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# Matthias Fahn:

# Minimum Wages and Relational Contracts

Munich Discussion Paper No. 2014-11

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## Minimum Wages and Relational Contracts<sup>\*</sup>

Matthias Fahn<sup>†</sup>

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#### Abstract

The need to give incentives is usually absent in the literature on minimum wages. However, especially in the service sector it is important *how well* a job is done, and employees must be incentivized to perform accordingly. Furthermore, many aspects regarding service quality cannot be verified, which implies that relational contracts have to be used to provide incentives. The present article shows that in this case, a minimum wage increases implemented effort, i.e., realized service quality, as well as the efficiency of an employment relationship. Hence, this paper can explain why productivity and service quality went up after the introduction of the British National Minimum Wage, and that this might actually have caused a more efficient labor market. Furthermore, several empirically observed implications of a (higher) minimum wage can be explained. It might reduce turnover of employees, have spillover effects on higher wages, and reduce wage dispersion.

Keywords: Minimum Wages, Relational Contracts.

JEL Classification: C73, D21, J24, J31.

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### 1 Introduction

Minimum wage laws and its positive or negative effects are one of the most controversially debated issues in economics. When trying to understand its consequences, however, only limited attention has been paid to how a minimum wage affects the provision of incentives.

In this article, I show that a minimum wage has a crucial impact on a firm's optimal choice of incentives and consequently on social welfare. If firms are forced to pay a higher wage than actually intended, they will require their employees to do a better job in return. In an environment where performance is not verifiable and labor turnover is high, an appropriate minimum wage can furthermore be efficiency-enhancing and increase the surplus of an employment relationship.

Minimum wages are especially relevant in the service sector. There in particular what matters is how well - and not only that - a job is done. The degree of service quality provided by employees is important for customer satisfaction and will have an impact on a firm's profits. Take employees of a fast food restaurant, who are supposed to be friendly to customers and careful when preparing the food. A cleaner can do a superficial job or clean everything thoroughly, and a nightwatchman might be more or less attentive. Then, it is necessary to incentivize workers, and the question arises how this should and can be done. As many aspects of service quality are highly subjective and cannot be precisely measured, it will generally be difficult to capture all relevant dimensions in an explicit, i.e., court enforceable, contract. Hence, relational contracts are needed to give incentives, and unsatisfactory performance is not detected and punished by a court. More precisely, relational contracts are used in settings where employees need incentives to perform a desired task but where it is impossible or at least very difficult to verify effort and output. This implies that all contingent compensation must be enforceable within an equilibrium of the dynamic game, and cooperation can only be sustained if the discounted future value of the employment relationship is sufficiently high.

The present article analyzes the impact of a minimum wage on the optimal choice of incentives within a relational contracting framework. A labor market with many homogeneous firms (also denoted as principals) and employees (or agents) exists, with excess supply of labor. Agents work if they believe they will be compensated, whereas principals only reward agents for their effort if reneging triggers sufficient punishment. However, agents can be costlessly replaced, and the market is not fully transparent in a sense that if turnover occurs, it is not possible to detect the reason, i.e., whether an agent is fired or leaves voluntarily. Thus, a firm cannot have an external - or market - reputation for honoring its promises and faces a commitment problem. Instead of making promised payments as a reward for previous effort, firms might have an incentive to renounce and replace employees. Therefore, agents know that they are not compensated for their effort unless replacement is sufficiently costly. This implies that an equilibrium with positive effort can only be enforced if (endogenous) turnover costs are present. A natural way to induce these costs (and preventing surplus destruction) is a social equilibrium where all new agents receive a rent which is at least as high as the payment promised to agents as a compensation for effort. However, firms are also exposed to these turnover costs whenever their employees leave for exogenous reasons. Although they have all bargaining power, firms are thus not able to capture the whole surplus of an employment relationship. Instead, they face a tradeoff between giving high incentives (induced by high wages) and reducing turnover costs (which also increase with equilibrium wages): Even if maximum incentives are enforceable, employers induce an effort level that is inefficiently low.

In this case, a minimum wage can reduce (and even completely offset) this inefficency. Assume the minimum wage binds and hence is higher than the payment needed to implement the (initial) profit-maximizing effort level. Then, additional rents are going to all employed agents. However, the principal is able to retrieve parts of these rents by demanding higher effort - which is possible because the minimum wage relaxes an agent's incentive compatibility constraint. Since effort is inefficiently low if firms are free in setting wages, the minimum wage increases the efficiency of an employment relationship.

There is evidence that a minimum wage increases productivity, and that this is driven by higher effort levels of employees. Galinda-Rueda and Pereira (2004) and Rizov and Croucher (2011) analyze how the introduction of a National Minimum Wage in Britain in 1999 affected labor productivity. Both find a positive and significant effect - in particular in the service sector. In addition, several surveys attempt to provide a better understanding of the specific channels that induced the observed productivity increases. Indeed, these surveys find that a substantial amount of firms responded to the minimum wage by inducing higher effort of workers or by providing higher service quality (Low Pay Commission, 2001, or Heyes and Gray, 2003).

In a next step, I test the robustness of the main result - that a minimum wage increases effort and efficiency - in alternative settings. In one extension, I relax the assumption that a principal can fully observe an employed agent's effort. Instead, she can just use the resulting output as an imperfect signal. If a minimum wage is sufficiently high, it will still cause higher effort and efficiency levels. Furthermore, asymmetric information can make it optimal to use termination in equilibrium to provide incentives. The reason is that if only contingent payments are used, compensation after a good outcome has to be higher than after a low outcome. As the lower wage must not undercut a minimum wage, compensation after observing the good outcome has to be adjusted accordingly to maintain incentives. Then, replacing an agent after a low outcome instead of paying him the minimum wage can become an alternative. This increases turnover costs (which are still required to keep the principal from reneging) but induces stronger incentives. Hence, turnover levels are generally higher when a minimum wage is present. However, a marginal increase of the minimum wage will at some point induce less turnover, which is driven by the positive impact of the wage floor on effort: When agents work harder, the likelihood of the output being low - and correspondingly the probability of a termination - goes down.

These results are supported by empirical evidence. Industries facing a minimum wage are usually characterized by high turnover levels. I show that this does not have to be an exogenuously given property but can also be driven by a firm's consideration to give incentives optimally. Furthermore, the negative marginal impact of a higher minimum wage on turnover has been documented by Portugal and Cardoso (2006), Dube et al. (2007) and Dube et al. (2011).

#### **Related Literature**

An important and considerable amount of research deals with employment effects of minimum wages. The hypothesis derived from the standard textbook model of a labor market - that a binding minimum wage leads to job losses - is now seriously questioned. Empirical studies like Card and Krueger (1994), Katz and Krueger (1992), Machin and Manning (1994) and most recently Dube et al. (2010) suggest that the employment effect of a minimum wage is not necessarily negative and might even be slightly positive. Other articles (for overviews see Brown, 1982, or Neumark and Wascher, 2007) still claim that a minimum wage destroys jobs.

Several theoretical models have been developed to explain the observed patterns. Bhashkar and To (1999) develop a model of monopsonistic competition where a minimum wage raises employment per firm but causes firms to exit the market, whereas other models focus on the importance of match specific human capital (Miller, 1984, or Flinn, 1986). Generally rentcreating search frictions are used as an explanation for the seemingly counterintuitive outcome that a minimum wage does not necessarily destroy jobs (based on Burdett and Mortensen, 1998, see also Card and Krueger, 1995, Flinn, 2006, or Dube et al., 2011).

However, these articles abstract from incentives, which have been given almost no attention in the relevant literature. Exceptions are Kadan and Swinkels (2009, 2010) and Rebitzer and Taylor (1995). Kadan and Swinkels (2009, 2010) analyze the effect of a wage floor in a standard moral hazard setting. They show that a minimum wage generally has a negative impact on induced effort levels. Different from my setting, they assume that agents are risk averse, effort cannot be observed, and an explicit contract is feasible. Then, a higher wage floor (i.e. payments that have to be made for the lowest output realization) generally increases the marginal costs of inducing effort, reducing total incentives given to employees. However, the non-verfiability of certain activities will often render explicit contracts infeasible, especially in the service sector where minimum wage laws are particularly important. Rebitzer and Taylor (1995) develop an efficiency wage model where a minimum wage makes it easier for firms to prevent a given number of employees from shirking. Thereby, the authors can explain positive employment effects of a minimum wage, however do not take the impact on a worker's productivity into account.

While I focus on the impact of a minimum wage on the quality of the work provided, quantity aspects have been analyzed as well. Strobl and Walsh (2011), for example, use a competitive model of the labor market to show that a minimum wage can either increase or decrease the hours worked by an employee. I abstract from the amount of hours worked which are verifiable - and instead focus on the usually non-verifiable aspect of service quality.

Finally, this article relates to the literature on relational contracts. MacLeod and Malcomson (1989) and Levin (2003) are probably the most prominent contributions to relational contracting in a setting with just one principal and one agent, and show that optimal contracts can take a rather simple form. Within a market setting, i.e. when agents are replaceable, MacLeod and Malcomson (1998) derive the necessity of endogenous turnover costs as a feature of productive employment relationships, Yang (2011) shows that exogenous turnover costs can under some conditions increase social welfare, while Board and Meyer-ter-Vehn (2013) characterize a market equilibrium if on-the-job-search is possible. In addition, Yang (2013) shows that it is optimal to backload wages in a relational contracting setting where agents can be replaced and can either be of a productive or an unproductive type.

