identical and homothetic* preferences are sufficient for an increase in real national income to imply a welfare improvement.

Foster, Majumdar, and Mitra (1990) want to know *when do Lorenz-type comparisons of expenditure distributions have anything to say about the associated levels of social welfare.* They first examine the case of an exchange economy, where prices are endogenously set according to relative supply and demand as the initial allocation varies. The question is whether the social welfare levels of equilibrium allocations might be reflected in Lorenz comparisons of equilibrium income distributions. They find that apart from certain notable special cases – *two agent economies* or *quasi-homothetic preferences* – the distributions of income at the competitive equilibria provide no information about social welfare. On the other hand, if one considers the general market economy common to analyses of real national income (see e.g. Sen (1976)), where each agent's bundle can be supported as a utility-maximizing point given market prices, then it is possible to derive conclusions about welfare changes starting with the distributions of incomes evaluated at the current market prices. More precisely, it is shown that social welfare at the equilibrium is greater than the social welfare generated by another allocation whenever the actual equilibrium income distribution generalised Lorenz dominates the distribution of income arising from the second allocation and evaluated at the current prices. Madden (1996) considers production economies and investigates *when Pareto efficient allocations and competitive equilibria are undominated for particular ethical quasi-orderings* like Suppes-Sen dominance and generalised Lorenz dominance. For instance, it is proven that the Pareto-optimal competitive equilibria of a productive economy typically fail to satisfy the minimal equity demand of generalised Lorenz optimality. Even worse, an artisan economy – similar to that retained in this paper – is shown to have competitive equilibria that are Suppes-Sen dominated even though they are Pareto efficient.

While the above papers are interested in the properties of the equilibrium allocation under different institutional arrangements and the inferences that can be made for social welfare, they do not try to relate particular modifications of the economic circumstances – like the distribution of endowments between the agents or their preferences – with the resulting changes in the equilibrium allocations. Notable exceptions are the papers by Brett and Weymark (2008) and Simula (2007) in the field of optimal taxation that seek to identify the *impact on the optimal allocation of taxes of changes in the distribution of productivities* when agents have identical quasilinear preferences. The complexity of the optimal taxation model does not permit the implications of such transformations like increased dispersion of productivities to be investigated as it is done in this paper. It is however possible to give indications about the

effect of the increase in one agent's productivity on the optimal solution. Brett and Weymark (2008) sign the directions of change in everyone's optimal consumptions and optimal marginal tax rates in response to such a change for quasilinear-in-leisure preferences, while Simula (2007) performs a similar exercise for quasilinear-in-consumption preferences.

1.5. Organization of the Paper

We introduce in Section 2 the model that we will use throughout the paper. It is an oversimplified economy where each agent decides in isolation the amount of her time she will devote to labour, which given her productivity or talent determines her consumption. No trade is allowed and an agent can only consume what she produces. Since there is no taxation, consumption equals gross income, that is the number of hours worked times productivity plus possibly an exogenous income. Section 3 is devoted to the presentation of the criteria for appraising changes in the allocation of individual talents. We make a distinction between those modifications of the distribution of talents that reduce dispersion and those that improve efficiency. Section 4 contains our main results concerning the identification of the properties of the consumption function that ensure that consumption inequality decreases as a result of less dispersed talents. We adopt the standard practice that consists in using the relative Lorenz criterion for making inequality comparisons. The relative inequality approach has been challenged by some authors (see in particular Kolm (1976)) and alternatives to the relative Lorenz quasi-ordering have been proposed. We provide in Section 5 the counterparts of the characterization results of the previous section when consumption inequality is evaluated by means of the absolute Lorenz quasi-ordering. We identify in Section 6 the properties of the utility functions that constitute the counterparts of the consumption elasticities conditions in the particular case where preferences are linear in working time. Section 7 concludes the paper summarizing our main findings, pointing at limitations and suggesting avenues for further research. Finally, Section 8 contains the proofs of our main results, while Appendix A provides the list of the utility functions used in our examples and figures.

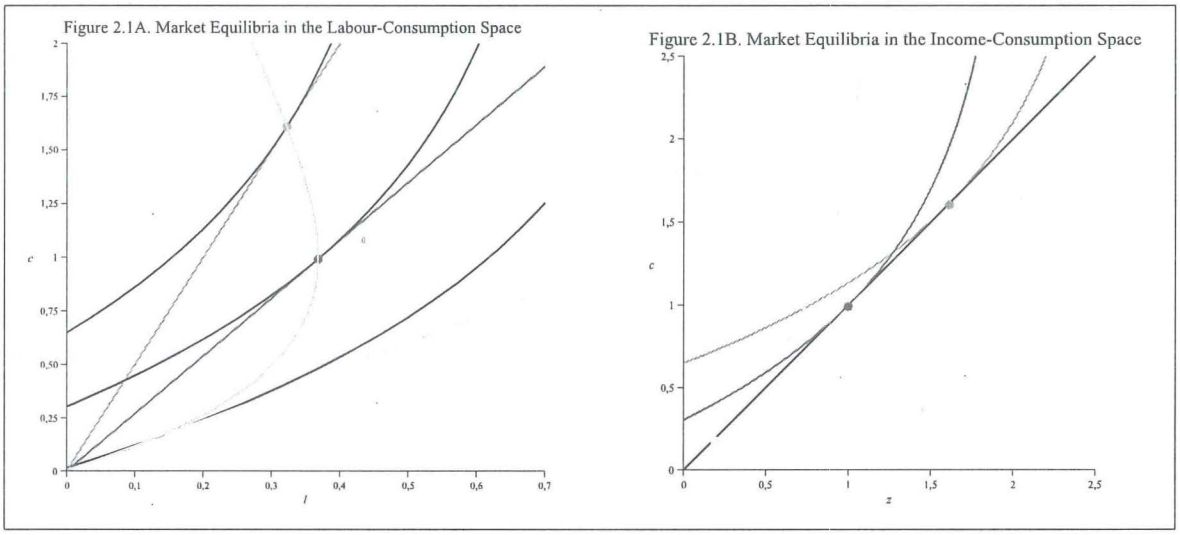
2. Notation and Preliminary Definitions

2.1. The Stylized Economy

Building upon Ebert and Moyes (2007), we consider an *artisan economy* with n individuals ($n \geqq 2$) and two commodities: *consumption* c and *labour time* ℓ with $0 \leqq \ell \leqq T$, where T represents the maximum amount of leisure available. All the individuals belonging to the same society have identical preferences over the consumption-labour space represented by an ordinal *direct utility function* $u(c, \ell)$ which is assumed to be (i) twice differentiable, (ii) increasing in consumption and decreasing in labour time, and (iii) stricly quasi-concave with respect to the consumption-labour time bundle. *Gross income* z is determined by *productivity* or equivalently *talent* $w > 0$, labour time ℓ and *exogenous income* m according to the formula $z = g(\ell; w, m) = w\ell + m$. Upon substitution into the direct utility function, we obtain the *personalised utility function* $U(c, z; w, m) := u(c, (z - m)/w)$. It follows from the properties of the direct utility function that $U(c, z; w, m)$ is (i) twice differentiable, (ii) increasing in consumption and decreasing in gross income, and (iii) stricly quasi-concave with respect to the consumption-gross income bundle. As is common practice in the literature, we further impose that $U(c, z; w, m)$ verifies the Spence-Mirrlees condition according to which

$$MRS(c, z; w, m) := -\frac{U_z(c, z; w, m)}{U_c(c, z; w, m)} \text{ is decreasing in } w, \tag{2.1}$$

on the appropriate domain. We find it convenient for later use to indicate by $\mathscr{U}$ the set of direct utility functions such that the above properties are satisfied and we further note that $\mathscr{U}$ is closed under increasing and twice differentiable transformations.



2.2. The Market Equilibria

The optimisation program of an individual endowed with productivity w and receiving exogenous income m is:

$$P(U,w,m) \qquad (c,z) \ \max \ U(c,z;w,m) \text{ s.t. } c \leqq z \ \text{ and } \ \frac{z-m}{w} \leqq T.$$

Given the personalised utility function $U(c,z;w,m)$, we denote as $W(U)$ the range of productivities $w > 0$ such that the optimization problem above has a *unique interior* solution and the Spence-Mirrlees condition is verified. We indicate by $C(w,m)$ and $Z(w,m)$ the solution of problem $P(U,w,m)$: the consumption and labour income of an individual depend on her productivity w and her exogenous income m. It follows from our assumptions that the consumption function $C(w,m)$ and the gross income function $Z(w,m)$ are increasing in w and m (see e.g. Ebert and Moyes (2007, Lemma 1)). We also note that, since utility is increasing in consumption and since there is no taxation, we necessarily have $C(w,m) = Z(w,m)$. One can immediately derive the labour supply function $L(w,m) = (Z(w,m) - m)/w$, which is not necessarily monotonic with respect to productivity. While the members of the same society have similar tastes and values, they may differ in their productivities and we let $\mathbf{w} := (w_1, \ldots, w_n) \in \mathbb{R}^n_{++}$ stand for a typical allocation of productivities where by assumption $w_1 \leqq w_2 \cdots \leqq w_n$. Given $U \in \mathscr{U}$, we indicate by $\mathbb{W}(U)$ the set of productivity allocations $\mathbf{w} := (w_1, \ldots, w_n)$ such that $w_i \in W(U)$, for all $i = 1, 2, \ldots, n$. The distributions of consumption and gross income, solutions to the optimisation program $P(U,w,m)$ for the n individuals, are denoted as

$$Z(\mathbf{w},m) := (Z(w_1,m), \ldots, Z(w_n,m)), \text{ and} \tag{2.2a}$$

$$C(\mathbf{w},m) := (C(w_1,m), \ldots, C(w_n,m)), \tag{2.2b}$$

respectively. The corresponding distribution of labour time is then given by

$$L(\mathbf{w}, m) := (L(w_1, m), \ldots, L(w_n, m)). \tag{2.3}$$

Assuming that individuals have different exogenous incomes – inherited wealth from their parents for instance – would open the route to the exploration of the impact on the distribution of consumption of changes in the joint distribution of talents and wealth. For the time being we assume throughout that $m = 0$ and without risk of confusion we denote respectively as $Z(\mathbf{w}) := Z(\mathbf{w}, 0)$, $C(\mathbf{w}) := C(\mathbf{w}, 0)$ and $L(\mathbf{w}) := L(\mathbf{w}, 0)$ the distributions of gross income, consumption and working time at the agents' equilibria. We have represented in Figures 2.1A and 2.1B the equilibrium allocations for three different levels of productivity but the same preferences in the labour-consumption space and in the gross income-consumption space, respectively. It can be seen that for these preferences labour supply is not monotonically increasing with productivity.

3. Appraising Modifications of the Distribution of Talents

One may conceive of different types of modifications in the allocation of productivity that affect the distribution of individual well-being. Here we focus on two particular aspects of the distribution of productivities that we expect might play an important role in explaining the shape of the distribution of consumption: *dispersion* and *efficiency*. Other things equal, it is clear that, if all agents in the economy have the same productivity, then there is no room for consumption inequality. Therefore, it is the fact that agents differ in their productivities that explains why inequalities in consumption arise in our stylised world. Taking this a step further, we reasonably expect that, the more concentrated – or equivalently the less dispersed – are agents' productivities, the more equal the distribution of consumption. In order to measure the concentration of productivities we appeal to the dispersive quasi-ordering that originated in the statistical literature (see e.g. Shaked and Shanthikumar (1994, Section 2.B)). Before we turn to the precise definition of the dispersive quasi-ordering, it is convenient to introduce a further piece of notation. Given a vector $\mathbf{u} \in \mathbb{R}^n$, we denote as $\mathbf{u}_{(\,)} := (u_{(1)}, \ldots, u_{(n)})$ its non-decreasing rearrangement defined by $u_{(1)} \leqq u_{(2)} \leqq \cdots \leqq u_{(n)}$. Then we have:

DEFINITION 3.1. Given two distributions of productivity $\mathbf{w}^*, \mathbf{w}^\circ \in \mathbb{R}^n_{++}$, we will say that $\mathbf{w}^*$ *is weakly less dispersed than* $\mathbf{w}^\circ$, which we write $\mathbf{w}^* \geq_{LD} \mathbf{w}^\circ$, if and only if

$$w^*_{(j)} \big/ w^*_{(i)} \leqq w^\circ_{(j)} \big/ w^\circ_{(i)}\,, \ \forall\, i = 1, 2, \ldots, j-1, \ \forall\, j = 2, 3, \ldots, n. \tag{3.1}$$

The asymmetric and symmetric components of $\geq_{LD}$ are defined in the usual way and indicated by $>_{LD}$ and $\sim_{LD}$, respectively. While it is open to debate, we feel that the dispersive quasi-ordering is more suited for comparing the degree of concentration of productivity levels than more standard inequality measures.

For reasons that will become clear later, it is useful to consider those modifications of the allocation of productivities that increase or decrease the productive efficiency of the economy. The following criterion, that allows us to compare distributions of productivities from the efficiency point of view, is quite natural. [2]

[2] At some point one would be tempted to make inferences about the economy overall productivity and the way individual productivity is allocated among the agents. While it is natural to consider that overall productivity increases when the distribution of individual productivity becomes more efficient, things are more intricate when one is interested in those changes that leave overall productivity constant. Such modifications of the distribution of individual productivities are particularly important for the evaluation of social welfare (for more on this, see Ebert and Moyes (2010)).

DEFINITION 3.2. Given two distributions of productivity $\mathbf{w}^*, \mathbf{w}^\circ \in \mathbb{R}^n_{++}$, we will say that $\mathbf{w}^*$ *is weakly more efficiently distributed than* $\mathbf{w}^\circ$, which we write $\mathbf{w}^* \geq_{ME} \mathbf{w}^\circ$, if and only if

$$w^*_{(i)} \geqq w^\circ_{(i)}, \ \forall i = 1, 2, \ldots, n. \tag{3.2}$$

Equivalently, we will say that $\mathbf{w}^\circ$ *is less efficiently distributed than* $\mathbf{w}^*$, something we write $\mathbf{w}^\circ \geq_{LE} \mathbf{w}^*$.

We define the asymmetric and symmetric components of $\geq_{ME}$ in the usual way, which we indicate by $>_{ME}$ and $\sim_{ME}$, respectively. [3] On some occasion we will be willing to investigate the joint impact on the distribution of consumption of less dispersed and more or less efficiently allocated productivities. Our first additional criterion combines considerations for more efficiency and less dispersion.

