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The interaction of direct and indirect risk selection

Normann Lorenz

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The interaction of direct and indirect risk selection

Normann Lorenz*

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Abstract

This paper analyzes the interaction of direct and indirect risk selection in health insurance markets. It is shown that direct risk selection – using measures unrelated to the benefit package like selective advertising or ‘losing’ applications of high risk individuals – nevertheless has an influence on the distortions of the benefit package caused by indirect risk selection. Direct risk selection (DRS) may either increase or decrease these distortions, depending on the type of equilibrium (pooling or separating), the type of DRS (positive or negative) and the type of cost for DRS (individual-specific or not). Regulators who succeed in reducing DRS by, e.g., banning excessive advertising or implementing fines for ‘losing’ applications, may therefore (unintentionally) mitigate or exacerbate the distortions of the benefit package caused by indirect risk selection. It is shown that the interaction of direct and indirect risk selection also alters the formula for optimal risk adjustment.


Keywords: Risk selection, risk adjustment, discrete choice.

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1 Introduction

Risk selection is considered to be one of the main problems in regulated health insurance markets. If there is community rating, so that insurers are not allowed to charge premiums according to risk, they will make profits with some individuals, and losses with others. Insurers who act on these incentives to attract profitable and repel unprofitable individuals are said to be engaged in risk selection.\(^1\)

Two forms of risk selection can be distinguished: direct risk selection (DRS) and indirect risk selection (IRS).\(^2\) With DRS, insurers know that a particular individual or group of individuals is characterized by non-average risk. DRS is therefore targeted at an individual the insurer has identified to either be a high or a low risk type (like, e.g., a hypochondriac) or at a group of individuals the insurer knows to have non-average expected cost (like, e.g., a certain age group or individuals living in a high cost area). Usually, the measures taken for DRS are not related to the benefit package (i.e., the medical services) offered, like selective advertising or ‘losing’ applications of high risk individuals.\(^3\) It has been shown that potential profits associated with successful DRS can be substantial.\(^4\)

With IRS, insurers do not know which particular individual is of high or low risk, but only know that there are different risk types in the population. The measures taken to engage in IRS usually consist of distorting the benefit package, so that it is attractive for low risks, but not for high risks. Several studies have shown that the incentives for IRS can be severe, and that insurers do indeed act on these incentives.\(^5\)

Regulators can counteract the incentives for both DRS and IRS by implementing a risk adjustment scheme, setting transfers to (and from) insurers depending on some signals which are informative about individuals’ expected cost. There is a huge literature on risk adjustment, dealing primarily with the formula that is used to calculate these transfers. In almost all risk adjustment schemes, this formula is based on a regression of actual health care expenditures on a set of explanatory variables like age, gender and morbidity. Most of the literature has been concerned with improving this underlying regression by, e.g., including additional variables or altering the grouping algorithm for diagnoses in morbidity based risk adjustment, so that a larger part of the variance of actual expenditures is explained. The larger the explained part of the variance, the closer the transfers are to actual cost, and the lower the incentives for risk selection should be.

There is only a small literature that departs from this statistical approach. Initiated by the very influential study of Glazer and McGuire (2000) on optimal risk adjustment, this literature explicitly models insurers’ incentives for risk selection and determines how a regulator can mitigate or even eliminate these incentives. In this first study, they have shown that a regulator can increase the effectiveness of a risk adjustment scheme by distorting the payments as calculated from conventional, regression-based risk-adjustment: there has to be

\[^1\]See van de Ven and Ellis (2000).
\[^2\]See Breyer et al. (2011).
\[^3\]van de Ven and van Vliet (1992) provide an extensive list of measures insurers may use for risk selection; for differential treatment of low and high risks’ applications see Bauhoff (2012).
\[^4\]See Shen and Ellis (2002).
overpayment for signals which are correlated with high risks and underpayment for signals which are correlated with low risks.\footnote{See also Glazer and McGuire (2002) and Jack (2006).} Optimal risk adjustment has also been derived for a setting where individuals differ in their elasticity to switch insurers or where insurers are allowed to vary their premium in some dimension, as is the case with age in the insurance exchanges in the US.\footnote{For the first setting, see Bijlsma et al. (2011), and for the second, McGuire et al. (2013) and Shi (2013).} For conventional, regression-based (instead of optimal) risk adjustment it has been shown that more precise risk adjustment may decrease welfare if there is imperfect competition and that it may – depending on the cost structure of risk selection – increase the extent of risk selection.\footnote{See Lorenz (2013) and Brown et al. (2011), respectively. In the empirical part of their study, Brown et al. (2011) find such an increase in the extent of risk selection for the Medicare Advantage program in the U.S.; however, there has been some disagreement on this finding, see Newhouse et al. (2012).}

An important implicit assumption in these studies on optimal risk adjustment (and in all the literature on risk selection) is that IRS and DRS are two distinct problems, which are independent in the sense that DRS has no impact on the distortions of the benefit package caused by IRS. Since DRS regards activities which are unrelated to the benefit package (like selective advertising or ‘losing applications’ of the high risks), such an assumption seems convincing.

In this study, it is shown that this assumption only holds for a special case of DRS, but that in general (the degree of) DRS has an influence on the distortions of the benefit package caused by IRS. DRS may either decrease or increase these distortions, depending on whether insurers try to attract the low risks (positive DRS) or to repel the high risks (negative DRS), whether a pooling or a separating equilibrium emerges, and whether the cost for DRS is individual-specific or not.\footnote{We will explain in greater detail what is meant by the term individual-specific cost in the next section.} A regulator who succeeds in reducing DRS by, e.g., banning excessive advertising or charging a fine for ‘losing’ applications of high risk individuals, may therefore unintentionally mitigate or exacerbate the distortions of the benefit package.

<table>
<thead>
<tr>
<th>cost</th>
<th>positive DRS</th>
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<td>pooling equilibrium</td>
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<td>non-individual-specific</td>
<td>no effect</td>
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<td>individual-specific</td>
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Table 1 gives an overview of the results: Positive and negative DRS with individual-specific cost create opposite effects in the pooling equilibrium, but similar effects in the separating equilibrium. In the pooling equilibrium, DRS only has an effect if cost is individual-specific; in the separating equilibrium, the distortion of the benefit package can also be affected for...
non-individual-specific cost. Interestingly, except for negative DRS with individual-specific cost in the pooling equilibrium, DRS always decreases the distortion (if there is an effect at all); however, as we will show in the following sections, the positive impact on the distortion is not caused by the same mechanism in the different settings.

We finally show that the mechanisms which lead to these results also have an effect on the formula for optimal risk adjustment developed by Glazer and McGuire (2000): the overpayment for a signal that indicates a high risk has to be larger or smaller than without DRS, depending on whether there is positive or negative DRS, respectively.

