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**Wirtschaftswissenschaftliche Fakultät  
der Eberhard-Karls-Universität Tübingen**

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and  
Endogenous Growth**

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# **Dual Labor Markets, Unemployment and Endogenous Growth**

Manfred Stadler

## **Abstract:**

This paper presents a general-equilibrium model of innovation, endogenous growth, and unemployment in a disaggregated economy. Unemployment is analyzed within a dual labor market setting, where the labor market is consisting of a primary high-wage and a secondary low-wage sector. The non-competitive wage differential between these sectors is explained either by union wage bargaining or by efficiency wages. Consumer goods production and innovative activities take place in the competitive secondary sector, while intermediate goods are produced in the non-competitive markets of the primary sector. We endogenize both the economy's growth rate and the equilibrium unemployment rate. It can be shown in comparative static analyses, that the relationship between unemployment and growth can be positive or negative, depending on the explanatory factors under consideration.

JEL classification: E24, J21, O33

Keywords: Innovation-based growth; Dual labor markets; Unemployment

## 1. Introduction

One of the most intriguing models of endogenous growth is that based on vertical innovations, in which technological change results from stochastic processes of intermediate goods' quality improvements along some market specific 'quality ladders'. The frequency of these innovations is driven by intentional R&D activities of private firms under dynamic competition. Grossman/Helpman (1991a,b) and Aghion/Howitt (1992) were the first to incorporate the underlying concept of R&D races, as developed by Loury (1979) and Lee/Wilde (1980) in Industrial Organization theory, into dynamic general-equilibrium models of innovation-based growth. During the nineties, the quality ladder approach has proven to be a suitable basic model for valuable extensions in several directions. Segerstrom (1991) has analyzed the impact of imitation processes when patent protection for an innovation is not perfect. Cheng/Dinopoulos (1996) have shown that such models are additionally able to account for stochastic growth and business cycles. Segerstrom/Anant/Dinopoulos (1990), Grossman/Helpman (1991a) and Dinopoulos/Oehmke/Segerstrom (1993) have taken an international view by studying the relationships between innovation-based growth and trade in open economies. However, all these variants of the quality ladder model have in common that they assume a Walrasian labor market and, hence, neglect any kind of unemployment. An important exception to this unattractive feature of the quality ladder model - and most other models of endogenous growth (see, e.g. Barro/Sala-i-Martin 1995) - is the model of Aghion/Howitt (1991, 1994). This model combines an innovation-based growth mechanism with a labor matching process as studied by Pissarides (1990) to account for frictional unemployment. It is assumed that the innovation process is accompanied by lay-offs of workers who first have to re-adapt to new technologies before being recruited by technologically advanced firms. The main results of this matching model are the existence of a positive equilibrium rate of unemployment and the possibility of an inverted U-shaped relationship between steady-state growth and equilibrium unemployment in a process of creative destruction.

In the present paper, we propose an alternative approach to model endogenous growth and equilibrium unemployment. There is a large variety of non-Walrasian labor market models not yet studied in the context of an innovation-based growth model. Wage bargaining and efficiency wage models are undoubtedly among the most prominent candidates for explaining long-run unemployment with wage rigidities. Recently, van Schaik/de Groot (1995) and de Groot (1996) have developed a convincing framework for analyzing such labor market imperfections within a specific productivity growth model which decisively builds on the theory of dual labor markets. This approach dates back to the work of Harris/Todaro

(1970) and has been extended by Calvo (1978) and McDonald/Solow (1985).<sup>1</sup> According to this theory, the labor market can be segmented into a high-wage primary sector and a low-wage secondary sector. While the primary sector is characterized by positive profits, the secondary sector is assumed to be competitive and the wage rate is determined by market clearing. We follow these models in assuming a dual labor market, characterized by a non-competitive wage differential between the two sectors. The wage differential will be explained either by trade unions, bargaining for rents in the primary sector, or by the existence of efficiency wages. We attempt to analyze both concepts within a unified treatment of our extended version of the quality ladder model.

The paper is organized as follows. Section 2 presents the producers' and consumers' behavior in the disaggregated economy. In Section 3 we determine the flow equilibrium of unemployment in the primary labor market. Section 4 solves the model and identifies the determinants of innovation, growth and unemployment. Finally, Section 5 concludes the paper.

## 2. The Behavior of Firms and Households

We consider an economy in which there are three sectors of production. In the competitive consumer goods sector, firms produce a homogeneous output good using labor and a variety of differentiated intermediate goods or components. All the components are produced by firms in non-competitive intermediate markets with labor as the only input factor. Finally, the quality of these intermediates can be upgraded in stochastic processes of sequential improvements as a result of intentional innovative activities by firms employing labor in a separate research sector. Each of the three sectors will now be characterized in detail.

