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Firm heterogeneity, labour marketing pooling, and the formation of local labour markets

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**Wirtschaftswissenschaftliche Fakultät
der Eberhard-Karls-Universität Tübingen**

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Pooling, and the Formation of
Local Labour Markets**

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Firm Heterogeneity, Labour Marketing Pooling, and the Formation of Local Labour Markets

by

Konrad Stahl and Uwe Walz*

Abstract

We investigate the simultaneous impact of output and input market imperfections on the characteristics and size of regional economic activity. Firms, producing imperfect substitutes, face product specific demand shocks which translate into firm-specific shocks. The more different the products are the less correlated the shocks. Then, larger regions provide a better hedge for firms. With heterogeneity on the output side labour demand of firms tends to become also more heterogenous, leading to higher retraining costs for workers in the case of a job loss. Since adjustment costs prove to be higher in larger regions, workers demand higher wages there, imposing thereby higher labour costs on firms. We analyze this trade-off and ask for the resulting firm characteristics in different regions, the interregional wage dispersion, as well as the impact on the regional distribution of economic activity.

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1 Introduction

Empirical studies suggest that industries as well as industry-related employment are concentrated in space, and that differences in industry structure among local labour markets are substantial (see e.g. Topel (1991), Neumann and Topel (1991)). These studies indicate that the structure of local labour markets is an important determinant for regional economic activity. Given a certain degree of mobility of workers, however, the structure of local labour markets is not exogenously given but rather depends on migration decisions which themselves are based on industry characteristics, expected wages and employment possibilities in the regions under consideration. That is, regional industrial structure and the structure of local labour markets are highly interdependent.

The purpose of this paper is to analyze this interdependence in a world of uncertainty. Firms face stochastic product demand, leading to uncertain labour demand and employment possibilities. Since firms produce imperfect substitutes these product market specific shocks translate into firm specific shocks. The more heterogeneous the products of firms, the more pronounced the firm specific character of shocks. Through profit maximizing output choice the disturbances on the output side are transmitted to firms' labour demand and hence, to the local labour market. Workers face the risk of losing their job. In this case these workers have to be retrained in order to meet the required skill characteristics of the new employer (or their productivity is lower with the new firm).

The degree of spatial concentration of firms and workers is governed by the interaction of an agglomeration advantage and a disadvantage, both resulting directly or indirectly from the output demand fluctuations. The agglomeration advantage is due to the imperfect correlation of product market specific shocks. It allows firms to realize an efficiency gain in the larger local labour market, as positive shock can be absorbed more easily since firms experiencing a positive shock can hire workers laid off by other firms. Hence any adjustment to output demand shocks can be accommodated

more easily. The agglomeration advantage is the more pronounced the more heterogeneous firms are and the less correlated product market specific shocks. However, there is also an agglomeration disadvantage. The reshuffling of labour across firms comes at a cost, as it requires retraining, and this the more, the more heterogeneous the firms. Retraining costs increase with firm heterogeneity and labour market size. If the retraining costs are borne by the mobile labourers, they require, on average, higher wages to compensate for these costs. In all, a larger region becomes less attractive because labour costs increase.

Our approach thereby builds upon and extends the labour market pooling argument. This argument due to Marshall (1920) was used again by David and Rosenbloom (1990) and especially Krugman (1991) to explain the agglomeration of workers and firms of an industry in a single region. Whereas the David and Rosenbloom (1990) analysis lacks a microeconomic underpinning, Krugman's (1991) work popularized the labour market pooling argument with a model containing a rather simple microeconomic structure. He argues that firm specific shocks provide a strong argument for (complete) concentration of economic activity.

There is, however, no microeconomic reason for firm specific shocks in Krugman's framework. Demand shocks can only be firm specific if firms produce heterogeneous output. If so, there is *a priori* no reason to assume that this output can be produced with homogeneous labour. There are certainly reasons for firm specific shocks even within a population of firms producing homogeneous goods and employing homogeneous labour; for instance, due to management failure. But in order to receive an empirically relevant labour market pooling argument some degree of firm heterogeneity has to be allowed for. This is the starting point of our analysis. We introduce firm heterogeneity and investigate the impact of the degree on firm heterogeneity on the labour market pooling argument.