DEFINITION 3.3. Given two distributions of productivity $\mathbf{w}^*, \mathbf{w}^\circ \in \mathbb{R}^n_{++}$, we will say that $\mathbf{w}^*$ *is more efficiently distributed and less dispersed than* $\mathbf{w}^\circ$, which we write $\mathbf{w}^* \geq_{MELD} \mathbf{w}^\circ$, if and only if

$$\mathbf{w}^* \geq_{ME} \mathbf{w}^\circ \ \text{ and } \ \mathbf{w}^* \geq_{LD} \mathbf{w}^\circ. \tag{3.3}$$

For our second additional criterion, less dispersion goes hand in hand with less efficiently allocated productivities.

DEFINITION 3.4. Given two distributions of productivity $\mathbf{w}^*, \mathbf{w}^\circ \in \mathbb{R}^n_{++}$, we will say that $\mathbf{w}^*$ *is less efficiently distributed and less dispersed than* $\mathbf{w}^\circ$, which we write $\mathbf{w}^* \geq_{LELD} \mathbf{w}^\circ$, if and only if

$$\mathbf{w}^* \geq_{LE} \mathbf{w}^\circ \ \text{ and } \ \mathbf{w}^* \geq_{LD} \mathbf{w}^\circ. \tag{3.4}$$

We define and denote the asymmetric and symmetric components of $\geq_{MELD}$ and $\geq_{LELD}$ in the usual way.

4. Relative Inequality and Comparisons of Distributions of Well-Being

We are interested in the comparisons of the distributions of the agents' consumptions at the market equilibria of two different artisan economies. We consider successively three cases: (i) both economies have the same preferences but different distributions of individual productivities, (ii) the economies have different preferences but the distributions of individual productivities are identical, and (iii) the economies differ both in terms of their preferences and distributions of productivities. We first have to make precise how we compare the distributions of consumption from an inequality point of view.

4.1. The Measurement of Relative Inequality

As we indicated in the Introduction, we want to obtain results that do not depend on a particular inequality index but hold rather for a large spectrum of value judgements. Focusing on relative inequality in this section, we therefore follow the standard practice and appeal to the relative Lorenz quasi-ordering in order to make comparisons of inequality within the

[3] The reader will rightly notice that our definition of a (weakly) more efficient distribution of productivities is actually nothing else than first order stochastic dominance or equivalently quantile dominance.

same society or across different societies. The ordinate of the *relative Lorenz curve* of the consumption distribution $\mathbf{c} := (c_1, \ldots, c_n)$ at $p = k/n$ is given by

$$RL\left(\frac{k}{n}; \mathbf{c}\right) := \frac{1}{n} \sum_{j=1}^{k} \frac{c_{(j)}}{\mu(\mathbf{c})}, \ \forall k = 1, 2, \ldots, n, \tag{4.1}$$

where $\mu(\mathbf{c})$ is the *arithmetic mean* of distribution $\mathbf{c}$. A consumption distribution is then considered as less unequal than another if its relative Lorenz curve lies nowhere below that of the latter, which we formally state as follows:

DEFINITION 4.1. Given two consumption distributions $\mathbf{c}^*, \mathbf{c}^\circ \in \mathbb{R}^n_{++}$, we will say that $\mathbf{c}^*$ *relative Lorenz dominates* $\mathbf{c}^\circ$, which we write $\mathbf{c}^* \geq_{RL} \mathbf{c}^\circ$, if and only if

$$RL\left(\frac{k}{n}; \mathbf{c}^*\right) \geqq RL\left(\frac{k}{n}; \mathbf{c}^\circ\right), \ \forall k = 1, 2, \ldots, (n-1). \tag{4.2}$$

Recourse to the relative Lorenz criterion ensures that a distribution cannot be considered better than another if it is ranked below the latter by at least one reasonable (relative) inequality index. In other words, all inequality indices that are deemed relevant must agree on the ranking of the pair of distributions under comparison for one distribution to dominate another according to the relative Lorenz quasi-ordering. The class of inequality indices consistent with the relative Lorenz quasi-ordering is quite large and contains most of the inequality measures currently used such as the Gini index, the Atkinson-Kolm-Sen (AKS) family of indices as well as the generalised entropy family. All these indices have the property that a progressive transfer – the operation consisting of transferring part of the consumption of a rich individual to a poorer one – reduces inequality. Another important property – to which we will return to in the next section – is the fact that equiproportionate additions leave inequality unchanged. For more details the reader is referred to the surveys by Foster (1985) or Moyes (1999) and the references therein.

4.2. Identical Preferences and Different Distributions of Productivities

What is the impact on consumption inequality of a change in the dispersion of productivities when the agents have the same given preferences? At first sight one might expect that in such a situation less dispersed productivities would lead automatically to a reduction of consumption inequality. However contrary to what intuition suggests consumption inequality does not necessarily decrease as a result of more concentrated productivities. A reduction in the dispersion of individual productivities may actually generate an unambiguous increase in consumption inequality even in the case of well-behaved preferences.

EXAMPLE 4.1. Let $n = 2$ and choose $\mathbf{w}^\circ = (1.80, 3.24)$ and $\mathbf{w}^* = (1.50, 2.50)$. We have $w_1^* < w_1^\circ$, $w_2^* < w_2^\circ$ and $w_2^\circ/w_1^\circ = 1.800 > 1.666 = w_2^*/w_1^*$, hence $\mathbf{w}^* >_{LELD} \mathbf{w}^\circ$. Choosing the utility function $u^2(c, \ell) = c - e^\ell$, we obtain at the market equilibrium, $\mathbf{c}^\circ = (1.058, 3.808)$ and $\mathbf{c}^* = (0.608, 2.290)$. [4] This implies that

$$c_2^\circ / c_1^\circ = 3.599 < 3.766 = c_2^* / c_1^*,$$

and, appealing to Lemma 8.3, we conclude that $C(\mathbf{w}^\circ) >_{RL} C(\mathbf{w}^*)$.

[4] We insist on the fact that all the utility functions considered in our examples verify the properties imposed in Section 2.2.

This example demonstrates that the standard properties of the utility function – and beyond that the properties of the preference ordering – do not guarantee that less dispersed productivities will always result in more equally distributed consumptions. However the reader might not be totally convinced because the transformation of the initial distribution of productivities is rather particular as the reduction in dispersion is accompanied by a decrease in the productivities of both agents.

EXAMPLE 4.2. Leave the preferences as defined in the previous example but choose now $\mathbf{w}^\circ = (0.50, 2.00)$ and $\mathbf{w}^* = (0.60, 2.39)$. We observe that $w_1^* > w_1^\circ$, $w_2^* > w_2^\circ$ and $w_2^\circ/w_1^\circ = 4.00 > 3.98 = w_2^*/w_1^*$, hence $\mathbf{w}^* >_{MELD} \mathbf{w}^\circ$. At the market equilibrium, we have $\mathbf{c}^\circ = (1.000, 2.732)$ and $\mathbf{c}^* = (1.130, 3.148)$, which implies

$$c_2^\circ / c_1^\circ = 2.732 < 2.785 = c_2^* / c_1^*,$$

thus $C(\mathbf{w}^\circ) >_{RL} C(\mathbf{w}^*)$.

Our second example eliminates any doubts that less dispersed productivities may lead to more unequally distributed consumptions. Put together, these two examples make it clear that the standard properties of the utility function – and beyond that the properties of the preference ordering – do not guarantee that a reduction in the dispersion of productivity endowments always results in more equally distributed consumption levels. This suggests that for this to be the case additional restrictions have to be imposed on the agents' common preference ordering. However, the identification of these additional properties of the preference ordering is a difficult task and we adopt an indirect approach that consists in deriving the conditions that the consumption function needs to satisfy for inequality to decrease as a result of less dispersed productivities.

The elasticity of the consumption function with respect to productivity will prove to be the key factor for signing the impact on consumption inequality of changes in the distribution of productivities. To simplify notation and for latter use we denote as $\eta(C, w) := C'(w)\, w/C(w)$ the elasticity of the consumption function. Our first result identifies those consumption functions with the property that consumption is more equally distributed among the agents as productivity becomes more concentrated.

Proposition 4.1. *Let $u \in \mathscr{U}$ and $n \geqq 2$. The following two statements are equivalent:*

(a) For all $\mathbf{w}^*, \mathbf{w}^\circ \in \mathbb{W}(U)$; $\mathbf{w}^* \geq_{LD} \mathbf{w}^\circ$ implies $C(\mathbf{w}^*) \geq_{RL} C(\mathbf{w}^\circ)$.

(b) $\eta(C, w)$ is constant in w, for all $w \in W(U)$.

Proposition 4.1 indicates the precise condition that the consumption function has to fulfill for our expectations to be verified. This condition, which requires that the consumption elasticity with respect to productivity be constant, is particularly restrictive. It is actually equivalent to requiring that

$$\frac{C(\lambda w^*)}{C(w^*)} = \frac{C(\lambda w^\circ)}{C(w^\circ)}, \ \forall\, \lambda \geqq 1, \ \forall\, w^*, w^\circ, \lambda w^*, \lambda w^\circ \in W(U), \tag{4.3}$$

a functional equation whose solution (Aczel, 1966, Chapter 3) is

$$C(w) = \beta w^\eta \ (\beta > 0, \eta > 0), \ \forall\, w \in W(U). \tag{4.4}$$

For sure the meaning of the requirement of a constant consumption elasticity might look rather abstruse at first sight. The equivalent condition (4.3) is helpful in this respect by making more explicit how demanding condition (b) of Proposition 4.1 is. According to the

former, a proportional change in an agent's productivity must imply a proportional change in her consumption but not necessarily of the same extent. [5]

So far we have not made use of information about the fact that productivities might be more efficiently distributed in one society than in another. Here we are interested in the impact on consumption inequality of a change in the dispersion of productivities but now accompanied by unambiguous increases or decreases in those productivities. To what extent does the introduction of additional information about efficiency in the distribution of productivities affect the preceding result? Imposing further restrictions in addition to the fact that productivities are less dispersed will result in weaker conditions that have to be satisfied by the consumption functions. A direct implication is that the class of consumption functions we are looking for is likely to enlarge. But even if this were the case, what this class looks like is still an open question. The following result provides the answer for the case where less dispersion goes hand in hand with more efficiently distributed productivities.

Proposition 4.2. *Let $u \in \mathscr{U}$ and $n \geqq 2$. The following two statements are equivalent:*

(a) For all $\mathbf{w}^*, \mathbf{w}^\circ \in \mathbb{W}(U)$; $\mathbf{w}^* \geq_{MELD} \mathbf{w}^\circ$ implies $C(\mathbf{w}^*) \geq_{RL} C(\mathbf{w}^\circ)$.

(b) $\eta(C, w)$ is non-increasing in w, for all $w \in W(U)$.

Proposition 4.2 indicates exactly how the class of consumption functions expands when one imposes the additional requirement that productivities are more efficiently distributed. Condition (b) of Proposition 4.2 is reminiscent of the concept of *increasing average progression* encountered in the taxation literature (see e.g. Jakobsson (1976), Le Breton, Moyes, and Trannoy (1996), Lambert (2001)). We note that for a differentiable consumption function this condition can be stated equivalently as

$$\frac{C(\lambda w^*)}{C(w^*)} \leqq \frac{C(\lambda w^\circ)}{C(w^\circ)}, \ \forall\, w^*, w^\circ \in W(U), \ \forall\, \lambda \geqq 1, \tag{4.5}$$

according to which the increase in an agent's consumption caused by a proportional increase in her productivity must be less than proportional to the latter. If now we want consumption inequality to decrease when productivities are less dispersed but also less efficiently distributed, then we obtain the following result that does not come as a surprise.

Proposition 4.3. *Let $u \in \mathscr{U}$ and $n \geqq 2$. The following two statements are equivalent:*

(a) For all $\mathbf{w}^*, \mathbf{w}^\circ \in \mathbb{W}(U)$; $\mathbf{w}^* \geq_{LELD} \mathbf{w}^\circ$ implies $C(\mathbf{w}^*) \geq_{RL} C(\mathbf{w}^\circ)$.

(b) $\eta(C, w)$ is non-decreasing in w, for all $w \in W(U)$.

The non-decreasingness of the consumption function elasticity guarantees that consumption inequality will always decrease as the agents' productivities become less dispersed and at the same time less efficiently distributed.

It is tempting to relate the conditions identified in Proposition 4.2 (resp. Proposition 4.3 to the concavity (resp. convexity) of the consumption function. Actually, the concavity of the consumption function and the requirement that its elasticity be non-increasing are independent properties. Computing the derivative of the consumption elasticity, condition (b) of Propositions 4.2 reduces to

$$\eta(C', w) \leqq \eta(C, w) - 1, \ \forall\, w \in W(U). \tag{4.6}$$

[5] The only case where consumption increases or decreases in the same proportion as productivity is when $\eta(C, w) = 1$ or in other words when the consumption function is proportional to productivity.

In the absence of additional restrictions, there is no logical relationship between the concavity of the consumption function and the fact it has a non-increasing elasticity. [6] Similarly, convexity of the consumption function does not guarantee that its elasticity is non-decreasing. An example is provided by the utility function $u^2(c,\ell) := c - e^{\ell}$: we have $C(w) = w \ln w$ which is convex but $\eta(C,w)$ is decreasing. Thus, under convex consumption, less dispersed and less efficiently allocated productivities would not necessarily result in a decrease in consumption inequality. Thus the convexity or concavity of the consumption function is no impediment for its elasticity to increase or decrease, and conversely.

