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Can trading volume explain option prices?

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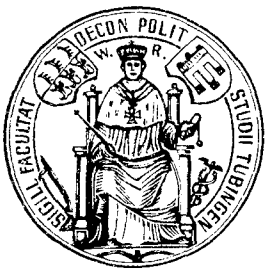

Tübinger Diskussionsbeiträge

Wirtschaftswissenschaftliche Fakultät
der Eberhard-Karls-Universität Tübingen

Can Trading Volume Explain Option Prices?

Hartmut Nagel
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1. Introduction[1]

During the last decade, a lot of work was done to extend the Black-Scholes option pricing model by allowing for a stochastic variance.[2] However, these extended models still have the problem that the variance is not observable. A common procedure for estimating this variable is to do it implicitly.[3] In this paper we follow a different approach by taking a first step towards an option valuation model which does not explicitly make use of unobservable state variables. Instead of using a stochastic variance variable directly, we assume that the variance of stock returns is determined by the trading activity in the stock or the options market, respectively. As we will see, this is consistent with many empirical studies which report a positive relationship between the volume and volatility of individual securities. To our knowledge, this is the first time that an option pricing model uses trading volume in order to model the stochastic nature of the stock return variance. The major focus of our work is to combine recent theoretical work on option pricing models with results from the statistical literature on volume, volatility and stock returns.

The rest of the paper is organized as follows: chapter 2 reviews the relevant statistical literature. In chapter 3, we present the two models which we use to test whether variances can be replaced by volume. In the next three chapters, we then focus on the question of whether it is possible to estimate the parameters of the models using the Generalized Method of Moments (GMM). To do this within a controlled environment, we generate time series by running Monte Carlo simulations and estimate the parameters from these time series using GMM. By doing so we also obtain more information about the small sample properties of test statistics for our specific setup of the GMM procedure. This is important since the properties of the GMM estimators are only known asymptotically. The results of the empirical analysis follow in chapter 7. This includes a discussion of the assumptions and estimation problems

[1]We would like to thank the Deutsche Kapitalmarktdatenbank for providing us with the data. Financial support of the Deutsche Forschungsgemeinschaft is gratefully acknowledged.

[2]See for example [Hull/White (1987)], [Johnson/Shanno (1987)], [Wiggins (1987)], [Scott (1987)], [Hull/White (1988)], [Chesney/Scott (1989)], [Scott (1991)], [Melino/Turnbull (1990)], [Stein/Stein (1991)], [Heston (1993)], [Scott (1997)] and [Bakshi/Chen (1997)].

[3]See e.g. [Nandi (1996)] and [Bakshi et al. (1997)].

regarding the market price of risk, as well as in- and out-of-the-sample investigations where both models are compared with the simple Black-Scholes model. Chapter 8 summarizes and concludes.

2. Empirical facts

As many studies have shown, there exists a relationship between stock prices and trading volumes. It has been observed for trading volume data from stock markets but also applies to data from options markets. Here we review some of the main findings which have motivated our work.

One of the earliest works is provided by [Ying (1966)], who examines the relationship between the S&P 500 and the volume of stock sales on the NYSE. He reports contemporaneous and delayed dependences of the index on trading volume. One of his conclusions is that "... any model that attempts to isolate prices from volumes or vice versa will inevitably yield incomplete if not erroneous results."[4] In [Crouch (1970a)] and [Crouch (1970b)], the author observes a positive relationship between the absolute values of daily price changes and daily volumes for both market indices and individual stocks. The positive correlation between volume and absolute price changes or volume and price changes per se has been confirmed by many studies. Surveys on the extensive literature on this subject can be found in [Karpoff (1987)], [Gallant et al. (1992)], and [McCarthy/Najand (1993)]. More recent studies which come to the same conclusions are for example [LeBaron (1993)], and [Majnoni/Massa (1996)]. [LeBaron (1993)] uses the log of daily squared returns of the CRSP index and log transformed turnover volumes on NYSE stocks and observes a strong contemporaneous correlation between these time series. Regarding the direction of this relationship he concludes that "... there is a channel from volume to future volatility".[5] The data examined by [Majnoni/Massa (1996)] consist of stock returns and trading volumes from the Milan Stock Exchange. Regarding the interaction between returns and volume, their tests confirm

[4] [Ying (1966)], p. 676.

[5] [LeBaron (1993)], p. 9.

the assumption that trading volume affects returns and returns affect volumes. Finally, they conclude that trading volume has its own informational content.[6]

Other studies focus on the relationship between the trading volume of derivatives and the returns of the underlying assets. [Rogalski (1978)] observes significant positive contemporaneous correlations between the warrant volume and stock price changes for 7 of 10 stocks on a monthly basis. [Easley et al. (1994)] examine the informational role of transaction volume in the options market. Their data consist of stock price changes for the 50 most actively traded firms on the CBOE and the trading volume of the corresponding options aggregated over 5 minute intervals. They obtained the best results by segregating the option volume in "positive" (long calls and short puts) and "negative" (short calls and long puts) positions when running regressions. Using this approach, it is the contemporaneous volume which explains most of the stock price changes. Based on their empirical findings, they conclude that "there clearly remains much to be learned, and much benefit to be gained, from a greater understanding of volume in asset markets."[7] Strong evidence for dependencies between price changes and option volumes for the S&P 100 has also been found by [Chatrath et al. (1995a)] and [Chatrath et al. (1995b)].[8]

3. The models

We model the stock price behavior in the usual way by using a geometric Brownian motion of the form

$$dS(t) = \mu S(t)dt + \sqrt{v(t)}S(t)dw_1(t). \tag{3.1}$$

The stock price is denoted by $S(t)$. It has an expected instantaneous rate of return μ and a stochastic instantaneous variance $v(t)$. The latter follows a mean reverting square-root process which is given as

$$dv(t) = \kappa\left[\theta - v(t)\right]dt + \sigma\sqrt{v(t)}dw_2(t). \tag{3.2}$$

[6] [Majnoni/Massa (1996)], p. 29 and p. 33.

[7] [Easley et al. (1994)], p. 35.

[8] Even though [Chatrath et al. (1995b)] focus on the impact of lagged variables, their Table III shows that the contemporaneous regression coefficient is the most significant one.

Here, θ is the long-run target variance and κ the mean reversion parameter. The volatility of this process is given by σ, which can be interpreted as the "volatility of variance". One of the essential features of this model is that the two-dimensional Brownian motion $[w_1(t), w_2(t)]$ is correlated according to

$$dw_1(t)dw_2(t) = \rho dt.$$

Based on this model, [Heston (1993)] provides a closed-form solution for European options. For the remainder of this paper we will refer to this model as the *stochastic variance model.* It will serve as a benchmark which enables us to measure the impact of using volume as a state variable.

In order to replace the unobservable state variable $v(t)$ by the observable variable volume $h(t)$, we assume a simple linear relationship:

$$v(t) = \alpha + \beta h(t). \tag{3.3}$$

Hence, we obtain

$$dS(t) = \mu S(t)dt + \sqrt{\alpha + \beta h(t)}S(t)dw_1(t) \tag{3.4}$$

and, using Ito's lemma,

$$dh(t) = \frac{1}{\beta}\left[\kappa\left[\theta - \alpha - \beta h(t)\right]dt + \sigma\sqrt{\alpha + \beta h(t)}dw_2(t)\right]. \tag{3.5}$$

It can be shown that equations (3.4) and (3.5) lead to a slightly modified version of Heston's call option pricing formula. We will refer to it as the *stochastic volume model*:

$$C(S,h,t) = SP_1 - Ke^{-r(T-t)}P_2 \qquad \text{with}$$

$$P_j(S,h,t;\ln K) = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty Re\left[\frac{e^{-i\phi \ln K} f_j(S,h,t;\phi)}{i\phi}\right]d\phi$$

$$f_j(S,h,t;\phi) = e^{C_j(T-t)+D_j(T-t)(\alpha+\beta h)+i\phi \ln S}$$

$$C_j(t;\phi) = ri\phi(T-t) + \frac{\kappa\theta}{\sigma^2}\left\{(b_j - \rho\sigma i\phi + d)(T-t) - 2\ln\left(\frac{1-ge^{d(T-t)}}{1-g}\right)\right\}$$

$$
\begin{aligned}
D_j(t;\phi) &= \frac{b_j - \rho\sigma i\phi + d}{\sigma^2}\left[\frac{1-e^{d(T-t)}}{1-ge^{d(T-t)}}\right] \\
g &= \frac{b_j - \rho\sigma\phi i + d}{b_j - \rho\sigma\phi i - d} \\
d &= \sqrt{(\rho\sigma i\phi - b_j)^2 - \sigma^2\left(2u_j i\phi - \phi^2\right)} \\
u_1 &= \frac{1}{2},\ u_2 = -\frac{1}{2},\ b_1 = \kappa + \lambda - \rho\sigma,\ b_2 = \kappa + \lambda \\
j &= 1,2.
\end{aligned}
$$

The parameter λ denotes the market price of variance risk induced by $dw_2(t)$.

4. Monte Carlo Simulations

Applying the transformation $y(t) = \ln S(t)$ to equation (3.1) leads to

$$
dy(t) = \left(\mu - \frac{v(t)}{2}\right)dt + \sqrt{v(t)}dw_1(t). \tag{4.1}
$$

We have to transform the continuous-time model into its discrete-time counterpart to run Monte Carlo simulations. In order to value non-path-dependent options, it is sufficient to simulate trajectories which merely need to have final values which are consistent with a given distribution, e.g. the normal distribution. In our case we want to use the whole sample paths to apply the GMM procedure. Therefore, we need a discretization that leads to so-called strong approximations of the trajectories.[9] The Milstein approximation has this property. Its general form is as follows:

$$
dx = adt + bdw \Rightarrow x_t = x_{t-1} + a\Delta t + b\Delta w_t + \frac{1}{2}b\frac{db}{dx}\left[(\Delta w_t)^2 - \Delta t\right].
$$

For the *stochastic variance model,* we derive for the discretized stock return process:[10]

$$
dy(t) = \left(\mu - \frac{v(t)}{2}\right)dt + \sqrt{v(t)}dw_1(t)
$$

[9] See [Hofmann et al. (1992)], p. 169f. and [Kloeden/Platen (1992)].

[10] For equation (4.1), the Milstein approximation leads to the same result as the Euler approximation.

$$\begin{aligned} y_t &= y_{t-1} + \left(\mu - \frac{v_{t-1}}{2}\right)\Delta t + \sqrt{v_{t-1}}\Delta w_{1,t} \\ &= y_{t-1} + \left(\mu - \frac{v_{t-1}}{2}\right)\Delta t + \varepsilon_{1,t} \end{aligned} \tag{4.2}$$

$$\varepsilon_{1,t}\,|v_{t-1} \sim N\left[0, v_{t-1}\Delta t\right],$$

and for the discretized variance process:

$$dv(t) = \kappa\left[\theta - v(t)\right]dt + \sigma\sqrt{v(t)}dw_2(t)$$

$$\begin{aligned} v_t &= v_{t-1} + \left(\kappa\left[\theta - v_{t-1}\right] - \frac{1}{4}\sigma^2\right)\Delta t + \sigma\sqrt{v_{t-1}}\Delta w_{2,t} + \frac{1}{4}\sigma^2\left(\Delta w_{2,t}\right)^2 \qquad (4.3) \\ &= v_{t-1} + \left(\kappa\left[\theta - v_{t-1}\right] - \frac{1}{4}\sigma^2\right)\Delta t + \varepsilon_{2,t} \\ &= v_{t-1} + \kappa^*\left[\theta - v_{t-1}\right] - \frac{1}{4}\sigma^{*2} + \varepsilon_{2,t} \qquad (4.4) \end{aligned}$$

$$\tfrac{4}{\sigma^{*2}}\left(\varepsilon_{2,t} + v_{t-1}\right)|v_{t-1} \sim \chi_1'^2\left(\tfrac{4}{\sigma^{*2}}v_{t-1}\right).^{11}$$

If we used the Euler approximation to derive the discrete variance process, $\varepsilon_{2,t}$ would have a normal distribution according to $\varepsilon_{2,t}\,|v_{t-1} \sim N\left[0, \sigma^{*2}v_{t-1}\right]$. Locally, this would not be consistent with the stochastic process given in equation (3.2). There the conditional distribution of $v(t+\Delta t)$ given $v(t)$ belongs to the class of non-central χ^2-distributions[12], as is the case for (4.3).

For the *stochastic volume model,* the derivation of the discrete stochastic processes proceeds as described above. This leads to the following system of equations, using the transformation $y = ln(S)$:

$$dy(t) = \left(\mu - \frac{\alpha + \beta h(t)}{2}\right)dt + \sqrt{\alpha + \beta h(t)}dw_1(t) \tag{4.5}$$

[11] See Appendix.