## 2 Model Setup

The economy consists of a mass 1 of small, identical firms ("principal", "she") and a mass of N > 1 identical employees ("agent", "he"). Principals and agents are risk neutral. The time horizon is infinite, time is discrete (periods are denoted t = 1, 2, ...), and all players share a common discount factor  $\delta$ . All players are either part of a match or not. At the beginning of each period t, every unmatched principal can offer a contract to exactly one unmatched agent. This offer consists of a legally enforceable wage payment  $w_t$  and the promise to pay a discretionary bonus  $b_t \geq 0$ . A principal who does not make an offer or gets rejected consumes his outside utility  $\overline{\pi}$  in the respective period, where I make the normalization  $\overline{\pi} = 0$ .

If an agent receives no offer from a principal or rejects an offer, he consumes his exogenuous outside utility, which is set to zero (note that an agent's endogenous outside utility - which reflects the possibility of finding a job with a positive rent and is introduced below - can actually be positive). All employed agents then consume  $w_t$  and choose effort  $e_t \in [0, 1]$ . This generates output  $y_t = \theta$  with probabability  $e_t$ , and output  $y_t = 0$  with probability  $(1 - e_t)$ . While output is directly consumed by the principal, an agent faces effort costs  $c(e_t)$ , with c', c'' > 0, and  $\lim_{e \to 1} c(e) = \infty$ . Then, the principal has the choice to pay  $b_t$ , followed by an exogenous shock which makes some agents leave the market (for example because the partner found a job somewhere else). With probability  $(1 - \gamma)$ , each agent - no matter whether employed or not - leaves the market and remains for another period with probability  $\gamma$ . As low-wage industries tend to have high turnover rates (Brown et al, 1982), exogenous turnover is a prominent phenomenon in the present setting.<sup>1</sup>

If an agent exits the market for exogenous reasons, he leaves for good and receives a payoff normalized to zero from then on. Note that this assumption is without loss of generality even when an agent expects a positive utility while being on the market. Furthermore, the number of employees remains fixed over time, hence  $(1 - \gamma)N$  new agents enter the market in every period. At the end of period t, an employed agent who has not left the market for exogenous reasons can get a contract offer by his current employer, consisting of the wage  $w_{t+1}$  and bonus  $b_{t+1}$ . If the agent accepts it, the match continues for another period. In any other case, i.e. if the principal does not make an offer or the agent does not accept, both enter the matching market in the next period.

The timing within a period t is summarized in the following graph:

$\begin{array}{c} \mathbf{Employed} \ \mathbf{A} \\ \mathbf{gets} \ w_t \end{array}$		P can pay bonus		
Matching of unmatched P with unmatched A	$\begin{array}{c} \mathbf{Employed} \ \mathbf{A} \\ \mathbf{supplies} \ e_t \end{array}$	v	A leaves with prob. $\gamma$	$\gamma {f N}~{f new}~{f agents}~{f enter}$

Using  $d_t^P \in \{0, 1\}$  to describe whether a principal is in a relationship in period t and, the payoff stream of an arbitrary principal at the beginning of a period t equals

$$\Pi_t = \mathbf{E}\left[\sum_{\tau=t}^{\infty} \delta^{\tau-t} d_{\tau}^P \left(e_{\tau}\theta - w_{\tau} - b_{\tau}\right)\right].$$

Using  $d_t^A \in \{0, 1\}$  to describe whether an agent is in a relationship (by construction, this implies that  $d_t^A = 0$  once an agent left the market for exogenous reasons), an arbitrary agent receives

$$U_t = \mathbb{E}\left[\sum_{\tau=t}^{\infty} \delta^{\tau-t} d_{\tau}^A \left(w_{\tau} + b_{\tau} - c(e_{\tau})\right)\right].$$

#### Information and Equilibrium Concept

I assume the following information structure. Within a match, information is symmetric. This implies that effort and output can be observed by both players. However, neither effort nor output are verifiable, i.e., no explicit contract based on them can be written.

All players outside a match ("the market") cannot observe anything that happens within a

<sup>&</sup>lt;sup>1</sup>Thereby, I deviate from MacLeod and Malcomson (1998), who assume that firms may leave or enter the market and the number of firms is determined by a zero-profit condition. In my setting firms can make profits, for example because they have local monopoly power on product markets.

match, especially not an employed agent's effort choice.<sup>2</sup> Hence, the market is not transparent and a deviation by one player is not detected by parties outside the respective relationship. In reality, information is generally not transmitted from past to current employees, and the experience made within a firm cannot be shared. However, even if agents were able to communicate, the content would not necessarily be credible - a worker who misbehaved and consequently got fired would usually not report his own mistakes but rather blame the firm, especially if prospective new employers had access to this information as well.<sup>3</sup>

This implies that although information within a match is symmetric, this paper deals with an infinitely repeated game with imperfect public monitoring. Any employee has no information concerning a principal's actions in past relationships but can use the fact that he received an offer (i.e., that an agent either left the firm or got fired) as an imperfect signal. Therefore, I follow the literature on imperfect public monitoring and use the solution concept of a pure-strategy public perfect equilibrium (PPE) to deal with an individual employment relationship: Each player's actions only depend on the public history they share with the respective partner. Put differently, the offer a principal makes to a new agent is independent of whether the previously employed worker has left for exogenous reasons or not; the same is true for the agent's decision concerning acceptance or rejection of a received offer, and concerning first-period effort. When a match has been active for a while, players' actions in addition depend on the events that were observed within the respective relationship.<sup>4</sup>

In addition, a player's strategy will depend on the strategies of all market participants, even though the market is not transparent. Hence, the market as a whole will be in a social equilibrium, as for example described by Ghosh and Ray (1996), Kranton (1996), or MacLeod and Malcomson (1998). This implies that a social norm can determine the exact outcome, as long as it is also an equilibrium of the dynamic game.

Taken together, each firm decides about its wage offer and whether to continue an employment relationship in a way that maximizes  $\Pi_t$  in each period t, given publicly observable past events, as well as strategies of agents and all other firms on and off equilibrium. Each agent decides about whether to accept a wage offer and subsequently chooses his effort in a way that maximizes  $U_t$  in each period, given past events observed by herself and the principal (especially past wage offers), and given on- and off-equilibrium strategies of firms and all other agents. Finally, I focus on symmetric equilibria, and assume that principals have no

 $<sup>^{2}</sup>$ Whether or not the only verifiable component - the wage payment - can be observed by the market is not relevant.

<sup>&</sup>lt;sup>3</sup>MacLeod and Malcomson (1998, p. 393) state that "... the use of reputation depends on there being reliable sources of information about why a separation occurred - the word of mouth of parties to the match is unlikely to be reliable since, if reputation is valuable, neither has an incentive to admit to cheating. Many employers are unwilling to provide information about former employees... Only in markets where the reasons for seeking a new match are well known are reputation effects really effective."

<sup>&</sup>lt;sup>4</sup>Below, I also analyze the case of asymmetric information where a principal cannot observe the agent's effort choice; then, the equilibrium concept remains the same in the sense that within a relationship, actions only depend on what has been observed by both players.

preferences concerning the identity of agents.

To simplify the following analysis, I further restrict strategies to be *contract-specific* in the sense of Board and Meyer-ter-Vehn (2013). This implies that actions of the firm and workers do not depend on the identity of the worker, calendar time, or history outside the current relationship. This assumption is without loss of generality for all periods except the first period of the whole game. In section 5 below, I relax this assumption and show that my results are not affected qualitatively when strategies are not restricted to be contract-specific.

## 3 Profit-Maximizing Equilibria Without a Minimum Wage

In this section, I solve for a social equilibrium that maximizes a principal's payoff - taking the behavior of all other principals as given (in section 5 below, I take a brief look at the equilibrium that maximizes joint profits). The selection of this particular equilibrium can be justified by the firms having all bargaining power. Hence, a firm with a vacant employment opportunity can make a take-it-or-leave-it offer to an arbitrary agent. Furthermore, Manning (2003) presents evidence that employers actually set wages in markets where a minimum wage is relevant. He states that "for the average worker in a non-union setting, this does seem to be the appropriate assumption" (p. 4).

#### **3.1** Payoffs and Constraints

The focus on contract-specific strategies makes calendar time irrelevant, which allows to omit the *t*-subscript. However, the timing *within* a specific relationship might matter, for example whether it is in an early or a later stage. Hence, I use the subscript  $\tau = 1, 2, ...$  to denote the period a given relationship is in.

Given that agents do not get fired and always accept employment offers in equilibrium, an employed agent's discounted payoff stream in the  $\tau$ 's employment period is denoted  $U_{\tau}$  and equals

$$U_{\tau} = w_{\tau} - c(e_{\tau}) + b_{\tau} + \delta \gamma U_{\tau+1}, \tag{1}$$

where  $e_{\tau}^*$  is equilibrium effort in empoyment period  $\tau$ . Future payoffs  $\delta U_{\tau+1}$  only enter with probability  $\gamma$ , since the agent might leave the market for exogenous reasons (with probability  $(1 - \gamma)$ ), then receiving a payoff of zero.

