van der Werf, Edwin

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Unilateral climate policy, asymmetric backstop adoption and carbon leakage in a two-region Hotelling model

Edwin van der Werf

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Institut für Volkswirtschaftslehre
Universität Oldenburg, D-26111 Oldenburg
Unilateral climate policy, asymmetric backstop adoption, and carbon leakage in a two-region Hotelling model*

Edwin van der Werf†
Netherlands Environmental Assessment Agency (PBL)

Abstract

We study backstop adoption and carbon dioxide emission paths in a two-region model with unilateral climate policy and non-renewable resource consumption. The regions have an equal endowment of the internationally tradable resource and a backstop technology. We first study the case of a unilateral stock constraint (e.g. a 450 ppmv carbon dioxide concentration target), and show that the non-abating region makes the final switch to the backstop before the abating region does, though the latter region has two disjoint phases of backstop use if its marginal cost is sufficiently low. Furthermore, we show that the abating region has an inverse N-shaped emission path, with growing emissions in the period for which the ceiling is binding. In addition, there is a phase in which this region has a positive carbon price, but higher emissions than the non-abating region. With a global intertemporal carbon budget instead of a stock constraint, the order of definite backstop adoption is reversed and the abating region's emissions are always lower. We also show that unilateral climate policy does not lead to international carbon leakage.

JEL Classification: F18, O13, Q32, Q54

Keywords: climate policy, non-renewable resources, backstop technology, carbon leakage, unilateral climate policy

1 Introduction

Although climate change is a global problem, only a subset of countries currently imposes policies to reduce harmful greenhouse gas (notably carbon dioxide, CO₂) emissions. It is well-known that, in response to such unilateral policies, non-abating regions might increase their emissions, for example because the world price of fossil fuels falls due to reduced demand from the abating countries. At the same time, it seems inevitable that substitution towards non-fossil energy sources will take place, either because the stocks of oil and gas will decrease in the coming decennia, or because a price on CO₂ emissions makes the use of fossil fuels more expensive in regions with climate policy.

In this paper, we model carbon dioxide emissions and adoption of a clean backstop technology in a two-region model where emissions stem from the use of a physically exhaustible non-renewable resource. We show that the abating region need not be the first one to adopt the

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†Netherlands Environmental Assessment Agency (PBL), PO Box 303, 3720 AH Bilthoven, The Netherlands. Telephone: +31 30 274 2324. E-mail: edwin.vanderwerf@pbl.nl. The author is also affiliated with University of Oldenburg, Germany, and is a CESifo Research Network Fellow.
clean technology, but that the order of definite backstop adoption depends on the type of unilateral policy imposed. With a unilaterally imposed ceiling on the total amount of CO2 in the atmosphere, the abating region can have two disjoint phases of backstop technology use if the marginal cost of the backstop is sufficiently low.

Since carbon dioxide emissions stem largely from the use of non-renewable fossil fuels, climate policy needs to be studied in the context of exhaustibility of energy resources as introduced by Hotelling (1931). Several papers have studied the adoption of a clean backstop technology in this context, but only using closed economy models. Both Hoel and Kverndokk (1996) and Tahvonen (1997) study adoption of a backstop technology in a closed economy with optimal climate policy, i.e. using a damage function where damage stems from the stock of CO2 in the atmosphere. In both papers, use of the backstop technology will initially be zero, but positive at a later date, if initial marginal damage is not too high. In Hoel and Kverndokk (1996), the non-renewable resource is not physically exhaustible, but its use declines over time as the unit extraction costs increase with cumulative extraction. With this model design, resource use will be positive for all finite t. Tahvonen (1997) however, treats the fossil fuel as a physically exhaustible, non-renewable resource. He shows that if the marginal cost of the backstop is sufficiently high, the resource stock will be exhausted in finite time, and ultimately the economy only uses the backstop.

Chakravorty, Magné and Moreaux (2006) study the case of a stock constraint, rather than optimal climate policy, with a physically exhaustible resource. The authors assume an imposed ceiling on the stock of carbon dioxide in the atmosphere. At some point in time, emissions from the use of the resource become smaller than natural decay of the stock of pollution, the stock starts to decline and hence the ceiling is no longer binding. With a stationary demand curve, the backstop will be used during the period for which the ceiling is binding, only if its marginal cost is sufficiently low. However, in this case it will be used jointly with the non-renewable resource, until the resource stock gets exhausted. If initial demand is sufficiently high but declines rapidly over time because of an inward-shifting demand curve, the backstop will be used in two disjoint phases, provided its marginal cost is sufficiently low. In this case it will first be used, together with the resource, when the ceiling becomes binding, with increasing resource use until the backstop becomes too expensive. The second phase is the end phase when resource stocks are exhausted. Chakravorty, Moreaux and Tidball (2008) study a similar problem as that in Chakravorty et al. (2006), but assume the presence of two fossil fuels that differ in their carbon content. Regarding backstop technology, however, they assume that it will only be used after the imposed ceiling ceases to be binding, due to very high marginal cost of the backstop.

As we study backstop adoption and carbon dioxide emissions in a two-region model, we are also able to study carbon leakage. Carbon leakage occurs when emission reductions by some countries induce an increase in emissions by other countries, for example due to a lower world price for fossil fuels (see e.g. Bohm, 1993), relatively cheaper carbon-intensive goods in non-abating countries (see e.g. Perroni and Rutherford, 1993), or lower marginal damage from emissions in non-abating regions (see e.g. Hoel, 1991). Focussing on the first two channels of carbon leakage, the simulation literature has found that the percentage of emission reduction that gets offset by the increase in emissions by countries outside the Kyoto Protocol generally ranges from 2 to 41% (see for example Burniaux and Oliveira Martins, 2000). Babiker (2005) even finds a leakage rate of 130% for one of his scenarios. That is, in response to an emission reduction in some countries, other countries increase their emissions by an even larger amount, such that global emissions increase and global warming actually accelerates.

None of these papers on leakage model fossil fuels as being non-renewable. As suggested by Sinn (2008), when resource owners continue to extract the planned amounts of the resource in
order to exploit their entire resource stocks over time, this will lead to a lower resource price, a leakage rate of 100%, and zero effectiveness of unilateral emission reductions. Thus far, only two papers have discussed unilateral climate policy in the context of non-renewable resources. The first is a paper by Hoel (2008), which discusses the effect of a decrease in the price of a backstop technology on global emissions, when countries differ in their exogenous carbon tax. However, it does not discuss carbon leakage from unilateral climate policy, as such. The second paper is Eichner and Pethig (2009). In that paper, the authors study carbon leakage in a three-region, two-period model without a backstop technology, for the case of an exogenous and unilateral flow constraint. They establish conditions under which global emissions increase in response to a tightening of the constraint.

The present paper discusses both the adoption of a backstop technology and carbon leakage, in a model in which consumption only stems from energy use. We study two types of unilaterally imposed climate policy. First, we study the case of a stock constraint, or atmospheric CO₂ concentration target, which can be used to achieve the ‘ultimate objective’ of the United Nations Framework Convention on Climate Change, which is “stabilization of greenhouse gas concentrations in the atmosphere at a level that would prevent dangerous anthropogenic interference with the climate system.” (Article 2, UNFCCC) However, because of scientific uncertainty on how concentration levels map into temperature levels, Allen, Frame, Huntingford, Jones, Lowe, Meinshausen and Meinshausen (2009) suggest to restrict global cumulative emissions over time. Such a policy implicitly imposes a global cumulative carbon budget, whereby some of the world’s stocks of fossil fuels will have to be left in the ground. We therefore also study the effects of this type of policy, when unilaterally imposed, on backstop technology adoption and emission paths.

We assume that the resource is internationally tradable, and resource owners in the two regions in the model (one abating, one non-abating) take the world-price of the resource as given. We show that, in case of a unilateral stock constraint, the emissions path of the abating region follows an inverse N-shaped path, with rising emissions in the period for which the ceiling is binding. Furthermore, the abating region has two disjoint phases of backstop technology use if the marginal cost of the backstop is sufficiently low. It is even possible for the abating region to have an upward jump in emissions and energy consumption at the instant at which the non-abating region switches to the backstop. Moreover, with a unilateral stock constraint, the non-abating region makes the definite switch to the backstop ahead of the abating region, and there is always a period in which the abating region faces a positive price on emissions while its emission levels still are higher than those in the non-abating region, despite a common world price for the non-renewable resource.