[12] See e.g. [Cox et al. (1985)] and [Chen (1995)]. For the properties of the non-central χ^2-distribution, see [Johnson/Kotz (1970)], chapter 28.

$$
\begin{aligned}
y_t &= y_{t-1} + \left(\mu - \frac{\alpha + \beta h_{t-1}}{2}\right)\Delta t + \sqrt{\alpha + \beta h_{t-1}}\Delta w_{1,t} \qquad (4.6)\\
&= y_{t-1} + \left(\mu - \frac{\alpha + \beta h_{t-1}}{2}\right)\Delta t + \varepsilon_{1,t}
\end{aligned}
$$

$$
\varepsilon_{1,t}\,|h_{t-1} \sim N\left[0, (\alpha + \beta h_{t-1})\,\Delta t\right]
$$

$$
\begin{aligned}
h_t &= h_{t-1} + \frac{1}{\beta}\left(\kappa\left[\theta - \alpha - \beta h_{t-1}\right] - \frac{1}{4}\sigma^2\right)\Delta t + \frac{\sigma}{\beta}\sqrt{\alpha + \beta h_{t-1}}\Delta w_{2,t} \qquad (4.7)\\
&\quad + \frac{1}{4\beta}\sigma^2\left(\Delta w_{2,t}\right)^2\\
&= h_{t-1} + \frac{\kappa^*}{\beta}\left[\theta - \alpha - \beta h_{t-1}\right] - \frac{1}{4\beta}\sigma^{*2} + \varepsilon_{2,t}
\end{aligned}
$$

$$
\frac{4}{\sigma^{*2}}\left(\beta\varepsilon_{2,t} + \alpha + \beta h_{t-1}\right)|h_{t-1} \sim \chi_1'^2\left(\frac{4}{\sigma^{*2}}\left(\alpha + \beta h_{t-1}\right)\right).
$$

In equations (4.2), (4.3), (4.6), and (4.7), which we use to simulate the sample paths, the variable t represents the discrete points within the time interval $[1, ..., T]$. We choose time intervals of one day which correspond to a value of .004 for Δt. To avoid any dependence from the starting values, we only use the second half of the simulated sample paths which have an overall length of $T = 1000$ data points. In other words, the trajectories consist of 500 data points, therefore representing a time horizon of two years. That is the same size as the empirical time series used in this study. We apply the GMM procedure to 5000 simulated time series to get reliable results. This enables us to draw conclusions on how the estimates are distributed.

5. The Moments

We apply the Generalized Method of Moments, developed by [Hansen (1982)], to estimate the parameters. In order to ease computational burden, we use the Euler approximation to derive the moment restrictions. As the results from the Monte Carlo simulations will show, the parameters can be estimated when using the Euler-based moments.

For the *stochastic variance model,* the discrete-time Euler approximation of (3.2) is given by

$$\begin{aligned} v_t &= v_{t-1} + \kappa(\theta - v_{t-1})\Delta t + \sigma\sqrt{v_{t-1}}\Delta w_{2,t} \\ &= v_{t-1} + \kappa\theta\Delta t - \kappa v_{t-1}\Delta t + \epsilon_{2,t} \\ &= v_{t-1} + a_{var} - \kappa^*_{var} v_{t-1} + \epsilon_{2,t}, \qquad t = 1, ..., T. \end{aligned} \tag{5.1}$$

$\epsilon_{1,t}$ and $\epsilon_{2,t}$ are described by a bivariate normal distribution of the form

$$\underline{\varepsilon}_t \,|v_{t-1} \sim N(\underline{0}, \Sigma), \qquad \text{with } \underline{\varepsilon}_t = \begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{pmatrix}, \Sigma = \begin{pmatrix} v_{t-1}\Delta t & \rho\sigma^*_{var} v_{t-1}\sqrt{\Delta t} \\ \rho\sigma^*_{var} v_{t-1}\sqrt{\Delta t} & \sigma^{*2}_{var} v_{t-1} \end{pmatrix}.$$

Next we derive the moments which are applied in the estimation procedure. Using the discrete-time approximation of the model as given in equations (4.2) and (5.1), we define

$$u_{1,t} = y_t - y_{t-1} - \mu\Delta t - \frac{v_{t-1}}{2}\Delta t, \qquad t = 1, ..., T$$

and

$$u_{2,t} = v_t - v_{t-1} - a_{var} + \kappa^*_{var} v_{t-1}, \qquad t = 1, ..., T$$

with

$$E\left[u_{1,t}\,|v_{t-1}\right] = E\left[u_{2,t}\,|v_{t-1}\right] = 0, \quad E\left[u^2_{1,t}\,|v_{t-1}\right] = v_{t-1}\Delta t, \quad E\left[u^2_{2,t}\,|v_{t-1}\right] = \sigma^{*2}_{var} v_{t-1}.$$

We choose the subsequent specifications for the moments:

$$f_t(\varphi) = \begin{bmatrix} u_{1,t} \\ u_{2,t} \\ u_{1,t}y_{t-1} \\ u_{2,t}v_{t-1} \\ u^2_{1,t} - v_{t-1}\Delta t \\ u^2_{2,t} - \sigma^{*2}_{var} v_{t-1} \\ \left(u^2_{1,t} - v_{t-1}\Delta t\right) y_{t-1} \\ \left(u^2_{2,t} - \sigma^{*2}_{var} v_{t-1}\right) v_{t-1} \\ u_{1,t}u_{2,t} - \rho\sigma^*_{var} v_{t-1}\sqrt{\Delta t} \\ \left(u_{1,t}u_{2,t} - \rho\sigma^*_{var} v_{t-1}\sqrt{\Delta t}\right) y_{t-1} \\ \left(u_{1,t}u_{2,t} - \rho\sigma^*_{var} v_{t-1}\sqrt{\Delta t}\right) v_{t-1} \end{bmatrix}, \quad \varphi = \begin{bmatrix} \mu \\ \rho \\ a_{var} \\ \kappa^*_{var} \\ \sigma^*_{var} \end{bmatrix}.$$

For the *stochastic volume model,* we also use the Euler approximation to construct the moment restrictions. For the purpose of comparison, we use the same type of moments. The discrete-time approximations of (4.5) and (3.5) are given by

$$y_t = y_{t-1} + \left(\mu - \frac{\alpha + \beta h_{t-1}}{2}\right)\Delta t + \varepsilon_{1,t}, \quad t = 1, ..., T$$

$$\begin{aligned} h_t &= h_{t-1} + \frac{1}{\beta}\left[\kappa\left(\theta - \alpha - \beta h_{t-1}\right)\Delta t + \sigma\sqrt{\alpha + \beta h_{t-1}}\Delta w_{2,t}\right] \\ &= h_{t-1} + \frac{\kappa\theta}{\beta}\Delta t - \frac{\kappa}{\beta}\Delta t\left(\alpha + \beta h_{t-1}\right) + \varepsilon_{2,t} \\ &= h_{t-1} + a_{vol} - \kappa^*_{vol}\left(\alpha + \beta h_{t-1}\right) + \varepsilon_{2,t}, \quad t = 1, ..., T. \end{aligned}$$

$\varepsilon_{1,t}$ and $\epsilon_{2,t}$ are distributed according to

$$\underline{\varepsilon}_t \,|h_{t-1} \sim N(\underline{0}, \Sigma),$$

$$\text{with } \underline{\varepsilon}_t = \begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{pmatrix}, \Sigma = \begin{pmatrix} \left(\alpha + \beta h_{t-1}\right)\Delta t & \rho\sigma^*_{vol}\left(\alpha + \beta h_{t-1}\right)\sqrt{\Delta t} \\ \rho\sigma^*_{vol}\left(\alpha + \beta h_{t-1}\right)\sqrt{\Delta t} & \sigma^{*2}_{vol}\left(\alpha + \beta h_{t-1}\right) \end{pmatrix}.$$

The empirical values of the residuals can be calculated as follows:

$$u_{1,t} = y_t - y_{t-1} - \mu\Delta t - \frac{\alpha + \beta h_{t-1}}{2}\Delta t, \qquad t = 1, ..., T$$

$$u_{2,t} = h_t - h_{t-1} - a_{vol} + \kappa^*_{vol}\left(\alpha + \beta h_{t-1}\right), \qquad t = 1, ..., T$$

with

$$E\left[u_{1,t}\,|h_{t-1}\right] = E\left[u_{2,t}\,|h_{t-1}\right] = 0$$

$$E\left[u^2_{1,t}\,|h_{t-1}\right] = \left(\alpha + \beta h_{t-1}\right)\Delta t, \quad E\left[u^2_{2,t}\,|h_{t-1}\right] = \sigma^{*2}_{vol}\left(\alpha + \beta h_{t-1}\right).$$

This leads to the following moment restrictions:

$$f_t(\varphi) = \begin{bmatrix} u_{1,t} \\ u_{2,t} \\ u_{1,t}y_{t-1} \\ u_{2,t}h_{t-1} \\ u_{1,t}^2 - (\alpha + \beta h_{t-1})\,\Delta t \\ u_{2,t}^2 - \sigma_{vol}^{*2}(\alpha + \beta h_{t-1}) \\ \left(u_{1,t}^2 - (\alpha + \beta h_{t-1})\,\Delta t\right) y_{t-1} \\ \left(u_{2,t}^2 - \sigma_{vol}^{*2}(\alpha + \beta h_{t-1})\right) h_{t-1} \\ u_{1,t}u_{2,t} - \rho\sigma_{vol}^{*}(\alpha + \beta h_{t-1})\sqrt{\Delta t} \\ \left(u_{1,t}u_{2,t} - \rho\sigma_{vol}^{*}(\alpha + \beta h_{t-1})\sqrt{\Delta t}\right) y_{t-1} \\ \left(u_{1,t}u_{2,t} - \rho\sigma_{vol}^{*}(\alpha + \beta h_{t-1})\sqrt{\Delta t}\right) h_{t-1} \end{bmatrix}, \quad \varphi = \begin{bmatrix} \mu \\ \rho \\ a_{vol} \\ \kappa_{vol}^{*} \\ \sigma_{vol}^{*} \\ \alpha \\ \beta \end{bmatrix}.$$

Of course, the number of moments and in particular their specification are the most crucial decisions to be made when using GMM. According to [Andersen/Sørensen (1996)], the optimal number of moments "... depends critically on sample size."[13] This is due to the fact that the estimates from GMM are merely asymptotically efficient. Thus, the properties of estimates from different sample sizes can vary. Hence, it is our opinion that only by applying GMM to simulated time series, can we study the properties of the estimates for a given model and a given length of the time series.

Assuming that our specification described above is true, $\mathrm{E}[f_t(\varphi)]$ has to be zero. Using empirical data, we get the corresponding sample counterpart $g_T(\varphi)$ which is defined as

$$g_T(\varphi) = \frac{1}{T}\sum_{t=1}^{T} f_t(\varphi) \;, \qquad t = 1, ..., T.$$

Minimizing the quadratic form

$$J_T(\varphi) = g_T'(\varphi)\, W_T(\varphi) g_T(\varphi),$$

where $W_T(\varphi)$ is a positive-definite symmetric weighting matrix, enables us to estimate the parameters μ, ρ, a_{var}, κ_{var}^{*}, and σ_{var}^{*}. Using the approach advocated by [Newey/West (1987)],

[13] [Andersen/Sørensen (1996)], p. 329.

$W_T(\varphi)$ depends on the estimated values of φ. In a first step, we use $W_T(\varphi) = I$ to obtain an inefficient estimate of φ. In a second step, we use this estimate to get an efficient estimate of φ. This is done recursively until the procedure converges.[14]

Due to the results in [Nagel/Schöbel (1996)], the number of sample autocovariances to be used in the calculation of $W_T(\varphi)$ is chosen to be 3.[15]

All calculations are done with GAUSS, using the Davidon-Fletcher-Powell optimization algorithm. We want to emphasize that we use an unconstrained optimization routine, i.e. we do not restrict the estimated parameter values in any way. By doing so, we can check whether the setup of the models is consistent with empirical data.

6. GMM results from simulated data

6.1. Stochastic variance model

In order to run the simulations, we use the following parameter values which are chosen to be consistent with estimates from empirical data:

S_o	$\sqrt{v_o}$	μ	ρ	κ	$\sqrt{\theta}$	σ	dt
100	.15	.1	$-.1$	4	.15	.20	.004

Table 6.1: Parameter values used for running Monte Carlo simulations.

In order to make the results easier to compare we not only report results for the originally estimated parameters, but also for the transformed ones which are used as input variables of the models. In Table 6.2 these are $\widehat{\kappa}$, $\sqrt{\widehat{\theta}}$, and $\widehat{\sigma}$. We do not estimate them directly, since the use of a reduced form of the model as described in (4.4) leads to better estimations.

As Table 6.2 shows, we are able to estimate the parameter values with sufficient precision. On average, all estimates except of $\widehat{\kappa}^*_{var}$ and $\widehat{a}_{vol}$ are almost exactly at their true values. The parameter which leads to the most significant problems in regard to its estimation is κ^*_{var}.

[14] For details see e.g. [Ferson/Foerster (1994)] and [Hansen et al. (1996)].

[15] The same number of lags is chosen by [Bühler/Grünbichler (1996)], who also use empirical time series of length $T = 500$.

The first problem arises from its biased estimation. On average, the values of $\widehat{\kappa}^*_{var}$ are too large.[16] But since the sensitivity of the model with respect to κ is low, this problem is of secondary importance. The next problem arises from the fact that $\widehat{\kappa}^*_{var}$ can become negative. This is due to the fact that the time series consist of only 500 data points. Thus, it can happen that a given trajectory shows the behavior of a random walk with a negative trend rather than the characteristics of a mean-reverting process. The smaller the true value of κ is, the more often its estimate is negative. One possibility to avoid this problem is to use longer time series. But as it is shown in [Nagel/Schöbel (1996)], longer empirical time series do not necessarily lead to more precise estimations because the parameter values themselves are not constant over time. The latter result can also be found in [Bühler/Grünbichler (1996)].

