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## Testing the bivariate mixture hypothesis using German stock market data

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**Wirtschaftswissenschaftliche Fakultät  
der Eberhard-Karls-Universität Tübingen**

**Testing the Bivariate Mixture Hypothesis  
Using German Stock Market Data**

Robert Jung und Roman Liesenfeld



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der Eberhard-Karls-Universität Tübingen

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**Using German Stock Market Data**

Robert Jung und Roman Liesenfeld

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### **Abstract**

According to the bivariate mixture hypothesis (BMH) as proposed by Tauchen and Pitts (1983) and Harris (1986,1987) the daily price changes and the corresponding trading volume on speculative markets follow a joint mixture of distributions with the unobservable number of daily information events serving as the mixing variable. Using German stock market data of 15 major companies the distributional properties of the BMH is tested employing maximum-likelihood as well as generalized method of moments estimation techniques. In addition to providing a new approach for the pointwise estimation of the latent information arrival rate based on the maximum-likelihood method, we investigate the time-series properties of the BMH. The major results can be summarized as follows: (i) the distributional characteristics of the data (esp. leptokurtosis and skewness in the distribution of price changes and volume respectively) cannot be explained satisfactorily by the BMH; univariate mixture models for price changes and trading volume separately reveal a possible specification error in the model; (ii) a univariate normal mixture model can account for the observed distributional characteristics of price changes; (iii) the estimated process of the latent information rate cannot fully explain the time-series characteristics of the data (esp. the volatility clustering or ARCH-effects).

## 1. Introduction

This paper contributes to the rapidly growing field in empirical finance that is concerned with the modelling of highly nonlinear dependencies in daily price change data from speculative markets. In numerous studies for various types of speculative markets, it has been shown that the empirical distribution of daily price changes is leptokurtic, meaning that it is excessively peaked around zero and that it exhibits fatter tails than the normal distribution. An appealing explanation for this departure from the shape of a Gaussian distribution was provided by Clark (1973). In his model the variance of the daily price change is determined by the number of price relevant information arriving in the market. If the information arrives randomly from day to day, the daily price changes are realizations of random variables with different variances. A mixture of (normal) distributions with different variances serves consequently as a suitable statistical model for the distribution of the price changes.

In order to obtain additional information about the pricing process Tauchen and Pitts (1983), Harris (1986, 1987) and Andersen (1994) incorporate the quantitative side of the market into the class of mixture models. Using basically the same assumptions as Clark they proposed a joint distribution for daily price changes and daily trading volume, resulting in a mixture of bivariate distributions. The decisive point is the fact that price changes and trading volume are endogenous variables, simultaneously directed by the exogenous information arrival process. Tauchen and Pitts as well as Harris (1987) used a simple sequential Walrasian equilibrium framework to derive a bivariate normal distribution for the trading volume and the price changes conditional on the daily number of information. Andersen developed a market microstructure model with liquidity trading and asymmetric information resulting in a Poisson distribution for the trading volume conditional on the number of information events. The theoretical implications of these bivariate mixture models are successful in accounting for the observed leptokurtosis in the distribution of price changes as well as in explaining the positive skewness in the distribution of volume and the documented positive contemporaneous correlation between trading volume and the volatility of price changes<sup>1</sup>.

A parallel approach which is also consistent with a fat tailed empirical distribution of price changes, is based on the autoregressive conditionally heteroscedastic (ARCH) model of Engle (1982) and Bollerslev's (1986) generalized ARCH (GARCH) model. In this class of models the conditional variance of price changes is a simple function of past information contained in previous price changes. An autoregressive structure in the variance specification of price changes allows for the persistence of volatility shocks, enabling the model to capture the frequently observed clustering of similar-sized price changes, the so-called ARCH-effects. In the search of the origin of these ARCH-effects in financial price change series Lamoureux and Lastrapes (1990), Locke and Sayers (1993) and Bauer, Nieuwland and Verschoor (1994) analyze whether these effects can be attributed to a corresponding time series behavior of the information arrival process in Clark's mixture model. Inserting the contemporaneous trading volume as a proxy for the unobservable information arrival

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<sup>1</sup>See Karpoff (1987) for a survey of the literature dealing with the relationship between price changes and trading volume.

rate in the conditional variance specification of (G)ARCH models shows that this variable has significant explanatory power regarding the variance of the price changes and that previous price changes contain negligible additional information about the variance of the price changes if volume is included. A disadvantage of this procedure is the fact that it treats volume as an exogeneous variable. But if volume and price change volatility are in fact jointly determined, a specification ignoring this fact possibly suffers from a simultaneity bias<sup>2</sup>. We therefore argue along the line of Andersen (1994) and Lamoureux and Lastrapes (1994) that it seems to be necessary to analyze the origin of ARCH-effects in a setting, in which trading volume is treated as an endogenous variable. This is the case in the bivariate mixture model of Tauchen and Pitts and consequently this approach serves as a framework for our investigation.

The main purpose of this study is to evaluate whether the bivariate mixture hypothesis (BMH) in the Tauchen-Pitts-Harris specification is able to capture the observed distributional patterns of daily price changes and trading volume for the German stock market data and whether the time series behavior of the latent information arrival process in this model can be regarded as the origin of the ARCH-effects. The paper is organized as follows: In section 2 the BMH is presented together with its implications for the *unconditional* moments of the price changes and trading volume. Section 3 briefly describes the data used in this study. In section 4 the distributional aspects of the BMH are discussed. Results of implied maximum likelihood (ML) estimates of the unconditional moments of the mixture model are presented (section 4.1) together with an explicit specification test of the BMH based on the generalized method of moments (GMM) procedure (section 4.2). Section 4.3 deals with univariate mixture models for trading volume and price changes separately in order to detect possible sources of misspecification in the bivariate setting. In section 5 the time series properties of the BMH are investigated. For this purpose the latent information arrival rate is estimated on basis of the joint distribution of price changes and trading volume with the ML-method. Section 6 contains the conclusions.

## 2. The Bivariate Mixture Hypothesis

Using the model developed primarily by Tauchen and Pitts it is possible to specify the joint distribution of the daily price changes and the corresponding volume of trading in financial markets. Given a set of information the traders demand for a single security is a linear function of the difference between their reservation prices and the current market price. Within the day a series of events take place, each event generates information with direct relevance for the pricing of the security. Every piece of such price relevant information leads the traders to adjust their reservation prices. Applying the equilibrium condition the resulting change in the market clearing price is the average of all traders reservation price changes.

If the reservation price changes are assumed to possess a distribution which is independent and identical with respect to all traders and with respect to all events and is

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<sup>2</sup>Lamoureux and Lastrapes (1990) are very well aware of this problem.

symmetric around zero it is possible to show that the price change and the trading volume corresponding to an event are uncorrelated. Furthermore it follows that the post event price changes and volumes are realizations of random variables that are independently and identically distributed for all events<sup>3</sup>. The total price change ( $dP_t$ ) and the total trading volume ( $V_t$ ) for a given day  $t$  is the sum over within-day price changes and trading volumes each of which occurs as a result of a new informational event. For a sufficiently large number of events on day  $t$  the joint distribution of daily price changes and trading volume follow approximately a bivariate normal distribution conditional on the daily number of events  $I_t$

$$\begin{pmatrix} dP_t | I_t \\ V_t | I_t \end{pmatrix} \sim N \left( \begin{bmatrix} \mu_{dp} I_t \\ \mu_v I_t \end{bmatrix}; \begin{bmatrix} \sigma_{dp}^2 I_t & 0 \\ 0 & \sigma_v^2 I_t \end{bmatrix} \right), \quad (1)$$

with  $\sigma_{dp} > 0$ ,  $\mu_v > 0$ ,  $\sigma_v > 0$ .

The number of daily information arrivals  $I_t$  is assumed to be a random variable. It follows then, that the *unconditional* joint distribution of  $dP_t$  and  $V_t$  is a mixture of independent bivariate normals with  $I_t$  serving as the mixture variable. This is the reason, why (1) is called the bivariate mixture model (BMM).

The implications for the *unconditional* moments of price changes and trading volume from the bivariate mixture model can be summarized as follows<sup>4</sup>:

- The excess kurtosis of the marginal *unconditional* distribution of the price changes ( $\kappa(dP)$ ) is positive and given by

$$\kappa(dP) = \frac{3\sigma_{dp}^4(\mu^2 + m_2) + 6\mu_{dp}^2\sigma_{dp}^2(m_3 + m_2) + \mu_{dp}^4 m_4}{(\sigma_{dp}^2\mu + \mu_{dp}^2 m_2)^2} - 3, \quad (2)$$

where  $\mu, m_2, m_3$  and  $m_4$  denote the mean, the second, the third and the fourth central moment of the mixing variable  $I_t$  respectively. If the mean of the price changes is approximately equal to zero ( $\mu_{dp} \approx 0$ ) the right hand side of (2) reduces to  $3m_2/\mu^2$  which is clearly positive.

- If the distribution of the mixing variable is skewed to the right (i.e.  $m_3 \geq 0$ ) the *unconditional* distribution of  $V$  is also skewed to the right with the skewness coefficient

$$\varsigma(V) = \frac{3\mu_v\sigma_v^2 m_2 + \mu_v^3 m_3}{(\sigma_v^2\mu + \mu_v^2 m_2)^{3/2}}. \quad (3)$$

Clearly  $\varsigma(V) > 0$  as long as  $m_3 \geq 0$ .

- The *unconditional* covariance between the squared price changes and the trading volume serves as a measure for the price change volatility-volume relationship and is given by

$$\text{Cov}(dP_t^2, V_t) = \mu_v \sigma_{dp}^2 m_2 + \mu_v \mu_{dp}^2 (m_3 + 2\mu m_2). \quad (4)$$

The right hand side of (4) is positive and equal to  $\mu_v \sigma_{dp}^2 m_2$  as long as  $\mu_{dp} \approx 0$ .

<sup>3</sup>For further details see Tauchen and Pitts (1983) pp. 490-491 and Harris (1987) p. 129.

<sup>4</sup>For a complete description of the predictions of the BMM regarding conditional and unconditional moments of price changes and trading volume see Harris (1987).