With a unilaterally imposed global cumulative carbon budget however, disjoint backstop use is not possible, whereas the order of definite backstop adoption is reversed compared to the case of a stock constraint. With this policy type, emissions in the abating region are always lower than emissions in the non-abating region.

For either type of policy, unilateral emission reductions are 100% effective, as the non-abating region does not adjust its emissions in response to climate policy in the abating region.

The remainder of the paper is organized as follows. First we introduce the basic model, with unilateral climate policy modelled as a ceiling on the stock of carbon dioxide in the atmosphere. In section 3, we present our results regarding emissions by the non-abating region. In section 4, we study the order of backstop adoption by the two regions, and the path of carbon dioxide emissions by the abating region. We then study both of these for an alternative type of policy: a unilaterally imposed global intertemporal carbon budget. We summarize and discuss our results in section 6.
2 The model

We model two regions $i \in \{A, N\}$ that are identical in their endowments and preferences. Each region holds a stock of a non-renewable resource of size $X_0$. Instantaneous utility $U(q^i(t))$ is strictly concave function, where $q^i$ is the amount of energy that is consumed in region $i$. Utility only comes from the consumption of energy, where energy from a polluting non-renewable resource $x$ and a clean backstop technology $y$ are perfect substitutes: $q^i(t) = x^i(t) + y^i(t)$. The backstop is available at constant marginal costs $c$ and is only domestically available.\(^1\) Since the non-renewable resource can be freely traded between the two regions, there is one world price. All agents take all prices as given. Throughout the paper we will use the social planner's solution to the problem under scrutiny. As there are no market failures within each region, the decentralized economies give the same optimal extraction and consumption paths.

We assume that region $A$ (for 'Abating'; $N$ indicates the non-abating region) unilaterally imposes a ceiling on the stock of carbon dioxide in the atmosphere. This stock of pollution is indicated by $Z(t)$; it grows due to emissions from the two countries, where one unit of resource consumption causes one unit of emissions, and declines due to natural decay: $Z(t) = \sum_t x^i(t) - \delta Z(t)$.\(^2\) Here and in the remainder of the paper, a dot over a variable denotes the time derivative of that variable. We indicate the restriction on the stock of pollution, as unilaterally imposed by region $A$, by $\bar{Z}$.

The problem for each region reads:

\[
\max \int_0^\infty U\left(q^i(t)\right) e^{-\rho t} dt; \quad (1)
\]

subject to:

\[
q^i(t) = x^i(t) + y^i(t); \quad (2)
\]

\[
x^i(t) = -\dot{X}^i(t); \quad (3)
\]

where region $A$ in addition faces the restriction:

\[
Z(t) \leq \bar{Z} \forall t; \quad (4)
\]

\[
\dot{Z}(t) = -\delta Z(t) \forall t; \quad Z(0) = Z_0 \leq \bar{Z} \text{ given}. \quad (5)
\]

The rate of pure time preference is indicated by $\rho$, and $x^i$ is the amount of the resource that is extracted in (supplied by) region $i$.

Following the literature, we assume that the resource cannot be stored (or that storage costs are prohibitively high):

\[
x^A(t) + x^N(t) = x^A(t) + x^N(t). \quad (6)
\]

Then, at each point in time, each region has to decide how much of the resource it should supply to the world market, and how much it should consume of the resource and of energy provided by the backstop technology. If, at some point in time, domestic extraction and consumption differ, the region builds up or brings down a stock of claims on the resource of the other region. This gives the following flow budget constraint for each region:

\[
\dot{M}^i(t) = r(t) M^i(t) + p(t) \left( x^A(t) - x^i(t) \right); \quad M^i(0) = 0, \quad (7)
\]

\(^1\)One could think of the backstop as being solar energy, which is hardly traded internationally. See e.g. Chakravorty et al. (2006, 2008).

\(^2\)Exponential decay is a rough but commonly used approximation of the natural uptake of carbon dioxide in the atmosphere by natural sinks like forests and the oceans. See e.g. Withagen (1994), Tahvonen (1997), Chakravorty et al. (2006, 2008).
where $M^i$ is region $i$'s stock of claims on the other region, $r(t)$ is the interest rate (which is determined on international markets and is taken as given by each region), and $p(t)$ is the price of the resource on the world-market. Combined with the preferences and endowments described above, this gives the following Lagrangians:

\[
\mathcal{L}^N(t) = U\left(q^N(t)\right) - \lambda^N_X(t)x^N(t) - cy(t) + \lambda^N_M(t)\left(r(t)M^N(t) + p(t)\left(x^N(t) - x^M(t)\right)\right) + \gamma^N_X(t)x^N(t) + \gamma^N_Y(t)y^N(t);
\]

\[
\mathcal{L}^A(t) = U\left(q^A(t)\right) - \lambda^A_X(t)x^A(t) - cy(t) + \lambda^A_M(t)\left(r(t)M^A(t) + p(t)\left(x^A(t) - x^M(t)\right)\right) - \tau(t)\left(\sum_i x^i(t) - \delta Z(t)\right) + \mu(t)\left(Z - Z(t)\right) + \gamma^A_X(t)x^A(t) + \gamma^A_Y(t)y^A(t)
\]

where, for region $l$, $\lambda^l_X$ is the co-state variable to (3), $\lambda^l_M$ is the co-state variable to (7), and the $y$s are Lagrange multipliers to the non-negativity constraints. For region $A$, $\tau$ is the co-state variable to (5) and $\mu$ is the Lagrange multiplier to the restriction in (4). Note that we changed the sign of $\tau$ so that we can interpret it as a (shadow) price for carbon dioxide emissions. For the remainder of the paper, we normalize $\lambda^A_M(0) \equiv 1$. The first-order conditions give

\[
\frac{\partial \mathcal{L}^N(t)}{\partial x^N(t)} = 0, \quad U'\left(q^N(t)\right) = \lambda^N_M(t)p(t) - \gamma^N_X(t);
\]

\[
\frac{\partial \mathcal{L}^A(t)}{\partial x^A(t)} = 0, \quad U'\left(q^A(t)\right) = \lambda^A_M(t)p(t) + \tau(t) - \gamma^A_X(t);
\]

\[
\frac{\partial \mathcal{L}^i(t)}{\partial y^i(t)} = 0, \quad \lambda^i_M(t)p(t) = \lambda^i_X(t)\gamma^i(t);
\]

\[
\frac{\partial \mathcal{L}^i(t)}{\partial x^i(t)} = 0, \quad U'\left(q^i(t)\right) = c - \gamma^i_X(t);
\]

\[
\frac{\partial \mathcal{L}^i(t)}{\partial Z(t)} - \tau(t) = -\rho \tau(t), \quad \tau(t) = (\rho + \delta)\tau(t) - \mu(t).
\]

Equations (10)-(11) state that marginal benefit from energy use, has to equal marginal costs, whereby $\lambda^M_M(t)$ converts the world price of the resource into units of utility. Equation (12) relates the world resource price to the local scarcity rent. Equation (14) gives the Hotelling rule that the scarcity rent of the resource has to grow at the rate of time preference. Equations (15)-(16) are the equations of motion for the co-state variables to the stock of resource claims and the stock of pollution, respectively. Equation (16) shows that as long as the ceiling on the stock of carbon dioxide is not yet binding, the carbon price has to grow. We define the amount of energy consumed when $U'(c) = c$ by $\bar{q}$. If in addition only the backstop is used, then $q^i(t) = \bar{q} = \bar{y}$.