Our model is also related to other theoretical approaches to the local labour market. In a series of papers, Kim (1989, 1990, 1991) analyzes the potential benefits of larger local labour markets when both firms and workers are heterogeneous. In his model

firms' decide upon their technological characteristics by locating in product space. The technological alternatives are located around a circle on which workers' abilities are uniformly distributed. Among other things, he shows that competition for workers becomes more intensive in larger regions and that workers have an incentive to invest more (less) in specific (general) human capital. Wages turn out to be higher in the larger local labour market. Hesley and Strange (1990) adopt a similar setting for two regions. They argue that mobile workers expect a better match if migrating to the larger market. Thisse and Zhenou (1995) using a related set up address a different issue. They ask for the socially optimal size and financing division of human capital formation costs between individual workers and the government. They also show that workers tend to be better off in larger labour markets and experience a higher wage there.

We model the interactions of firms and workers as a three-stage game. In the first stage, firms choose to enter the one of two labour markets in which expected profits are highest. In the second stage, workers also decide about their participation in one of the labour markets, based upon the difference between expected wages and expected retraining costs. In the third stage, product market shocks are revealed and firms decide upon their optimal output level as well as on labour demand. If firms face a positive output demand shock they hire workers initially employed by other firms.² Since these workers do not perfectly meet their skill requirements, they need to be retrained. Retraining costs, borne by the employees increase with the heterogeneity between firms. Together with output levels wages are determined such that the labour market clears. Hence, there is always full employment.

Thus, our model leads us to an explanation for the empirically observable interregional differences in wages and industry structure. More specifically, we first set forth a novel rationalization of the observation that wages increase in the size of the region

²Basically this is a short-cut of a multi-period output game in which transitory shocks occur each period. In order to facilitate the analysis we restrict the model to a single period in which output levels are chosen after shocks have occurred and are observed.

as specified by number of firms and workers: since expected risk of losing the job and thus incurring retraining costs is larger in the larger region, mobile workers demand higher wages in order to be compensated for this higher risk. Due to this wage effect, it turns out that output and firm size are lower in the larger region. Furthermore, we show that the degree of spatial concentration of industry depends very much on industry characteristics. By parameterizing the degree of heterogeneity between firms' output, we demonstrate that for an industry with firms producing rather homogenous goods labour market pooling indeed takes place, by which in equilibrium all firms locate in one region. The labour market pooling argument vanishes completely, however, if firms in a particular industry are sufficiently heterogeneous.

The paper is organized as follows. In the next section we outline the basic structure of the model. In the third section we derive labour market as well as goods-market equilibria after demand shocks have been observed. In the fourth section we address the residence decisions of workers and their impact on expected regional wages and firm size. In the fifth section we look at firms' locational decisions and the resulting spatial structure. We ask which type of industry tends to concentrate partially or completely, and in which such a tendency does not exist. In the concluding section we discuss the main results and potential extensions.

2 The Basic Model

Our model economy consists of two regions populated by \bar{L} workers, each endowed with one unit of inelastically supplied labour. Workers have to decide in which region m ($m = A, B$) to reside. This decision is costlessly implemented.

There are n ($n > 2$) firms producing differentiated goods for the world market. Uncertainty arises in the form of product specific demand shocks. Let ϵ_i denote a stochastic variable measuring the shock in the market for the good produced by firm

i. It faces the inverse output demand function

$$p_i = a - x_i - k \sum_{j \neq i}^n x_j + \epsilon_i, \quad i, j = 1, \dots, n, \quad (1)$$

where p_i is the price it receives for its products and x_i and x_j reflect the output levels of the firm i and its j -th competitor. The parameter a reflects the size of the market. The degree of substitutability is measured by the parameter k ($0 \leq k \leq 1$). The larger k , the more homogeneous the goods produced by the firms. With $k = 1$, goods are perfect substitutes; with $k = 0$ each firm holds a monopoly position and does not face any competition from the other firms in its output market.

The shocks in the different product markets are identically distributed. The shock variable is distributed with expectation $E(\epsilon_i) = 0$ and variance $\text{var}(\epsilon_i) = \sigma^2$. The correlation between shocks in different product markets is assumed to be a function of their relative substitutability. The symmetric covariance between demand shocks faced by firm i and its j -th competitor is given by $\text{cov}(\epsilon_i, \epsilon_j) = k\sigma^2$. Hence, shocks are perfectly correlated if firms produce homogeneous goods ($k = 1$), and zero correlated if the product markets are unrelated ($k = 0$). If firms produce identical goods, shocks hit firms in identical manners. If they produce differentiated goods product market shocks are transmitted into firm-specific shocks. Arguably, this covariance structure reflects the idea of firm, i.e., product specific demand shocks.