Another interesting point is the interdependence between the estimates of κ^*_{var} and a_{var}. Table 6.2 clearly shows that both estimates are too large on average. Looking at $\widehat{\kappa}$ and $\widehat{\theta}$, things are different. In particular for $\widehat{a}_{var}$, the bias is reduced by some 50% to about .8% for $\sqrt{\widehat{\theta}}$. Since $\widehat{a}_{var}$ has to be transformed according to $\theta = \frac{a_{var}}{\kappa^*_{var}}$ to obtain $\widehat{\theta}$, the bias in the estimation of a_{var} is obviously mainly due to the problems with the estimation of κ^*_{var}. This is confirmed by [Nagel/Schöbel (1996)], who estimate the parameter θ separately. Their results show that the estimation of θ is unbiased.[17] Another point we would like to mention is that negative values of $\widehat{a}_{var}$ only occur when κ^*_{var} is negative as well.[18]

As already stated, the estimated parameter values are not restricted in any way. Therefore, it is theoretically possible to get values for $\widehat{\rho}$ which are not within the interval $[-1, 1]$. As we can see in Table 6.2 this never occurs. Hence, our choice of moments appears to be very robust in this regard.

Using GMM, one also obtains estimates for the standard deviations of the estimated parameter values. They can be used to calculate t-statistics to test for the significance of

[16] This problem is confirmed by [Ball/Torous (1996)], who apply the GMM to simulated interest rates which follow a square-root process as proposed by [Cox et al. (1985)]. Their results also show that $\widehat{\kappa}$ is upward biased.

[17] The same is found by [Ball/Torous (1996)]. To estimate the long-run level θ, they also use a reduced model. Although their estimates of κ are upward biased, the resulting values of $\widehat{\theta}$ are not.

[18] Here negative values of $\widehat{\kappa}^*_{var}$ are obtained 7 times. For 3 of these estimations, the corresponding values of $\widehat{a}_{var}$ are negative as well.

	True value	Mean	Minimum	Maximum	Standard deviation	Mean of estimated standard deviations
$\widehat{\mu}$	.1000	.0979	−.3280	.5136	.1112	.1013
$\widehat{\rho}$	−.1000	−.0999	−.2756	.0714	.0469	.0433
$\widehat{\kappa}^*_{var}$	.0016	.0261	−.0102	.1137	.0127	.0100
$\widehat{a}_{var}$	.0004	.0006	−.0001	.0035	.0003	.0003
$\widehat{\sigma}^*_{var}$	.0126	.0125	.0111	.0143	.0004	.0004
$\widehat{\kappa}$	4	6.5320	−2.5625	28.4372	3.1770	–
$\sqrt{\widehat{\theta}}$	.1500	.1488	.0854	.5009	.0200	–
$\widehat{\sigma}$	.2000	.1981	.1749	.2260	.0066	–

Table 6.2: Summary statistics of parameter values estimated from simulated time series.

the parameter values. Table 6.2 shows that the estimated standard deviations all perform reasonably well compared to the actual calculated standard deviation of the estimates.[19]

A test which is often applied when using GMM is the χ^2 test for goodness of fit of the overidentifying restrictions. The test statistic is distributed according to $\chi^2(q)$. q is the difference between the number of moments M minus the number of parameters to be estimated N. For the stochastic variance model, $M = 11$ and $N = 5$, i.e. the test statistic's appropriate distribution is of type $\chi^2(6)$. [Andersen/Sørensen (1996)] show that the test statistic and its p-value depend crucially on the number of moments and the length of the time series. Using their results, we report and discuss the main characteristics of the χ^2 test in the context of our setup.

In theory, p-values of test statistics are uniformly distributed. As Figure 6.1 shows, this is not the case for the p-values of the χ^2 test when applying our test procedure. The distribution of the p-values is skewed to the left. This could be avoided by using shorter time series or more moments, which leads to a rightward shift in the distribution of the p-values. But reducing the length of the time series results in a decreased quality of the estimations. Since this is of primary concern, we reject this approach. An increase in the number of moment restrictions can be harmful because this has to be chosen in accordance with the sample size. Including too many moments "... is generally not optimal ... if the sample size is limited"[20]. Since we

[19] For a more detailed examination of the properties of the t-statistics, see [Nagel/Schöbel (1996)].

[20] [Andersen/Sørensen (1996)], p. 349.

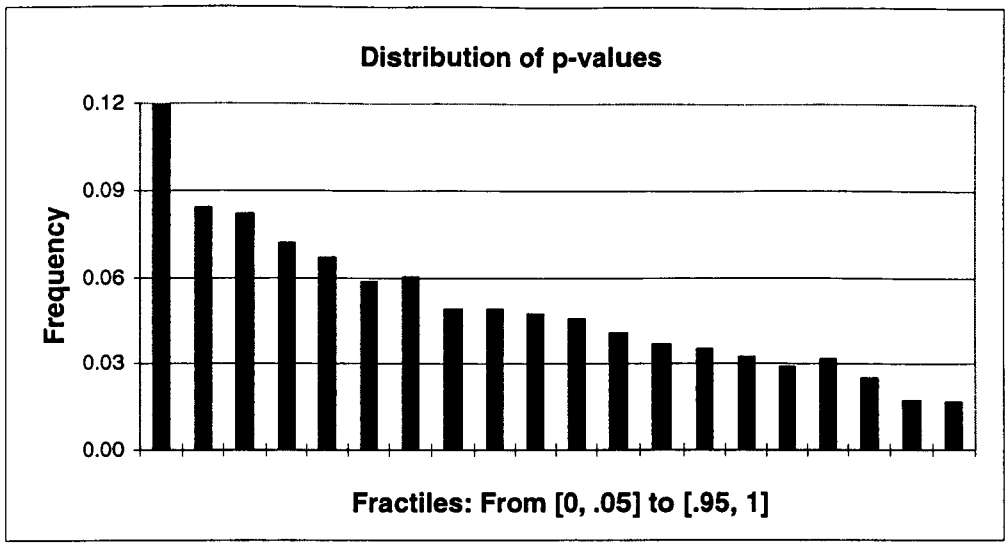

Figure 6.1: Distribution of p-values based on the χ^2 test of overidentifying restrictions for the stochastic variance model.

work with short time series, we restrict ourselves to moments which reflect the main features of the models under consideration.

In order to provide additional insight into the distribution of the p-values, we refer to Table 6.3. We can see that 2.8% of all time series lead to p-values which are within the interval [0, 0.01]. If the conditions for the χ^2 test were optimal, we would expect 1% of the time series to result in p-values of this magnitude. Sorting the p-values in increasing order, the last column shows the 50th, 250th, 500th, 1000th, and 2500th p-value. In other words, these are the small sample p-values for the given quantiles. The mean of the p-values is .367. If the

p^*	Percentage of time series with $p \leq p^*$	small sample p-value for the p^* quantile
.01	2.8	.0032
.05	11.9	.0187
.10	20.3	.0402
.20	35.7	.0979
.50	68.9	.3148

Table 6.3: p-values obtained from the χ^2 test for goodness of fit of the overidentifying restrictions.

p-values were uniformly distributed, the mean should be at about .5. Hence, the χ^2 test for goodness of fit of the overidentifying restrictions is conservative in the context of our setup. This means that the hypothesis that our model correctly describes the data is rejected too often. Keeping these results in mind, we can still use the p-values to interpret the results.

6.2. Stochastic volume model

In contrast to the stochastic variance model, we have to estimate 7 parameters. Their values are chosen to correspond to the simulations of the stochastic variance model.

S_o	h_o	μ	ρ	κ	$\sqrt{\theta}$	σ	α	β	dt
100	1	.1	$-.1$	4	.15	.20	0	.0225	.004

Table 6.4: Parameter values used for running Monte Carlo simulations.

The most important parameter is β since its precise estimation is crucial to the calculation of κ, θ, and σ. As Table 6.5 shows, there is a tendency toward underestimating β. On the other hand, $\widehat{\alpha}$ is too large on average, although, compared to the corresponding standard deviation,

	True value	Mean	Minimum	Maximum	Standard deviation	Mean of estimated standard deviations
$\widehat{\mu}$	.1000	.0998	$-.3102$	.5126	.1085	.1021
$\widehat{\rho}$	$-.1000$	$-.0982$	$-.2518$	.0744	.0465	.0442
$\widehat{\kappa}^*_{vol}$	.7111	1.3036	$-.1541$	5.7916	.7126	.6044
$\widehat{a}_{vol}$	.0160	.0273	.0014	.1348	.0151	.0122
$\widehat{\sigma}^*_{vol}$	.5622	.5668	.4667	.6749	.0277	.0265
$\widehat{\alpha}$	.0000	.0011	$-.0184$	.0197	.0039	.0035
$\widehat{\beta}$	.0225	.0207	.0075	.0386	.0042	.0038
$\widehat{\kappa}$	4	6.4147	$-.7103$	23.6244	2.9730	–
$\sqrt{\widehat{\theta}}$	.1500	.1468	.0944	.6965	.0224	–
$\widehat{\sigma}$	.2000	.1851	.0644	.3683	.0377	–

Table 6.5: Summary statistics of parameter values estimated from simulated time series.

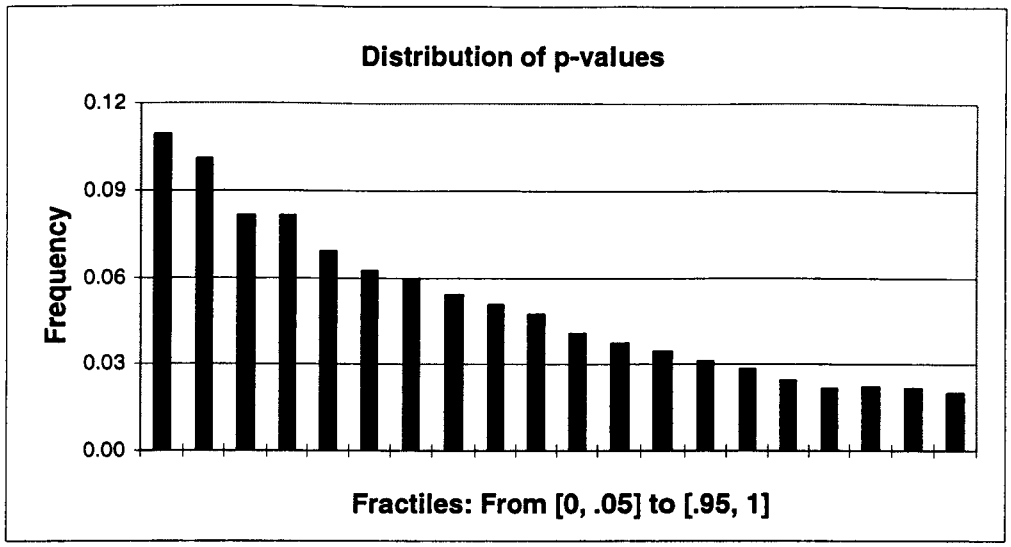

Figure 6.2: Distribution of p-values based on the χ^2 test of overidentifying restrictions for the stochastic volume model.

this error is marginal. We also observe that the estimation of σ^*_{vol} is very precise on average, but due to the underestimation of β the values of $\widehat{\sigma}$ are slightly too low. For $\widehat{\kappa}^*_{vol}$ and $\widehat{a}_{vol}$, we face the same problems as in case of the stochastic variance model. But in contrast to $\widehat{a}_{var}$ we never get negative values for $\widehat{a}_{vol}$ and the number of negative estimates of κ^*_{vol} is reduced to 4.

As for the stochastic volume model, the distribution of the p-values calculated using the χ^2 test of overidentifying restrictions is skewed to the left. Figure 6.2 and Table 6.6 show that the main characteristics are very similar, although the shift itself is less pronounced.

p^*	Percentage of time series with $p \leq p^*$	small sample p-value for the p^* quantile
.01	1.9	.0056
.05	10.9	.0237
.10	21.0	.0444
.20	37.3	.0948
.50	71.7	.2953

Table 6.6: p-values obtained from the χ^2 test for goodness of fit of the overidentifying restrictions.

7. Empirical Results

7.1. Data Description

The empirical work is based on data from the German stock market. The option prices which we seek to explain are those of options on the German stock index DAX. Two aspects make them very attractive to empirical research: the DAX is a performance index, i.e. it is adjusted for dividend payments, and DAX-options are of European type.

