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Dynamic efficiency and price leadership in the DAX-future and the DAX cash market: An empirical investigation

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**Wirtschaftswissenschaftliche Fakultät
der Eberhard-Karls-Universität Tübingen**

**Dynamic Efficiency and Price Leadership
in the DAX-Future and the Dax Cash Market:
An Empirical Investigation**

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Abstract

This paper provides results of an empirical study, addressing both the question of the efficiency of the DAX-Future market concerning its risk transfer function and the question of price leadership. For determining the pricing relationship between the futures and the cash market, co-integration analysis and nonlinear least squares estimation methods are applied. Results indicate that the pricing relationship is closely related to the cost-of-carry model and co-integrated. It will be shown that the cash market leads the futures market.

I. Introduction

A large number of empirical studies addresses the question of the efficiency of futures markets concerning themselves firstly with the risk transfer function, and secondly with the question of price leadership for the cash markets. Regarding the risk transfer function it must be shown that, in the long run, futures and cash prices do not move in different directions, but rather return to a stable relationship. The faster the deviations from this relation diminish, the greater the efficiency of futures markets will be. Concerning price leadership of the futures markets, it must be shown that, the reaction of cash markets to the above mentioned deviations is stronger compared to the reaction of the futures markets. A theoretical model addressing these two features was developed by Garbade and Silber (1983). Their basic pricing relation is comprised in the cost-of-carry model. The cost-of-carry model represents the adequate theoretical price relation between futures and cash prices under the assumptions of perfect markets

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and deterministic interest rates.¹ This model has been widely used to investigate the pricing efficiency of futures markets.²

Only few authors³ have explicitly recognized the relationship between the Garbade and Silber model and the co-integration analysis, which was developed, among others, by Engle and Granger (1987). They show that co-integration is related to error-correction models. The essence of error correction models lies in the fact that adjustments in a dependent variable depend on the extent to which an explanatory variable deviated from an equilibrium relationship with the dependent variable.⁴ The Garbade and Silber model belongs to this family of error-correction models. The main difference between empirical studies using co-integration analysis and those using solely the Garbade and Silber model is that, in the former, the long-run relationship between futures and cash prices is estimated, while in the latter the cost-of-carry model is used as the long-run relationship. To investigate price leadership, the former use estimated mispricing series while the latter use mispricing series computed on the basis of the cost-of-carry relation.

Price leadership is usually determined by investigating the lead-lag relationships based on Granger-Sims-type causality tests using solely returns data.⁵ However, if the cash

¹ For a derivation see Ross (1978), Cox, Ross and Ingersoll (1981); for the application to stock index futures see Cornell and French (1983).

² Empirical studies based on the Garbade and Silber model are e. g. the original study by Garbade and Silber (1983), Schwarz and Laatsch (1991) for the S&P500, and Khoury and Yourougou (1991) for various grain markets. Other studies using the cost-of-carry model are e. g. Figlewski (1984), MacKinlay and Ramaswamy (1988), Merrick (1989), Yadav and Pope (1990). The cost-of-carry model has also been used to test the pricing relation of the DAX-Future-market. See Bamberg and Röder (1992) and Prigge and Schlag (1992). The majority of these studies investigate the possibility of arbitrage profits.

³ Yadav and Pope (1991) and Antoniou and Garrett (1993) explicitly recognize the relation between co-integration and the Garbade and Silber model. The latter estimate the Garbade and Silber model. Within the last year several empirical studies about the interdependence of cash and futures markets using co-integration analysis have been published but do not address the Garbade and Silber model. Examples are Wahab and Lashgari (1993) for the S&P500 and the FT-SE 100, Crowder and Hamed (1993) for oil futures, Ghosh (1993) for the S&P 500, Fortenbery and Zapata (1993) for grain futures. There also have been several empirical studies using the Garbade and Silber (1983) model that do not address the co-integration analysis.

⁴ For a detailed analysis of issues concerning co-integration and error correction models see Banerjee et al (1993).

⁵ See Granger (1969), Sims (1972). These kinds of tests are applied e.g. in Stoll and Whaley (1990) and Grünbichler, Longstaff and Schwartz (1993).

and futures prices follow random walks, or, more generally, are integrated of order 1, and if they are co-integrated, which means that a linear combination of the two series is stationary, an error correction model will be the adequate specification of a model investigating leadership.⁶ In this case mispricing terms should be included in the regression equations to determine price leadership.

The purpose of this paper is to examine the pricing relationship between the DAX-Future, the futures contract on the German stock index DAX (Deutscher Aktien Index), and the DAX. Daily data from eleven contracts based on a single equation cointegration approach will be used. Measures for testing the efficiency of the futures markets are suggested. These measures are closely related to those derived in the Garbade and Silber model.

The remainder of the paper is organized as follows. In Section II, the essentials of the Garbade and Silber model are represented. As an extension of the model the stochastic behavior of the mispricing series is determined in the presence of transaction costs and adequate measures of efficiency are discussed. In Section III, the applied econometric techniques are introduced. Nonlinear least squares is utilized to determine simultaneously the pricing relationship and the stochastic behavior of the mispricing term. Section IV describes the data. In Section V, the empirical results are presented. It is shown, that cash and futures prices are co-integrated, and that the pricing relationship is close to the cost-of-carry model. If daily data are analyzed, the cash market leads on average the futures market. Further, the question whether the mispricing behavior depends on the time to maturity will be addressed. In Section VI, the findings are summarized and concluding remarks are made.

⁶ For a definition of the terms integration, co-integration, error correction mechanisms see Engle and Granger (1987).

II. The Garbade and Silber Model

Garbade and Silber (1983) develop a dynamic equilibrium model of simultaneous cash and futures prices. Their model is based on the actions of three types of market participants:

- cash market traders with an elasticity of demand denoted by A,
- futures market participants with an elasticity of demand denoted by B,
- arbitrageurs with an elasticity of demand denoted by H.

The fundamental relationship between the cash and the futures price is determined by the cost-of-carry model:

$$f_t^* = i_t + r \cdot (T - t) \quad (1)$$

where f_t^* is the natural logarithm of the futures price, i_t is the natural logarithm of the cash price, r is the risk-free rate of interest and T is the expiration date of the futures price.

