

Henkel, Joachim; Stahl, Konrad O.; Walz, Uwe

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**Wirtschaftswissenschaftliche Fakultät
der Eberhard-Karls-Universität Tübingen**

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in a Spatial Economy**

Joachim Henkel, Konrad Stahl and Uwe Walz



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Coalition Building in a Spatial Economy

by Joachim Henkel^a, Konrad Stahl^b and Uwe Walz^{c, d}

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Abstract

We analyze the possibility and consequences of coalition-formation amongst suppliers of retail services. We first provide a framework in which producers of substitutes have an incentive to cluster in market places in order to attract consumers dispersed in space. Owing to spatial externalities, the resulting spatial equilibrium can be welfare suboptimal. We characterize regimes in which we find too little and those in which there is too much agglomeration of firms. We analyze the role of coalitions of firms (e.g. initiated by a land developer) in this framework and show that such coalitions can overcome the suboptimality of the decentralized spatial allocation.

Key words: endogenous agglomerations, monopolistic competition, coalition building

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^aBain, Germany

^bUniversity of Mannheim, ZEW, and CEPR, London

^cUniversity of Tuebingen and CEPR, London

^dCorresponding Author: Department of Economics, University of Tuebingen, Mohlstr. 36, Germany.

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1 Introduction

The spatial allocation of economic activity is governed by the interaction of agglomerative and disagglomerative forces. These forces are generated from what one might call spatial externalities. An example of an agglomerative force is the case in which the profitability of a location to a firm may be enhanced if additional households demanding its non-tradable supply decide to locate there. However, upon locating individual agents such as these households typically do not take such an enhancement into account. Owing to such spatial externalities central to regional and urban economics analysis, suboptimality of decentralized spatial allocation decisions is the rule rather than the exception.

Towards improving on allocation decisions in space, the intervention of regional or urban government authorities is typically called for. These authorities are to internalize the spatial externalities, in order to restore the optimality of the spatial allocation. However, there are many obstacles to determine an optimal allocation and to implement it. Information problems are probably most important. It is also not clear whether regional and urban government authorities have an incentive to target the decentralized optimal allocation. It might rather be in their parochial interest to even worsen decentralized allocation decisions from a welfare point, e.g., by attracting economic activity to the location under their jurisdictions (see, for example, Walz and Wellisch (1996)).

In this paper, we consider an alternative way to cope with the suboptimality of spatial allocation decisions. Rather than delegating this task to government authorities, we consider the possibility of coalition formation amongst private economic agents and their effects on spatial allocation decisions. We start from our earlier observation that some of the central market failures are due to spatial externalities not internalized by decentralized individual decision making. In addition we make use of the idea that coalitions of agents do have an incentive to internalize these externalities at least partially. Towards this, we analyze the behavior of coalitions in a specific spatial context

and investigate whether coalition formation can improve on the spatial allocation. If this is the case, then it might be preferable from a public policy point of view to allow for, or even to strengthen, the incentives for coalition formation. Indeed, we will show that some market failures can be resolved at least partially within the private sector. Below we focus on the retailing sector as an empirically relevant example to resolve the allocation problem. There, shopping center developers take such a coordination function.

We model the emergence of retailing markets involving specialized sellers of differentiated products, by combining two branches of the literature in which agglomeration incentives of firms are analyzed. On one hand we apply aspects of industrial organization approaches, in which the location decisions of small numbers of firms in space are analyzed via game theoretic methods. They usually adopt a continuous space Hotelling type framework (e.g. Eaton and Lipsey (1979, 1982), Stahl (1982a,b), de Palma et al. (1985), Economides (1986), Stahl (1987), Economides and Siow (1988), Gehrig (1994), Schulz and Stahl (1996), and Gehrig et al. (1997)). In these models, consumers are continuously distributed in space. On the other hand we make use of regional economics approaches in which monopolistically competitive firms select from a discrete set of locations. Typically, Dixit and Stiglitz' (1977) by now classic framework is adopted (see e.g. Fujita (1988), Rivera-Batiz (1988), Krugman (1991a, 1992), and Walz (1996, 1997)).

We present a monopolistic competition model where consumers are distributed uniformly along a line, and thus are differentiated by income net of their transactions costs of patronizing a market. For simplicity of exposition, firms are restricted to locate at the line's end points. We concentrate on the formation of agglomerations by specialized (one product) retailers. Upon patronizing one of these points called market places, at a cost purely dependent on distance, consumers buy the utility maximizing commodity bundle. Just as in Dixit and Stiglitz (1977) or Stahl (1982b), consumers are differentially attracted to the market place offering a larger commodity bundle. We allow for an endogenous determination of the number of firms in each agglomeration.

Our basic approach resembles the work of Church and Gandal (1992) who analyze the emergence of technical standards. We modify their approach in a number of ways (e.g. with respect to the formulation of transport costs and the structure of the game) and adopt it to a spatial economics setting.

We derive conditions under which in equilibrium one single agglomeration arises and/or two (symmetric) agglomerations coexist. We show that as consumers' income increases and thus access costs become relatively less important, a single agglomeration is the likely equilibrium outcome. The same tendency arises with decreasing substitutability between the goods. We also characterize situations in which multiple equilibria arise. Equilibria with two agglomerations of different size are not found.

We also determine the welfare preferred equilibria under a simple additive welfare criterion. For parameter regimes under which multiple equilibria occur, the one actually arising may be welfare-inferior. In this case, there is room for the welfare-increasing coalitions formation considered then. Welfare-improvement via coalitions is most promising if a welfare inferior single agglomeration equilibrium arises. In this case a coalition of firms has an incentive to voluntarily defect from the single agglomeration equilibrium and establish the second market place. Welfare improving coalition formation is more demanding when it comes to changing a symmetric two market equilibrium into a welfare-superior single market one. There, only in a few cases does coalition formation represent a way to overcome the suboptimal market outcome.

The paper is structured as follows. Section 2 contains the model description. In section 3 we deduce the equilibrium allocations. The welfare comparison of equilibria is conducted in section 4. Section 5 contains the analysis of coalition formation and its welfare effects. We summarize and conclude with section 6.

2 The Model

The geographical space considered here is by a line of unit length. Possible market places are restricted to be located at the ends of the line. We refer to these markets

as 0 and 1. A continuum of consumers of mass M is uniformly distributed along the line. Each consumer is endowed with income Y spent on transportation to one of the markets and on purchasing commodities there. Transportation costs tz are linear in distance z . Income is large enough to cover the expenses for the longest possible trip, i.e. $Y > t$. Each consumer maximizes utility

$$U(x_1, \dots, x_n) = \left(\sum_{i=1}^n x_i^\alpha \right)^{\frac{1}{\alpha}} \quad 0 < \alpha < 1, \quad (1)$$

where n denotes the number of goods consumed and x_i , $i = 1, \dots, n$, represents the quantity bought of good i . The parameter α describes the degree of substitutability between the goods: the larger α , the closer substitutes they are. Consumers shop only at one market place. The demand of a consumer living at z for good i ($i = 1, \dots, n_k$) at location k ($k \in \{0, 1\}$) is

$$x_{ik} = \frac{p_{ik}^{-\frac{1}{\beta}-1}}{\sum_{j=1}^{n_k} p_{jk}^{-\frac{1}{\beta}}} (Y - t|z - k|), \quad (2)$$

where $\beta = \frac{1}{\alpha} - 1$. p_{ik} stands for price of good i at location k . The term $(Y - t|z - k|)$ equals total consumption expenditures (income minus transportation costs) of the consumer. Using (1) and (2) the indirect utility function of a consumer living at z and shopping at location k is given by

$$V(pz, k) = q_k(Y - t|z - k|), \quad (3)$$

where $q_k := \left(\sum_{i=1}^{n_k} q_{ik}^{-1/\beta} \right)$ represents the inverse of the local market price index multiplied by n_k^β .