We use data that covers the period from January 2, 1992 until December 23, 1994 to estimate the parameters by aid of GMM. As a first input variable for the GMM estimations, we use the daily DAX values reported at 1.30 p.m. In order to generate the time series necessary for the estimation of the stochastic variance model, we first calculate daily returns of the DAX. In a second step, we calculate the annualized variance of the returns over the last 2, 10, 22, and 33 days. Thus, we have 4 time series to test the stochastic variance model.[21] They are labeled *Var2, Var10, Var22,* and *Var33.*

With regard to volume data, we face the problem that the DAX is not a traded asset. Hence, we can only measure a proxy of its daily volume. Since the DAX consists of the 30 most important and liquid German stocks which account for more than 75% of the daily trading activities, we use aggregated volume data for all German stocks. More precisely, we use the daily number of transactions in German stocks as reported in the Handelsblatt. This is motivated by the results of [Jones et al. (1994)]. They show that a significant part of stock return volatilities can be explained by the transactions per day, rather than by the trading volume itself. A possible explanation is given in [Chan et al. (1995)]. They report that not the volume per transaction, but rather the number of transactions shows a U-shaped form during a day.[22] This is exactly what is found in many studies examining intraday volatilities.[23] Similar results are reported by [Abhyankar et al. (1997)], who also find that

[21] We also used the squared daily stock returns as a time series describing the variance. But in this case the GMM procedure did not converge.

[22] [Kirchner/Schlag (1996)] examine German stocks and find the same pattern for the frequency of transactions.

[23] See e.g. [Wood et al. (1985)], [Gwilym et al. (1997)], and [Abhyankar et al. (1997)]. [Röder (1996)] and [Kirchner/Schlag (1996)] confirm this result for the variance of intraday returns of the DAX and the intraday

the average transaction volume is relatively flat during the day.[24] [Kirchner (1996)] confirms these results for the trading volume of German stocks.

We also incorporate the trading volume of the options market when examining the role of trading activities on option prices. We use the daily average number of DAX options traded at the DTB. The latter are derived as follows: first, we calculate the sum of all traded call option contracts across the different exercise prices. The same is done for the put options. Next, we divide the results by the number of exercise prices. By doing so, we account for the fact that the number of exercise prices and hence the total number of option contracts varies over time. In a final step, we calculate the sum of the daily average number of call and put options, which leads to a time series for daily option volume. We also calculate the moving averages of the stock and option volume time series over the last 2, 10, 22, and 33 trading days. In other words, we have a total of 10 time series to measure trading activities. We label the time series which are computed from the number of transactions *NoT1, NoT2, NoT10, NoT22,* and *NoT33.* Equivalently, we label the option volume time series *OV1, OV2, OV10, OV22,* and *OV33.*[25] These time series are transformed such that the mean of every time series is one. This is necessary to obtain convergence of the GMM procedure.[26]

We use empirical time series with 500 data points to estimate the parameters. The comparison of the theoretical and empirical option prices is made for 1994. More precisely, it is the period from February 10, 1994 until December 23, 1994, which is made up of 220 trading days.[27] The total number of call (put) option prices for this period is 18341 (18341). We only use call (put) prices which obey the restriction $C \geq S - Ke^{-r(T-t)}$ ($P \geq Ke^{-r(T-t)} - S$).[28]

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volatility of German stocks, respectively.

[24] [Abhyankar et al. (1997)], p. 360.

[25] Possible problems which can arise from a *simultaneous equations bias* as described e.g. in [Hamilton (1994)] are not considered here.

[26] Although this procedure seems to be a bit arbitrary, it is correct. This is due to the fact that all parameters, including β, are estimated simultaneously. Hence, different linear transformations of the original time series only lead to different estimates of β. Using the transformed data with the corresponding $\widehat{\beta}$ leads to the same option prices, regardless of how the original time series are transformed.

[27] The first 33 trading days in 1992 are used to calculate the first data points for the time series *Var33*, *NoT33*, and *OV33*.

[28] The maximum value of $Ke^{-r(T-t)}$ for put prices is never exceeded for our sample.

After accounting for these minimum values, we have a total of 17275 calls (17831 puts) left. To avoid problems which could occur due to short times to maturity, we also exclude all options which have less than 14 days left before expiration. This further reduces the number of calls (puts) to 15654 (16057).

In order to calculate the theoretical option prices, we use daily IBIS-DAX data reported at 5 p.m. This is because at that time the Deutsche Terminbörse (DTB) calculates the settlement prices of all available DAX-options which are based on the DAX at that time. By using these option prices we avoid the well-known problems with non-synchronous data. In addition the daily trading volume is known at that time, too. The interest rates which we use are the 1-, 3-, and 6-month interbank interest rates which are inter- and extrapolated, respectively, to match the given times to maturity of the options.

7.2. Parameter estimations from historical data

As described in the previous chapter, we use different time series to estimate the parameters. Thus, we first want to look at the estimation results for these time series. The right column of Table 7.1 reports the average p-values of the χ^2 test of overidentifying restrictions. Based on these values, we cannot reject the hypothesis that the data is correctly described by the models. Using the small sample p-values given in Tables 6.6 and 6.3 the level of significance is at least 10% for all time series.

As for the parameter values themselves, we first want to have a look at the values of $\widehat{\kappa}$ and $\widehat{\sigma}$. Table 7.2 shows that decreasing the number of days used to calculate a single data point of the time series leads to larger values of $\widehat{\kappa}$ and $\widehat{\sigma}$. This can be explained by the fact that taking the average of more data results in time series which show a higher persistence. But what are the implications of this result? Assuming that equation 3.2 (3.5) correctly describes the *instantaneous* variance (volume) process, one should expect the time series *Var2 (NoT1, OV1)* to be most consistent with the model. But having a large value of κ implies that the current value of v is almost irrelevant to the option price, since the variance (volume) will be pulled back to its long-run level θ at a very high rate. The same problem arises with regard to σ. Although large values of $\widehat{\kappa}$ are accompanied by large values of $\widehat{\sigma}$, comparative

	$\widehat{\mu}$	$\widehat{\rho}$	$\widehat{\kappa}^*_{var}$, $\widehat{\kappa}^*_{vol}$	$\widehat{a}_{var}$, $\widehat{a}_{vol}$	$\widehat{\sigma}^*_{var}$ / $\widehat{\sigma}^*_{vol}$	$\widehat{\alpha}$	$\widehat{\beta}$	p
Var2	.1397	.0068	.6697	.0132	.2035	–	–	.481
	(.0414)	(.0214)	(.0146)	(.0012)	(.0107)			
Var10	.1381	.0206	.0647	.0016	.0367	–	–	.606
	(.0499)	(.0596)	(.0069)	(.0002)	(.0007)			
Var22	.1193	−.0546	.0178	.0004	.0157	–	–	.482
	(.0476)	(.0564)	(.0057)	(.0001)	(.0007)			
Var33	.1631	−.0440	.0098	.0003	.0111	–	–	.663
	(.0554)	(.0779)	(.0038)	(.0001)	(.0004)			
NoT1	.1424	.1520	9.0866	.1962	.9431	−.0055	.0262	.355
	(.0619)	(.0288)	(2.0457)	(.0521)	(.0236)	(.0093)	(.0082)	
NoT2	.1233	.0889	2.8638	.0606	.5709	−.0120	.0324	.411
	(.0562)	(.0251)	(.5826)	(.0139)	(.0201)	(.0096)	(.0087)	
NoT10	.1254	.0663	.1722	.0034	.1525	−.0191	.0390	.229
	(.0612)	(.0180)	(.0549)	(.0011)	(.0044)	(.0048)	(.0039)	
NoT22	.1304	.0854	.0065	.0001	.0806	−.0151	.0352	.145
	(.0573)	(.0113)	(.0336)	(.0007)	(.0023)	(.0078)	(.0068)	
NoT33	.1439	.0660	.0246	.0004	.0542	−.0138	.0341	.104
	(.0464)	(.0250)	(.0575)	(.0013)	(.0015)	(.0182)	(.0166)	
OV1	.1473	.0160	15.1367	.3200	2.0638	−.0028	.0267	.314
	(.0543)	(.0427)	(1.3520)	(.0341)	(.0799)	(.0028)	(.0023)	
OV2	.1581	.0211	5.5872	.1173	1.3152	−.0075	.0318	.249
	(.0463)	(.0387)	(.6603)	(.0173)	(.0298)	(.0006)	(.0012)	
OV10	.1487	.0102	.5342	.0114	.3334	−.0114	.0360	.166
	(.0373)	(.0135)	(.1337)	(.0033)	(.0115)	(.0051)	(.0045)	
OV22	.1302	−.0231	.0962	.0026	.1422	−.0161	.0409	.081
	(.0440)	(.0201)	(.0566)	(.0013)	(.0059)	(.0050)	(.0042)	
OV33	.1651	.0260	.0833	.0024	.1019	−.0100	.0348	.066
	(.0540)	(.0231)	(.0583)	(.0013)	(.0029)	(.0038)	(.0035)	

Table 7.1: Estimated parameter values from empirical time series for the stochastic variance and stochastic volume model. The standard deviations are given in parenthesis.

	$\widehat{\kappa}$	$\widehat{\theta}$	$\widehat{\sigma}$
Var2	167.4298	.0198	3.2179
	(3.6516)	(.0014)	(.1695)
Var10	16.1784	.0245	.5796
	(1.7136)	(.0013)	(.0108)
Var22	4.4473	.0250	.2486
	(1.4218)	(.0016)	(.0106)
Var33	2.4613	.0268	.1748
	(.9476)	(.0030)	(.0070)
NoT1	55.4216	.0214	.3922
	(2.5391)	(.0015)	(.1285)
NoT2	21.9711	.0210	.2949
	(1.2038)	(.0013)	(.0891)
NoT10	1.6936	.0199	.0943
	(.5946)	(.0038)	(.0115)
NoT22	.0855	−.0576	.0450
	(.2743)	(.8697)	(.0098)
NoT33	.0965	.0252	.0289
	(.4261)	(.0283)	(.0132)
OV1	100.835	.0211	.8725
	(10.7775)	(.0011)	(.0995)
OV2	44.3428	.0209	.6618
	(4.2902)	(.0013)	(.0206)
OV10	4.7495	.0214	.1907
	(1.0406)	(.0026)	(.0304)
OV22	.9823	.0128	.0922
	(.5606)	(.2105)	(.0133)
OV33	.6988	.0226	.0562
	(.4654)	(.1874)	(.0068)

Table 7.2: Estimated parameter values from empirical time series for the stochastic variance and stochastic volume model. The standard deviations are given in parenthesis.

statics analysis reveals that the latter cannot offset the influence of $\widehat{\kappa}$. In other words, if κ is large, the corresponding value of σ has only a marginal impact on the option price. The only parameter which still matters is θ, since it determines the average level of v and h, respectively. One possible explanation for the behavior of these time series can be found in [Merville/Pieptea (1989)]. They show that "... the mean-reverting hypothesis is correct, but in addition (we) find an important discrete noise component in the volatility process."[29]

Looking at the variance time series which are more likely to fit to the corresponding model, we see that, on average, the values of $\widehat{\rho}$ are negative. This implies a negative correlation between the percentage changes of the DAX and its variance. Such a relationship is often reported in other empirical studies as well.[30] Using volume data, things are different. With the exception of the time series *OV22,* the average values of $\widehat{\rho}$ are positive. This is also a result that is often found in empirical works.[31]

A closer look at the values of θ reveals that for *NoT22* and *OV22* we obtain very small values as compared to the other time series. In particular the negative value for *NoT22* is not consistent with the model. This is due to the very small values of $\widehat{\kappa}$. From chapter 3.4., we already know that the estimation of a small κ can result in a negative $\widehat{\kappa}$. Hence, in a second step we check whether all estimated parameter values obey the restrictions $-1 \leq \widehat{\rho} \leq 1$, $\widehat{\kappa} > 0$, $\widehat{\theta} > 0$, $\widehat{\sigma} > 0$ and $\widehat{\alpha} + \widehat{\beta} h > 0$. If one of the daily estimated parameters violates these boundaries, the estimation results of this day are excluded from the sample for this time series. The resulting average values of the parameter estimates are reported in Table 7.3. Here all values of $\sqrt{\widehat{\theta}}$ are within a reasonable range of about 14% to 18% p.a. This includes the results for the volume time series. In other words, our choice of the transformation process as given in equation (3.3) leads to estimates of $\sqrt{\theta}$ which correspond to the results of the variance time series. This is an important result regarding the plausibility of the chosen transformation. The right column of Table 7.3 reports the number of days for which the above restrictions are not violated. Looking at the results for $\widehat{\kappa}$ in Table 7.2 and the number of days in Table 7.3, one can easily see that smaller values of $\widehat{\kappa}$ lead to more violations of

[29] [Merville/Pieptea (1989)], p. 195.

[30] See e.g. [Schmalensee/Trippi (1978)], [Christie (1982)], and [Nelson (1991)].

[31] See e.g. [Karpoff (1987)] and [Majnoni/Massa (1996)].