Assuming that the reservation prices of cash and futures markets - the prices that would motivate a market participant to change his holdings - follow random walk processes,

Garbade and Silber develop the following model:

$$\begin{bmatrix} i_t \\ g_t \end{bmatrix} = \begin{bmatrix} 1-a & a \\ b & 1-b \end{bmatrix} \cdot \begin{bmatrix} i_{t-1} \\ g_{t-1} \end{bmatrix} + \begin{bmatrix} e_{i,t} \\ e_{g,t} \end{bmatrix} \quad (2)$$

with

$$a = B \cdot H / (A \cdot B + A \cdot H + B \cdot H), \text{ where } 0 \leq a \leq 1,$$

$$b = A \cdot H / (A \cdot B + A \cdot H + B \cdot H), \text{ where } 0 \leq b \leq 1,$$

⁷ For dividend paying shares the theoretical relationship will be

$$f_t^* = i_t + (r - d) \cdot (T - t),$$

where d is dividend yield, if the dividend is paid continuously.

and

$$e_{i,t} = w_t + (1-a) \cdot w_{i,t} + a \cdot w_{g,t} \quad (2a)$$

$$e_{g,t} = w_t + b \cdot w_{i,t} + (1-b) \cdot w_{g,t} \quad (2b)$$

where

$$e_{i,t} \sim N(0, + (1-a)^2 \cdot \sigma_{wi}^2 + a^2 \cdot \sigma_{wg}^2),$$

$$e_{g,t} \sim N(0, \sigma_w^2 + b^2 \cdot \sigma_{wi}^2 + (1-b)^2 \cdot \sigma_{wg}^2),$$

$$\text{Cov}[e_{i,t}, e_{g,t}] = \sigma_w^2 + (1-a) \cdot b \cdot \sigma_{wi}^2 + a \cdot (1-b) \cdot \sigma_{wg}^2.$$

g_t is the 'cash equivalent price', defined as $g_t = f_t - r \cdot (T - t)$, where f_t is the logarithm of the futures price at time t , w_t represents a price change component common to all participants at time t , $w_{i,t}$ and $w_{g,t}$ are price change components idiosyncratic to the index value or to the futures price respectively. Each component is assumed to be independently and identically normally distributed with zero mean and variances σ_w^2 , σ_{wi}^2 and σ_{wg}^2 . The correlation between the components is assumed to be zero. Equation (2) represents a bivariate random walk whose character depends on the elasticity of demand H , i. e. the supply of arbitrage services.

If there are no arbitrage services, which means $H = 0$, it follows that

$$a = B \cdot H / (A \cdot B + A \cdot H + B \cdot H) = 0$$

$$b = A \cdot H / (A \cdot B + A \cdot H + B \cdot H) = 0.$$

The dynamics of the cash and futures prices can be described as two independent random walks. Using the futures market for hedging a long-term cash position will render ineffective because in the long run the two prices will move in different directions and will never come together.

If the elasticity of the supply of arbitrage services is $H = \infty$, it follows that

$$\lim_{H \rightarrow \infty} a = \lim_{H \rightarrow \infty} B / (A \cdot B / H + A + B) = B / (A + B)$$

$$\lim_{H \rightarrow \infty} b = \lim_{H \rightarrow \infty} A / (A \cdot B / H + A + B) = A / (A + B).$$

As a result the two markets are perfectly integrated. This can easily be shown by taking a closer look at the dynamic behavior of the mispricing term u_t , defined as $u_t = f_t - i_t - r \cdot (T-t)$.

The stochastic process for the mispricing term can be derived from (2)

$$u_t = (1-a-b) \cdot u_{t-1} + e_t \quad (3)$$

where $e_t = e_{g,t} - e_{i,t}$, with $e_t \sim N(0, (1-a-b)^2 \cdot (\sigma_{wi}^2 + \sigma_{wg}^2))$. With $H = \infty$, the mispricing will be zero. Deviations from the theoretical pricing relationship do not exist. The futures market can be used to perfectly hedge any cash market position. If $H = 0$, the mispricing will follow a random walk. Under these circumstances there is no effective instrument for hedging a cash position in the long run. The higher the elasticity of supply of arbitrage services the smaller the term $(1-a-b)$, and the higher the effectivity of the futures market as a risk management instrument. The term $(1-a-b)$ represents an adequate measure for the efficiency of the futures market concerning its risk transfer function. If the parameter value is close to zero today's mispricing has no impact on tomorrow's mispricing.

A low value for $(1-a-b)$ also leads to a low variance of the mispricing term. The variance of the mispricing term as another measure for the efficiency of the futures markets is closely related to the value $(1-a-b)$. A low variance of the mispricing term is desired because it reduces the risk of future use of the futures market e.g. for unwinding hedging positions.

The close link between $(1-a-b)$ and the variance of the mispricing term is loosened if transaction costs are taken into account. Several authors, e.g. Gould (1988) or Merrick (1989), address the role of transaction costs for the determination of fair futures prices. They define a window for fair futures prices where no arbitrage profits can be made. If transaction costs C exist, two different regimes for the dynamic behavior of the mispricing term follow. If the futures price is inside the arbitrage window, no arbitrage services will be supplied; if it is outside the window, arbitrageurs' actions will move the futures price into the window. Whenever $|u_t| \leq C$, $a=0 \wedge b=0$ follows; whenever $|u_t| > C$, $a \neq 0 \vee b \neq 0$ follows. Inside the window u_t follows a random walk, outside the window u_t follows a stationary process. The value of $(1-a-b)$ changes with each regime. If transaction costs are taken into account, $(1-a-b)$ cannot be considered as the sole measure for efficiency. The variance of the mispricing term must be added to obtain a solid base for the determination of efficiency. If the arbitrage window is sufficiently large and the price change sufficiently small, it is theoretically conceivable that, over a restricted period of time, the mispricing term follows a random walk. This can happen if the variances of the idiosyncratic components of the price changes ($\sigma_{wi}^2 + \sigma_{wg}^2$) are small.

The model (2) can also be represented in terms of price changes. The error-correction form can easily be seen. The question of price leadership can be answered by looking at the coefficients a and b . If $a < b$, then the cash market leads the futures market, vice versa.⁸

$$\Delta i_t = a \cdot u_{t-1} + e_{i,t} \quad (4a)$$

$$\Delta f_t = -r - b \cdot u_{t-1} + e_{g,t} \quad (4b).$$

⁸ For a detailed analysis see Garbade and Silber (1983), p.292-293.