Propositions 4.1, 4.2 and 4.3 make clear that the efficiency dimension of the transformation of the distribution of productivity plays a crucial role in the determination of the class of the consumption functions that ensure that consumption inequality decreases when productivities are less dispersed. Conditions (b) of Propositions 4.1, 4.2 and 4.3 constrain the underlying

Table 4.1: Propositions 4.1, 4.2 and 4.3 in a glance

	$C(\mathbf{w}^*) \geq_{RL} C(\mathbf{w}^\circ)$	
$\mathbf{w}^* \geq_J \mathbf{w}^\circ$	Consumption Function	Utility Function
$\mathbf{w}^* \geq_{LD} \mathbf{w}^\circ$	$C(w) = \beta w^\eta \ (\beta, \eta > 0)$	$u^1(c,\ell) = c - \dfrac{\ell^2}{2}$ $u^{14}(c,\ell) = -(\ell+3)e^{-\frac{c-3}{\ell+3}-1}$
$\mathbf{w}^* \geq_{MELD} \mathbf{w}^\circ$	$\eta(C,w) \downarrow w$	$u^2(c,\ell) = c - e^{\ell} \ (w > 1)$ $u^{15}(c,\ell) = -(\ell+3)e^{-\frac{c-2}{\ell+3}-1} \ (w > \frac{1}{3})$
$\mathbf{w}^* \geq_{LELD} \mathbf{w}^\circ$	$\eta(C,w) \uparrow w$	$u^3(c,\ell) = \ln c - \dfrac{1}{c} - \ell$ $u^{16}(c,\ell) = -(\ell+3)e^{-\frac{c-4}{\ell+3}-1}$

preference orderings and this raises immediately two questions. Do there exist preference orderings that generate consumption functions satisfying these conditions? If so, what do these orderings look like or how can one be sure that they are compatible with the conditions identified in Propositions 4.1, 4.2 and 4.3? As far as the first question is concerned, a look at Table 4.1 should convince the reader that the conditions the consumption function needs to satisfy for inequality to decrease when productivities are more concentrated are not totally unrealistic. There exist non-pathological utility functions – and therefore preferences – that generate such consumption patterns. It is also worthnoting that these elasticity conditions are compatible with a large class of preference orderings comprising among other things separable preferences and quasilinear preferences. The question of whether these conditions are plausible or likely to be met in practice is something that remains to be investigated but that lies outside the scope of this paper. [7]

[6] For instance, if one imposes the further restriction that $0 \leqq \eta(C,w) < 1$, then condition (4.6) implies that C is concave.

[7] Leaving aside the fact that productivities are difficult to observe, the major problem is that the consumption patterns we witness are determined jointly by the tax system and the distribution of productivities. This will make it difficult for the econometrician to separate those changes in the distribution of consumption arising from modifications in the allocation of productivities from those resulting from modifications of the tax system.

4.3. Different Preferences and Identical Distributions of Productivities

We consider now the impact on consumption inequality of a modification of the preference ordering which as before we assimilate with a modification of the consumption function.

EXAMPLE 4.3. Given $n = 2$, choose $\tilde{\mathbf{w}} = (1.50, 2.50)$ and let

$$u^\circ(c, \ell) = u^3(c, \ell) = \ln c - \frac{1}{c} - \ell \quad \text{and}$$

$$u^*(c, \ell) = u^6(c, \ell) = -e^{-c} - \ell.$$

At the market equilibrium, we obtain $\tilde{\mathbf{c}}^\circ = (2.186, 3.265)$ and $\tilde{\mathbf{c}}^* = (0.405, 0.916)$, which implies

$$\tilde{c}_2^\circ / \tilde{c}_1^\circ = 1.493 < 2.261 = \tilde{c}_2^* / \tilde{c}_1^*,$$

thus $C(\tilde{\mathbf{w}}^\circ) >_{RL} C(\tilde{\mathbf{w}}^*)$.

The substitution of the utility function u^* for the utility function u° has resulted in an unambiguous increase in consumption inequality for a specific allocation of productivities. Actually it is possible to find a distribution of productivities, where the opposite conclusion is obtained.

EXAMPLE 4.4. Choosing now $\hat{\mathbf{w}} = (3.50, 5.00)$, the market equilibria allocations for $u^\circ = u^3$ and $u^* = u^6$ become respectively $\hat{\mathbf{c}}^\circ = (4.311, 5.854)$ and $\hat{\mathbf{c}}^* = (1.252, 1.609)$. This implies

$$\hat{c}_2^\circ / \hat{c}_1^\circ = 1.357 > 1.284 = \hat{c}_2^* / \hat{c}_1^*,$$

thus $C(\hat{\mathbf{w}}^\circ) >_{RL} C(\hat{\mathbf{w}}^*)$.

The two above examples demonstrate that anything can happen in the absence of some appropriate condition that must be satisfied by the utility functions u^* and u°. Our next result identifies the condition that has to verified for consumption inequality to decrease *whatever* the allocation of productivities among the agents.

Proposition 4.4. *Let u^* and $u^\circ \in \mathscr{U}$ and $n \geqq 2$. The following two statements are equivalent:*

(a) For all $\mathbf{w} \in \mathbb{W}(U^*) \cap \mathbb{W}(U^\circ)$; $C^*(\mathbf{w}) \geq_{RL} C^\circ(\mathbf{w})$.

(b) $\eta(C^*, w) \leqq \eta(C^\circ, w)$, for all $w \in W(U^*) \cap W(U^\circ)$.

Proposition 4.4 indicates the way the consumption function has to be adjusted in order that consumption inequality is reduced for any arbitrary allocation of productivity. Condition (b) in Proposition 4.4 is reminiscent of the concept of a *more progressive tax schedule* encountered in the taxation literature (see e.g. Jakobsson (1976), Le Breton *et al.* (1996), Ebert and Moyes (2002)). It can be stated equivalently as

$$\frac{C^*(\lambda w)}{C^*(w)} \leqq \frac{C^\circ(\lambda w)}{C^\circ(w)}, \ \forall\, w \in W(U^*) \cap W(U^\circ), \ \forall\, \lambda \geqq 1. \tag{4.7}$$

This means that the relative increase in consumption caused by a proportional increase in productivity is smaller under C^* than under C°. Here again it is possible to provide pairs of utility functions such that the corresponding consumption functions satisfy condition (b) of Proposition 4.4: two such pairs are presented in Table 4.2. Here again we note that these elasticity conditions do not impose stringent constraints on the underlying preference orderings which comprise non-separable preferences as well as separable preferences. In particular, preferences linear in *either* consumption *or* labour are equally admissible.

Table 4.2: Proposition 4.4 in a glance

$C^*(\mathbf{w}) \geq_{RL} C^\circ(\mathbf{w})$	
CONSUMPTION FUNCTIONS	UTILITY FUNCTIONS $u^*(c,\ell)$ AND $u^\circ(c,\ell)$
$\eta(C^*,w) \leqq \eta(C^\circ,w)$	$u^1(c,\ell) = c - \frac{\ell^2}{2}$; $\quad u^5(c,\ell) = 2\sqrt{c} - \ell$
	$u^3(c,\ell) = \ln c - \frac{1}{c} - \ell$; $\quad U^9(c,\ell) = c - \frac{5}{2}\left[\ell e^{\frac{-1}{\ell}} - \int_1^{+\infty} \frac{e^{-t\ell}}{t}\, dt\right]$

4.4. Different Preferences and Different Distributions of Productivities

Finally we turn to the general case where both preferences and the distributions of productivities are allowed to differ. Suppose that productivities become less dispersed: what changes in preferences guarantee that consumption inequality is reduced as a result? Or, what transformations of the distribution of productivity ensure that the inequality reducing impact of the change in the preferences is preserved? Our first example might suggest that there is no guarantee in the absence of a suitable adjustment of preferences that less dispersed productivities give rise to more equally distributed consumption levels.

EXAMPLE 4.5. Choose $\tilde{\mathbf{w}}^\circ = (1.50, 3.00)$, $\tilde{\mathbf{w}}^* = (1.35, 2.60)$, and

$$u^\circ(c,\ell) = u^3(c,\ell) = \ln c - \frac{1}{c} - \ell \text{ and}$$
$$u^*(c,\ell) = u^6(c,\ell) = -e^{-c} - \ell.$$

Observe that $\tilde{w}_1^* < \tilde{w}_1^\circ$, $\tilde{w}_2^* < \tilde{w}_2^\circ$ and $\tilde{w}_2^\circ/\tilde{w}_1^\circ = 2.000 > 1.925 = \tilde{w}_2^*/\tilde{w}_1^*$, hence $\mathbf{w}^* >_{LELD} \mathbf{w}^\circ$. At the market equilibrium, we have $\tilde{\mathbf{c}}^\circ = (2.186, 3.791)$ and $\tilde{\mathbf{c}}^* = (0.300, 0.955)$, which implies

$$\tilde{c}_2^\circ/\tilde{c}_1^\circ = 1.734 < 3.183 = \tilde{c}_2^*/\tilde{c}_1^*,$$

thus $C(\tilde{\mathbf{w}}^\circ) >_{RL} C(\tilde{\mathbf{w}}^*)$.

On the other hand, our second example suggests that the choice of the initial distribution of productivities and the way it is made more concentrated are important.

EXAMPLE 4.6. Choose $\hat{\mathbf{w}}^\circ = (3.50, 4.50)$, $\hat{\mathbf{w}}^* = (4.30, 5.40)$, and let u° and u^* as defined in the previous example. Observe that $\hat{w}_1^* > \hat{w}_1^\circ$, $\hat{w}_2^* > \hat{w}_2^\circ$ and $\hat{w}_2^\circ/\hat{w}_1^\circ = 1.285 > 1.255 = \hat{w}_2^*/\hat{w}_1^*$, hence $\mathbf{w}^* >_{MELD} \mathbf{w}^\circ$. At the market equilibrium, we have $\hat{\mathbf{c}}^\circ = (4.311, 5.342)$ and $\hat{\mathbf{c}}^* = (1.458, 1.686)$, which implies

$$\hat{c}_2^\circ/\hat{c}_1^\circ = 1.239 > 1.156 = \hat{c}_2^*/\hat{c}_1^*,$$

thus $C(\hat{\mathbf{w}}^*) >_{RL} C(\hat{\mathbf{w}}^\circ)$.

Taken together these two examples seem to indicate that, in the absence of appropriate restrictions on the way the distribution of productivity and the preferences are altered simultaneously, almost anything can occur. Clearly, the results of Sections 4.2 and 4.3 provide clues about the restrictions that will guarantee that consumption inequality will decrease in particular cases. For instance, it is a direct consequence of Propositions 4.2 and 4.4 that, for $C^*(\mathbf{w}^*) \geq_{RL} C^\circ(\mathbf{w}^\circ)$ whenever $\mathbf{w}^* \geq_{MELD} \mathbf{w}^\circ$, it is sufficient that

$$\eta(C^*,w) \leqq \eta(C^\circ,w),\ \forall\ w > 0, \text{ and} \tag{4.8}$$

$$\eta(C^*,w) \text{ is non-increasing in } w,\ \forall\ w > 0. \tag{4.9}$$

Indeed, assuming that $\mathbf{w}^* \geq_{MELD} \mathbf{w}^\circ$ and invoking Proposition 4.2 and condition (4.9), we conclude that $C^*(\mathbf{w}^*) \geq_{RL} C^*(\mathbf{w}^\circ)$. Condition (4.8) and Proposition 4.4 ensure that $C^*(\mathbf{w}^\circ) \geq_{RL} C^\circ(\mathbf{w}^\circ)$, and transitivity of the relative Lorenz quasi-ordering makes the argument complete. Similarly, Propositions 4.2 and 4.4 ensure that, if condition (4.8) holds and

$$\eta(C^\circ, w) \text{ is non-decreasing in } w, \ \forall\ w > 0, \tag{4.10}$$

then $C^*(\mathbf{w}^*) \geq_{RL} C^\circ(\mathbf{w}^\circ)$ whenever $\mathbf{w}^* \geq_{LELD} \mathbf{w}^\circ$.

However, while these conditions are sufficient for consumption inequality to decrease, they are far from being necessary. A glance at Example 4.6 suggests that there might be no need for less dispersed and more efficiently distributed productivities to imply more equal consumption levels that the elasticity of the dominated consumption function C° is non-decreasing. The elasticity of C° is not monotonically non-decreasing and that this does not prevent $C^*(\mathbf{w}^*)$ from relative Lorenz dominating $C^\circ(\mathbf{w}^\circ)$. This suggests that conditions (4.9) and (4.10) might be unnecessary and that it should be possible to propose weaker restrictions that still guarantee that consumption inequality decreases when productivities become more concentrated and preferences change. The next result provides the answer in the absence of any information about the direction of the change in efficiency implied by the increase in the concentration of productivities.

Proposition 4.5. *Let u^* and $u^\circ \in \mathscr{U}$ and $n \geqq 2$. The following two statements are equivalent:*

(a) For all $\mathbf{w} \in \mathbb{W}(U^*) \cap \mathbb{W}(U^\circ)$; $\mathbf{w}^* \geq_{LD} \mathbf{w}^\circ$ implies $C^*(\mathbf{w}^*) \geq_{RL} C^\circ(\mathbf{w}^\circ)$.

(b) $\eta(C^*, w) \leqq \eta(H, w) \leqq \eta(C^\circ, w)$, for all $w \in W(U^*) \cap W(U^\circ)$ and some differentiable function H with constant elasticity.

Proposition 4.5 confirms that for consumption inequality to decrease as a result of less dispersed productivities, it is necessary that the elasticity of C^* is not greater than the elasticity of C° everywhere. But it also underlines the fact that this is not sufficient: upper and lower bounds are respectively imposed on the elasticities of C^* and C°, and these bounds are equal and constant over the admissible range of productivities. Condition (b) of Proposition 4.5 actually imposes the condition that the curves representing the elasticities of the consumption functions can be separated by an horizontal line. A look at Figure 4.2, where we have represented the elasticities of a sample of utility functions, will convince the reader of how stringent this condition is. On the other hand, Proposition 4.5 also confirms that it is not necessary that C^* or C° verify conditions (4.9) or (4.10) for consumption inequality to decrease.

So far we have not made use of the fact that the productivities might be more efficiently distributed in one society than in another. We are interested here in the impact on consumption inequality of a change in the dispersion of productivities accompanied by an increase in efficiency.

Proposition 4.6. *Let u^* and $u^\circ \in \mathscr{U}$ and $n \geqq 2$. The following two statements are equivalent:*

(a) For all $\mathbf{w} \in \mathbb{W}(U^*) \cap \mathbb{W}(U^\circ)$; $\mathbf{w}^* \geq_{MELD} \mathbf{w}^\circ$ implies $C^*(\mathbf{w}^*) \geq_{RL} C^\circ(\mathbf{w}^\circ)$.

(b) $\eta(C^*, w) \leqq \eta(H, w) \leqq \eta(C^\circ, w)$, for all $w \in W(U^*) \cap W(U^\circ)$ and some differentiable function H with non-increasing elasticity.