Firms produce with labour only, and are endowed with the simple production function

$$x_i = L_i, \quad (2)$$

where L_i denotes the number of workers employed by the i th firm. Region i 's typical firm's profit function is

$$\Pi_i^m = (a - x_i - k \sum_{j \neq i} x_j + \epsilon_i - w^m)x_i, \quad (3)$$

where w^m denotes the wage rate in region m . As the number of firms is exogenously fixed in the present version of our model, we disregard entry costs.

If a worker is hired after having been laid off by another firm, he needs to undergo retraining in order to acquire the skills demanded at his new job. That retraining cost is assumed to be the larger, the more different the goods produced by the old and new firms. We denote these adjustment costs by $C(k)$ with $\partial C/\partial k < 0$. We describe these adjustment costs in more detail in section 4.

As indicated before, we model the interaction of firms and workers as a three-stage game. In its first stage, firms decide on their place of location on the basis of expected profits. In the second stage, workers decide on the location. We assume away migration costs and costs of trade. Furthermore, we suppose that wages adjust in the third stage to clear labour markets so that full employment always prevails. This implies that in the second stage, utility-maximizing workers will base their choice of residence on a comparison of the expected wages net of expected retraining costs. They migrate towards the higher net wage region.

In the final stage, demand shocks are observed by all agents, and output and wages are simultaneously determined. Upon revelation of demand, firms decide in a Cournot-Nash manner on their profit-maximizing output levels. After shocks are revealed in stage 2, we can rewrite the profit function as:

$$\Pi_i^m = (a - x_i - k \sum_{j \neq i} x_j + e_i - w^m)x_i, \quad (4)$$

whereby e_i is the actual realization of the stochastic variable ϵ_i .

As usual, the model is solved backwards. Thus, we first look at output and wage determination after the shocks are revealed, given firms' and workers' location decisions. We then analyze the workers choice of residence and finally, the locational choice of firms before the shocks have occurred.

3 Output and Wage Determination

Output levels and wages are determined after product market specific shocks have occurred and firms have observed them given they themselves and the workers have

chosen a location. Wages are determined such that the local labour market clears (there is always full employment). There is perfect competition in local labour markets.³

Once shocks are revealed, profit maximization on the basis of (4) yields the following first-order condition for the typical firm i located in m

$$a - 2x_i^m - k \sum_{j \neq i} x_j + e_i - w^m = 0. \quad (5)$$

Solving this condition simultaneously for all n firms yields, after tedious calculations (which are explained in more detail in the appendix), the Cournot-Nash equilibrium output levels

$$x_i^m = \frac{(a + e_i - w^m)(2 + k(n - 2)) - (n - 1)ak + (n^m - 1)w^m k + n^q w^q k - \sum_j e_j k}{(2 - k)(2 + k(n - 1))} \quad m, q = A, B; m \neq q. \quad (6)$$

Labour market equilibrium requires that total labour demand just matches total labour supply in region m , L^m , which is exogenous in the third stage. Hence, we can write the labour market equilibrium in the two regions as

$$L^m = \sum_i^{n^m} x_i, \quad m = A, B. \quad (7)$$

Plugging (6) into (7) and assuming $L^m > 0$ ($m = A, B$) yields after some rearrangements:

$$\frac{L^m}{n^m} ((2 - k)(2 + k(n - 1))) - a(2 - k) - \frac{\sum_i^{n^m} e_i}{n^m} (2 + k(n - 2)) + \frac{\sum^{n^m} \sum^n e_j k}{n^m}$$

³Abstracting from market power on the input side of firms considerably facilitates the analysis. Conversely, allowing for imperfect competition in input as well as output markets makes the discussion highly intractable. To the best of our knowledge, this task was nowhere undertaken yet. However, there are arguments supporting our assumption. As long as the number of firms in a region is sufficiently large, it is unproblematic. With a small number of firms in a region, these firms could exert market power. But workers foresee this possibility of exploitation and take this into account when migrating. That is, the gains from market power would not accrue to firms. Hence, there are good reasons to argue that this assumption would not drastically alter our qualitative results (see also Krugman (1991) on this issue).