III. Econometric Methods

In the following, the econometric methods which are used in this empirical study will be presented. These methods represent the basis for testing the cost-of-carry model, i.e. the fundamental equilibrium relationship of the Garbade and Silber model, the estimation of (1-a-b), and the variance of the mispricing term σ_u^2 .

Several different procedures for estimating the parameters of models like (2) have been proposed. Ghosh (1993) uses the two-step procedure proposed by Engle and Granger (1987) to investigate the dynamic relationship between cash and futures markets. After determining the order of integration for futures and cash prices, the long-run relationship between cash and futures prices is estimated by ordinary least squares. Estimated mispricing terms are obtained and checked for the existence of unit roots. Then the model of the form (4a) and (4b) is estimated by using estimated mispricing terms, thus addressing the question of dependency of the markets. This procedure comes close to the one used in Garbade-and-Silber-based empirical studies. One disadvantage of the two-step procedure is given by the fact that the estimation of the coefficients determining the long-run relationship can be biased. Another disadvantage is that generally no inference can be made for these coefficient values. These disadvantages are not shared by the technique used in this paper, a single estimation technique as suggested by Phillips and Loretan (1991).⁹ This technique can be asymptotically equivalent to full-system-maximum-likelihood estimation, the system approach discussed in Johansen and Juselius (1990). The system approach has been applied in the index-futures studies by Wahab and Lashgari (1993) and Antoniou and Garrett (1993). The single equation technique is easier to implement, and inference can proceed in the usual manner with asymptotic normal t-ratios. The core of the proposal by Phillips and Loretan is represented by the use of dynamic specification of the model and its estimation by nonlinear least squares. The Garbade and Silber model can easily be written in system form, as analyzed by Phillips and Loretan. It should be noted that for the

⁹ See Phillips and Loretan (1991), pp.420-425.

empirical study cost-of-carry $r \cdot (T-t)$ was computed with actual daily interest rates. Therefore cost-of-carry is expressed as $r_t \cdot (T-t)$. This leads to the following system with the additional parameters o and p .

$$f_t = o \cdot i_t + p \cdot r_t \cdot (T-t) + u_{1t} \quad (5a)$$

$$\Delta i_t = u_{2t} \quad (5b)$$

with

$$\begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix} = \begin{bmatrix} (1-a-b) \cdot (f_{t-1} - o \cdot i_{t-1} - p \cdot r_{t-1} \cdot (T-t+1)) + (1-a-b) \cdot (w_{i,t} - w_{g,t}) \\ a \cdot (f_{t-1} - o \cdot i_{t-1} - p \cdot r_{t-1} \cdot (T-t+1)) + w_t + a \cdot w_{i,t} + (1-a) \cdot w_{g,t} \end{bmatrix}$$

The regression equation which will be estimated by nonlinear least squares is

$$f_t = o \cdot i_t + p \cdot r_t \cdot (T-t) + d \cdot (f_{t-1} - o \cdot i_{t-1} - p \cdot r_{t-1} \cdot (T-t+1)) + v_t \quad (6),$$

where v_t is i.i.d. with zero mean and variance $(1-a-b) \cdot (\sigma_{w_i}^2 + \sigma_{w_g}^2)$.¹⁰ In the Garbade and Silber model, the coefficient $o=1$, $p=1$ and $d=(1-a-b)$. The advantage of estimating the long-run equilibrium parameter p over using computed cash equivalent prices is that the use of wrong opportunity cost measures in the latter context can lead to the inclusion of a deterministic trend into the mispricing series. Estimating $(1-a-b)$ from equation (3) without taking into account a deterministic trend could lead to the false conclusion that the mispricing term follows a random walk process, whereas, in fact it follows a trend-stationary process.¹¹ If $o=1$ a value of p of less than one indicates an underpricing of the futures contract compared to the used opportunity cost measure. It

¹⁰ Phillips and Loretan (1991), p. 424, suggest the inclusion of leads of Δi in the single equation specification to cope with the feedback from u_1 to u_2 . This has to be done to obtain errors that form a martingale difference sequence with respect to the past history of u_1 and the full history of u_2 , which is important for estimator efficiency, unbiasedness and for inference. The dynamic single equation was also estimated using leads. However, the results do not differ from the ones documented here.

¹¹ Let $f_t = s_t + r \cdot (T-t)$, r being the true opportunity measure and u_t being the true mispricing. If r^* is used for computing the cash equivalent price, then the mispricing will be $u_t^* = (r - r^*) \cdot (T-t)$. See Perron (1988), p.317, for the problem of trend-stationary vs. difference stationary processes.

should be noted that the true opportunity costs are not known. Costs associated with borrowing or lending of shares reduce the opportunity costs, transaction costs lead to an interval of possible opportunity costs.

For answering the question of price leadership, the following equations will be estimated using ordinary least squares,

$$\Delta i_t = c_i + a \cdot u_{t-1} + v_{i,t}$$

$$\Delta f_t = c_f - b \cdot u_{t-1} + v_{g,t},$$

where $v_{i,t}$ and $v_{g,t}$ are random components with zero mean and constant variance, c_i and c_f are constants. If the value of a (b) is significantly greater than zero, while b (a) is not significantly different from zero, the futures market (cash market) leads the cash market (futures market). If both a and b are significantly greater than zero, there is a bidirectional influence. If in this case a is significantly greater than b , the futures markets leads the cash market and vice versa.

IV. Data

The German stock index DAX is a value-weighted index consisting of the shares of the 30 largest firms traded on the Frankfurt Stock Exchange, the major stock exchange in Germany. The DAX nearly represents 60% of the total market capitalization of domestic firms listed on the Stock Exchanges and over 65% of the trading volume of German share markets. The DAX is special in being a performance index, i.e. in the computation of the index, cash dividend payments¹² are treated as if they were reinvested in the stock that

¹² The gross dividends are taxed with a 36% tax rate. The shareholder obtains an after-tax cash dividend plus a tax credit at the level of the pre-paid tax.

paid the dividends. Once a year the DAX is rebalanced to avoid ever-increasing weights of dividend paying shares.¹³

Trading in DAX-Future-contracts started ~~on~~ 21. November 1990. There are four DAX-Future contracts maturing each year. The maturity months are March, June, September and December. While the DAX-Future contract is only screen-traded on the German futures and option exchange DTB (Deutsche Terminbörse), the DAX-shares are both floor-traded on several German Stock Exchanges and screen-traded on IBIS II (Integriertes Börsenhandels- und Informations-System).¹⁴ The DTB is a fully-computerized exchange with an automatic order matching system. While the Frankfurt Stock Exchange is open only three hours each business day, 10:30 a.m. to 1:30 p.m., the computer exchanges open one to two hours earlier and close at 5 p.m.

