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An option pricing approach to investment in innovations in a competitive environment

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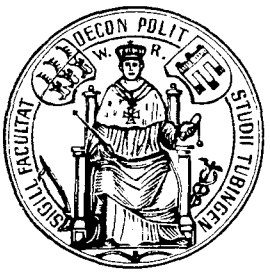
An Option Pricing Approach to Investment in Innovations in a Competitive Environment



Tübinger Diskussionsbeiträge

Wirtschaftswissenschaftliche Fakultät
der Eberhard-Karls-Universität Tübingen

An Option Pricing Approach to Investment in Innovations in a Competitive Environment

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Abstract

In this paper it is illustrated how option-based valuation can be used to determine whether and when a firm should patent and adopt an innovation if the arrival time of competitors is stochastic. Four distinct strategies are derived: Apply for a patent without introducing the new technology right away, patent the innovation and invest immediately, initiate the new project without patent protection, or defer the decision. It is shown how competition and the level of patent fee determines the strategy to be pursued and the maximum amount of R&D expenditures.

1. Introduction

Option pricing theory can be applied to many real investment decisions. Similar to options on financial securities real options involve the right, not the obligation, to acquire or sell an asset for a fixed price at a fixed date (European type option) or in a time interval (American type option). Pindyck (1991), Dixit and Pindyck (1994) and Trigeorgis (1996) give excellent surveys on the existing literature on real options. Several authors have noted that an innovation investment is a real option. The current investment is viewed as a link in a chain of future projects opening up growth opportunities (Grenadier and Weiss (1997)). The firm thus has a call option on the value generated from the innovation plus future growth options with a striking price equivalent to the investment cost. This option is of the perpetual American type since the firm has no deadline when to decide to invest. If the value of the project develops favorably the firm invests and if the project is not yet lucrative the firm 'waits and sees'. However, when a rival firm develops close substitutes to the innovation of the firm then the value of the firm's innovation investment diminishes so that the option value becomes zero. Thus, one must take into account the possible arrival of competitive firms when valuing real options and determining optimal investment strategies. Therefore, the effect of competition on the investment decision is analyzed in this paper.

Applying financial option pricing theory to real investment decisions has its limitations since no market is as perfect and as complete as the capital market. Preference free option valuation models rely on the redundancy of the options to be replicated. Whenever it is possible to construct a selffinancing replicating portfolio consisting of the riskfree security and the underlying that yields a payoff which exactly equals the payoff of the option the initial endowment required for this portfolio must be the value of the option in the absence of arbitrage opportunities. Real projects which form the underlying in the real options framework are usually not traded. Hence, arbitrage pricing theory and thus preference free option valuation theory is not applicable when valuing real options. In the following, riskneutrality of the firm is assumed, so that all assets are priced so as to yield an expected rate of return equal to the riskfree rate.

The analogy with financial options not only is not exact with regard to the ability to replicate the option with traded assets but also with regard to the exclusiveness of the option. The opportunity to introduce a new technology is not proprietary to the firm. Trigeorgis (1996) has shown that competition in the market may force the company to invest early so that the flexibility value of the deferred investment strategy is eroded. The non-exclusiveness of the option is incorporated into the following analysis by modelling the project value process as a mixed-jump-diffusion-process where a down-jump occurs when competition

enters the market. Hence, competitive arrivals are regarded as exogenously given and strategic interactions between competing firms are not accounted for.

The paper is organized as follows. Section 2 presents the basic assumptions. In section 3 the options approach to investment in innovation is studied in the case of a proprietary investment opportunity that is protected by enforceable patents. Section 4 studies the optimal timing of project initiation in a competitive environment. In section 5 an options approach is presented for the optimal timing of patents and investment. A firm that developed a new product has the possibility to apply for a patent in the competitive market. The firm anticipates the introduction by competitors of close substitute products and is thus eager to preempt the rivals. The model yields four distinct strategies for the firm depending on the patent fee and the investment cost.