### 2.1 *Production of Consumer Goods*

Consumer goods are produced by competitive firms in a final goods sector according to the constant returns to scale Cobb-Douglas technology

$$(1) \quad Y = F^\alpha L_y^{1-\alpha}, \quad 0 < \alpha < 1,$$

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<sup>1</sup> Dickens/Lang (1988) and Saint-Paul (1996) discuss the recent reemergence of segmented labor market theory.

where  $F$  denotes an index of a variety of differentiated intermediate goods and  $L_y$  represents labor employment in the consumer goods sector. Since the final goods market is competitive, the output price  $p_y$ , which is taken as given by each firm, equals the marginal cost of production. The unit cost function, dual to (1), is given by

$$(2) \quad p_y = (p_F/\alpha)^\alpha (w/(1-\alpha))^{(1-\alpha)},$$

where  $p_F$  denotes the price index of intermediates and  $w$  denotes the wage rate for workers in the consumer goods sector.

The intermediate input factor  $F$  is assumed to represent a quality index of several input components,  $x(j)$ , each produced in a single intermediate market  $j$ ,  $j=1,\dots,n$ . The intermediates substitute only imperfectly for one another. The potential quality grades of the components are arrayed along the rungs of a known quality ladder. Each new generation of an intermediate good provides a  $\lambda$  times higher quality, where  $\lambda > 1$  is assumed to be exogenous and identical in all intermediate markets. The quality improvements result from successful R&D activities undertaken in a separate sector to be characterized below. If the lowest quality available at time  $t=0$  is normalized to one, the highest available quality of component  $j$  is given by  $\lambda^{m(j)}$ , where  $m(j) = 0,1,2,\dots$  denotes the number of sequential quality improvements in market  $j$  up to the present. At each moment in time, the highest quality components in all intermediate markets define the state-of-the-art. The quality grades within each market are perfect substitutes as inputs to consumer goods production. Again using a constant returns to scale Cobb-Douglas technology, the intermediate input index can be specified as

$$F = \exp \left\{ \sum_{j=1}^n \ln \left[ \sum_{\ell(j)=0}^{m(j)} \lambda^{\ell(j)} x_{\ell(j)} \right]^{1/n} \right\}.$$

It will be shown below that, due to the pricing strategies of the intermediate goods producers, only the top-of-the-line components will be produced and used in the consumer goods production, so that the index function reduces to

$$(3) \quad F = \exp \left\{ \sum_{j=1}^n \ln \left[ \lambda^{m(j)} x(j) \right]^{1/n} \right\}.$$

The unit cost function for the intermediate input index is then given by

$$(4) \quad p_F = \exp \left\{ \sum_{j=1}^n \ln \left[ n \lambda^{-m(j)} p(j) \right]^{1/n} \right\}.$$

Profit maximizing behavior of consumer goods producers implies a labor demand of

$$(5) \quad L_y = (1 - \alpha)p_y F^\alpha L_y^{1-\alpha} / w$$

and intermediate goods demands in the markets  $j=1, \dots, n$  of

$$(6) \quad x(j) = (\alpha / n)p_y F^\alpha L_y^{1-\alpha} / p(j) .$$

The next step is to derive the optimal pricing strategies of the intermediate goods producers. As in Barro/Sala-i-Martin (1995, Ch. 7) and in Aghion/Howitt (1998, Ch. 2), it will turn out that it depends decisively on the (relative) size of the markup factor  $(\alpha/n)$ , whether incumbent firms in the intermediate sector engage in monopoly or in limit pricing.

## 2.2 *Production of Intermediate Goods*

In each intermediate market, goods are produced subject to a common constant returns to scale technology with labor  $L_x(j)$  as the only input factor. Thus, a technological leader, whose innovation is perfectly protected by an infinitely lived patent, can realize a profit stream of

$$(7) \quad \pi(j) = (\alpha / n)p_y F^\alpha L_y^{1-\alpha} - w_x L_x(j) ,$$

where  $w_x$  is the prevailing wage rate in this sector which will be determined below. For now, we assume the linear production function

$$(8) \quad x(j) = L_x(j) .$$

If the technological leaders' profit maximization is unconstrained by rivals in the same market - a scenario which usually defines 'drastic' innovations (see, e.g. Aghion/Howitt 1998, Ch. 2) - the first order condition with respect to labor input yields the demand functions

$$(9) \quad L_x^D = (\alpha / n)^2 p_y F^\alpha L_y^{1-\alpha} / w_x$$

and hence the monopoly prices

$$(10) \quad p^D = w_x n / \alpha$$

and flow profits

$$(11) \quad \pi^D = (n / \alpha - 1)w_x L_x = (\alpha / n)(1 - \alpha / n)p_y F^\alpha L_y^{1-\alpha}$$



