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# Claude Hillinger: The Case for Utilitarian Voting

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# THE CASE FOR UTILITARIAN VOTING

Claude Hillinger\*

## ABSTRACT

Utilitarian voting (UV) is defined in this paper as any voting rule that allows the voter to rank all of the alternatives by means of the scores permitted under a given voting scale. Specific UV rules that have been proposed are approval voting, allowing the scores 0, 1; range voting, allowing all numbers in an interval as scores; evaluative voting, allowing the scores -1, 0, 1.

The paper deals extensively with Arrow's impossibility theorem that has been interpreted as precluding a satisfactory voting mechanism. I challenge the relevance of the ordinal framework in which that theorem is expressed and argue that instead utilitarian, i.e. cardinal social choice theory is relevant for voting. I show that justifications of both utilitarian social choice and of majority rule can be modified to derive UV. The most elementary derivation of UV is based on the view that no justification exists for restricting voters' freedom to rank the alternatives on a given scale.

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Key words: approval voting, Arrow's impossibility theorem, cardinal collective choice, evaluative voting, majority rule, range voting, utilitarian voting

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## 1. INTRODUCTION

Under utilitarian voting (UV) a voter can score each alternative with one of the scores permitted by a given voting scale. There is no further restriction on how the alternatives may be scored. Three specific utilitarian voting methods have been proposed: approval voting (AV), evaluative voting (EV) and range voting RV. While these will be discussed, the main focus of the paper is on utilitarian voting generally. I argue that UV is superior to traditional voting rules that restrict how alternatives may be scored.

Any claim regarding the superiority of a voting rule, or class of such rules, must confront Arrow's claim, expressed in his impossibility theorem, that no satisfactory method of collective choice can exist. That Arrows theorem precludes the existence of a satisfactory voting rule appears still to be the dominant view among voting theorists. In their introduction to a recent symposium on voting theory, Levin and Nalebuff (1995, p.3) put it as follows:

One can speculate on why alternatives to plurality rule have had such a difficult time being adopted. Part of the cause may be Arrow's general possibility theorem. Arrow (1951) demonstrates that any voting system applied to an unrestricted collection of voter preferences must have some serious defects; we must always choose between flawed alternatives.

A large literature on Arrow's conditions exists, but has not produced a successful challenge to the impossibility theorem. This paper takes a different approach. My starting point is the recognition that any mathematical theory, or formalism, is first of all no more than that. The claim that the formalism is about an empirical phenomenon, such as collective choice, can only be substantiated by examining the conceptualization that led to the formalism. Arrow formalized the problem of social choice by assuming that individuals have weak preference orderings over socially relevant alternatives. Let  $R_i$  be the preference ordering of the  $i$ th voter,  $i=1, \dots, N$  and  $R$  the corresponding social ordering. The problem of social choice is then formalized as that of finding a suitable mapping  $O: (R_1, \dots, R_N) \rightarrow R$ . I refer to this type of mappings as *ordinal*. Formal social choice procedures that are in actual use, including voting procedures are not ordinal. Instead, they are mappings from sets of numbers to a set of numbers. Let  $v_i = (s_{i1}, \dots, s_{ij})$  be the vote of the  $i$ th voter and  $s_{ij}$  the score he gives to alternative  $j$ . The aggregate vote is  $v = (s_1, \dots, s_j)$ , with  $s_j$  the aggregate score for alternative  $j$ . The mapping  $C: (v_1, \dots, v_N) \rightarrow v$  is *cardinal*. Each cardinal mapping implies an ordinal mapping, but since an infinity of cardinal mappings imply the same ordinal mapping, the reverse implication does not exist.

There is no logical reason why ordinal conditions should not be applied to cardinal mappings. However, the fact that all voting procedures are cardinal, should at least raise the question of the relevance of ordinal conditions.

The paper has the following structure: Section 2 asks of how the problem of voting should be conceptualized. This includes the question of what other activities voting is related to and how these have been structured. Theories relevant to voting are also reviewed. I argue that *prima facie* the utilitarian theory of collective choice, rather than the ordinal theory is relevant. Some basic definitions required subsequently are given in Section 3. Section 4 gives a simple and pragmatic derivation of UV. It is based on the idea that lifting the restrictions on the scoring of the alternatives that characterize traditional voting methods leads directly to UV. Section 5 examines the arguments advanced by Arrow and in the subsequent literature in favor of the ordinal approach. The lack of progress in both social choice and voting theories is examined in Section 6 and attributed to faulty conceptualizations in both cases. Section 7 examines both Arrow's conditions and the Condorcet criterion in the context of UV. In all fields other than voting, the aggregation of judgments proceeds by freely attaching scores

from a given scale to the objects being evaluated. In Section 8 I argue that voting should follow this example. Section 9 describes UV as being simply an application of traditional utilitarianism. Section 10 argues that rational delegates to a constitutional convention would unanimously opt for UV. Section 11 examines May's derivation of majority rule and d'Aspremont and Grevers' derivation of utilitarian collective choice and show that both can be used to justify UV. The framework can also be used to relate other voting methods to the theory of decision making under uncertainty. It is shown in particular that plurality voting (PV) corresponds to maximizing the probability of the largest gain, irrespective of the risk. Preference distortion and strategic voting with traditional voting methods are discussed in Section 12. The choice between the three UV methods that have been proposed is discussed in Section 13. Utilitarian voting in the historical Republic of Venice is the subject of Section 14. The paper concludes with a call for the empirical study of alternative voting methods.