- Furthermore the BMM implies a positive first-order autocovariance of squared price changes if the latent information arrival rate  $I_t$  has a positive first-order autocovariance as well. This is due to the fact that

$$\text{Cov}(dP_t^2, dP_{t-1}^2) = \sigma_{dp}^4 \text{Cov}(I_t, I_{t-1}). \quad (5)$$

Thus the observed persistence in price change volatility may well be the result of a corresponding time series behavior of the latent mixture variable.

The remainder of this study is devoted to answering two important questions: are these theoretical predictions which are only qualitative in nature reconfirmed by the German stock market data and is the BMM in the Tauchen-Pitts-Harris specification capable of fully capturing the empirical regularities inherent in the data. The former is relatively easy to show while the latter is of considerable complexity, thus requiring intensive treatment.

### 3. Data Description

Our data set consists of daily closing prices ( $P_t$ ) and daily numbers of shares traded for 15 German stocks listed in the DAX index<sup>5</sup> and were obtained from the Karlsruher Kapitalmarktdatenbank (KKMDB). The sample period starts on 2.01.1990 and ends on 31.5.1994 giving a sample size of over 1100. We adjusted the data for effects of dividends and capital changes. In order to make our results comparable to other studies we did not use the simple price difference  $P_t - P_{t-1}$  but the log-differences  $\log P_t - \log P_{t-1}$  instead.

We detrended the data using a procedure originally proposed by Gallant, Rossi and Tauchen (1992) for two reasons: firstly we are not interested in explaining the long-run behavior of price volatility and trading volume but rather the day to day movements and we wanted to remove a possible non-stationarity in the data, because estimation and testing procedures to be employed subsequently rely on the stationarity assumption. For the volume data this procedure essentially comprises of the following three steps: In a first step volume is regressed on a linear and a quadratic time trend. In a second step the logarithm of the resulting residuals are again regressed on a linear and a quadratic time trend. The residuals obtained from this second regression are finally transformed in order to ensure that the adjusted data's mean and variance is exactly the same as that of the unadjusted data. Essentially this procedure is designed to remove a possible significant trend both in the mean (step one) and in the variance (step two) of the data. For the price change series it is only necessary to remove a possible trend in the variance and to do the linear transformation. It can reasonably be assumed that after detrending price changes and trading volume in this way that both series are stationary<sup>6</sup>.

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<sup>5</sup>The DAX index is the leading German stock market index comprising of 30 stocks of major German companies. Appendix I gives a list of the 15 companies included in this study together with the corresponding abbreviations.

<sup>6</sup>Unlike Gallant, Rossi and Tauchen (1992) we do not correct the data for the well documented systematic calendar effects such as the January effect, the Monday effect and the weekend effect. In the framework of the bivariate mixture model these effects may be viewed as the results of a specific behavior of the information arrival process and should therefore not be removed from the data.



The summary statistics for the detrended data are given in Table 1. It is already well known that all the empirical regularities found in financial markets data by many other studies can be found in the German stock market data as well. Concentrating on the important higher moments one can see that the excess kurtosis of the empirical distribution of the price changes  $\hat{\kappa}(dP)$  exceeds the value of zero for all 15 stocks. The Kiefer-Salmon kurtosis statistic  $K_{dp}$  to test for excess kurtosis as well as the Kiefer-Salmon joint test statistic  $SK_{dp}$  to test normality against skewness and/or excess kurtosis are both given in the table<sup>7</sup>. Both statistics lead to a rejection of the null hypothesis at any reasonable significance level for all 15 stocks in our study. In order to detect ARCH effects in the price change series the Box-Pierce statistic  $Q_{dp^2}(20)$  of squared price changes are calculated for 20 lags<sup>8</sup>. The results indicate strong persistence in the volatility of the price changes for nearly all stocks.

For the volume data we find a positive empirical skewness coefficient  $\hat{\zeta}(V)$  as well as a highly significant Kiefer-Salmon skewness statistic  $S_v$  for all 15 stocks<sup>9</sup>.

Finally, for all cases there is a significant positive contemporaneous relationship between price change volatility and trading volume. As a descriptive measure the correlation coefficient between squared price changes and trading volume  $\rho(dP_t^2, V_t)$  is presented. Its significance is proved using Fisher's Z-test. All these statistics show that the price change and volume data exhibit properties, which are consistent with the theoretical predictions of the BMH described in section 2.

## 4. Empirical Results Concerning the Distributional Aspects of the BMM

### 4.1. Maximum-Likelihood Estimation

Following the suggestion of Tauchen and Pitts (1983) we estimate the parameters of the BMM (1) using the maximum-likelihood method. This is only possible if a distributional assumption concerning the unobservable mixing variable  $I_t$  is introduced. Since the log-normal distribution has been employed quite successfully in several other studies<sup>10</sup> we adopt this specification.

Once the distribution of the information rate  $f_I(I_t)$  is specified the latent variable  $I_t$ , which enters the model in a highly nonlinear fashion, can be integrated out of the mixture

<sup>7</sup>See Kiefer and Salmon (1983) for the derivation of both test statistics and the proofs that under the null hypothesis of a normal distribution for the price changes  $K_{dp}$  follows a chi-square distribution with one degree of freedom and the  $SK_{dp}$  statistic follows a chi-square distribution with two degrees of freedom.

<sup>8</sup>Under the iid hypothesis  $Q_{dp^2}(20)$  is asymptotically  $\chi^2_{(20)}$  distributed.

<sup>9</sup> $S_v$  follows a  $\chi^2_{(1)}$  distribution under the null hypothesis.

<sup>10</sup>Clark (1973), Tauchen and Pitts (1983) as well as Richardson and Smith (1994) have tried other distributions like the Poisson and the inverted gamma, but in all studies find strong evidence in favor of the lognormal. Furthermore, the lognormality of  $I_t$  implies a lognormal price change variance, which is in line with diffusion models for a stochastic volatility variable (see e.g. Taylor (1994).)

model (1) giving us an *unconditional* joint distribution of the observable variables  $dP_t$  and  $V_t$

$$f_{dP,V}(dP_t, V_t) = \int_I f_{dP,V}(dP_t, V_t | I_t) \cdot f_I(I_t) dI_t. \quad (6)$$

Since conditional on the mixing variable price changes and the corresponding trading volume are stochastically independent their joint distribution conditional on  $I_t$  is given by

$$f_{dP,V}(dP_t, V_t | I_t) = f^N(dP_t : \mu_{dp} I_t, \sigma_{dp}^2 I_t) \cdot f^N(V_t : \mu_v I_t, \sigma_v^2 I_t), \quad (7)$$

where  $f^N(\cdot)$  denotes the density of a normal distribution.

It should be emphasized at this stage of the analysis that within the mixture model framework the only source of autocorrelation in the volatility of the price changes is a possible serial dependence in the latent variable  $I_t$ , as already derived in (5). Application of the ML-method to the *unconditional* joint distribution of  $dP_t$  and  $V_t$  after integrating out the latent mixing variable  $I$  as given in (6) however makes it impossible to distinguish between an uncorrelated and an autocorrelated rate of information arrival<sup>11</sup>. From this perspective the distribution  $f_I(I_t)$  can be regarded as the *unconditional* distribution resulting from *conditional* distributions of  $I_t$  given the history of the series  $f_I(I_t | I_{t-1}, I_{t-2}, \dots)$ . This leads to inefficient but probably consistent parameter estimates, if  $I_t$  is stationary over time.

Due to the latent character of the mixing variable, the bivariate mixture model is invariant with respect to an arbitrary transformation of the scale of  $I_t$ . By setting its mean denoted by  $\mu$  to one, the model can be normalized and the following parameters can be estimated and identified:  $\tilde{\mu}_{dp} = \mu_{dp} \mu$ ,  $\tilde{\sigma}_{dp} = \sigma_{dp} \sqrt{\mu}$ ,  $\tilde{\mu}_v = \mu_v \mu$ ,  $\tilde{\sigma}_v = \sigma_v \sqrt{\mu}$  and  $\theta$ , the parameter characterizing the variation in the rate of information arrivals<sup>12</sup>.

The resulting log-likelihood function is of the following form:

$$\log L(\tilde{\mu}_{dp}, \tilde{\sigma}_{dp}, \tilde{\mu}_v, \tilde{\sigma}_v, \theta | dP_t, V_t) = \sum_{t=1}^T \log [f_{dP,V}(dP_t, V_t | \cdot)], \quad (8)$$

where

$$\begin{aligned} f_{dP,V}(dP_t, V_t | \cdot) &= \int_{-\infty}^{+\infty} [2\pi\tilde{\sigma}_{dp}^2 \exp(\theta z - \theta^2/2)]^{-\frac{1}{2}} \exp \left\{ -\frac{[dP_t - \tilde{\mu}_{dp} \exp(\theta z - \theta^2/2)]^2}{2\tilde{\sigma}_{dp}^2 \exp(\theta z - \theta^2/2)} \right\} \\ &\times [2\pi\tilde{\sigma}_v^2 \exp(\theta z - \theta^2/2)]^{-\frac{1}{2}} \exp \left\{ -\frac{[V_t - \tilde{\mu}_v \exp(\theta z - \theta^2/2)]^2}{2\tilde{\sigma}_v^2 \exp(\theta z - \theta^2/2)} \right\} \\ &\times (2\pi)^{-\frac{1}{2}} \exp(-\frac{z^2}{2}) dz. \end{aligned}$$

The log-likelihood function is made up of  $T$  infinite integrals one for each observation pair of  $dP_t$  and  $V_t$ . There is no way to arrive at a closed form solution for these integrals. Consequently the ML-estimation has to be based on numerical integration procedures<sup>13</sup>.

<sup>11</sup>Note that when applying the ML-method it is impossible to take into account a serial dependence of the variable  $I_t$ , since  $I_t$  is latent.

<sup>12</sup>The parameter  $\theta$  is related to the coefficient of variation of  $I_t$  in the following way:  $\theta = [\ln(m_2/\mu^2 + 1)]^{1/2}$ .