in all intermediate markets.<sup>2</sup> The price setting equation (10) shows that the technological leader's monopoly price is given by a constant markup on the prevailing wage rate in this sector. The profit stream (11) can be realized by the market leader, until he is replaced by the next innovator in the market. It can be shown that, due to the replacement effect, the incumbent firm will never invest in R&D devoted to a superior innovation in its own market. This implies that all innovations are carried out by entrants who then find themselves exactly one step ahead of the former leaders that they have displaced in the continuing process of creative destruction. Thus, the monopoly price (10) would allow the closest competitor to price at most at  $w_x n / (\alpha \lambda)$ . It follows that, if the condition  $n / \alpha < \lambda$  holds, the second best quality (and all lower quality) producers cannot compete against each leader's unconstrained monopoly price. Since this inequality will hold only if the innovation size  $\lambda$  is large, monopoly pricing requires drastic innovations. In the case of non-drastic innovations ( $\lambda < n / \alpha$ ), the technological leaders are restricted in their pricing behavior. They have to set prices that are sufficiently below the monopoly prices so that the closest follower cannot compete without realizing negative flow profits. As before, it can be shown that market leaders undertake no research targeted to improve the quality of their own intermediate products because the incremental gain of a two-step quality advantage to an incumbent is strictly smaller than the gain of a one-step quality advantage to an external innovator. Therefore, the optimal limit-pricing strategy is given by

$$(12) \quad p^N = \lambda w_x,$$

which implies labor demands

$$(13) \quad L_x^N = (\alpha / n \lambda) p_y F^\alpha L_y^{1-\alpha} / w_x$$

and flow profits

$$(14) \quad \pi^N = (\lambda - 1) w_x L_x = (\alpha / n) (1 - 1 / \lambda) p_y F^\alpha L_y^{1-\alpha}$$

in all intermediate markets. In the basic quality ladder model of Grossman/Helpman (1991a,b) with a continuum of intermediate markets, monopoly pricing cannot occur because the corresponding infinite number  $n$  of markets implies infinite monopoly prices which are always restricted by the pricing strategies of the lower quality competitors in the same market. Within our more general model we are able to follow Barro/Sala-i-Martin (1995, Ch. 7) in accounting for both important types of pricing behavior using a unified treatment. A comparison of (10) and (12) shows that in both cases the intermediate goods prices are fixed

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<sup>2</sup> Since these equilibrium equations equally hold for all technologically leading firms in the sector, independently of market  $j$ ,  $j=1, \dots, n$ , indexes are omitted for simplicity.

markups on the prevailing wage rate. The only difference is that in the case of non-drastic innovations the innovation size  $\lambda$  replaces  $n/\alpha$  as an indicator of market power of incumbent firms. Either way, only the highest available quality of each type of intermediate good is produced and used as an input by consumer goods producers and, hence, continual leapfrogging in the technological leadership positions occurs.

In standard models of innovation-based growth, the wage rate  $w_x$  equals the one in the consumer goods sector,  $w$ , since labor is homogeneous and the wage is determined by Walrasian labor market clearing (see, e.g. Grossman/Helpman 1991a). However, there is convincing empirical evidence that wages significantly and persistently differ across the sectors of an economy and that this wage structure cannot be explained solely by sectoral differences in human skills, working conditions or other specific characteristics of workers. Further, higher wages are to be observed especially in industries characterized by some degree of market power (see, Dickens/Katz 1987, Krueger/Summers 1988, Gera/Grenier 1994). In view of the long lasting unemployment phenomenon in modern economies, this evidence strongly suggests that labor markets should not be modeled in a Walrasian sense, not even in the long run. To account for this evidence, we adopt the idea of dual labor markets in the modern version of McDonald/Solow (1985) which was recently integrated into a productivity growth model by van Schaik/de Groot (1995) and de Groot (1996). According to this concept, the labor market is segmented into a high-wage primary sector and a low-wage secondary sector. The primary sector is characterized by high profits and thus corresponds to the intermediate goods sector in our model. The competitive secondary sector comprises jobs in the consumer goods production and in R&D for which the wage is determined by market clearing. The higher wages in the primary sector will be explained either by trade unions, bargaining for rents in the high-profit intermediate sector, or by efficiency wage considerations. We attempt to analyze both important concepts of labor market theory using a unified treatment within the structure of our innovation-based growth model.