## 2. HOW SHOULD THE PROBLEM OF VOTING BE CONCEPTUALIZED?

In voting theory as well as in the more abstract theory of collective choice, the problem is usually described as being that of the aggregation of preferences. Just what 'preference' means is usually not elaborated. Before turning to this issue, it is useful to point out that for Condorcet who stood at the beginning of the mathematical theory of voting, as well as for earlier theorists, the problem was that of choosing the correct alternative, for example the best qualified candidate. This was viewed as a cognitive problem rather than as one of subjective preference.<sup>1</sup>

'Preference' refers to a choice between alternatives. If an individual chooses an alternative *a* over *b*, that means that he *expects* the subjective experience that results from the choice of *a* to be superior to that resulting from the choice of *b*. An expectation is always more or less uncertain, even if the experience follows directly upon the choice. In the case of political elections the uncertainty is very large. The voter is uncertain about how a politician, once elected, will act; he is also uncertain about the consequences of various actions; finally, he cannot be sure in advance of how a given outcome will affect his psychological state of well being. I will sometimes refer to individuals' evaluations of alternatives as 'judgments', rather than 'preferences', the reason being that the former term is more general and can be applied to situations in which it would be inappropriate to speak of preference. For example, teachers rate students in relation to their grasp of some subject matter. The note given reflects the teacher's judgment, not his preference in the sense of, for example, his liking a particular student better than another. When notes are aggregated, over exams to give a final grade, or over course grades to determine if a student qualifies for graduation, this is an aggregation of judgments. In deciding how to aggregate judgments in a given field, such as voting, it is of interest to study the aggregation of judgments in other fields and this will be done in the present paper.

In addition to looking directly at how the aggregation of judgments is actually performed in different fields, it is clearly desirable to look at existing theories of how such aggregations should be performed. In the voting context, voting theory and the theory of collective choice are obviously relevant. In relation to the latter, we should deal with the fact that there are two broad theories, the ordinal theory in the mold of Arrow and the cardinal theory associated most prominently with the work of Harsanyi.<sup>2</sup> These may also be termed the 'negative' and the 'positive' theories, since Arrow presented an impossibility result, while the cardinal theorists presented consistent sets of axioms. While the ordinal theory has dominated voting theory, a principal claim of the present paper is that the cardinal theory is relevant for voting.

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<sup>1</sup> Early voting theory is discussed and the most important documents are reproduced in McLean and Urken (1995).

<sup>2</sup> Harsanyi (1953, 1955, 1976).

### 3. SOME DEFINITIONS

Let  $s_{ij}$  be the *score* given by the  $i$ th voter to the  $j$ th alternative,  $i = 1, \dots, N$ ,  $j = 1, \dots, J$ . The scores are real numbers in a specified interval:  $\underline{S} \leq s_{ij} \leq \bar{S}$ . The interval defined by the upper and lower bounds is a *voting scale*  $VS = (\underline{S}, \bar{S})$ . A particular VS may involve further restrictions on admissible scores; these will be discussed presently. The *ballot* of the  $i$ th voter is the vector of scores he gives to the alternatives:  $b_i = (s_{i1}, \dots, s_{iJ})$ . The *total score* of the  $j$ th alternative is  $s_j = \sum_i s_{ij}$ . The *total vote* is the vector  $b = (s_1, \dots, s_J)$ . Alternative  $h$  will be a winner if and only if  $s_h \geq s_j$ , for all  $j$ .

The above definitions cover virtually all voting methods, except for the fact that traditional methods place restrictions on how the scores made available by a given voting scale may be used. Thus, plurality voting has the two permissible scores, (0,1), and the restriction that the 1 can be assigned to only one alternative, 0 being assigned to the rest. Such voting methods will be termed *restricted*. All non restricted voting methods are will be termed *utilitarian*, this terminology being taken from collective choice theory.

The best known utilitarian voting method is *approval voting* (AV) for which  $VS = (0,1)$ . For *scale voting* (SC) a score can be any real number such that  $\underline{S} \leq s_{ij} \leq \bar{S}$ . Evaluative voting, advocated by the present author, has  $VS = (-1, 0, 1)$ . I will compare these alternatives, in Section 13, but the main focus of the paper is on utilitarian voting in general.