<sup>13</sup>We used a 20 point Gauss-Hermite quadrature to compute the integrals. The optimization itself was carried out using the GAUSS procedure MAXLIK. Both Newton-Raphson and DFP algorithms were alternatively applied to crosscheck the results. Method of moment estimators served as starting points of the iteration process. To ensure global concavity, different starting values have been used as well.

The results of the ML-estimation of the parameters of the normalized mixture model are presented in table 2. These estimates are of the expected sign and seem to be reasonable. With ML-estimates of the parameter at hand it is possible to calculate the implied estimates of the *unconditional* moments of  $dP_t$  and  $V_t$ , which can be compared with their corresponding sample moments given in table 1. The necessary formulas to compute the implied moments are given in Appendix II and the results are presented in table 3. Since it is not possible to conduct a specification test of the mixture model using the ML-results directly<sup>14</sup>, the comparison between the results in table 3 and table 1 serves as a first check of our specification. While the difference between estimates moments and sample moments for means as well as standard deviations of both price changes and trading volume are almost negligible for higher order moments they are often substantial. While the estimated excess kurtosis of price changes  $\kappa(dP)$  is positiv for all stocks in our sample it is quite below their sample counterparts. The same applies to the estimated skewness of the trading volume  $\varsigma(V)$ . In the last column of table 3 the implied estimates of the *unconditional* correlation coefficient between squared price changes and trading volume is given. It is positive in every case and for almost all stocks quite close to the sample correlation as given in table 1.

To illustrate the downward bias of the implied estimates of the excess kurtosis compared to the sample value for the price change variable, a graph for the (randomly choosen) BMW stock of the empirical frequency distribution of price changes and the corresponding fitted marginal *unconditional* density resulting from the ML-estimates of the BMM is presented in figure 1. The location and the scale of the empirical distribution is fairly well approximated by the fitted density, but there are noteable problems in the tails of the distribution. The empirical distribution exhibits data points in the extreme tail areas, which are not captured by the fitted distribution. It should be noted however that the empirical excess kurtosis is dominated by a very few data points (two or three) in the very extrem tail areas<sup>15</sup>. This demonstrates the downward bias of the implied estimates of the excess kurtosis compared to the sample value. Figure 2 gives the empirical and the estimated marginal distribution for the trading volume. The observed strong positive skewness in the volume data can be approximated quite well by the estimated distribution. It is also remarkable that the fitted density tends to be polymodal a result of fitting small clusters of data points by mixing normal distributions with different means. To summarize the results of the ML-estimates one can conclude that the BMM (1) has shortcomings as far as the adequate description of the distributional patterns of the *unconditional* joint distribution of trading volume and price changes is concerned.

## 4.2. Generalized Method of Moments Estimation

So far we did not present a rigorous specification test of the BMM. As mentioned above, such a test based on the likelihood principle is not readily available. Hence, following a

<sup>14</sup>For the information matrix test proposed by White (1980) for example, the third derivatives of the likelihood function are needed. Since we had to rely on numerical approximations of these functions the third derivatives are not available.

<sup>15</sup>One of these extrem values for nearly all stocks is reported for the day of the Gorbachev putsch in August 1991 with negative returns of up to 10 percent.

recent proposal by Andersen (1994) and Richardson and Smith (1994), we applied the J-test, based on Hansen's (1982) Generalized Method of Moments (GMM) procedure, in order to test the adequacy of the model specification. The test requires only weak assumptions like stationarity and ergodicity of the price changes and volume series. It rests on the idea that the mixture model places certain testable restrictions on the unconditional moments of  $dP_t$  and  $V_t$ . The hypothesis of a correct model specification will be rejected if at least some of the moment restrictions are systematically violated. In a way the test compares the implied moment estimates with the corresponding sample moments. We have used a similar but more heuristical approach based on the ML-estimates above. Exploiting the fact that given the specification is correct, the selected sample moments converge in probability to the corresponding (*unconditional*) expected values resulting from the underlying model.

A critical step in the practical application of the generalized method of moments is the choice of a set of moment restrictions. For our study we focused on those moment restrictions which reflect the typically mentioned stylized facts characterizing the data on stock markets<sup>16</sup>. Thus the set of moment restrictions is made up of the first four *unconditional* moments of  $dP_t$ , the first three moments of  $V_t$  and the *unconditional* covariance between  $dP_t^2$  and  $V_t$  resulting from the BMM (1) and the lognormality assumption for the mixing variable<sup>17</sup>. After normalizing the mixture model by setting  $\mu = 1$  we employed the following set of moment restrictions:

$$\begin{aligned}
E[dP_t] &= \tilde{\mu}_{dp} \\
E[V_t] &= \tilde{\mu}_v \\
E[(dP_t - \tilde{\mu}_{dp})^2] &= \tilde{\sigma}_{dp}^2 + \tilde{\mu}_{dp}^2 \tilde{m}_2 \\
E[(V_t - \tilde{\mu}_v)^2] &= \tilde{\sigma}_v^2 + \tilde{\mu}_v^2 \tilde{m}_2 \\
E[(dP_t - \tilde{\mu}_{dp})^3] &= 3\tilde{\mu}_{dp}\tilde{\sigma}_{dp}^2\tilde{m}_2 + \tilde{\mu}_{dp}^3(\tilde{m}_2^3 + 3\tilde{m}_2^2) \\
E[(V_t - \tilde{\mu}_v)^3] &= 3\tilde{\mu}_v\tilde{\sigma}_v^2\tilde{m}_2 + \tilde{\mu}_v^3(\tilde{m}_2^3 + 3\tilde{m}_2^2) \\
E[(dP_t - \tilde{\mu}_{dp})^4] &= 3\tilde{\sigma}_{dp}^4(1 + \tilde{m}_2) + 6\tilde{\mu}_{dp}^2\tilde{\sigma}_{dp}^2(\tilde{m}_2^3 + 3\tilde{m}_2^2 + \tilde{m}_2) \\
&\quad + \tilde{\mu}_{dp}^4(6(1 + \tilde{m}_2) + (1 + \tilde{m}_2)^6 - 3 - 4(1 + \tilde{m}_2)^3) \\
E[(dP_t^2 - (\tilde{\mu}_{dp}^2(\tilde{m}_2 + 1) + \tilde{\sigma}_{dp}^2))(V_t - \tilde{\mu}_v)] &= \tilde{\sigma}_{dp}^2\tilde{\mu}_v\tilde{m}_2 + \tilde{\mu}_{dp}^2\tilde{\mu}_v(\tilde{m}_2^3 + 3\tilde{m}_2^2 + 2\tilde{m}_2) .
\end{aligned} \tag{9}$$

The vector of unknown parameters is given by  $(\tilde{\mu}_{dp}, \tilde{\sigma}_{dp}, \tilde{\mu}_v, \tilde{\sigma}_v, \tilde{m}_2)$ , where  $\tilde{m}_2 = m_2/\mu^2 = \exp(\theta^2) - 1$ . Thus the system of moment restrictions consists of eight equations and five parameters leaving us with three over-identifying restrictions to be tested. The parameters are estimated by minimizing a quadratic form of the distance between the sample moments and their theoretical counterparts over the parameter space<sup>18</sup>. Hansen's J-statistic

<sup>16</sup>In principle there is an infinite number of moment restrictions resulting from the bivariate normal mixture model, which could be used.

<sup>17</sup>For the ML-estimation a distributional assumption for  $I_t$  is inevitably necessary, whereas for the GMM-procedure it is not. In contrast to the studies by Andersen and Richardson/Smith we explicitly include moment restrictions resulting from the lognormality assumption into our model since we are interested in getting results, which are comparable to those of the ML-estimations.

The resulting additional restrictions concerning the standardized higher moments of  $I_t$  are given in Appendix II.

<sup>18</sup>To get consistent but inefficient GMM-estimates of the parameter vector in a first step the identity matrix is used as the weighting matrix in the quadratic form. In the second step we employed the Newey

based on three over-identifying restrictions is under the null-hypothesis of a correct model specification asymptotically  $\chi^2$ -distributed with three degrees of freedom.

Table 4 presents the results of the GMM-estimation. Several points are noteworthy. The parameter values are very close to the estimates obtained from the ML-estimation, the only exception being the  $\tilde{\sigma}_v$ -parameter. For four stocks (BAS, BAY, HFA, MMW) the estimates of this parameter are zero indicating as noted by Lamoureux and Lastrapes (1994), that the GMM estimator for  $\tilde{\sigma}_v$  may contain little information about the true value and should not be regarded as an evidence against the model<sup>19</sup>. The estimated standard errors of the parameters are notably larger than the standard errors obtained from the ML-estimation. This is especially true for the  $\tilde{\sigma}_v$  parameter and the  $\tilde{m}_2$  parameter. This result is not surprising given the fact that the ML-estimation rests on the entire information about the assumed shape of the *unconditional* joint distribution of  $dP$  and  $V$ , whereas the GMM procedure is based only on certain aspects of the distributional assumption in form of the selected moment restrictions. Therefore less a priori information is taken into account by the GMM estimation and it is therefore less efficient.

The result of the J-test is given in columns seven and eight of table 4. Choosing a level of significance of 5-percent for five out of the fifteen stocks contained in our study the null hypothesis of a correct model specification has to be rejected. We conclude from this that the bivariate mixture model in the specification used in this study is able to capture the distributional properties of price change and trading volume series from the German stock market only to a limited extent. It should however be noted that the obtained result is only valid given the selected set of *unconditional* moments in (9). Choosing another set of moment restrictions would shift the scope of the test away from those aspects of the model that we are interested in (kurtosis of the price changes, skewness of the trading volume and correlation between volatility of price changes and volume). Among other things this would imply the incorporation of moments of a growing order. Since the influence of extreme data points and outliers on higher order moments can be very drastic, the thus obtained results would have to be interpreted with great caution. That it is not a sensible strategy to incorporate all possible moment restrictions is also the result of an extensive Monte Carlo study conducted by Andersen and Sørensen (1994). For an univariate stochastic volatility model the authors are able to show that there exists a trade off between the number of moments incorporated and the quality of the objective function used in the GMM procedure for estimation and testing purposes<sup>20</sup>. The optimal number and the optimal set of moment restrictions for the bivariate mixture model still has to be developed, but this is left for future work.