### 2.2.1 *Union Wage Bargaining*

In the first version of our model, we assume that in each firm of the primary sector a trade union is operating. At each point in time, the union and the respective firm divide the rents, generated by the market power of each firm in the intermediate sector, according to a Nash bargaining game. The union only represents the interests of insiders currently working in the firm, but not those of unemployed outsiders. According to the right-to-manage approach, unions and firms negotiate only over the wage, whereafter firms choose their employment

levels according to their demand functions (9) and (13), respectively.<sup>3</sup> The Nash product of this bargaining game is given by

$$(15) \quad \Gamma = \max_{w_x} [(w_x - w)L_x^i]^\beta [\pi^i]^{1-\beta}, \quad 0 < \beta < 1, \quad i=D, N,$$

subject to the flow profit functions (11) and (14). The parameters  $\beta$  and  $(1-\beta)$  represent the unions' and the firms' bargaining power respectively. In the case no agreement is reached, the workers can join the secondary sector without friction and receive the lower wage rate  $w$ . Therefore, the gain per worker of an agreement is the difference between the wage rates in the primary and secondary sector. The unions' objective is, accordingly, to maximize the difference of labor income for the employed workers inside the primary sector compared to those in the secondary sector. The firms' objective is, of course, to maximize their flow profits since there is no outside profit if no agreement is reached. Using the expression

$$(\partial L_x^i / \partial w_x) / (L_x^i / w_x) = -1 / (1 - \alpha / n)$$

for the wage elasticity of labor demand in the intermediate sector which can be derived from (9) in the case of drastic innovations as well as from (13) in the case of non-drastring innovations, maximization of the Nash product (15) yields a wage differential  $\omega := w_x / w > 1$  between the primary and the secondary sector that can be calculated as

$$(16) \quad \omega = 1 + \beta(n / \alpha - 1)$$

for both types of innovation. Either way, the wage differential is solely determined by exogenous factors. It equals one if firms have all the bargaining power ( $\beta=0$ ), but increases with the unions' bargaining power which is accompanied by unemployment.

### 2.2.2 Efficiency Wages

In the previous section, bargaining power of trade unions was responsible for a non-competitive wage differential between the sectors of the dual labor market. An alternative explanation builds on the emergence of efficiency wages. All the efficiency wage models have in common that there are economic reasons - such as retaining workers to avoid costly labor turnovers (Salop 1979), motivating workers to prevent shirking (Shapiro/Stiglitz 1984) or recruiting qualified workers taking into account the adverse selection problem (Weiss 1980) - for firms to pay wages that are in excess of market clearing wages. Akerlof (1982, 1984) and Akerlof/Yellen (1986, 1988, 1990) proposed in their gift exchange version of the

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<sup>3</sup> An interesting alternative to this right-to-manage approach is to assume efficient bargaining where unions and firms negotiate over both wage rates and employment levels (see Oswald 1985).

efficiency wage theory, that workers tend to raise their productivity if they are offered a 'fair' wage. The assumption that efficiency wages operate especially in the high-profit sector can best be justified by this sociological approach of a mutual gift exchange between employers and workers. Empirical studies, e.g. by Krueger/Summers (1988) have revealed that there is an interindustry wage structure that is indeed best explained by that kind of efficiency wages in segmented labor markets. The simplest way to account for such effort behavior is to modify the intermediate goods production functions (8) to

$$(17) \quad x(j) = \varepsilon(w_x) L_x(j), \quad \varepsilon'(w_x) > 0, \quad \varepsilon''(w_x) < 0,$$

where  $\varepsilon$  denotes the workers' effort which depends positively on the (relative) wage rate. If there are unemployed workers in the sector, the incumbent firms can not only choose the amount of labor but also the wage rate freely. When firms are unconstrained in their optimization of profits (7) in the case of drastic innovations, the first-order conditions are given by

$$(18) \quad (\alpha/n)^2 p_y F^\alpha L_y^{1-\alpha} \varepsilon(w_x) / x(j) - w_x = 0$$

and

$$(19) \quad (\alpha/n)^2 p_y F^\alpha L_y^{1-\alpha} \varepsilon'(w_x) L_x^D(j) / x(j) - L_x^D(j) = 0,$$

which can be combined to yield the well-known Solow (1979) - condition

$$(20) \quad w_x \varepsilon'(w_x) / \varepsilon(w_x) = 1.$$

The first-order condition (18) implies a monopoly pricing rule

$$(21) \quad p^D = w_x n / (\alpha \varepsilon),$$

which corresponds to (10) for  $\varepsilon=1$ . The resulting profit function is identical to (11). In the case of non-drastic innovations, firms engage in limit pricing. A Nash-equilibrium is determined by a wage rate that minimizes the marginal costs of producing one unit of the intermediate input, i.e.,  $w_x / \varepsilon(w_x)$ , which again yields the Solow condition (20), and further by a limit price setting

$$(22) \quad p^N = \lambda w_x / \varepsilon$$

which equals (12) for  $\varepsilon=1$ . The corresponding flow profit function is given by (14). As in the case with union wage bargaining, we are able to endogenize the wage differential between the

primary and the secondary sector. For example, we can follow van Schaik/de Groot (1995) and make use of an adjusted Akerlof (1982) specification<sup>4</sup>