$$= n^q w^q k - w^m (2 + k(n^q - 1)). \quad (8)$$

Solving the system of two equations (one for each region) given in (8), we obtain:

$$\begin{aligned} w^m = & a - \frac{L^m}{n^m} (2 + k(n^m - 1)) - \frac{L^q}{n^q} n^q k \\ & + \Gamma \left(\frac{2 + k(n^m - 1)}{n^m} \left(\sum_i^{n^m} e_i (2 + k(n - 2)) + \sum \sum^n e_j k \right) \right. \\ & \left. + k \left(\sum_i^{n^q} e_i (2 + k(n - 2)) + \sum \sum^n e_j k \right) \right), \end{aligned} \quad (9)$$

with $\Gamma = 1/((2 - k)(2 + k(n - 1)))$.

Using the expressions for equilibrium wages in (6), we find after some calculations a surprisingly simple expression for the equilibrium output levels:

$$x_i^m = \frac{L^m}{n^m} + \frac{1}{(2 - k)n^m} \left(e_i (n^m - 1) - \sum_{j \neq i}^{n^m} e_j \right). \quad (10)$$

Hence, we get for expected output, $E x_i^m = \frac{L^m}{n^m}$. In the following we start from this expected or average output of each firm. Suppose, for instance, that in a first real time period (which is not explicitly modelled) each firm produces and employs workers according to this average level of output. We then analyze explicitly the deviation from this level of output and employment.

With the equilibrium values of wages and output, we can now turn to the location and residence decisions of firms and workers and the resulting spatial equilibrium.

4 Workers' Choice of Local Labour Markets

In choosing their place of residence, workers take the outcome of the wage and output game into account. Since residence decisions are undertaken before shocks are realized, (risk-neutral) workers decide on the basis of expected values.

Workers migrate towards the region offering the highest level of expected wage minus expected costs of retraining. Using (9), we obtain as the expected wage in

region m

$$Ew^m = a - \bar{L}k - \frac{L^m}{n^m}(2 - k). \quad (11)$$

In addition, workers have to consider their expected retraining costs. These costs are influenced by various factors. The first is the heterogeneity of firms on the output side. We first postulate a positive relationship between heterogeneity of firms on the output side measured by the parameter k and heterogeneity of firms' labour demand measured by k_I , i.e.: $k_I = ak$. For simplicity we focus on the case with $a = 1$. Hence, the more heterogenous the firms' products the more different their technological input requirements. Also, the more heterogenous the firms' technological requirements, the larger the retraining costs workers have to bear when losing and seeking alternative employment. If firms are completely identical ($k = 1$), we let the retraining costs be zero, while they are largest if $k = 0$. Second, expected retraining costs are a function of the probability for a worker of losing his job. Hence, we have to look for the expected negative deviation of realized output (and hence labour demand) from the expected value of output. Let V_i^m denote that deviation for a typical firm i in region m . Expected retraining costs per worker are then specified as $EC_i^m = \frac{cV_i^m(1-k)}{L^m/n^m}$, where c reflects the costs of training for a worker exclusive of the factor reflecting firm heterogeneity. The nominator describes the expected costs for workers per firm hit by a lay off. These are divided by the average number of workers employed in the representative firm. In all, this specification reflects in a simple way the idea that with increasing expected deviation and increased heterogeneity of firms (decreasing k) adjustment costs per firm increase.

In order to compute V_i^m we define a variable y measuring the deviation of output from its expected value after shocks have been observed (see (10)):

$$y_i^m = \frac{1}{(2 - k)n^m} \left(e_i(n^m - 1) - \sum_{j \neq i}^{n^m} e_j \right). \quad (12)$$

We assume that y_i^m is a truncated normally distributed variable.⁴ The truncation

⁴This is, for instance, the case if the ϵ_i are truncated normally distributed variables, too.