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and West (1987) matrix as a consistent estimate of the optimal weighting matrix. To determine the bandwidth parameter we followed Andrews (1991), who proposed a data dependent choice procedure based on univariate AR(1) estimations for each moment restriction separately.

<sup>19</sup>In a small Monte Carlo study whose design and results is not reproduced here we were able to show that whenever the true value of  $\tilde{\sigma}_v$  is much smaller than the value of  $\tilde{\mu}_v$  a considerable share of  $\tilde{\sigma}_v$ - estimates exhibits a value of zero, even if the true value is indeed positive. This simulation result is reflected in the fact that for the four stocks with a zero GMM-estimate of  $\tilde{\sigma}_v$  the magnitude of ML-estimates of  $\tilde{\sigma}_v$  is small relative to the value of ML-estimates of  $\tilde{\mu}_v$ .

<sup>20</sup>In our study we also used several other sets of moment restrictions. Our results indicate that higher order moments (e.g. fourth moment of  $V_i$  or higher cross products of  $dP_i$  and  $V_i$ ) have a strong tendency to dominate the results obtained.

### 4.3. Two Univariate Mixture Models

In the BMM the conditional marginal distributions of price changes and trading volume are given by two normal distributions respectively. The means and variances of these distributions are directly proportional to the same mixing variable. Therefore price changes and volume depend in a symmetric fashion on the information arrival rate. As noted by Richardson and Smith (1994) one possible interpretation of the shortcomings of the bivariate mixture model may be that the symmetric dependence of the two observable variables on the same mixing variable is not the correct specification even if a mixture model is adequate. Thus one way to analyze the reasons for the misspecification is to look at the unconditional marginal distributions of  $dP_t$  and  $V_t$  separately. From (1) it follows that these distributions can be written as

$$\begin{aligned} f_{dP}(dP_t) &= \int_I f_{dP}^N(dP_t | I_t) \cdot f_I(I_t; \theta) dI_t \\ f_V(V_t) &= \int_I f_V^N(V_t | I_t) \cdot f_I(I_t; \theta) dI_t. \end{aligned} \quad (10)$$

This specification allows us to separately test the adequacy of the normal mixture hypothesis for price changes and trading volume. Furthermore, it should be noted that both price changes and trading volume are still linked to the same mixing variable  $I_t$ . But estimating the two mixture models separately allows us to abstract from the assumption of a symmetric impact of  $I_t$  on  $dP_t$  and  $V_t$ . According to the BMM the estimated values for the parameter  $\theta$  measuring the standardized variation in the daily number of price relevant informations should not differ systematically in the two univariate models.

Again using the lognormality assumption for the mixing variable (i.e.  $I_t \sim \text{LN}(\theta)$ ) as in the bivariate specification above, the log-likelihood function for the price changes has the following form:

$$\log L(\tilde{\mu}_{dp}, \tilde{\sigma}_{dp}, \theta_{dp} | dP_t) = \sum_{t=1}^T \log[f_{dP}(dP_t | \cdot)], \quad (11)$$

where

$$\begin{aligned} f_{dP}(dP_t | \cdot) &= \int_{-\infty}^{+\infty} \left[ \left( 2\pi\tilde{\sigma}_{dp}^2 \exp\{\theta_{dp} \cdot z - \theta_{dp}^2/2\} \right)^{-\frac{1}{2}} \times \right. \\ &\quad \left. \times \exp\left\{ -\frac{(dP_t - \mu_{dp} \cdot \exp\{\theta_{dp} \cdot z - \theta_{dp}^2/2\})^2}{2\tilde{\sigma}_{dp}^2 \exp\{\theta_{dp} \cdot z - \theta_{dp}^2/2\}} \right\} \right] \times \left[ (2\pi)^{-\frac{1}{2}} \exp\left\{ -\frac{z^2}{2} \right\} \right] dz. \end{aligned}$$

For the trading volume  $V_t$  the corresponding log-likelihood function  $\log L(\tilde{\mu}_v, \tilde{\sigma}_v, \theta_v | V_t)$  is of the same form<sup>21</sup> as (11). The structure of the likelihood problem remains the same as compared to the bivariate form of (8) and therefore the same remarks concerning the estimation procedure apply here as well.

The results of the univariate mixture models are summarized in table 5. Compared to the results of the bivariate model the parameter estimates of  $\tilde{\mu}_{dp}, \tilde{\sigma}_{dp}, \tilde{\mu}_v, \tilde{\sigma}_v$  are almost

<sup>21</sup>For the parameter  $\theta$  we introduce the subscripts (v) and (dp) respectively in order to differentiate between the estimation results for the two univariate models.

identical. This is not the case for the parameter  $\theta$ . The univariate specification reveals quite a distinct effect of  $I_t$  on the  $dP_t$  and  $V_t$ . Since the estimated values of  $\theta_{dp}$  are significantly larger for all stocks than the estimated values of  $\theta_v$  it seems that far more variation in the mixing variable is needed to explain the stochastic behavior of price changes than that of trading volume. This can be interpreted as a further hint for a possible misspecification of the BMM. Furthermore in the estimation of the bivariate model the variation of trading volume dominates the estimation of  $\theta$  as can be seen from the fact that  $\theta_v$  in the marginal model is in contrast to  $\theta_{dp}$  very close to  $\theta$  in the bivariate model.

In order to investigate the adequacy of the univariate specifications we calculated the implied *unconditional* moments resulting from the ML-estimates of the parameters of the univariate models. Comparing the sample moments with these estimated unconditional moments (see table 6) reveals that with respect to the excess kurtosis in  $dP_t$  the univariate specification leads to significantly better predictions as compared to the bivariate model for all 15 stocks. The reason for this improvement stems from the fact that the estimated  $\theta_{dp}$  values almost solely determines the excess kurtosis of  $dP_t$  through  $\kappa(dP) \approx 3 [\exp(\theta_{dp}^2) - 1]$  (as long as  $\tilde{\mu}_{dp} \approx 0$ ). For the BAY stock, for example the value of the  $\theta_{dp}$  is more than twice as high as compared to the bivariate estimate of  $\theta$ . This leads to an increase in the explained excess kurtosis of  $dP_t$  from 0.565 to 3.591 which is sufficiently close to the sample value of 3.730. As shown in table 6, the estimated *unconditional* moments of the trading volume do not differ substantially from those of the bivariate case. This is due to the fact, that the estimates for  $\theta_v$  are very close to the estimates of  $\theta$  in the bivariate model.

As in the bivariate case we applied Hansen's J-test to test the mixture of distribution model along with the lognormality assumption for  $I_t$  on the two univariate models. For this purpose we selected the following set of moment restrictions for  $dP_t$ :

$$\begin{aligned}
E[dP_t] &= \tilde{\mu}_{dp} \\
E[(dP_t - \tilde{\mu}_{dp})^2] &= \tilde{\sigma}_{dp}^2 + \tilde{\mu}_{dp}^2 \tilde{m}_2 \\
E[(dP_t - \tilde{\mu}_{dp})^3] &= 3\tilde{\mu}_{dp}\tilde{\sigma}_{dp}^2\tilde{m}_2 + \tilde{\mu}_{dp}^3(\tilde{m}_2^3 + 3\tilde{m}_2^2) \\
E[(dP_t - \tilde{\mu}_{dp})^4] &= 3\tilde{\sigma}_{dp}^4(1 + \tilde{m}_2) + 6\tilde{\mu}_{dp}^2\tilde{\sigma}_{dp}^2(\tilde{m}_2^3 + 3\tilde{m}_2^2 + \tilde{m}_2) \\
&\quad + \tilde{\mu}_{dp}^4(6(1 + \tilde{m}_2) + (1 + \tilde{m}_2)^6 - 3 - 4(1 + \tilde{m}_2)^3)
\end{aligned} \tag{12}$$

For the trading volume the analogous set of moment restrictions apply.

Given the three unknown parameters and the four moments restrictions, we have one over-identifying restriction to be tested. Under the null hypothesis of a correct model specification, the test statistic asymptotically follows a chi-square distribution with one degree of freedom. The resulting p-values for the tests are reproduced in table 5, where it can be seen that in the univariate case the model specification for the price changes  $dP_t$  cannot be rejected since the p-values lie well above the critical 5 percent level for all stocks in our sample. So one can conclude that the univariate normal mixture model fits the observed distributional characteristics of price changes reasonably well. With respect to trading volume we have to reject the null hypothesis of a correct univariate specification for the same stocks (BAS, DGS, BAY and HFA) as we have done for the bivariate mixture

model itself. Together with the fact that the p-values for volume are substantially lower than for price changes this indicates that the BMM is rejected because of a specification error in the conditional marginal distribution of volume.

Since the BMM is developed from a simple Walrasian type of equilibrium model, in which trading is only caused by the arrival of new information, our results with respect to the bivariate model as well as to the univariate specifications cast doubt on the validity of this theoretical framework. From the empirical point of view at least another conditional marginal distribution for trading volume is necessary. Recently, Andersen (1994) developed a modified version of the bivariate mixture model using a theoretical microstructure framework with liquidity trading and asymmetric informations. His model employs a Poisson distribution for the trading volume conditional on the daily rate of information instead of a normal distribution we are using. Furthermore, it explicitly considers a part of the trading volume which is not related to the daily number of informational events. This part of the demand for trade is caused by unique liquidity desires. The empirical results concerning this modified version of the BMM does not indicate any misspecification using US stock market data. Thus our findings for the German stock market seem to be consistent with the results of Andersen (1994). But from a theoretical point of view it is not clear whether the modified version of Andersen's mixture model is adequate for the German stock market which operates without a market-maker system, an essential ingredient of Andersen's microstructure framework. This question is left for future work.

The findings of this section can be summarized as follows: We found evidence in favor of the hypothesis that the distributional characteristics of price changes can be captured by a univariate normal mixture model. Furthermore there is evidence against the hypothesis that price changes and volume are linked to the mixing variable  $I_t$  in a symmetric fashion as implied by the BMM.