2. The Model

In the following, it is assumed that a firm has a new technology, called project, which creates a net operating cash flow CF in perpetuity. The firm faces uncertainty about the magnitude of future cash flows. It is assumed that the profit flow CF evolves according to the following geometric Brownian motion with instantaneous drift α and instantaneous volatility σ:

$$dCF = \alpha CF\,dt + \sigma CF\,dz.$$

This implies that future profit flows are lognormally distributed and the expected value grows at a trend rate α. This trend reflects efforts in R&D. The volatility is exogenously specified and could result from changing input prices, changing demand curves or price structures.

Let r denote the riskfree interest rate. Provided that this interest rate exceeds the trend rate, the present value of the project V, i.e. the present value of expected cash inflows[1] under riskneutrality is given by

$$V_0 = \int_0^\infty E(CF)e^{-rt}dt = \int_0^\infty CFe^{(\alpha-r)t}dt = \frac{CF}{r-\alpha}. \qquad (2.1)$$

If the trend exceeds the interest rate, the project value is infinite, so that only the former case is considered.

[1]Future growth opportunities are not explicitly incorporated in the following analysis.

Since V is a constant multiple of CF the value of the project follows geometric Brownian motion as well:

$$dV = \alpha V dt + \sigma V dz.$$

The cost of introducing the new technology to the market I is presumably deterministic. The revenue from adoption of the project thus is $V - I$.

Investment may be postponed so that management can decide not to invest if the project value turns unfavorable. However, deferral also has its disadvantages since during the postponement period the firm misses the net operating cash inflows CF.

3. Patent Protection

In this section a firm is considered that has a proprietary technology protected by patents so that the resulting monopoly rents of the project can not be eroded and are thus not temporary. If the patent never expires the firm has the right to acquire V at the cost I whenever it seems favorable according to the price movement. The net value of the investment opportunity when exercising this right equals $V - I$. This right is analogous to a perpetual American call option with underlying V and striking price I. The forgone cash flows during the postponement period are analogous to a forgone continuous dividend yield, so that early exercise of the option may be optimal. It is possible to obtain a closed-form solution for these types of options.

The change in value of an option $W(V)$ that depends on the underlying V is according to Itô's Lemma:[2]

$$dW = W_V \alpha V dt + W_V \sigma V dz + \frac{1}{2} W_{VV} V^2 \sigma^2 dt$$

The expected value of the option at any time T is thus given by:

$$E(W(V,T)) = W(V,0) + \int_0^T \alpha V W_V dt + \int_0^T \frac{1}{2} \sigma^2 V^2 W_{VV} dt. \qquad (3.1)$$

[2] F_i denotes the partial derivative of F with respect to i: $F_i = \frac{\partial F}{\partial i}$.

Under riskneutrality the present value of the option equals the expected value discounted at the riskfree interest rate:

$$W(V,0) = E(W(V,T))e^{-rT}.$$

This can be written as

$$E(W(V,T)) = W(V,0)e^{rT} = W(V,0) + \int_0^T rWdt. \tag{3.2}$$

Combining (3.1) and (3.2) the following time independent differential equation (DE) for the value of the option is derived:

$$\alpha V W_V + \frac{1}{2}\sigma^2 V^2 W_{VV} - rW = 0.$$

The option value and thus the value of the deferrable investment opportunity is determined by solving this DE subject to the following boundary conditions:

$$\begin{aligned} W(0) &= 0 \\ W(V^*) &= V^* - I \qquad \text{(Value-Matching)} \\ W_V(V^*) &= 1 \qquad \text{(Smooth-Pasting).} \end{aligned} \tag{3.3}$$

The first condition states that once the value of the project reaches zero the option is terminated and thus worthless. This is due to the fact that zero is an absorbing barrier for the project value. The second boundary condition is the value-matching condition. The essence of the investment timing strategy is to find for every decision moment a critical project value V^*. At this critical value the value from postponing the investment further equals the net value of the project. As soon as the trigger value is exceeded, management invests. The smooth-pasting condition guarantees the optimality of the trigger price V^*.