The consumer indifferent between shopping at location 0 and 1 is located at z^* . With (3) we can express z^* by: ²

$$z^* = \frac{y(m^\beta - 1) + 1}{m^\beta + 1}. \quad (4)$$

²Of course, this expression and the one above only hold if the r.h.s. of (4) lies in $[0, 1]$; otherwise, z^* is either 0 or 1.

with $y := Yt^{-1} > 1$ and $m := (q_0/q_1)^{1/\beta}$. The market demand for good i at location 0 and location 1, respectively, is then

$$X_{i0} = \frac{p_{i0}^{-\frac{1}{\beta}-1}}{\sum_{j=1}^{n_0} p_{j0}^{-\frac{1}{\beta}}} \left(M \int_0^{z^*} (Y - zt) dz \right) , \quad (5)$$

$$X_{i1} = \frac{p_{i1}^{-\frac{1}{\beta}-1}}{\sum_{j=1}^{n_1} p_{j1}^{-\frac{1}{\beta}}} \left(M \int_{z^*}^1 (Y - (1-z)t) dz \right) . \quad (6)$$

Our firms trade one good each and, incur a fixed cost F and constant variable costs c . A firm trading good i at location k maximizes profits

$$G_{ik} = (p_{ik} - c)X_{ik} - F, \quad (7)$$

by choosing the optimal price

$$p = \frac{c}{\alpha} \equiv c(\beta + 1) , \quad (8)$$

which is derived under the assumption of a large number of firms implying that strategic interactions are neglected. Backward substitution of (8) implies $m = n_0/n_1$, such that (4) can easily be reinterpreted: the larger the relative number of goods provided in location 0, the larger its market area, due to consumers' preference for variety: $\partial z^*/\partial m > 0$. Furthermore, (4) reveals that the larger Y , or the smaller t , the more attractive the location with the larger number of goods, as income lost for transportation purposes becomes relatively less important: $\partial z^*/\partial y > (<)0$ and $\partial z^*/\partial t < (>)0$ as $m > (<)1$.

Inserting (5), (6) and (8) in (7) gives us the typical firm's profits at the respective location:³

$$G_0(n_0, n_1) = \frac{\beta M z^*}{(\beta + 1)n_0} \left(Y - \frac{t z^*}{2} \right) - F \quad (9)$$

and

$$G_1(n_0, n_1) = \frac{\beta M (1 - z^*)}{(\beta + 1)n_1} \left(Y - \frac{t(1 - z^*)}{2} \right) - F . \quad (10)$$

³We drop the firm index, as firms operating at one location perform symmetrical.

In principle we model the interaction between firms and consumers as a three stage non-cooperative game. In its first stage, firms decide simultaneously about entry and location. In its second, firms choose their prices, whilst in its last stage, consumers choose the locations to shop. Since we have already analyzed the last two stage we concentrate now on the first stages and the resulting spatial equilibria.

3 The number of marketplaces

3.1 Equilibrium candidates

In our model, there are two basic types of configurations characterized by (n_0, n_1) the number of firms in either marketplace: those in which firms locate at both ends of the line - we call this a fragmented configuration; and those with concentration of all firms at on of the ends - we call this an agglomerated configuration. A necessary condition for the former to constitute an equilibrium is that firms profits are equalized across both markets. We can express the profit differential with the help of (4), (9), and (10) as ⁴

$$G_0 - G_1 = \frac{\beta M t}{2(\beta + 1)n_0(m^\beta + 1)^2} \left[(y(m^\beta - 1) + 1)(y(m^\beta + 3) - 1) + m((y - 1)m^\beta - y)((3y - 1)m^\beta + y) \right] \quad (11)$$

Let $g(m)$ denote the expression in square brackets.⁵ $g(m) = 0$ constitutes the necessary equilibrium condition. We indicate in appendix 1 that this condition is satisfied for at most three values. These imply either a symmetric (fragmentation) two agglomeration equilibrium with firms located in equal numbers at both ends of the line; or an agglomeration equilibrium with all firms located at one of its ends.

⁴Just as equation (4), the following equation is only correct if the r.h.s. of (4) lies in $[0, 1]$. However, it can be shown that $G_0 - G_1 = 0$ can arise only for $z^* \in (0, 1)$.

⁵We henceforth neglect the integer problem and treat n_0 and n_1 as continuous variables.

3.2 The symmetric two agglomeration equilibrium

For a configuration involving n_k firms at location k , to be an equilibrium, no firm can have an incentive to deviate. Suppose \bar{n}_0 and \bar{n}_1 firms decide to enter at location 0 and 1, respectively, and profits are the same in both locations, i.e. $g(\bar{m}) = 0$ (with $\bar{m} = \bar{n}_0/\bar{n}_1$), and driven to zero: $G_0(\bar{n}_0, \bar{n}_1) = G_1(\bar{n}_0, \bar{n}_1) = 0$. Then, (\bar{n}_0, \bar{n}_1) constitutes an equilibrium if no firm has an incentive to switch location nor a potential entrant finds entry profitable.

In appendix 2 we prove:

Proposition 1 *i.) There is a function $\beta_b(y)$ with $\partial\beta_b/\partial y < 0$, such that for given y , there is unique Nash equilibrium involving an identical number of firms at both locations iff $\beta < \beta_b(y)$. Hence, a symmetric fragmentation equilibrium is the likelier the lower the typical consumers income, the higher her transportation costs, and the closer the substitutability between the goods.*

ii.) In the symmetric fragmentation equilibrium, $n_0 = n_1 = \frac{\beta M}{2(\beta+1)F}(Y - t/4) =: n^{(2)}$ firms will locate in either market place.

3.3 Single agglomeration equilibria

We now address the possibility that the concentration of all firms in a single location ($m = 0$ or $m = \infty$) constitutes an equilibrium. Let $\mu := Mt/F$.

Appendix 3 contains the proof for

Proposition 2 *(i) For sufficiently large μ , there exists a function $\beta_g(y, \mu)$ with $\partial\beta_g/\partial y < 0$, such that for given y , $\beta \geq \beta_g(y, \mu)$ implies that there is an agglomeration equilibrium at one of the line's ends. Hence, an agglomeration Nash equilibrium is the likelier the higher the typical consumers' income, the lower her transportation costs, and the poorer the substitutability between the goods.*

(ii) If $\beta \in [\beta_g(y, \mu), \beta_b(y)[$, an agglomeration and a symmetric fragmentation equilibrium coexist.

(iii) In that regime, consumers enjoy a larger variety of goods with an agglomeration equilibrium. However, those living further away have to bear the burden of higher transport costs.

Figure 1 shows the function $\beta_g(y, \mu)$ as the lower curve where, as an example, $\mu = 500$ has been chosen.⁶ The second curve is the upper boundary $\beta_b(y)$ of the area of stability of a symmetric configuration with zero profits. Hence, the parameter space is divided into three regimes.⁷ In region \mathcal{A} , a symmetric fragmentation configuration constitutes an equilibrium, while an agglomeration configuration does not. In \mathcal{B} , both configurations are equilibria, while in \mathcal{C} an agglomeration configuration is an equilibrium and a symmetric fragmentation configuration is not.