With the exception of the first contract maturing in December 1990 the following eleven contracts are investigated in the empirical study. The samples start with the March 1991 contract and ends with the September 1993 contract. For the empirical study daily DAX-Future transaction prices nearest to the closing time of the Frankfurt Stock Exchange and closing DAX-values are used. They were provided by the KKMDB (Karlsruher Kapitalmarktdatenbank). For the computation of the cost-of-carry, the quotations of the daily averages of the one-month-interbanking-offering-rates were taken from the "Handelsblatt".

Concerning the pricing relationship and the behavior of the mispricing series the data set is analyzed on a contract-by-contract basis, which has the advantage of showing changes of the pricing relationship and the mispricing behavior over time. To reduce the effects produced by nonsynchronous data¹⁵, each contract data set starts with a maturity of no

¹³ For details on the computation of the DAX see Loistl (1991) pp.72-86 or Gießelbach (1989) pp.261-265.

¹⁴ For more details on the microstructure of DAX share and the DAX-Future market see Grünbichler, Longstaff and Schwartz (1993), pp.8-11.

¹⁵ Miller, Muthuswamy and Whaley (1993) show that observed negative autocorrelation of basis changes of index futures, where the basis ($F_t - I_t$), F_t being the futures price and I_t being the index value, could be mainly a statistical illusion arising from infrequent trading of many shares in the index. This is a problem

longer than 4 months. For three quarters of the investigated time, the contract is nearest-to-deliver. The price leadership is determined by equations (4a) and (4b). To reduce the possibilities of spurious results arising from small samples and especially arising from extreme price change values, a single series is constructed from the eleven contracts for both futures price changes and mispricing terms. The breaking point for changing the relevant contract is 5 days before maturity of the nearest-to-deliver. Regressions are carried out in a manner that futures price changes of one contract are not regressed on mispricing terms of a different contract.

V. Empirical Results

a) The pricing equation and mispricing behavior

The regression results for the pricing equation are presented in table I. A full report is given in the appendix.¹⁶ It should be noted that the regression equation is estimated without using a constant term. Therefore the residuals are tested for having a zero mean. The null hypothesis of zero mean cannot even be rejected at very low significance levels. The t-values are between 0 and 0.01. For two contracts, March 1992 and 1993, serial correlation in the residuals is detected at high significance levels. Using more lags of the mispricing terms in the case of the March 1992 and no lags for the March 1993 contract does not change the estimates for α and β .

arising especially from the use of intra-daily data. The intra-day basis is closely related to the mispricing terms used here. In intra-day data, cost-of-carry does not change. All of the shares in the DAX trade more than once a day. Grünbichler, Longstaff and Schwartz (1993) report the average time between trades for the firms in the DAX index as being 4.94 minutes. Therefore the impact of nonsynchronous trading on the positive autocorrelation in daily index value changes is extremely small and can be ignored. The same is true for the negative autocorrelation of daily futures price changes arising from bid/ask spreads as shown by Roll (1984).

¹⁶ As a prerequisite logarithmic index values and futures prices have been tested for unit roots. Augmented-Dickey-Fuller tests and Phillips-Perron tests have been applied. The results indicate, that the index values and the futures prices are integrated of order one.

All estimated values for α are very close to one, but only four out of eleven are not significantly different from one. If a DAX index value of 1800 points and a time to maturity of zero is assumed, the maximum and minimum difference between the theoretical futures prices and the futures prices using estimated coefficient values will be between -2.5 to 3.5. The minimum tick for the DAX-Future is a half point. Since the value of α is not constantly smaller or larger than one, the empirical findings do not suggest a rejection of a value of exactly one for α .

The estimated value of β is in nine contracts at the 99%-level significantly different from one. It is always smaller than one. On average, futures contracts seem to be undervalued,¹⁷ which can be due to the use of a false opportunity cost measure. It should be noted that the real opportunity costs are not known, and that the one-month interest rate is just a proxy. Another reason for such an undervaluation can lie in the existence of dividends. Since the DAX is a performance index, and dividend payments are included, an undervaluation can only be a consequence of the tax credit connected with dividend payments. Over 90% of the dividends of the DAX shares are paid in the spring and summer months. If tax credits matter, June and September contracts should be more undervalued than the rest. However, this is not the case.¹⁸ Since a high value of α can compensate a low value of β average mispricing is computed using both values simultaneously. As a result, undervaluation decreases with some slight exceptions constantly until June 1993, and then increases again.

The estimated values of d show that the DAX-Future price and the DAX value are co-integrated.¹⁹ Only for the June 1993 contract the hypothesis of no co-integration, $d=1$, is

¹⁷ For the first half of 1991, Prigge and Schlag (1992), and for the full year of 1991, Bamberg and Röder (1992), obtain the same result using the cost-of-carry model to compute mispricing.

¹⁸ It was tested if dividend paying days have an influence on the behavior of mispricing changes. If undervaluation results from the existence of tax credits connected with dividend payments, there should be an impact on the mispricing change. If the tax rate used for tax credit is too low compared to the arbitrageurs' tax rate, the mispricing change should be positive; if it is too high, mispricing change should be negative on these days. There was no significant influence; consequently the results are not reported here.

¹⁹ The critical values were taken from MacKinnon (1991). The variant "2 variables with constant and trend" has been applied.

not rejected. It is interesting to note that this contract is also the most undervalued one. On average 63% of current day's mispricing is eliminated on the following business day. In three cases - the March 92, the March 93 and the September 93 contract - the mispricing series take on the feature that d was not significantly different from zero. However, stability of the value d lacks for the different contracts. There is no hint that the risk transfer function of the futures market improves with time. If the standard errors for d given in the appendix are used to test the difference from the mean of 0.37, three out of eleven estimated coefficients will be different from 0.37 at the 95% level. 11 observations do not suffice to draw any conclusion from this result.