$$\varepsilon = -\beta_1 + \beta_2(w_x/w)^{\beta_3}, \quad 0 < \beta_2(1-\beta_3) < \beta_1, \quad 0 < \beta_3 < 1,$$

which yields, by applying the Solow-condition (20), a constant effort level

$$\varepsilon = \beta_1\beta_3/(1-\beta_3)$$

and a constant wage differential

$$(23) \quad \omega = [\beta_1/(\beta_2(1-\beta_3))]^{1/\beta_3}.$$

After these considerations, we are left with two important results. Firstly, the union wage bargaining and the efficiency wage versions of our model obviously lead to the same type of the leaders' temporary flow profit functions, (11) in the case of drastic and (14) in the case of non-drastic innovations. Secondly, the only difference between the two versions of the dual labor market theory is due to alternative foundations of the wage differential between the sectors. Both versions have in common, however, that this wage differential is rigid and constant over time. Having this in mind, we only have to distinguish between drastic and non-drastic innovations without referring to the underlying cause of unemployment in the analyses below. We now turn to the innovative behavior of rival firms in the research sector.

### 2.3 *Innovative Activity and Creative Destruction*

The quality of intermediate goods can be upgraded by a sequence of innovations, each of which builds upon its predecessors. A successful innovation opens up the opportunity to search for the next innovation. This effect is assumed to be external to all firms in the R&D sector and, hence, implies an important positive spillover effect from innovations since newcomers can join a R&D race in order to create the technologies for the state-of-the-art products, without having taken all of the rungs of the quality ladder themselves. The winner of each R&D race is granted a patent and realizes as technological leader a temporary profit stream as expressed in (11) and (14) in the cases of drastic and non-drastic innovations respectively, until it is replaced by the winner of the next race. The arrival of innovations in

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<sup>4</sup> Akerlof (1982) additionally accounts for the wages of rival firms and the unemployment rate in the effort function. In equilibrium, wages of all firms are identical. In our model, this holds within the primary sector, too. Unemployment is not part of our specification since all workers are free to join the secondary sector without friction.

each market  $j$  follows a Poisson process whose hazard rate is assumed to be given by the linear specification

$$(24) \quad h(j) = \mu L_h(j),$$

where  $L_h(j)$  is the labor input in the research sector devoted to a vertical innovation in an intermediate market  $j$ , and  $\mu$  denotes the labor productivity in R&D. Thus, each research firm's probability of innovative success is proportional to its labor input.<sup>5</sup> Since we assume perfect competition in the R&D sector with a zero expected discounted profit, the research sector belongs - according to the above classification - to the secondary sector. This means that R&D workers receive the competitive wage rate  $w$ . At a flow cost of  $wL_h(j)dt$  over the time interval of length  $dt$ , each firm participating in the present patent race can attain the value  $V$  of a successful innovator who becomes the technological leader in the market  $j$  with probability  $\mu L_h(j)dt$ . Thus, maximization of  $V\mu L_h(j)dt - wL_h(j)dt$  with respect to labor input implies an infinite R&D investment if  $V > w/\mu$ , and no R&D at all if  $V < w/\mu$ . With free entry into the patent races the former case cannot occur. The latter case implies a no-growth trap. The unique steady-state equilibrium with positive but finite R&D activities requires

$$(25) \quad V = w/\mu.$$

Each research firm participating in a R&D race does not realize any revenues and, therefore, needs to borrow or to issue equity claims on a perfect capital market. As usual in the innovation-based growth theory, we assume that research firms finance their R&D investments by issuing equity claims that pay nothing in the event that the R&D effort fails but pay the income stream associated with an intermediate market leadership if the firm succeeds and wins the patent race. Arbitrage in the capital market ensures that the expected rate of return to the equity owners of an incumbent firm equals the instantaneous interest rate  $r(t)$  on a riskless loan. Over the time interval of length  $dt$ , a leader's equity owners receive profits  $\pi dt$  with  $\pi$  being paid out continuously as dividends. In addition, they enjoy capital gains  $\dot{V}dt$  if the R&D efforts targeted at the leader's intermediate product fail in  $dt$ . This event occurs with probability  $(1 - h(j)dt)$ . With probability  $h(j)dt$ , however, one of the targeted research efforts will succeed, the leader will be replaced by an entrepreneur, and the equity owners will suffer a total capital loss of  $V$ . Taking the limit as time length  $dt$  goes to zero, the no-arbitrage condition in each market  $j$  can be written as

$$(26) \quad \pi^i + \dot{V} - h^i V = rV, \quad i=D, N.$$

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<sup>5</sup> By extending the basic quality ladder model, Jones (1995) and Stokey (1995) use a more general form of the innovation production function which allows for decreasing returns to scale in R&D.

The next step will be to determine the interest rate by analyzing the consumer behavior in the household sector.