The complementary slackness conditions are:

\[
x^i(t) \geq 0; \quad y^i(t) \geq 0; \quad x^i(t)\gamma^i_X(t) = 0;
\]

\[
y^i(t) \geq 0; \quad y^i(t)\gamma^i_Y(t) = 0;
\]

\[
\mu(t) \geq 0; \quad Z - Z(t) \geq 0; \quad \mu(t)\left[Z - Z(t)\right] = 0.
\]
Finally, the transversality conditions
\begin{align}
\lim_{t \to -\infty} \lambda^L_M(t) e^{-\rho t} &\geq 0; \\
\lim_{t \to -\infty} \lambda^L_X(t) e^{-\rho t} &\geq 0; \\
\lim_{t \to -\infty} \tau(t) e^{-\rho t} &\geq 0, \\
\lim_{t \to -\infty} \lambda^L_M(t) X^l(t) e^{-\rho t} &\geq 0; \\
\lim_{t \to -\infty} \lambda^L_X(t) X^l(t) e^{-\rho t} &\geq 0; \\
\lim_{t \to -\infty} \tau(t) X^l(t) e^{-\rho t} &\geq 0,
\end{align}

have to hold. From integration of (14), combined with (20), and (10)-(13) and the complementary slackness conditions (17)-(18), then follows that region i's resource stocks have to be depleted when $\lambda^l_M(t) p(t) = \lambda^l_X(t)$ equals $c$. We define this instant, at which region $i$ switches to the backstop, as $t = T^l_i$, hence from (14) we have $\lambda^l_X(0) e^{\rho T^l_i} = c$.

We now briefly discuss the case in which neither region faces climate policy at any point in time. Taking the time derivative of (10) and (12) and combining the results with each other and with (14), we find after some rearrangement
\begin{equation}
\ddot{q}^l(t) = -\frac{\rho}{\eta(q^l(t))},
\end{equation}

whenever $x^l(t) = q^l(t)$. Here and in the remainder of the paper, a hat over a variable denotes its growth rate. We denote equilibrium variables for this laissez-faire economy, in which both countries never face climate policy, by an asterisk *. We define $\eta(q^l(t)) = -U'' q^l(U') > 0$ as the coefficient of relative risk aversion, or the inverse of the elasticity of intertemporal substitution.

In both countries, consumption under laissez-faire declines over time. From (14), (15) and the time derivative of (12) follows that the world-price of the resource $p(t)$ grows at the rate of interest. Given our normalization $\lambda^A_M(0) = 1$ and the fact that both countries face the same world price for the resource, $\lambda^A_X(0)$ has to be set such that it exactly exhausts its resource stock at the instant of the switch to the backstop. With a unilaterally imposed restriction on the stock of pollution by $A$, this region has to set $\lambda^A_X(0)$ and $\tau(0)$ such that it exhausts its resource stock and meets the pollution constraint.

Since, under laissez-faire, consumption in each region declines at each point in time, aggregate supply has to decrease as well. Although global supply equals global demand at each point in time (equation (6)), the extraction rates of the individual countries and the pattern of trade cannot be determined: resources from the two countries are considered as perfect substitutes in consumption, while each region's planner is indifferent to how much to extract at each point in time.

### 3 Emissions by the non-abating region

In this section, we study how the non-abating region responds to an emission reduction in region $A$, and to what extent this can lead to carbon leakage. Carbon leakage occurs if the non-abating region increases its emissions in response to an emission reduction in region $A$, for example because of a lower world price for fossil fuels. Formally, there is carbon leakage at instant $t$ if $x^A(t) < x^*^A(t)$ and $x^N(t) > x^*^N(t)$, where $x^A(t)$ is the amount of emissions in the abating region, when $\tau(t) > 0$.

We first prove the following useful intermediate result:

**Lemma 1.** Suppose two regions are described by equations (1)-(7). Then $\int_{0}^{\infty} x^l(t) dt = X_0$. That is, over time each region consumes an amount of the non-renewable resource equal to its own endowment.
Proof: Since, in each region, energy consumption switches to the backstop in finite time, it follows from (7), (15) and (21) that \( \lim_{t \to \infty} M(t) = 0 \). Combining (12) in growth rates, (14), and (15), gives \( \tilde{\rho}(t) = r(t) \). Combining this result with (7) and \( \lim_{t \to \infty} M(t) = 0 \) gives \( \int_0^\infty x(t) dt = X_0 \).

The value of the interest rate is undetermined: given that both the return on assets and the growth rate of the world price for the resource is equal to \( r(t) \), the findings above are immaterial to the exact value of the interest rate.

With this result in hand, we can show the effect of unilateral climate policy on emissions in the non-abating region:

**Proposition 1.** Suppose two regions are described by equations (1)-(7). Then \( x^N(t) = x^A(t) \) \( \forall t \).

That is, the non-abating region will not change its emission path in response to a change in emissions in the abating region.

Proof. From Lemma 1 and the fact that all results are immaterial to \( r(t) \) follows that (23) holds for \( N \) irrespective of what \( A \) does. Since neither \( X_0 \) nor the time horizon has changed, \( x^N(t) = x^A(t) \) \( \forall t \).

This immediately gives the following result on carbon leakage:

**Corollary 1.** Suppose two regions are described by equations (1)-(7). Then unilateral climate policy will not lead to any carbon leakage.

Since the non-abating region does not change its emissions path in response to climate policy in the other region, carbon leakage is zero. This result shows that the simplest extension of the standard closed-economy non-renewable resource model towards a two-region model, leads to a result that goes against the suggestion by Sinn (2008) that unilateral climate policy might be 100% ineffective: Corollary 1 states that unilateral climate policy is 100% effective. We find this surprising result by extending the standard closed-economy cake-eating model (for example used in Withagen, 1994, Tahvonen, 1997, Chakravorty et al., 2006, 2008) to include a second, identical region - except for its climate policy. Note that the result is independent of whether a backstop technology is available.

The result is driven by the fact that the two regions will consume an amount equal to their own resource stock in order to meet their transversality conditions. The non-abating region follows a laissez-faire consumption and emission path, which are determined by (23), initial stocks and the time horizon. Since neither of these is affected by the unilateral climate policy, the non-abating region does not adjust its consumption and emission path, hence carbon leakage is zero at each point in time.

In the next sections, we study the effect of unilateral climate policy on backstop adoption in the two regions and on emissions in the abating region.

### 4 Emission paths and backstop adoption with a unilateral stock constraint

In this section we take a stock constraint, for example a 450 ppm CO2 concentration level, as the policy target for the abating region. For the rest of the section, we focus on the policy-relevant cases and presume that \( X_0, Z_0, \tilde{Z}, \delta \) and \( c \) are such that the following assumptions hold:
Assumption 1. \( \exists T(X_0, Z_0, Z, \delta, c) \equiv \{ t | \lim_{|T| \rightarrow T} Z(T)X_0, Z_0, Z, \delta, c < Z; Z(T)X_0, Z_0, Z, \delta, c = Z \} > 0, \) and \( \tau(t) > 0 \) for a strictly positive interval of time.

That is, the stock of carbon dioxide in the atmosphere reaches the unilaterally imposed ceiling at some point in time \( T > 0 \) and stays there for a non-zero period of time.\(^3\)

Assumption 2. \( x^N(T(X_0, Z_0, Z, \delta, c)) \leq \delta Z. \)

That is, the unilateral stock constraint is feasible.

Assumption 3. \( x^N(t = 0|X_0, Z_0, Z, \delta, c) < x^N(t = 0|X_0, c) = x_{I}^{f}(t = 0|X_0, c). \)

In words: initial emissions in the abating region are lower than under laissez-faire. This implies that the drop in the initial scarcity rent in \( A \), compared to laissez-faire, is larger than the initial shadow price for \( CO_2 \) emissions. Note that the last equality follows from Proposition 1.