We are not aware of any theoretical study that explicitly models the interaction of direct and indirect risk selection. Some of the results regarding the distortions caused by IRS have been derived under perfect competition, but DRS seems incompatible with such a setting where individuals are perfectly informed about all benefit packages and premiums, and always choose the insurer who offers the best benefit package-premium combination. We therefore derive our results within a discrete choice model, which can easily capture different levels of competition. To keep the model simple, we consider the case that the benefit package is one-dimensional, but the results can just as well be derived for a multi-dimensional benefit package and when the shadow price approach of Frank et al. (2000) is employed; they also hold for a setting where the premium is set by a regulator and not by insurers.

The remainder of this paper is organized as follows: In Section 2 we introduce the basic discrete choice model and show how DRS can be incorporated in such a model. We derive the impact of risk-type-specific DRS on the benefit package in the pooling equilibrium in Section 3. We determine how the results are modified if we consider the (probably more realistic) case that DRS is targeted at a signal that is correlated with risk type in Section 4. We derive the implications for the optimal risk adjustment formula in Section 5. The separating equilibrium is analyzed in Section 6. We briefly illustrate our results with an example in Section 7 and Section 8 concludes.

2 The Model

2.1 Basic model without DRS

Individual preferences regarding the benefit-premium bundle are given by

\[ u = p_r v(m) - R, \]

where \( R \) denotes the premium and \( m \) the level of medical services (measured in monetary terms). \( p^r \) is the probability of becoming ill, and there are two risk types \( r = H, L \), with \( p^H > p^L \); the share of \( L \)-types is \( \lambda \). The utility of receiving medical treatment, \( v(m) \), is

\[ \text{Eggleston (2000) derives the optimal mix of supply and demand side cost sharing for a setting with a single (semi-altruistic) HMO that can influence the level of medical services (according to the outcome of a patient-provider bargaining process) and can dump a share of high risks at some cost; however, there is no competition as there is only one provider.} \]

\[ \text{See Lorenz (2013).} \]
increasing at a decreasing rate, i.e., \( v'(m) > 0 \) and \( v''(m) < 0 \). The efficient level of medical services is implicitly defined by \( v'(m^{EB}) = 1 \).

There are \( n \) insurers \( j \), each offering a benefit-premium bundle \( \{m^j, R^j\} \). Individuals’ decision of which insurer to choose may, however, not only depend on these benefit-premium bundles, but also on some other factors, like perceived friendliness of personnel, location, or which insurer was recommended by family and friends. In a discrete choice model, these other factors are captured by augmenting individuals’ utility as given by (1) by an individual- and insurer-specific utility component \( \varepsilon^j_i \); the utility of an individual \( i \) (being of risk type \( r \)) when choosing an insurer \( j \) therefore is

\[
 u_i(m^j, R^j) = p^r v(m^j) - R^j + \varepsilon^j_i. \tag{2}
\]

If we assume \( \varepsilon^j_i \) to be i.i.d. extreme value, the logit model with its analytically tractable choice probabilities arises. Denoting risk type \( r \)’s utility of the benefit-premium bundle offered by insurer \( j \) by

\[
 V^j_r = p^r v(m^j) - R^j,
\]

and specifying the variance of \( \varepsilon^j_i \) as \( Var(\varepsilon^j_i) = \sigma^2 \pi^2 / 6 \), the probability of individual \( i \) choosing a particular insurer \( k \) is

\[
 Prob(i \text{ chooses } k) = Prob(V^k_r + \varepsilon^k_i > V^l_r + \varepsilon^l_i \forall l \neq k) = \frac{\sum_{j=1}^n e^{V^j_r / \sigma}}{\sum_{j=1}^n e^{V^j_r / \sigma}}. \tag{3}
\]

We denote this probability by \( P^k_r \); it is also insurer \( k \)’s market share among the individuals of risk type \( r \). \( P^k_r \) is increasing in \( V^k_r \): a higher share of individuals of risk type \( r \) will choose insurer \( k \), if this insurer offers a higher level of medical services or charges a lower premium.

The variance of the additional utility component, \( Var(\varepsilon^j_i) = \sigma^2 \pi^2 / 6 \), is a measure of the level of competition in this health insurance market. If \( \sigma \) is small, all the \( \varepsilon^j_i \) are very similar and therefore only play a minor role for which insurer is chosen: Offering an only somewhat higher utility level than all the other insurers will, in this case, attract a large share of all individuals; this implies a high level of competition. If, on the other hand, \( \sigma \) is large, the other factors besides the benefit level and the premium – captured by large positive and large negative \( \varepsilon^j_i \) – are rather important, so that insurers, when increasing their premium (or reducing their benefit level), only lose a small share of their insured; a large level of \( \sigma \) therefore corresponds to a low level of competition. As shown by Lorenz (2013) who derives the equilibria for this basic model without DRS, the level of competition determines which type of equilibrium emerges: if the level of competition is high (i.e., \( \sigma \) is small), a separating equilibrium similar (but not identical) to the Rothschild-Stiglitz equilibrium under perfect competition arises if the level of competition is low (i.e., \( \sigma \) is large), there will be a pooling equilibrium.

\[\text{12}\text{See Train (2009, p. 40).}\]

\[\text{13}\text{See Zweifel et al. (2009), chapter 7, for the Rothschild-Stiglitz equilibrium.}\]
2.2 Positive DRS

We consider positive DRS to be an activity each insurer is engaged in which generates some cost and increases the probability of being chosen by the individual (or group of individuals) the activity is targeted at. We model this increase in the probability of being chosen to stem from an increase in the utility the individual receives, which may either be real (as, e.g., with a discount for a fitness club membership) or just perceived (as with advertising). We denote the cost by $a^j$ and the increase in utility by $g(a^j)$, where $g(a^j)$ is increasing and concave (satisfying the Inada-conditions). With positive DRS, the (perceived) utility of individual $i$ choosing an insurer $j$ therefore is

$$u_i(m^j, R^j) = p^r v(m^j) - R^j + g(a^j) + \varepsilon_i^j,$$

so that insurer $k$’s market share is given by\(^\text{14}\)

$$P^k_r = \frac{e^{\frac{V^k + g(a^k)}{\sigma}}}{\sum_j e^{\frac{V^j + g(a^j)}{\sigma}}}.$$

In Section 3, where we derive the equilibrium, it will turn out important to distinguish two cases regarding the cost $a^j$: non-individual-specific and individual-specific cost. With non-individual-specific cost, total cost for DRS of an insurer $j$ is independent of the number of individuals choosing this insurer. The prime example for this case is selective advertising, where cost does not increase if an additional individual chooses insurer $j$. With individual-specific cost, total cost for DRS of an insurer $j$ is proportional to the number of individuals choosing this insurer. The prime example here are additional benefits which the regulator (or society) considers not to be part of a ‘normal’ basic benefit package insurers are supposed to provide, like discounts for fitness club memberships or special counseling services for minor or life-style related health problems (e.g., nutrition counseling). In this case, total cost of DRS increases if an additional individual chooses insurer $j$.