#### 2.4 Households

In the dynastically structured household sector, we assume that consumers with identical preferences maximize their discounted utility over an infinite time horizon. For simplicity, we use the time separable intertemporal utility function

$$U(C) = \int_0^{\infty} e^{-\rho t} \ln C \, dt ,$$

where  $\rho$  is the common subjective rate of time preference and the intertemporal elasticity of substitution equals one. In equilibrium, the consumption level  $C$  equals the final goods production  $Y$ . Since there will be equilibrium unemployment in our model, we assume each household as being composed out of a continuum of members. This enables us to maintain the usual concept of a representative household which maximizes utility subject to its intertemporal budget constraint

$$\int_0^{\infty} e^{-rt} p_y C \, dt \leq A(0) .$$

The constraint requires that the representative household's discounted value of spending must not exceed the discounted value of its labor income plus the value of its present wealth, summarized by the variable  $A(0)$ .<sup>6</sup> Solving this intertemporal optimization problem yields the Keynes-Ramsey rule (see, e.g. Grossman/Helpman 1991a, Ch. 2)

$$(27) \quad \dot{C}/C = r - \dot{p}_y / p_y - \rho .$$

The optimal time path of consumer spending not only applies to a representative household but also to the aggregate economy. It proves convenient to impose a normalization of consumer goods prices so that nominal consumer spending remains constant through time. Accordingly we set

$$(28) \quad E := p_y Y = 1$$

for aggregate spending to serve as numéraire. Then (27) implies that

$$(29) \quad r = \rho .$$

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<sup>6</sup> In a Walrasian labor market where wage differentials and unemployment cannot occur, per capita labor income would equal the common wage rate.

The interest rate equals the rate of time preference which is assumed to be constant over time.

### 3. Equilibrium Unemployment in the Dual Labor Market

As discussed above, union bargaining behavior and efficiency wages are plausible reasons to justify non-competitive wage differentials in the primary (intermediate) sector. Since labor is assumed to be homogeneous, all workers would like to be employed in the intermediate sector. However, the demand for labor in this sector is restricted due to the non-competitive wage differentials as derived in (16) for the case of union wage bargaining and in (23) for the case of efficiency wages. Thus, at the beginning of his career or upon being laid off, a worker faces two alternatives. Either he can accept a job in the secondary sector without any frictions or he can decide to join the pool of unemployed and wait for a job opening in the primary sector. In equilibrium, of course, the present discounted stream of labor income must be equal in both states. This is only possible if the probability of entering the primary sector from the secondary sector is lower than that from unemployment. Indeed, this is a crucial assumption in such versions of the dual labor market theory (see, e.g. McDonald/Solow 1985). To keep the model as simple as possible, we assume that there is no equilibrium flow from the secondary to the primary sector. Indeed, it may be optimal for firms in the primary sector to hire only from the pool of unemployed, using the acceptance of temporary unemployment as a device to screen out the less highly motivated workers (for empirical evidence, see Burda 1988).<sup>7</sup> Further, there must be a positive probability for finding a job in the primary sector. This in turn implies the occurrence of layoffs. In the following, we assume a constant layoff rate  $\delta$  and a constant job finding rate  $\kappa$ . While the layoff rate is assumed to be exogenously given, the job finding rate will be determined endogenously.<sup>8</sup> Under these assumptions, all possible transitions among states are Poisson processes and, hence, the value functions of the different states do not depend on how long a worker has already been in a certain state. Let  $W_i$  denote the expected value of discounted labor income from the present moment forward of a worker in state  $i=y,x,u$ , i.e., in the secondary sector, in the primary sector, or unemployed. Using the method of dynamic programming, these value functions can be written as

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<sup>7</sup> This means that workers in the secondary sector cannot proceed to a primary job without first going through the channel of unemployment. This rather restrictive assumption may be rationalized by positive signaling effects of unemployment, while the acceptance of a job in the secondary sector is conceived as being a bad signal (see McCormick 1990).

<sup>8</sup> In their matching model, Aghion/Howitt (1994) equalize the separation rate with the innovation rate. This implies, however, that innovators prefer unemployed workers to trained specialists. Another possibility is to identify the separation rate with the death rate of the individuals of the dynasty.