We show in Appendix A that Assumptions 1-3 imply that \( x^N(T(X_0, Z_0, Z, \delta, c)) > \frac{1}{2} \delta Z \) and \( c > c \equiv \{ c | \bar{y} = \delta Z \}. \) For future reference it is useful to define:

- \( \bar{c} \equiv \{ c | \bar{y} = \delta Z - x^N(T)X_0, Z_0, Z, \delta, c) \}, \) i.e. if \( c = \bar{c} \), the marginal cost of the backstop is such that the associated amount of (backstop) energy consumption is equal to the maximally allowed emissions for region \( A \), at the instant at which the stock of \( CO_2 \) in the atmosphere reaches the ceiling; and

- \( \bar{c} \equiv \{ c | T_{H}(X_0, Z_0, Z, \delta, c) = T_{N}^B(X_0, c) \} \) where \( T_{H} \) is the instant from which onward \( \tau(t) = 0, \) i.e. if \( c = \bar{c} \), the ceiling on the stock of carbon dioxide in the atmosphere ceases to be binding at the instant at which the non-abating region switches to the backstop.

These definitions allow us to study the effects of 'high', 'intermediate' and 'low' marginal costs of backstop technology on its adoption and emission paths.

4.1 Case 1: high marginal cost of the backstop, \( c \geq \bar{c} \)

In the current case we assume that marginal costs of the backstop are sufficiently high, such that, when the ceiling is reached – given \( X_0, Z_0, Z \) and \( \delta - \) energy provision by the backstop is smaller than (or equal to) the difference between fossil-energy use allowed by natural uptake and fossil-energy use (and hence emissions) in the non-abating region. Note that here and in the remainder of the paper, at each point in time, \( x^N(t) \) is solely determined by the initial resource stock \( X_0 \), marginal cost of the backstop \( c \), time preference parameter \( \rho \), and the exact shape of the utility function (via equation (23)).

Define \( t = T_{C} \) as the instant at which emission paths of the two regions cross. Emissions in the abating region, and its adoption of the backstop technology, can then be described as follows:

**Proposition 2.** Suppose two regions are described by equations (1)-(7), and \( c \geq \bar{c} \). Then

a. The emission path of the abating region has an inverse N-shape: emissions decline until the stock of carbon dioxide reaches the ceiling at \( t = T \), then they rise until the constraint ceases to be binding, after which emissions decline again; if \( c < \infty \), emissions are zero once the switch to the backstop has been made;
b. There exist unique \( t = T_C \) and \( t = T_H \), with \( T_C < T_H \):

c. \( x^A(t) > x^N(t) \) \( \forall t \in (T_C, T_b^A) \), with \( T_b^A = \infty \) if \( c = \infty \); that is, after \( t = T_C \) and as long as \( x^A(t) > 0 \), emissions in region \( A \) are higher than those in region \( N \), even though initially (up to \( t = T_H \)) the abating region still has effective climate policy (\( r(t) > 0 \));

d. If \( c < \infty \), \( T_H < T_b^N < T_b^A \) i.e. the non-abating region adopts the backstop earlier than the abating region, but only after the ceiling ceases to be binding.

Proof. See Appendix B.

The energy consumption paths of the laissez-faire economy, the abating economy with a stock constraint, and the non-abating region, are illustrated in Figure 1 for the case of finite marginal cost of the backstop; the corresponding paths of the (shadow) prices are illustrated in Figure 2.\(^4\) For \( t < T_b^I \), the respective region's consumption path is also its emission path.

The emissions path for the abating region has an inverse N-shape, that is, it contains a cycle in which resource consumption and emissions first go down until the stock of carbon dioxide reaches the ceiling, then increases until this constraint ceases to be binding, and then declines following a 'standard' cake-eating consumption path until the switch toward the backstop has been made. The new and perhaps surprising result is that emissions in the regulated region increase during the period for which the constraint is actually binding. This result reflects the interaction between the two regions via the stock constraint: as region \( A \) wants global emissions not to be larger than \( \delta Z \) once the stock of carbon dioxide reaches the ceiling, it cannot

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\(^4\) The position of \( q^A(0) \) relative to \( \delta Z / 2 \) is arbitrarily chosen.
emit more than the difference between natural uptake and emissions in the non-abating region. Since emissions in the latter region decline over time, the abating region can increase its emissions during the phase in which it wants global emissions to be constant and equal to natural uptake.

The emission paths cross at \( t = T_c \), at which instant both countries emit an amount equal to half the natural uptake. At \( t = T_H \), the constraint ceases to be binding. Between \( T_C \) and \( T_H \), the abating region still has a price on emissions but has higher emissions than the non-abating region. Its emissions are also higher during the phase in which neither region abates (until region \( A \) switches to the backstop). This is due to the increased abundance of the resource endowment as perceived by the abating region, as can be seen from equations (10) and (12).

When \( \tau(t) = 0 \), resource consumption is determined by the scarcity rent \( \lambda^A(t) \). Since, during some period of time, the abating region can consume less of the resource than under laissez-faire, more is available during periods of time in which it does not face a positive carbon price, which is equivalent to having a larger resource stock at \( t = T_H \), compared to laissez-faire. As a consequence, the scarcity rent of the abating region is lower than under laissez-faire, and hence lower than the rent of the non-abating region. Assuming \( c \) is finite, \( \lambda^A \) equals the marginal cost of the backstop technology at a later instant than the scarcity rent of the non-abating region. As a consequence, the latter region adopts the backstop before the abating region does (\( T_b^N < T_b^A \)).

### 4.2 Case 2: 'intermediate' marginal cost of the backstop, \( c \in (\tilde{c}, \bar{c}) \)

Next we discuss the case where the marginal cost of the backstop is sufficiently low, so that the abating region will adopt it in the period for which the ceiling on the stock of carbon dioxide in the atmosphere is binding, and energy consumption in region \( A \) is continuous at each point in
time. Emissions in the abating region, and its adoption of the backstop, can then be described as follows:

Proposition 3. Suppose two regions are described by equations (1)-(7), and \( c \in (\bar{c}, \overline{c}) \). Then

a. The emission path of the abating region has an inverse N-shape with a downward jump at \( t = T \): emissions decline until the stock of carbon dioxide reaches the ceiling at \( t = T \) at which instant they jump down; they rise until the constraint ceases to be binding, after which emissions decline again; emissions are zero once the switch to the backstop has been made;

b. The abating region has two distinct phases in which the backstop is used: initially it only uses the non-renewable; at \( t = T \) the non-renewable is partly substituted by the backstop; as long as the ceiling is binding it gradually uses more of the resource until at \( t = T_S < T_H \) only the non-renewable is used; the second phase is the end-phase in which only the backstop is used;

c. There exist unique \( t = T_C \) and \( t = T_H \), with \( T_C < T_H < T_{b_n} \);

d. \( x^A(t) > x^N(t) \) \( \forall t \in (T_C, T_{b_A}) \); that is, after \( t = T_C \) and until the final switch to the backstop by region \( A \), emissions in region \( A \) are higher than those in region \( N \), even though initially (up to \( t = T_H \)) the abating region still has effective climate policy (\( r(t) > 0 \)).

e. \( T_H < T_{b_N} < T_{b_A} \), that is, the non-abating region adopts the backstop earlier than the abating region, but only after the ceiling ceases to be binding.

Proof. See Appendix C. \( \square \)
The energy consumption paths of the laissez-faire economy, the abating economy with a stock constraint, and the non-abating region for the case \( c \in (\bar{c}, \bar{c}) \) are illustrated in Figure 3.\(^5\) As long as \( q^A(t) > \bar{y} \), emissions equal consumption. The corresponding paths of the (shadow) prices are illustrated in Figure 4.

When \( c \) is sufficiently low, the abating region starts to use the backstop at the instant at which the ceiling is reached. However, since the ceiling has to be binding for a positive period of time (Assumption 1) and \( x^A(T) > \frac{1}{2} \delta Z \), \( x^A(T) > 0 \) so the backstop is used jointly with the non-renewable. As the price on emissions declines over time, and as emissions by the non-abating region decline, the abating region can increase its emissions and from some \( T_b^{N} < T_c < T_H \) onward the backstop is no longer used. However, the scarcity rent in the abating region increases over time (see (14)), and at \( T_b^{A} > T_H \) it switches to the backstop again, and its resource stock is exhausted. Again, the non-abating region switches to the backstop before the abating region does, as the increased perceived abundance of the resource endowment in region \( A \) causes its scarcity rent to fall, relative to laissez-faire. Note that energy consumption by the abating region is continuous at each point in time: it is the composition of energy consumption that jumps at \( t = T \).