Table II reports the mean and the standard deviation of the mispricing term. The mean is not different from zero mean, as indicated by the t -values. If realistic DAX values are used, only the June 1993 contract will have a mean close to the minimum tick of the DAX-Future contract. It is exactly the June 1993 contract for which the hypothesis of no co-integration cannot be rejected. However, it should be noted that this contract does not have the highest standard error. On the other hand the March 1992 contract has the highest variance while not having a value d significantly different from zero. The close relationship between d and σ_u stated by the Garbade and Silber model cannot be supported empirically. There is no evidence that transaction costs are responsible for this finding. The relation between d and σ_u is rather complex. While u_t denotes relative mispricing, the mispricing series shown in figure 1 are expressed in index points. Relative mispricing can easily be converted to index points by multiplying u_t with F_t .

Table I: *Summary of the regression results for the pricing equation and the mispricing behavior*

regression equation: $f_t = o \cdot i_t + p \cdot r_t \cdot (T-t) + d \cdot (f_{t-1} - o \cdot i_{t-1} - p \cdot r_{t-1} \cdot (T-t+1)) + v_t$

*Hypothesis: Pricing equation is equal to cost-of-carry-relation $H_0: o = 1, p = 1,$
no serial correlation for mispricing series $H_0: d = 0$*

, **, * significantly different from the null hypothesis
at the 90%, 95%, 99%-level*

a: d is significantly smaller than one at the 99%-level

N: Number of observations

R²: Coefficient of determination

∅ mispricing: $I_t \cdot e^{r \cdot (T-t)} - I_t^0 \cdot e^{p \cdot r \cdot (T-t)}$, where $I_t = 1800, r = 0.08/360$ and $(T-t) = 45$

	N	o	p	∅ mispricing	d	R ²
March 91	59	1.00028 *	0.31 ***	-8.80	0.41 ^a ***	0.9980
June 91	74	0.99991	0.64 ***	-7.71	0.33 ^a ***	0.9979
Sept. 91	82	0.99999	0.74 ***	-4.88	0.47 ^a ***	0.9978
Dec. 91	74	0.99983 ***	0.85 ***	-5.16	0.37 ^a ***	0.9978
March 92	80	1.00023 **	0.69 ***	-2.58	0.18 ^a	0.9933
June 92	79	1.00012 *	0.73 ***	-3.29	0.50 ^a ***	0.9932
Sept. 92	82	0.99997	0.93	-1.75	0.60 ^a ***	0.9996
Dec. 92	84	1.00014 **	0.82 ***	-1.35	0.26 ^a **	0.9971
March 93	81	0.99993 *	0.97 *	-1.36	-0.08 ^a	0.9989
June 93	80	0.99982	0.62 ***	-9.26	0.83 ***	0.9967
Sept. 93	78	0.99899 **	0.84 ***	-4.27	0.18 ^a	0.9993

Table II: *Descriptive statistics for the mispricing term*

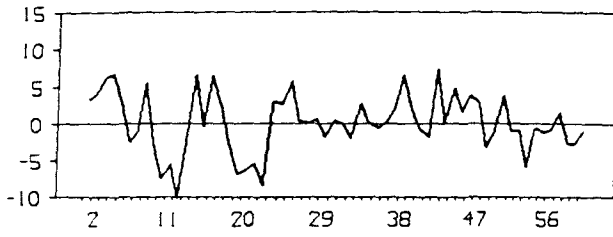
N: *Number of observations*
 \bar{u} : *Mean of the mispricing series*
 $t(\bar{u}=0)$: *t-statistics for mean = 0*
 σ_u : *standard deviation of mispricing series*

	N	$\bar{u} \cdot 10^3$	$t(\bar{u}=0)$	σ_u	d
March 91	60	0.0621	0.17	0.0028	0.41
June 91	75	0.0873	0.45	0.0017	0.33
Sept. 91	83	-0.0750	-0.45	0.0015	0.47
Dec. 91	75	-0.0300	-0.20	0.0013	0.37
March 92	81	-0.0249	-0.07	0.0031	0.18
June 92	80	-0.0871	-0.61	0.0013	0.50
Sept. 92	83	-0.1893	-0.85	0.0020	0.60
Dec. 92	85	0.0460	0.27	0.0016	0.26
March 93	82	-0.0058	-0.04	0.0013	-0.08
June 93	81	0.3153	1.11	0.0025	0.83
Sept. 93	79	-0.0269	-0.17	0.0014	0.18

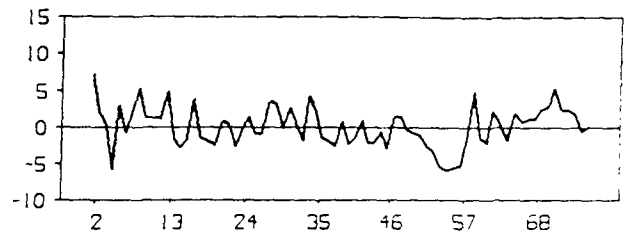
To analyze the dependence of mispricing behavior on the time to maturity, the coefficient d is estimated using equation (3). The values for d are obtained using a rolling sample with 30 observations for each contract. The first sample for each contract, with exception of the first contract, starts roughly 120 days before maturity and ends roughly 80 days before maturity. For the following samples, the last observation is dropped and the next is included. Figure 2 shows the results. No systematic behavior of d due to time to maturity can be observed. For six out of eleven contracts, d seems to tend to zero with decreasing time to maturity. However, there are also hump-shaped and increasing movements.