$$(30) \quad \rho W_y = w$$

$$(31) \quad \rho W_x = w_x + \delta(W_u - W_x)$$

$$(32) \quad \rho W_u = \kappa(W_x - W_u),$$

since both wage rates  $w$  and  $w_x$  will be constant in the steady-state equilibrium. A worker in the secondary sector receives the present discounted value of an infinite stream of labor income  $w$ , where, according to (29), the discount rate equals the constant rate of time preference. A worker in the primary sector receives a stream of higher wages  $w_x = \omega w > w$  until he is laid off and joins the pool of unemployed. In a short time interval of length  $dt$ , this event occurs with probability  $\delta dt$ . A worker currently unemployed faces a probability of  $\kappa dt$  of finding a job in the primary sector within time  $dt$ . As already noted, in equilibrium it must hold that the value of a job in the primary sector equals the value of being unemployed,  $W_y = W_u$ . Then, we can solve (30), (31) and (32) simultaneously for the three value functions to yield a constant primary sector job finding rate<sup>9</sup>

$$(33) \quad \kappa = \frac{(\rho + \delta)}{\omega - 1},$$

which depends positively on the rate of time preference and the layoff rate, but negatively on the wage differential between the sectors. Together with the flow-equilibrium condition  $\kappa U = \delta n L_x$ , (33) determines the amount of unemployed workers as

$$(34) \quad U = \frac{\delta n (\omega - 1)}{(\rho + \delta)} L_x.$$

With a diminishing wage differential,  $\omega=1$ , there is no unemployment and the model simplifies to the basic quality ladder model with a Walrasian labor market. However, the larger the difference between the wage rates in the primary and the secondary sector, the larger is the share of unemployed workers with respect to aggregate employment in the primary intermediate sector, either due to union wage bargaining or due to efficiency wage considerations.

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<sup>9</sup> In an extended version of this flow model, van Schaik/de Groot (1995) assume an additional turnover rate from the secondary to the primary sector. This specification does not qualitatively change the results, but complicates the following analysis of the labor allocation over the sectors.

#### 4. Sectoral Labor Allocation, Growth and Unemployment

To solve the complete model, we impose a non-Walrasian equilibrium condition on the labor market:

$$(35) \quad L = L_y + nL_x^i + nL_h^i + U, \quad i=D, N.$$

The constant labor force  $L$  of the economy is either employed in consumer goods production, in intermediate goods production, in R&D, or is unemployed. Using (1), (28) and (29), the labor demand function (5) in the consumer goods sector is given by

$$(36) \quad L_y = (1 - \alpha) / w .$$

The labor demands in the intermediate sector, (9) and (13), are given by

$$(37) \quad nL_x^D = \alpha^2 / (n\omega w)$$

in the case of drastic innovations and

$$(38) \quad nL_x^N = \alpha / (\lambda\omega w)$$

in the case of non-drastic innovations. The innovation rates can be derived from the no-arbitrage condition (26). Since the wage rate  $w$  must not change in a steady-state equilibrium, we can use the incumbent firms' value (25), the interest rate (29) and the flow profit functions (11) and (14) to find

$$(39) \quad h^D = (\alpha / n)(1 - \alpha / n)\mu / w - \rho$$

and

$$(40) \quad h^N = (\alpha / n)(1 - 1/\lambda)\mu / w - \rho ,$$

respectively. Taking into account the innovation rate equation (24), the labor demands in the R&D sector, corresponding to (39) and (40), are

$$(41) \quad nL_h^D = \alpha(1 - \alpha / n) / w - n\rho / \mu$$

and

$$(42) \quad nL_h^N = \alpha(1 - 1/\lambda) / w - n\rho / \mu .$$

Inserting the various labor demand functions and the unemployment equation (34) into the resource constraint (35) yields the (inverse) wage rate

$$w^{-1} = (L + n\rho/\mu)\psi; \quad \psi := \left[ \frac{\alpha^2}{n} \left( \frac{\rho + \delta\omega}{(\rho + \delta)\omega} \right) + \left( 1 - \frac{\alpha^2}{n} \right) \right]^{-1}$$

in the case of drastic innovations which can be substituted into (39) to derive the equilibrium innovation rates

$$(43) \quad h^D = (\alpha/n)(1 - \alpha/n)(\mu L + n\rho)\psi - \rho.$$

The results in the case of non-drastic innovations are quite similar. The equilibrium (inverse) wage rate

$$w^{-1} = (L + n\rho/\mu)\tilde{\psi}; \quad \tilde{\psi} := \left[ \frac{\alpha}{\lambda} \left( \frac{\rho + \delta\omega}{(\rho + \delta)\omega} \right) + \left( 1 - \frac{\alpha}{\lambda} \right) \right]^{-1}$$

can be substituted into (40) to yield

$$(44) \quad h^N = (\alpha/n)(1 - 1/\lambda)(\mu L + n\rho)\tilde{\psi} - \rho.$$

Using comparative static analyses, we can show that both innovation rates  $h^D$  and  $h^N$  depend positively on the labor force  $L$  and on the productivity  $\mu$  of labor in R&D. In the case of non-drastic innovations, the innovation rate  $h^N$  in addition depends positively on the innovation size  $\lambda$ . So far, the results coincide with those resulting from the basic quality ladder model with a Walrasian labor market. In contrast, however, the effect of the patience of households, reflected by the rate of time preference  $\rho$ , is now ambiguous. The additional effects from labor market imperfections are characterized by a positive impact of the wage differential  $\omega$  and a negative effect of the layoff rate  $\delta$  on the pace of innovations. An increasing wage differential, caused by an increase in the unions' bargaining power or efficiency wage considerations, shifts labor resources from the primary into the secondary sector, thereby increasing R&D activities. The opposite effect occurs with an increase in the lay-off rate, thereby reducing research.