An agent currently unemployed but on the market at the beginning of a period receives a job offer with probability  $\mu \equiv \frac{(1-\gamma)}{N-\gamma}$ . Thus, an agent's *endogenous* reservation utility  $\overline{U}$  (in contrast to an agent's *exogenous* outside utility which is set to zero) equals

$$\overline{U} = \mu U_1 + \delta (1 - \mu) \gamma \overline{U}$$

As strategies are independent of calendar time and because of market intransparency,  $\overline{U}$  is constant over time. Furthermore, the actions of a single player have no impact on the value of  $\overline{U}$ , and it is taken as given by all players. This also implies that collusion among principals is precluded, an issue briefly discussed in section 5.

In equilibrium, several constraints have to be satisfied for any agent who is part of a match. First of all, an agent must prefer to be employed rather than not. This implies that the utilities  $U_{\tau}$  must exceed the endogenous reservation utility  $\overline{U}$ , which is captured by an agent's individual rationality (IRA) constraints,

$$U_{\tau} \ge \overline{U} \quad \forall \tau \quad .$$
 (IRA)

Furthermore, given  $U_{\tau}$  and  $\overline{U}$ , it must be in the interest of an employed agent to actually choose equilibrium effort  $e_{\tau}^*$  in period  $\tau$ , i.e. his incentive compatibility (IC) constraint must be satisfied. Here, I assume that an agent who does not exert  $e_{\tau}^*$  is fired,<sup>5</sup> - recall that effort is observable and cheating will therefore never happen in equilibrium - and re-enters the job market in the subsequent period. If an agent deviates and chooses an effort level different from equilibrium effort, he will obviously set e = 0 (or put differently: if satisfied for  $e_{\tau} = 0$ , (IC) also holds for any other effort level). Thus, (IC) equals

$$c(e_{\tau}^*) \le b_{\tau} + \delta \gamma \left( U_{\tau+1} - \overline{U} \right). \tag{IC}$$

Hence, an agent can be compensated for his effort either by a bonus paid at the end of period  $\tau$  or by future payoff streams, in particular involving fixed wage payments in future periods.

A principal's payoff in the  $\tau$ 's period of a relationship is denoted  $\Pi_{\tau}$  and equals

$$\Pi_{\tau} = e_{\tau}^* \theta - w_{\tau} - b_{\tau} + \delta \left[ \gamma \Pi_{\tau+1} + (1-\gamma) \Pi_1 \right] \quad \forall \tau \ .$$

As the current relationship is only continued with probability  $1 - \gamma$ , the payoff of starting a new relationship,  $\Pi_1$ , enters future payoffs with probability  $\gamma$ . Each principal faces the following constraints. First of all, starting a new or maintaining an ongoing employment relationship should be better than completely shutting down. This gives a principal's individual rationality (IRP) constraint

$$\Pi_{\tau} \ge 0 \quad \forall \tau. \tag{IRP}$$

Furthermore, each principal must have an incentive to honor his promises and compensate an employed agent accordingly. Because compensation can either consist of a bonus paid at

 $<sup>^5{\</sup>rm Following}$  Abreu (1998), the most severe punishment after cheating can be used to characterize any equilibrium.

the end of a period or future wage payments, two sets of constraints are necessary. Generally, if a principal reneges and refuses to either pay the promised bonus or offer a new contract with an appropriate wage, I make the standard assumption that all trust is lost in the specific relationship, the employed agent does not believe the firm's promises anymore and is not willing to exert positive effort from then on. However, agents can be replaced, and information regarding the principal's past behavior cannot be transmitted to potential future employees. This prevents the use of multilateral relational contracts (Levin 2002), i.e., a principal cannot be punished for misbehavior in the past by agents not involved back then. After a deviation, a principal hence will fire the agent and start a new relationship (shutting down will not be optimal in an equilibrium where a principal chooses to run the business in the first place because of (IRP)).

A dynamic enforcement (DE) constraint makes sure that paying the bonus at the end of period  $\tau$  is optimal for the principal - compared to reneging and starting a new empoyment relationship in period  $\tau + 1$ . Hence,

$$-b_{\tau} + \delta \left[ \gamma \Pi_{\tau+1} + (1-\gamma) \Pi_1 \right] \ge \delta \Pi_1, \tag{DE}$$

where I have to take the possibility into account that the principal might also have to start a new employment relationship after paying the bonus - namely when the agent leaves the market for exogenous reasons. Note that although there is excess supply of labor and firms can fill vacancies immediately, bonuses can be self-enforced in this market - profits from starting a new relationship,  $\Pi_1$ , just have to be sufficiently small.

At the end of the period, agents who are employed and did not leave the market are supposed to receive an offer by their current employer for the subsequent period. The nonreneging (NR) constraint guarantees that it is optimal for a principal to actually make this offer, in contrast to fire the current employee, enter the matching market in the subsequent period and start a new employment relationship.

Hence, (NR) constraints are

$$\Pi_{\tau} \ge \Pi_1 \quad \forall \tau. \tag{NR}$$

Given the constraints just derived, my objective is to find the social equilibrium that maximizes a principal's profit,  $\Pi_1$ . However, the problem can be substantially simplified. First of all, it is possible to only use wage payments to compensate agents for their effort, and set  $b_{\tau} = 0$  in all periods.

**Lemma 1**: Without loss of generality, it is possible to set  $b_{\tau} = 0$  in all periods  $\tau \ge 1$ .

*Proof*: Assume there is a profit-maximizing equilibrium with  $\Pi_1 \ge 0$  (then, (NR) implies

 $\Pi_{\tau} \geq 0 \,\forall \tau$  as well) that contains a period  $\tau'$  with  $b_{\tau'} > 0$ . Now, decrease  $b_{\tau'}$  by an  $\varepsilon$  sufficiently small for  $b_{\tau'}$  to remain positive, and increase  $w_{\tau'+1}$  by  $\frac{\varepsilon}{\delta\gamma}$ . This has no impact on  $\Pi_1$  (since the changes in  $b_{\tau'}$  and  $w_{\tau'+1}$  cancel out), but increases  $U_{\tau'+1}$  by  $\frac{\varepsilon}{\delta\gamma}$  and reduces  $\Pi_{\tau'+1}$  by  $\frac{\varepsilon}{\delta\gamma}$  (payoffs in other periods stay unchanged as well).

Thus, (IC) and (DE) are unaffected, whereas (IRA) in period  $\tau' + 1$  is relaxed. Concerning (NR) in period  $\tau' + 1$ , note that (DE) for period  $\tau'$  is  $b_{\tau'} \leq \delta \gamma (\Pi_{\tau'+1} - \Pi_1)$ . Hence, a positive  $b_{\tau'}$  implies  $\Pi_{\tau'+1} > \Pi_1$ , and (NR) remains satisfied. Q.E.D..

Bonus payments and fixed wages are substitutes, and replacing one by the other has no impact on the principal's profits. Note that the same would be true if there was an additional possibility for principals to reward agents, namely a bonus paid immediately after realization of the shock that some agents leave the market. There are two reason why using wage payments instead of bonuses is more convenient. On the one hand, a minimum wage affects total wage payments within a period. On the other hand, the (NR) constraint might make it necessary to pay a fixed wage in the first period of a relationship, even if only bonus payments are used later.

Proposition 1 has several consequences. Given (NR) constraints, (DE) constraints are not needed anymore, and an (IRP) constraint is only needed for period 1. Furthermore, (IRA) can be omitted for all periods  $\tau \ge 2$  because the fact that no bonus payments are used implies - in combination with (IC) constraints - that  $U_{\tau} - \overline{U} > 0$  in all periods  $\tau \ge 2$ .

In a next step, I show that stationary contracts are optimal in all periods  $\tau \ge 2$ , i.e. that effort and wages are constant over time.

**Lemma 2**: Equilibrium effort in all periods  $\tau$  satisfies  $e_{\tau}^* \leq e^{FB}$ , where  $e^{FB}$  is the surplusmaximizing, first-best effort level  $e^{FB}$  and characterized by  $\theta = c'$ .

Furthermore, a profit-maximizing social equilibrium is stationary in a sense that it is optimal to have  $w_{\tau} = w_{\tau'}$  and  $e_{\tau} = e_{\tau'}$  in all periods  $\tau, \tau' \geq 2$ .

The proof to Lemma 2 can be found in Appendix A.

The intuition for this result is straightforward. If effort in one period is higher than in another, it will generally be possible to increase the lower effort at no cost for the principal besides compensating the agent for his additional effort. This would increase the principal's profits as long as effort does not exceed its efficient level.

Concerning wages, it does not matter for an agent whether he is compensated for his effort immediately (i.e., by the wage paid in the subsequent period) or by a (credible) promise of future payments. Each non-stationary contract can thus be replaced by a stationary one, namely by averaging out changes in promised continuation payoffs. We only have to consider the first period of a new relationship independently, as it includes turnover costs that keep firms from reneging.

**Proposition 1**: Turnover must be costly for the firm. This implies that either  $w_1 > 0$  or  $e_1 < e^*$ , where  $e^*$  is equilibrium effort in all periods  $\tau \ge 2$ .

*Proof*: Using previously derived results that effort and wages are constant in all periods  $\tau \geq 2$ , and denoting equilibrium wages  $w^*$ , the (NR) constraint in all these periods becomes

$$(e^*\theta - w^*) \ge e_1\theta - w_1. \tag{NR}$$

Q.E.D.