2.3 Negative DRS

Like positive DRS, we model negative DRS as an activity that generates some cost, but decreases the probability of being chosen by a particular individual (or group of individuals). We denote the cost of negative DRS by $b^j$ and the utility decrease by $f(b^j)$, where $f(b^j)$ is increasing and concave (satisfying the Inada-conditions). With negative DRS, the (perceived) utility of individual $i$ choosing an insurer $j$ therefore is

$$u_i(m^j, R^j) = p^r v(m^j) - R^j - f(b^j) + \varepsilon_i^j,$$

and insurer $k$’s market share is

$$P^k_r = \frac{e^{\frac{V^k - f(a^k)}{\sigma}}}{\sum_j e^{\frac{V^j - f(a^j)}{\sigma}}}.$$

\(^{14}\)To simplify the notation, we do not introduce different symbols for $P^k_j$ for the case of no, positive or negative DRS; we will, however, always make clear to which case we refer.
Unlike with positive DRS, it is difficult to imagine some activity where the cost an insurer incurs for negative DRS is independent of the number of individuals choosing this insurer. ‘Negative advertising’ might be an example, where an insurer informs about some undesirable feature of its offer, like scrupulous utilization reviews, but this and similar examples may seem rather far-fetched.\footnote{For matters of completeness we nevertheless briefly consider this case when we derive the equilibrium.} We think it is more realistic to consider negative DRS to be an activity insurers are engaged in during the application process, so that the cost depends on the number of individuals applying at the insurer.\footnote{In Section\ref{sec:app} we argue that the main effects should be similar if negative DRS does not occur during the application process, but is targeted at individuals who already hold a contract with the insurer.} Activities which fall into this category are that insurers require additional (unnecessary) paper work or involve the high risk individuals in lengthy phone calls in which they try to persuade (or even urge) these individuals to choose a different insurer.\footnote{After a German sickness fund operating mainly in high cost areas went bankrupt in 2011, members of this fund who then had to apply at other funds received phone calls in which some of the insurers told them that they could not continue their drug therapy or disease management program should they not choose a different insurer; see, e.g., Spiegel (2011).}

The number of individuals applying at an insurer could be equal to or larger than the number of individuals eventually choosing the insurer. The first case applies if individuals know about the level of DRS of all insurers, or (correctly) infer from the first encounter with an insurer who is engaged in negative DRS that all the other insurers will be so as well (at the same level).

The second case applies if individuals do not know about DRS and – after having experienced negative DRS at the first insurer – still believe that the other insurers are not engaged in DRS. If the first insurer was, say, insurer $k$, and at least one of the other insurers offered a higher utility, i.e., if

$$\max(V_{r}^l + \varepsilon_{i}^l \quad \forall \ l \neq k) > V_{r}^k - f(b^k) + \varepsilon_{i}^k,$$

individual $i$ would apply at a second insurer, where he would then learn that also this second insurer is engaged in DRS. Still assuming that the remaining insurers are not, individual $i$ may apply at a third insurer, and so on. If, during this process, individual $i$ applies at each insurer that seems to offer a higher utility (because the individual does not yet know about the DRS-activities of this insurer), it is then straightforward to show that individual $i$ will apply at insurer $k$ (at some point during this process) if

$$V_{r}^k + \varepsilon_{i}^k > V_{r}^l - f(b^l) + \varepsilon_{i}^l \quad \forall \ l \neq k.$$

In this case, the share of individuals applying at a particular insurer $k$ is

$$\tilde{P}_r^k = \frac{e^{V_{r}^k/\sigma}}{e^{V_{r}^k/\sigma} + \sum_{j \neq k} e^{V_{r}^j - f(b^j)/\sigma}}. \quad \tag{8}$$

If individuals apply at more than one, but less than all insurers before they (correctly) infer that all insurers are engaged in negative DRS, the share of individuals applying at insurer $k$ will be some $\hat{P}_r^k$ greater than $P_r^k$ as given by (7), but smaller than $\tilde{P}_r^k$ as given by (8). As we show in Section\ref{sec:app}, the equilibrium for $\hat{P}_r^k$ will be very similar to the one for $\tilde{P}_r^k$.\footnote{For matters of completeness we nevertheless briefly consider this case when we derive the equilibrium.}
We think that this case, where insurers incur cost for negative DRS for a share \( \hat{P}_{kr} \) (or \( \tilde{P}_{kr} \)) that is larger than \( P_{kr} \), also describes a setting where insurers pay insurance brokers for steering high risk individuals to other insurers. In such a setting, the payment will – at least to a certain degree – depend on the number of high risks the insurance broker is dealing with, which, if he is successful, is larger than the number of individuals eventually choosing the insurer.

To sum up: For negative DRS we consider three cases: Cost either occurs for all individuals or only for the share of individuals applying at the insurer, where this share may either be the same as the one choosing the insurer, (i.e., \( P_{jr} \)), or a larger share, (i.e., \( \hat{P}_{kr} \) or \( \tilde{P}_{kr} \)).

### 3 The pooling equilibrium with DRS against the risk type

In this section, we derive the impact of DRS if it is targeted at the risk type; we will consider the (probably more realistic) case that it is targeted at a signal that is (less than perfectly) correlated with risk type in Section 4. We begin by briefly deriving the equilibrium without DRS, so that it can be compared with the equilibria for the different settings with DRS.

#### 3.1 The pooling equilibrium without DRS

It turns out to be easier to derive the equilibrium if we formulate the objective of insurer \( k \) in terms of \( V_{L}^{k} \) and \( m^{k} \) (instead of \( R^{k} \) and \( m^{k} \)); to do so, we express \( V_{H}^{k} \) as

\[
V_{H}^{k} = V_{L}^{k} + (p^{H} - p^{F})v(m^{k}).
\]

Solving \( V_{L}^{k} = p^{L}v(m^{k}) - R^{k} \), and substituting in \( \pi_{r}^{k} = R^{k} - p r m^{k} \), profit per individual (of risk type \( r \)) is given by

\[
\pi_{r}^{k} = p^{L}v(m^{k}) - V_{L}^{k} - p r m^{k}.
\]

Normalizing the mass of individuals to one and assuming profit maximization, the objective of insurer \( k \) is

\[
\max_{V_{L}^{k}, m^{k}} \pi^{k} = \lambda P_{k}^{L} \pi_{L}^{k} + (1 - \lambda) P_{H}^{k} \pi_{H}^{k}.
\]  

(9)