Furthermore, we prove in Appendix 4 that an asymmetric fragmented configuration does not constitute an equilibrium:

Proposition 3 *A configuration (n_0, n_1) with $g(m) = 0$, $g'(m) > 0$ implies incentives for some firms to deviate. Hence, there is no asymmetric fragmentation equilibrium, since $g'(m) > 0 \forall m$ with $g(m) = 0$ and $m \neq 1$.*

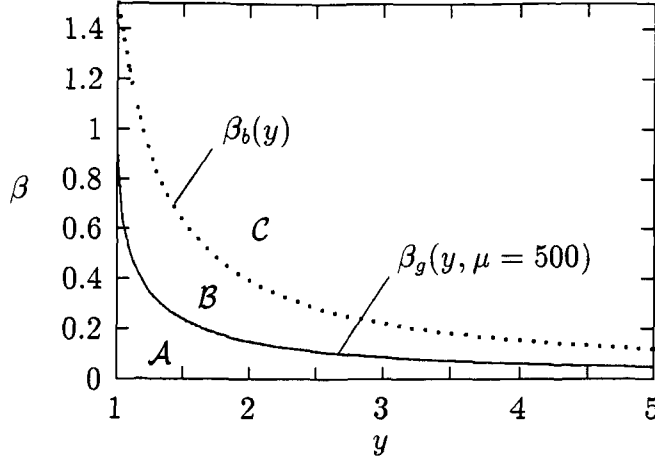
Hence, Figure 1 completely describes the equilibrium regimes in our game.

In concluding the discussion of equilibrium regimes, observe finally that a replication of the consumer sector leaves $\beta_b(y)$ unchanged, but moves $\beta_g(y, \mu)$ towards the axes. With further replication, the parameter region in which the symmetric fragmentation equilibrium is the only one disappears and the agglomeration equilibrium always arises as at least one equilibrium type. This is due to the fact that a larger number of consumers increases the absolute difference between the equilibrium number of goods supplied in one market and in two markets, making the agglomeration disadvantage of transport costs increasingly unimportant. The same happens when the firms' entry cost F is reduced: the firms need only a smaller market share to cover their cost

⁶Our numerical analysis provides ample evidence that for sufficiently large μ a unique $\beta_g(y, \mu)$ -curve exists that fulfills $\beta_g(y, \mu) < \beta_b(y)$.

⁷Any other value of μ yields qualitatively the same result, provided it is not too small. A higher value of μ shifts the boundary between areas \mathcal{A} and \mathcal{B} downwards.

Figure 1: Areas of fragmentation (\mathcal{A}, \mathcal{B}) and agglomeration equilibria ($\mu = 500$)



which implies an increase in the regime in which the agglomerated configuration is an equilibrium. The effect of independent variations of t cannot be studied as easily, as y varies in t as well. However, it can be shown that the effect of a change in t via μ is more than compensated by the effect via γ . Hence the regime in which both the agglomerated and the fragmented equilibria exist increases, and that in which only the fragmented one obtains decreases with a decrease in consumers' transactions costs.

4 Welfare aspects

We now turn to a welfare comparison of the symmetric fragmentation and the agglomeration configuration. Towards this we employ a simple additive welfare function in which all consumers' surpluses are weighed equally. Producer surplus does not arise, as profits are zero in both types of equilibria due to free entry.

We derive in appendix 5

Proposition 4 (i) *The welfare superior of the two types of configurations is always an equilibrium.*

(ii) *There exists a $\beta_w(y) \in]\beta_g(y, \mu), \beta_b(y)[$ such that for any given y , $\beta \geq \beta_w(y)$ implies welfare preferability of the symmetric fragmentation equilibrium, and $\beta < \beta_w(y)$*

implies welfare preferability of the agglomeration equilibrium. Hence, the decentralized solution leads to a welfare-inferior solution if, with $\beta \in]\beta_g, \beta_w[$ ($\beta \in [\beta_w, \beta_b[$) an agglomeration (symmetric fragmentation) equilibrium emerges.

Why do welfare inferior equilibria arise at all? The reasoning is as follows. In their respective location decisions, firms do not take properly into account consumers' interests. They only consider the marginal consumer (via the market size effect), while the interests of the remaining consumers are left aside. Specifically, suppose that the symmetric fragmentation equilibrium is welfare preferred, but consider a firm's move that increases the asymmetry between the agglomerations. This increases utility for those customers who live close to the larger location, and decreases it for those far away who still prefer the smaller location. For a relatively low heterogeneity of the goods and low income, the second effect dominates such that the concentration of firms tends to be less preferable. Conversely, let the agglomeration equilibrium be welfare preferred, and let a deviating firm's move increase the symmetry between the locations. Then, at relatively low degrees of substitutability, or high consumer income, the utility increase for those enjoying increased variety is outweighed by the disadvantages to the customers consuming less variety, such that the symmetric fragmentation equilibrium tends to be welfare inferior.

Before turning to the analysis of coalitions we should note that the size of the regime in which an agglomeration equilibrium is welfare inferior increases with population density. This is due to the fact that β_w is invariant in μ whereas β_g decreases with μ (see our discussion in 3.3). Hence, our analysis confirms very well with the intuition that it is not feasible to achieve welfare-improving decentralization of economic activity when population size increases.

5 Coalitions

In many real cases an urban developer or a big investor in a (planned) shopping mall coordinates the actions of single firms. In the non-cooperative game-theoretical

framework developed in the preceding sections, such a coordinating agent was assumed away. Now, we consider the possibility of coalitions of firms. We proceed in two steps. In the first one, we analyze the incentives of coalitions to deviate from a decentralized equilibrium as described in the preceding sections. In the second one, we ask for the welfare effects of such deviations.

It seems straightforward to assume that coalition building becomes more difficult and costly when the size of the coalition increases. One might think about transaction costs involved to getting agents to agree on joint behavior, and costs to set up enforceable contracts. Therefore, we use the size of the smallest coalition altering profitably its location as a measure for the stability of a configuration. The smaller the critical coalition, the easier it is for the intermediary coordinating the location decisions of firms to organize a sufficient number of firms to switch location jointly. We assume the existence of binding and enforceable contracts.

More specifically we use a modified version of Aumann's (1959) strong Nash equilibrium (SNE) to analyze the stability of configurations against deviations of coalitions of a given maximal size. A strategy profile s^* characterizes a SNE, if no partition J of the set of players and a strategy profile \tilde{s}_J exist, such that *each member* of J will strictly gain by deviating from s_J^* to \tilde{s}_J . Thereby, gains are not transferable among players. In the present context this does, however, not restrict the generality of the analysis, due to symmetry between the players.⁸ By definition a SNE in Aumann's sense is stable against deviations of all feasible coalitions. However, considering only coalitions of maximal size k , $1 \leq k \leq N$, we pursue matters based on the following

Definition [k-SNE] *Suppose the existence of a game $\Gamma = (I, (S^i)_{i \in I}, (G^i)_{i \in I})$ with $I = \{1, \dots, N\}$. Then the strategy profile $s^* \in \prod_{i \in I} S^i$ is a k -SNE if and only if for all $J \subseteq I$ with $|J| \leq k$ (i.e. for all coalitions with at most k members) and for all $s_J \in \prod_{j \in J} S^j$ no $i \in J$ exists such that $G^i(s_J, s_{-J}^*) > G^i(s^*)$.*

⁸We show later on that the only deviations of interest are the ones in which the strategy of all coalition members is the same before and after deviation. Due to the symmetry between firms, deviating is strictly profitable for all or for none.

According to this definition the set of all 1-SNEs is equivalent to the set of Nash-equilibria. In the case of N players the definitions of N -SNE and Aumann's SNE coincide. Since – by definition – the set of k -SNEs is a subset of all Nash-equilibria, only the Nash-equilibria derived in section 3 are candidates for a k -SNE.

The analysis of k -SNEs is facilitated by the observation that it suffices to look at coalitions which only include firms moving jointly from one location to the other (see the proof in Appendix 6). Including into the coalition entrants and/or firms switching location in the other direction weakens the profitability of the coalition.