Turnover costs are necessary due to the intransparency of the labor market, rendering it impossible for firms to establish a market reputation for keeping their promises. Without turnover costs, firms would have no incentives to keep their promises, and equilibria with positive effort would not exist. To see this point, assume that principals did not face these costs and were able to completely extract the surplus generated within an employment relationship. Then, the wage in the first period of an employment relationship,  $w_1$ , would be zero (and  $e_1 = e^*$ ), whereas wages paid in later periods had to be strictly positive to compensate agents for past effort. Hence, firms would always renounce, fire an agent at the end of the period instead of offering a subsequent contract, and employ a new agent. This turnover can either manifest in a rent going to new agents -  $w_1 > 0$  - or a destruction of surplus ( $e_1 < e^*$ ). Of course, any combination satisfying the (NR) constraint ( $e^*\theta - w^*$ )  $\geq e_1\theta - w_1$  also works, as well as other (observable) means of money burning conducted by the principal (see MacLeod, 2003, for a discussion of several options to conduct money burning).

An interesting aspect of this result is that a productive *current* relationship can only be sustained if starting any relationship in the *future* is costly for the respective principal although neither principals nor agents are able to observe anything that happens outside the matches they are part of. Hence, the social equilibrium requires some norm that determines how any relationship starts. If this norm did not exist, the static Nash equilibrium with no effort would be the unique outcome of the game.

There, Lemma 1 implies that either surplus must be destroyed by a voluntary reduction of first-period effort, or a rent must be given to newly employed agents. Both approaches have been used in comparable models.<sup>6</sup> I follow MacLeod and Malcomson (1998) and only use the

 $<sup>^{6}</sup>$ Kranton (1996), Ghosh and Ray (1996), for example analyze a double-sided moral hazard setting with many players who differ in their types - i.e., whether they are generally willing/able to cooperate or not.

first-period wage  $w_1$  to induce appropriate turnover costs (hence,  $e_1 = e^*$ ). The reasons are twofold. On the one hand, this equilibrium does not destroy any surplus but only reflects a redistribution from principals to agents. On the other hand, the introduction of a minimum wage - which is analyzed below - forces the principals to pay a certain wage level already in the first period of an employment relationship. However, any other form of turnover costs as described in the previous paragraph would leave the main result of this paper (see Proposition 3 below) unaffected.

Now, the (NR) constraint equals  $w_1 \ge w$ , and newly employed agents receive an upfront payment which has to be at least as high as the wage workers with longer tenure get as a compensation for past effort. These costs cannot be avoided in equilibrium. Instead, principals will be exposed to them all the time an agent leaves for exogenous reasons and consequently has to be replaced. As turnover costs must increase with equilibrium compensation, firms face a trade-off between giving optimal incentives and reducing turnover costs. This induces them to voluntarily reduce the feasible effort level and thus the relationship surplus:

**Proposition 2:** Given  $\Pi_1 > 0$ , profit-maximizing per period effort level  $e^*$  is characterized by

$$c' = \delta \gamma \theta. \tag{2}$$

Proof: The (NR) constraint  $w_1 \ge w$  will bind. If it did not bind, the upfront payments could be slightly reduced without violating any constraint. As (IC) binds as well (see the proof to Lemma 2), we have  $w_1 = w = \frac{c(e^*)}{\delta\gamma} + (1 - \delta\gamma)\overline{U}$ . Plugging these values into profits -  $\Pi_1 = \Pi = \frac{e^*\theta - w}{1-\delta} = \frac{e^*\theta - \frac{c(e^*)}{\delta\gamma} - (1-\delta\gamma)\overline{U}}{1-\delta}$  - the problem becomes

$$\max_{e} \Pi_{1} = \frac{e\theta - \frac{c(e)}{\delta\gamma} - (1 - \delta\gamma)\overline{U}}{1 - \delta}$$
(3)

s.t.

Social equilibria there contain a probation phase with reduced effort in the onset of a relationship to learn about a partner's type. Similarly, Watson (1999) shows that "starting small" is optimal in the case of two-sided incomplete information. MacLeod and Malcomson (1998) in a setting more similar to the present one propose a social equilibrium with efficiency wages where newly employed agents receive a rent. There, the fixed wage paid in every period has the status of a fair wage in this market - if it is not offered at the onset of an employment relationship, the respective employer is not regarded trustworthy.

$$\frac{c(e^*)}{\delta\gamma} \ge 0 \tag{IRA}$$

$$e^*\theta - \frac{c(e^*)}{\delta\gamma} - (1 - \delta\gamma)\overline{U} \ge 0.$$
 (IRP)

(IRA) will always be satisfied and hence can be omitted. (IRP) is satisfied by assumption to avoid non-trivial solutions without production. As  $\overline{U}$  is not affected by a single principal's actions, the profits of a firm are maximized if effort is characterized by  $\theta - \frac{c'}{\delta\gamma} = 0$ . Q.E.D.

Because efficient effort is characterized by  $c' = \theta$ , equilibrium effort is inefficiently low. This inefficiency is not induced by the future surplus being too low (which I assume it is not) but by the inability of principals to establish an external reputation. The necessary turnover costs make principals face a tradeoff between surplus maximization and the minimization of turnover costs: When giving stronger incentives and consequently increasing the surplus, a principal also raises the rent she has to give to agents. Hence, utility is not perfectly transferable, unlike in standard relational contracts setting with one principal and one agent, see e.g. Levin (2003). There, the provision of incentives can be separated from surplus distribution, and it is thus convenient to focus on equilibria that maximize total surplus. In my setting, social equilibria that maximize profits (which are my focus because firms represent the short side of the market) yield a different outcome with respect to effort and efficiency than equilibria with other underlying objective functions.<sup>7</sup>

## 4 The Impact of a Minimum Wage on Effort and Surplus

If the labor market is characterized by voluntary turnover, a lack of transparency, and nonverifiability of the quality with which employees execute their tasks, a binding minimum wage can counteract efficiency losses induced by an employer's inability to establish an external reputation:

**Proposition 3**: Assume total wage payments in a period cannot be lower than a minimum wage  $\overline{w}$ . If the minimum wage binds and profits are still positive, effort is higher than in a situation without a minimum wage and increases in  $\overline{w}$ . The total surplus created within a relationship increases in  $\overline{w}$  as long as  $\overline{w}$  is not too large. Finally, a minimum wage reduces firms' profits, whereas agents benefit.

*Proof*: When a minimum wage  $\overline{w}$  is present, the constraint  $w \geq \overline{w}$  is added to the max-

<sup>&</sup>lt;sup>7</sup>For example, efficient effort could be obtained in equilibria where agents received a higher rent.

imization problem. Obviously, a minimum wage only has an impact if it binds, i.e. if it is higher than  $\frac{c(\tilde{e})}{\delta\gamma} + (1 - \delta\gamma)\overline{U}$ , where  $\tilde{e}$  is the equilibrium effort level characterized by  $c' = \delta\gamma\theta$ , and  $\overline{U}$  is an agent's associated endogenous outside option. If  $\overline{w}$  is lower, a marginal increase does not affect effort and surplus. Hence, assume that  $\overline{w} \geq \frac{c(\tilde{e})}{\delta\gamma} + (1 - \delta\gamma)\overline{U}$ .

In this case, (IC) as well as (NR) constraints remain binding: If (IC) did not bind, firms could implement more effort at no cost. If (NR) did not bind, firms could reduce  $w_1$  without violating any constraint, thereby increasing profits. Furthermore, the concavity of the problem implies that wages are identical to the minimum wage and not higher. Hence, a binding minimum wage determines equilibrium effort, which is then characterized by  $\overline{w} = \frac{c(e^*)}{\delta\gamma} + (1 - \delta\gamma)\overline{U}$ , with  $\overline{U} = \mu U_1 + \delta(1 - \mu)\gamma\overline{U}$  and  $U_1 = \overline{w} + \delta\gamma\overline{U}$ . There, the last expressions show that agents benefit from a binding minimum wage.

Now,  $\overline{w}(1-\mu)\delta\gamma = c(e^*)$ , where  $\mu = \frac{(1-\gamma)}{N-\gamma}$ , and  $\frac{de^*}{d\overline{w}} = \frac{\delta\gamma(1-\mu)}{c'} > 0$ , establishing that effort is higher with a binding minimum wage than without, and further increasing in  $\overline{w}$ .

The per period surplus created by an employed agent equals  $s \equiv e^*\theta - c(e^*)$ . Therefore,  $\frac{ds}{dw} = \frac{de^*}{dw}(\theta - c')$  which is positive as long as  $\theta \ge c'$ .

The impact on profits for relationship starting after the introduction of a minimum wage is given by  $\frac{d\Pi_1}{dw} = \frac{1}{1-\delta} \frac{de^*}{dw} (\theta - \frac{c'}{\gamma} \frac{1}{1-\mu}) < 0.$ Q.E.D.

This result is driven by the binding (IC) constraint. If firms are forced to pay a higher wage than intended, they are able to get something back by requiring those agents who want to keep their jobs to work harder. As firms enforced inefficiently low effort before, a binding minimum wage can increase the surplus of each employment relationship. If the minimum wage is too high, though, the surplus of an employment relationship goes down. Nevertheless, implemented effort unambigously increases in  $\overline{w}$ .

Although firms induce higher effort and hence get something back, they do not receive a net benefit from a minimum wage, but additional rents are shifted to agents. This result is in line with empirical results found by Holzer et al. (1991) or Draca et. al (2011). All of my results are only valid as long as the principals' (IR) constraints do not bind, and each firm makes positive profits, for example because of some local monopoly power on product markets. If the constraint became binding, a minimum wage would lead some firms to leave the market.