## 5. Empirical Results Concerning the Time Series Properties of the BMM

So far the main focus has been on the distributional aspects of the BMM. An issue of at least equal importance is the time series property of the mixture model. An important question in this context is whether the observed time series behavior of the data is due to the corresponding time series properties of the latent mixture variable. We especially want to investigate whether a positive serial correlation in the daily number of information is indeed the origin of the so called ARCH-effects in the price change series. As can be seen from relation (5), this explanation is consistent with the theoretical implications of the bivariate mixture model.



Empirical evidence on this issue can be gathered employing the following simple regression model:

$$dP_t^2 = \alpha + \gamma I_t + \sum_{i=1}^{20} \beta_i \cdot dP_{t-i}^2 + \epsilon_t, \quad (13)$$

where the squared price changes serve as a measure for the price change variance on day  $t$ . This approach is adopted from the work of Engle (1982) where (13) with the parameter  $\gamma$  set to zero is used to conduct a test on the existence of persistence in the price change volatility. Persistence in the price change volatility is said to exist if the  $\beta$ -coefficients are significantly different from zero. The inclusion of the daily number of information as an explanatory variable in our setting should render the  $\beta$ -coefficients statistically insignificant if the serial correlation in the latent mixing variable  $I_t$  is the only source of the ARCH-effects. A straightforward application of the regression model (13) to the data is impossible due to the unobservable nature of  $I_t$ . While there have been several proposals for adequate proxy variables in the literature recently the results are not quite unanimous. Lamoureux and Lastrapes (1990) and Bauer et al. (1994) for example were able to show that after controlling for the contemporaneous trading volume the volatility persistence is removed from the price change data. Locke and Sayers (1993) report a significant variance persistence even after controlling for the number of daily information using different proxy variables like trading volume, number of recorded transactions and quoted number of price changes.

We pursued a somewhat different approach in our study and estimated the series of the unobservable variable  $I_t$  exploiting both the information contained in the price change series as well as the trading volume series. The estimate of the  $I_t$  series is based on the joint conditional distribution of  $dP_t$  and  $V_t$  given the value of the latent mixing variable for day  $t$

$$f_{dP,V}(dP_t, V_t | \tilde{I}_t; \tilde{\mu}_{dp}, \tilde{\mu}_v, \tilde{\sigma}_{dp}, \tilde{\sigma}_v) = f^N(dP_t | \tilde{I}_t; \tilde{\mu}_{dp}, \tilde{\sigma}_{dp}) \cdot f^N(V_t | \tilde{I}_t; \tilde{\mu}_v, \tilde{\sigma}_v), \quad (14)$$

where  $f^N(\cdot)$  again denotes the density of a normal distribution and  $\tilde{I}_t$  is the normalized mixture variable  $I_t/\mu$ . Substitution of the parameters  $(\tilde{\mu}_{dp}, \tilde{\mu}_v, \tilde{\sigma}_{dp}, \tilde{\sigma}_v)$  by their corresponding ML-estimates<sup>22</sup> based on the *unconditional* joint distribution of  $dP_t$  and  $V_t$  as given in section 4.1, the following log-likelihood function results:

$$\begin{aligned} \log L(\tilde{I}_t | dP_t, V_t, \hat{\mu}_{dp}, \hat{\mu}_v, \hat{\sigma}_{dp}, \hat{\sigma}_v) = & -\log(2\pi) - \frac{1}{2} \left[ \log(\hat{\sigma}_{dp}^2 \tilde{I}_t) + \frac{(dP_t - \hat{\mu}_{dp} \tilde{I}_t)^2}{\hat{\sigma}_{dp}^2 \tilde{I}_t} + \right. \\ & \left. + \log(\hat{\sigma}_v^2 \tilde{I}_t) + \frac{(V_t - \hat{\mu}_v \tilde{I}_t)^2}{\hat{\sigma}_v^2 \tilde{I}_t} \right]. \end{aligned} \quad (15)$$

Estimation of the information arrival rate for each day separately is carried out by maximizing (15) with respect to  $\tilde{I}_t$ . Set up in this particular way, the unknown value of the mixing variable is treated as a parameter to be estimated and the estimate of  $\tilde{I}_t$  is the value with the highest probability given the pair of observations  $(dP_t, V_t)$  and the ML-estimates

<sup>22</sup>One could also use the GMM estimates of the parameters, since for a correct model specification both ML and GMM procedures are consistent. But since the finite sample properties of the ML estimates seem to be better than of the GMM estimates, we based the estimation of  $I_t$  on the former ones.

$(\hat{\mu}_{dp}, \hat{\mu}_v, \hat{\sigma}_{dp}, \hat{\sigma}_v)$ . Hence the estimation of the value of  $I_t$  at a single day  $t$  is based on two observations. Repeating this procedure for all pairs of  $(dP_t, V_t)$  contained in our sample gives us the entire series  $\{\tilde{I}_t\}_{t=1}^T$ .

This procedure of extracting the latent mixture variable is similar to that proposed by Lamoureux and Lastrapes (1994), who investigate the sources of the ARCH effects in a bivariate mixture framework without any restrictions concerning the joint distribution of price changes and volume conditional on the mixing variable. Since ML-estimation is not feasible without distributional assumptions, Lamoureux and Lastrapes (1994) define the estimate of  $\tilde{I}_t$  as that value, which sets the observed  $dP_t^2$  and  $V_t$  as close as possible to their corresponding conditional means  $E[dP_t^2|\tilde{I}_t]$  and  $E[V_t|\tilde{I}_t]$  respectively, resulting from the BMM. As noted by Lamoureux and Lastrapes (1994), even if for the estimation of a single  $\tilde{I}_t$  only one pair of observations  $(dP_t, V_t)$  is used, the total sample information is regarded by using the ML estimates of the parameters. In addition to that one can expect that if the model specification is correct the estimated series of mixing variable maps the time series behavior and the distributional pattern of the true latent variable, even if the errors in the estimates of the single  $\tilde{I}_t$ 's may be large.

Naturally the question about the precision of our estimated  $\tilde{I}_t$  series arises. Using a Monte Carlo framework we tried to gain at least a small insight into this subject. Our setting is as follows: using an AR(1)-process for the logarithms of the mixing variable together with a normally distributed error term we generated an observable information arrival series according to

$$\log I_t = \delta_0 + \delta_1 \log I_{t-1} + \delta_2 v_t, \quad (16)$$

where  $v_t \sim \text{iid}N(0,1)$ . This is the standard lognormal stochastic volatility process as used e.g. by Taylor (1994) and Andersen (1994). Now the obtained series is used as an input into the BMM (1) giving us  $T$  pairs of generated  $(dP_t, V_t)$  values which can be used to obtain an estimated  $I_t$  series according to our procedure discussed above. Comparing the thus obtained estimated series with the now known  $I_t$  series allows us to make at least a qualitative statement about our proceeding. Note that the assumptions about the data generating process of  $I_t$  together with the bivariate mixture framework is consistent with the observed persistence in the price change volatility and with the lognormality assumption for the unconditional distribution of the mixing variable used for the ML and the GMM-estimation in section 4.

To keep the paper short, we only present one typical time series plot of the estimated series and the true series of the mixing variable obtained from our Monte Carlo study<sup>23</sup> in figure 3. The estimated series maps the structure of the true series very well, even if the former exhibits more volatility than the latter. We therefore argue that an estimated series for the daily number of information can be regarded as a reasonable proxy variable.

Applying the discussed pointwise estimation procedure we extracted an estimated  $\tilde{I}_t$  series for every single stock in our sample and estimated the regression (13) substituting

<sup>23</sup>For the Monte Carlo study we used a sample size of 1100 and following parameter values:  $\mu_{dp} = 0$ ,  $\sigma_{dp} = 0.01$ ,  $\mu_v = 0.0003$ ,  $\sigma_v = 0.0014$ . For the data generating process of  $I_t$  in equation (16) we set  $\delta_0 = 0.25$ ,  $\delta_1 = 0.95$ ,  $\delta_2 = 0.196$ , which leads to the following values of identifiable parameters in the BMM:  $\tilde{\mu}_{dp} = 0$ ,  $\tilde{\sigma}_{dp} = 0.013$ ,  $\tilde{\mu}_v = 0.054$ ,  $\tilde{\sigma}_v = 0.018$  and  $\theta = 0.62$ .

$I_t$  by the extracted series. As a test of insignificance of variance persistence we used the F-test of the joint hypotheses that all coefficient other than  $\alpha$  and  $\gamma$  are zero. Under the Null hypotheses the test statistic is F-distributed with  $(20, T - 22)$  degrees of freedom. The results are reported in table 7. For every single stock the coefficient  $\gamma$  of the extracted series is highly significant, but it appears insufficient in explaining the ARCH-effects in the price change series. In fact, for all stocks in which ARCH effects are evident the persistence parameters  $\beta_1, \dots, \beta_{20}$  remain statistically significant after controlling for the extracted series of the mixing variable of the BMM. Thus even if the process of daily number of information, which is extracted exploiting the information from the volume series has explanatory power concerning the daily volatility, it can not explain the observed dynamic structure of volatility.

## 6. Conclusions

The aim of this study was to evaluate the ability of the bivariate mixture model (BMM) in the Tauchen-Pitts-Harris specification to capture the well known stylized facts in price change and trading volume data for the German stock market. While the BMM is in principle able to explain the most important distributional patterns (e.g. the excess kurtosis in the price changes or the positive skewness in the trading volume) a comparison between the implied *unconditional* moments obtained from a ML-estimation and the corresponding sample moments reveal non negligible shortcomings in the model specification. This finding is confirmed by the tests we conducted based on the generalized method of moments (GMM). Using univariate mixture models for price changes and trading volume separately we were able to show that the impact of the latent information arrival rate on the two variables is far from being symmetric as implied by the BMM.

Of at least equal importance are the dynamic properties of the BMM. The key question in this context is whether the ARCH-effects in the price change series can solely be contributed to the positive serial correlation in the information arrival process. Due to the latent character of this process a formal test of this issue has to be based on a proxy variable for the information arrival. We developed a new procedure for obtaining a suitable proxy variable based on the ML-technique. It allows us to estimate the information arrival rate for each trading day separately given the price change and trading volume and ML-estimates of the parameters of the BMM. Our results suggest that although the proxy for the information arrival process has some explanatory power concerning the daily volatility in the price change series, there remains to be a considerable amount of dynamics unexplained by the BMM.