5.1 Coalition Building in the Presence of a Single Market Place

By definition, a single firm cannot profitably deviate from an equilibrium $(n^{(1)}, 0)$ (which implies zero profits). However, the joint change of location of 50% of firms from $(n^{(1)}, 0)$ is always profitable (see Stahl (1982a)): the coalition receives, as before its move, half of total revenues, which have, however, increased. The increase in revenues is due to an increase in total consumption expenditures from $M(Y - t/4)$ to $M(Y - t/2)$ as a consequence of lower total transport costs. Hence, the configuration $(n^{(1)}, 0)$ can never be a $n^{(1)}$ -SNE and consequently does not constitute a SNE either.

The crucial question is: for which k and which parameter combination is $(n^{(1)}, 0)$ a k -SNE? We can restrict the analysis to the case of a location change of a coalition from 0 to 1. Suppose a fraction r of firms chooses a coordinated location change from 0 to 1, the market place with initially zero firms. The condition that this move leaves the firms not worse off is

$$G_1((1 - r)n^{(1)}, rn^{(1)}) \geq 0 . \quad (12)$$

Recalling that equality in (12) $m := (1 - r)/r$ is equivalent to,

$$\frac{Mt\beta(m + 1)}{2n^{(1)}(\beta + 1)(m^\beta + 1)^2} (m^\beta(1 - y) + y) (m^\beta(3y - 1) + y) - F = 0 . \quad (13)$$

This equation defines

$$\beta_1(y, r) = \left[\ln \left(\frac{r}{1-r} \right) \right]^{-1} \ln \left(\frac{2y-1}{\sqrt{y^2-2ry+r}} - 1 \right). \quad (14)$$

Location changes are profitable if $\beta < \beta_1(y, r)$. (cf. Appendix 7). The index delineates coalition building with *one* market place, in contrast to coalition building in the presence of *two* market places (cf. subsection 5.2). Due to (A.8), $n^{(1)}$, M , t and F can be eliminated from $\beta_1(y, r)$. For the limit values $r \rightarrow 0$ and $r \rightarrow 0.5$ we get:

$$\lim_{r \rightarrow 0} \beta_1(y, r) = 0, \quad \lim_{r \rightarrow 0.5} \beta_1(y, r) = \infty \forall y. \quad (15)$$

Figure 2 illustrates the function $\beta_1(y, r)$ for different values of r . For instance, with parameter combinations below the lowest curve, a coordinated change of location from the agglomerated Nash equilibrium is profitable if more than 10% of the firms cooperate. With more than 40% cooperating, a change in location is even profitable in a part of parameter space where a Nash-equilibrium with two market places does not exist ($\beta \in [\beta_b(y), \beta_1(y, 0.4)]$).

In Appendix 7 we prove

Proposition 5 *Consider the configuration $(n^{(1)}, 0)$. For all $r \in]0, 0.5[$ and all $y > 1$ there exists a $\beta_1(y, r)$ such that $G_1((1-r)n^{(1)}, rn^{(1)}) \geq 0 \Leftrightarrow \beta \leq \beta_1(y, r)$. That is, a coalition of size $rn^{(1)}$ can change to location 1 without incurring losses if and only if $\beta \leq \beta_1(y, r)$. Furthermore, $\partial \beta_1(y, r) / \partial y < 0$, and $\lim_{y \rightarrow \infty} \beta_1(y, r) = 0$ holds.*

Therefore, the configuration $(n^{(1)}, 0)$ represents for the parameter values (y, β) a $(rn^{(1)})$ -SNE if and only if no $\tilde{r} \leq r$ exists such that $\beta < \beta_1(y, \tilde{r})$. Numerical evidence indicates that $\partial \beta_1(y, r) / \partial r > 0$. If this is the case, then $(n^{(1)}, 0)$ is a $(rn^{(1)})$ -SNE if and only if $\beta \geq \beta_1(y, r)$.

The economic intuition behind the effects of r and y on the profitability of defection is the following. With decreasing income, transport costs become relatively more important for consumers living further away from the larger market place. Hence, attracting these consumers to the second (smaller) market places is facilitated making

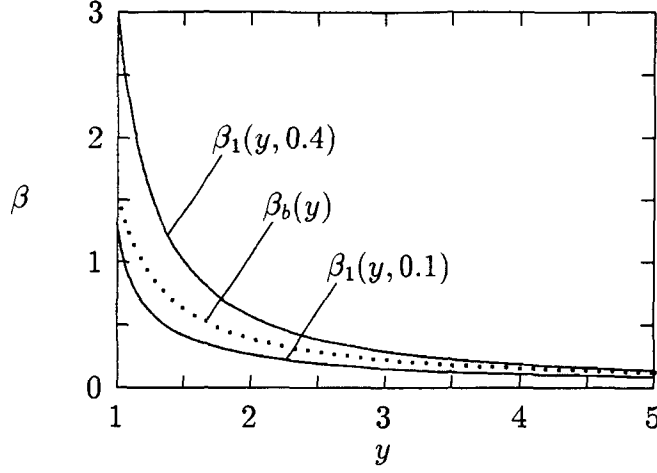


Figure 2: Below $\beta_1(y, r)$: profitable change of location from 0 to 1 of a coalition of size $r = 0.1$ $r = 0.4$, respectively, starting from $(0, n^{(1)})$.

the defection of a coalition of given size more likelier to be profitable. On one hand a coalition of larger size (larger r) is able to attract more consumers (a larger market area). But on the other hand, competition increases with r in the second location. However, our numerical analysis indicates that the first, profitability enhancing effect dominates the second, which operates in the opposite direction.

5.2 Coalition Building with Two Market Places

For which k , y and β is the symmetric fragmentation configuration with zero profits $(n^{(2)}, n^{(2)})$ a k -SNE? A coalition of size $rn^{(2)}$ can profitably switch location from 0 to 1 if

$$G_1((1-r)n^{(2)}, (1+r)n^{(2)}) > 0,$$

with $m = (1-r)/(1+r)$. This leads to the function

$$\beta > \beta_2(y, r) := \left[\ln \left(\frac{1+r}{1-r} \right) \right]^{-1} \ln \left(\frac{2(2y-1)}{\sqrt{(2y-1)^2 - r(4y-1)}} - 1 \right). \quad (16)$$

Location changes are profitable if $\beta > \beta_2(y, r)$ (cf. Appendix 8) This is a necessary condition; it is sufficient if $z^* \in]0, 1[$. As soon as the market of decreasing size at 0 can

no longer attract consumers ($z^* = 0$), (16) is not valid. Instead $G_1((1-r)n^{(2)}, (1+r)n^{(2)}) = \beta(\beta+1)^{-1}Mt(y-1/2)/(n^{(2)}(1+r)) - F$, such that $G_1((1-r)n^{(2)}, (1+r)n^{(2)}) > 0$ together with part ii.) of Proposition 1 leads to the condition

$$y > y_{\min}(r) := \frac{3-r}{4(1-r)} \Leftrightarrow r < r_{\max}(y) := \frac{4y-3}{4y-1}. \quad (17)$$

The condition $r < r_{\max}(y)$ can readily be understood: the number of firms $n^{(2)}(1+r)$ at the larger marketplace after the move is limited by consumers' total net income, which decreases due to increased transportation costs.

Together, the conditions (16) and (17) are necessary and sufficient. The limit values of $\beta_2(y, r)$ for minimal and maximal (sensible) coalition size are:

$$\lim_{r \rightarrow 0} \beta_2(y, r) = \frac{4y-1}{2(2y-1)^2} \equiv \beta_b(y), \quad (18)$$

$$\lim_{r \rightarrow (4y-3)/(4y-1)} \beta_2(y, r) = \ln\left(\frac{y}{y-1}\right) [\ln(2(2y-1))]^{-1}. \quad (19)$$

Figure 3 illustrates the restrictions (16) and (17) for $r = 80\%$, together with $\beta_b(y)$ (dotted). The formation of a coalition and deviation from the initial equilibrium is profitable to the right of the limiting curves.