Figure 1: *Mispricing series (in Index points)*

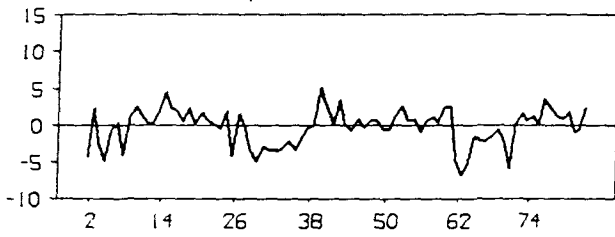
March 91



June 91



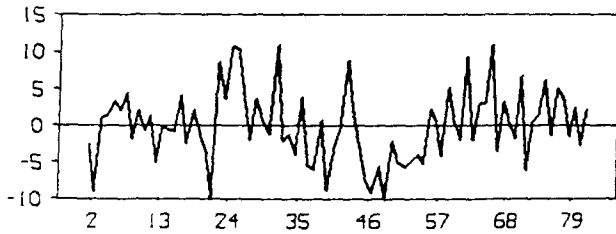
September 91



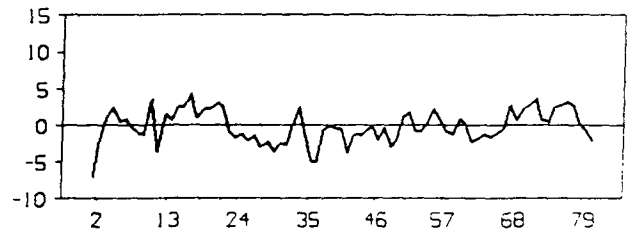
December 91



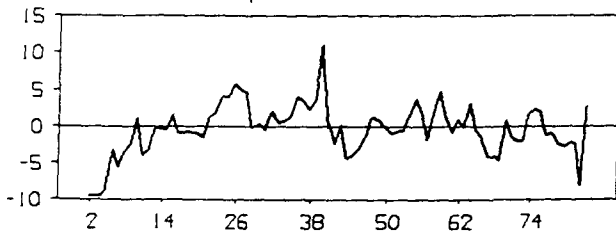
March 92



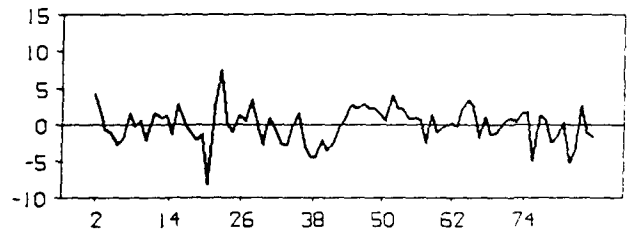
June 92



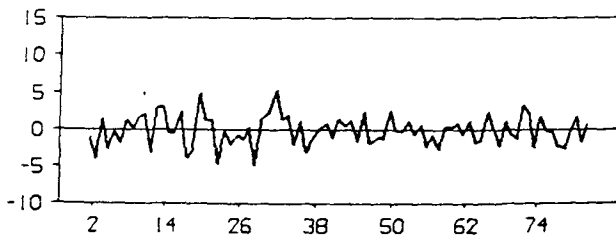
September 92



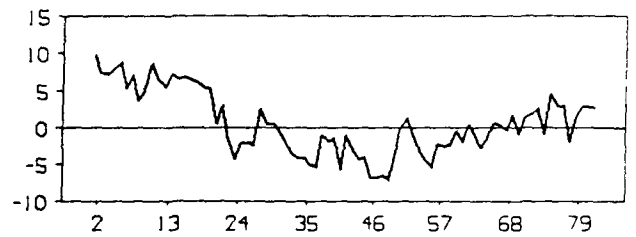
December 92



March 93



June 93



September 93

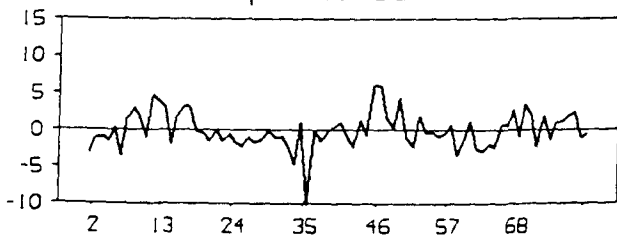
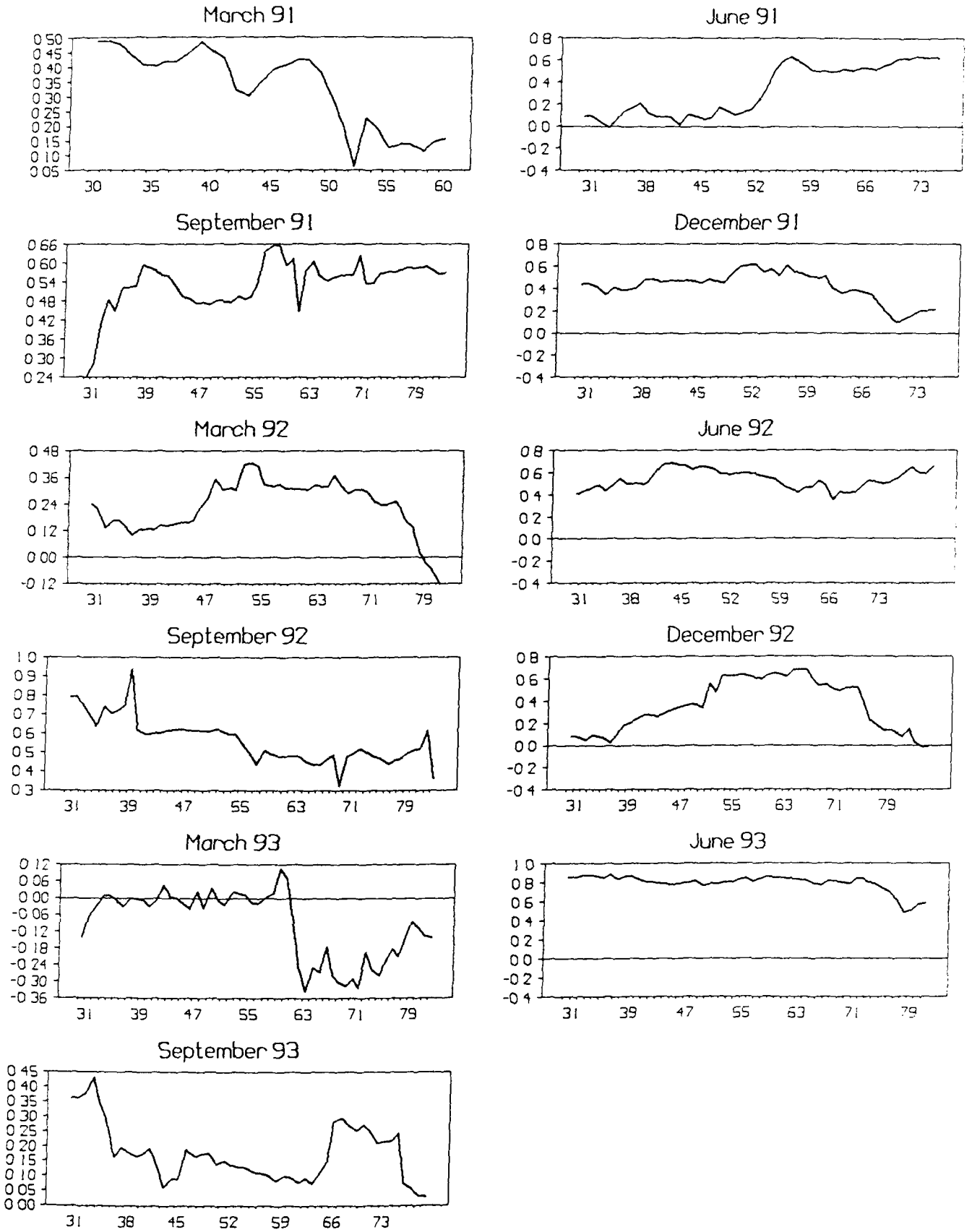