avoids negative output levels or those exceeding total employment. For the first two moments of y_i^m we find (see Hamerle et al. (1989)):

$$E(y_i^m) = \frac{1}{(2-k)n^m} \left(Ee_i(n^m - 1) - \sum_{j \neq i}^{n^m} Ee_j \right) = 0, \quad (13)$$

and

$$\text{var}(y_i^m) = \mathbf{I} \Sigma \mathbf{I}' = \left(1 - \frac{1}{n^m} \right) \frac{\sigma^2(1-k)}{(2-k)^2}, \quad (14)$$

whereby \mathbf{I} denotes a row vector with n^m elements of the following form:

$\mathbf{I} = \left(\frac{(n^m-1)}{n^m(2-k)}, \frac{-1}{n^m(2-k)}, \dots, \frac{-1}{n^m(2-k)} \right)$. The corresponding column vector is represented by \mathbf{I}' . The $n^m \times n^m$ matrix Σ is the variance-covariance matrix with σ^2 s on the main diagonal and $k\sigma^2$ as the off-diagonal elements.

Using the results from (13) and (14) and taking into account that y is normally distributed, we find the density function of y ,

$$f(y_i^m) = \frac{1}{\sqrt{2\pi\sigma_m^2}} e^{-\frac{y^2}{2\sigma_m^2}},$$

with $\sigma_m^2 := \text{var}(y_i^m)$.

We approximate V_i^m by:⁵

$$V_i^m = \int_0^\infty y_i^m f(y_i^m) dy_i^m. \quad (15)$$

Plugging the above density function into (15) yields (see e.g. Bronstein/Semendjajew (1987, p. 66))

$$\begin{aligned} V_i^m &= \int_0^\infty \frac{y_i^m}{\sqrt{2\pi\sigma_m^2}} e^{-\frac{y^2}{2\sigma_m^2}} \\ &= \left(\sqrt{1 - \frac{1}{n^m}} \right) \sqrt{\frac{\sigma^2(1-k)}{(2-k)^2}} \frac{1}{\sqrt{2\pi}}. \end{aligned} \quad (16)$$

Hence, expected adjustment costs increase with the number of firms in the region. The reason is that a larger number of firms in the region and a larger local labour

⁵This is a reasonable approximation if individual shocks are rather small compared to total output of firms which we assume to be the case, see Johnson et al. (1991).

market allow firms to adjust output more radically in response to demand shocks. On average, more workers are laid off and rehired. This implies that the individual laborer's expected adjustment costs are higher.

Finally, we allow for a further agglomeration disadvantage by assuming that workers' retraining costs are proportional to the regional per worker endowment with an immobile factor (human capital being specific to this sector), i.e. $c^m = \bar{c}/h^m$ with $h^m = H/L^m$ and H denoting the identical endowment of the regions with the specific factor. This reflects the notion that in order to increase their flexibility workers invest in general human capital (a process which we do not consider explicitly). In this process, workers make use of local training facilities (the local human capital stock). The degree of flexibility, each worker achieves is a function of the regional endowment with human capital per worker. The larger h^m the higher the degree of flexibility and the lower c^m in region m .

Taking all this together gives us the following expression for expected retraining costs for a worker residing in region m :

$$EC_i^m = n^m \left(\sqrt{1 - \frac{1}{n^m}} \right) \frac{\sigma(1-k)^{3/2} \bar{c}H}{(2-k) \sqrt{2\pi}}. \quad (17)$$

The migration equilibrium condition for an interior solution can be written as :

$$Ew^A - EC^A = Ew^B - EC^B. \quad (18)$$

Taking (11) and (17) we find for for region A and B, respectively,

$$\frac{L^A}{n^A} = \frac{\bar{L}}{n} + \frac{(1-k)^{3/2} \sigma H \bar{c} \sqrt{2\pi} (n - n^A)}{(2-k)^2 n} \left(n^B \sqrt{1 - \frac{1}{n^B}} - n^A \sqrt{1 - \frac{1}{n^A}} \right) \quad (19)$$

and

$$\frac{L^B}{n^B} = \frac{\bar{L}}{n} - \frac{(1-k)^{3/2} \sigma H \bar{c} n^A}{(2-k)^2 \sqrt{2\pi} n} \left(n^B \sqrt{1 - \frac{1}{n^B}} - n^A \sqrt{1 - \frac{1}{n^A}} \right). \quad (20)$$

This implies that – for strictly positive given (n^A, n^B) – there is a unique migration equilibrium in the second stage of the game.