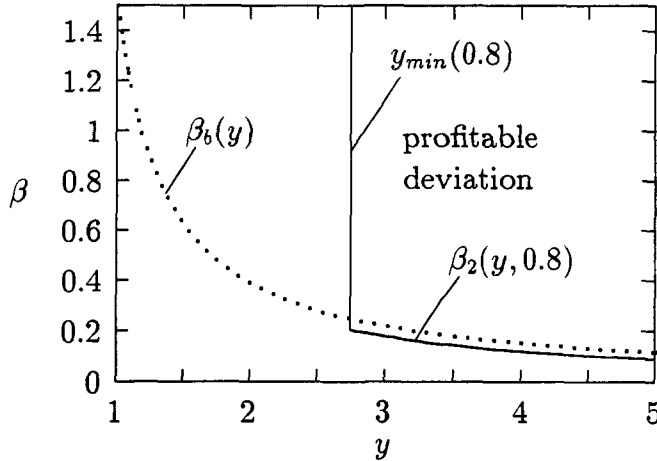


Figure 3: Above and to the right of $y_{\min}(0.8)$ and $\beta_2(y, 0.8)$: profitable deviation of a coalition of 80% of $n^{(2)}$, starting from $(n^{(2)}, n^{(2)})$.

Given that μ is not too small there exists a parameter region below $\beta_b(y)$ in which multiple equilibria arise.

In Appendix 8 we show

Proposition 6 *Consider a configuration $(n^{(2)}, n^{(2)})$. For any $r \in]0, 1[$ there exists a $\beta_2(y, r)$ and a $y_{\min}(r)$, such that $G_1((1-r)n^{(2)}, (1+r)n^{(2)}) \geq 0 \Leftrightarrow \beta \geq \beta_2(y, r) \wedge y > y_{\min}(r)$. That is, a coalition of size $rn^{(2)}$ can profitably switch to location 1 if and only if $\beta > \beta_2(y, r)$ and $y > y_{\min}(r)$. Furthermore $\partial\beta_2(y, r)/\partial y < 0$ and $\lim_{y \rightarrow \infty} \beta_2(y, r) = 0$.*

That β_2 decreases with y (or that for given β a profitable defection of a coalition of size r becomes more likely with increasing y) is just the consequence of the fact that with increasing y , transport costs become relatively less important. This makes it easier for firms in the larger market to attract more consumers living further away. In turn, this implies higher profits in the larger market. Defection of a coalition of a given size becomes more profitable.

Given the parameter values (y, β) the configuration $(n^{(2)}, n^{(2)})$ constitutes an $(rn^{(2)})$ -SNE iff no $\tilde{r} \leq r$ exists such that $y > y_{\min}(\tilde{r})$ and $\beta > \beta_2(y, \tilde{r})$.

A drawback of the SNE-concept is that it often fails to exist.⁹ However, in the game considered here the configuration $(n^{(2)}, n^{(2)})$ may represent a SNE. This is the case if for parameters (y, β) and for all $r \in]0, 1[$ either $y \leq y_{\min}(r)$ or $\beta \leq \beta_2(y, r)$. The set of parameter values for which a SNE exists is definitely not empty. This can be seen by observing that the condition for a profitable deviation of a coalition of size $rn^{(2)}$, namely inequality (16), converges in the limit $\beta \rightarrow 0$ to the inequality $m > 1$, which cannot be fulfilled as $m = (1-r)/(1+r) < 1$. Therefore, for given y $(n^{(2)}, n^{(2)})$ is a SNE if β is sufficiently small.

5.3 Welfare aspects of coalition formation

Analyzing welfare aspects in section 4, we found that in a certain portion of the (y, β) parameter space both types of equilibria coexist. In the regime $\beta \in [\beta_g, \beta_w[$ -

⁹See, for example, Bernheim *et al.* (1987).

call it B_1 – the agglomeration equilibrium is suboptimal. The same is true for the fragmentation equilibrium in the regime $\beta \in [\beta_w, \beta_b]$ called B_2 . What implications does the possible formation of coalitions have on the implementation of welfare preferred equilibria? More precisely, given that a certain configuration is a Nash-equilibrium, under which conditions is it a k -SNE? Does the formation of coalitions favor welfare optimal equilibria? It does indeed, as the following proposition shows for the case of agglomeration equilibria.

Proposition 7 (i) *For any $y > 1$ and any $r \in]0, 0.5[$ there exists a function $\tilde{\beta}_1(y, r)$ such that a configuration $(n^{(1)}, 0)$ is a $(rn^{(1)})$ -SNE iff $\beta \geq \tilde{\beta}_1(y, r)$ and $\beta \geq \beta_g(y, \mu)$. The function $\tilde{\beta}_1(y, r)$ is monotonically increasing in r , $\lim_{r \rightarrow 0} \tilde{\beta}_1(y, r) = 0$, and $\lim_{r \rightarrow 0.5} \tilde{\beta}_1(y, r) = \infty$.*

(ii) *If, for given y , the formation of coalitions of the maximum relative size r makes an agglomeration equilibrium disappear that is superior to the fragmentation configuration (i.e. one in area B_2 or C , $\beta \geq \beta_w(y)$), then it makes all inferior ones (those in area B_1 , $\beta \in [\beta_g(y, \mu), \beta_w(y)[$) disappear.*

The proof of this proposition is contained in Appendix 9.

Hence, the formation of coalitions is welfare improving in the sense that if the maximum relative size r of coalitions is increased from zero, then, for given y , the “first” concentrated equilibria to disappear are those that are suboptimal compared to the symmetric equilibrium.

In order to grasp the economic intuition behind this result, it is helpful to recall the reasons for the suboptimality of the decentralized uncoordinated spatial allocation. The suboptimality is due to the fact that by deciding upon their location, individual firms consider only the marginal consumer and the marginal market area effect, but disregard the impact on all other consumers and firms. The coalition, in turn, partially internalizes this external effect, by taking the market area effect on all coalition members into account. In doing so, the coalition gains most if the coordination problem is rather pronounced, i.e. if the spatial allocation in the uncoordinated Nash equilibrium is suboptimal.

The question arises if we can obtain a similar result for symmetric equilibria. The situation here is more complicated, since we have two conditions for a profitable deviation of a coalition ((16) and (17)) instead of just one. Furthermore, the limit behaviour of $\beta_2(y, r)$ towards the minimum and maximum value of r ((18) and (19)) is not as simple as that of $\beta_1(y, r)$ (15). Nevertheless, we can prove the following

Proposition 8 (i) *For any $y > 1$ and any $r \in]0, r_{max}(y)[$ there exists a function $\tilde{\beta}_2(y, r)$ such that a configuration $(n^{(2)}, n^{(2)})$ is a $(rn^{(2)})$ -SNE iff $\beta \leq \tilde{\beta}_2(y, r)$ and $\beta < \beta_b(y, \mu)$. The function $\tilde{\beta}_2(y, r)$ is continuous and monotonically decreasing in r , $\lim_{r \rightarrow 0} \tilde{\beta}_2(y, r) = \beta_b(y)$, and $\tilde{\beta}_2(y, r_{max}(y)) \leq \ln(y/(y-1))/\ln(2y-1)$.*

(ii) *If, for given y , the formation of coalitions of the maximum relative size $r \in]0, r_{max}(y)[$ makes a symmetric equilibrium disappear that is superior to the concentrated configuration (i.e. one in area \mathcal{A} or \mathcal{B}_1 , $\beta \leq \beta_w(y)$), then it makes all inferior ones (those in area \mathcal{B}_2 , $\beta \in]\beta_w(y), \beta_b(y)[$) disappear.*

The proof of this proposition is presented in Appendix 10.