4.3 Case 3: low marginal cost of the backstop, \( c \in [\underline{c}, \bar{c}] \)

When the marginal cost of the backstop is lower than or equal to \( \bar{c} \), given Assumptions 1-3, energy consumption and hence utility of the non-abating region is no longer continuous:

**Proposition 4.** Suppose two regions are described by equations (1)-(7), and \( c \in [\underline{c}, \bar{c}] \). Then

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\(^5\)The position of \( q^A(0) \) relative to \( \delta Z/2 \) is arbitrarily chosen.
a. The emission path of the abating region has an inverse N-shape with a downward jump at \( t = T \) and an upward jump at \( t = T_b^N \); emissions decline until the stock of carbon dioxide reaches the ceiling at \( t = T \) at which instant they jump down; they rise until the non-abating region switches to the backstop, at which instant energy consumption and emissions (t.) (for sufficiently small \( \delta \) or \( Z \) or sufficiently large \( X_0 \) or \( Z_0 \), all ceteris paribus) jump up to \( \delta Z \) and stay at this level until the constraint ceases to be binding and then decline again, or (ii.) otherwise jump up to a level below or equal to \( \delta Z \) and immediately start declining (i.e. \( T_b^N = T_H \)); emissions are zero once the switch to the backstop is made;

b. The abating region has two distinct phases in which the backstop is used: initially it only uses the non-renewable; at \( t = T \) the non-renewable is partly replaced by the backstop; then

i. if \( c \geq c' = \{ c | \frac{\delta Z}{\delta} = \frac{1}{2} \} \), it gradually uses more of the resource until \( y^A \) continuously goes to zero at \( T_b^N < T_b^L \), or

ii. if \( c < c' \), it gradually uses more of the resource until \( y^A \) jumps to zero at \( t = T_b^N < T_H \);

the second phase is the end-phase in which only the backstop is used;

c. \( x^A(t) > x^N(t) \) \( \forall t \in (T_b^N, T_b^A) \) with \( T_b^N < T_H < T_b^A \); that is, after the switch to the backstop by region \( N \) and until the final switch to the backstop by region \( A \), emissions by region \( A \) are higher than those by region \( N \) (which equal zero), even though initially (up to \( t = T_H \)) the abating region still has effective climate policy; if \( c \in (c', \bar{c}) \), then \( 3T_c < T_b^N \) such that the emission paths cross, and \( x^A(t) > x^N(t) \) \( \forall t \in (T_c, T_b^A) \).

d. At \( t = T_b^N \), the shadow price for emissions in \( A \) jumps down.

Proof. See Appendix D.

The energy consumption paths of the laissez-faire economy, the abating economy with a stock constraint, and the non-abating region are illustrated in Figure 5, for the case \( \bar{c} < c < c' < \bar{c} \).

The corresponding paths of the (shadow) prices are illustrated in Figure 6.

When \( U'(q) < c' \), the level of utility is higher than the level of utility from consuming an amount of energy equal to half of the decay of CO\(_2\) in the atmosphere. As a consequence, the emission paths of the two regions cannot cross while both have positive emissions: if they did, then this would give a utility level that is lower than that of consuming energy from the backstop. However, the abating region must have higher emissions than the non-abating region for some period of time, for otherwise it would not exhaust its resource stock, which cannot be optimal (violates (20)). Then, given \( c \in (c', \bar{c}) \), emissions in the abating region have to jump up at the instant at which the non-abating region switches to the backstop, to a level smaller than (if the constraint no longer binds) \( \delta Z \). This upward jump in emissions is induced by a downward jump in the shadow price for carbon emissions.

How can a jump in energy consumption, and hence in utility, be optimal for the non-abating region, even when forward-looking agents know when \( t = T_b^N \)? Technically, the abating region has to optimize over two distinct periods, with two versions of (5): it optimizes for \( t \geq T_b^N \) with \( \dot{Z} = x^A(t) \) and \( Z(t) \leq \bar{Z} \) for \( Z(T_b^N) = \bar{Z} \) and \( X^A(T_b^N) \) given, and optimizes for \( t \in (0, T_b^N) \) with \( \dot{Z} = x^A(t) + x^N(t) \), \( x^N(t) > 0 \) and \( Z(t) \leq \bar{Z} \) for \( Z_0 \) and \( X_0 \) given.\(^7\) As the emission path of the non-

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\(^6\)The position of \( q^A(0) \) and \( q^N(0) \) relative to \( \delta Z \) is arbitrarily chosen.

\(^7\)In essence, the problem is one where at \( t = 0 \) it is known that at a known later date the restriction on emissions in region \( A \) will get alleviated. Di Maria, Smulders and Van der Werf (2008) study a related problem: announcement of a future constraint on extraction and emissions in a closed economy with multiple resources. They find that at the instant of the policy announcement, extraction and emissions jump up, whereas they jump down at the instant of implementation.
5 Emission paths and backstop adoption with a unilaterally imposed intertemporal global carbon budget

An important drawback of a policy aimed at a stabilization level of atmospheric CO₂ concentration as studied in the previous section, is that the eventual equilibrium global mean temperature remains uncertain as it is uncertain how concentration levels map to temperature increases. Allen et al. (2009) therefore propose to restrict global emissions in the 1750-2500 period to 1 trillion tonnes carbon (1 TtC, 3.67 trillion tonnes CO₂). Such a global intertemporal carbon budget implies that some of the stocks of fossil fuels, for example coal, will have to remain un-exploited. The idea behind this proposal is that policy targets based on limiting cumulative emissions of carbon dioxide are likely to be more robust to scientific uncertainty than emission rate or concentration targets. The authors find that total anthropogenic emissions of 1 TtC, about half of which has already been emitted since industrialization began, results in a most likely peak in carbon-dioxide-induced warming of 2°C above pre-industrial temperatures.

We can implement this proposal in our model in the following way. The global carbon budget from $t = 0$ (the year 2009, say) onwards is given by $\bar{X}$. Note that natural uptake is not taken

\[ q'(t), q''(t) \]

\[ g''(t) = g''(t) \]

\[ \delta Z \]

\[ y \]

\[ x^A \text{ only} \quad \text{joint use in } A \quad x^A \text{ only} \quad x^A \text{ only} \quad y^A \text{ only} \]

\[ T_b^N \quad T_b^N \quad T_b^N \quad T_b^N \]

\[ \text{Figure 5: Energy consumption paths for the case } c \in [c, \bar{c}]; c < c' < \bar{c}. \]
into account, so $\delta = 0$. In that case the total amount of CO$_2$ that can be accumulated in the atmosphere by region $A$ is given and equal to $(\bar{X} - X_0) \in (0, X_0)$. As a consequence, the abating region has to consume less than its total endowment. The social planner in region $A$ can simply implement this by maximizing intertemporal utility subject to the resource constraint $\int_0^{T^A} x^A(t) dt \leq \bar{X} - X_0 < X_0$. Consequently, there is no price on carbon emissions: the smaller resource endowment simply induces a higher resource scarcity rent in the abating region, relative to laissez-faire. The effects of the carbon budget on emissions and adoption of the backstop technology are as follows:

**Proposition 5.** Suppose two regions are described by equations (1)-(7), $\delta = 0$, and $X^A_0 = \bar{X} - X_0 < X^N_0 = X_0$. Then

a. $x^A(t) < x^N(t) \forall t < T^N_b$, that is, the abating region has lower emissions than the non-abating region;

b. $T^A_b < T^N_b$ that is, the abating region adopts the backstop earlier than the non-abating region.