The total value of the deferrable real investment opportunity is[3]

$$W(V) = \begin{cases} AV^{\beta} & \text{for } V \leq V^* \\ V - I & \text{for } V > V^* \end{cases} \tag{3.4}$$

where

$$\beta = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left(\frac{\alpha}{\sigma^2} - \frac{1}{2}\right)^2 + 2\frac{r}{\sigma^2}} \tag{3.5}$$

and where

$$V^* = \frac{\beta}{\beta - 1} I \qquad A = \left(\frac{\beta - 1}{\beta}\right)^{\beta - 1} \frac{1}{\beta} \left(\frac{1}{I}\right)^{\beta - 1}. \tag{3.6}$$

The ability to defer investment is valuable to managers. This is neglected by the net present value method. The value of the investment opportunity exceeds $\max(V - I, 0)$. As long as the project value is below the trigger price the value of the option to postpone is above the net project value. The firm initiates the project when the value of waiting is less than the value of forgone cash flows. Thus, contrary to the net present value method, management invests when $V \geq \beta / (\beta - 1) I > I$.

The value of the investment opportunity increases with σ and so does the critical price V^*. Thus, uncertainty increases the value of a firm's investment opportunities even if the firm is riskneutral due to the upside potential that higher uncertainty creates while the downside loss is limited because the new technology need not be adopted. The value increases with the drift of the project value since a larger drift implies lower forgone cash flows and it decreases with the interest rate because this rate reflects the opportunity cost. The larger the drift and the lower the interest rate the higher is the tendency to postpone investment.

4. Competition and No Patent Protection

In this section an investment opportunity in a new technology that is not protected by an enforceable patent is analyzed. The firm has a competitive advantage

[3] Cf. Dixit and Pindyck (1994), p. 152. Pindyck (1991), p. 1121.

in realizing the project. However, economic rents from the new product introduction are eroded as soon as several potential competitors catch up and enter the industry. The project value and thus the value of the investment opportunity then drops to zero. An exponential probability distribution for the arrival time of competitors is assumed. This can be reflected by a modification of the stochastic process for the cash inflows. The profit flow now follows a mixed jump-diffusion-process of the following type:[4]

$$dCF = \alpha CF dt + \sigma\, CF dz - CF dq$$

where

$$dq = \begin{cases} 0 & \text{with probability } (1 - \lambda dt) \\ 1 & \text{with probability } \lambda dt. \end{cases}$$

dq is the increment of a Poisson process and assumed to be independent of dz. The instantaneous probability that competitors enter the market the next moment given that they have not entered before is expressed by the constant hazard rate[5] λdt. If the jump occurs, monopolistic rents will be eroded. A version of Ito's Lemma for mixed Poisson and diffusion processes[6] yields the following stochastic process for the project value V^C under competition

$$dV^C = \alpha V^C dt + \sigma V^C dz - V^C dq \tag{4.1}$$

where

$$V^C = \frac{CF}{r + \lambda - \alpha}. \tag{4.2}$$

If the jump occurs, the project value drops to zero. Since this is an absorbing barrier the value stays there forever, so that the investment opportunity becomes

[4]See Merton (1990), p. 92.

[5]See Stadler (1991), p. 297. The expected arrival time of competition decreases with an increasing hazard rate, so that this rate is regarded as a measure of intensity of rivalry.

[6]See Merton (1990), p. 92.

worthless. The firm is free to choose whether or not and when to adopt the project V^C at the cost I. To solve for the optimal investment strategy and the value of the investment opportunity $W^C(V^C)$ a procedure analogous to the one above is used. The dynamics of the option $W^C(V^C)$ are given by

$$dW^C = W_V^C \alpha V^C dt + W_V^C \sigma V^C dz + \frac{1}{2} W_{VV}^C \left(V^C\right)^2 \sigma^2 dt - W^C dq.$$

The expected value of the option at any time T is

$$\begin{aligned} E(W^C(V^C,T)) &= W^C(V^C,0) + \int_0^T \alpha W_V^C V^C dt + \int_0^T \frac{1}{2} W_{VV}^C \sigma^2 \left(V^C\right)^2 dt \\ &\quad - \int_0^T W^C \lambda dt. \end{aligned}$$

Combining this equation and (3.2) leads to the following differential equation for the option value:

$$\alpha V^C W_V^C + \frac{1}{2}\sigma^2 \left(V^C\right)^2 W_{VV}^C - (r+\lambda) W^C = 0.$$