Proposition 8 closely resembles Proposition 7: again, if the maximum relative size r of coalitions is increased from zero, then, for given y , the “first” symmetric equilibria to disappear are those that are suboptimal compared to the concentrated equilibrium. In this sense here, too, the possibility of coalition formation has a welfare improving effect.

For given y and coalitions of any size possible, a symmetric configuration with zero profits is stable against deviations of coalitions iff $\beta \leq \tilde{\beta}_2(y, r_{max}(y))$ and $\beta < \beta_b(y)$. Numerical evidence suggests that for $y \leq y_1 \approx 1.984$ we have $\tilde{\beta}_2(y, r_{max}(y)) = \beta_b(y)$, and hence no symmetric equilibrium is affected by coalition formation. In the range $y_1 < y \leq y_2 \approx 3.852$ we found $\tilde{\beta}_2(y, r_{max}(y)) \in [\beta_w(y), \beta_b(y)[$, which means that only suboptimal symmetric equilibria disappear due to coalition formation. Finally, for $y > y_2$, we have $\tilde{\beta}_2(y, r_{max}(y)) < \beta_w(y)$: all suboptimal equilibria and part of those superior to the agglomeration configuration disappear when coalitions of any size are possible.

Taking the above findings together we can state that the possibility of coalition formation, by destroying suboptimal equilibria “more easily” (i.e. with smaller maximum size of the coalition) than optimal ones, favours an efficient outcome of the game and, hence, has a welfare improving effect.

6 Summary

We analyzed in this paper the endogenous formation of market places by i.) uncoordinated decisions and ii.) by coalitions of firms. The central trade-off discussed in the present model is between agglomeration advantages to consumers arising from increased product variety, and the agglomeration disadvantages to the firms that increased competition. More specifically, the larger the number of products available in an agglomeration, the larger the access costs consumers are willing to incur to participate in this marketplace, increasing thereby the market area captured by the agglomeration. However, the more firms are located at the same location, the smaller the market share per firm and the stiffer competition.

Both, agglomerative as well as disagglomerative force are accompanied by spatial external effects. Hence, we find that in cases in which multiple equilibria coexist, equilibria resulting from the uncoordinated decisions of agents can be suboptimal. It turns out that with a suboptimal agglomeration equilibrium allowing for (or even promoting) coalition formation seems to have the most clear cut welfare enhancing effects (e.g. coordinated by an investor or a land-developer). Coordinated action will destroy the welfare-inferior decentralized equilibria first. In a setting with welfare-inferior symmetric fragmentation equilibria welfare improving activities are also possible.

Hence our model suggests that coordinated action via a coalition formation is a potential alternative to local authorities trying to improve on spatial allocation decisions. The story told here can be naturally varied and applied to a specific context. For instance, one could again start with an inefficient entry and location equilibrium, and then ask for the minimal size (in terms of number of commodities) a multiproduct

seller should take to distort that equilibrium towards the welfare preferred one.¹⁰ The essence of the story remains unchanged: almost by definition, coalitions have a greater incentive to internalize spatial externalities than individual agents making uncoordinated decisions. This leaves room for private sector solution even in the presence of spatial externalities. However, a final caveat is in order. So far, we considered only the private coordination of location decisions, and not one involving, for instance, product selection, or private decisions. All these decisions undertaken by specialized sellers generate externalities. In particular, the unilateral introduction of a product, or a unilateral price decrease lead to an expansion of the market size captured by a market place.¹¹ In contrast to the coalition of specialized sellers considered in our paper, the aforementioned multiproduct seller will have the potential to internalize these externalities, at the cost of (partially) monopolizing the (local) market. In fact, while we have discussed here the interaction between sellers of final commodities and consumers all of the principal arguments can be recasted in a context of interacting firms. Thus there is much room for the analysis of an intriguing subject, namely that of finding an optimal structure for the organization of industry in space.

¹⁰Observe that the resulting move to the new equilibrium would involve the exit of some specialized firms.

¹¹These externalities are described in detail in Stahl (1987) or Schulz and Stahl (1996).

Appendix

Appendix 1: Derivation of $\{m\}$ such that $g(m) = 0$

Note that $g(0) = (1 - y)(3y - 1) < 0$ (see (11) and $\lim_{m \rightarrow \infty} g(m) = +\infty > 0$). Furthermore, we find $g(1) = 0$, which follows from symmetry. If goods are poor substitutes (large β), $g(m)$ intersects the m -axis exactly once, namely, at $m = 1$.¹² Conversely, if goods are close substitutes (small β), we find a local maximum in $(0, 1)$ (and a corresponding minimum in $(1, \infty)$). In this case, $g(m)$ intersects the m -axis three times; namely at $m_1^* < 1$, at $m_2^* > 1$, with $m_1^* = 1/m_2^*$ (in either case from below), and at $m_3^* = 1$ (from above). For $y \geq 1 + 2^{-1/2}$ we can show that $g(m)$ has either one, or three roots.¹³ While we have not been able to extend the proof to the full range of y , we derived ample numerical evidence that the result holds for any $y \in]1, \infty[$.

Appendix 2: Proof of Proposition 1

ad i.) We first note that profits per firm at, say, location 0 are the higher the fewer firms there are at the other location. Hence, if further entry at that location is profitable, then all the more profitable is a switch from the other location. When looking for profitable deviations from a possible equilibrium configuration, we can thus restrict our attention to players switching locations.

For strictly positive \bar{n}_0, \bar{n}_1 to constitute an equilibrium, it is necessary and sufficient that

$$G_i(\bar{n}_0, \bar{n}_1) = 0 \text{ for } i = 0, 1,$$

$$G_0(\bar{n}_0 + 1, \bar{n}_1 - 1) \leq 0, \text{ and} \tag{A.1}$$

$$G_1(\bar{n}_0 - 1, \bar{n}_1 + 1) \leq 0. \tag{A.2}$$

Inequalities (A.1) and (A.2) stipulate that a firm, upon changing from a zero profit

¹²By symmetry it suffices to look at the range $m \in [0, 1]$ towards analyzing the zeros of $g(m)$.

¹³A proof is available upon request.

location earns negative profits at its new location. Let $\bar{n} := (\bar{n}_0 + \bar{n}_1)/2$, $\Delta := (\bar{n}_0 - \bar{n}_1)/2$. We rewrite conditions (A.1) and (A.2) in differential notation as¹⁴

$$\frac{\partial G_0(\bar{n}, \Delta)}{\partial \Delta} < 0, \quad (\text{A.3})$$

$$\frac{\partial G_1(\bar{n}, \Delta)}{\partial \Delta} > 0. \quad (\text{A.4})$$

Let us now turn first to the situation where the number of firms is the same in both locations. Clearly, this is a candidate for an equilibrium, since $g(1) = 0$ (see (11)). In this case, conditions (A.3) and (A.4) are equivalent. Using $z^*(n, \Delta = 0) = 0.5$ and

$$\left. \frac{\partial z^*(n, \Delta)}{\partial \Delta} \right|_{\Delta=0} = \frac{\beta}{n} \left(y - \frac{1}{2} \right), \quad (\text{A.5})$$

the necessary and sufficient condition for an equilibrium is

$$\left. \frac{\partial G_0(n, \Delta)}{\partial \Delta} \right|_{\Delta=0} = \frac{\beta t}{8(\beta + 1)n^2} [2\beta(2y - 1)^2 - (4y - 1)] < 0. \quad (\text{A.6})$$

We can now deduce from eq. (A.6) the region in the (y, β) -parameter plane where a symmetric fragmentation configuration constitutes an equilibrium as the area below the function

$$\beta_b(y) = \frac{4y - 1}{2(2y - 1)^2}. \quad \text{Q.e.d.} \quad (\text{A.7})$$

ad ii.) Plugging (4) in (9) and (10) yields for a zero-profit situation with an equal number of firms in both locations the expression in Proposition 1.