The FOCs are given by

\[
\frac{\partial \pi^{k}}{\partial V_{L}^{k}} = \lambda \left[ \frac{P_{L}^{k}(1 - P_{L}^{k})}{\sigma} \pi_{L}^{k} - P_{L}^{k} \right] + (1 - \lambda) \left[ \frac{P_{H}^{k}(1 - P_{H}^{k})}{\sigma} \pi_{H}^{k} - P_{H}^{k} \right] = 0
\]

(10)

\[
\frac{\partial \pi^{k}}{\partial m^{k}} = \lambda P_{L}^{k} \left[ p^{L}v'(m^{k}) - p^{L} \right] + (1 - \lambda) P_{H}^{k} \left[ p^{L}v'(m^{k}) - p^{H} \right] + (1 - \lambda) \frac{P_{H}^{k}(1 - P_{H}^{k})}{\sigma} (p^{H} - p^{F})v'(m^{k}) \pi_{H}^{k} = 0,
\]

where we use the property that the derivative of \( P_{r}^{k} \) with respect to \( V_{r}^{k} \) can be expressed in terms of \( P_{r}^{k} \) itself as

\[
\frac{\partial P_{r}^{k}}{\partial V_{r}^{k}} = \frac{P_{r}^{k}(1 - P_{r}^{k})}{\sigma}.
\]
Using the fact that in equilibrium $P_j^j = \frac{1}{n} \forall \; j$, condition (10) yields
\[
\lambda \pi_L^k + (1 - \lambda) \pi_H^k = \frac{n \sigma}{n - 1}. \tag{12}
\]
Average profit per insured decreases in $n$ and increases in $\sigma$: a higher level of competition (large $n$, small $\sigma$) decreases profit. Solving (12) for $\pi_H^k$ and substituting in (11) yields the condition determining the distortion of the benefit level:
\[
\left[1 - \frac{\lambda(1 - \lambda)(p_H^k - p_L^k)^2}{n \sigma} \right] v'(m^k) = 1. \tag{13}
\]
Because the fraction is positive, it is immediately apparent that $v'(m^k) > 1$, so that $m^k$ is distorted below the efficient level $m^{FB}$. As is to be expected, the distortion increases in the difference $p_H^k - p_L^k$.

In the following, we will compare this condition with the respective conditions for the different cases of DRS in order to determine whether the distortion increases or decreases due to DRS. To make clear which case of cost we refer to, we will always explicitly state the insurer’s objective.

### 3.2 The pooling equilibrium with positive DRS

#### 3.2.1 Non-individual-specific cost

With positive DRS against the risk type and non-individual-specific cost, insurer $k$’s objective is given by
\[
\max_{V_L^k, m^k, a^k} \pi^k = \lambda \left( P_L^k \pi_L^k - a^k \right) + (1 - \lambda) P_H^k \pi_H^k, \tag{14}
\]
where $P_L^k$ contains $g(a^k)$ and $g(a^j)$ as given in equation (5). A positive equilibrium level of $a^k$ is implicitly defined by
\[
\frac{\partial \pi^k}{\partial a^k} = \lambda \left[ \frac{P_L^k(1 - P_L^k)}{\sigma} g'(a^k) \pi_L^k - 1 \right] = 0. \tag{15}
\]
However, because the FOCs with respect to $V_L^k$ and $m^k$ are identical to (10) and (11), condition (13) still holds, so that the distortion of $m^k$ does not change.

**Result 1.** In the pooling equilibrium, the distortion of the benefit level is unaffected by positive DRS if cost for DRS is non-individual-specific.

We comment on this result in the following section, where we compare it with the equilibrium if cost is individual-specific.
### 3.2.2 Individual-specific cost

With positive DRS and individual-specific cost, insurer $k$’s objective reads as

$$\pi^k = \lambda P^k_L \left( \pi^k_L - a^k \right) + (1 - \lambda) P^k_H \pi^k_H,$$  \hspace{1cm} (16)

where $P^k_L$ is again given by (5). While the FOC with respect to $m^k$ is still given by (11), the positive level of $a^k$ (implicitly defined by condition (39), given in Appendix A.1) now enters the FOC with respect to $V^k_L$, which can be simplified to yield

$$\lambda(\pi^k_L - a^k) + (1 - \lambda)\pi^k_H = \frac{n\sigma}{n - 1}.$$  \hspace{1cm} (17)

If cost is individual-specific, it is average net profit including $a^k$ that equals $\frac{n\sigma}{n - 1}$. Solving for $\pi^k_H$ and substituting in (11), we have

$$v'(m^k) = \frac{1}{1 - \lambda(1 - \lambda)(p^H - p^L) m^k + \lambda(1 - \lambda)(p^H - p^L) a^k}.$$  \hspace{1cm} (18)

Since the last term in the brackets $[\cdot]$ is positive, compared with no DRS, $v'(m^k)$ has to decrease: $m^k$ increases, so the distortion is reduced. More generally, the larger the equilibrium level of $a^k$, the larger $m^k$.

**Result 2.** *In the pooling equilibrium, the distortion of the benefit level decreases in the level of positive DRS if cost for DRS is individual-specific.*

The incentive to distort the benefit level $m$ (with or without DRS) arises because profit per high risk is lower than profit per low risk, where the degree of the distortion depends on the difference between these two profits, which in the case without DRS is given by

$$\pi_L - \pi_H = (p^H - p^L)m.$$  \hspace{1cm} [18]

With positive DRS, profit per low risk decreases because insurers waste part of $\pi_L$ on the expenditures for DRS; this reduces the difference between net profits (including $a$), and thereby the incentive to distort $m$.

This is different for the case of non-individual-specific cost considered in the last section. Although this cost decreases total profit, it does not specifically decrease profit per $L$-type, so that the difference between the profits remains the same; therefore, positive DRS has no influence on the distortion of the benefit level if cost is non-individual-specific.

### 3.3 The pooling equilibrium with negative DRS

We now turn to the case of negative DRS, where we consider the three different types of cost discussed in Section 2.3.

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18See the second term in the brackets in condition (13), which contains $(p^H - p^L)m$. 
3.3.1 Non-individual-specific cost

With negative DRS and non-individual-specific cost, insurer $k$’s objective is given by

$$\max_{V^k_L, m^k, b^k} \pi^k = \lambda P^k_L \pi^k_L + (1 - \lambda) \left( P^k_H \pi^k_H - b^k \right),$$  \hspace{1cm} (19)

where $P^k_H$ contains $f(b^k)$ and $f(b^j)$ as given in (7). Like with positive DRS, if cost is non-individual-specific, the FOC with respect to $V^k_L$ and $m^k$ are not affected by $b^k$ and are identical to (10) and (11), so that the distortion of the benefit level is the same as without DRS.