**Proof.** See Appendix E

Figures 7 and 8 present the consumption and (shadow) price paths for a global intertemporal carbon budget. The higher scarcity rent in region $A$ causes emissions in this region to be lower than under laissez-faire (and, hence, lower than those by the non-abating region) at each point in time. Secondly the unilateral global budget induces asymmetric adoption of the backstop technology. Since the growth rate of the rent is the same for both regions, a higher scarcity rent in region $A$ means that it equals the marginal cost of the backstop sooner, and the abating region adopts the clean backstop technology before the non-abating region does. This result
Figure 7: Energy consumption paths for a global intertemporal carbon budget.

is opposite to what we found in the previous section: with a unilateral stock constraint, it is the non-abating region that makes the definite switch to the backstop first. Hence, the order in which the regions make the definite switch to the backstop depends on the type of unilateral climate policy imposed.

6 Summary and discussion

We have studied backstop technology adoption in a two-region model, when carbon dioxide emissions stem from consumption of a non-renewable resource, using the simplest possible model: we have split the integrated world economy of the standard cake-eating model into two identical regions with price-taking resource owners and a backstop technology, and studied the effects of unilateral emission reductions. First, we studied the case where the abating region imposes a unilateral stock constraint. That is, the total amount of carbon dioxide in the atmosphere is not allowed to be larger than a certain amount (450 ppm CO\textsubscript{2}, say), but because of the natural uptake of carbon dioxide by the oceans and the terrestrial biosphere, the abating region can still exhaust its entire resource stock over time, as long as it postpones its emissions. We have shown that, for this type of unilateral policy, the non-abating region permanently switches to the backstop before the abating region does. However, if the cost of the backstop is sufficiently low, there is a phase for which the ceiling is binding and the abating region jointly uses the non-renewable and the backstop, and hence the abating region has two disjoint periods of backstop use. In addition we have shown that, generally, emissions in the abating region will follow an inverse N-shaped path, with rising emissions in the period for which the ceiling is binding. Furthermore there will be a period in which the abating region has active climate policy, but higher emissions than the non-abating region.
As there is scientific uncertainty on how CO₂ concentration levels map into temperature increases, we have looked at a binding policy where the abating region decides that, over time, no more than a given amount of carbon dioxide should be emitted globally; that is, the abating region imposes a (feasible) intertemporal global carbon budget. Since the non-abating region exhausts its resource stocks anyway, the abating region is not allowed to exhaust its entire stock over time, and hence its scarcity rent is higher than under laissez-faire. When introducing a backstop technology, this means that the abating region switches to this technology before the non-abating region does, contrary to the case of a unilaterally imposed stock constraint.

Furthermore, we have shown that unilateral climate policy does not affect the emission path of the non-abating region. As a consequence, unilateral emission reductions are 100% effective. This result is driven by the intertemporal budget restriction of each region. When consumption deviates from extraction, and, hence, some of the resource is exported or imported, a stock of claims on the other region’s resource stock grows or diminishes. Since, at any point in time, both the resource price and the value of the stock of claims (with zero instantaneous net trade) grow at the rate of interest, and since each region has to meet its intertemporal budget constraint, both regions consume and emit an amount equal to their initial resource endowments.

As our model is relatively simple, we now discuss how the results from the model containing a stock constraint are affected by four changes to our model. We discuss different types of climate policy, the use of a second input in final good production, a non-stationary demand curve, and extraction costs.

Instead of a unilateral stock constraint, the abating region could introduce a constraint on the flow of its emissions, or it could impose optimal policy, with a trade-off between consumption from energy and damages from the stock of CO₂ in the atmosphere. With either of these policies, the order of definite backstop adoption will be the same as under a stock constraint,
as in all cases the abating region has a lower scarcity rent than the non-abating region, due to the increased perceived abundance after the imposition of climate policy. Neither the inverse N-shape in the emission path of the abating region, nor disjoint backstop use, can be found for the case of a unilateral flow constraint, as emissions by the abating region will be constant when constrained, and then declining. For the case of optimal climate policy, Tahvonen (1997) has shown, for a closed (world) economy, that when the stock of pollution is allowed to first increase and then decrease (which is the outcome of the recent climate negotiations in Copenhagen) and the third derivative of the strictly convex damage function (which is additively separable from utility from resource consumption) is non-negative, global emissions decrease monotonically. Keeping marginal damages as in Tahvonen's result, a declining emission path for the non-abating region in a two-region world then implies that the emission path of the abating region can have any shape, depending on the exact slopes of the path of marginal damages and the path of emissions for the non-abating region. Consequently, neither an inverse N-shaped emission path for the abating region, nor disjoint backstop use for this region, can be ruled out.

A second change to our model could be the introduction of a second input, as an imperfect substitute for the energy good $q$, in the production of a final good; for example a fixed factor such as inelastically supplied labour (see e.g. Eichner and Pethig, 2009), or a stock of capital. In both cases, each region would have a second source of income, the level of which is affected by the amount of the resource used in production. Hence, unilateral climate policy introduces income effects, such that the abating region has a lower return on the second input and, therefore, a smaller intertemporal budget than the other region. Consequently, the abating region consumes an amount smaller than its own endowment, resulting in a higher marginal utility of income and higher scarcity rent. As a result it may make the final switch to the backstop before the non-abating region, as in the case of a global carbon budget, and international carbon leakage might be positive for some period of time. The inverse N-shaped emissions path in the abating region, however, is likely to be unaffected by the introduction of a second, imperfectly substitutable input. The introduction of a second, imperfectly substitutable, input, and the possibility of non-zero carbon leakage, is an interesting line of future research.

In case of a stock constraint, the rising emissions in the abating region in the period during which the constraint is binding, directly follows from the fact that emissions in the non-abating region decline due to a stationary demand curve and an ever-increasing scarcity rent. This raises the question of what happens when the demand curve is non-stationary. We currently see increasing demand for fossil fuels in most developed and developing countries, partly due to increasing living standards within the latter countries, and due to an increasing world population. However, the pace at which living standards increase (notably in China) cannot be indefinitely sustained, whereas world population is expected to shrink in the middle or second-half of this century. Chakravorty et al. (2006) study backstop adoption in the case of a closed economy with a ceiling on the stock of carbon dioxide in the atmosphere, and find that disjoint use of the backstop (which we find with a unilaterally imposed ceiling with $c < \overline{c}$) might occur in the case of an inward-shifting demand curve with not too high marginal cost of the backstop. When the demand curve in region $A$ does not shift outward too much, or the demand curve in $N$ does not shift inward too much, the order of backstop adoption is unaffected, since then the scarcity rents of one region would not have to change too much in the direction of that of $\overline{c}$.

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8 The third derivative of an iso-elastic function, for example, is positive.

9 Lutz, Sanderson and Scherbov (2001) find that there is a 55% chance of the world population reaching its peak by 2075 and around 85% chance that this occurs by the end of the century. In the 'low' and 'medium' scenarios of United Nations (2004), which differ in their assumptions on fertility rates, world population starts to decline between 2025 and 2050, and between 2075 and 2100 respectively.
the other. The inverse N-shaped consumption and emission path of the abating region holds as long as the demand curve in the non-abating region does not shift out at too high a pace. If it does, then emissions in region $N$ rise over time, so emissions in $A$ cannot rise when the ceiling is binding. An inward-shifting demand curve in $N$, however, steepens the rising emission path in region $A$. Shifting demand curves do not affect our result regarding zero carbon leakage. Again, as each region's intertemporal budget restriction is unaffected, each consumes an amount equal to its own endowment.

A final extension that is worthwhile to discuss is the introduction of stock-dependent extraction costs. In this case, when more of a resource stock is extracted (or less is available), the higher the costs will be of extracting a unit of the resource. Hence, introduction of stock-dependent extraction costs, per se, does not affect our model results, as each region starts with the same endowment. Given our uniform world price for the resource, extraction in both regions is the same at each point in time, therefore resource stocks and extraction costs are the same as well. Stock-dependent extraction costs only matter if initial stocks are different, and, hence, a second asymmetry between the two regions (next to the introduction of climate policy) is introduced in our model. Clearly, in this case, the resource of the region with lowest extraction costs is used, until unit extraction costs are equal for both countries. We leave the introduction of this and further asymmetries (for example level of development or technology, or region sizes) to future research.