¹⁴While in (A.1) and (A.2) equality is sufficient to guarantee that there are no incentives to switch locations, this is not the case in differential notation: even if a single firm, as compared to the whole marketplace, is small enough to justify the differential approach, the gains from a switch of location may be positive, if small in second order, when the derivatives in (A.3) or (A.4) vanish.

¹⁶It turns out that in the case of two symmetric markets, the condition $g' < 0$ is equivalent to (A.6). This is due to the fact that total profits remain the same with a marginal deviation from the symmetric solution. Hence, the deterioration of profits in the agglomeration the defecting firms switches to, relative to the profit situation in the agglomeration it defects from, guarantees that defection is unprofitable. In general, however, (A.6) implies $g' < 0$, but not vice versa. Hence, $g' > 0$ is a sufficient condition for a configuration not to be an equilibrium; see Proposition 3.

Appendix 3: Derivation of Proposition 2

W.l.o.g. consider the situation $(0, n^{(1)})$, where $n^{(1)}$ is determined from the zero profit condition

$$n^{(1)} = \frac{\beta}{\beta + 1} M(Y - t/2)/F = \frac{\beta\mu}{\beta + 1} \left(y - \frac{1}{2}\right) , \quad (\text{A.8})$$

with $\mu := Mt/F$. No single firm changes location if this yields negative profits. As $z^* = 0$ if $n^{(1)} - 1 \geq (y/(y - 1))^{1/\beta}$, this is always the case if the number of firms is sufficiently large. The defecting firm is not able to attract consumers and thus will not be able to cover its fixed costs.¹⁷ For simplicity of exposition we restrict the defecting firm to set its price to $p = c/\alpha$.¹⁸ From (9) we obtain as the condition for an unprofitable deviation of a single firm

$$\beta\mu z^* \left(y - \frac{z^*}{2}\right) \leq \beta + 1 , \quad (\text{A.9})$$

where z^* is implicitly defined by (4) and (A.8).

Equality in (A.9) implicitly defines a function $\beta_g(y, \mu)$, such that an agglomerated configuration with zero profits is an equilibrium iff $\beta \geq \beta_g(y, \mu)$.

ad ii.) Combing part ii.) of Proposition (1) and part i.) of Proposition (2) gives us ii.)

ad iii.) $n^{(1)} > n^{(2)}$ (see Proposition (2) and (A.8)). *Q.e.d..*

Appendix 4: Proof of Proposition 3

Suppose there exists an asymmetric solution involving $\tilde{m} := \tilde{n}_0/\tilde{n}_1$ and $G_0(\tilde{n}_0, \tilde{n}_1) = G_1(\tilde{n}_0, \tilde{n}_1) = 0$. W.l.o.g. look at $\tilde{n}_0 < \tilde{n}_1$. Hence, $z^* < 0.5$. With a deviation of

¹⁷The condition for $n^{(1)}$ to be “large” is, however, different from the condition justifying the approximation $p = c/\alpha$. From a numerical analysis we obtained that in a considerable portion of the parameter space a single firm, defecting from an agglomeration of 100 firms, will obtain positive profits when defecting to location 0, while the approximation $p = c/\alpha$ is less than one percent off the correct value. Hence, it is not inconsistent to maintain $p = c/\alpha$, whilst taking the finiteness of n_1 into account when considering a defecting firm.

¹⁸It would do better when correctly optimizing its price. While this would slightly diminish the area of stability of a single agglomeration in parameter space, the results would remain qualitatively unchanged.

a single firm from location 1 to location 0, m increases and the marginal consumer moves closer towards the center (see (4)). Hence, a smaller fraction of total income is spent on transportation, leading to larger overall profits for the given total number of firms. With $g'(m) > 0$ the profit differential between the two agglomerations becomes positive: $G_0(\tilde{n}_0 + 1, \tilde{n}_1 - 1) - G_1(\tilde{n}_0 + 1, \tilde{n}_1 - 1) > 0$. With increasing overall profits, $G_0(\tilde{n}_0 + 1, \tilde{n}_1 - 1) + G_1(\tilde{n}_0 + 1, \tilde{n}_1 - 1) > 0$. It follows that $G_0(\tilde{n}_0 + 1, \tilde{n}_1 - 1) > 0$. Hence, $g'(m) > 0$ is a sufficient condition for profitable deviation and, thus, for the instability of the corresponding configuration. Since $g'(m) > 0 \quad \forall m \neq 1$, asymmetric equilibria do not exist. Q.e.d.

Appendix 5: Derivation of Proposition 4

Denote by $W^{(1)}$ and $W^{(2)}$ the total welfare in the case of one and two locations, respectively. Total welfare under the alternative equilibrium regimes is easily calculated from (3)¹⁹

$$W^{(1)} = \frac{M(n^{(1)})^\beta}{c(\beta + 1)} \left(Y - \frac{t}{2} \right) = \frac{M^{1+\beta} \beta^\beta}{c(\beta + 1)^{(\beta+1)} F^\beta} \left(Y - \frac{t}{2} \right)^{1+\beta}, \quad (\text{A.10})$$

and

$$W^{(2)} = \frac{M(n^{(2)})^\beta}{c(\beta + 1)} \left(Y - \frac{t}{4} \right) = \frac{M^{1+\beta} \beta^\beta}{2^\beta c(\beta + 1)^{(\beta+1)} F^\beta} \left(Y - \frac{t}{4} \right)^{1+\beta}. \quad (\text{A.11})$$

Hence, the condition for the agglomeration equilibrium to be welfare superior (inferior) to the fragmentation equilibrium is

$$2^\beta \left(y - \frac{1}{2} \right)^{1+\beta} > (<) \left(y - \frac{1}{4} \right)^{1+\beta}, \quad (\text{A.12})$$

which can be solved for β :

$$\beta > (<) \beta_w(y) =: \frac{\ln \left(1 + \frac{1}{4y-2} \right)}{\ln \left(2 - \frac{2}{4y-1} \right)}. \quad (\text{A.13})$$

¹⁹Strictly speaking, (A.10) and (A.11) show the consumers' aggregate utility. However, as there is no outside good, the utility derived from not shopping at all can be considered as zero.

The curve $\beta_w(y)$ separating the two welfare regimes divides region \mathcal{B} into two parts. It is decreasing in y , and for $y \rightarrow \infty$ it runs asymptotically to $(y4 \ln 2)^{-1}$. In all, there are two instances in which the free market outcome can be welfare-inferior: in regime $\mathcal{B}_1(\beta \in [\beta_g, \beta_w])$ the agglomeration equilibrium is welfare-inferior and in $\mathcal{B}_2(\beta \in [\beta_w, \beta_b])$ the same is true for the symmetric fragmentation equilibrium. *Q.e.d.*

Appendix 6:

Due to symmetry among firms a strategy profile may be described by a triple $s := (n_0, n_1, n_v)$, with $n_0 + n_1 + n_v = N$ and $n_\tau (\tau = 0, 1, v)$ describing the number of firms choosing location 0 or 1, respectively, or stay out of the market altogether. Once again for symmetry reasons a coalition of size k is described by a triple $(\tilde{n}_0, \tilde{n}_1, \tilde{n}_v)$, with $0 \leq \tilde{n}_0 \leq n_0, 0 \leq \tilde{n}_1 \leq n_1, 0 \leq \tilde{n}_v \leq n_v$ und $\tilde{n}_0 + \tilde{n}_1 + \tilde{n}_v = k$. A deviation of a coalition from the strategy profile s can be described with the help of the matrix

$$\mathcal{M} := \begin{pmatrix} \tilde{n}_{00} & \tilde{n}_{01} & \tilde{n}_{0v} \\ \tilde{n}_{10} & \tilde{n}_{11} & \tilde{n}_{1v} \\ \tilde{n}_{v0} & \tilde{n}_{v1} & \tilde{n}_{vv} \end{pmatrix},$$

whereby, for example, \tilde{n}_{01} denotes the number of coalition members using initially the strategy 0 and choosing, after deviation, strategy 1. Hence, we find for the sum of the first (second, third) row of \mathcal{M} $\tilde{n}_0, (\tilde{n}_1, \tilde{n}_v)$.