The additional requirement that productivities are more efficiently distributed in addition to the fact that they are less dispersed allows us to relax the conditions that need to be fulfilled by the consumption functions C^* and C°. There are still upper and lower bounds imposed on the elasticities of C^* and C°, but now these bounds are not constant but decline with the level of productivity. If it happens that productivities are less efficiently distributed while at the same time they are more concentrated, then we obtain the following result.

Figure 4.1. Consumption Elasticities for Different Preference Orderings

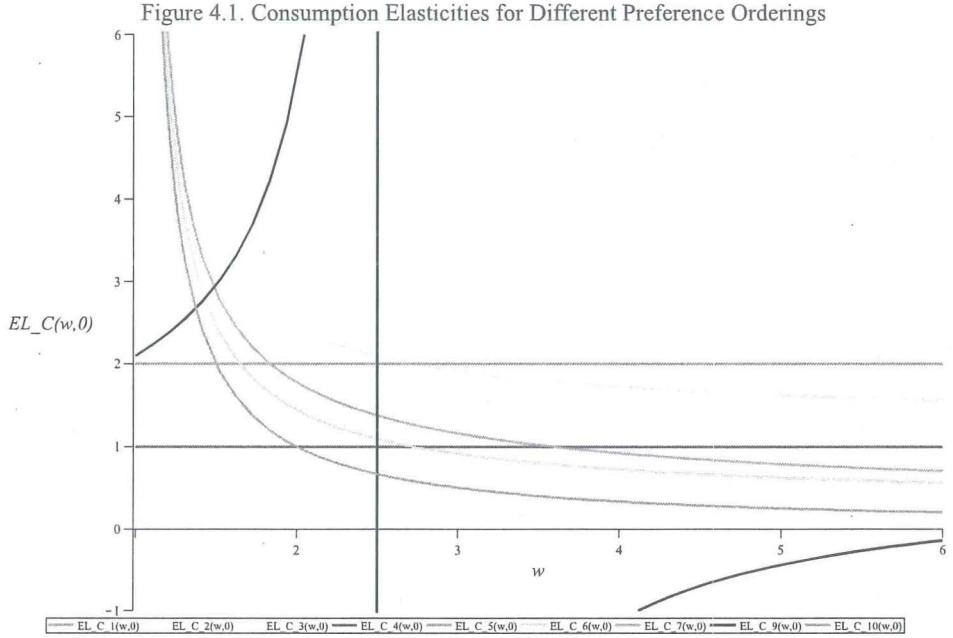


Table 4.3: Propositions 4.5, 4.6 and 4.7 in a glance

	$C^*(\mathbf{w}^*) \geq_{RL} C^\circ(\mathbf{w}^\circ)$	
$\mathbf{w}^* \geq_J \mathbf{w}^\circ$	CONSUMPTION FUNCTION	UTILITY FUNCTIONS $u^*(c,\ell)$ AND $u^\circ(c,\ell)$
$\mathbf{w}^* \geq_{LD} \mathbf{w}^\circ$	$\eta(C^*,w) \leqq \eta(H,w) \leqq \eta(C^\circ,w)$	$u^4(c,\ell) = \ln c - \ell$; $u^5(c,\ell) = 2\sqrt{c} - \ell$
	$H(w) = \beta w^\eta$ $(\beta, \eta > 0)$	$u^3(c,\ell) = \ln c - \frac{1}{c} - \ell$; $u^2(c,\ell) = c - e^\ell$ $(w > 1)$
$\mathbf{w}^* \geq_{MELD} \mathbf{w}^\circ$	$\eta(C^*,w) \leqq \eta(H,w) \leqq \eta(C^\circ,w)$	$u^4(c,\ell) = \ln c - \ell$; $u^5(c,\ell) = 2\sqrt{c} - \ell$
	$\eta(H,w) \downarrow w$	$u^4(c,\ell) = \ln c - \ell$; $u^2(c,\ell) = c - e^\ell$ $(w > 1)$
$\mathbf{w}^* \geq_{LELD} \mathbf{w}^\circ$	$\eta(C^*,w) \leqq \eta(H,w) \leqq \eta(C^\circ,w)$	$u^4(c,\ell) = \ln c - \ell$; $u^5(c,\ell) = 2\sqrt{c} - \ell$
	$\eta(H,w) \uparrow w$	$u^3(c,\ell) = \ln c - \frac{1}{c} - \ell$; $u^2(c,\ell) = c - e^\ell$ $(w > 1)$

Proposition 4.7. *Let u^* and $u^\circ \in \mathscr{U}$ and $n \geqq 2$. The following two statements are equivalent:*

(a) For all $\mathbf{w} \in \mathbb{W}(U^*) \cap \mathbb{W}(U^\circ)$; $\mathbf{w}^* \geq_{LELD} \mathbf{w}^\circ$ implies $C^*(\mathbf{w}^*) \geq_{RL} C^\circ(\mathbf{w}^\circ)$.

(b) $\eta(C^*,w) \leqq \eta(H,w) \leqq \eta(C^\circ,w)$, for all $w \in W(U^*) \cap W(U^\circ)$ and some differentiable function H with non-decreasing elasticity.

The necessary and sufficient conditions identified in Propositions 4.5, 4.6 and 4.7 impose constraints on the preference orderings of the two economies under comparison. While it is a difficult exercise to uncover the precise meaning of these restrictions for the shape of the preference orderings, it is easy to convince oneself of the existence of preferences that generate consumption functions satisfying these requirements (see Section 6 below). We provide in Table 4.3 instances of pairs of utility functions that fulfill each of these conditions. The elasticities of the consumption functions generated by the utility functions of Table 4.3 – as well as some other utility functions used throughout the paper – are depicted in Figure 4.2.

5. Absolute Inequality and Comparisons of Distributions of Well-Being

The inequality criterion The concept of relative inequality has been challenged by certain authors (see in particular Kolm (1976)) and this has given rise to different alternatives to the relative Lorenz quasi-ordering in the literature (see among others Bossert and Pfingsten (1990), Krtscha (1994), Del Rio and Ruiz-Castillo (2000)). Here we follow Kolm (1976)'s original suggestion and take the view that it is the absolute rather than the relative differences in consumption levels that matter. There are admittedly other possibilities and it is in principle possible to adapt the results in this section to conform with these alternative views. The analogue to the relative Lorenz quasi-ordering when one subscribes to Kolm (1976)'s proposal is the so-called absolute Lorenz criterion. While the standard relative Lorenz criterion compares the cumulated consumption shares of the agents, the absolute Lorenz criterion is concerned with the cumulated consumption shortfalls from mean consumption of the agents in the economy (see Moyes (1987)). More precisely, the ordinate of the absolute Lorenz curve of the consumption distribution $\mathbf{c} := (c_1, \ldots, c_n)$ at $p = k/n$ is given by

$$AL\left(\frac{k}{n}; \mathbf{c}\right) := \frac{1}{n} \sum_{j=1}^{k} \left[c_{(j)} - \mu(\mathbf{c})\right], \ \forall k = 1, 2, \ldots, n. \tag{5.1}$$

It represents the amount of consumption needed on average in order to make sure that all individuals get a consumption level equal to the average consumption in the economy. The absolute Lorenz quasi-ordering is based on the comparison of the absolute Lorenz curves and it is formally defined as follows.

DEFINITION 5.1. Given two consumption distributions $\mathbf{c}^*, \mathbf{c}^\circ \in \mathbb{R}^n_{++}$, we will say that $\mathbf{c}^*$ *absolute Lorenz dominates* $\mathbf{c}^\circ$, which we write $\mathbf{c}^* \geq_{AL} \mathbf{c}^\circ$, if and only if

$$AL\left(\frac{k}{n}; \mathbf{c}^*\right) \geqq AL\left(\frac{k}{n}; \mathbf{c}^\circ\right), \ \forall k = 1, 2, \ldots, (n-1). \tag{5.2}$$

Less dispersed productivities and consumption inequality As in Section 4 we would like to know in what circumstances less dispersed productivities give rise to less unequally distributed consumption levels when the preference ordering is given. More generally, we are interested in those adjustments of the preferences that guarantee that a reduction in the dispersion of productivities translates into a reduction in consumption inequality. Since the conditions we obtain are to a large extent straightforward adaptations of the conditions derived in Section 4, we avoid presenting a long list of propositions and rather summarise the main results by means of a few tables. It has been shown in Section 4 that the elasticity of the consumption function is the key factor for appraising the impact on relative inequality of changes in the distribution of productivities. The derivative of the consumption function with respect to the logarithm of productivity will play a similar role when the focus is on absolute inequality. To simplify notation and for latter use we denote as $\xi(C, w) := C'(w)\, w$ the derivative of the consumption function with respect to the logarithm of productivity. Table 5.1 sets out the properties that the consumption function must satisfy for consumption absolute inequality to decline as a result of less dispersed productivities when the preference ordering is fixed. When the consumption function is linear in the logarithm of productivity, consumption inequality is guaranteed to decrease as productivities become less dispersed. This particular consumption function is the solution of the functional equation

$$C(\lambda w^*) - C(w^*) = C(\lambda w^\circ) - C(w^\circ), \ \forall\ w^*, w^\circ \in W(U), \ \forall\ \lambda \geqq 1, \tag{5.3}$$

Table 5.1: Identical Preferences and Different Distributions of Productivities

	$C(\mathbf{w}^*) \geq_{AL} C(\mathbf{w}^\circ)$	
$\mathbf{w}^* \geq_J \mathbf{w}^\circ$	CONSUMPTION FUNCTION	UTILITY FUNCTION
$\mathbf{w}^* \geq_{LD} \mathbf{w}^\circ$	$C(w) = \alpha + \beta \ln w \ (\alpha \in \mathbb{R}, \beta > 0)$	$u^6(c,\ell) = -e^c - \ell \quad (w > 1)$ $u^{11}(c,\ell) = -e^{\frac{c}{2}} - \ell \quad (w > 2)$
$\mathbf{w}^* \geq_{MELD} \mathbf{w}^\circ$	$\xi(C,w) \downarrow w$	$u^{10}(c,\ell) = c - \frac{c^2}{8} - \ell \quad (0 < c \leqq 4, w > 1)$
$\mathbf{w}^* \geq_{LELD} \mathbf{w}^\circ$	$\xi(C,w) \uparrow w$	$u^3(c,\ell) = \ln c - \frac{1}{c} - \ell$ $u^{13}(c,\ell) = -\frac{1}{4} e^{\sqrt{c}} - \ell$

(see Aczel (1966, Chapter 3)), which indicates that the (absolute) differences in consumption are not affected by proportional increases in productivity. Taking the derivative of (5.3), we obtain equivalently

$$\xi(C, \lambda w) = \xi(C, w), \ \forall\, w \in W(U), \ \forall\, \lambda \geqq 1, \tag{5.4}$$

hence the derivative of $C(w)$ with respect to the logarithm of w is constant. The necessary and sufficient conditions for consumption inequality to decrease, whatever the distribution of productivities, when preferences change are summarised in Table 5.2.

Table 5.2: Different Preferences and Identical Distributions of Productivities

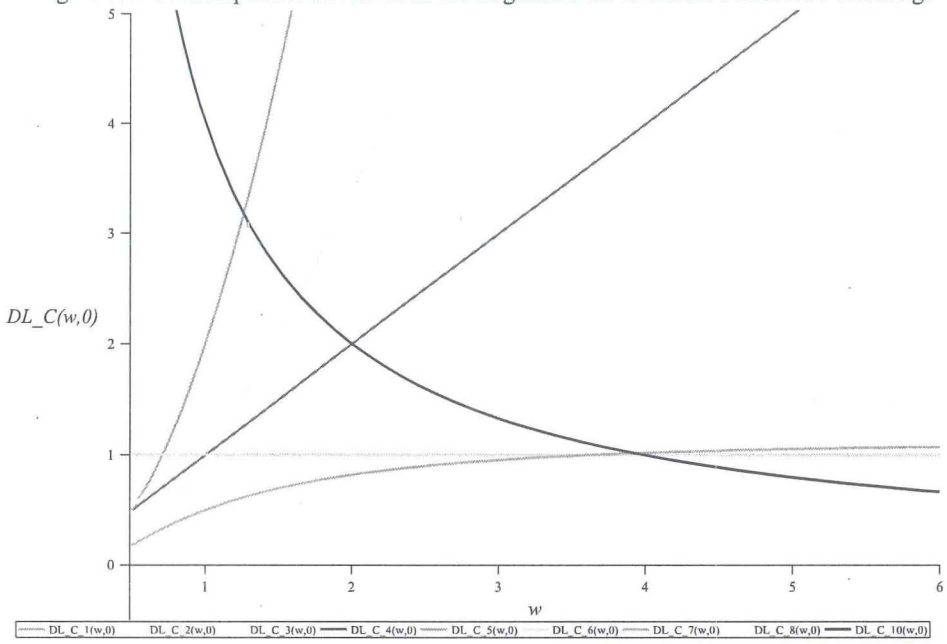
$C^*(\mathbf{w}) \geq_{AL} C^\circ(\mathbf{w})$	
CONSUMPTION FUNCTIONS	UTILITY FUNCTIONS $u^*(c,\ell)$ AND $u^\circ(c,\ell)$
$\xi(C^*, w) \leqq \xi(C^\circ, w)$	$u^7(c,\ell) = -e^{-c} - e^{\ell} \quad (w > 1); \quad u^6(c,\ell) = -e^c - \ell \quad (w > 1)$ $u^4(c,\ell) = \ln c - \ell; \quad u^5(c,\ell) = 2\sqrt{c} - \ell$

Taken together, Tables 5.1 and 5.2 make it clear that the derivative of the consumption function with respect to the logarithm of productivity is the key variable that determines the direction of the change in consumption inequality when preferences or the distribution of productivity are modified. The general case where changes affect simultaneously the preferences and the distributions of productivity is easily dealt with and we skip the presentation of the necessary and sufficient conditions for inequality reduction in consumption which are *mutatis mutandis* similar to those obtained in the previous section. The derivatives of the logarithm of the consumption functions generated by the utility functions in Tables 5.1 and 5.2 – as well as some other utility functions used throughout the paper – are depicted in Figure 5.1. It should be noted that the conditions that guarantee that consumption inequality decreases under appropriate transformations of the distribution of productivities are not independent. Suppose that $\xi(C,w)$ is non-increasing, which upon using the fact that $C(w)$ is increasing and positive, is equivalent to

$$\eta(C', w) \leqq -1, \ \forall\, w \in W(U). \tag{5.5}$$

Then clearly (5.5) implies (4.6) and $\eta(C,w)$ is non-increasing in w. By the same token, if $\eta(C,w)$ is non-decreasing in w, then so is $\xi(C,w)$.