Figure 2: Parameter value d related to the time to maturity

b) The question of price leadership

To reduce the possibility of spurious correlations resulting from high daily index returns, index and related futures returns which are in absolute terms higher than 2.25% are eliminated. From the total sample consisting of 676 observations, 24 observations are excluded. As a result from the exclusion, the returns were not found to be significantly different from normality. Three subsamples are constructed to control the stability of the results. Table III shows the regression results. As far as the total sample is concerned, the cash market leads the futures market. While a is not significantly different from zero, b is 0.45 and significantly different from zero at the 99%-level. Similar results are obtained for the 1991 and 1993 contracts. For the 1992 contracts, however, the futures market leads the cash market.

Table III: *Summary of regression results concerning the question of price leadership*

$$\begin{aligned} \text{Regression equations: } \Delta i_t &= c_i + a \cdot u_{t-1} + v_{i,t} \\ \Delta f_t &= c_f - b \cdot u_{t-1} + v_{g,t} \end{aligned}$$

Hypothesis: $H_0: a = 0, b = 0$

, **, * significantly different from the null hypothesis at the 90%, 95%, 99%-level
The figures in parentheses are standard errors.*

	a	R ²	b	R ²
all contracts	0.16 (0.16)	0.002	0.45 (0.17) ***	0.011
1991 contracts	-0.47 (0.31)	0.010	1.00 (0.32) ***	0.042
1992 contracts	0.67 (0.23) ***	0.034	0.01 (0.24)	0.000
1993 contracts	0.04 (0.35)	0.000	0.56 (0.36)	0.013

The results based on the full sample as well as on the 1991 and 1993 sample are in contrast to the empirical findings of Grünbichler, Longstaff and Schwartz (1993), who

analyze the year 1991. Using 5-minute data and Granger-Sims-type causality tests they come to the conclusion that the DAX-Future leads the index by about fifteen to twenty minutes. Schwarz and Laatsch (1991) analyze the Major Market Index (MMI), which is a price-weighted index composed of 20 of the most actively traded stocks on the New York Stock Exchange. The characteristics, i.e. a small number of shares and a large trading volume, are shared by the DAX. Using the Garbade and Silber model for an empirical analysis, Schwarz and Laatsch obtain similar results for d using daily data. Different contracts imply different results concerning price leadership. Solely the increase in trading volume of futures indicates that the futures market leads the cash market. When examining five-minute data, Schwarz and Laatsch come to the result that the futures market leads the cash market. Leading cash markets on a daily basis and leading futures markets on an intra-daily basis are not a special feature of the DAX-Future market. Grünbichler, Longstaff and Schwartz (1993) give the following reasons for the leadership of the futures market or of cash markets, respectively:²⁰

- trading costs are lower in the DAX-Future market, so that informed traders may find it possible to trade on the basis of less significant pieces of information, thus accelerating the price discovery process in the futures market.
- computerized trading speeds up the process of information collection, of demand execution and order execution.
- there are information lags in a floor-trading market .
- screen-trading markets are not as transparent as floor trading markets in the sense that traders may not know with whom they are trading. This attracts informed traders.
- an explanation for the lead of the cash markets is given according to Subrahmanyam (1991).²¹ If an informed trader has firm specific information, it may be optimal to trade directly in the shares of the firm rather than trading in the index. In this case, the cash market leads the futures market. The impact of this feature should not be underestimated. It is easier to aggregate firm-specific

²⁰ Grünbichler, Longstaff and Schwartz (1993), pp.4-8.

²¹ Subrahmanyam (1991), pp.41-43.

information into an index than to disaggregate the index to obtain firm-specific information. This could be the reason for the leadership of the cash market when lower frequency is analyzed.

Therefore lead-lag relationship between the cash and the futures market can be bidirectional. It should not be forgotten that screen-trading is not the sole characteristic of the DAX-Future market. DAX shares are also screen-traded.

VI. Conclusion

In this paper, the DAX-Future market has been analyzed to provide answers to the questions of pricing relationship, mispricing behavior and price leadership. This is done with the use of co-integration analysis on the basis of the theoretical model of Garbade and Silber. The pricing relationship between the DAX-Future and the DAX is found to be close to the one suggested by the cost-of-carry model. On average, the DAX-Future is undervalued. On average, 63% of the current day's mispricing was eliminated the following day, which induces co-integration of the DAX-Future and the DAX. The desirable result of a 100%-elimination of the current day's mispricing on the following day is obtained for three contracts. There is no indication that the DAX-Future market improves its risk transfer function with decreasing time to maturity or in later contracts, respectively. Over the whole period, the cash market leads the futures market. When subsamples are used ambiguous results are obtained. Similar results for the price leadership of the cash market for daily data are also reported by Wahab and Lashgari (1993) for the S&P 500 and the FT-SE 100, and by Schwarz and Laatsch (1991) for the MMI. Using Granger-Sims-type causality tests and 5-minute data, Grünbichler, Longstaff and Schwartz (1993) find that the DAX-Future market leads the cash market. Schwarz and Laatsch obtain similar results for 5-minute data. For weekly data, they show that the cash market leads the futures market. Therefore, one question remains: Why does the lead of futures markets in high frequency data diminish and turn into a lead of the cash market in low frequency data?