The endogenously derived innovation rates which are equal across all intermediate sectors not only determine the pace of quality improvements of specific components but also the endogenous growth rate of the aggregate economy. In each intermediate market  $j$ , the probability that the current technology will take  $m(j)$  rungs up the quality ladder in a time interval of length  $dt$  follows the Poisson distribution

$$f(m(j)) = e^{-h^i(j)dt} (h^i(j)dt)^{m(j)} / m(j)!, \quad i=D, N.$$

Thus, starting from time  $t=0$ , one can derive the expected number of innovations in each market as  $h^i(j)t=ht$ ,  $j=1,\dots,n$ . Whereas the innovation process in any particular intermediate market is erratic and stochastic, technological progress at the aggregate level is smooth if the number  $n$  of intermediate markets is large. Inserting the derived expectation values into (3) and (4), and differentiating with respect to time, yields the growth rates of the intermediate input index and the intermediate input price index as:

$$\dot{F}/F = -\dot{p}_F/p_F = h^i \ln \lambda, \quad i=D,N.$$

According to the production function (1) and the dual cost function (2), the endogenous growth rate of the economy is then determined by

$$(45) \quad g = \dot{Y}/Y = -\dot{p}_y/p_y = \alpha h^i \ln \lambda, \quad i=D,N.$$

Thus, the growth rate of the economy depends in the same way on the exogenous factors as discussed with the innovation rates. In addition, there is now a positive impact of innovation size  $\lambda$  on growth even if innovations are drastic.

Inserting the equilibrium secondary sector wages in (37) and (38) respectively, and making use of (34) finally gives the unemployment rates

$$(46) \quad u^D = \frac{(\alpha^2/n)(1+n\rho/\mu L)\delta(\omega-1)}{(\alpha^2/n)(\rho+\delta\omega) + (1-(\alpha^2/n))(\rho+\delta)\omega}.$$

in the case of drastic innovations and

$$(47) \quad u^N = \frac{(\alpha/\lambda)(1+n\rho/\mu L)\delta(\omega-1)}{(\alpha/\lambda)(\rho+\delta\omega) + (1-\alpha/\lambda)(\rho+\delta)\omega}$$

in the case of non-drastic innovations. In both cases, unemployment rates depend positively on the layoff rate and the wage differential but negatively on the labor force and the productivity of R&D. In the case of drastic innovations, unemployment is independent of innovation size, but it depends negatively on it if innovations are non-drastic.

As expected, the rate of growth and the rate of unemployment do not always move in the same direction. Whether there exists a trade-off or not, crucially depends on the explanatory factors whose variations are considered. In particular, if unions' bargaining power or the importance of efficiency wages in the primary sector increase, both growth and unemployment rates will increase, too. However, larger labor forces, R&D productivities and innovation sizes fasten the growth rate but tend to reduce unemployment.

## 5. Conclusion

In this paper, we developed a dynamic general-equilibrium model of endogenous innovation-based growth with a dual labor market which is able to explain equilibrium unemployment due to union wage bargaining or efficiency wages. The relationship between unemployment and growth is shown to depend crucially on the considered explanatory factors. Changes of the bargaining power of unions or of the importance of efficiency wages imply a positive relationship, while variations of the labor force, the layoff rate, the innovation size and the productivity of R&D imply a negative relationship. Of course, these comparative static results decisively depend on the definition of the primary and secondary sector of the economy and should be interpreted with caution. Results are certainly not robust if the R&D sector would be classified into the primary sector. However, such an alternative classification would not harmonize with the usual distinctions of competitive and non-competitive sectors in the disaggregated growth models which formed the basis for our investigations.

The theoretical analysis of the mutual relationships between innovation, growth and unemployment is still in its infancy. While earlier investigations focused either on innovation-based growth, neglecting imperfections in the labor market, or on unemployment, neglecting the effects of innovative activities, the intention of this paper was to join these important streams of inquiry to develop a more comprehensive model of innovation, growth, and unemployment.

Because the model relies on a large number of simplifying assumptions, the derived results should be viewed as suggestive. Of course, the technological development of a growing economy is much more complicated than can be studied within such formal models which are necessarily highly abstract and stylized. Nevertheless, the presented model is able to capture some empirical evidence about growth and unemployment fairly well. Due to the fundamental importance of the topic, it seems worthwhile to integrate some further aspects such as capital accumulation, imperfect financial markets, or international trade into the model to achieve an even more comprehensive model of innovation, growth, and unemployment.

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