Figure 5.1. Consumption Derivatives in the Logarithm for Different Preference Orderings



6. Quasilinear Preferences

So far we have identified the properties of the consumption functions that guarantee that inequality in consumption decreases as a result of particular transformations of the distribution of productivities. We have also specified the changes in the consumption function that imply more equal consumption when individual productivities are fixed and when they are less dispersed. We also provided evidence by means of examples that there exist preference orderings that generate consumption functions with the desired properties in our artisan economy. However, neither the theoretical results nor these examples are informative about what the underlying preference orderings must look like and how they have to be altered. Although one can always associate a unique preference ordering to any consumption function, it may be impossible in practice to derive an explicit representation of this ordering. A first difficulty originates in the fact that it is not always possible to obtain the expenditure function in a closed form starting from the demand system. A second problem is that, even if one succeeded in deriving the expenditure function starting from the demand system and in recovering the indirect utility function, then it is difficult to interpret the *indifference curves* in the prices-income space and relate these with the standard indifference curves (see however Blackorby, Primont, and Russell (1978)).

Even though they are routinely used in many areas, quasilinear preferences are quite restrictive and one may therefore be skeptical about their use here. A close inspection of our examples suggests that quasilinear preferences are sufficiently flexible for accommodating all of the possible situations that we are interested in. There is then no loss of generality – at least as far as the questions addressed in this paper are concerned – in restricting attention to such preferences. Furthermore it is always possible in this case to solve explicitly the artisan's optimisation problem and to derive the explicit form of the consumption function. In the particular case where the utility function is linear in labour time, the consumption function has the property that it does not depend on exogenous income. Suppose that preferences can be represented by $u(c, \ell) = v(c) - \ell$, where the consumption utility function v is increasing and strictly concave. Maximising $u(c, \ell)$ under the budget constraint $c = w\ell + m$ and setting

$m = 0$, we get the necessary and sufficient first order condition $v'(c) = 1/w$, from which we derive the consumption function

$$c = v'^{-1}\left(\frac{1}{w}\right) =: C(w). \tag{6.1}$$

We consider successively the cases where we are interested in reductions of relative and absolute consumption inequality.

Relative inequality Using (6.1), the consumption elasticity can be rewritten as

$$\eta(C,w) = \frac{1}{\eta(C^{-1},c)} = \frac{\dfrac{1}{v'(c)}}{-\dfrac{v''(c)\,c}{v'(c)^2}} = \frac{-1}{\dfrac{v''(c)\,c}{v'(c)}} = -\frac{v'(c)}{v''(c)\,c}. \tag{6.2}$$

Differentiating (6.2), we obtain

$$\eta_w(C,w) \equiv \frac{\partial \eta(C,w)}{\partial w} = -\frac{v''(c)\,c\left[v''(c)\,\dfrac{dc}{dw}\right] - v'(c)\left[(v'''(c)\,c + v''(c))\,\dfrac{dc}{dw}\right]}{[v''(c)\,c]^2}. \tag{6.3}$$

Using the fact that consumption increases with productivity, we finally deduce that

$$\eta_w(C,w) \left\{ \begin{matrix} < \\ = \\ > \end{matrix} \right\} 0 \text{ iff } \frac{v''(c)\,c}{v'(c)} - \frac{v'''(c)\,c}{v''(c)} \left\{ \begin{matrix} > \\ = \\ < \end{matrix} \right\} 1. \tag{6.4}$$

Conditions (6.4) are reminiscent of the notions of decreasing, constant and increasing relative risk aversion where the difference between relative risk aversion and relative prudence plays a crucial role. Table 6.1 provides a summary indicating the connections between the properties of the utility function and those of the corresponding consumption function. Suppose next

Table 6.1: Consumption relative inequality and the properties of the utility function

Ranking of Consumption Vectors	Consumption Function Elasticity	Consumption Utility Function
A. Changes in the distribution of talents		
$C(\mathbf{w}^*) \geq_{RL} C(\mathbf{w}^\circ)$	$\eta_w(C,w) \geqq 0$	$\dfrac{v'''(c)\,c}{v''(c)} - \dfrac{v''(c)\,c}{v'(c)} \leqq 1$
$C(\mathbf{w}^\circ) \geq_{RL} C(\mathbf{w}^*)$	$\eta_w(C,w) \leqq 0$	$\dfrac{v'''(c)\,c}{v''(c)} - \dfrac{v''(c)\,c}{v'(c)} \geqq 1$
B. Changes in the preferences		
$C^*(\mathbf{w}^*) \geq_{RL} C^\circ(\mathbf{w}^\circ)$	$\eta_w(C^*,w) \geqq \eta_w(C^\circ,w)$	$-\dfrac{v^{*\prime\prime}(c)\,c}{v^{*\prime}(c)} \leqq -\dfrac{v^{\circ\prime\prime}(c)\,c}{v^{\circ\prime}(c)}$
$C^\circ(\mathbf{w}^\circ) \geq_{RL} C^*(\mathbf{w}^*)$	$\eta_w(C^*,w) \leqq \eta_w(C^\circ,w)$	$-\dfrac{v^{*\prime\prime}(c)\,c}{v^{*\prime}(c)} \geqq -\dfrac{v^{\circ\prime\prime}(c)\,c}{v^{\circ\prime}(c)}$

that there exists a differentiable and increasing function $H(w)$ with positive values such that

$$\eta_w(C^*,w) \leqq \eta_w(H,w) \leqq \eta_w(C^\circ,w), \ \forall\, w \in W(U^*) \cap W(U^\circ). \tag{6.5}$$

Let $c = H(w)$ and consider the function $v : W(U^*) \cap W(U^\circ) \longrightarrow \mathbb{R}$ defined by

$$v(c) = \int_0^c \frac{1}{H^{-1}(s)}\, ds. \tag{6.6}$$

We note that $v(c)$ is increasing since $v'(c) = 1/H^{-1}(c) > 0$ and since $w > 0$ by assumption. Furthermore

$$v''(c) = -\frac{1}{[H^{-1}(c)]^2\, H'(w)} < 0, \tag{6.7}$$

since $H(w)$ is increasing. Finally, maximising $u(c, \ell) = v(c) - \ell$ under the budget constraint $c = w\ell$, we get the first order condition

$$v'(c) = \frac{1}{w} = H^{-1}(c), \tag{6.8}$$

which proves that $H(w)$ is the consumption function generated by the utility function $u(c, \ell) = v(c) - \ell$. Then one can immediately derive the restrictions on the consumption utility function corresponding to the conditions that the elasticity of H is non-decreasing or non-increasing.

Absolute inequality Using (6.1), the derivative of the consumption function in the logarithm can be rewritten as

$$\xi(C, w) = \frac{C^{-1}(c)\, c}{\xi(C^{-1}, c)} = \frac{\dfrac{1}{v'(c)}}{-\dfrac{v''(c)}{v'(c)^2}} = \frac{-1}{\dfrac{v''(c)}{v'(c)}} = -\frac{v'(c)}{v''(c)}. \tag{6.9}$$

Upon differentiation we obtain

$$\xi_w(C, w) \equiv \frac{\partial \xi(C, w)}{\partial w} = -\frac{v''(c)\, v''(c)\, \dfrac{dc}{dw} - v'(c)\, v'''(c)\, \dfrac{dc}{dw}}{[v''(c)\, c]^2} \gtreqless 0. \tag{6.10}$$

Since consumption increases with productivity, we deduce that

$$\xi_w(C, w) \left\{ \begin{array}{c} < \\ = \\ > \end{array} \right\} 0 \quad \text{iff} \quad \frac{v''(c)}{v'(c)} - \frac{v'''(c)}{v''(c)} \left\{ \begin{array}{c} > \\ = \\ < \end{array} \right\} 0. \tag{6.11}$$

For the derivative of the consumption function in the logarithm to be decreasing, constant and increasing with productivity it is necessary and sufficient that the consumption utility function exhibit decreasing, constant and increasing relative risk aversion, respectively. This amounts to imposing the restriction that the difference between absolute risk aversion and absolute prudence is non-positive (see e.g. Eeckhoudt, Gollier, and Schlesinger (1996)). In Table 6.2 we present the properties of the utility functions that guarantee that absolute inequality in consumption decreases when productivities are less dispersed and when tastes vary. Suppose next that there exists a differentiable and increasing function $H(w)$ with positive values such that

$$\xi_w(C^*, w) \leqq \xi_w(H, w) \leqq \xi_w(C^\circ, w), \ \forall\ w \in W(U^*) \cap W(U^\circ). \tag{6.12}$$

Here again consider the function $v : W(U^*) \cap W(U^\circ) \longrightarrow \mathbb{R}$ defined by (6.6): it is increasing and concave. By construction $H(w)$ solves the optimisation problem of an agent with utility function $u(c, \ell) = v(c) - \ell$ and talent w. Then one immediately derives the restrictions on the consumption utility function corresponding to the conditions that the derivative in the logarithm of H is non-decreasing or non-increasing.

Table 6.2: Consumption absolute inequality and the properties of the utility function

Ranking of Consumption Vectors	Consumption Derivative in the Logarithm	Consumption Utility Function
A. Changes in the distribution of talents		
$C(\mathbf{w}^*) \geq_{AL} C(\mathbf{w}^\circ)$	$\xi_w(C,w) \geqq 0$	$\frac{v'''(c)}{v''(c)} - \frac{v''(c)\,c}{v'(c)} \leqq 0$
$C(\mathbf{w}^\circ) \geq_{AL} C(\mathbf{w}^*)$	$\xi_w(C,w) \leqq 0$	$\frac{v'''(c)}{v''(c)} - \frac{v''(c)\,c}{v'(c)} \geqq 0$
B. Changes in the preferences		
$C^*(\mathbf{w}^*) \geq_{AL} C^\circ(\mathbf{w}^\circ)$	$\xi_w(C^*,w) \geqq \xi_w(C^\circ,w)$	$-\frac{v^{*\prime\prime}(c)}{v^{*\prime}(c)} \leqq -\frac{v^{\circ\prime\prime}(c)}{v^{\circ\prime}(c)}$
$C^\circ(\mathbf{w}^\circ) \geq_{AL} C^*(\mathbf{w}^*)$	$\xi_w(C^*,w) \leqq \xi_w(C^\circ,w)$	$-\frac{v^{*\prime\prime}(c)}{v^{*\prime}(c)} \geqq -\frac{v^{\circ\prime\prime}(c)}{v^{\circ\prime}(c)}$

7. Concluding Remarks

For an artisan economy, where all agents have the same preferences and no exogenous income, we have sought to establish the properties of the consumption function that secure a reduction in inequality under different scenarios. It has been shown that the elasticity of the consumption function is the key variable for determining the impact on relative inequality of changes in both the productivity endowments and the preferences. The derivative of the consumption function in the logarithm proved to play a similar role when the focus is on absolute inequality. While there is in principle a one-to-one relationship between the consumption function and the preference ordering, it is in most cases impossible to recover the direct utility function starting with the consumption function. Here we took an intermediate path by restricting our attention to the case where preferences are linear in labour time. This enabled us to identify the properties of the corresponding utility functions that ensure that the elasticity and the derivative in the logarithm of the consumption function have the desired properties. More precisely, it has been shown that the degree of *relative risk aversion* of the consumption utility function is determining in the case of relative inequality, while *absolute risk aversion* is the key parameter when one is interested in absolute inequality.

We are aware of the fact that our model is very crude as a description of the real world and among the different limitations we see in our stylised economy some appear – at least to the authors of the paper – to be more important than others. First, we insist on the fact that the assumption of no exogenous income is important in the derivation of our results. Things become more intricate when one drops this restriction: for a number of utility functions, the condition of a monotonic consumption elasticity obtained under the assumption that $m = 0$ no longer holds when this is not the case. In practice agents differ not only in terms of their talents but also with respect to their non-labour incomes and this heterogeneity is likely to be reflected in the way consumption is distributed among the agents at the market equilibrium. This calls for a multidimensional approach in order to control for changes in the joint distribution of talents and non-labour incomes. [8] It is probably more difficult to account for the heterogeneity

[8] For quasilinear-in-labour preferences, consumption is independent of exogenous income, which implies that the way non-labour income is allocated among the population in the artisan economy has no impact on consumption inequality. When preferences are quasilinear-in-consumption, this is no longer true and the monotonicity properties of the elasticity of consumption with respect to productivity may well depend on the amount of non-labour income. Choosing for instance $u^2(c,\ell) = c - e^c$, we get $C(w,m) = m + w \ln w$ and

in agents' preferences even though this is no doubt a key question to be addressed in future work.

By definition, the artisan economy that we considered has the property that the agents cannot improve their situations by transferring parts of their endowments. Certainly the assumption that the agents behave in complete isolation is a strong requirement. Substituting a more general production economy for our artisan economy might well invalidate our results. So one way to go might be to consider a model where the agents do not put all their effort in the private production but contribute to the overall production in proportion to their talents and to investigate what this would imply for inequality in consumption.

A third limitation arises in the fact that no taxation is allowed for in our model mainly – we must admit – for convenience. The introduction of taxation is mostly important when one is willing to estimate the agents common consumption function from microdata and test whether the different conditions we have identified are met or not. [9] We have shown elsewhere else (see Ebert and Moyes (2007)) that it is extremely difficult to sign the redistributive impact of income taxation when the agents can adjust their labour time to income tax changes. Bringing together these two models leaves little room for definite results even though we recognise that we have not made attempts in this direction.

8. Proofs of the Results

8.1. Improvements in Relative Inequality

The following result (see Marshall, Olkin, and Proschan (1967, Theorem 2.4)), which we provide a proof of for completeness, will be used repeatedly in subsequent proofs.