The same boundary conditions (3.3) apply as before. The solution is of the form (3.4), but with V replaced by V^C and β replaced by β^C, so that A is replaced by A_C:

$$\begin{aligned} \beta^C &= \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left(\frac{\alpha}{\sigma^2} - \frac{1}{2}\right)^2 + 2\frac{r+\lambda}{\sigma^2}} \\ A_C &= \left(\frac{\beta^C - 1}{\beta^C}\right)^{\beta^C - 1} \frac{1}{\beta^C} \left(\frac{1}{I}\right)^{\beta^C - 1} \end{aligned} \tag{4.3}$$

The impact of random competitive arrivals on the value of the investment opportunity can be viewed as larger opportunity costs. The option value thus decreases with the hazard rate. Similarly, the project itself becomes less valuable with an increasing hazard rate, c.p. While deferring investment management forgoes the expected profit flows $(r + \lambda - \alpha) V^C$. Hence, the firm initiates the project earlier with an increasing conditional probability of competitive arrival.

5. Competition and Potential Patent Protection

In this section a firm is assumed to have achieved a technological breakthrough in creating a new product. The firm is aware of a competing rival who may launch the new product or a close substitute before it is ready to do so. However, through patenting the firm acquires an exclusive option to invest in the project. Management can thus preempt a competitor's entry by making a commitment of capital, i.e. the patent fee P which not only consists of notarial fees but may also cover bonus for the staff. Innovative activities do not end after patenting the new product. The firm proceeds with some incremental cost-reducing innovations. Hence, its objective now is to find the optimal patenting date and the optimal investment date to maximize its profits.

When confronted with the availability of patent application, in a first step the firm must determine the project value at which it is optimal to stop the search stage and patent the innovation. There is a trade-off between the benefits of waiting and the danger of competitive damage. In a second step the optimal investment strategy for the firm now being a monopolist has to be derived.

The firm holds a perpetual option to apply for a patent at a cost P. By that it receives the embedded option to invest in the patented project at a cost I. This is analogous to a perpetual American option on the investment option with a striking price equivalent to P. In order to derive an analytical solution for the problem it is assumed that the patent never expires. Thus, the firm holds a perpetual American option W on a perpetual American option C^I. The underlying of the latter option is the patent protected project V and the striking price equals the investment cost I. Since competitive arrivals do not influence cash flows given patent protection the underlying V of the option $C^I(V)$ follows a pure diffusion process. The option value is therefore given by (3.4)

$$C^I(V) = \begin{cases} AV^\beta & \text{for } V \leq V^* \\ \\ V - I & \text{for } V > V^*. \end{cases}$$

As soon as the project value exceeds the trigger price V^* (cf. (3.6)) the option is exercised.

The above investment option is obtained when the option to patent the new technology is exercised at a striking price P. This option to patent expires if competitors introduce close substitutes. Hence, even though the option has an infinite time to maturity if no jump occurs it has a finite life if competition en-

ters. The option thus has a stochastic lifetime with the instantaneous conditional probability λdt that the option is killed.

The value of the option to patent is derived by solving the following differential equation

$$\alpha V W_V + \frac{1}{2}\sigma^2 V^2 W_{VV} - (r+\lambda)W = 0$$

subject to the boundary conditions

$$\begin{aligned} W(0) &= 0 \\ W(V^{**}) &= C^I(V^{**}) - P \qquad \text{(Value-Matching)} \\ W_V(V^{**}) &= C^I_V(V^{**}) \qquad \text{(Smooth-Pasting),} \end{aligned}$$

where V^{**} denotes the necessary level of the project value to complete the search and apply for a patent. The following value function is obtained:

$$W(V) = \begin{cases} A' V^{\beta C} & \text{for } V \leq V^{**} \\ C^I(V) - P & \text{for } V > V^{**}. \end{cases}$$

In order to determine the trigger price V^{**} and the value of waiting two different scenarios are analyzed.