3.3.2 Individual-specific cost

We now consider the case that the cost of negative DRS occurs for each individual applying at the insurer. If the number of individuals applying equals the number of individuals choosing the insurer, insurer $k$’s objective reads as

$$\pi^k = \lambda P^k_L \pi^k_L + (1 - \lambda) P^k_H \left( \pi^k_H - b^k \right),$$  \hspace{1cm} (20)

where $P^k_H$ is again as given by (7). Taking the positive level of $b^k$ (defined by condition (40) in Appendix A.1) into account, the FOC with respect to $V^k_L$ yields

$$\lambda \pi^k_L + (1 - \lambda) (\pi^k_H - b^k) = \frac{n \sigma}{n - 1},$$  \hspace{1cm} (21)

so that the distortion of $m$ is now determined by

$$\left[ 1 - \frac{\lambda (1 - \lambda) (p^H - p^L)^2}{n \sigma} \right] m^k - \frac{\lambda (1 - \lambda) (p^H - p^L)}{n \sigma} b^k \right] v'(m^k) = 1.$$  \hspace{1cm} (22)

Since the last term in the brackets is negative, $m^k$ decreases. With negative DRS, the distortion of the benefit level is larger than without DRS, and it increases in $b^k$.

Negative DRS therefore just creates the opposite effects compared to positive DRS. It increases the loss entailed by the high risks; this increases the difference in (net) profits between the two risk types, which increases the incentive to distort $m$.

3.3.3 Individual-specific cost for a number of applicants that is larger than the number of insured

We finally consider the case that cost for negative DRS occurs for each individual applying at the insurer, where the number of individuals applying is larger than the number of individuals choosing the insurer. Here we derive the result for $\tilde{P}^k_r$, but the formulas are identical if we replace $\tilde{P}^k_r$ by $\hat{P}^k_r$.

The insurer’s objective is given by

$$\pi^k = \lambda P^k_L \pi^k_L + (1 - \lambda) \left( P^k_H \pi^k_H - \tilde{P}^k_H b^k \right),$$  \hspace{1cm} (23)

where $P^k_H$ denotes the share of high risks choosing insurer $k$ (as given in (7)), and $\tilde{P}^k_H$ denotes the share of high risks which (at some point during the application process) apply
at insurer $k$ (as given by (8)). In this case, the positive equilibrium level of $b^k$ is defined by

$$\frac{\partial \pi^k}{\partial b^k} = (1 - \lambda) \left[ \frac{P^k_H (1 - P^k_H)}{\sigma} (-f'(b^k)) \pi^k_H - \tilde{P}^k_H \right] = 0. \quad (24)$$

With this positive level of $b^k$, condition (11) now reads as

$$\left[ 1 - \frac{\lambda(1 - \lambda)(p^H - p^L)^2}{\sigma n} m^k - \frac{\lambda(1 - \lambda)(p^H - p^L)}{\bar{p}} \tilde{P}^k_H (1 - \tilde{P}^k_H) \right] \nu'(m^k) = 1. \quad (25)$$

Since the last term in the brackets is negative, the distortion is increased. Note that as the share of individuals applying at insurer $k$ approaches 1, the last term in the brackets approaches zero: If (almost) all individuals apply at insurer $k$, cost for DRS is (almost) non-individual-specific, so that the distortion of $m$ is (almost) not affected by DRS. However, in the more realistic case that not all individuals apply at all insurers, $\tilde{P}^k_H < 1$, and the distortion is larger than without DRS.

We now summarize the results for negative DRS:

**Result 3.** In the pooling equilibrium, the distortion of the benefit level (i) is unaffected by negative DRS if cost for DRS is non-individual-specific and (ii) increases in the level of negative DRS if cost for DRS occurs for each individual applying at the insurer, regardless of whether the number of individuals applying is equal to or larger than the number of individuals choosing the insurer (as long as not all individuals apply at all insurers).

In the pooling equilibrium, positive and negative DRS create just the opposite effects on the distortion of the benefit level. As we show in Section 6, this is not the case for the separating equilibrium, where positive and negative DRS both reduce the distortion of the benefit level.

### 4 The pooling equilibrium with DRS against a signal that is correlated with risk type

In this section we consider the (probably more realistic) case that DRS is not targeted at the risk type itself, but at a signal that is (less than perfectly) correlated with risk type. In the following, we will consider a setting where in addition to the two risk types, there are two signal types $s = Y, O$, young and old. The shares of the four types of individuals, $\mu_{rs}$, are given in Table 2 where $\delta > 0$ captures the case of a positive correlation of high risk type and (old) age.

<table>
<thead>
<tr>
<th>$s$</th>
<th>$\mu_{LY}$</th>
<th>$\mu_{HY}$</th>
<th>$\mu_{LO}$</th>
<th>$\mu_{HO}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y$</td>
<td>$\lambda \eta + \delta$</td>
<td>$\lambda - \lambda \eta$</td>
<td>$\lambda (1 - \eta) - \delta$</td>
<td>$\lambda (1 - \eta) (1 + \delta)$</td>
</tr>
<tr>
<td>$O$</td>
<td>$\lambda (1 - \eta) - \delta$</td>
<td>$\lambda - \lambda \eta$</td>
<td>$\lambda (1 - \eta) - \delta$</td>
<td>$\lambda (1 - \eta) (1 + \delta)$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\sigma$</th>
<th>$M$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$</td>
<td>$1 - \lambda$</td>
</tr>
</tbody>
</table>
This formulation of a positive correlation has the advantage that increasing $\delta$ increases the level of correlation without altering the shares of the two risk types, $\lambda$ and $(1 - \lambda)$, and the shares of the two signal-types, $\eta$ and $(1 - \eta)$.

Since there is no effect on the distortion of the benefit level if cost is non-individual-specific, in the following only the case of individual-specific cost will be considered.

### 4.1 Positive DRS

With positive DRS of the young and individual-specific cost, insurer $k$’s objective reads as

$$
\pi^k = \sum_r \sum_s \mu_{rs} P^k_{rs} \pi^k_{rs} - \sum_r \mu_{rY} P^k_{rY} a^k,
$$

where only $P^k_{rY}$ contains $g(a^k)$ and $g(\alpha^j)$. Condition (II) now yields

$$
\left[ 1 - \frac{\lambda(1 - \lambda)(p^H - p^L)^2}{n\sigma' n - 1} m^k + \frac{(p^H - p^L)}{n\sigma' n - 1} \delta a^k \right] \nu'(m^k) = 1.
$$

Since the last term in the brackets is positive, $\nu'(m^k)$ has to decrease, so the distortion is reduced. However, for a given level of $a^k$ the reduction of the distortion is of course not as large as with DRS against the risk type itself, since $\delta < \lambda(1 - \lambda)$. As is to be expected, for a given level of $a^k$ the reduction of the distortion increases in the level of correlation (i.e., in $\delta$).