Subtracting (19) from (20) gives us:

$$\begin{aligned} \text{sign} \left(\frac{I^A}{n^A} - \frac{L^B}{n^B} \right) &= \text{sign} \left(-n^A \sqrt{1 - \frac{1}{n^A}} + n^B \sqrt{1 - \frac{1}{n^B}} \right) \\ &= \text{sign}(n^B - n^A) \end{aligned} \quad (21)$$

Together with (11) this reveals that for positive n^A and n^B wages are higher in the larger region whereas firm size is lower there. The intuition for this is straightforward. Larger expected adjustment costs in the larger region require that workers demand higher wages to be extracted into this region. Only with higher wages which compensate them for higher expected retraining costs, they are willing to settle in the larger region. This implies that profit-maximizing output for firms is lower (see (7)).

5 Firm location and spatial equilibrium

We now turn to the first stage of the game inducing the firms' location decisions. Firms will locate in the region in which they expect to realize the highest expected profits. Denote by $E\Pi_i^m$ these profits of firm i if located in region m . (3) and (5) imply

$$E\Pi_i^m = E((x_i^m)^2). \quad (22)$$

With the help of (10) we can rewrite this expression as

$$E\Pi_i^m = \left(\frac{L^m}{n^m} \right)^2 + \frac{1}{(2-k)^2} \left(1 - \frac{1}{n^m} \right) \{ \sigma^2(1-k) \}. \quad (23)$$

The second term reflects the influence of stochastic demand, i.e. the product specific shocks. This term is increasing in the number of firms located in the respective region. Hence, the larger the absolute number of firms located in that region (with a given L^m/n^m ratio and for $k < 1$) the more attractive it becomes for a firm to locate there. This just reflects the fact that a larger number of firms and a larger local labour market facilitates the absorption of a shock on individual product demand as long as demand shocks are not perfectly correlated between products. In the case of a positive

demand shock on its products, the respective firm can rely on workers laid off by firms experiencing a negative demand shock. The larger the local labour market, the more elastic labour supply perceived by the individual firms: the wage rate reaction to shocks is less pronounced in the larger market, which allows firms to take advantage of a (positive) shock. It follows that realized output at the firm level fluctuates more in the larger region, which increases its profits from positive shocks. This effect calls for labour market pooling, i.e. for a concentration of firms and workers in one region.

In (23), the expression in curly brackets represents the difference between variance and covariance. It reveals that the more heterogeneous the firms' products, i.e., the smaller k , the more pronounced the positive effect of a larger local labour market on firms' profits. It is in this respect that increased heterogeneity in the firms' products fosters the regional concentration of economic activity in an industry. However, there is a second channel through which k influences the second term in (23), namely via the first denominator. This effect works through competition in the output market. It weakens the labour market pooling effect with a decreasing k . At any rate, the covariance effect dominates as can be seen by differentiating the second expression in (23).

A distribution $((n^A)^*, (n^B)^*)$ of firms across labour markets is an equilibrium in the first stage is given if no single firm has an incentive to deviate from the equilibrium configuration. Hence, for $((n^A)^*, (n^B)^*)$ to constitute an equilibrium, it is necessary and sufficient that

$$E\Pi^A((n^A)^*, (n^B)^*) - E\Pi^B((n^A)^*, (n^B)^*) = 0 \quad , \text{ and} \quad (24)$$

$$E\Pi^j((n^j)^* + 1, (n^i)^* - 1) - E\Pi^i((n^i)^*, (n^j)^*) \leq 0 \quad \text{for } i, j = A, B, \quad (25)$$

with $(n^i)^* > 1$. With a corner solution $((n^i)^* = n$ und $(n^j)^* = 0)$ only (25) applies.

Plugging (19) and (20) in (22) we find firms' expected maximal profits in each region as a function of the interregional distribution of firms only. Taking the difference

between these expected profits gives us

$$\begin{aligned}
h(n^A) &:= E\Pi^A - E\Pi^B = 2\frac{\bar{L}}{n} \left(\frac{(1-k)^{3/2}\sigma H\bar{c}}{(2-k)^2\sqrt{2\pi}} \right) \left((n-n^A)\sqrt{1-\frac{1}{n-n^A}} - n^A\sqrt{1-\frac{1}{n^A}} \right) \\
&+ \left(\frac{(1-k)^{3/2}\sigma H\bar{c}}{(2-k)^2\sqrt{2\pi}} \right)^2 \left((n-n^A)\sqrt{1-\frac{1}{n-n^A}} - n^A\sqrt{1-\frac{1}{n^A}} \right)^2 \left(\frac{n-2n^A}{n} \right) \\
&+ \frac{\sigma^2(1-k)}{(2-k)^2} \left(\frac{1}{n-n^A} - \frac{1}{n^A} \right). \tag{26}
\end{aligned}$$