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A Proof that Assumptions 1-3 imply \( x^N(T(X_0, Z_0, Z, \delta, c)) > \frac{1}{2} \delta \bar{Z} \) and \( c > \bar{c} \)

To ease notation in this and following appendices, we write \( x^N(t) \) for \( x^N(t|X_0, c) \), and \( x^A(t) \) for \( x^A(t|X_0, Z_0, Z, \delta, c) \).

First, we prove that Assumption 1 implies \( x^N(T) > \frac{1}{2} \delta \bar{Z} \). Suppose \( x^N(T) < \frac{1}{2} \delta \bar{Z} \). Then region \( A \) can consume the same amount, without violating the constraint, and follow a laissez-faire path like \( N \). But then \( x^N(T) + x^A(T) < \delta \bar{Z} \) and the constraint is not binding at \( T \), which violates Assumption 1. Now suppose \( x^N(T) = \frac{1}{2} \delta \bar{Z} \). Then \( A \) can consume the same amount, without violating the constraint, and follow a laissez-faire path like \( N \). But then \( x^N(T) + x^A(T) = \delta \bar{Z} \), and from (23) and (5), \( T = T_H \), which violates Assumption 1. Hence \( x^N(T) > \frac{1}{2} \delta \bar{Z} \).

Next, suppose \( c < \bar{c} \). Assumptions 1 and 2 imply \( x^N(T) \leq \delta \bar{Z} \). But \( c < \bar{c} \) implies \( x^N(T) = 0 \). Then \( x^A(T) = \delta \bar{Z} \). But \( c < \bar{c} \) implies \( x^A(T) = 0 \), hence \( \bar{Z} = 0 \) and the ceiling cannot be binding. This violates Assumption 1, hence we cannot have \( c < \bar{c} \).

Before we prove that we cannot have \( c = \bar{c} \), we state and prove two Lemmas.

**Lemma 2.** \( T^N_b \geq T \).

**Proof.** Suppose \( T^N_b < T \). Then \( \delta \bar{Z} > x^A(t) > \delta \bar{Z} \forall t \in (T^N_b, T) \) and \( \dot{x}^A(t) > 0 \) to reach \( x^A(t) = \delta \bar{Z} \). But since \( \mu < 0 \forall t < T, \bar{t} > 0 \), so with (14), the time derivative of (11) and \( U'' < 0 \) we find \( \dot{x}^A(t) < 0 \forall t < T \). So, we cannot have \( T^N_b < T \).  

Lemma 3. \( y^A(t) = 0 \forall t < T. \)

Proof. Suppose \( y^A(t) > 0 \) for some \( t < T. \) Then either \( c < c' \), which we just ruled out, or \( A \) is indifferent between \( x \) and \( y \), so \( r > 0 \) and \( \tau > 0 \) for \( \lambda_X^A(t) + \tau(t) = c \) (from (11) and (13)). But then \( \mu > 0 \), which cannot hold for \( t < T. \) So, we cannot have \( y^A(t) > 0 \) for some \( t < T. \) \( \Box \)

Now suppose \( c = c' \). Define \( t = T_H \) as the instant at which the constraint (4) ceases to be binding. Then \( x^A(t) \geq \delta \bar{Z} \) so \( T_H = T^A_b \), and \( T^N_b = T. \) From Lemma 3 then follows that \( \lambda_X^A(T) + \tau(T) = \lambda_X^N(T) = c. \) From (14) and (16) follows that the LHS of the latter equality grows faster than the middle part for all \( t < T. \) But then \( x^A(t) > x^N(t) \forall t < T \) which violates Assumption 3. Hence we cannot have \( c = c' \), which completes the proof that Assumptions 1-3 imply \( c > c. \)

B Proof of Proposition 2

Since \( X_0 \) is finite and \( \delta > 0, Z(t) < \bar{Z} \) in finite time. Equations (16), (19), and (22) then imply that \( \tau = 0 \) from this instant onward. Hence \( T_H \) exists and is finite; since \( T \) is unique (see footnote 3), \( T_H \) is unique too. Optimality requires (9) to be continuous. With (11), (12), (14), and (15), this implies that \( q^A \) must be continuous at \( t = T_H. \) Since \( \tau(t) = 0 \) implies \( \lambda_X^A(t) = -\rho/\eta(x^A(t)) < 0 \) (see derivation of (23)), \( \lambda_X^A(t) < 0 \forall t \in (T_H, T^N_b) \) for \( c < \infty; \lambda_X^A(t) < 0 \forall t > T_H \) for \( c < \infty. \) For \( t > T^A_b, \)

\( x^A(t) = 0 \) so emissions are zero.

For all \( t < T, \mu(t) = 0, \) so from (11), (12), (14), and (16) and \( \frac{U(t)}{\sigma} < 0 \) follows that \( \lambda_X^A(t) < 0 \forall t < T. \)

If \( c > \bar{c}, \) \( \bar{y} < \delta \bar{Z} - x^N(t). \) If \( c = \bar{c}, \) \( \bar{y} = \delta \bar{Z} - x^N(t). \) For all \( t \in [T, T_H], \)

\( x^N(t) + \lambda_X^A(t) = \delta \bar{Z}, \) while (23) holds for \( N \) with \( \lambda_X^N(t) = x^N(T). \) Hence \( \lambda_X^A(t) > 0 \forall t > T_H \) and \( \lambda_X^A(t) = 0 \forall t < T^A_b. \) This completes the proof of part a.

By assumption, \( \bar{y} > x^A(0) < x^N(0) = x^A(0). \) From Lemma 1, part a of this proposition, and continuity of (9) then follows that \( \exists t = T_C < T_H, \) with \( x^A(T_C) \) being continuous. This proves part b.

Part c follows from a and b and the definition of \( T_H. \)

From part c follows that \( x^A(t) > x^N(t) \) if \( \tau(t) = 0. \) From (14) and \( \lambda_X^A(T^N_b) = c \) then follows that if \( c < \infty, T^A_b > T^N_b, \) which proves part d.

C Proof of Proposition 3

The proof is identical to that of Proposition 2, except that in addition we need to prove the downward jump in emissions at \( t = T, \) the use of the backstop during part of the phase in which the ceiling is binding, and the role of \( \bar{c}. \)

For all \( t \in [T, T_H], \)

\( \lambda_X^A(t) = \delta \bar{Z} - x^N(t), \) for otherwise the constraint (4) would be violated. Since continuity of (9) implies that \( \lambda_X^A(T) \) is continuous, and by assumption \( \bar{y} > \delta \bar{Z} - x^N(T), \)

\( \lambda_X^A(T) \) jumps down to \( \delta \bar{Z} - x^N(T) \) and \( \lambda_X^A(T) \) jumps from zero to \( \bar{y} - \delta \bar{Z} + x^N(T). \) Note that the economy is indifferent between the two energy sources as \( \lambda_X^A(T) + \tau(T) = c. \)

From (23), (4) and (5) follows that \( \lambda_X^A(t) > 0 \forall t \in (T, T_H). \) Then \( T_S \in (T, T_H) \) at which \( x^A(T_S) = \bar{y}. \)

Suppose \( c = c' = \{c|\bar{y} = \frac{1}{2}\delta \bar{Z} \}. \) Then, under Assumptions 1-3, at \( t = T^N_b, \) region \( A \) has not yet exhausted its resource stock. However, it is not possible to have the constraint binding (which requires \( x^A(T^N_b) = \delta \bar{Z} \)) and simultaneously have \( q^A(T^N_b) \) continuous. If we increase \( c \) from \( \bar{c} \) to \( c', \) such that Assumptions 1-3 still hold, we will find a \( c = \bar{c}, c' < \bar{c} < \bar{c}. \) For \( c' \in (c, \bar{c}), \)

\( T^N < T_H \) and \( q^A(T^N_b) \) cannot be continuous, as will be shown in Appendix D.