### 4.2 Negative DRS

Solving the FOCs for insurer $k$’s objective

$$
\pi^k = \sum_r \sum_s \mu_{rs} P^k_{rs} \pi^k_{rs} - \sum_r \mu_{rO} P^k_{rO} b^k,
$$

where now only $P^k_{rO}$ contains $f(b^k)$ and $f(\beta^j)$, we have

$$
\left[ 1 - \frac{\lambda(1 - \lambda)(p^H - p^L)^2}{n\sigma' n - 1} m^k - \frac{(p^H - p^L)}{n\sigma' n - 1} \delta b^k \right] \nu'(m^k) = 1.
$$

This shows that negative DRS yields again just the opposite effect of positive DRS. We summarize these results as follows:

**Result 4.** In the pooling equilibrium, the distortion of the benefit level (i) decreases in the level of positive DRS of a signal that is correlated with low risk and (ii) increases in the level of negative DRS against a signal that is correlated with high risk if cost for DRS is individual-specific. A higher level of correlation (a higher $\delta$) increases the effect of a given level of DRS on the distortion of the benefit level.
5 Implications of DRS on optimal risk adjustment in the pooling equilibrium

We now discuss the implications of the interaction of direct and indirect risk selection for optimal risk adjustment. As shown by Glazer and McGuire (2000), if a regulator does not observe individuals’ risk type, but only a signal that is correlated with risk type (like age), it is still feasible to eliminate the distortion of the benefit level by overpaying for a signal that indicates a high risk, and underpaying for a signal that indicates a low risk. In our setting with two risk types and two age groups introduced in the last section, this requires the payment for the old to be larger than the average cost of the old, and vice versa for the young.

A concern, already raised by Glazer and McGuire (2000) themselves, is that such over- and underpayments create incentives for DRS regarding the signal, but so far, it has not been analyzed whether this has an influence on optimal risk adjustment. In this section we show that their formula which eliminates the distortion of the benefit level has indeed to be modified if there is DRS regarding the signal. In the following, we will first determine the optimal (over-)payments without DRS, and then derive the modification if there is DRS.

5.1 Optimal risk adjustment without DRS

With risk adjustment, each insurer receives a payment of $RA_O$ for each insured that is old; these payments are financed by a risk adjustment fee $RAF$ which each insurer has to pay for each insured (including the old). The balanced budget constraint requires this fee to be

$$RAF = (1 - \eta)RA_O.$$  

The insurer’s objective with risk adjustment is given by

$$\pi_k = \sum_r \mu_{rY} P_{rY}^{kY} \left( \pi_{rY}^k - RAF \right) + \sum_r \mu_{rO} P_{rO}^{kO} \left( \pi_{rO}^k - RA + RA_O \right).$$ (30)

Solving the FOCs and using the balanced budget constraint, condition (13) now reads as

$$\left[ 1 - \frac{(p_H - p_L)}{n-1} \left( \lambda(1-\lambda)(p_H - p_L)m^k - \delta RA_O \right) v'(m^k) \right] v'(m^k) = 1,$$ (31)

so that the distortion of $m$ is eliminated for

$$RA_O = \frac{\lambda(1-\lambda)}{\delta}(p_H - p_L)m.$$ (32)

The lower $\delta$, i.e., the lower the correlation of old age and high risk, the larger $RA_O$ has to be. If there is perfect correlation, the share of the low risks equals the share of the young, so $\eta = \lambda$; in addition, the mass of individuals in the lower left and the upper right corner in Table 2 has to be zero. With $\eta$ replaced by $\lambda$, this requires $\delta = \lambda(1-\lambda)$, so $RA_O = (p_H - p_L)m$, which is just the cost difference between the two risk types. With less than perfect correlation, $\delta < \lambda(1-\lambda)$, so there is overpayment. We now show how this overpayment is modified by DRS. Again only the case of individual-specific cost is considered, as there is no effect on the overpayment if cost is non-individual-specific.
5.2 Optimal risk adjustment with positive DRS

With optimal risk adjustment there is overpayment for the old, so this is the group positive DRS will be targeted at. The insurer’s objective in this case is given by

$$\max_{V_k^k, m^k, a^k} \pi^k = \sum_r \mu_{rY} P_{rY}^k \left( \pi_{rY}^k - RA\right) + \sum_r \mu_{rO} P_{rO}^k \left( \pi_{rO}^k - RA + RO - a^k \right),$$

where $P_{rO}^k$ contains $g(a^k)$ and $g(a^j)$. Taking the positive level of $a^k$ (defined by the FOC with respect to $a^k$) into account, condition (31) now reads as

$$\left[ 1 - \left( \frac{pH - pL}{n-H} \right) \left( \lambda(1-\lambda)(pH - pL) m^k - \delta(RO - a^k) \right) \right] v'(m^k) = 1, \quad (34)$$

which requires

$$RA_O = \frac{\lambda(1-\lambda)}{\delta}(pH - pL) m^k + a^k. \quad (35)$$

Because part of the overpayment for the old is spent on positive DRS, the optimal payment for the old has to be raised by exactly these expenditures, so that the net difference in payments – including $a^k$ – equals the amount as given by (32).

5.3 Optimal risk adjustment with negative DRS

The case of negative DRS is again just the opposite of positive DRS. With extensive underpayment of the young, they will entail a loss, so negative DRS will be targeted at this signal type. Solving the FOC for insurer $k$’s objective (stated in Appendix A.2) yields an optimal risk adjustment payment of

$$RA_O = \frac{\lambda(1-\lambda)}{\delta}(pH - pL) m - b^k. \quad (36)$$

Since insurers waste money on negative DRS against the young, the optimal risk adjustment payment has to be decreased by these expenditures so that the net difference in payments equals the amount as given by (32).

We summarize these findings for optimal risk adjustment as follows:

**Result 5.** If cost for DRS is non-individual-specific, optimal risk adjustment is not affected by DRS. With individual-specific cost, if there is positive (negative) DRS regarding a signal that is used for risk adjustment and that is correlated with high (low) risk, the optimal overpayment of the signal that is correlated with high risk has to be increased (decreased) by the expenditures for DRS.

These results show that the formula for optimal risk adjustment derived by Glazer and McGuire (2000) is not invalidated by DRS: there is still overpayment for a signal that is correlated with high risk, and underpayment for a signal correlated with low risk. Also, DRS does not invalidate the claim that optimal risk adjustment can implement the efficient benefit level. However, the formula to derive the efficient benefit level has to be modified...
and include insurers’ expenditures on DRS if cost is individual-specific. Whether these expenditures are negligible or significant is an empirical matter, but, e.g., the findings of Starc (2014), who reports that insurers spend a large part of potential profits on marketing and insurance brokers, indicate that these expenditures may be substantial.