There are two candidate types for a spatial equilibrium: either a corner solution with all firms located in one either of two regions, or an equilibrium with firms being distributed in positive numbers among the two regions. We call the former type a pooled, and the latter type a fragmented equilibrium.

In view of (24), a glance on (26) reveals that the symmetric constellation with $n_s^A = n_s^B = n/2$ is a candidate for the latter type ($h(n_s^A) = 0$). Assuming for simplicity differentiability of h , the sufficient condition corresponding to (25) is $\partial h(n^A)/\partial n^A < 0$ if evaluated at $n^A = n/2$.

Differentiating (26) and evaluating at $n^A = n/2$ gives us

$$\begin{aligned}
\frac{\partial(E\Pi^A - E\Pi^B)}{\partial n^A} &= \left\{ -2\frac{\bar{L}}{n} \left(\frac{(1-k)^{3/2}\sigma H\bar{c}}{(2-k)^2} \right) (2(1-2/n)^{0.5} + \right. \\
&\quad \left. + (1/n)(1-2/n)^{-0.5}) \right\} + \frac{\sigma^2(1-k)}{(2-k)^2} (8n^{-2}). \tag{27}
\end{aligned}$$

There are two effects, pointing into different directions. The first effect is represented by the term in curly brackets in eq. (27). It reflects labor retraining costs which firms have to bear indirectly that increase with firm size. This calls for a symmetric equilibrium. The countervailing effect represented in the second expression in eq. (27) stands against a symmetric equilibrium. It reflects the pooling advantages due to product heterogeneity.

With a corner solution we find a similar pattern as with the symmetric solution. Inserting $n^A = 1$ as well as $n^B = n-1$ into (26) yields (the reverse case with $n^A = n-1$,

$n^B = 1$ is just symmetric):⁶

$$\begin{aligned}
h(n^A = 1) = & 2 \frac{\bar{L}}{n} \left(\frac{(1-k)^{3/2} \sigma H \bar{c}}{(2-k)^2} \right) ((n-1)(n-2))^{0.5} + \\
& + \left(\frac{(1-k)^{3/2} \sigma H \bar{c}}{(2-k)^2} \right)^2 \frac{(n-1)(n-2)^2}{n} \\
& - \frac{\sigma^2 (1-k) (n-2)}{(2-k)^2 (n-1)}.
\end{aligned} \tag{28}$$

Once again we find two effects pointing into different directions. Without the indirect burden of retraining costs shown in the first two terms, the advantage of a corner solution would dominate and all firms would settle in one region. There, the more elastic labour supply schedule would enhance their possibility of taking advantage of the product specific shocks. However, this effect has to be contrasted by larger adjustment costs in the larger region. Firms have to compensate workers in order to induce them to migrate in the second stage of the game towards the larger region. The net effect is *a priori* not clear-cut.

Taking all these effects together we find three potential patterns for $h(n^A)$, which are displayed in figure 1.⁷ In the first and third case, respectively, only one type of equilibria arises, namely the symmetric fragmentation equilibrium when the firms' products are very heterogenous, (see Figure 1a), and a pooling equilibrium of firms and workers in a single region (see Figure 1c) when products are quite homogenous. In the second case involving intermediate values of k and portrayed in Figure 1b we find multiplicity of equilibria in the sense that both a corner solution as well as a symmetric interregional distribution of firms and workers may arise.

Insert Figure 1 about here

⁶If n is sufficiently large, $h(n^A) < 0$ corresponds to (25).

⁷We have ample numerical evidence towards non-existence of a fragmentation equilibrium, involving $(n^A)^* \neq n/2$. with e.g. $0 < (n^A)^* < n/2$. We have, however, not been able yet to construct an analytical proof of non-existence of such equilibria.