At first, $V^* \leq V^{**}$ is considered. Under this scenario the firm exercises its real option to invest immediately when exercising the option to patent. This is effectively the same as undertaking the project in the absence of competition at a cost $I + P$. The two options merge to one American option with stochastic life on the patent protected project with a striking price equivalent to the investment cost plus the patent fee:

$$W(V) = \begin{cases} A_I V^{\beta C} & \text{for } V \leq V^{**} = V_I^* \\ V - I - P & \text{for } V > V^{**} = V_I^* \end{cases} \tag{5.1}$$

where

$$V_I^* = \frac{\beta^C}{\beta^C - 1}(I + P) \quad A_I = \left(\frac{\beta^C - 1}{\beta^C}\right)^{\beta^C - 1} \frac{1}{\beta^C} \left(\frac{1}{I + P}\right)^{\beta^C - 1}.$$

This scenario ($V^* \leq V_I^*$) is based on large patent fees:

$$P \geq \frac{\beta^C - \beta}{(\beta - 1)\beta^C} I. \tag{5.2}$$

Hence, the patent fee must exceed the investment cost multiplied by a positive constant that may be above 1.

The factor increases with the hazard rate and the trend rate α and decreases with the interest rate. The higher the volatility the larger must be the patent fee in order to prompt investment and patent application simultaneously because high volatility leads to a large value of the option to wait to invest. Given large patent fees it is never optimal to apply for a patent just to eliminate competition. Whenever the project reaches patent maturity it reaches market maturity simultaneously.

The higher the volatility the larger is the value of the option to patent and invest, because higher volatility implies a higher upside potential, and the higher must be the value of the project so that it is initiated. The larger the riskfree interest rate, the larger is the opportunity cost of holding the option alive. Thus, the larger the interest rate the lower is the option value and the incentive to postpone filing for a patent and initiating the project. Furthermore, the option value increases with the trend rate and decreases with the hazard rate. An increasing trend rate and decreasing hazard rate induces later patenting and project initiation.

If the trigger value that prompts investment is above the critical value that leads to patent application $V^* \geq V^{**}$, then patenting is not followed by investment immediately. The patent costs are sunk but protect from rival damage. If no patent is filed for the firm faces the risk of the option suddenly becoming worthless. Therefore management may be justified in patenting relatively early to preempt exogenous anticipated competitive arrivals even if the project has not yet reached market maturity. The following option value is obtained under this scenario

$$W'(V) = \begin{cases} A_P V^{\beta^C} & \text{for } V \leq V^{**} = V_P^* \\ AV^\beta - P & \text{for } V > V^{**} = V_P^* \end{cases} \tag{5.3}$$

where

$$A_P = P\frac{\beta}{\beta^C-\beta}\left(\frac{\beta^C-\beta}{\beta^C}\frac{A}{P}\right)^{\frac{\beta^C}{\beta}}.$$

The critical value V_P^*, at which it is optimal to apply for a patent is given by

$$V_P^* = \left(\frac{\beta^C(\beta-1)}{\beta^C-\beta}\frac{P}{I}\right)^{\frac{1}{\beta}}\left(\frac{\beta}{\beta-1}\right)I.$$

A necessary condition for $V^* \geq V_P^*$ is the violation of condition (5.2).

When competitive entry is frequent it is optimal to preempt any competitive arrivals through the early commitment of capital P. Even if this fee is quite large, under intensive rivalry it might be optimal to apply for a patent and put it to sleep, because market maturity is not yet reached. With an increasing hazard rate the value of the option on the option decreases and the trigger value that induces patent application decreases.

The higher the volatility the higher is the value of the option to invest and the longer the project is postponed. Thus, the underlying of the option to patent and therefore the option value increases with increasing volatility. The impact of an increasing volatility on the trigger price V_P^* for the option to patent is ambiguous. On the one hand, higher volatility leads to a higher value of the underlying and thus to a more urgent demand for patent protection. On the other hand, earlier patenting means forgoing more upside potential of the underlying. An increasing interest rate or decreasing trend rate leads to a decreasing value of the option to invest and therefore to a decreasing value of the option on the option to invest. Since the underlying of the option to patent decreases with the interest rate, a larger project value is required to incur filing for a patent. Contrary, the trigger price decreases with an increasing trend rate.

A firm that faces the problem of determining the optimal amount of fixed R&D expenditures when the value of the project grows continuously at a deterministic trend but also evolves stochastically according to a Brownian motion because of minor technological improvements or drawbacks and when the success of rivals leads to termination of the project should determine the value of the above option. Depending on whether (5.2) is satisfied or not the value is given by (5.1) or (5.3). This value is an upper boundary for the fixed R&D expenditures due to the finite expiration date of a patent in real life.