Lemma 8.1. *Let $n \geqq 2$ and $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n_{++}$ such that $x_1 \leqq x_2 \leqq \cdots \leqq x_n$ and $y_1 \leqq y_2 \leqq \cdots \leqq y_n$. Then $x_1/y_1 \geqq x_2/y_2 \geqq \cdots \geqq x_n/y_n$ implies that $\mathbf{x} \geq_{RL} \mathbf{y}$.*

PROOF. By definition for $\mathbf{x} \geq_{RL} \mathbf{y}$ we need have

$$\frac{\sum_{j=1}^{k} x_j}{\sum_{i=1}^{k} x_i + \sum_{i=k+1}^{n} x_i} \geqq \frac{\sum_{j=1}^{k} y_j}{\sum_{i=1}^{k} y_i + \sum_{i=k+1}^{n} y_i}, \ \forall\, k = 1, 2, \ldots, n-1. \tag{8.1}$$

This can be equivalently rewritten as

$$\frac{\sum_{i=1}^{k} y_i + \sum_{i=k+1}^{n} y_i}{\sum_{j=1}^{k} y_j} \geqq \frac{\sum_{i=1}^{k} x_i + \sum_{i=k+1}^{n} x_i}{\sum_{j=1}^{k} x_j}, \ \forall\, k = 1, 2, \ldots, n-1, \tag{8.2}$$

which simplifies to

$$\frac{\sum_{i=k+1}^{n} y_i}{\sum_{j=1}^{k} y_j} \geqq \frac{\sum_{i=k+1}^{n} x_i}{\sum_{j=1}^{k} x_j}, \ \forall\, k = 1, 2, \ldots, n-1. \tag{8.3}$$

Upon developing we obtain

$$\frac{y_{k+1}}{\sum_{j=1}^{k} y_j} + \cdots + \frac{y_n}{\sum_{j=1}^{k} y_j} \geqq \frac{x_{k+1}}{\sum_{j=1}^{k} x_j} + \cdots + \frac{x_n}{\sum_{j=1}^{k} x_j}, \ \forall\, k = 1, 2, \ldots, n-1. \tag{8.4}$$

---

one can check that $\eta(C, w) = w\,(1 + \ln w)/(m + w \ln w)$ is increasing on $(1, +\infty)$ when $m = 0$ and decreasing otherwise.

[9] Indeed, the consumption patterns we observe are determined in part by the agents' preferences and in part by the constraints they face which comprise among other things the tax schedule.

For the inequalities (8.4) to be verified it is sufficient that

$$\frac{\sum_{j=1}^{k} x_j}{x_h} \geqq \frac{\sum_{j=1}^{k} y_j}{y_h}, \ \forall\, h = k+1, k+2, \ldots, n, \ \forall\, k = 1, 2, \ldots, n-1. \tag{8.5}$$

Again a sufficient condition for (8.5) to hold is that $x_j/x_h \geqq y_j/y_h$, for all $j = 1, 2, \ldots, k$, all $h = k+1, \ldots, n$ and all $k = 1, 2, \ldots, n-1$, which follows from our assumption that $x_i/y_i \geqq x_{i+1}/y_{i+1}$, for all $i = 1, 2, \ldots, n-1$. □

While all our results involve conditions based on the elasticity of the consumption function, we will dispense with differentiability in the proofs noting that

$$\eta(C, w^\circ) \left\{ \begin{array}{c} > \\ = \\ < \end{array} \right\} \eta(C, w^*), \ \forall\, w^\circ < w^* \ (w^\circ, w^* \in W(U)), \tag{8.6}$$

is actually equivalent to

$$\frac{C(\lambda w^\circ)}{C(w^\circ)} \left\{ \begin{array}{c} > \\ = \\ < \end{array} \right\} \frac{C(\lambda w^*)}{C(w^*)}, \ \forall\, \lambda \geqq 1, \ \forall\, w^\circ < w^* \ (w^\circ, w^*, \lambda w^\circ, \lambda w^* \in W(U)), \tag{8.7}$$

in the case of a differentiable consumption function. Finally, the next result borrowed from Aczel (1966, Chapter 3) will prove useful later on.

Lemma 8.2. *The only solution to the functional equation*

$$\frac{C(\lambda w^*)}{C(w^*)} = \frac{C(\lambda w^\circ)}{C(w^\circ)}, \ \forall\, w^\circ < w^*, \ \forall\, \lambda \geqq 1, \tag{8.8}$$

is given by

$$C(w) = \beta w^\eta \ (\beta > 0, \eta > 0), \ \forall\, w > 0. \tag{8.9}$$

Proof of Proposition 4.1. Since it is obvious that a constant elasticity is sufficient for condition (a) to hold, we only prove the necessity part of the proposition. Suppose that C is not isoelastic or equivalently in virtue of Lemma 8.2 that condition (8.8) is violated, in which case there are two possibilities to be considered.

CASE 1: $C(\lambda w^\circ)/C(w^\circ) < C(\lambda w^*)/C(w^*)$, for some $w^\circ < w^*$ and some $\lambda \geqq 1$. Choose $\mathbf{w}^\circ := (w^\circ, \ldots, w^\circ, w^*)$, $\mathbf{w}^* := (\lambda w^\circ, \ldots, \lambda w^\circ, \lambda w^*)$, and let $C(\mathbf{w}^\circ) := (C(w^\circ), \ldots, C(w^\circ), C(w^*))$ and $C(\mathbf{w}^*) := (C(\lambda w^\circ), \ldots, C(\lambda w^\circ), C(\lambda w^*))$. Then $\mathbf{w}^* \geq_{LD} \mathbf{w}^\circ$, but

$$\frac{k\, C(\lambda w^\circ)}{(n-1)\, C(\lambda w^\circ) + C(\lambda w^*)} < \frac{k\, C(w^\circ)}{(n-1)\, C(w^\circ) + C(w^*)}, \tag{8.10}$$

for all $k = 1, 2, \ldots, n-1$, hence $\neg\, [C(\mathbf{w}^*) \geq_{RL} C(\mathbf{w}^\circ)]$.

CASE 2: $C(\lambda w^\circ)/C(w^\circ) > C(\lambda w^*)/C(w^*)$, for some $w^\circ < w^*$ and some $\lambda \geqq 1$. Choosing now $\mathbf{w}^\circ := (\lambda w^\circ, \ldots, \lambda w^\circ, \lambda w^*)$ and $\mathbf{w}^* := (w^\circ, \ldots, w^\circ, w^*)$, we have $\mathbf{w}^* \geq_{LD} \mathbf{w}^\circ$. Letting $C(\mathbf{w}^\circ) := (C(\lambda w^\circ), \ldots, C(\lambda w^\circ), C(\lambda w^*))$ and $C(\mathbf{w}^*) := (C(w^\circ), \ldots, C(w^\circ), C(w^*))$, we obtain

$$\frac{k\, C(w^\circ)}{(n-1)\, C(w^\circ) + C(w^*)} < \frac{k\, C(\lambda w^\circ)}{(n-1)\, C(\lambda w^\circ) + C(\lambda w^*)}, \tag{8.11}$$

for all $k = 1, 2, \ldots, n-1$, and we conclude that $\neg\, [C(\mathbf{w}^*) \geq_{RL} C(\mathbf{w}^\circ)]$. □

Proof of Proposition 4.2.

(b) $\Longrightarrow$ (a). Consider two distributions of productivities $\mathbf{w}^\circ := (w_1^\circ, \ldots, w_n^\circ)$ and $\mathbf{w}^* := (w_1^*, \ldots, w_n^*)$. We have to show that, if

$$\frac{C(\lambda w^\circ)}{C(w^\circ)} \geqq \frac{C(\lambda w^*)}{C(w^*)},\ \forall\ w^\circ < w^*,\ \forall\ \lambda \geqq 1, \tag{8.12}$$

then $\mathbf{w}^* \geq_{LD} \mathbf{w}^\circ$ and $\mathbf{w}^* \geq_{ME} \mathbf{w}^\circ$ imply that $C(\mathbf{w}^*) \geq_{RL} C(\mathbf{w}^\circ)$. We have

$$\begin{aligned}
\frac{C(w_i^*)}{C(w_i^\circ)} = \frac{C((w_i^*/w_i^\circ)\, w_i^\circ)}{C(w_i^\circ)} &\geqq \frac{C((w_i^*/w_i^\circ)\, w_{i+1}^\circ)}{C(w_{i+1}^\circ)} \quad \text{[invoking (8.12) since } \mathbf{w}^* \geq_{ME} \mathbf{w}^\circ] \\
&\geqq \frac{C((w_{i+1}^*/w_{i+1}^\circ)\, w_{i+1}^\circ)}{C(w_{i+1}^\circ)} \quad \text{[since } \mathbf{w}^* \geq_{LD} \mathbf{w}^\circ \text{ and } C \text{ is non-decreasing]} \\
&= \frac{C(w_{i+1}^*)}{C(w_{i+1}^\circ)},\ \forall\ i = 1, 2, \ldots, n-1,
\end{aligned} \tag{8.13}$$

and thanks to Lemma 8.1, we conclude that $C(\mathbf{w}^*) \geq_{RL} C(\mathbf{w}^\circ)$.

(a) $\Longrightarrow$ (b). The proof is analogous to that of Case 1 in Proposition 4.1 and it is omitted. □

Proof of Proposition 4.3.

(b) $\Longrightarrow$ (a). Consider two distributions of productivities $\mathbf{w}^\circ := (w_1^\circ, \ldots, w_n^\circ)$ and $\mathbf{w}^* := (w_1^*, \ldots, w_n^*)$. We have to show that, if

$$\frac{C(\lambda w^\circ)}{C(w^\circ)} \leqq \frac{C(\lambda w^*)}{C(w^*)},\ \forall\ w^\circ < w^*,\ \forall\ \lambda \geqq 1, \tag{8.14}$$

then $\mathbf{w}^* \geq_{LD} \mathbf{w}^\circ$ and $\mathbf{w}^* \geq_{LE} \mathbf{w}^\circ$ imply that $C(\mathbf{w}^*) \geq_{RL} C(\mathbf{w}^\circ)$. We have

$$\begin{aligned}
\frac{C(w_i^\circ)}{C(w_i^*)} = \frac{C((w_i^\circ/w_i^*)\, w_i^*)}{C(w_i^*)} &\leqq \frac{C((w_i^\circ/w_i^*)\, w_{i+1}^*)}{C(w_{i+1}^*)} \quad \text{[invoking (8.14) since } \mathbf{w}^* \geq_{LE} \mathbf{w}^\circ] \\
&\leqq \frac{C((w_{i+1}^\circ/w_{i+1}^*)\, w_{i+1}^*)}{C(w_{i+1}^*)} \quad \text{[since } \mathbf{w}^* \geq_{LD} \mathbf{w}^\circ \text{ and } C \text{ is non-decreasing]} \\
&= \frac{C(w_{i+1}^\circ)}{C(w_{i+1}^*)},\ \forall\ i = 1, 2, \ldots, n-1,
\end{aligned} \tag{8.15}$$

and we deduce from Lemma 8.1 that $C(\mathbf{w}^*) \geq_{RL} C(\mathbf{w}^\circ)$.

(a) $\Longrightarrow$ (b). The proof goes is analogous to that of Case 2 in Proposition 4.1 and it is omitted. □

Here again we find it convenient to express our original conditions involving the elasticities of the consumption function in terms of restrictions that do not involve derivatives. Indeed, we note that, if C is differentiable, then

$$\eta(C^*, w) \left\{ \begin{array}{c} > \\ = \\ < \end{array} \right\} \eta(C^\circ, w),\ \forall\ w \in W(U^*) \cap W(U^\circ), \tag{8.16}$$

is actually equivalent to requiring that

$$\frac{C^*(\lambda w)}{C^*(w)} \left\{ \begin{array}{c} > \\ = \\ < \end{array} \right\} \frac{C^\circ(\lambda w)}{C^\circ(w)}, \ \forall\, w, \lambda w \in W(U^*) \cap W(U^\circ), \ \forall\, \lambda \geqq 1. \tag{8.17}$$

Proof of Proposition 4.4.

(b) $\Longrightarrow$ (a). Given any arbitrary distribution of productivities $\mathbf{w} := (w_1, \ldots, w_n)$, we have to show that, if

$$\frac{C^*(\lambda w)}{C^*(w)} \leqq \frac{C^\circ(\lambda w)}{C^\circ(w)}, \ \forall\, w \in W(U^*) \cap W(U^\circ), \ \forall\, \lambda \geqq 1, \tag{8.18}$$

then $C^*(\mathbf{w}) \geq_{RL} C^\circ(\mathbf{w})$. Since by definition $w_{i+1} \geqq w_i$, for all $i = 1, 2, \ldots, n-1$, it follows from (8.18) that

$$\frac{C^*(w_{i+1})}{C^*(w_i)} = \frac{C^*\left(\left(\frac{w_{i+1}}{w_i}\right) w_i\right)}{C^*(w_i)} \leqq \frac{C^\circ\left(\left(\frac{w_{i+1}}{w_i}\right) w_i\right)}{C^\circ(w_i)} = \frac{C^\circ(w_{i+1})}{C^\circ(w_i)}, \ \forall\, i = 1, 2, \ldots, n-1, \tag{8.19}$$

and we deduce from Lemma 8.1 that $C^*(\mathbf{w}) \geq_{RL} C^\circ(\mathbf{w})$.

(a) $\Longrightarrow$ (b). Suppose that $C^*(\lambda w)/C^*(w) > C^\circ(\lambda w)/C^\circ(w)$, for some $\lambda \geqq 1$ and some $w \in W(U^*) \cap W(U^\circ)$ such that $\lambda w \in W(U^*) \cap W(U^\circ)$. Choosing $\mathbf{w} := (w, \ldots, w, \lambda w)$, we obtain

$$\frac{k\, C^*(w)}{(n-1)\, C^*(w) + C^*(\lambda w)} < \frac{k\, C^\circ(w)}{(n-1)\, C^\circ(w) + C^\circ(\lambda w)}, \tag{8.20}$$

for all $k = 1, 2, \ldots, n-1$, and we conclude that $\neg\, [C^*(\mathbf{w}) \geq_{RL} C^\circ(\mathbf{w})]$. □

Although we have not exploited this idea in the paper it is worth mentioning that one can prove along a similar reasoning that, if $C^{*\prime}(w)\, w/C^*(w) \geqq C^{\circ\prime}(w)\, w/C^\circ(w)$, for all $w \in W(U^*) \cap W(U^\circ)$, then $C^\circ(\mathbf{w}) \geq_{RL} C^*(\mathbf{w})$, for all $\mathbf{w} \in \mathbb{W}(U^*) \cap \mathbb{W}(U^\circ)$, and conversely.

Proof of Proposition 4.5.

(b) $\Longrightarrow$ (a). This follows from combination of Propositions 4.1 and 4.4 (see the discussion in Section 4.4).