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## Appendix I: List of the companies in the sample

DAX firms					
Symbol	Firms full name	Symbol	Firms full name	Symbol	Firms full name
BAS	BASF	DBC	Deutsche Babcock	LIN	Linde
BAY	Bayer	DGS	Degussa	MAN	MAN
BMW	BMW	HEN	Henkel	MET	Metallgesellschaft
CON	Continental	HFA	Hoechst	MMW	Mannesmann
DAI	Daimler-Benz	KAR	Karstadt	PRS	Preussag

## Appendix II: Implied estimates of the unconditional moments

Since  $I_t$  is assumed to be lognormally distributed, the following standardized moments result:

$$\begin{aligned}
 \tilde{m}_2 &\equiv \frac{m_2}{\mu^2} = \exp(\theta^2) - 1 \\
 \tilde{m}_3 &\equiv \frac{m_3}{\mu^3} = \tilde{m}_2^3 + 3\tilde{m}_2^2 \\
 \tilde{m}_4 &\equiv \frac{m_4}{\mu^4} = (1 - \tilde{m}_2)^6 + 6(1 + \tilde{m}_2) - 3 - 4(1 + \tilde{m}_2)^3.
 \end{aligned}$$

Using the implied estimates of these moments it is possible to calculate:

$$\begin{aligned}
 E(dP) &= \tilde{\mu}_{dp} \\
 \text{Var}(dP) &= \tilde{\sigma}_{dp}^2 + \tilde{\mu}_{dp}^2 \tilde{m}_2 \\
 \varsigma(dP) &= \frac{3\tilde{\mu}_{dp} \tilde{\sigma}_{dp}^2 \tilde{m}_2 + \tilde{\mu}_{dp}^3 \tilde{m}_3}{(\tilde{\sigma}_{dp}^2 + \tilde{\mu}_{dp}^2 \tilde{m}_2)^{3/2}} \\
 \kappa(dP) &= \frac{3\tilde{\sigma}_{dp}^4 (1 + \tilde{m}_2) + 6\tilde{\mu}_{dp}^2 \tilde{\sigma}_{dp}^2 (\tilde{m}_3 + \tilde{m}_2) + \tilde{\mu}_{dp}^4 \tilde{m}_4}{(\tilde{\sigma}_{dp}^2 + \tilde{\mu}_{dp}^2 \tilde{m}_2)^2} - 3.
 \end{aligned}$$

The moments of the *unconditional* marginal distribution of  $V_t$  are calculated in an analogous way.

For the *unconditional* correlation between squared price changes and volume the following formula applies:

$$\rho(dP^2, V) = \frac{\tilde{\mu}_v \tilde{\sigma}_{dp}^2 \tilde{m}_2 + \tilde{\mu}_v \tilde{\mu}_{dp}^2 (\tilde{m}_3 + 2\tilde{m}_2)}{(\text{Var}(dP^2) \text{Var}(V))^{1/2}},$$

with:

$$\text{Var}(dP^2) = \tilde{\sigma}_{dp}^4 (3\tilde{m}_2 + 2) + \tilde{\mu}_{dp}^2 \tilde{\sigma}_{dp}^2 (6\tilde{m}_3 + 16\tilde{m}_2 + 4) + \tilde{\mu}_{dp}^4 (\tilde{m}_4 + 3\tilde{m}_3 + 6\tilde{m}_2 + (\tilde{m}_2 + 1)^2 + 1).$$

**Table 1: Empirical properties of the Volume and price change data**

<i>Stocks</i>	Price changes				Volume		Correlation
	$\hat{\kappa}_{dP}$	$K_{dP}$	$SK_{dP}$	$Q_{dP2}(20)$	$\hat{\zeta}_V$	$S_V$	$\rho(dP_t^2, V_t)$
BAS	5.738	1503.4	1535.3	39.3	5.342	5213.2	0.2182***
BMW	7.528	2601.9	2620.2	21.2	3.767	2606.6	0.2636***
CON	2.511	289.5	293.0	58.9	11.477	24193.0	0.1137***
DAI	3.882	691.5	691.9	141.8	1.225	275.4	0.2413***
DBC	9.632	4255.6	4269.7	65.6	7.670	10795.5	0.1584***
DGS	3.314	502.9	506.3	74.2	2.467	1114.3	0.1905***
BAY	3.730	638.7	640.1	50.4	7.394	10041.8	0.2397***
HFA	4.157	794.8	797.2	51.9	4.928	4463.9	0.2089***
MAN	8.746	3509.3	3609.5	36.3	2.205	892.5	0.1939***
HEN	10.949	5505.2	5695.9	40.6	2.873	1515.8	0.1500***
KAR	8.731	3493.7	3562.2	67.9	2.723	1359.3	0.1902***
LIN	10.969	5519.4	5592.9	42.0	4.246	3308.4	0.1691***
MMW	22.017	22218.1	22219.7	171.1	3.228	21915.3	0.2025***
MET	7.625	2660.1	2687.6	62.2	3.799	2640.9	0.3356***
PRS	7.996	2930.4	3080.5	12.5	4.958	4507.6	0.1571***
*** significant at the level of 1 percent							

**Table 2: Maximum likelihood estimates of the parameters of the bivariate normal mixture model. Standard errors are given in parentheses.**

<i>Stocks</i>	$\bar{\mu}_{dp}$	$\bar{\sigma}_{dp}$	$\bar{\mu}_v$	$\bar{\sigma}_v$	$\theta$	LL
BAS	0.0000 (0.0004)	0.0129 (0.0003)	0.3808 (0.0049)	0.0576 (0.0028)	0.4978 (0.0050)	3525.8
BMW	0.0003 (0.0004)	0.0147 (0.0002)	0.1232 (0.0023)	0.0298 (0.0045)	0.6145 (0.0103)	4548.0
CON	-0.0001 (0.0005)	0.0189 (0.0005)	0.1210 (0.0031)	0.0596 (0.0023)	0.7205 (0.0123)	3920.6
DAI	0.0000 (0.0004)	0.0144 (0.0003)	0.6850 (0.0096)	0.1447 (0.0116)	0.4053 (0.0143)	2924.6
DBC	0.0000 (0.0006)	0.0196 (0.0005)	0.0626 (0.0011)	0.0141 (0.0008)	0.6518 (0.0093)	5001.3
DGS	0.0000 (0.0005)	0.0164 (0.0004)	0.0592 (0.0012)	0.0149 (0.0009)	0.5869 (0.0147)	5219.1
BAY	0.0001 (0.0004)	0.0129 (0.0003)	0.5653 (0.0065)	0.0732 (0.0045)	0.4155 (0.0040)	3262.8
HFA	0.0001 (0.0004)	0.0142 (0.0003)	0.3274 (0.0051)	0.0621 (0.0036)	0.5302 (0.0075)	3612.1
MAN	-0.0001 (0.0005)	0.0154 (0.0004)	0.1118 (0.0016)	0.0211 (0.0015)	0.4938 (0.0089)	4701.9
HEN	0.0001 (0.0004)	0.0118 (0.0003)	0.0463 (0.0008)	0.0114 (0.0008)	0.5453 (0.0127)	5908.5
KAR	-0.0001 (0.0004)	0.0140 (0.0003)	0.0609 (0.0010)	0.0128 (0.0008)	0.5452 (0.0102)	5458.6
LIN	0.0000 (0.0003)	0.0116 (0.0003)	0.0404 (0.0009)	0.0102 (0.0006)	0.6178 (0.0193)	6042.9
MMW	0.0001 (0.0005)	0.0160 (0.0004)	0.3806 (0.0055)	0.0653 (0.0039)	0.5167 (0.0092)	3391.6
MET	-0.0010 (0.0007)	0.0229 (0.0006)	0.0627 (0.0017)	0.0250 (0.0010)	0.8907 (0.0204)	4674.2
PRS	0.0001 (0.0005)	0.0167 (0.0004)	0.1268 (0.0024)	0.0333 (0.0018)	0.6045 (0.0109)	4369.5

**Table 3:** Implied estimates of the unconditional moments of price changes and trading volume resulting from the ML estimation of the bivariate normal mixture model. Sample values are given in parentheses.

<i>Stocks</i>	Price changes			Volume			Correlation
	$\mu(dP)$	$\sigma(dP)$	$\kappa(dP)$	$\mu(V)$	$\sigma(V)$	$\zeta(V)$	$\rho(dP^2, V)$
BAS	0.0000 (0.0000)	0.0129 (0.0141)	0.8435 (5.738)	0.3808 (0.4076)	0.2099 (0.3288)	1.6623 (5.342)	0.3023 (0.2182)
BMW	0.0003 (0.0003)	0.0147 (0.0158)	1.3780 (7.528)	0.1231 (0.1255)	0.0885 (0.1037)	2.1731 (3.767)	0.3473 (0.2636)
CON	-0.0001 (-0.0001)	0.0188 (0.0186)	2.0418 (2.511)	0.1209 (0.1310)	0.1162 (0.1927)	2.4792 (11.47)	0.3522 (0.1137)
DAI	-0.0000 (-0.0001)	0.0144 (0.0153)	0.5354 (3.882)	0.6850 (0.6846)	0.3235 (0.3138)	1.1876 (1.225)	0.2373 (0.2413)
DBC	0.0000 (0.0001)	0.0195 (0.0201)	1.5880 (9.632)	0.0625 (0.0627)	0.0476 (0.0573)	2.4203 (7.670)	0.3668 (0.1584)
DGS	0.0000 (0.0000)	0.0164 (0.0165)	1.2338 (3.314)	0.0591 (0.0598)	0.0407 (0.0448)	2.0036 (2.467)	0.3319 (0.1905)
BAY	0.0001 (0.0001)	0.0129 (0.0138)	0.5654 (3.730)	0.5653 (0.5895)	0.2561 (0.4102)	1.3200 (7.394)	0.2597 (0.2397)
HFA	0.0001 (0.0001)	0.0142 (0.0151)	0.9738 (4.157)	0.3273 (0.3378)	0.1965 (0.2825)	1.7798 (4.928)	0.3134 (0.2089)
MAN	-0.0000 (-0.0001)	0.0154 (0.0164)	0.8282 (8.746)	0.1118 (0.1145)	0.0624 (0.0705)	1.6044 (2.205)	0.2940 (0.1939)
HEN	0.0001 (0.0001)	0.0118 (0.0122)	1.0391 (10.94)	0.0462 (0.0468)	0.0295 (0.0327)	1.7890 (2.873)	0.3114 (0.1500)
KAR	-0.0001 (-0.0001)	0.0139 (0.0144)	1.0384 (8.731)	0.0608 (0.0613)	0.0380 (0.0424)	1.8329 (2.723)	0.3179 (0.1902)
LIN	0.0000 (0.0000)	0.0115 (0.0122)	1.3941 (10.96)	0.0404 (0.0404)	0.0293 (0.0330)	2.1785 (4.246)	0.3469 (0.1691)
MMW	0.0001 (0.0001)	0.0160 (0.0180)	0.9182 (22.01)	0.3805 (0.3796)	0.2204 (0.2387)	1.7324 (3.228)	0.3093 (0.2025)
MET	-0.0009 (-0.0001)	0.0229 (0.0232)	3.6807 (7.625)	0.0627 (0.0593)	0.0733 (0.0661)	4.2108 (3.799)	0.4372 (0.3356)
PRS	0.0001 (0.0001)	0.0166 (0.0170)	1.3232 (7.996)	0.1268 (0.1278)	0.0905 (0.1080)	2.0883 (4.958)	0.3387 (0.1571)