	$\widehat{\rho}$	$\widehat{\kappa}$	$\sqrt{\widehat{\theta}}$	$\widehat{\sigma}$	$\widehat{\alpha}$	$\widehat{\beta}$	# of days
Var2	.0068	167.4298	.1405	3.2179	–	–	220
	(.0214)	(3.6516)	(.0052)	(.1695)			
Var10	.0206	16.1784	.1564	.5796	–	–	220
	(.0596)	(1.7136)	(.0041)	(.0108)			
Var22	−.0546	4.4473	.1581	.2486	–	–	220
	(.0564)	(1.4218)	(.0050)	(.0106)			
Var33	−.0440	2.4613	.1635	.1748	–	–	220
	(.0779)	(.9476)	(.0091)	(.0070)			
NoT1	.1520	55.4216	.1461	.3922	−.0055	.0262	220
	(.0288)	(2.5391)	(.0051)	(.1285)	(.0093)	(.0082)	
NoT2	.0889	21.9711	.1450	.2949	−.0120	0324	220
	(.0251)	(1.2038)	(.0044)	(.0891)	(.0096)	(.0087)	
NoT10	.0663	1.6936	.1405	.0943	−.0191	.0390	220
	(.0180)	(.5946)	(.0134)	(.0115)	(.0048)	(.0039)	
NoT22	.0790	.3022	.1784	.0458	−.0157	.0361	101
	(.0047)	(.1557)	(.1503)	(.0098)	(.0081)	(.0068)	
NoT33	.0762	.3487	.1396	.0221	−.0042	.0255	149
	(.0131)	(.1081)	(.0185)	(.0055)	(.0073)	(.0063)	
OV1	.0160	100.835	.1453	.8725	−.0028	.0267	220
	(.0427)	(10.7775)	(.0038)	(.0995)	(.0028)	(.0023)	
OV2	.0211	44.3428	.1446	.6618	−.0075	.0318	220
	(.0387)	(4.2902)	(.0044)	(.0206)	(.0006)	(.0012)	
OV10	.0102	4.7495	.1461	.1907	−.0114	.0360	220
	(.0135)	(1.0406)	(.0091)	(.0304)	(.0051)	(.0045)	
OV22	−.0229	1.0479	.1798	.0930	−.0164	.0411	208
	(.0193)	(.5021)	(.0898)	(.0132)	(.0050)	(.0042)	
OV33	.0290	.7973	.1797	.0562	−.0101	.0347	198
	(.0215)	(.3763)	(.1086)	(.0071)	(.0039)	(.0036)	

Table 7.3: Estimated parameter values from empirical time series for the stochastic variance and stochastic volume model. In order to calculate the reported average values, only estimates which obey the restrictions $-1 \leq \widehat{\rho} \leq 1$, $\widehat{\kappa} > 0$, $\widehat{\theta} > 0$, $\widehat{\sigma} > 0$ and $\widehat{\alpha} + \widehat{\beta} h > 0$ are used. The standard deviations are given in parenthesis.

the model restrictions. We want to emphasize that, for $\widehat{\rho}$ and $\widehat{\sigma}$, not only the average values but also every daily estimation obeys the relevant boundaries.

7.3. Implicit parameter estimations

In order to apply the valuation formulas of the stochastic volume and variance models, respectively, we also have to estimate the parameter λ, which represents the *market price of risk* which is induced by dw_2. In the derivation of the closed-form solutions, we assume that the risk premium associated with this Wiener process is proportional to $v(t)$ and $\alpha + \beta h(t)$, respectively. As [Cox et al. (1985)] have shown, this can be done by assuming that the representative consumer has a logarithmic utility function. In the context of such a general equilibrium model, λ can mathematically be interpreted as the covariance between changes in the variance and percentage changes in optimally invested wealth.[32] Since λ cannot be estimated from the data used so far, we estimate it implicitly using at-the-money calls and puts, respectively. The same options are used to estimate the daily implicit volatilities of the Black-Scholes model. In order to account for their maturity bias, the implicit estimations are done for each of the 5 different maturities which were available during the observation period. These maturities are 1, 2, 3, 6 and 9 months.

The estimation results for at-the-money calls are given in Table 7.4 and those for at-the-money puts are reported in Table 7.5. We observe that all average values of $\widehat{\lambda}$ are negative. Having a look at the standard deviations, it is clear that the single estimations are within a broad range as compared to the mean values. Although all of the latter are negative, single estimations lead to positive values of $\widehat{\lambda}$. This confirms the results of [Kapadia (1995)] and [Guo (1996)]. [Guo (1996)] estimates λ for PHLX currency options. He also obtains positive and negative values for $\widehat{\lambda}$. This is consistent with the interpretation of λ as a covariance as described above. [Kapadia (1995)] examines call options on the S&P 100 index and estimates λ for a stochastic variance model which has a specification similar to the one given in equations (3.1) and (3.2). His estimates of λ vary from 15 to -148.8, leading to an average value of -54.5. Hence, option pricing models which assume that the volatility risk is not priced by the

[32] See [Cox et al. (1985)] and [Schöbel (1995)].

	Maturity in days				
	$14-32$	$28-60$	$56-95$	$91-179$	$182-270$
BS-IV	.1892	.1934	.1913	.1909	.1829
	(.0274)	(.0209)	(.0167)	(.0128)	(.0097)
Var2	−85.70	−84.53	−79.67	−78.17	−69.26
	(40.42)	(22.84)	(18.74)	(14.20)	(12.06)
Var10	−13.11	−10.62	−7.77	−6.59	−4.85
	(17.75)	(6.28)	(3.59)	(2.18)	(1.62)
Var22	−10.56	−6.21	−3.93	−2.86	−1.70
	(14.45)	(5.29)	(3.13)	(1.94)	(1.27)
Var33	−7.59	−5.22	−3.07	−2.03	−1.04
	(12.99)	(5.33)	(2.95)	(1.72)	(1.11)
NoT1	−34.00	−28.63	−25.12	−23.98	−20.53
	(23.03)	(10.97)	(8.37)	(6.28)	(5.30)
NoT2	−25.82	−17.59	−13.05	−11.13	−8.99
	(19.42)	(7.75)	(4.74)	(3.18)	(2.38)
NoT10	−21.40	−11.82	−6.82	−4.24	−2.39
	(19.08)	(7.30)	(3.94)	(2.51)	(1.39)
NoT22	−23.58	−10.93	−6.36	−3.51	−1.93
	(13.73)	(4.91)	(2.30)	(1.15)	(.63)
NoT33	−12.47	−7.68	−4.47	−2.62	−1.38
	(12.65)	(4.88)	(2.63)	(1.26)	(.67)
OV1	−49.94	−45.94	−43.41	−42.79	−37.32
	(27.89)	(15.44)	(13.16)	(10.59)	(8.99)
OV2	−31.55	−23.76	−20.93	−19.88	−17.04
	(19.10)	(8.05)	(5.83)	(4.43)	(3.59)
OV10	−15.02	−9.61	−5.98	−4.20	−2.69
	(12.90)	(6.79)	(3.80)	(2.52)	(1.49)
OV22	−14.03	−7.39	−4.29	−2.74	−1.38
	(12.78)	(5.30)	(3.22)	(2.30)	(1.37)
OV33	−11.04	−6.66	−3.84	−2.40	−1.17
	(10.84)	(4.54)	(2.66)	(1.80)	(1.05)

Table 7.4: Estimated average values of the implied Black-Scholes volatilities and of λ for the stochastic variance and stochastic volume model, estimated from at-the-money calls. The standard deviations are given in parenthesis.

	Maturity in days				
	$14-32$	$28-60$	$56-95$	$91-179$	$182-270$
BS-IV	.2023	.2073	.2064	.2058	.2037
	(.0372)	(.0239)	(.0183)	(.0142)	(.0103)
Var2	−98.12	−96.99	−93.23	−91.22	−88.82
	(43.21)	(20.41)	(15.13)	(11.98)	(9.64)
Var10	−18.45	−13.79	−10.36	−8.62	−7.52
	(18.39)	(6.66)	(3.47)	(1.95)	(1.41)
Var22	−15.04	−8.61	−5.60	−3.93	−2.85
	(16.38)	(5.67)	(3.10)	(1.83)	(1.24)
Var33	−12.03	−7.54	−4.62	−2.97	−1.98
	(14.95)	(5.46)	(2.87)	(1.65)	(1.06)
NoT1	−41.09	−34.11	−30.49	−28.89	−27.66
	(23.19)	(9.26)	(6.08)	(4.76)	(4.07)
NoT2	−31.13	−20.87	−15.78	−13.33	−11.95
	(19.15)	(6.75)	(3.63)	(2.48)	(1.91)
NoT10	−25.63	−13.94	−8.20	−5.04	−3.12
	(18.34)	(6.59)	(3.37)	(2.28)	(1.30)
NoT22	−27.23	−12.96	−7.43	−4.13	−2.49
	(14.76)	(4.41)	(2.12)	(1.05)	(.59)
NoT33	−17.18	−10.37	−6.04	−3.45	−2.03
	(13.11)	(4.86)	(2.26)	(1.16)	(.60)
OV1	−59.10	−54.35	−52.24	−51.19	−49.94
	(27.95)	(12.39)	(9.37)	(8.13)	(7.70)
OV2	−37.88	−28.43	−25.29	−23.77	−22.70
	(18.82)	(6.51)	(4.15)	(3.29)	(2.88)
OV10	−19.47	−11.98	−7.60	−5.23	−3.76
	(13.67)	(6.26)	(3.48)	(2.29)	(1.38)
OV22	−18.39	−9.57	−5.68	−3.53	−2.11
	(12.88)	(4.90)	(2.91)	(2.13)	(1.26)
OV33	−15.66	−8.91	−5.24	−3.18	−1.88
	(12.00)	(4.22)	(2.30)	(1.65)	(.94)

Table 7.5: Estimated average values of the implied Black-Scholes volatilities and of λ for the stochastic variance and stochastic volume model, estimated from at-the-money puts. The standard deviations are given in parenthesis.

market as proposed e.g. by [Hull/White (1987)], [Scott (1987)], and [Wiggins (1987)] are not consistent with these findings. These models obviously lack an important model parameter. We also observe that the absolute values decrease when more data is used to generate the time series. This is consistent with the definition of λ as the market price of risk since these time series are less volatile. In contrast to [Kapadia (1995)] and [Guo (1996)], we also examine whether there are maturity or moneyness biases when estimating λ. Looking first at the estimates for different times to maturity, we can observe that this factor is obviously crucial to the magnitude of $\widehat{\lambda}$. The longer the time to maturity, the smaller the absolute values of $\widehat{\lambda}$ are. This is not consistent with the model assumptions. The same tendency can be found for the implicit volatilities of the Black-Scholes model. With the exception of the implicit volatility for the nearest-to-deliver options, all others decrease for longer times to maturities. These patterns are given for both types of options, calls and puts. Comparing Tables 7.4 and 7.5, we observe that the absolute values of the estimates from put options are larger than those from call options. This implies that put options are more expensive than call options. This confirms the results of [Whaley (1986)], [Taylor/Xu (1994)], and [Harvey/Whaley (1992)], who examine futures, currency, and stock index options, respectively. A possible explanation for this phenomenon is given in [Harvey/Whaley (1992)]. They conclude that buying "... index puts is a convenient and inexpensive form of portfolio insurance. Excess buying pressure of puts (relative to calls) may cause prices to increase ...".[33]

As many empirical examinations have shown, the implicit Black-Scholes volatilities are lowest for at-the-money options which leads to the well-known *volatility smile*. One of the explanations for this empirical fact that can be found in the literature is that the Black-Scholes model does not account for stochastic volatility.[34] If this is really the main reason for the existence of the volatility smile, we should expect the implicit estimates of λ to be the same for different moneyness categories. In order to check for this, we calculate the implicit market prices of risk for all options and all time series. It turns out that the behavior is quite similar to the one of the Black-Scholes implied volatilities. The smallest absolute values of

[33] [Harvey/Whaley (1992)], p. 56.

[34] See e.g. [Ball/Roma (1994)] and [Renault/Touzi (1996)], who prove that the volatility smile can be explained in the presence of a stochastic volatility process as proposed by [Hull/White (1987)].

$\widehat{\lambda}$ are observed for options which have a moneyness that is slightly below one. For in- and out-of-the-money options, the absolute values of $\widehat{\lambda}$ increase.

This raises the question as to whether the magnitudes of the observed biases are equal. This cannot be answered by comparing the implicit estimates of σ and λ directly, since they differ not only in their absolute values, but also in their units. That is why we proceed as follows: in a first step, we calculate option prices for the stochastic variance and stochastic volume model using the at-the-money estimates of λ for all different moneyness classes. Next the implicit Black-Scholes volatilities are calculated from these option prices. If the magnitude of the moneyness bias in the case of $\widehat{\lambda}$ is the same as for the Black-Scholes model, these implicit volatilities should all be of the same size. If the models which we examine can explain part of the Black-Scholes moneyness bias, we expect the volatility smile to be existent but not as pronounced as for the original option prices. That is exactly what we observe. In order to visualize these findings, representative results are depicted in Figures 7.1 to 7.3. The Black-Scholes implied volatilities are depicted in Figure 7.1 and the market prices of risk for the same options based on the parameter estimates of *Var33* are shown in Figure 7.2. A comparison between the implied Black-Scholes volatilities based on the original option prices

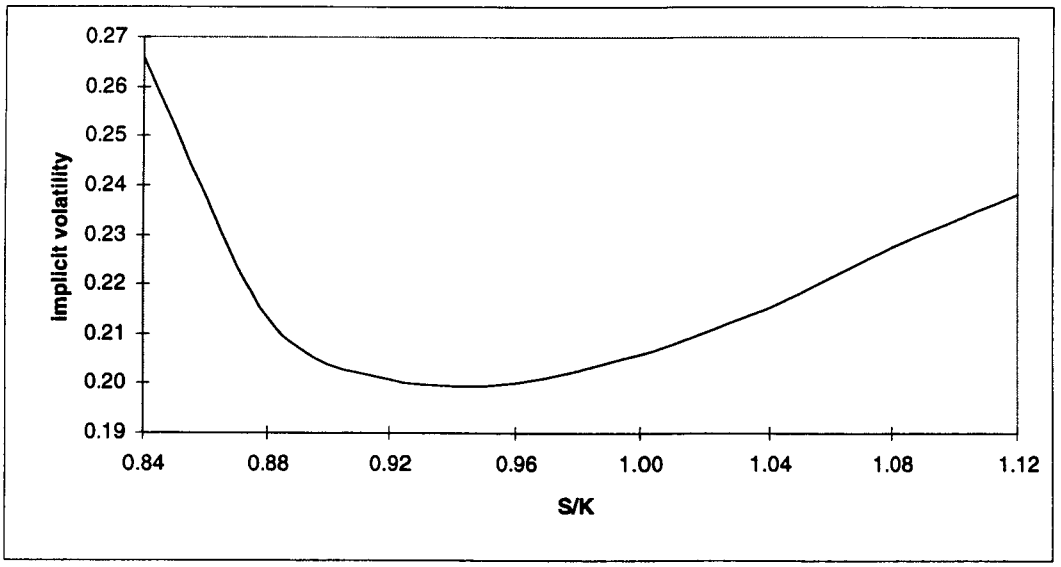

Figure 7.1: Implicit Black-Scholes volatilities calculated out of 3rd-nearest-to-deliver put options.