There is evidence that the productivity of firms indeed goes up after the introduction of a minimum wage, and that these productivity gains are particularly significant for firms in the service industry. Galindo-Rueda and Pereira (2004) analyze how British firms responded to the introduction of a National Minimum Wage in 1999. They find a positive one-off effect on labor productivity (measured as gross output relative to employment), which in addition is only observed in the service sector and not in manufacturing.

Rizov and Croucher (2011) conduct a further study on the effect of the British National Minimum Wage. They compute a structural estimation of production functions within disaggregate 4-digit industries, controlling for supply and demand factors that affect firms. Hence, a minimum wage potentially affects firm productivity through the input price channel. They find that productivity substantially went up after the introduction - and subsequent increases - of the minimum wage, again with a substantially higher impact in service industries than in manufacturing.

However, both studies can only speculate on the factors that cause the observed productivity increases. In general, productivity might go up because of reductions in employment or working hours (which however was not observed in both studies), the adjustment of prices, or issues like training, changes in the organizational structure of firms, or - as is the point of this paper - the provision of more effort and hence a higher service quality.

Several studies attempt to fill this gap, conducting extensive surveys in which managers were asked how they responded to the introduction of the British National Minimum Wage. Manning et. al (2003) focus on workers in the residential care homes industry. They find that the effect of the minimum wage on worker effort is positive, however not significantly different from zero. The British Low Pay Commission (Low Pay Commission, 2001) - which is supposed to analyze the impact of the British National Minimum Wage and make recommendation concerning potential increases - initiated many research projects to study the exact impact of the National Minimum Wage. They find that 30 % of all firms in the surveys responded by improving the quality of provided services. In one of the involved projects, Heyes and Gray (2003) conduct a survey of small-scale enterprises in the Yorkshire and Humberside region, with a special focus on service industries (motor services, retail, care homes, hairdressing and hospitality). There, 61 % of the firms state that "Increasing workers' level of effort" was an important or very important response to the minimum wage. The point "Improving quality of products and/or services" is regarded as important or very important by 63% of the respondents.

### 5 Robustness

In this section, I relax some assumptions and show that the main results continue hold.

#### 5.1 Asymmetric Information

In many instances, it will not be possible or simply too expensive for firms to continuously monitor an employee's actions. Then, firms might not be able to observe an agent's effort but only the resulting output  $y_t$ . In that case, the impossibility for firms to create an external reputation also leads to an imposed effort level that is inefficiently small, and a sufficiently high minimum wage can increase effort and surplus.

Furthermore, under asymmetric information it can be optimal to fire the agent after a low outcome if the minimum wage is binding. Without a termination threat, incentives are solely given by two wage levels - a high wage denoted  $w^+$  after  $y = \theta$ , and a low wage  $w^-$  after y = 0. Since  $w^-$  cannot be below  $\overline{w}$ , a higher minimum wage - for a given effort level - also triggers an increase in  $w^+$ . However, principals are forced to pay the high wage to any new agent - otherwise, they would always renege after a good outcome.

Firing an agent after a low outcome (and only rewarding the agent after a high outcome) generates two effects. On the one hand, expected turnover costs increase for a given wage, as agents do not only leave the firm for exogenous reasons anymore. On the other hand, the agent's payoff after a low outcome is reduced, and it becomes cheaper to provide incentives.

In the following, the endogenously determined probability of a continuation of the relationship after a low output was observed (conditional on the agent not leaving for exogenous reasons) is denoted  $\alpha$ . However, I assume that  $\alpha \in \{0, 1\}$ , hence an agent is either always fired after a low outcome, or never. This is mainly done for concreteness, as allowing for intermediate values would not give additional results.<sup>8</sup>

The main results are presented in Proposition 4, a more elaborate analysis can be found in Appendix B. There, I derive the stationary equilibrium in contract-specific strategies that maximizes each principal's profits at the beginning of a new employment relationship.<sup>9</sup>

**Proposition 4**: Assume effort is an agent's private information, but output is observable to both parties. Then, implemented effort in profit-maximizing equilibria is inefficiently low if no minimum wage is present. If a minimum wage is present and as long as a principal's profits are positive, there exists a threshold  $\overline{w}^{\#}$  such that  $\alpha = 1$  for  $\overline{w} \leq \overline{w}^{\#}$  and  $\alpha = 0$  for  $\overline{w} > \overline{w}^{\#}$ .

Effort is affected by the minimum wage in the following way:

- If  $\overline{w} \leq \overline{w}^{\#}$  and the minimum wage binds, i.e. determines  $w^-$ , there exists a threshold  $\overline{w}'$  such that  $\frac{de^*}{d\overline{w}} < 0$  for  $\overline{w} \leq \overline{w}'$  and  $\frac{de^*}{d\overline{w}} = 0$  for  $\overline{w} > \overline{w}'$ .
- If  $\overline{w} > \overline{w}^{\#}$  and the minimum wage binds, then  $\frac{de^*}{d\overline{w}} > 0$

<sup>&</sup>lt;sup>8</sup>In an earlier version of this paper, I showed that intermediate values might or might not be part of an equilibrium. Furthermore,  $\alpha$  could only adopt intermediate values if a public randomization device existed. Just using mixed strategies would not be sustainable, since new employees are supposed to receive  $w^+$ . Hence, a principal would always have an incentive to keep an agent after a low output and pay  $w^-$ .

<sup>&</sup>lt;sup>9</sup>Note that the restriction to stationary contracts is not without loss of generality here. Instead of terminating the relationship with the same probability in every period, the principal could make  $\alpha_t$  contingent on the whole respective output history. Fong and Li (2010) provide a complete characterization of optimal relational contracts in non-market relationships (i.e., with just one principal and one agent), where the agent faces a limited liability constraint and effort is binary.

The proof to Proposition 4 can be found in Appendix B.

Inefficiently low effort is still driven by a principal's inability to establish an external reputation, hence new agents must receive an rent. Different from before, a minimum wage that binds - but is sufficiently small for  $\alpha = 1$  to be optimal - has no positive impact on effort. First, effort is slightly reduced and then remains unaffected by a further increase in  $\overline{w}$ . If the minimum wage is sufficiently large, firing an agent after a low outcome becomes optimal. This increases his incentives to exert effort, since staying in an employment relationship is strictly better than getting fired and receiving  $\overline{U}$ . This reduction in the costs of providing incentives at some point exceeds the higher turnover costs. When  $\alpha = 0$  is optimal, a further increase in  $\overline{w}$  increases effort and - up to some point - also the surplus of an employment relationship.

Proposition 4 implies that a minimum wage not only has an impact on effort levels, but also on turnover and wage compression within an industry. If a minimum wage exists, turnover should generally be higher than otherwise. However, if the minimum wage is sufficiently high, an additional increase will trigger less turnover. The latter is implied by the minimum wage's positive impact on effort, which makes a realization of the low output - and thus a layoff - less likely.

There exists a considerable amount of empirical evidence that these outcomes are indeed observed when a minimum wage is present. Generally, industries where a minimum wage is relevant - like the fast food industry - are characterized by high turnover levels (see Brown et al., 1982). Although low wage industries are generally considered to face high turnover, some of this might be driven by a firm's optimal provision of incentives. The negative marginal impact of a minimum wage increase on turnover has been well established empirically. Portugal and Cardoso (2006) find that separations of teenage workers in Portugal decreased after a minimum wage increase, while Dube et al. (2007) observe that average tenure rose substantially in restaurants in San Francisco. The most recent contribution is Dube et al. (2011), who also find strong evidence that turnover rates for teenagers and restaurant workers fall after a minimum wage increase.

Furthermore, a minimum wage induces wage compression. As long as the minimum wage is so low that agents remain employed after a low output, wages paid after a low and those after a high output realization are different. When the minimum wage is sufficiently high and agents get fired when y = 0, all employed agents receive the same wage in every period. An early empirical contribution exploring wage compression is Grossman (1983), later followed by Katz and Krueger (1998), who find that the minimum wage has induced wage compression in the Texas fast food industry. Furthermore, Lee (1999) provides evidence that the substantial decline of real minimum wages in the US was mainly responsible for a sharp increase of the wage dispersion among low income workers in the eighties.

Finally, spillover effects exist, i.e. minimum wages also have an impact on higher wages.

As long as  $\alpha = 1$ , a minimum wage also affects the high wage  $w^+$ , which has to be adjusted upwards to keep incentives constant. For evidence on spillover effects see Card and Krueger (1995), or Neumark and Wascher (2008). Several reasons have been provided, for example that firms substitute low wage with high wage workers or that an adjustment of wages is necessary to maintain differentials between high and low skilled workers (Grossman, 1983). In the present setting setting, spillover effects occur even with homogenous workers, and I neither need fairness perceptions (Falk et al., 2006) nor a more advanced bargaining concept (Dittrich and Knabe, 2010) to derive this result.

#### 5.2 First Period of the Game

The restriction to contract-specific strategies in the main setting is not completely without loss of generality, and conditioning strategies on calendar time could increase a firm's profits. The reason is that *future* turnover costs are needed to persuade agents *today* that promises are going to be honored. In the first period of the whole game, though, a principal has not been able to renege yet. Hence, there is no need to give the first employed agent a rent, and costs only accrue after this first agent has left for exogenous reasons.

In this section, I show that relaxing the assumption concerning contract-specific strategies and letting period t = 1 be treated differently from later ones is associated with higher effort levels as long as no minimum wage is present. However, exogenous turnover will still make it optimal for principals to induce inefficiently low effort.