6 DRS in the separating equilibrium

6.1 The separating equilibrium without DRS

The separating equilibrium arises for a high level of competition (i.e., a low level of $\sigma$) and is similar, but not identical, to the Rothschild-Stiglitz equilibrium under perfect competition, see Lorenz (2013). Both equilibria can be found in Figure 1.

Under perfect competition, the separating equilibrium consists of contract $B$, chosen by the high risks, and contract $A_1$, chosen by the low risks, where contract $A_1$ is at the intersection of the iso-profit line for contracts chosen by the $L$-types (which has slope $p^L$) and the indifference curve of the $H$-types associated with contract $B$, so that the incentive compatibility constraint is satisfied; in addition, both iso-profit lines pass through the origin and are in this case zero-profit lines.

With imperfect competition, some insurers offer contract $B$ designated for the $H$-types, and the remaining insurers offer a contract similar to $A_1$, designated for the $L$-types. To simplify the exposition, we will refer to these insurers as insurers of type $B$ and type $A$, respectively. Compared with the case of perfect competition, under imperfect competition the iso-profit lines are shifted upwards, reflecting the profit per individual (see condition (12)). In addition, contract $A_1$ no longer is the equilibrium contract designated for the $L$-types. If contracts $B$ and $A_1$ were offered, $V^A_H = V^B_H$ and all the $H$-types for which $V^A_H > V^B_H$ prefer contract $B$.

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19 The separating equilibrium for a low level of $\sigma$ exists under the same condition as the Rothschild-Stiglitz equilibrium: the share of low risks must not be too large.

20 See Zweifel et al. (2009), chapter 7.
the largest additional utility component for one of the insurers of type $A$ is larger than the largest additional utility component for one of the insurers of type $B$, would prefer $A_1$ over $B$. With a considerable share of $H$-types choosing contract $A_1$, insurers of type $A$ would suffer a loss. Insurers of type $A$ therefore have to modify their contract so that $V^A_H$ is reduced and none (or almost none) of the $H$-types choose their contract.

The shaded area in Figure 1 indicates by how much insurers of type $A$ have to reduce $V^A_H$. This shaded area represents the density $P^A_H(1 - P^A_H)\frac{1}{\sigma}$ corresponding to the distribution function $P^A_H = P^A_H(V^A_H|V^B_H)$, which is one of the insurer $A$’s market share as a function of $V^A_H$, given $V^B_H$. The darkness of the shaded area is a measure of this density. Above the shaded area, both the distribution and the density are zero; none of the $H$-types choose a contract $A$ (with a high premium $R^A$ and thus a low level of utility $V^A_H$) that is too far above the indifference curve associated with contract $B$. The first (few) $H$-types are attracted by a contract at the upper boundary of the shaded area; these are the $H$-types with a particularly large $\varepsilon^A_i$ compared to $\varepsilon^B_i$. Moving contract $A$ further into the shaded area increases the share of $H$-types choosing this contract, where the increase of the share is given by the density at this point (represented by the darkness of the shaded area). A contract below the shaded area would be chosen by all the $H$-types, so $P^A_H = 1$ and $P^A_H(1 - P^A_H)\frac{1}{\sigma} = 0$.

For a more detailed derivation see Lorenz (2013), where it is shown that in equilibrium, insurers of type $A$ offer contract $A_3$, which is somewhat above the iso-profit line and somewhat inside the shaded area, so that a small share of the $H$-types choose the contract designated for the $L$-types. This contract is offered by $n^A$ insurers of type $A$, while contract $B$ is offered by $n^B$ insurers of type $B$, so that a profit equality constraint $\pi^A = \pi^B$ and $n^A + n^B = n$ is satisfied.

6.2 The separating equilibrium with positive DRS

With positive DRS of the low risks and individual-specific cost, the objective of one of the insurers of type $A$ equals the objective as given in (16), with $k$ replaced by $A$. Since in the separating equilibrium $P^A_H \neq \frac{1}{n}$, the FOC with respect to $V^A_H$ now yields

$$\lambda(\pi^A_L - a^A) + (1 - \lambda)\frac{n^2}{n - 1}P^A_H(1 - P^A_H)\pi^A_H = \lambda \frac{n\sigma}{n - 1} + (1 - \lambda)\frac{n^2}{n - 1}P^A_H.$$

As is apparent, the direct effect of a positive $a^A$ (compared to $a^A = 0$ and holding $m$ constant) is an increase in the premium $R^A$, because $\pi^A_L$ and $\pi^A_H$ have to increase, so that condition (37) is still satisfied. In Figure 2(a), this increase in $R^A$ can be shown by an upward shift of the iso-profit line associated with the insurers of type $A$. With the iso-profit line shifted upwards, insurers of type $A$ can increase the benefit level before attracting the same share of $H$-types as without DRS. Because a higher level of $m^A$ (accompanied by the according increase of the premium by $p^L \Delta m^A$) increases the utility of the low risks, insurers will offer this higher level under (imperfect) competition, so that the new equilibrium will be a contract like $A_4$. Since positive DRS reduces the attractiveness of the contract offered by insurers of type $A$ for the $H$-types in the premium-dimension, they can increase the attractiveness of their contract in the benefit level-dimension.

**Result 6.** In the separating equilibrium, the distortion of the benefit level is reduced if there is positive DRS of the low risks and cost is individual-specific.
6.3 The separating equilibrium with negative DRS

In this last case we consider negative DRS of insurers of type A against the $H$-types. The effect caused by this kind of DRS is different from all the other effects derived so far, which have all been due to changes in risk type-specific profits per individual. Here, an effect on the distortion of the benefit level arises because not all insurers engage in DRS against the $H$-types, as there is no incentive for insurers of type $B$ to do so. The share of $H$-types choosing an insurer of type $A$ is therefore given by

$$P_k^A = \frac{e^{\frac{V_A - f(b^A)}{\sigma}}}{n^A e^{\frac{V_A - f(b^A)}{\sigma}} + n B e^{\frac{V_B}{\sigma}}}.$$  (38)

The larger $b^A$, the less attractive for the high risks is the contract offered by insurers of type $A$; this, c.p., reduces $P_k^A$ and therefore allows these insurers to offer a higher benefit level. Figure 2 (b) shows by how much $m^A$ can be increased; there, $A^L_5$ denotes a contract offered by insurers of type $A$ as perceived by the $L$-types, while $A^H_5$ denotes the same contract as perceived by the $H$-types. Compared to $A^L_5$, $A^H_5$ is shifted upwards by $f(b^A)$, the utility decrease of negative DRS (measured in monetary terms). The larger $b^A$, the larger $f(b^A)$, and therefore the larger $m^A$ can be without increasing the share of $H$-types choosing an insurer of type $A$.

The effect just described occurs regardless of whether cost for DRS is individual-specific or not. Therefore, with negative DRS against the $H$-types, there exists one case where DRS influences the distortion caused by IRS even if cost is non-individual-specific.