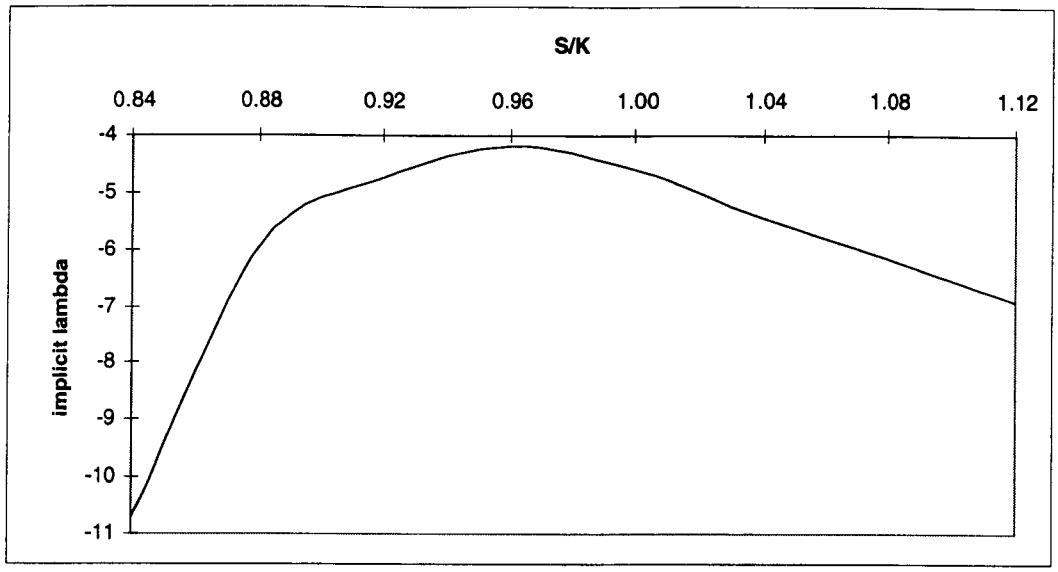

Figure 7.2: Implicit market price of risk λ calculated out of 3rd-nearest-to-deliver put options using the parameter estimates of *Var33*.

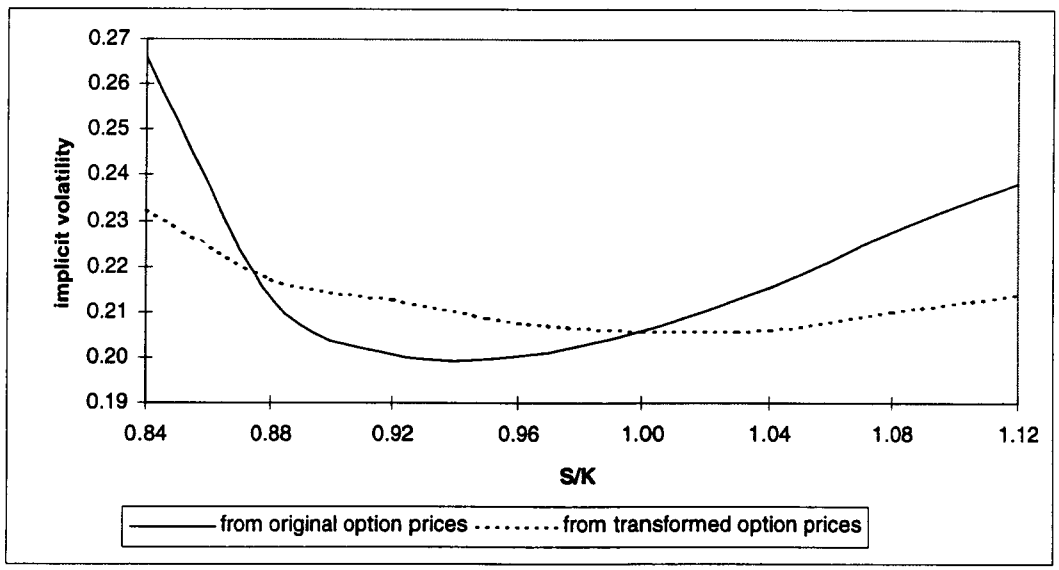

Figure 7.3: Implicit Black-Scholes volatilites from original and transformed 3rd-nearest-to-deliver put options. The transformation is done by calculating option prices for the stochastic variance model using the parameter estimates of *Var33* and the corresponding at-the-money value of λ for all different moneyness classes.

and those which are calculated from transformed option prices is given in Figure 7.3. The transformed option prices are derived as described above.

We want to emphasize that the behavior of $\widehat{\lambda}$ is the same for the stochastic variance and the stochastic volume model.

7.4. In-the-sample option prices

The main purpose of option pricing models is to explain empirical option prices. But since the intrinsic value, which is defined as $S - e^{-r\tau}K$ for calls and $e^{-r\tau}K - S$ for puts, is known, this problem is reduced to explaining the difference between the empirical option price minus its intrinsic value. This difference is often referred to as the *time value* of the option. Hence, we want to focus on the question as to how well the time values are described by the models under consideration.

Using the given parameter estimates, we can calculate the theoretical time value for the different models and time series. This is done by using S, K, r, $T-t$, the estimated parameters for a given day, and the spot value of the time series which describes the variance (volume) as input variables for the stochastic variance (volume) model. The resulting theoretical time values are compared to the empirical ones. The root mean squared error of relative time value differences serves as criterion for the quality of the theoretical results. It is defined as

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\frac{tv_i^{emp} - tv_i^{th}}{tv_i^{emp}}\right)^2}.$$

Here, the empirical time value is denoted by tv_i^{emp} and the theoretical value by tv_i^{th}. N is the number of options within a given moneyness class aggregated over all maturities.

Figure 7.4 shows the average empirical time values computed from all call options to provide an idea of the magnitude of the time values for our data.

As reported in the previous chapter, we have different samples for the given time series. Hence, we cannot compare the results directly. Therefore, we use the Black-Scholes model as a benchmark. In other words, we calculate the Black-Scholes option prices and the corresponding RMSE's for the different samples. Next we look at the differences between the RMSE's of the Black-Scholes model and the RMSE's of the other models. Positive differ-

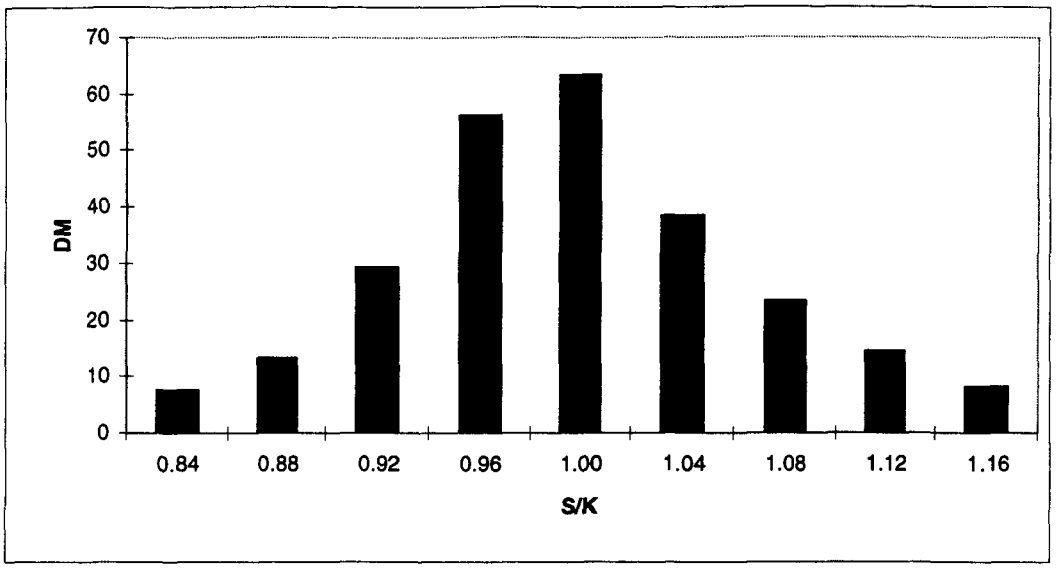

Figure 7.4: Empirical time values computed from call options.

ences mean that the Black-Scholes model performs better. The opposite applies when the differences are negative. Here it is important to remember that the Black-Scholes option prices are calculated via implicitly estimated volatilities, whereas only one of the necessary parameters of the other models is calculated implicitly. All others are estimated from historical data. But as a number of studies have shown, Black-Scholes implied volatilities normally lead to better volatility forecasts than historical volatilities.[35] Hence, due to the differences in estimating the parameters, the Black-Scholes model has an advantage over the other models.

The main question that we want to answer is how well empirical time values can be explained by using the different models and time series, respectively. In a first step, we examine in-the-sample option prices, that is, parameter estimates and option prices are given for the same day.

In the case of the stochastic variance model, Figure 7.5 shows that the results based on the time series *Var33* clearly performs best. There is also an obvious tendency that the RMSE's become smaller for the time series which are calculated from more data. For *NoT*, things are not quite as obvious. Although *NoT1* leads to the smallest RMSE differences for

[35] See eg. [Latané/Rendleman (1976)], [Chiras/Manaster (1978)], and [Beckers (1981)].

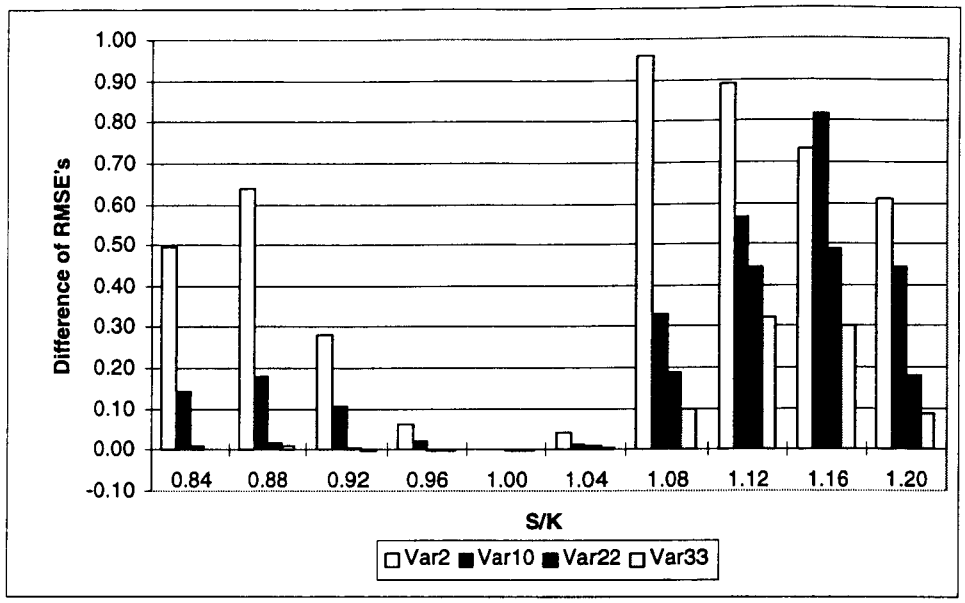

Figure 7.5: Differences between the RMSE's calculated with the Black-Scholes model and the RMSE's calculated with the stochastic variance model.