The larger the riskfree interest rate, the larger is the opportunity cost, and the less the firm is willing to spend on R&D. With large interest rates the firm initiates the project immediately after patenting. The larger the volatility the more is invested in R&D. A lower volatility more often leads to simultaneous patent application and investment. Otherwise patenting is done solely for the purpose of preempting competitors even if it is not clear that the new technology will be used. The larger the trend rate of the project and the smaller the probability of rivals entry into the market the more is spend on R&D.

So far, three distinct strategies depending on the patent fee and the project value were considered: Patent and acquire the option to invest, patent and initiate the project immediately thereafter, or postpone the decision. However, there is one more strategy: Launch the new product without any patent protection. The induced patent fee is saved at the disadvantage of the danger of competitive entry. Management thus faces the risk of the project suddenly becoming worthless.

A firm that either follows this strategy or postpones the decision holds a perpetual American option on the project without patent protection. The dynamics of the value of this project are given by (4.1), because the value may suddenly drop to zero. The present value of perpetual cash flows that are eroded as soon as potential competitors enter the market V^C can be expressed as a constant fraction of the present value of patent protected cash flows V. The factor ω decreases with an increasing hazard rate. Combining (2.1) and (4.2) leads to

$$V_0^C = \frac{r-\alpha}{r-\alpha+\lambda} V_0 \equiv \omega V_0.$$

The opportunity to invest in the unpatented project thus is analogous to an American call option with stochastic lifetime on a certain share ω of the patent protected project with a striking price equivalent to I. Besides obtaining the portion of the patented project the firm acquires an option with stochastic life when exercising the option. This option C^P is written on the difference between the patent protected project value and the unprotected project value and the exercise price equals the patent fee. Having initiated the unprotected project always leaves the option to apply for a patent later for the purpose of obtaining protection at the cost P.

It is optimal to pursue this strategy if

$$P \geq \frac{\lambda}{r-\alpha} I.$$

Hence, under large opportunity costs and a small hazard rate it can be optimal to enter the market without becoming a monopolist. When following this strategy, in addition to the direct benefits of the investment the firm obtains the option to convert from an unpatented project to a patented project. The value of this option which may expire suddenly is given by

$$C^P(V) = A_{IWP} V^{\beta^C}$$

where

$$A_{IWP} = \left(\frac{\beta^C - 1}{\beta^C}\right)^{\beta^C - 1} \frac{1}{\beta^C} (1-\omega)^{\beta^C} \left(\frac{1}{P}\right)^{\beta^C - 1}.$$

Exercising this option yields $1 - \omega$ shares of the project at the cost P. The option is the more valuable the lower the interest rate and the larger the drift and the volatility. The effect of an increasing probability of competitive entry on the option value is ambiguous. The value increases with the hazard rate at low hazard rates because the value of the additional share of the project that is received when exercising the option increases. On the other hand, the larger the hazard rate the higher is the probability that the option value suddenly drops to zero. This has a negative effect which dominates the positive effect given large hazard rates. The option is exercised when the project value is above the critical price

$$V^*_{IWP} = \left(\frac{\beta^C}{\beta^C - 1}\right)\left(\frac{1}{1-\omega}\right) P.$$

The higher the probability of competitive entry the lower is the value that triggers patent application. The critical value increases with the interest rate and decreases with the trend rate, because an increasing interest rate and decreasing trend leads to a lower value of the underlying of the option.

The following value for the option to invest without obtaining patent protection and thus the following upper boundary for fixed R&D expenditures in this scenario is derived:

$$W''(V) = \begin{cases} A_{IP}V^{\beta^C} & \text{for } V \leq V_{IP}^* \\ \omega V - I + A_{IWP}V^{\beta^C} & \text{for } V > V_{IP}^* \end{cases}$$

where

$$A_{IP} = \left(\frac{\beta^C - 1}{\beta^C}\right)^{\beta^C - 1} \frac{1}{\beta^C} \left((1-\omega)^{\beta^C} \left(\frac{1}{P}\right)^{\beta^C - 1} + \omega^{\beta^C} \left(\frac{1}{I}\right)^{\beta^C - 1}\right)$$

and where the critical value that triggers investment is given by

$$V_{IP}^* = \left(\frac{\beta^C}{\beta^C - 1}\right)\left(\frac{1}{\omega}\right) I.$$