**Result 7.** In the separating equilibrium, negative DRS against the high risks reduces the distortion of the benefit level of the low risks regardless of whether cost for negative DRS is individual-specific or not.
7 Example

In this section, we briefly illustrate our results with an example, for which we assume \( v(m) = \ln(m) \), \( p^L = 0.2 \) and \( p^H = 1 \), so that one of the risk types is chronically ill and \( m^{FB} = 1 \). For the two functions capturing DRS, we assume \( g(a) = \alpha \sqrt{a} \) and \( f(b) = \beta \sqrt{b} \). Increasing \( \alpha \) (or \( \beta \)) makes DRS more effective, as \( g'(a) \) increases in \( \alpha \) (and \( f'(b) \) increases in \( \beta \)). A higher level of \( \alpha \) (or \( \beta \)) therefore increases the level of DRS chosen in equilibrium.

7.1 The pooling equilibrium

For the pooling equilibrium, we assume \( \sigma = 0.2 \) and \( \lambda = 0.9 \). Figure 3 shows the distortion of the benefit level for different levels of \( \alpha \) and \( \beta \) if DRS is targeted at the risk type. For positive DRS, \( m(\alpha) \) increases in \( \alpha \), as expenditures for DRS of the low risks reduce the difference in type-specific profits. The opposite holds for negative DRS, where \( m(\beta) \) decreases in \( \beta \).

![Figure 3: m(\alpha) for positive and m(\beta) for negative DRS in the pooling equilibrium](image)

7.2 Optimal risk adjustment

For the example regarding optimal risk adjustment we keep \( \sigma = 0.2 \), but now consider the symmetric case of \( \lambda = 0.5 \), \( \eta = 0.5 \) and \( \delta = 0.125 \). Without DRS, the efficient benefit level is implemented by \( RA_O = 1.6 \); this is just twice the cost difference between the two risk types, since \( \lambda(1 - \lambda) \frac{1}{2} = 2 \) (recall condition (32)). Figure 4(a) shows the equilibrium level of \( m \) for positive and negative DRS targeted at signal age (for \( RA_O = 1.6 \)): With positive DRS, \( m(\alpha) < m^{FB} \), since part of the overpayment is spent on DRS; with negative DRS, \( m(\beta) > m^{FB} \). Figure 4(b) shows the optimal risk adjustment payment \( RA_O \) necessary to achieve the efficient benefit level: With positive DRS, the overpayment has to be increased, i.e., \( RA_O(\alpha) > 1.6 \); with negative DRS, \( RA_O(\beta) < 1.6 \) suffices to achieve \( m^{FB} \).

---

21These are the only functions which rationalize the use of least squares regressions to determine the risk adjustment payments as is done in basically all risk adjustment schemes; see Lorenz (2014).

22The equilibria cannot be determined analytically, but have been calculated using numerical optimization; the Mathematica-script is available from the author upon request.
7.3 The separating equilibrium

For this last example we still assume $\lambda = 0.5$, but now choose a much smaller level of $\sigma$, so that a separating equilibrium emerges; in Figure 5 we present the results for $\sigma = 0.03$. Figure 5(a) shows the increase of $m^A$, the benefit level of the contract designated for the low risks, if there is positive DRS. Figure 5(b) captures the case of negative DRS against the high risks, where again $m^A$ increases in the level of DRS; if negative DRS is very effective (very large $\beta$), $m^A$ is close to the efficient level $m^{FB} = 1$: If it is very easy to repel the high risks with direct risk selection, there is no need to perform indirect risk selection by distorting the benefit level.

Figure 5: Distortion of the benefit level designated for the low risks in the separating equilibrium with DRS

8 Conclusion

In this paper, the interaction of direct and indirect risk selection has been analyzed. It has been shown that direct risk selection, using measures unrelated to the benefit package, nevertheless has an influence on the distortions of the benefit package caused by indirect risk selection. Two mechanisms have been identified for being responsible for this influence:
First, if cost for DRS is individual-specific, DRS selectively reduces the profit per individual of the risk type DRS is targeted at. Positive DRS therefore reduces and negative DRS increases the difference in profits between the low and the high risks. Since the degree of the distortion depends on the difference between these two profits, positive DRS reduces the distortion of the benefit level, while negative DRS increases it. In addition, it has been shown that the effect on type- or signal-specific profits also has an impact on the optimal risk adjustment formula: With positive DRS, overpayments have to be larger than without DRS, and smaller, if there is negative DRS.

The second mechanism is important for the separating equilibrium, where only insurers offering the contract for the low risks engage in negative DRS. This reduces the utility high risk individuals receive if they choose a contract designated for the low risks, which allows insurers offering such a contract to increase the benefit level without attracting a too large share of the high risks. Negative DRS therefore reduces the distortion in the separating equilibrium, regardless of whether cost is individual-specific or not.

We have derived these results for a setting where negative DRS occurs during the application process, but we think that the main mechanisms also hold if negative DRS is targeted at the high risks who already hold a contract with the insurer. If an insurer makes its benefit package more attractive for the high risks, a larger share of them will choose this insurer; this will increase the cost of negative DRS, even if the activity of risk selection and the cost associated with it occur only later (when the insurer tries to induce the high risks to switch to another insurer). With negative DRS, high risks are more expensive than without DRS; anticipating these additional cost, insurers will therefore not make their contract as attractive for this risk group as they would without DRS. Therefore, negative DRS will increase the distortion in such a setting as well.
A Appendix

A.1 FOCs for insurers’ objective

The FOC of (16) with respect to $a^k$ is

$$
\frac{\partial \pi^k}{\partial a^k} = \lambda \left[ \frac{P_L^k(1 - P_L^k)}{\sigma}g'(a^k) \left( \pi_L^k - a^k \right) - P_L^k \right] = 0. \tag{39}
$$

The FOC of (20) with respect to $b^k$ is

$$
\frac{\partial \pi^k}{\partial b^k} = (1 - \lambda) \left[ \frac{P_H^k(1 - P_H^k)}{\sigma}(-f'(b^k)) \left( \pi_H^k - b^k \right) - P_H^k \right] = 0. \tag{40}
$$

A.2 Insurers’ objective with risk adjustment and negative DRS

Insurer $k$’s objective if there is risk adjustment and negative DRS is given by

$$
\pi^k = \mu_{LY} P_{LY}^k \left( \pi_{LY}^k - RA - b^k \right) + \mu_{HO} \left( \pi_{HO}^k - RA + RO \right) \tag{41}
+ \mu_{LO} P_{LO}^k \left( \pi_{LO}^k - RA + RO \right) + \mu_{HY} P_{HY}^k \left( \pi_{HY}^k - RA - b^k \right),
$$

where $P_{rY}^k$ contains $f(b^k)$ and $f(b^j)$. 

22
References