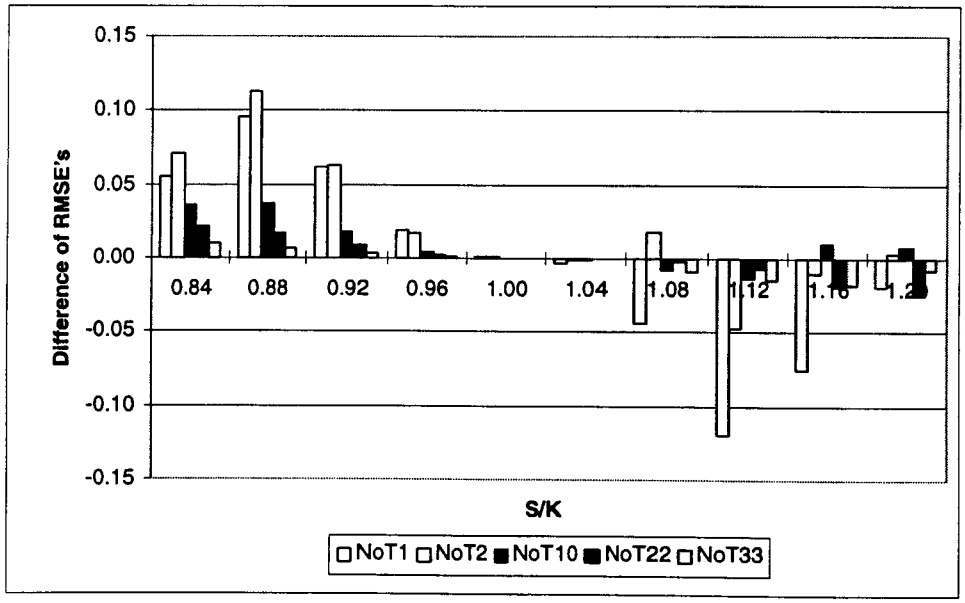

Figure 7.6: Differences between the RMSE's calculated with the Black-Scholes model and the RMSE's calculated with the stochastic volume model.

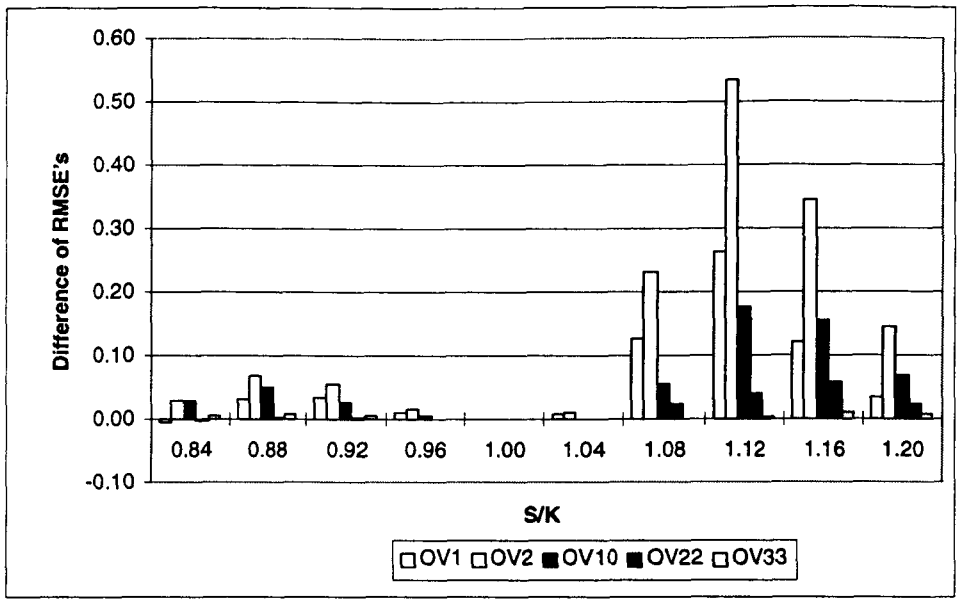

Figure 7.7: Differences between the RMSE's calculated with the Black-Scholes model and the RMSE's calculated with the stochastic volume model.

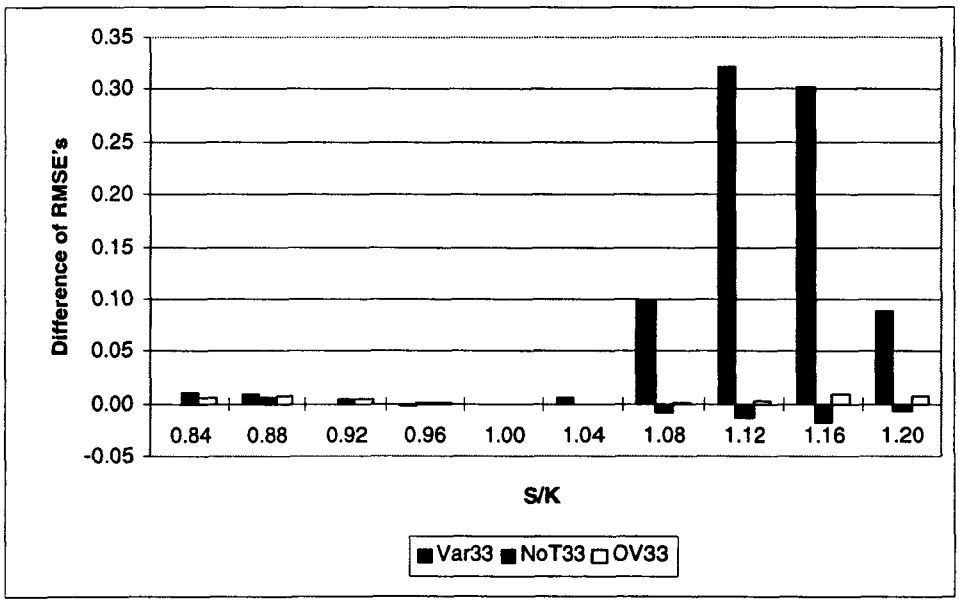

Figure 7.8: Differences between the RMSE's calculated with the Black-Scholes model and the RMSE's calculated with the stochastic variance and volume model, respectively. The calculations are based on in-the-sample call options.

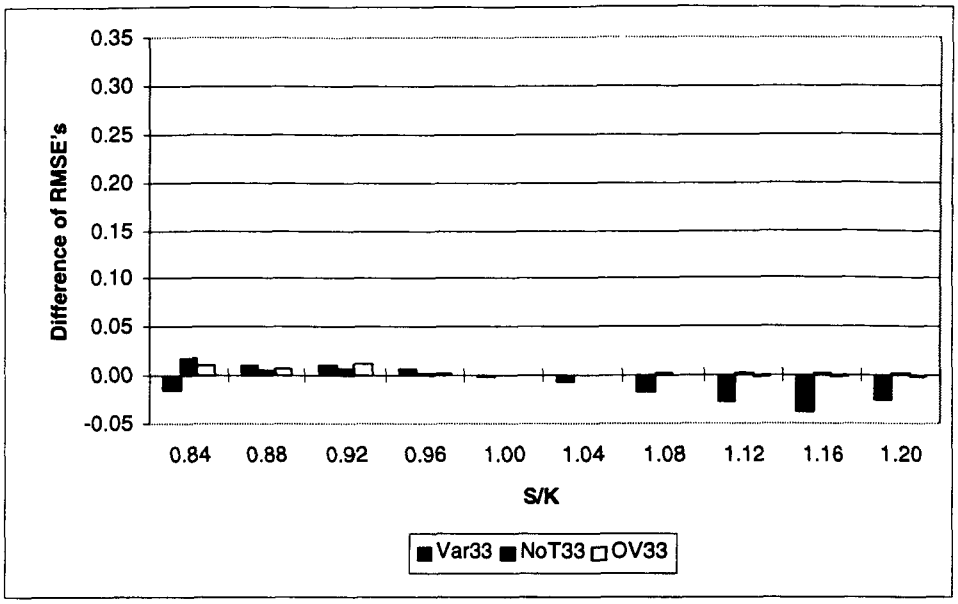

Figure 7.9: Differences between the RMSE's calculated with the Black-Scholes model and the RMSE's calculated with the stochastic variance and volume model, respectively. The calculations are based on in-the-sample put options.

in-the-money options, it also has significantly larger RMSE differences for out-of-the-money options as, for example, *NoT33*. A look at Figure 7.7 shows that, except for the magnitude of the data, things are similar to Figure 7.6. Here *OV33* clearly leads to the smallest values. Due to these results, we want to restrict ourselves to the time series *Var33*, *NoT33*, and *OV33* for the remainder of the paper. A direct comparison of these 3 time series is depicted in Figures 7.8 and 7.9. With regard to the ability of the used models to explain in-the-sample option prices, we can state that the performance of all models is very similar. The only exception is the in-the-money call options case for *Var33*. For these options, the stochastic variance model leads to the largest deviations from the empirical call prices.

7.5. Out-of-the-sample option prices

Until now we have restricted our study to in-the-sample option prices. But even more important is the question as to how well our model can explain out-of-the-sample time values. In order to examine this aspect, we identify the constant model parameters and use them to compute the option prices of the one- and five-days-ahead trading days, respectively. By

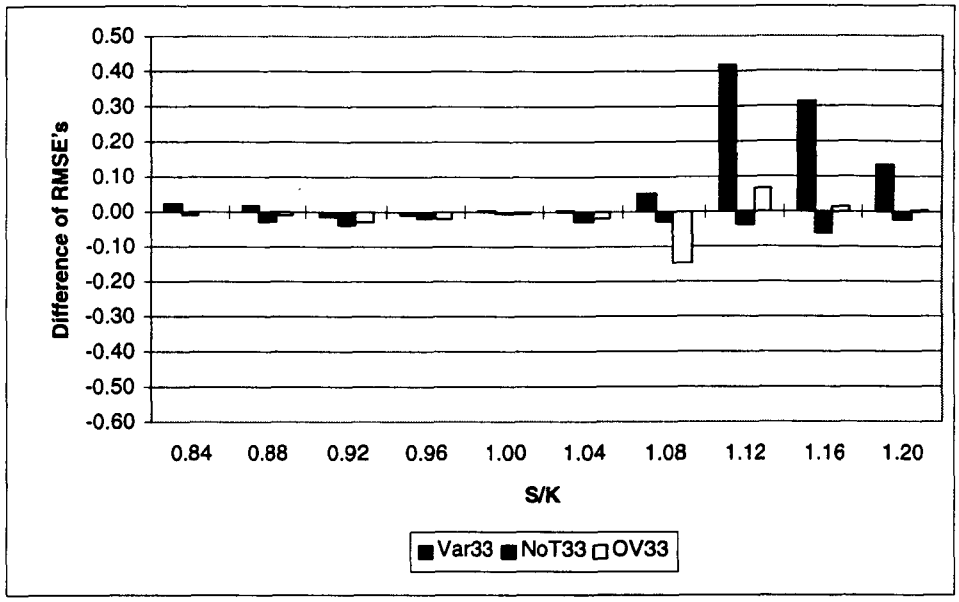

Figure 7.10: Differences between the RMSE's. The calculations are based on one-day-out-of-the-sample call options.

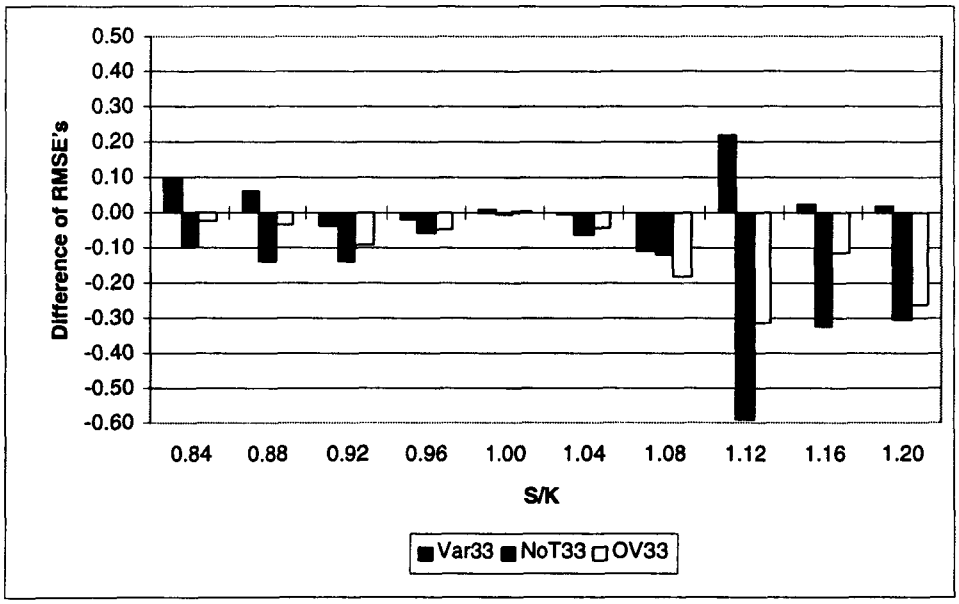

Figure 7.11: Differences between the RMSE's. The calculations are based on five-days-out-of-the-sample call options.