Table 4:

GMM estimates of the parameters of the bivariate normal mixture model. Standard errors are given in parentheses.  $\chi^2$ -statistics and p-values of the Hansen's J-test.

<i>Stocks</i>	$\tilde{\mu}_{dp}$	$\tilde{\sigma}_{dp}$	$\tilde{\mu}_v$	$\tilde{\sigma}_v$	$\tilde{m}_2$	$\chi^2_{(3)}$	p-value	S-opt <sup>a)</sup>
BAS	0.0003 (0.0004)	0.0128 (0.0006)	0.3810 (0.0116)	0.0000 (--)	0.3513 (0.1112)	8.0943	0.0441	11
BMW	0.0004 (0.0005)	0.0143 (0.0006)	0.1226 (0.0053)	0.0094 (0.0674)	0.5989 (0.1291)	7.0860	0.0692	9
CON	0.0000 (0.0007)	0.0179 (0.0005)	0.1265 (0.0051)	0.1073 (0.0118)	0.4687 (0.1091)	4.7098	0.1943	1
DAI	0.0000 (0.0005)	0.0140 (0.0006)	0.6841 (0.0169)	0.0482 (0.0965)	0.2134 (0.0292)	6.1071	0.1065	10
DBC	-0.0002 (0.0006)	0.0184 (0.0006)	0.0611 (0.0017)	0.0246 (0.0078)	0.3898 (0.1454)	6.3329	0.0965	1
DGS	0.0002 (0.0005)	0.0151 (0.0005)	0.0593 (0.0019)	0.0120 (0.0042)	0.5044 (0.0536)	12.738	0.0052	4
BAY	0.0002 (0.0004)	0.0125 (0.0004)	0.5692 (0.0127)	0.0000 (--)	0.2985 (0.1150)	10.857	0.0125	2
HFA	0.0001 (0.0004)	0.0139 (0.0005)	0.3227 (0.0088)	0.0000 (--)	0.4552 (0.0704)	13.354	0.0039	2
MAN	0.0000 (0.0005)	0.0157 (0.0006)	0.1133 (0.0029)	0.0084 (0.0175)	0.3487 (0.0374)	3.7822	0.2859	4
HEN	0.0002 (0.0004)	0.0113 (0.0004)	0.0465 (0.0012)	0.0087 (0.0071)	0.4069 (0.0872)	3.6554	0.3011	3
KAR	0.0000 (0.0004)	0.0136 (0.0004)	0.0598 (0.0016)	0.0149 (0.0047)	0.3309 (0.0623)	9.9428	0.0191	3
LIN	0.0000 (0.0004)	0.0111 (0.0004)	0.0392 (0.0012)	0.0123 (0.0042)	0.4096 (0.0916)	5.9961	0.1112	5
MMW	0.0002 (0.0005)	0.0156 (0.0008)	0.3738 (0.0107)	0.0000 (--)	0.3151 (0.0882)	4.8008	0.1869	10
MET	-0.0008 (0.0007)	0.0215 (0.0010)	0.0577 (0.0275)	0.0294 (0.0067)	0.8607 (0.1997)	4.4803	0.2141	3
PRS	0.0003 (0.0005)	0.0159 (0.0006)	0.1254 (0.0049)	0.0421 (0.0170)	0.4355 (0.1151)	3.9435	0.2676	6

a) used bandwidth parameter for the Newey and West (1987) matrix calculated by the data dependent choice procedure of Andrews (1991) based on univariate AR(1) estimations for each moment restriction.

Table 5:

Maximum likelihood estimates of the unconditional marginal distributions of price changes and trading volume and the p-values of the Hansen's J-test. Standard errors are given in parentheses.

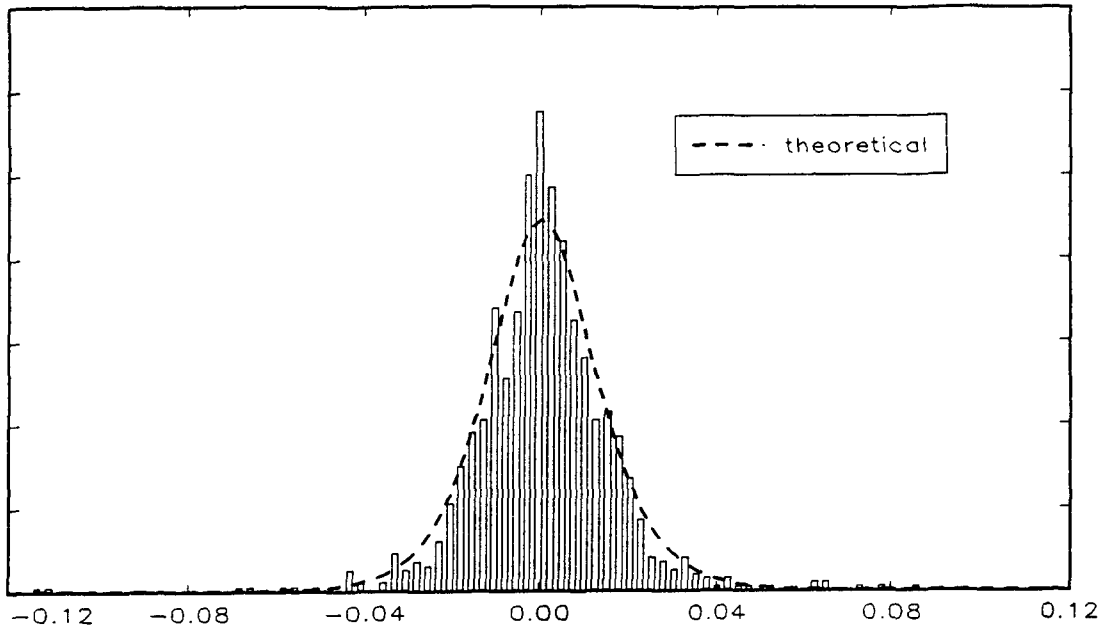
Stocks	Price changes					Volume				
	$\tilde{\mu}_{dp}$	$\tilde{\sigma}_{dp}$	$\theta_{dp}$	LL	p-value	$\tilde{\mu}_v$	$\tilde{\sigma}_v$	$\theta_v$	LL	p-value
BAS	0.0000 (0.0004)	0.0138 (0.0005)	0.9620 (0.0692)	3214.7	0.2214	0.3838 (0.0050)	0.0581 (0.0028)	0.4962 (0.0063)	276.7	0.0313
BMW	0.0003 (0.0005)	0.0154 (0.0006)	0.9744 (0.0812)	3119.3	0.3567	0.1235 (0.0031)	0.0290 (0.0020)	0.6155 (0.0145)	1386.3	0.0562
CON	-0.0002 (0.0006)	0.0187 (0.0006)	0.8401 (0.0725)	2874.9	0.5162	0.1202 (0.0032)	0.0567 (0.0030)	0.7260 (0.0214)	1037.2	0.3323
DAI	0.0000 (0.0005)	0.0153 (0.0005)	0.9280 (0.0703)	3110.4	0.8741	0.6847 (0.0094)	0.1783 (0.0133)	0.3601 (0.0218)	-205.8	0.3298
DBC	0.0000 (0.0006)	0.0195 (0.0007)	0.8542 (0.0863)	2826.4	0.6978	0.0600 (0.0013)	0.0148 (0.0010)	0.5806 (0.0134)	2131.3	0.3199
DGS	0.0000 (0.0005)	0.0165 (0.0006)	0.8807 (0.0778)	3010.6	0.5858	0.0595 (0.0026)	0.0138 (0.0006)	0.5944 (0.0318)	2200.0	0.0222
BAY	0.0001 (0.0004)	0.0127 (0.0005)	0.8871 (0.0685)	3228.9	0.6144	0.5654 (0.0074)	0.0749 (0.0050)	0.4150 (0.0063)	1.8	0.0062
HFA	0.0001 (0.0004)	0.0150 (0.0005)	0.9724 (0.0692)	3147.1	0.5895	0.3284 (0.0050)	0.0624 (0.0028)	0.5281 (0.0063)	425.6	0.0004
MAN	-0.0001 (0.0005)	0.0160 (0.0006)	0.8608 (0.0808)	3042.6	0.3265	0.1117 (0.0017)	0.0202 (0.0012)	0.4929 (0.0081)	1629.0	0.0820
HEN	0.0001 (0.0004)	0.0118 (0.0005)	0.8750 (0.0879)	3387.1	0.1978	0.0461 (0.0008)	0.0094 (0.0005)	0.5587 (0.0101)	2538.3	0.3035
KAR	-0.0001 (0.0004)	0.0140 (0.0005)	0.7653 (0.0790)	3171.1	0.4288	0.0611 (0.0013)	0.0125 (0.0006)	0.5466 (0.0125)	2256.2	0.0681
LIN	0.0000 (0.0004)	0.0118 (0.0005)	0.8898 (0.0871)	3383.1	0.4569	0.0394 (0.0007)	0.0104 (0.0005)	0.5726 (0.0108)	2617.0	0.1118
MMW	0.0001 (0.0005)	0.0167 (0.0007)	0.9098 (0.0953)	3007.2	0.9671	0.3705 (0.0063)	0.0626 (0.0044)	0.4721 (0.0105)	349.9	0.1503
MET	-0.0009 (0.0007)	0.0226 (0.0009)	1.0069 (0.0762)	2693.4	0.8182	0.0628 (0.0023)	0.0244 (0.0009)	0.8868 (0.0340)	1915.3	0.0718
PRS	0.0001 (0.0005)	0.0166 (0.0006)	0.8894 (0.0752)	3005.8	0.1431	0.1266 (0.0031)	0.0305 (0.0014)	0.6077 (0.0146)	1356.7	0.1540