**Lemma 3**: Assume strategies do not have to be contract-specific. As long as no (binding) minimum wage is present, equilibrium effort is characterized by

$$c' = \gamma \theta.$$

*Proof*: To avoid confusion with section 3, where I used time subscripts to describe the tenure of a specific employment relationship, I use the subscript 0 for the first period of the game. No subscripts are needed for all later periods, since all other results regarding stationarity remain valid.

The payoff of a principal in the first period of the game is  $\Pi_0 = e_0\theta - w_0 + \delta\Pi = e_0\theta - w_0 + \delta \frac{e^*\theta - w}{1 - \delta}$ , whereas an employed agent receives  $U_0 = w_0 - c(e_0) + \delta\gamma U = w_0 - c(e_0) + \delta\gamma \frac{w - c(e^*)}{1 - \delta\gamma}$ . Since an agent's (IC) constraint solely depends on future payments, effort in the first period of the game will be the same as later, i.e.,  $e^0 = e^*$ . Furthermore, an agent's outside option in the first period equals  $\overline{U}_0 = \delta\gamma \overline{U}$ .

As before, (IC) as well as (NR) constraints bind in equilibrium, with the exception that no (NR) constraint is needed for the first period of the game. Plugging the resulting values into

the payoff functions, and noting that an (IRA) constraint is only needed for the first period of the game (in all later periods, it is automatically satisfied given (IC)), yields the remaining constraints

$$w_0 \ge 0 \tag{IRA 0}$$

$$e^*\theta - \frac{c(e^*)}{\delta\gamma} - (1 - \delta\gamma)\overline{U} \ge 0.$$
 (IRP)

(IRA 0) has to bind in equilibrium, as otherwise a reduction would increase  $\Pi_0$  without violating any constraint. Hence,  $w_0 = 0$ , and principals implement the effort level that maximizes  $\Pi_0 = \frac{e^*\theta - \frac{c(e^*)}{\gamma} - \delta(1-\delta\gamma)\overline{U}}{1-\delta}$ . As long as no minimum wage is present, effort is thus given by  $\gamma\theta - c' = 0$ . Q.E.D.

If the first period of the game can be treated differently from later ones, effort is higher than before (there,  $\delta\gamma\theta - c' = 0$ ). However, effort is still inefficiently low, and the impact of the minimum wage remains unaffected - if it is sufficiently high, it triggers an increase in effort and surplus within an employment relationship.

#### 5.3 Collusion

The social equilibrium derived before maximizes a principal's profits, taking the behavior of all other principals as given. This implies that firms do not consider the impact of their choices on agents' outside options and on rents other firms have to pay. If the social equilibrium is characterized by contracts that maximize firms' joint utilities (subject to all constraints), implemented effort in the absence of a minimum wage will be lower than before.

**Lemma 4**: In the social equilibrium maximizing joint profits, equilibrium effort is given by

$$c' = \theta \delta \gamma (1 - \mu).$$

*Proof*: If the objective is to maximize firms' joint utilities, all other properties of the problem remain unaffected, and equilibrium effort maximizes  $\Pi = \frac{e\theta - \frac{c(e)}{\delta\gamma} - (1 - \delta\gamma)\overline{U}}{1 - \delta} = \frac{e\theta - \frac{c(e)}{\delta\gamma(1 - \mu)}}{1 - \delta}$ , giving the optimality condition stated above. Q.E.D.

Under collusion, effort is lower than before (where effort is characterized by  $c' = \delta \gamma \theta$ ). The reason is that lower effort and consequently lower wages also reduce an agent's outside option.

In this case, potential benefits of a minimum wage would be higher than before.

Such an equilibrium might be induced by collusion amongst firms, or by assuming that all of them are owned by one player. As long as information concerning the behavior of agents is not shared between the firms, the owner would optimally set wages that take the impact on an agent's outside option into account. Such a market might exist in the fast food industry - a market where minimum wages are particularly relevant - where many restaurants are run by franchisees who are relatively independent when running their outlets. Coordination on lower wages than individually optimal would be beneficial for the chain as a whole. Then, a minimum wage has an even bigger potential to increase the productivity of employment relationships.

## 6 Conclusion

Incentives should not be neglected when analyzing the impact of a minimum wage. How well a job is actually done is important in the service sector - where minimum wage laws are especially relevant - and employees need to be incentivized to perform accordingly. If relevant aspects of performance like the friendliness towards customers cannot be verified, relational contracts must be used to give incentives. As firms face a commitment problem in the case of intransparent labor markets, they enforce inefficiently low service quality. If forced to pay a higher wage than actually intended, they also require higher levels of effort. Thus, a minimum wage can increase service quality and even the efficiency of many occupations. There is evidence that the proposed mechanism indeed plays a role, however more empirical research is certainly needed.

## Appendix A

#### **Omitted Proofs**

**Lemma 2**: Equilibrium effort in all periods  $\tau$  satisfies  $e_{\tau}^* \leq e^{FB}$ , where  $e^{FB}$  is the surplusmaximizing, first-best effort level  $e^{FB}$  and characterized by  $\theta = c'$ .

Furthermore, a profit-maximizing social equilibrium is stationary in a sense that it is optimal to have  $w_{\tau} = w_{\tau'}$  and  $e_{\tau} = e_{\tau'}$  in all periods  $\tau, \tau' \geq 2$ .

*Proof*: The characterization of  $e^{FB}$  follows from maximizing the per-period surplus  $e\theta - c(e)$ .

For the remainder, it is convenient to use the results of Lemma 1 and rewrite the remaining constraints.

$$(IC)c(e_{\tau}^{*}) \leq \delta\gamma \left(\sum_{k=\tau+1}^{\infty} (\delta\gamma)^{k-(\tau+1)} (w_{k} - c(e_{k}^{*})) - \overline{U}\right), \forall\tau$$
$$(IRA)\sum_{k=\tau}^{\infty} (\delta\gamma)^{k-\tau} (w_{k} - c(e_{k}^{*})) \geq \overline{U}, \forall\tau$$
$$(NR)\sum_{k=\tau}^{\infty} (\delta\gamma)^{k-\tau} (e_{k}^{*}\theta - w_{k}) \geq e_{1}^{*}\theta - w_{1} + \sum_{\nu=2}^{\infty} (\delta\gamma)^{\nu-1} (e_{\nu}^{*}\theta - w_{\nu}), \forall\tau$$
$$(IRP)\sum_{\nu=1}^{\infty} (\delta\gamma)^{\nu-1} (e_{\nu}^{*}\theta - w_{\nu}) \geq 0$$

To see that  $e_{\tau}^* \leq e^{FB}$ , assume there is a period  $\tau'$  with  $e_{\tau'}^* > e^{FB}$ . Reducing  $e_{\tau'}^*$  by  $\varepsilon$ , and reducing  $w_{\tau'}$  accordingly to keep  $w_{\tau'} - c(e_{\tau'}^*)$  constant does not violate any constraint but increases  $\Pi_1$ . This increases  $\Pi_1$  and  $\Pi_{\tau}$ , however  $\Pi_{\tau}$  by more than  $\Pi_1$ , hence relaxes (NR) and (IRP). Since  $\Pi_1$  is increased, the initial equilibrium was not profit-maximizing.

Now, I show that (IC) constraints bind in all periods  $\tau \geq 1$ : Assume that the profitmaximizing equilibrium containts a period  $\tau'$  where (IC) does not bind. Then (IRA) might or might not bind. If it does not bind, reduce  $w_{\tau'+1}$  by a small  $\varepsilon > 0$  such that (IC) in period  $\tau'$ and (IRA) are still satisfied, as well as (IRA). This increases  $\Pi_1$  and  $\Pi_{\tau}$ , however  $\Pi_{\tau}$  by more than  $\Pi_1$ , hence relaxes (NR) and (IRP). Since  $\Pi_1$  is increased, the initial equilibrium was not profit-maximizing.

If (IRA) binds, the reduction of  $w_{\tau+1}$  can be accompanied by an increase in  $w_1$  such that (IRA) just remains binding.

Using this result and plugging  $c(e_{\tau}) = \delta \gamma \left( \sum_{k=\tau+1}^{\infty} (\delta \gamma)^{k-(\tau+1)} (w_k - c(e_k^*)) - \overline{U} \right)$  into payoffs gives

$$U_{\tau} = w_{\tau} + \delta \gamma \overline{U} \text{ and}$$

$$\Pi_{\tau} = -\frac{c(e_{\tau-1}^{*})}{\delta \gamma} - \overline{U} + \sum_{k=\tau}^{\infty} (\delta \gamma)^{k-\tau} (e_{k}^{*}\theta - c(e_{k}^{*})) + \Pi_{1} \frac{\delta(1-\gamma)}{1-\delta \gamma}$$

$$= -\frac{c(e_{\tau}^{*})}{\delta \gamma} - \overline{U} + \sum_{k=\tau+1}^{\infty} (\delta \gamma)^{k-(\tau+1)} (e_{k}^{*}\theta - c(e_{k}^{*})) + \frac{\delta(1-\gamma)}{1-\delta} \left[ -w_{1} + \sum_{\tau=1}^{\infty} (\delta \gamma)^{\tau-1} (e_{\tau}^{*}\theta - c(e_{\tau}^{*})) - \overline{U} \right]$$
all periods  $\tau \geq 2$  and constraints

for all periods  $\tau \geq 2$ , and constraints

$$(\text{IRA})w_{\tau} + \delta\gamma\overline{U} \geq \overline{U}, \forall\tau$$

$$(\text{NR}) - \frac{c(e_{\tau-1}^{*})}{\delta\gamma} + \sum_{k=\tau}^{\infty} (\delta\gamma)^{k-\tau} \left(1 - (\delta\gamma)^{\tau-1}\right) (e_{k}^{*}\theta - c(e_{k}^{*}))$$

$$\geq -w_{1} + \sum_{\nu=1}^{\tau-1} (\delta\gamma)^{\nu-1} (e_{\nu}^{*}\theta - c(e_{\nu}^{*})), \forall\tau \geq 2$$

$$(\text{IRP}) - w_{1} + \sum_{\tau=1}^{\infty} (\delta\gamma)^{\tau-1} (e_{\tau}^{*}\theta - c(e_{\tau}^{*})) - \overline{U} \geq 0.$$

In a next step, I show that effort is identical in all periods  $\tau \geq 2$ . Assume to the contrary that this is not the case and there is at least one period with an effort level different from the others. Hence, defining  $e_{max} = \max\{e_2^*, e_3^*, \dots\}$ , there is at least one period with effort strictly lower than  $e_{max}$ .