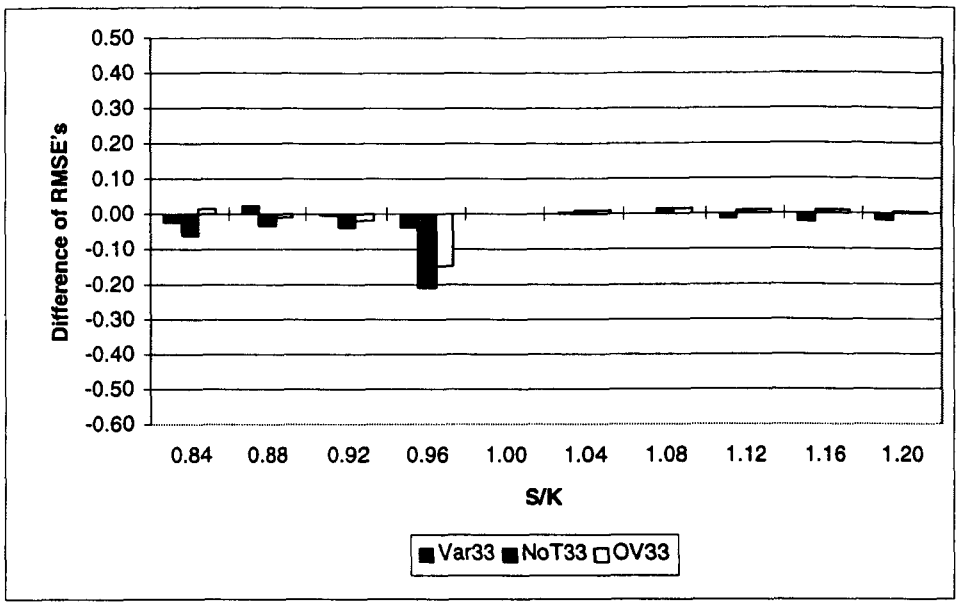

Figure 7.12: Differences between the RMSE's. The calculations are based on one-day-out-of-the-sample put options.

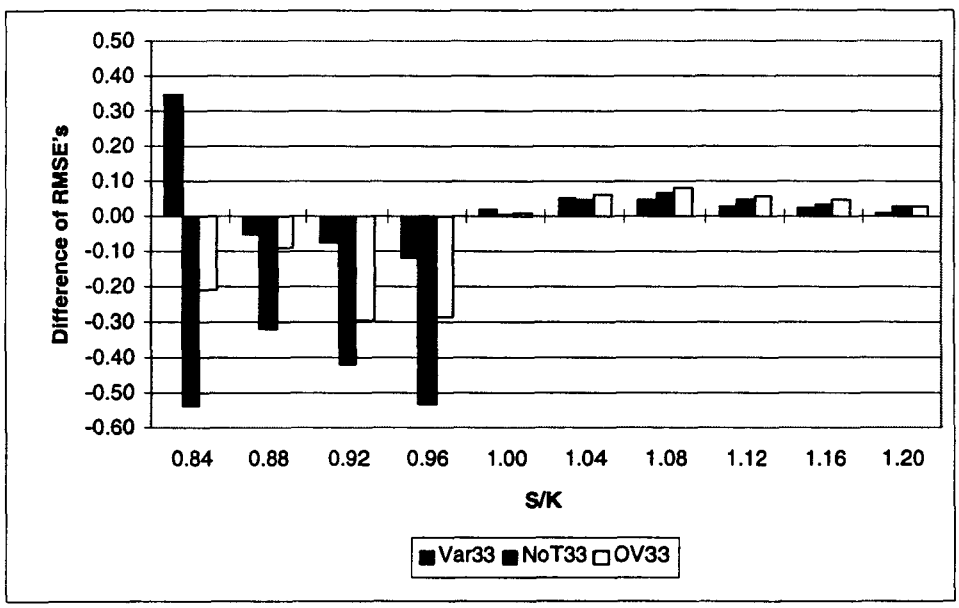

Figure 7.13: Differences between the RMSE's. The calculations are based on five-days-out-of-the-sample put options.

doing so, we proceed in a way which is consistent with the assumptions of the different models in regard to their variance processes. In case of the one-day-out-of-the-sample examination, this means, for the Black-Scholes model, that we use the implied volatility of the previous trading day. The parameters S, t, and r are taken from the current trading day. To test the other models, we proceed in a similar way. We take the estimates of ρ, κ, θ, σ, α, and β of the previous day and use them together with the current values of S, t, r, v, and h, respectively, to calculate the theoretical option prices.

Figures 7.10 and 7.11 show the results for call options with a lag of 1 and 5 days, respectively. Looking at the stochastic variance model, we see that on average the Black-Scholes model performs better. For the stochastic volume model things are different. It explains out-of-the-sample time values better than the Black-Scholes model. This is especially true for longer lags. As Figure 7.11 shows, the valuation bias of the Black-Scholes model can be reduced by up to 60 percentage points. It is interesting to note that both time series, *NoT33* and *OV33*, show the same kind of behavior. Although the ability to explain the empirical time values better than the Black-Scholes model is given for both time series, *NoT33* leads to the most significant improvements. That is, stock trading volume seems to have a greater explanatory power than option volume. On the other hand, the latter obviously explains at least part of the stochastic nature of the stock variance. Otherwise it could not lead to an improved option valuation.

Things are similar in the case of out-of-the-sample put prices. As Figures 7.12 and 7.13 show, the stochastic variance model performs somewhat better, especially for the one-day-out-of-the-sample puts. Here the mispricing of in-the-money options is smaller. But again, the stochastic variance model does not lead to significant improvements on the Black-Scholes model. As for the stochastic volume model, we observe a distinct decrease in the RMSE's for in-the-money puts. Once again, this is more pronounced for longer lags and the time series *NoT33*. These improvements clearly offset the only marginally higher RMSE's of the out-of-the-money puts since the latter have lower prices than in-the-money options. In other words, an increase in the accuracy of theoretical time values by a certain percentage leads to higher gains for in-the-money options when measured, for example, on a DM basis.

8. Summary and Conclusions

In this paper we address several questions regarding the stochastic nature of stock variances. We first modify the model of [Heston (1993)] in such a way that it explicitly incorporates trading volume as a state variable. This leads to a valuation formula which includes observable state variables only. Next, we examine whether it is possible to estimate the parameters of the original Heston model and our modified model from historical data using the Generalized Method of Moments. In order to determine whether this procedure is able to identify the parameters of the stochastic processes, we run Monte Carlo simulations. They show that only the estimation of the speed of adjustment parameter κ is biased. But since the sensitivity of the models to κ is very small, this problem is not of primary concern. More serious problems arise from the fact that the estimates of κ can become negative if its true value is positive but only slightly above zero. Furthermore, we find that the estimated standard deviations of the parameter estimates all perform reasonably well compared to the true standard deviations of the estimated parameters. With regard to the χ^2 test of overidentifying restrictions, we can state that it is very conservative when our choice of moments is used. Knowing this, one can still make use of this test by using modified p-values which can easily be derived from the estimation results.

We apply the GMM procedure to data from the German stock and option markets. The historical variances which are used to estimate the parameters of Heston's model are derived from time series of the German stock index DAX. These variances are not only used as input variables for the GMM procedure, but also for the model itself. That is, we use historical variances to verify the accuracy of the theoretical model.[36] Volume data used in this study are the number of transactions in German stocks and the daily average number of DAX-options traded at the DTB. We use these data to derive a total of 4 different variance and 10 different volume time series. The resulting parameter estimates reveal the same problem with regard to negative values of $\widehat{\kappa}$ as we have found for the simulated time series. Interesting results are found regarding the specification of v, the instantaneous variance of stock returns.

[36] In [Scott (1987)] and [Scott (1991)], the author also applies the method of moments, but uses the estimates to compute the daily volatilities *implicitly*.

The time series which should come closest to this definition results in parameter estimates which do not support the assumption that the instantaneous variance follows a pure mean-reverting process. They rather support the results of [Merville/Pieptea (1989)], who find that a noise component is superimposed on the mean-reverting process. Our estimates of ρ from empirical data support evidence from other studies stating that there is a negative (positive) correlation between returns and variance (trading volume).

The only parameter which cannot be estimated from historical time series is the market price of risk λ. Thus, as usual, we have to compute it implicitly from option prices. It turns out that λ shows a similar behavior as the implied Black-Scholes volatilities. For at-the-money options, we obtain the smallest absolute values. Since the units of λ and the implicit volatilities do not allow a direct comparison of the moneyness bias, we also calculate theoretical option prices using at-the-money values of λ and compute the corresponding implied Black-Scholes volatilities. Comparing these implied volatilities with the original ones, we find that the stochastic models can explain part of the Black-Scholes moneyness bias. The estimates of λ also reveal a maturity bias and show that put options are more expensive than call options. These new results are not consistent with the model assumptions, which imply that the value of λ ought to be the same across all moneyness classes, maturities, and option types.

Having estimated all model parameters we focus on the question as to which model better describes empirical option prices. Because the intrinsic value of an option can easily be computed without any model, the accuracy of an option pricing model only depends on its ability to explain the time value of an option. Thus, we do not compare empirical and theoretical option prices, but rather their time values. In order to measure the impact of stochastic variance and stochastic volume separately, we use the Black-Scholes valuation errors as a benchmark. Comparing these values with those of the Heston model enables us to identify the impact of stochastic variance on the explanatory power of the option pricing models under consideration. We then look at the differences in the results of the stochastic volume model and the original Heston model. These differences must be due to the use of volume instead of variance data. Since we employ different time series to estimate the

parameters, we first examine which one of them leads to the best results with respect to in-the-sample options only. It turns out that, within all classes of time series, those with the highest persistence, corresponding to the longest observation intervals used to calculate one data point, perform best. This again supports the above result of a superimposed noise component. We compare the ability of the different models to explain the time values of in-the-sample options. We find that, on average, all models perform equally well. This is reflected in the differences of the RMSE's which are all within a range of ± 4 percentage points. The only exception is the case of in-the-money call options when using the stochastic variance model. Here we have a distinct increase in the RMSE.

However, we want to emphasize that the setup of the estimation procedures for the different models clearly favors the Black-Scholes model because it is the only model whose parameters are either directly observable or implicitly estimated. The other models also incorporate parameters which are estimated from historical data. Bearing this in mind, the results indicate the potential of the stochastic models to perform better than the Black-Scholes model. This is confirmed by [Nandi (1996)] and [Bakshi et al. (1997)], who estimate all parameters implicitly.

The next question which we address is how the models perform when applied to out-of-the-sample options. We use a setup that takes the assumptions of the models into account. In the case of the Black-Scholes model, we have to assume, for example, that the implied volatilities are constant. In order to calculate theoretical option prices for the other models, we use lagged estimates of ρ, κ, θ, σ, α, and β together with the current values of S, t, r, v, and h, respectively. We choose lags of 1 and 5 days. Thus, we use parameters which are estimated for a specific day in the past to compute theoretical option prices for subsequent days. For the stochastic variance model, we do not find an improved ability to explain time values. On average this model is outperformed by the Black-Scholes model. Things are different for the stochastic volume model. With the exception of in-the-money puts, we obtain theoretical time values which provide significantly improved descriptions of their empirical counterparts. The magnitude of mispricing based on the Black-Scholes model can be reduced by up to 60 percentage points for a lag of 5 days. This result is even more

remarkable because, as mentioned above, it is based on parameters which are estimated from historical data. It also supports our choice regarding the relationship between volume and variance.

The use of different volume data for the stochastic volume model enables us to draw some conclusions about their importance for the valuation of options. First, using historical volume data, we can explain empirical time values better as compared to the use of historical variances. Second, volume data from the stock market outperforms data from the options market. Third, although data from the options market is inferior to those from the stock market, they still contain information which is important for the valuation of options.

Further research should be directed towards the refinement of alternative option pricing models which only incorporate observable variables. This includes modifications of the linear relationship between volume and variance which we have used. It would also be interesting to estimate the parameters of a stochastic variance model implicitly and investigate the relationship between the implied variance and observable volume data.

A. Appendix

$$\frac{1}{4}\sigma^2\Delta w^2 + \sigma\sqrt{v_{t-1}}\Delta w \equiv \varepsilon_t$$

$$\sigma^2\Delta w^2 + 4\sigma\sqrt{v_{t-1}}\Delta w = 4\varepsilon_t$$

$$\sigma^2\Delta w^2 + 4\sigma\sqrt{v_{t-1}}\Delta w + 4v_{t-1} = 4\varepsilon_t + 4v_{t-1}$$

$$\Delta w^2 + \frac{4}{\sigma}\sqrt{v_{t-1}}\Delta w + \frac{4}{\sigma^2}v_{t-1} = \frac{4}{\sigma^2}\left(\varepsilon_t + v_{t-1}\right)$$

$$\frac{\Delta w^2}{\Delta t} + \frac{4}{\sigma\sqrt{\Delta t}}\sqrt{v_{t-1}}\frac{\Delta w}{\sqrt{\Delta t}} + \frac{4}{\sigma^2\Delta t}v_{t-1} = \frac{4}{\sigma^2\Delta t}\left(\varepsilon_t + v_{t-1}\right)$$

$$\left(\frac{\Delta w}{\sqrt{\Delta t}} + \frac{2}{\sigma^*}\sqrt{v_{t-1}}\right)^2 = \frac{4}{\sigma^{*2}}\left(\varepsilon_t + v_{t-1}\right)$$

$$\left(\frac{\Delta w}{\sqrt{\Delta t}} + \frac{2}{\sigma^*}\sqrt{v_{t-1}}\right)^2 | v_{t-1} \sim \chi_1'^2\left(\frac{4}{\sigma^{*2}}v_{t-1}\right)$$ [37]

[37] See [Johnson/Kotz (1970)], p. 130.

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