The rest of the proof follows the proof of Proposition 2.
D Proof of Proposition 4

We solve the current problem for region A as a three-stage problem:

\[
\max W = \int_0^T U(\cdot)e^{-\rho t}dt + \int_T^{T_H} U(\cdot)e^{-\rho t}dt + \int_{T_H}^{\infty} U(\cdot)e^{-\rho t}dt. \tag{D.1}
\]

Denote these stages by I, II, and III, respectively. Clearly, in stage III, \( \tau(t) = 0 \), so we have a path of declining emissions, given \( X^A(T_H) \) and \( M^A(T_H) \), until \( t = T^*_b \), and backstop use afterwards. In stage II, the stock constraint is binding, so equations (5) and (19) reduce to the restriction

\[
x^A(t) = \delta Z - x^N(t), \tag{D.2}
\]

where region A takes \( x^N(t) \) as given. However, since \( c < (\gamma, \tilde{c}) \), \( x^N(t) \) jumps to zero at \( t = T^*_f \), and we effectively have to cut stage II in two periods: for \( t \in [T, T^*_f] \), (D.2) with \( x^N(t) \geq \tilde{y} \) is binding, whereas for \( t \in (T^*_f, T) \),

\[
x^A(t) = \delta Z. \tag{D.3}
\]

is binding. Denote the first of these two sub-stages by IIa and the second by IIb. Denote \( \theta^{IIa} \) the Lagrange multiplier to the restriction in (D.2) and \( \theta^{IIb} \) the Lagrange multiplier to the restriction in (D.3). We can associate a Lagrangian with each stage:

\[
\mathcal{L}^A(\cdot) = U(q^A(\cdot)) - \lambda^{III}_X(\cdot)x^A_1(\cdot) - cy(\cdot) + \lambda^{III}_M(\cdot) \left( r(\cdot)M^A(\cdot) + p(\cdot) \left( x^A(t) - x^A(t) \right) \right) - T^I I(t) \left( \sum x^I(t) - \delta Z(t) \right) + \mu^I(t) \left( Z - Z(t) \right) + \gamma^X_1(t)x^A(t) + \gamma^M_1(t)y^A(t) + V^O(X^A(O)(T), M^A(O)(T), Z), \tag{D.4}
\]

\[
\mathcal{L}^A(\cdot) = U(q^A(\cdot)) - \lambda^{IIIa}_X(\cdot)x^A_1(\cdot) - cy(\cdot) + \lambda^{IIIa}_M(\cdot) \left( r(\cdot)M^A(\cdot) + p(\cdot) \left( x^A(t) - x^A(t) \right) \right) + \theta^{IIa}(t) \left( \delta Z - x^N(t) - x^A(t) \right) + \gamma^X_1(t)x^A(t) + \gamma^M_1(t)y^A(t) + V^O(X^A(O)(T^*_b), M^A(O)(T^*_b), Z), \tag{D.5}
\]

\[
\mathcal{L}^A(\cdot) = U(q^A(\cdot)) - \lambda^{IIIb}_X(\cdot)x^A_1(\cdot) - cy(\cdot) + \lambda^{IIIb}_M(\cdot) \left( r(\cdot)M^A(\cdot) + p(\cdot) \left( x^A(t) - x^A(t) \right) \right) + \theta^{IIb}(t) \left( \delta Z - x^N(t) - x^A(t) \right) + \gamma^X_1(t)x^A(t) + \gamma^M_1(t)y^A(t) + V^O(X^A(O)(T_H), M^A(O)(T_H), Z), \tag{D.6}
\]

\[
\mathcal{L}^A(\cdot) = U(q^A(\cdot)) - \lambda^{III}(\cdot)x^A_1(\cdot) - cy(\cdot) + \lambda^{III}(\cdot) \left( r(\cdot)M^A(\cdot) + p(\cdot) \left( x^A(t) - x^A(t) \right) \right) - T^III(t) \left( x^A(t) - \delta Z(t) \right) + \mu^III(t) \left( Z - Z(t) \right) + \gamma^X_1(t)x^A(t) + \gamma^M_1(t)y^A(t), \tag{D.7}
\]

where \( V^O(X^A(O)(T), M^A(O)(T), Z) \) is the maximum value function for stages IIa to III, that is

\[
V^O(X^A(O)(T), M^A(O)(T), Z) = \int_T^{\infty} U(q^O(t))e^{-\rho t}dt,
\]

and similarly for the other \( V^O(\cdot) \) functions. In each stage, the stocks at the end of the preceding stage are taken as initial stocks. We are especially interested in what happens at \( t = T^*_b \). Following Theorem 7.2.1 of Léonard and Long (1992), the transversality condition for the stock of resources, for stage IIa, reads:

\[
\lambda^{IIIa}_X(T^*_b) = \frac{\partial V^O(\cdot)}{\partial X^A(T^*_b)}. \tag{D.8}
\]
For the stock of resource claims on the other region, we have

$$\lambda_M^{AIa}(T_b^N) = \frac{\partial V^O(\cdot)}{\partial M^A(T_b^N)}.$$ (D.9)

Following the derivation of equation (4.80) in Léonard and Long (1992), we find

$$\frac{\partial V^O(\cdot)}{\partial X^A(T_b^N)} = \lambda_M^{AIb}(T_b^N); \quad \frac{\partial V^O(\cdot)}{\partial M^A(T_b^N)} = \lambda_M^{AIb}(T_b^N).$$ (D.10)

Comparing these results with (D.8)-(D.9), we see that $\lambda_X^A$ and $\lambda_A^X$ are continuous at the border between stage IIa and IIb. Since $c \in (\xi, \bar{c})$, $x^N(t)$ jumps to zero at $t = T_b^N$ whereas $x^A(T_b^N)$ has to jump up, in order not to violate (20), which implies that $\theta(t)$ must jump down at $t = T_b^N$. For $\delta \bar{Z}$ or $c$ sufficiently low, ceteris paribus, $x^A(t)$ jumps up to $\delta \bar{Z}$ and $T_H > T_b^N$.

Now we can prove the proposition. Consumption has to be continuous at each point in time, except for $t = T_b^N$, as just shown. Then, given the definition of $\bar{c}$, the proof of part a follows the proof of Proposition 3a. The proof of part b follows the proof of Proposition 3b, whereby $c < c'$ implies that $T_C$ does not exist so $y^A(t)$ cannot go to zero continuously but jumps to zero at $T_b^N$ (see proof of Proposition 3). The proof of part c follows the proof of 3d. Part d has been shown above.

E Proof of Proposition 5

We need to find the sign of $dT_b^i / dx_0$. There will be a switch from consumption of non-renewable energy to energy from the backstop at $t = T_b^i$ such that $\lambda_X^A(0) e^{\rho T_b^i} = c$, from which follows that $T_b^i = (\ln(c/\lambda_X^A(0)))/\rho$, given the optimal initial scarcity rent $\lambda_X^A(0)$. The latter has to satisfy

$$\int_0^{T_b^i} x^i(t) dt = X_0 \Rightarrow \int_0^{T_b^i} \ln(\frac{\lambda_X^A(t)}{\lambda_X^A(0)}) U(t) dt = X_0 = 0. \quad (E.1)$$

Define the latter expression as the implicit function $F(\cdot)$. Then

$$\frac{d\lambda_X^A(0)}{dx_0} = -\frac{\partial F/\partial x_0}{\partial F/\partial \lambda_X^A(0)} = \left( -\frac{1}{\rho} \frac{1}{\lambda_X^A(0)} U^{-1}(c) + \int_0^{T_b^i} \ln(\frac{x^i(t)}{\lambda_X^A(0)}) e^{\rho t} \frac{1}{U''(x^i(t))} dt \right) < 0, \quad (E.2)$$

since $U'' < 0$. From $T_b^i = (\ln(c/\lambda_X^A(0)))/\rho$ we find

$$\frac{dT_b^i}{dx_0} = -\frac{1}{\rho \lambda_X^A(0)} < 0, \quad (E.3)$$

so $dT_b^i / dx_0 > 0$. This proves part b of the proposition.

The growth rate of consumption is as in (23). Since $\lambda_X^A(0)$ is higher than under laissez-faire, $x^A(t) < x^i(t) = x^N(t)$ $\forall t$, which proves part a.
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