(a) $\Longrightarrow$ (b). Let $\mathbf{w}^\circ := (w^\circ, \ldots, w^\circ, \lambda w^\circ)$ and $\mathbf{w}^* := (w^*, \ldots, w^*, \lambda w^*)$, where $\lambda > 1$ and $w^\circ, w^* \in W(U^*) \cap W(U^\circ)$ are arbitrary but such that $\lambda w^\circ, \lambda w^* \in W(U^*) \cap W(U^\circ)$. Clearly $\mathbf{w}^* \geq_{LD} \mathbf{w}^\circ$. Invoking condition (a), this implies that $C^*(\mathbf{w}^*) \geq_{RL} C^\circ(\mathbf{w}^\circ)$, which upon simplifying reduces to

$$\frac{C^\circ(\lambda w^\circ)}{C^\circ(w^\circ)} \geqq \frac{C^*(\lambda w^*)}{C^*(w^*)} \ [> 1]. \tag{8.21}$$

Taking the logarithm of (8.21) and letting λ go to one, we obtain

$$\lim_{\lambda \to 1} \frac{\ln C^\circ(\lambda w^\circ) - \ln C^\circ(w^\circ)}{\ln \lambda} \geqq \lim_{\lambda \to 1} \frac{\ln C^*(\lambda w^*) - \ln C^*(w^*)}{\ln \lambda} \ [> 0], \tag{8.22}$$

or equivalently since C° and C^* are differentiable

$$\frac{C^{\circ\prime}(w^\circ)\, w^\circ}{C^\circ(w^\circ)} \geqq \frac{C^{*\prime}(w^*)\, w^*}{C^*(w^*)} \ [> 0], \tag{8.23}$$

which holds for all $w^\circ, w^* \in W(U^*) \cap W(U^\circ)$. Then, there exists $\eta > 0$ such that

$$\frac{C^{\circ\prime}(w^\circ)\, w^\circ}{C^\circ(w^\circ)} \geqq \eta \geqq \frac{C^{*\prime}(w^*)\, w^*}{C^*(w^*)} \; [>0], \; \forall \; w^\circ, w^* \in W(U^*) \cap W(U^\circ). \tag{8.24}$$

Defining $H(w) := \beta w^\eta$ with $\beta > 0$ arbitrary, we deduce from (8.24) that

$$\frac{C^{\circ\prime}(w^\circ)\, w^\circ}{C^\circ(w^\circ)} \geqq \frac{H'(w^\circ)\, w^\circ}{H(w^\circ)} = \frac{H'(w^*)\, w^*}{H(w^*)} \geqq \frac{C^{*\prime}(w^*)\, w^*}{C^*(w^*)} \; [>0], \tag{8.25}$$

which holds for all $w^\circ, w^* \in W(U^*) \cap W(U^\circ)$. Thus we have identified a non-decreasing and isoelastic function H such that

$$\frac{C^{\circ\prime}(w)\, w}{C^\circ(w)} \geqq \frac{H'(w)\, w}{H(w)} \geqq \frac{C^{*\prime}(w)\, w}{C^*(w)}, \; \forall \; w \in W(U^*) \cap W(U^\circ), \tag{8.26}$$

which makes the proof complete. □

Proof of Proposition 4.6.

(b) $\Longrightarrow$ (a). This follows from combination of Propositions 4.2 and 4.4 (see the discussion in Section 4.4).

(a) $\Longrightarrow$ (b). Let $\mathbf{w}^\circ := (w^\circ, \ldots, w^\circ, \lambda w^\circ)$ and $\mathbf{w}^* := (w^*, \ldots, w^*, \lambda w^*)$, where $\lambda > 1$ and $w^\circ \leqq w^*$ $(w^\circ, w^* \in W(U^*) \cap W(U^\circ))$ are arbitrary but such that $\lambda w^\circ, \lambda w^* \in W(U^*) \cap W(U^\circ)$. Clearly $\mathbf{w}^* \geq_{LD} \mathbf{w}^\circ$ and $\mathbf{w}^* \geq_{ME} \mathbf{w}^\circ$. Invoking condition (a), this implies that $C^*(\mathbf{w}^*) \geq_{RL} C^\circ(\mathbf{w}^\circ)$. Arguing as in the proof of necessity in Proposition 4.5, we arrive at

$$\frac{C^{\circ\prime}(w^\circ)\, w^\circ}{C^\circ(w^\circ)} \geqq \frac{C^{*\prime}(w^*)\, w^*}{C^*(w^*)} \; [>0], \tag{8.27}$$

which *now* holds for all $w^\circ \leqq w^*$ $(w^\circ, w^* \in W(U^*) \cap W(U^\circ))$. It follows that

$$\frac{C^{\circ\prime}(w^\circ)\, w^\circ}{C^\circ(w^\circ)} \geqq h(w^\circ) := \sup \left\{ \frac{C^{*\prime}(w)\, w}{C^*(w)} \;\middle|\; w \geqq w^\circ \right\} \; [>0], \tag{8.28}$$

which is true for all $w^\circ \in W(U^*) \cap W(U^\circ))$. This implies in turn that

$$\frac{C^{\circ\prime}(w)\, w}{C^\circ(w)} \geqq h(w) \geqq \frac{C^{*\prime}(w)\, w}{C^*(w)}, \; \forall \; w \in W(U^*) \cap W(U^\circ)). \tag{8.29}$$

We note that by construction h is bounded and non-increasing over $W(U^*) \cap W(U^\circ)$. Consider the function

$$H(w) := \exp \int_{\underline{w}}^{w} \frac{h(s)}{s}\, ds, \; \forall \; w \in W(U^*) \cap W(U^\circ)), \tag{8.30}$$

where $\underline{w} = \inf\{W(U^*) \cap W(U^\circ)\}$. The function H has a non-increasing elasticity $h(w) = H'(w)\, w / H(w)$ and it is such that

$$\frac{C^{\circ\prime}(w)\, w}{C^\circ(w)} \geqq \frac{H'(w)\, w}{H(w)} \geqq \frac{C^{*\prime}(w)\, w}{C^*(w)}, \; \forall \; w \in W(U^*) \cap W(U^\circ), \tag{8.31}$$

which makes the proof complete. □

Proof of Proposition 4.7.

(b) $\Longrightarrow$ (a). This follows from combination of Propositions 4.3 and 4.4 (see the discussion in Section 4.4).

(a) $\Longrightarrow$ (b). Similar *mutatis mutandis* to the proof of necessity in Proposition 4.6. Let $\mathbf{w}^\circ := (w^\circ, \ldots, w^\circ, \lambda w^\circ)$ and $\mathbf{w}^* := (w^*, \ldots, w^*, \lambda w^*)$, where $\lambda > 1$ and $w^\circ \geqq w^*$ $(w^\circ, w^* \in W(U^*) \cap W(U^\circ))$ are arbitrary but such that $\lambda w^\circ, \lambda w^* \in W(U^*) \cap W(U^\circ)$. Clearly $\mathbf{w}^* \geq_{LD} \mathbf{w}^\circ$ and $\mathbf{w}^* \geq_{LE} \mathbf{w}^\circ$. Invoking condition (a), this implies that $C^*(\mathbf{w}^*) \geq_{RL} C^\circ(\mathbf{w}^\circ)$. Arguing as in the proof of necessity in Proposition 4.5, we arrive at

$$\frac{C^{\circ\prime}(w^\circ)\, w^\circ}{C^\circ(w^\circ)} \geqq \frac{C^{*\prime}(w^*)\, w^*}{C^*(w^*)} \; [> 0], \tag{8.32}$$

which *now* holds for all $w^\circ \geqq w^*$ $(w^\circ, w^* \in W(U^*) \cap W(U^\circ))$. It follows that

$$\inf \left\{ \frac{C^{\circ\prime}(w)\, w}{C^\circ(w)} \;\middle|\; w \geqq w^* \right\} =: h(w^*) \geqq \frac{C^{*\prime}(w^*)\, w^*}{C^*(w^*)} \; [> 0], \tag{8.33}$$

which is true for all $w^* \in W(U^*) \cap W(U^\circ))$. This implies in turn that

$$\frac{C^{\circ\prime}(w)\, w}{C^\circ(w)} \geqq h(w) \geqq \frac{C^{*\prime}(w)\, w}{C^*(w)}, \; \forall\, w \in W(U^*) \cap W(U^\circ)). \tag{8.34}$$

We note that by construction h is bounded and non-decreasing over $W(U^*) \cap W(U^\circ)$. Consider the function

$$H(w) := \exp \int_{\underline{w}}^{w} \frac{h(s)}{s}\, ds, \; \forall\, w \in W(U^*) \cap W(U^\circ)), \tag{8.35}$$

where $\underline{w} = \inf\{W(U^*) \cap W(U^\circ)\}$. The function H has a non-decreasing elasticity $h(w) = H'(w)\, w / H(w)$ and it is such that

$$\frac{C^{\circ\prime}(w)\, w}{C^\circ(w)} \geqq \frac{H'(w)\, w}{H(w)} \geqq \frac{C^{*\prime}(w)\, w}{C^*(w)}, \; \forall\, w \in W(U^*) \cap W(U^\circ), \tag{8.36}$$

which makes the proof complete. □

8.2. Improvements in Absolute Inequality

The following result, which indicates a sufficient condition for absolute Lorenz domination, is useful.

Lemma 8.3. *Let $n \geqq 2$ and $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$ such that $x_1 \leqq x_2 \leqq \cdots \leqq x_n$ and $y_1 \leqq y_2 \leqq \cdots \leqq y_n$. Then $x_1 - y_1 \geqq x_2 - y_2 \geqq \cdots \geqq x_n - y_n$ implies that $\mathbf{x} \geq_{AL} \mathbf{y}$.*

PROOF. By definition for $\mathbf{x} \geq_{AL} \mathbf{y}$ we need have

$$n \sum_{j=1}^{k} x_j - k \sum_{i=1}^{n} x_i \geqq n \sum_{j=1}^{k} y_j - k \sum_{i=1}^{n} y_i, \; \forall\, k = 1, 2, \ldots, n-1. \tag{8.37}$$

This can be equivalently rewritten as

$$\sum_{j=1}^{k} \left[\sum_{i=1}^{k} (x_j - x_i) + \sum_{i=k+1}^{n} (x_j - x_i) \right] \geqq \sum_{j=1}^{k} \left[\sum_{i=1}^{k} (y_j - y_i) + \sum_{i=k+1}^{n} (y_j - y_i) \right], \tag{8.38}$$

for all $k = 1, 2, \ldots, n-1$, which reduces to

$$\sum_{j=1}^{k} \sum_{i=k+1}^{n} (x_i - x_j) \leqq \sum_{j=1}^{k} \sum_{i=k+1}^{n} (y_i - y_j), \ \forall\, k = 1, 2, \ldots, n-1. \tag{8.39}$$

Upon developing we obtain

$$\sum_{j=1}^{k} (x_{k+1} - x_j) + \cdots + \sum_{j=1}^{k} (x_n - x_j) \leqq \sum_{j=1}^{k} (y_{k+1} - y_j) + \cdots + \sum_{j=1}^{k} (y_n - y_j), \tag{8.40}$$

for all $k = 1, 2, \ldots, n-1$. For the $(n-k)$ above inequalities to be verified it is sufficient that

$$\sum_{j=1}^{k} (x_h - x_j) \leqq \sum_{j=1}^{k} (y_h - y_j), \ \forall\, h = k+1, k+2, \ldots, n, \ \forall\, k = 1, 2, \ldots, n-1. \tag{8.41}$$

Again a sufficient condition for (8.41) to hold is that $x_h - x_j \leqq y_h - y_j$, for all $j = 1, 2, \ldots, k$, all $h = k+1, \ldots, n$ and all $k = 1, 2, \ldots, n-1$, which follows from our assumption that $x_i - y_i \geqq x_{i+1} - y_{i+1}$, for all $i = 1, 2, \ldots, n-1$. □

The results follow from Section 8.1 by substituting for the original consumption function $C(w)$ the function $\overline{C}(w) = f \circ C(w)$, with $f(s) = \exp(s)$. Indeed we obtain

$$\eta(\overline{C}, w) = \frac{f'(C(w))\, C(w)}{f(C(w))} \frac{C'(w)\, w}{C(w)} = C'(w)\, w = \xi(C, w), \ \forall\, w \in W(U), \tag{8.42}$$

which upon substitution in the relevant formula gives the desired results.

A. List of the Utility Functions Used in the Paper

We give below the list of the utility functions that we have used in the examples and also in the figures for the sake of illustration. The utility functions U^{EPA} and U^{STERN} are borrowed from Saha (1993) and Stern (1986), respectively.

Number	Utility Function
1	$U^1(c,\ell) = c - \frac{\ell^2}{2} \quad (w > 0)$
2	$U^2(c,\ell) = c - e^{\ell} \quad (w > 1)$
3	$U^3(c,\ell) = \ln c - \frac{1}{c} - \ell \quad (w > 0)$
4	$U^4(c,\ell) = \ln c - \ell \quad (w > 0)$
5	$U^5(c,\ell) = 2\sqrt{c} - \ell \quad (w > 0)$
6	$U^6(c,\ell) = -e^{-c} - \ell \quad (w > 1)$
7	$U^7(c,\ell) = -e^{-c} - e^{\ell} \quad (w > 1)$
8	$U^8(c,\ell) = 2\sqrt{c} - \frac{\ell^2}{2} \quad (w > 0)$
9	$U^9(c,\ell) = c - \frac{5}{2}\left[\ell\, e^{\frac{-1}{\ell}} - \int_1^{+\infty} \frac{e^{-t\ell}}{t}\, dt\right] \quad (0 < w < 1)$
10	$U^{10}(c,\ell) = c - \frac{c^2}{8} - \ell \quad (0 < c \leqq 4;\ w > 1)$
	$U^{\text{EPA}}(c,\ell) = a - e^{-b\,c^d} - \ell \quad (a = 0.00)$
11	$U^{11}(c,\ell) = U^{\text{EPA}}(c,\ell) \quad (b = 0.50;\ d = 1.00;\ w > 2)$
12	$U^{12}(c,\ell) = U^{\text{EPA}}(c,\ell) \quad (b = 0.25;\ d = 1.50;\ w > ?)$
13	$U^{13}(c,\ell) = U^{\text{EPA}}(c,\ell) \quad (b = 0.25;\ d = 0.50;\ w > 0)$
	$U^{\text{Stern}}(c,\ell) = \frac{\ell - b}{\chi}\, e^{\frac{\chi(c+a)}{\ell - b} - 1} \quad \left(a = \frac{\rho}{\chi} - \frac{\xi}{\chi^2};\ b = \frac{\xi}{\chi};\ \chi = -1;\ \xi = 3\right)$
14	$U^{14}(c,\ell) = U^{\text{Stern}}(c,\ell) \quad (\rho = 0;\ w > 0)$
15	$U^{15}(c,\ell) = U^{\text{Stern}}(c,\ell) \quad (\rho = -1;\ w > \frac{1}{3})$
16	$U^{16}(c,\ell) = U^{\text{Stern}}(c,\ell) \quad (\rho = +1;\ w > 0)$

References

Aczel, J. (1966). *Lectures on Functional Equations and Their Applications.* Academic Press, New-York.