**Table 6: Implied estimates of the unconditional moments of price changes and trading volume resulting from the ML estimation of the univariate normal mixture models. Sample values are given in parentheses.**

<i>Stocks</i>	Price changes			Volume		
	$\mu(dP)$	$\sigma(dP)$	$\kappa(dP)$	$\mu(V)$	$\sigma(V)$	$\zeta(V)$
BAS	0.0000 (0.0000)	0.0138 (0.0141)	4.5698 (5.738)	0.3838 (0.4076)	0.2109 (0.3288)	1.6547 (5.342)
BMW	0.0003 (0.0003)	0.0154 (0.0158)	4.7733 (7.528)	0.1235 (0.1255)	0.0886 (0.1037)	2.1879 (3.767)
CON	-0.0002 (-0.0001)	0.0187 (0.0186)	3.0783 (2.511)	0.1202 (0.1310)	0.1150 (0.1927)	2.5556 (11.47)
DAI	-0.0000 (-0.0001)	0.0153 (0.0153)	4.0979 (3.882)	0.6847 (0.6846)	0.3109 (0.3138)	0.9428 (1.225)
DBC	0.0000 (0.0001)	0.0195 (0.0201)	3.2231 (9.632)	0.0600 (0.0627)	0.0407 (0.0573)	1.9751 (7.670)
DGS	0.0000 (0.0000)	0.0165 (0.0165)	3.5159 (3.314)	0.0595 (0.0598)	0.0411 (0.0448)	2.0702 (2.467)
BAY	0.0001 (0.0001)	0.0137 (0.0138)	3.5914 (3.730)	0.5654 (0.5895)	0.2563 (0.4102)	1.3150 (7.394)
HFA	0.0001 (0.0001)	0.0150 (0.0151)	4.7251 (4.157)	0.3284 (0.3378)	0.1964 (0.2825)	1.7688 (4.928)
MAN	-0.0000 (-0.0001)	0.0160 (0.0164)	3.2948 (8.746)	0.1117 (0.1145)	0.0619 (0.0705)	1.6091 (2.205)
HEN	0.0001 (0.0001)	0.0118 (0.0122)	3.4528 (10.94)	0.0461 (0.0468)	0.0294 (0.0327)	1.9095 (2.873)
KAR	-0.0001 (-0.0001)	0.0140 (0.0144)	2.3892 (8.731)	0.0611 (0.0613)	0.0381 (0.0424)	1.8458 (2.723)
LIN	0.0000 (0.0000)	0.0118 (0.0122)	3.6217 (10.96)	0.0394 (0.0404)	0.0266 (0.0330)	1.9092 (4.246)
MMW	0.0001 (0.0001)	0.0167 (0.0180)	3.8655 (22.01)	0.3705 (0.3796)	0.1954 (0.2387)	1.5260 (3.228)
MET	-0.0009 (-0.0001)	0.0226 (0.0232)	5.3790 (7.625)	0.0628 (0.0593)	0.0728 (0.0661)	4.1844 (3.799)
PRS	0.0001 (0.0001)	0.0166 (0.0170)	3.6180 (7.996)	0.1266 (0.1278)	0.0899 (0.1080)	2.1349 (4.958)

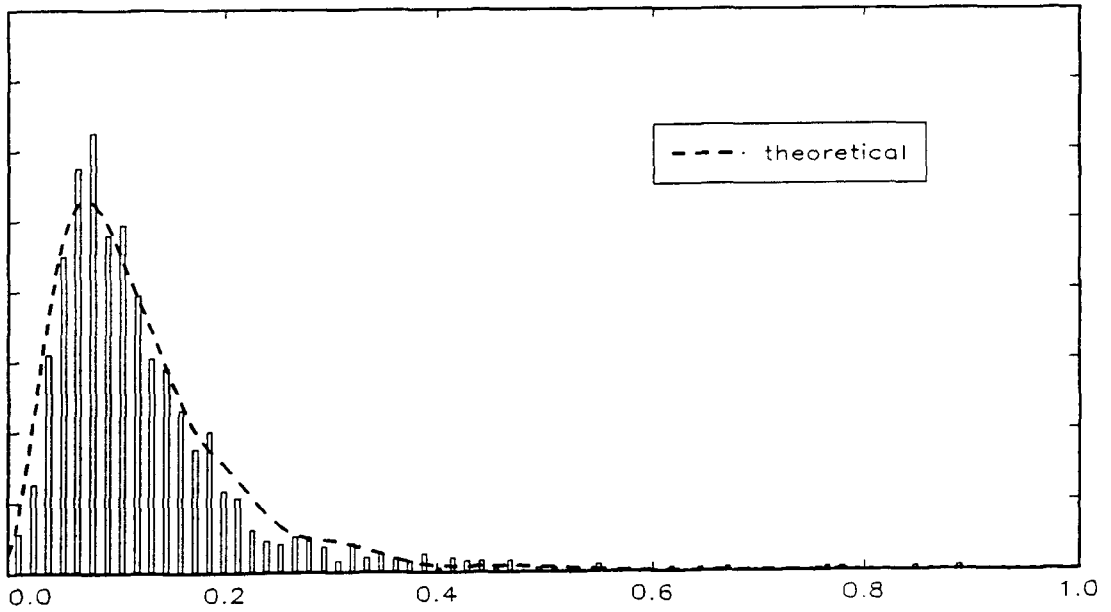
**Table 7:** The estimated coefficients of the extracted mixing variable and the F-statistic of the variance persistence test. Standard errors and probability values respectively are given in parentheses.

	$\gamma$	$F(20, T - 22)$
BAS	0.016 (0.002)	1.663 (0.034)
BMW	0.028 (0.002)	0.567 (0.935)
CON	0.009 (0.001)	2.631 (0.000)
DAI	0.038 (0.004)	3.384 (0.000)
DBC	0.030 (0.004)	3.277 (0.000)
DGS	0.020 (0.002)	2.506 (0.000)
BAY	0.016 (0.002)	1.930 (0.008)
HFA	0.015 (0.002)	1.897 (0.009)
MAN	0.033 (0.004)	1.187 (0.2569)
HEN	0.017 (0.002)	1.647 (0.036)
KAR	0.023 (0.003)	3.325 (0.000)
LIN	0.015 (0.002)	1.666 (0.032)
MMW	0.055 (0.007)	10.955 (0.000)
MET	0.065 (0.005)	1.324 (0.154)
PRS	0.020 (0.003)	0.789 (0.729)



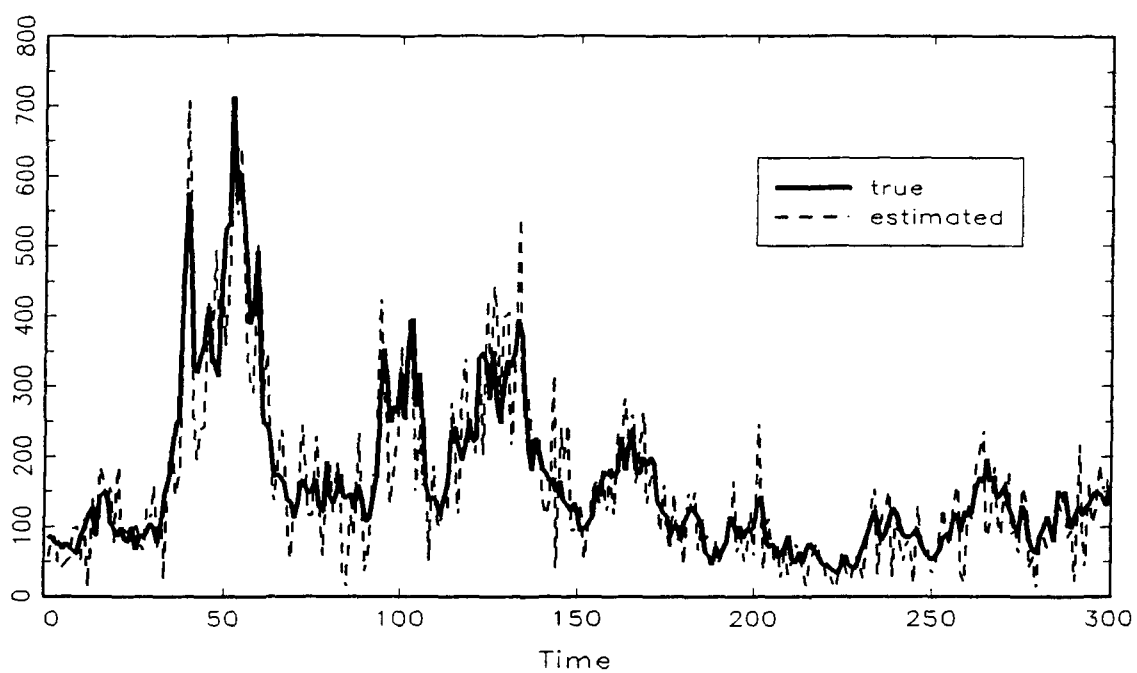
**Figure 1**

Theoretical and empirical distribution of price changes for BMW



**Figure 2**

Theoretical and empirical distribution of trading volume for BMW



**Figure 3**

Estimated and true mixing variable for a simulated data set