Now, take the first period where equilibrium effort is  $e_{max}$  and denote this period  $\overline{\tau} \geq 2$ . Then, either (A) effort in all subsequent periods is  $e_{max}$  as well, or (B), there is at least one period  $\tau' > \overline{\tau}$ , with  $e_{\tau'}^* < e_{max}$ .

In case (A), we have 
$$c(e^*)$$

$$\begin{split} \Pi_{\overline{\tau}} &= -\frac{c(e_{\overline{\tau}-1}^{*})}{\delta\gamma} - \overline{U} + \frac{(e_{max}\theta - c(e_{max}))}{1 - \delta\gamma} + \Pi_1 \frac{\delta(1 - \gamma)}{1 - \delta\gamma}, \text{ and} \\ \Pi_{\overline{\tau}+k} &= -\frac{c(e_{max})}{\delta\gamma} - \overline{U} + \frac{(e_{max}\theta - c(e_{max}))}{1 - \delta\gamma} + \Pi_1 \frac{\delta(1 - \gamma)}{1 - \delta\gamma}, \text{ all } k \ge 1 \end{split}$$

However, due to the definition of  $e_{max}$  and since not all effort levels are identical in periods  $\tau \geq 2, e_{\overline{\tau}-1}^* < e_{max}$ . Hence,  $\Pi_{\overline{\tau}} > \Pi_{\overline{\tau}+k}$ , i.e., the (NR) constraint does not bind in period  $\overline{\tau}$ .

Now, increase effort in period  $\overline{\tau} - 1$  by an  $\varepsilon > 0$ , thereby also increasing  $\Pi_1$  (since  $e^*_{\overline{\tau}-1} \leq$  $e^{FB}$ ). If (NR) was not binding in periods  $\overline{\tau} + k, k \geq 1$ , higher effort in period  $\overline{\tau} - 1$  increases  $\Pi_1$  without violating any constraint, and represents an improvement for principals. If (NR) was initially binding in periods  $\overline{\tau} + k, k \ge 1, w_1$  can be raised to a level keeping  $\Pi_1$  constant.

In case (B), note that

$$\begin{aligned} \Pi_{\overline{\tau}+1} &= -\frac{c(e_{max})}{\delta\gamma} - \overline{U} + \sum_{\substack{k=\overline{\tau}+1\\k=\overline{\tau}+1}}^{\infty} \left(\delta\gamma\right)^{k-(\overline{\tau}+1)} \left(e_k^*\theta - c(e_k^*)\right) + \Pi_1 \frac{\delta(1-\gamma)}{1-\delta\gamma} \\ &\leq -\frac{c(e_{max})}{\delta\gamma} - \overline{U} + \frac{(e_{max}\theta - c(e_{max}))}{1-\delta\gamma} + \Pi_1 \frac{\delta(1-\gamma)}{1-\delta\gamma}, \text{ where } \Pi_1 \text{ on the right hand side remains at} \end{aligned}$$

its original level.

Increase effort in any period following  $\overline{\tau} + 1$  where  $e^* < e_{max}$  by an  $\varepsilon > 0$ , which also increases  $\Pi_1$ . If (NR) was not binding there initially, this adjustment represents an improvement for principals. If it was binding,  $w_1$  can be raised to keep  $\Pi_1$  constant.

To show that wages are identical in all periods  $\tau \geq 2$ , just take a binding (IC) constraint,  $c(e) = \delta \gamma \left( \sum_{k=\tau+1}^{\infty} (\delta \gamma)^{k-(\tau+1)} (w_k - c(e^*)) - \overline{U} \right)$ , and note that all wages besides  $w_1$  neither enter  $\Pi_1$  nor any constraint. Take any equilibrium effort e and any accompanying equilibrium wage scheme  $(w_2, w_3, ...)$ . Then, it is possible to replace the latter by another wage scheme with constant elements, (w, w, ...), where  $w = \frac{c(e^*)}{\delta \gamma} + (1 - \delta \gamma)\overline{U}$ . Q.E.D.

## Appendix B - The Optimal Stationary Relational Contract Under Asymmetric Information

In each period t, the principal can observe  $y_t$  but not  $e_t$  (still, the output remains nonverifiable). Thus, the agent's compensation can only be based on past output levels. The wage an agent receives after a success in the previous period is denoted  $w_t^+$ , while  $w_t^-$  equals the compensation after  $y_{t-1} = 0$ . In the latter case, i.e. after observing a low outcome, the principal might also terminate the relationship, and fires the agent with probability  $1 - \alpha_t$ .

This implies the following timing. At the beginning of any period, the agent receives his wage. Then, effort is chosen and the output realized. Subsequently, the agent leaves for exogenous reasons with probability  $(1 - \gamma)$ . If  $y_t = \theta$ , remaining agents receive an offer where next-period's wage is  $w_t^+$ . If  $y_t = 0$ , remaining agents either get fired (which happens with probability  $(1 - \alpha_t)$ ) or receive an offer where next-period's wage is  $w_t^-$ . However, note that I restrict  $\alpha$  to the values 0 and 1 (for a general characterization, see an earlier version of this paper). This is solely done for simplicity, however could be justified by the inexistence of a public randomization device. Intermediate levels of  $\alpha$  cannot be supported by a mixed strategy, since the principal would always keep the agent after a low output - as new agents have to receive  $w^+$ , and  $w^+ > w^{-.10}$ 

The following constraints have to be satisfied: It may never be optimal to replace an agent instead of compensating him after a high output. Thus, the wage a new agent receives,  $w_1$ , has to be at least as high as  $w^+$  (obviously,  $w^+ \ge w^-$ ). This gives the (NR) constraint

$$w_1 \ge w^+.$$
 (NR)

After a success or when starting a new relationship, i.e., when the wage  $w^+$  has to be paid, the principal's payoff must be larger than her outside option, giving

$$\Pi^+ \ge 0, \tag{IRP}$$

where  $\Pi^+ = e^*\theta - w^+ + \delta \left[\gamma \left(e\Pi^+ + (1-e)\Pi^-\right) + (1-\gamma)\Pi_1\right]$  is a principal's payoff stream if output in the previous period was high.

For the remaining constraints, I denote an employed agent's payoff stream after a high output in the previous period  $U^+$  and after a low output  $U^-$ . Note that

$$U^{+} = w^{+} - c(e) + \delta\gamma \left[ eU^{+} + (1-e) \left( \alpha U^{-} + (1-\alpha)\overline{U} \right) \right] \text{ and } U^{-} = w^{-} + -c(e) + \delta\gamma \left[ eU^{+} + (1-e) \left( \alpha U^{-} + (1-\alpha)\overline{U} \right) \right].$$

Then, the agent's utility after a failure (again taking into account that  $w^- \leq w^+$ ) must not be below his outside option, implying

<sup>&</sup>lt;sup>10</sup>Note that this argument relies on the restriction to stationary strategies, which - as already pointed out - is not without loss of generality in the case of asymmetric information.

$$U^- \ge \overline{U}.$$
 (IRA)

An agent's effort is determined by his incentive compatibility (IC) constraint, with

$$e^* \in \operatorname{argmax} - c(e) + \delta\gamma \left[ eU^+ + (1-e) \left( \alpha U^- + (1-\alpha)\overline{U} \right) \right].$$
 (IC)

Finally,  $\alpha \in \{0, 1\}$ , and the wages paid by the principal must exceed a potential minimum wage.

For convenience, I set  $\overline{U} = 0$  in this section, i.e., assume that there is only one firm present. However, this has no qualitative impact on my results.

Then, the objective is to maximize profits from starting a new relationship, i.e.,

$$\max_{e,\alpha,w^+,w^-} \Pi_1 = e\theta - w_1 + \delta \left[ \gamma \left( e\Pi^+ + (1-e)\Pi^- \right) + (1-\gamma)\Pi_1 \right],$$

subject to (NR), (IRP), (IRA) and (IC).