Atkinson, A. B. (1970). On the measurement of inequality. *Journal of Economic Theory*, **2**, 244–263.

Blackorby, C., Primont, D., and Russell, R. R. (1978). *Duality, Separability, and Functional Structure: Theory and Economic Applications*. Elsevier North Holland, New-York.

Bossert, W. and Pfingsten, A. (1990). Intermediate inequality: Concepts, indices and welfare implications. *Mathematical Social Sciences*, **19**, 117–134.

Brett, C. and Weymark, J. A. (2008). The impact of changing skill levels on optimal nonlinear income taxes. *Journal of Public Economics*, **92**, 1765–1771.

Chipman, J. S. and Moore, J. C. (1973). Aggregate demand, real national income, and the compensation principle. *International Economic Review*, **14**, 153–181.

Chipman, J. S. and Moore, J. C. (1980). Real national income with homothetic preferences and a fixed distribution of income. *Econometrica*, **48**, 401–422.

Del Rio, C. and Ruiz-Castillo, J. (2000). Intermediate inequality and welfare. *Social Choice and Welfare*, **17**, 223–239.

Ebert, U. and Moyes, P. (2002). Welfare, inequality and the transformation of incomes. The case of weighted income distributions. In P. Moyes, C. Seidl, and A. F. Shorrocks, editors, *Inequalities: Theory, Experiments and Applications*, pages 9–50. Journal of Economics, Supplement 9.

Ebert, U. and Moyes, P. (2007). Income taxation with labour responses. *Journal of Public Economic Theory*, **9**, 653–682.

Ebert, U. and Moyes, P. (2010). Talents, preferences and social welfare. GREThA, Université Montesquieu Bordeaux IV, in progress.

Eeckhoudt, L., Gollier, C., and Schlesinger, H. (1996). Changes in background risk and risk taking behavior. *Econometrica*, **64**, 683–689.

Foster, J. E. (1985). Inequality measurement. In H. Young, editor, *Fair Allocation*, pages 38–61. American Mathematical Society, Providence, RI.

Foster, J. E., Majumdar, M. K., and Mitra, T. (1990). Inequality and welfare in market economies. *Journal of Public Economics*, **41**, 351–367.

Gravel, N. and Moyes, P. (2010). Utilitarianism and welfarism: Does it make a difference? GREThA, Université Montesquieu Bordeaux IV, in progress.

Heckman, J. J. (2008). Schools, skills, and synapses. *Economic Inquiry*, **46**, 289–324.

Jakobsson, U. (1976). On the measurement of the degree of progression. *Journal of Public Economics*, **5**, 161–168.

Kolm, S.-C. (1969). The optimal production of social justice. In J. Margolis and H. Guitton, editors, *Public Economics*, pages 145–200. Macmillan, London.

Kolm, S.-C. (1976). Unequal inequalities I. *Journal of Economic Theory*, **12**, 416–442.

Krtscha, M. (1994). A new compromise measure of inequality. In W. Eichhorn, editor, *Models and Measurement of Welfare and Inequality*, pages 111–120. Springer-Verlag, Heidelberg.

Lambert, P. J. (2001). *The Distribution and Redistribution of Income*. Manchester University Press, Manchester, third edition.

Le Breton, M., Moyes, P., and Trannoy, A. (1996). Inequality reducing properties of composite taxation. *Journal of Economic Theory*, **69**, 71–103.

Madden, P. (1996). Suppes-Sen dominance, generalised Lorenz dominance and the welfare economics of competitive equilibrium: Some examples. *Journal of Public Economics*, **61**, 247–262.

Marshall, A. W., Olkin, I., and Proschan, F. (1967). Monotonicity of ratios of means and other applications of majorization. In O. Shisha, editor, *Inequalities*, pages 177–190. Academic Press, New-York.

McCaig, B. and Yatchew, A. (2007). International welfare comparisons and nonparametric testing of multivariate stochastic dominance. *Journal of Applied Econometrics*, **22**, 951–969.

Mirrlees, J. A. (1971). An exploration in the theory of optimum income taxation. *Review of Economic Studies*, **38**, 175–208.

Moyes, P. (1987). A new concept of Lorenz domination. *Economics Letters*, **23**, 203–207.

Moyes, P. (1999). Stochastic dominance and the Lorenz curve. In J. Silber, editor, *Handbook of Income Inequality Measurement*, pages 199–222. Kluwer Academic Publishers, Boston/Dordrecht/London.

OECD (2006). *Going for Growth*. OECD Publishing, Paris.

Osberg, L. (2002). Time, money and inequality in an international perspective. Paper presented at the International Worskshop Income Distribution and Welfare, Università Luigi Bocconi, Milano, Italy, 30–31 May and 1 June 2002.

Saha, A. (1993). Expo-power utility: A "flexible" form for absolute and relative risk aversion. *American Journal of Agricultural Economics*, **75**, 905–913.

Sen, A. K. (1976). Real national income. *Review of Economic Studies*, **43**, 19–39.

Sen, A. K. (1997). *On Economic Inequality. Expanded Edition with a Substantial Annexe by James Foster and Amartya Sen*. Clarendon Press, Oxford, second edition.

Shaked, M. and Shanthikumar, G. J. (1994). *Stochastic Orders and their Applications*. Academic Press, Oxford.

Shorrocks, A. F. (1983). Ranking income distributions. *Economica*, **50**, 3–17.

Simula, L. (2007). Optimality conditions and comparative static properties of non-linear income taxes revisited. Paris School of Economics, LEA-INRA Working Paper No. 2007-15.

Stern, N. (1986). On the specification of labour supply functions. In R. Blundell, editor, *Unemployment, Search and Labour Supply*, pages 143–189. Cambridge University Press, Cambridge.

Bisher erschienen *

V-268-05 **Udo Ebert**, Ethical inequality measures and the redistribution of income when needs differ

V-269-05 **Udo Ebert**, Zur Messung von Risiko

V-270-05 **Roman Lokhov and Heinz Welsch**, Emissions Trading among Russia and the European Union: A CGE Analysis of Potentials and Impacts

V-271-05 **Heinz Welsch and Udo Bonn**, Is There a "Real Divergence" in the European Union? A Comment

V-272-05 **Martin Duensing**, Duale Einkommensteuer für Deutschland

V-273-05 **Udo Ebert and Georg Tillmann**, Distribution-neutral provision of public goods

V-274-05 **Heinz Welsch**, Kleines Land in Großer Welt: Der Beitrag Deutschlands, Österreichs und der Schweiz zur ökonomischen Literatur am Beispiel des Ausschusses für Umwelt- und Ressourcenökonomie

V-275-05 **Heinz Welsch**, The Welfare Costs of Corruption

V-276-05 **Heinz Welsch and Udo Bonn**, Economic Convergence and Life Satisfaction in the European Union

V-277-05 **Heinz Welsch**, The Welfare Effects of Air Pollution: A Cross-Country Life Satisfaction Approach

V-278-05 **Heinz Welsch,** Conflicts over Natural Resource Exploitation: A Framework and Cross-Country Evidence

V-279-05 **Udo Ebert and Heinz Welsch**, Environmental Emissions and Production Economics: Implications of the Materials Balance

V-280-06 **Udo Ebert**, Revealed preference and household production

V-281-06 **Heinz Welsch**, Is The"Misery Index" Really Flawed? Preferences over Inflation and Unemployment Revisited

V-282-06 **Heinz Welsch**, The Magic Triangle of Macroeconomics: How Do European Countries Score?

V-283-06 **Carsten Ochsen, Heinz Welsch**, The Social Costs of Unemployment: Accounting for Unemployment Duration

V-284-06 **Carsten Ochsen, Heinz Welsch**, Labor Market Institutions: Curse or Blessing

V-285-06 **Udo Ebert**, Approximating willingness to pay and willingness to accept for nonmarket goods

V-286-06 **Udo Ebert**, The evaluation of nonmarket goods: Recovering preferences in household production models

V-287-06 **Udo Ebert**, Welfare measurement in the presence of nonmarket goods: A numerical approach

V-288-06 **Heinz Welsch, Jan Kühling**, Using Happiness Data for Environmental Valuation: Concepts and Applications

V-289-06 **Udo Ebert and Georg Tillmann**, How progressive is progressive taxation? An axiomatic analysis

V-290-06 **Heinz Welsch**, The Social Costs of Civil Conflict: Evidence from Surveys of Happiness

V-291-06 **Udo Ebert and Patrick Moyes**, Isoelastic Equivalence Scales

V-292-07 **Tobias Menz, Heinz Welsch**, Carbon Emissions and Demographic Transition: Linkages and Projections

V-293-07 **Udo Ebert, Heinz Welsch**, Optimal Environmental Regulation: Implications of the Materials Balance

V-294-07 **Ole Christiansen, Dirk H. Ehnts and Hans-Michael Trautwein**, Industry Relocation, Linkages and Spillovers Across the Baltic Sea: Extending the Footloose Capital Model

V-295-07 **Ole Christiansen, Dirk H. Ehnts and Hans-Michael Trautwein**, Industry Relocation, Linkages and Spillovers Across the Baltic Sea: Extending the Footloose Capital Model (erneuerte Fassung zu V-294-07)

V-296-07 **Christoph Böhringer**, Combining Bottom-Up and Top-Down

V-297-07 **Christoph Böhringer and Carsten Helm**, On the Fair Division of Greenhouse Gas Abatement Cost

V-298-07 **Christoph Böhringer**, Efficiency Losses from Overlapping, Regulation of EU Carbon Emissions

V-299-07 **Udo Ebert**, Living standard, social welfare and the redistribution of income in a heterogeneous population

V-300-07 **Udo Ebert**, Recursively aggregable inequality measures: Extensions of Gini's mean difference and the Gini coefficient

V-301-07 **Udo Ebert**, Does the definition of nonessentiality matter? A clarification

V-302-07 **Udo Ebert**, Dominance criteria for welfare comparisons: Using equivalent income to describe differences in needs

V-303-08 **Heinz Welsch, Jan Kühling**, Pro-Environmental Behavior and Rational Consumer Choice: Evidence from Surveys of Life Satisfaction

V-304-08 **Christoph Böhringer and Knut Einar Rosendahl**, Strategic Partitioning of Emissions Allowances Under the EU Emission Trading Scheme

V-305-08 **Niels Anger, Christoph Böhringer and Ulrich Oberndorfer**, Public Interest vs. Interest Groups: Allowance Allocation in the EU Emissions Trading Scheme

V-306-08 **Niels Anger, Christoph Böhringer and Andreas Lange**, The Political Economy of Environmental Tax Differentiation: Theory and Empirical Evidence

V-307-08 **Jan Kühling and Tobias Menz**, Population Aging and Air Pollution: The Case of Sulfur Dioxide

V-308-08 **Tobias Menz, Heinz Welsch**, Population Aging and Environmental Preferences in OECD: The Case of Air Pollution

V-309-08 **Tobias Menz, Heinz Welsch**, Life Cycle and Cohort Effects in the Valuation of Air Pollution: Evidence from Subjective Well-Being Data

V-310-08 **Udo Ebert**, The relationship between individual and household welfare measures of WTP and WTA

V-311-08 **Udo Ebert**, Weakly decomposable inequality measures

V-312-08 **Udo Ebert**, Taking empirical studies seriously: The principle of concentration and the measurement of welfare and inequality

V-313-09 **Heinz Welsch**, Implications of Happiness Research for Environmental Economics

V-314-09 **Heinz Welsch, Jan Kühling**, Determinants of Pro-Environmental Consumption: The Role of Reference Groups and Routine Behavior

V-315-09 **Christoph Böhringer and Knut Einar Rosendahl**, Green Serves the Dirtiest: On the Interaction between Black and Green Quotas

V-316-09 **Christoph Böhringer, Andreas Lange, and Thomas P. Rutherford**, Beggar-thy-neighbour versus global environmental concerns: an investigation of alternative motives for environmental tax differentiation

V-317-09 **Udo Ebert,** Household willingness to pay and income pooling: A comment

V-318-09 **Udo Ebert,** Equity-regarding poverty measures: differences in needs and the role of equivalence scales

V-319-09 **Udo Ebert and Heinz Welsch**, Optimal response functions in global pollution problems can be upward-sloping: Accounting for adaptation

V-320-10 **Edwin van der Werf,** Unilateral climate policy, asymmetric backstop adoption, and carbon leakage in a two-region Hotelling model

V-321-10 **Jürgen Bitzer, Ingo Geishecker, and Philipp J.H. Schröder,** Returns to Open Source Software Engagement: An Empirical Test of the Signaling Hypothesis

V-322-10 **Heinz Welsch, Jan Kühling**, Is Pro-Environmental Consumption Utility-Maximizing? Evidence from Subjective Well-Being Data

V-323-10 **Heinz Welsch und Jan Kühling**, Nutzenmaxima, Routinen und Referenzpersonen beim nachhaltigen Konsum

V-324-10 **Udo Ebert**, Inequality reducing taxation reconsidered

V-325-10 **Udo Ebert**, The decomposition of inequality reconsidered: Weakly decomposable measures

V-326-10 **Christoph Böhringer and Knut Einar Rosendahl**, Greening Electricity More Than Necessary: On the Excess Cost of Overlapping Regulation in EU Climate Policy

V-327-10 **Udo Ebert and Patrick Moyes**, Talents, Preferences and Inequality of Well-Being

---

* Die vollständige Liste der seit 1985 erschienenen Diskussionspapiere ist unter http://www.vwl.uni-oldenburg.de/43000.html zu finden.