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Appendix

- DW : Durbin-Watson-test for first-order autocorrelation
H₀: DW = 2 no autocorrelation
- JB : Jarque-Bera-test for normality
H₀: JB = 0 normality
- LB(s) : Ljung-Box-test for serial correlation up to order s
H₀: LB(s) = 0 no serial correlation
- ARCH(s): Test for autoregressive conditional heteroskedasticity up to order s
H₀: ARCH(s) = 0 no autoregressive conditional heteroskedasticity

Table I: Summary of the regression results for the pricing equation and the mispricing behavior

regression equation: $f_t = o \cdot i_t + p \cdot r_t \cdot (T-t) + d \cdot (f_{t-1} - o \cdot i_{t-1} - p \cdot r_{t-1} \cdot (T-t+1)) + v_t$

Hypothesis: Pricing equation is equal to cost-of-carry-relation $H_0: o = 1, p = 1,$

no serial correlation for mispricing series $H_0: d = 0$

mean being not different from a zero mean $H_0: \bar{v}=0$

*, **, *** significantly different from the null hypothesis at the 90%, 95%, 99%-level

	March 91	June 91	Sept. 91	Dec. 91	March 92	June 92	Sept. 92	Dec. 92	March 93	June 93	Sept. 93
N	59	74	82	74	80	79	82	84	81	80	78
o	1.00028*	0.99991	0.99999	0.99983***	1.00023**	1.00012*	0.99997	1.00014**	0.99993*	0.99982	0.99990*
σ_o	0.00015	0.00007	0.00008	0.00006	0.00011	0.00006	0.00011	0.00006	0.00003	0.00022	0.00005
p	0.31***	0.64***	0.74***	0.85***	0.69***	0.73***	0.93	0.82***	0.97*	0.62***	0.84***
σ_p	0.08	0.03	0.03	0.03	0.05	0.03	0.05	0.02	0.01	0.12	0.03
d	0.41***	0.33***	0.47***	0.37***	0.18	0.50***	0.60***	0.26**	-0.08	0.83***	0.18
σ_d	0.12	0.11	0.10	0.11	0.11	0.09	0.09	0.11	0.11	0.07	0.11
residual analysis											
Σv_t^2	0.00038	0.00017	0.00014	0.00011	0.00073	0.00008	0.00019	0.00019	0.00013	0.00013	0.00014
R ²	0.9980	0.9979	0.9978	0.9978	0.9933	0.9932	0.9996	0.9971	0.9989	0.9967	0.9993
DW	2.02	2.02	1.90	2.13	2.08	1.91	2.04	1.98	1.97	2.44	2.09
$\bar{v} (\cdot 10^5)$	-0.463	-0.102	-0.205	0.038	0.146	-0.012	-0.063	-0.138	0.002	-0.150	0.040
t($\bar{v}=0$)	-0.014	-0.006	-0.014	0.003	0.004	-0.001	-0.004	-0.008	0.000	-0.010	0.003
Skewness	0.21	0.25	-0.39	-0.22	0.02	-0.25	0.43	-0.28	0.03	0.16	-0.86
Kurtosis	2.69	2.66	3.54	3.39	3.34	3.00	4.60	3.79	3.07	2.60	7.78
JB	0.61	1.09	2.90	1.07	0.35	0.82	10.04**	3.01	0.02	0.79	78.16***
LB(5)	3.13	4.64	5.04*	2.76	6.50**	4.42	4.17	5.65*	8.37**	5.60*	5.97*
LB(10)	6.55	5.44	12.93*	11.49	27.06***	8.46	9.48	11.92	12.97*	9.83	7.48
ARCH(1)	0.17	1.06	4.19**	0.02	0.05	5.98*	14.77***	5.39**	0.61	0.23	0.02
ARCH(5)	13.21**	4.75	5.10	1.74	3.00	8.68	21.00***	14.21**	9.55*	5.48	0.54

Table III: Summary of the regression results concerning the question of price leadership

Regression equations: $\Delta i_t = c_i + a \cdot u_{t-1} + v_{i,t}$ $\Delta f_t = c_f - b \cdot u_{t-1} + v_{g,t}$

Hypothesis: $H_0: a = 0, b = 0$ *, **, *** significantly different from the null hypothesis at the 90%, 95%, 99%-level

	all contracts		contracts 91		contracts 92		contracts 93	
	DAX	DAX-Future	DAX	DAX-Future	DAX	DAX-Future	DAX	DAX-Future
N	651	651	225	225	241	241	185	185
c	0.0004	0.0000	0.0005	0.0003	-0.0003	-0.0005	0.0009	0.0004
σ_c	0.0003	0.0003	0.0005	0.0006	0.0005	0.0005	0.0005	0.0006
a,b	0.16	0.45***	-0.47	1.00***	0.67***	0.01	0.04	0.56
$\sigma_{a,b}$	0.16	0.17	0.31	0.32	0.23	0.24	0.35	0.36
residual analysis								
Σv_t^2	0.038	0.040	0.014	0.015	0.013	0.014	0.010	0.010
R ²	0.002	0.011	0.010	0.042	0.034	0.000	0.000	0.013
DW	1.89	1.88	2.07	2.02	1.83	1.86	1.71	1.71
Skewness	0.05	0.12	0.25	0.31	-0.11	0.03	-0.12	-0.13
Kurtosis	3.17	3.28	2.95	3.18	3.35	3.38	3.28	3.20
JB	1.05	3.71	2.41	3.88	1.73	1.47	1.00	0.76
LB(5)	6.62*	6.77*	1.08	0.81	4.19	1.93	10.80***	8.90**
LB(10)	9.43	9.16	4.27	6.75	6.13	2.37	16.31**	14.42*
LB(20)	17.28	18.51	20.40	22.35	16.20	12.89	29.97**	27.44*
ARCH(1)	1.81	1.57	1.88	2.16	1.49	1.27	0.68	1.70
ARCH(5)	4.92	5.19	3.97	4.79	7.21	6.44	1.67	3.34