Against the background of our starting point it is crucial to understand the impact of firm heterogeneity on the different equilibria to occur. Investigating the respective terms in (27) as well as (28) reveals that with increasing homogeneity of firms' products, i.e., increasing k towards unity, both the agglomerative and the disagglomerative forces diminish. In this case, shocks are strongly correlated among firms, thus reducing pooling advantages and labour demand becomes less heterogenous among firms which reduces retraining costs and thus pooling advantages. However, it turns out that the negative impact of the parameter of firm heterogeneity is more pronounced on the disagglomerative than on the agglomerative force. This can be shown as follows. Let us define $d = ((1 - k)^{3/2})/((2 - k)^2)$ and $e = (1 - k)/((2 - k)^2)$. We need to show that d/e as well as d^2/e are decreasing with k in order to ensure that the symmetric (corner) solution becomes less (more) likely with an increase in k . This is straightforward: $\partial(d/e)/\partial k = \partial(1 - k)^{0.5}/\partial k < 0$ and $\partial(d^2/e)/\partial k = \partial((1 - k)^2)/((2 - k)^2))/\partial k < 0$ which proves our claim.

Hence, our analysis has shown that symmetric fragmentation as well as pooling equilibria may arise in our model world. The corner (symmetric) case is the more likely the more homogenous firms are.

6 Concluding Remarks

In this paper we consider the effects of output demand shocks on the formations of local labour markets and on regional industry structure. We provide a microeconomic underpinning for firm specific shocks as caused by variations in the demand for heterogenous products. We model the idea that heterogeneity of firms in product market has to be accompanied by heterogenous skill requirements of these firms and investigate the impact of this two-sided heterogeneity on regional wages, industry structure and industry size.

We give first a new rationalization for wages to increase in the size of the labour market. In equilibrium, workers require this as in larger regions, chances to be laid

off and thus incurring retraining costs are higher. This in turn induces a smaller firms size as firms facing higher labour costs there choose a lower level of output. Furthermore, we find conditions under which a symmetric fragmentational equilibrium involving an equal number of firms in both labour markets arises and those under which the complete concentration of economic activity in one region constitutes a spatial equilibrium. We also find a situation in which both of these equilibria may arise alternatively. The probability of a symmetric equilibrium increases with firms' heterogeneity whereas the corner solution becomes then less likely.

Our approach can be used as a starting point for further analysis. There are various straightforward directions for extensions: particularly interesting ones involve the consideration of asymmetries between the regions and the explicit treatment of unemployment in a variant of the model in which shocks are only incompletely absorbed via wage adjustment. One could then look at the effects of industry wide shocks on asymmetric firms in the different regions and hence, on differences in regional income and (un-)employment. The understanding of these effects is of considerable interest for local policy makers. By allowing for local unemployment one could investigate the effects of shocks not only on regional income but also on regional unemployment and ask for the relationship between local industry structure and local unemployment rates as well as for asymmetries in the impact of public policies on these. We leave these issues for future research.

Appendix

Derivation of Eq. (6)

We can rewrite eq. (5) in matrix form as:

$$\begin{pmatrix} 2 & k & k & \cdots & k \\ k & 2 & k & \cdots & k \\ \dots & \dots & \dots & \dots & \dots \\ k & k & k & 2 & k \\ k & k & k & \cdots & 2 \end{pmatrix} \begin{pmatrix} x_1^A \\ \dots \\ x_{n^A}^A \\ \dots \\ x_n \end{pmatrix} = \begin{pmatrix} a + e_1 - w^A \\ \dots \\ a + e_{n^A} - w^A \\ \dots \\ a + e_n - w^A \end{pmatrix} \quad (\text{A.1})$$

Solving this equation yields

$$x_i^m = \frac{(a + e_i - w^m)D_{11} - \sum_{j \neq i} (a + e_j - w^j)D_{12}}{2D_{11} - k(n-1)D_{12}}, \quad (\text{A.2})$$

whereby D_{ij} denotes the determinant of the minor corresponding to the element of the i -th row and the j -th column.

By noting that $D_{11} = (2/k + (n-2))D_{12}$, we can rewrite (A.2) after rearranging terms to

$$x_i^m = \frac{(a + e_i - w^m)(2 + k(n-2)) - (n-1)ak + (n^m - 1)w^m k + n^q w^q k - \sum_j e_j k}{(2-k)(2+k(n-1))}, \quad (\text{A.3})$$

which just equals eq. (6).

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