This critical price increases with volatility, drift, and hazard rate. An increasing volatility means more upside potential and an increasing drift lower forgone cash flows. The larger the interest rate, the larger are the opportunity costs of waiting and the lower is the trigger price. The impact of the probability of competitive entry on the option value is once again ambiguous. On the one hand, an increasing hazard rate may lead to a higher profit when the option to invest is exercised. On the other hand, there is a higher risk of the option becoming worthless. Since the first effect is reversed with an increasing hazard rate the option value usually decreases with an increasing probability of competitive arrival.

The above option value can also be written as

$$W''(V) = \begin{cases} A_C V^{C\beta^C} & \text{for } V \leq V_{IP}^* \\ V^C - I & \text{for } V > V_{IP}^* \end{cases} + \begin{cases} A_{IWP}V^{\beta^C} & \text{for } V \leq V_{IWP}^* \\ V - V^C - P & \text{for } V > V_{IWP}^* \end{cases}$$

where It is obvious that the firm holds an option written on the unprotected project value plus an option written on the difference between the patented project value and the unpatented project value.

Based on the level of the patent fee and its relation to the investment cost the following option values and strategies are thus distinguished:

If

$$P < \frac{\beta^C - \beta}{(\beta - 1)\beta^C} I$$

the firm neither invests nor patents as long as the project value is below the trigger price V_P^*. When the trigger price is exceeded, the firm applies for a patent and receives the proprietary option to invest at the cost I. The firm can safely postpone the project and invest only if the value develops favorably. The patent fee is small enough to apply for a patent just to preempt competitors even if it is put to sleep because the project has not yet reached market maturity. $W'(V)$ is the maximum amount the firm should spend on R&D.

If

$$P < \frac{\lambda}{r-\alpha} I \quad \text{and} \quad P \geq \frac{\beta^C - \beta}{(\beta - 1)\beta^C} I$$

management postpones the decision until the project value exceeds the critical value V_I^*. Then the firm patents the new technology and initiates the project immediately thereafter. The patent fee is so large that applying for a patent is only optimal in case of a market mature product. On the other hand, the patent fee is not large enough for the firm to refrain from patenting at the disadvantage of not being a monopolist. Fixed R&D expenditures should not exceed $W(V)$ in this scenario.

If

$$P \geq \frac{\lambda}{r-\alpha} I \quad \text{and} \quad P \geq \frac{\beta^C - \beta}{(\beta - 1)\beta^C} I$$

investment and patent application is deferred until the project value exceeds the trigger price V_{IP}^*. Then the firm launches the product while it keeps the option to apply for a patent later. The patent fee is very large so that patenting is optimal just in case of a large project value. In this scenario, fixed R&D expenditures amount to $W''(V)$.

6. Conclusion

In this paper the optimal investment and patenting strategy for a firm is presented.

There are four possible strategies for a firm that just developed a new product and is aware of competing rivals who may introduce the new product or a close substitute beforehand. When the value of the project is low, the volatility and the trend rate are large, and the interest rate is low then the firm postpones investment and patent application. Otherwise the optimal strategy depends on the magnitude of the patent fee and its relation to the investment cost. The above analysis suggests that with low patent fees it is optimal to preempt competitors

by patenting even if it is not clear that the patent is going to be used so that investment is still deferred. With medium patent fees the firm patents and simultaneously initiates the project as soon as the product reaches market maturity. With large patent fees the firm invests while taking the risk of economic rents being possibly eroded by competitors. Management is free to apply for a patent later to eliminate competition.

Investment is postponed if the patent fee exceeds $(\beta^C - \beta)/((\beta - 1)\beta^C)I$. The larger the patent fee the larger must be the project value to trigger investment. If the patent fee exceeds $\lambda/(r - \alpha)I$ investment is no longer postponed but firms have very little incentive to apply for a patent when entering the market. Revenue from patent fees for the patent agency are lowest in this case.

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