As before, (NR) will bind. Furthermore, the linearity of output realizations in effort makes it possible to use the first order approach to determine effort. Since

$$U^{-} = \frac{w^{-} - c(e^{*}) + \delta \gamma e^{*} \left(w^{+} - w^{-}\right)}{1 - \delta \gamma (e^{*} + (1 - e^{*})\alpha)} \text{ and } U^{+} = \frac{w^{+} - c(e^{*}) - \delta \gamma (1 - e^{*})\alpha \left(w^{+} - w^{-}\right)}{1 - \delta \gamma (e^{*} + (1 - e^{*})\alpha)}, \text{ effort is characterized by}$$

$$-c'(1 - \delta\gamma (e^* + \alpha(1 - e^*))) + \delta\gamma (w^+ - \alpha w^- - \alpha\delta\gamma (w^+ - w^-) - (1 - \alpha)c(e^*)) = 0$$

In the following, I determine optimal effort levels for  $\alpha = 0$  and for  $\alpha = 1$  separately and then compare profits.

(A) 
$$\alpha = 1$$

In this case,  $\Pi^+ = \frac{e^*\theta - w^+ + \delta\gamma(1-e)(w^+ - w^-)}{1-\delta}$ , and an agent's effort is characterized by  $-c' + \delta\gamma(w^+ - w^-) = 0$ .

This allows me to substitute  $w^+ = \frac{c'}{\delta\gamma} + w^-$  into the principal's problem, which gives the Lagrange function (taking into account that  $w^+ > w^-$  and hence  $w^+ > \overline{w}$ )

$$L = \frac{e^{*\theta - \frac{c}{\delta\gamma} + (1 - e^{*})c' - w^{-}}}{1 - \delta\gamma} + \lambda_{IRP}\Pi^{+} + \lambda_{IRA}\frac{w^{-} - c(e^{*}) + e^{*}c'}{1 - \delta\gamma} + \lambda_{MW}(w^{-} - \overline{w})$$

First order conditions are

$$\frac{\partial L}{\partial e} = \frac{\theta - \frac{c''}{\delta\gamma} + (1 - e^*)c'' - c'}{1 - \delta} + \lambda_{IRP}\frac{\partial\Pi^+}{\partial e} + \lambda_{IRA}\frac{ec''}{1 - \delta\gamma} = 0$$
(4)

and

$$\frac{\partial L}{\partial w^{-}} = \frac{-1}{1-\delta} + \lambda_{IRP} \frac{\partial \Pi^{+}}{\partial w^{-}} + \lambda_{IRA} \frac{1}{1-\delta\gamma} + \lambda_{MW} = 0.$$
(5)

Assuming  $\lambda_{IRP} = 0$ , condition (5) gives that either (IRA) or (MW) must bind. First, assume that  $\overline{w}$  is sufficiently small that (MW) is slack. Then,  $\lambda_{IRA} = \frac{1-\delta\gamma}{1-\delta}$  and

$$\theta - c' - c'' \frac{1 - \delta \gamma}{\delta \gamma} = 0.$$
(6)

Now, assume that (MW) binds. If (IRA) does not bind, (4) gives that effort is characterized by

$$\theta - c' - c'' \left(\frac{1 - \delta\gamma(1 - e^*)}{\delta\gamma}\right) = 0.$$
<sup>(7)</sup>

Since effort has to be continuous in  $\overline{w}$ , there will be a range where both, (MW) and (IRA) bind, and effort is at levels between those characterized by (6) and (7).

Hence, when the minimum wage becomes binding, effort first decreases (since  $\overline{w} = c(e^*)$  –  $e^*c'$  in this range), and then remains constant.

As long as the minimum wage does not bind,  $\frac{d\Pi_1}{d\overline{w}} = 0$ If it binds  $\frac{d\Pi_1}{d\overline{w}} = \frac{\frac{de^*}{d\overline{w}} \left(\theta - c'' - c' \frac{(1 - \delta\gamma(1 - e^*))}{\delta\gamma}\right) - 1}{1 - \delta}$ . In the range where  $\frac{de^*}{d\overline{w}} < 0$ ,  $\theta - c' - c'' \left(\frac{1 - \delta\gamma(1 - e^*)}{\delta\gamma}\right) < 0$ , hence  $\frac{d\Pi_1}{d\overline{w}} > -\frac{1}{1 - \delta}$ . When the minimum wage is sufficiently high for  $\frac{de}{d\overline{w}} = 0$ ,  $\frac{d\Pi_1}{d\overline{w}} = -\frac{1}{1 - \delta}$ .

(B)  $\alpha = 0$ 

In this case,  $\Pi^+ = \frac{e^*\theta - w^+}{1 - \delta}$ , and an agent's effort is characterized by  $-c'\left(1-\delta\gamma e^*\right)+\delta\gamma\left(w^+-c(e^*)\right)=0.$ 

This allows me to substitute  $w^+ = \frac{c'}{\delta\gamma} (1 - \delta\gamma e^*) + c(e^*)$  into the principal's problem, which gives the Lagrange function (taking into account that (IRA) cannot bind - an agent is always fired after a low outcome, but necessarily receives a rent when output was high and hence when newly hired)

$$L = \frac{e\theta - \frac{c'}{\delta\gamma}(1 - \delta\gamma e) - c(e)}{1 - \delta} + \lambda_{IRP}\Pi^+ + \lambda_{MW} \left(\frac{c'}{\delta\gamma}(1 - \delta\gamma e) + c(e) - \overline{w}\right)$$
  
The first order conditions equals

$$\frac{\partial L}{\partial e} = \frac{\theta - \frac{c''}{\delta\gamma} \left(1 - \delta\gamma e^*\right)}{1 - \delta} + \lambda_{IRP} \frac{\partial \Pi^+}{\partial e} + \lambda_{MW} \frac{c''}{\delta\gamma} \left(1 - \delta\gamma e^*\right) = 0 \tag{8}$$

Again, it is assumed that  $\lambda_{IRP} = 0$ .

When  $\lambda_{MW} = 0$ , effort is characterized by

$$\theta - \frac{c''}{\delta\gamma} \left(1 - \delta\gamma e^*\right) = 0. \tag{9}$$

If the minimum wage is sufficiently high for  $\lambda_{MW} > 0$ , effort is characterized by

$$\overline{w} = \frac{c'}{\delta\gamma} \left(1 - \delta\gamma e^*\right) + c(e^*). \tag{10}$$

As long as the minimum wage does not bind,  $\frac{de^*}{d\overline{w}} = \frac{d\Pi_1}{d\overline{w}} = 0$ . When the minimum wage binds,  $\frac{de^*}{d\overline{w}} = \frac{1}{c'' \frac{(1-\delta\gamma e^*)}{\delta\gamma}} > 0$  and  $\frac{d\Pi_1}{d\overline{w}} = \frac{\frac{de^*}{d\overline{w}}\theta - 1}{1-\delta} > -\frac{1}{1-\delta}$ . These considerations allow me to prove

**Proposition 4**: Assume effort is an agent's private information, but output is observable to both parties. Then, implemented effort in profit-maximizing equilibria is inefficiently low if no minimum wage is present. If a minimum wage is present and as long as a principal's profits are positive, there exists a threshold  $\overline{w}^{\#}$  such that  $\alpha = 1$  for  $\overline{w} \leq \overline{w}^{\#}$  and  $\alpha = 0$  for  $\overline{w} > \overline{w}^{\#}$ . Effort is affected by the minimum wage in the following way.

- If  $\overline{w} \leq \overline{w}^{\#}$  and the minimum wage binds, i.e. determines  $w^-$ , there exists a threshold  $\overline{w}'$  such that  $\frac{de^*}{d\overline{w}} < 0$  for  $\overline{w} \leq \overline{w}'$  and  $\frac{de^*}{d\overline{w}} = 0$  for  $\overline{w} > \overline{w}'$ .
- If  $\overline{w} > \overline{w}^{\#}$  and the minimum wage binds, then  $\frac{de^*}{d\overline{w}} > 0$

*Proof*: If no minimum wage is present  $\Pi_1(\alpha = 0) = \frac{e^*\theta - \frac{c'}{\delta\gamma}(1-\delta\gamma) - c(e^*) - c'(1-e^*)}{1-\delta}$  and  $\Pi_1(\alpha = 1) = \frac{e^*\theta - \frac{c'}{\delta\gamma}(1-\delta\gamma) - c(e^*)}{1-\delta}$ ; thus,  $\Pi_1(\alpha = 1) > \Pi_1(\alpha = 0)$  for a given effort level. Hence,  $\alpha = 1$  is optimal in the situation without a minimum wage. In this case, effort is characterized by condition (6) above and below its efficient level.

Furthermore, note that  $\Pi_1(\alpha = 1) = \frac{e^*\theta - \frac{c'}{\delta\gamma}(1-\delta\gamma)-c(e^*)}{1-\delta}$  is still the true in the region where the minimum wage binds, but  $\lambda_{IRA} > 0$  as well. This implies that  $\alpha = 0$  cannot be optimal as long as (IRA) binds.

To establish the threshold  $\overline{w}^{\#}$ , note that when the minimum wage binds but (IRA) is slack,  $\frac{d\Pi_1(\alpha=1)}{d\overline{w}} = -\frac{1}{1-\delta}$ . Furthermore,  $\frac{d\Pi_1}{d\overline{w}} \ge -\frac{1}{1-\delta}$ , with a strict inequality for  $\overline{w}$  sufficiently large. Since  $\alpha = 1$  is optimal for  $\overline{w}$  sufficiently small, the threshold as described in the Proposition exists.

The rest of the Proposition follows from the considerations in cases (A) and (B) above.

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