If there are coalition members who play the same strategy before and after deviation, their participation in the coalition is without consequence for the payoffs of the other players. That implies that we can restrict our search for profitable deviations of coalitions to the case $\tilde{n}_{00} = \tilde{n}_{11} = \tilde{n}_{vv} = 0$. A deviation is profitable for a coalition if *each* member of the coalition gains from the deviation.

Starting from a situation with zero profits, no coalition member can gain by abandoning market entry, since this would yield zero profits, too. Therefore, it suffices to consider the case $\tilde{n}_{0v} = \tilde{n}_{1v} = 0$.

Suppose $\tilde{n}_{01} \geq \tilde{n}_{10} > 0$ (or, equivalently, $\tilde{n}_{10} \geq \tilde{n}_{01} > 0$) and a profitable deviation of the coalition. Then, the deviation of the smaller coalition with $(\tilde{n}_{01} - \tilde{n}_{10})$ firms

switching from 0 to 1 and none from 1 to 0 is profitable, too. This enables us to focus on situations in which either $\tilde{n}_{01} = 0$ or $\tilde{n}_{10} = 0$.

If the deviation of a coalition with $\tilde{n}_{01} > 0$ and $\tilde{n}_{v0} > 0$, ($\tilde{n}_{10} > 0$ and $\tilde{n}_{v1} > 0$) is profitable, then this is even more the case with a deviation of a smaller coalition with no further market entry \tilde{n}_{v0} (\tilde{n}_{v1}). We can limit ourselves on matrices \mathcal{M} which contain either solely positive \tilde{n}_{01} and \tilde{n}_{v1} or solely positive \tilde{n}_{10} and \tilde{n}_{v0} .

Finally, suppose it is profitable for a coalition of \tilde{n}_{01} firms to deviate from location 0, with \tilde{n}_{v1} entrants choosing location 1. Then, deviation is even more profitable if $\min\{n_0, \tilde{n}_{01} + \tilde{n}_{v1}\}$ firms switch from location 0 to 1 and no market entry occurs.

It thus suffices to analyze matrices \mathcal{M} in which only \tilde{n}_{01} or \tilde{n}_{10} are positive. Q.e.d.

Appendix 7: Proof of Proposition 5

With the help of (A.8) we can rewrite (13) to

$$(m+1) \left(m^\beta (1-y) + y \right) \left(m^\beta (3y-1) + y \right) \geq (2y-1) \left(m^\beta + 1 \right)^2. \quad (\text{A.14})$$

From this, we obtain the limiting curve $\beta_1(y, r)$ (with $m = (1-r)/r$)

$$\beta_1(y, r) = \left[\ln \left(\frac{r}{1-r} \right) \right]^{-1} \ln \left(\frac{2y-1}{\sqrt{y^2 - 2ry + r}} - 1 \right). \quad (\text{A.15})$$

The coalition does not loose after its switch of location if $\beta \leq \beta_1(y, r)$. The limitation to $r < 0.5$ is straightforward since a change of location is always profitable with $r = 0.5$. Accordingly, $\beta_1(y, r)$ diverges with $r \rightarrow 0.5$ (from below) $\Leftrightarrow m \rightarrow 1$ (from above), since $\ln m \rightarrow 0$. The slope of the limiting curve therefore is

$$\frac{\partial \beta_1(y, r)}{\partial y} = \frac{y(1-2r) + r}{\ln \left(\frac{r}{1-r} \right) \left(\frac{r}{1-r} \right)^{\beta_1(y, r)} (y^2 - 2ry + r)^{3/2}} < 0, \quad (\text{A.16})$$

since $1-2r > 0$. Finally, we have to show that $\lim_{y \rightarrow \infty} \beta_1(y, r) = 0$. This follows straightforwardly from (A.15), since the argument of the second logarithmic term converges to 1 with $y \rightarrow \infty$. Q.e.d.

Appendix 8: Proof of Proposition 6

With the help of (10) and (lii)) we can rewrite $G_1((1-r)n^{(2)}, (1+r)n^{(2)}) \geq 0$ to

$$2(m+1) \left(m^\beta(1-y) + y \right) \left(m^\beta(3y-1) + y \right) \geq (4y-1) \left(m^\beta + 1 \right)^2 . \quad (\text{A.17})$$

With an equality sign in (A.17) the following limiting curve emerges $\beta_2(y, r)$ (with $m = (1-r)/(1+r)$):

$$\beta_2(y, r) = \left[\ln \left(\frac{1+r}{1-r} \right) \right]^{-1} \ln \left(\frac{2(2y-1)}{\sqrt{(2y-1)^2 - r(4y-1)}} - 1 \right) . \quad (\text{A.18})$$

After the change of location a coalition does not loose if $\beta \geq \beta_2(y, r)$. A further condition guarantees $y > y_{\min}(r)$ that total maximal revenues which can be achieved at the enlarged market place ($Mt(y - 1/2)$) suffice to cover fixed costs of $(1+r)n^{(2)}$ firms. We find for the slope of $\beta_2(y, r)$:

$$\frac{\partial \beta_2(y, r)}{\partial y} = -4 \frac{(2y-1)^2 + r}{\ln \left(\frac{1+r}{1-r} \right) \left(\frac{1+r}{1-r} \right)^{\beta_2(y, r)} ((2y-1)^2 - r(4y-1))^{3/2}} < 0 . \quad (\text{A.19})$$

Due to the fact that the argument of the second logarithmic term in (A.18) converges with $y \rightarrow \infty$ to 1, $\lim_{y \rightarrow \infty} \beta_1(y, r) = 0$ holds. Q.e.d.

Appendix 9: Proof of Proposition 7

Define

$$\tilde{\beta}_1(y, r) := \max_{\rho \in [0, r]} \beta_1(y, r) . \quad (\text{A.20})$$

By definition this function has all the features required by Proposition 7(i). Proposition 7(ii) follows directly from monotonicity of $\tilde{\beta}_1(y, r)$ in r and from the fact that, for given y , the points (y, β) in parameter space, where a suboptimal agglomeration equilibrium exists ($\beta < \beta_w(y)$), lie below those where the agglomeration equilibrium is optimal ($\beta \geq \beta_w(y)$). Q.e.d.

Appendix 10: Proof of Proposition 8

Define

$$\tilde{\beta}_2(y, r) := \min_{\rho \in [0, r]} \beta_2(y, r) . \quad (\text{A.21})$$

$\tilde{\beta}_2(y, r)$ fulfills by definition all the requirements of Proposition 8(i) (see (18) and (19)). The second part of the Proposition follows from monotonicity of $\tilde{\beta}_2(y, r)$ in r and from the fact that, for given y , the points (y, β) in parameter space, where a suboptimal fragmentation equilibrium exists ($\beta > \beta_w(y)$), lie above those where the symmetric equilibrium is optimal ($\beta \leq \beta_w(y)$). Q.e.d.

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