### 4. THREE CONDITIONS THAT IMPLY UV

This section contains a simple, pragmatic argument for UV. It is based on the idea that UV results simply from the lifting of restrictions imposed by traditional voting methods. More formally:

*Condition Use of a Voting Scale (UVS): A voting method must be based on a voting scale as defined above.*

*Condition Cardinal Aggregation (CA): The outcome of an election must be based on cardinal aggregation, i.e., the simple sums of scores for the various alternatives.*

*Condition Voter Sovereignty (VS): A voting rule must allow for voter sovereignty, so that each voter is free to assign to any alternative any of the scores provided by the voting scale.*

The three conditions define UV.

I can think of no argument against voter sovereignty and am unaware of any in the literature. This simple derivation is therefore quite powerful.

### 5. IS THERE A CASE FOR ORDINAL COLLECTIVE CHOICE?

The argument for ordinal social choice is generally expressed in terms of the alleged impossibility of interpersonal comparisons of utility. Arrow formulated it thus:

The viewpoint will be taken here that interpersonal comparison of utilities has no meaning and, in fact, that there is no meaning relevant to welfare comparisons in the measurability of individual utility. The controversy is well-known and hardly need be recited here. During the entire controversy, the proponents of measurable utility have been unable to produce any proposition of economic behavior which could be explained by their hypothesis and not by those of the indifference-curve theorists. 1 Indeed, the only meaning the concepts of utility can be said to have is their indications of actual behavior, and, if any course of behavior can be explained by a given utility function, it has been amply demonstrated that such a course of behavior can be equally well explained by any other utility function which is a strictly increasing function of the first. If we cannot have measurable utility, in this sense, we cannot have interpersonal comparability of utilities a fortiori. (Arrow, 1963, 1951, p.9).

Arrow is here referring to the static theory of consumer choice. Subsequent passages deal with the cardinal utility theory of von Neumann and Morgenstern. Arrow states that, since this theory is about an individual's choice among uncertain prospects, it is irrelevant to social choice. That is correct, but applies equally to the theory of consumer choice under certainty. The fact that this theory can do without cardinal utility has no implication for collective choice.

In a later section of his book Arrow explicitly considered additive utility and argued that it cannot serve as the basis of collective choice. He considers a choice between three alternatives and assumes that the individual utility functions are normalized so that the most preferred alternative gets a 1, the least preferred a 0.

It is not hard to see that the suggested assignment of utilities is extremely unsatisfactory. Suppose there are altogether three alternatives and three individuals. Let two of the individuals have the utility 1 for alternative x, .9 for y, and 0 for z; and let the third individual have the utility 1 for y, .5 for x and 0 for z. According to the above criterion, y is preferred to x. Clearly, z is a very undesirable alternative since each individual regards it as worst. If z were blotted out of existence, it should not make any difference to the final outcome; yet, under the proposed rule for assigning utilities to alternatives, doing so would cause the first two individuals to have utility 1 for x and 0 for y, while the third individual has utility 0 for x and 1 for y, so that the ordering by sum of utilities would cause x to be preferred to y. (p. 3).

This argument illustrates a fundamental blind spot in the work of Arrow and in collective choice theory that followed in his foot steps. *It is the failure to recognize the possibility of measuring along an independent scale.* Suppose the voting procedure used is range voting, the VS being the interval (0, 1). Then, under sincere voting, assumed by Arrow throughout, each alternative would have a defined value in this interval, independently of other alternatives. Alternatively, consider approval voting. An alternative in the interval (0, .5) on the voter's subjective utility scale would receive a 0 AV score, one in the interval (.5, 1) would receive a 1. Addition or removal of other alternatives would not change these scores.

There is another related blindness. It would appear that the primary challenge for anyone who argues against cardinal social choice is to confront it directly. The seminal contribution is Harsanyi (1955). In the second (1963) edition of his book, Arrow has the new Chapter 8 dealing with developments since the first edition; there is no mention of Harsanyi's work. Arrow gave a summary of his work at a symposium on *Human Values and Economic Policy* participated by economists and philosophers (Hook, 1967). Neither his, nor any other contribution mentions Harsanyi. Oddly, Samuelson (1967) in his comment on Arrow's paper conjectures that an impossibility theorem similar to that of Arrow can be proven for cardinal aggregation also!

Arrow was actually Harsanyi's thesis advisor at Stanford University in 1956-58. In a glowing foreword to Harsanyi's (1976) collected papers Arrow writes:

When John Harsanyi came to Stanford University as a candidate for the Ph.D., I asked him why he was bothering, since it was most unlikely that he had anything to learn from us. He was already a known scholar; in addition to some papers in economics, the first two papers in this volume had already been published and had dazzled me by their originality and their combination of philosophical insight and technical competence. (p.vii). The two papers referred to include the 1955 classic already mentioned. Regarding Harsanyi's contribution to social choice, Arrow writes:

Harsanyi has maintained consistently the importance of founding ethics as well as descriptive social science on the basis of the rational behavior of society and of individuals. The result has been a vigorous defense, rehabilitation, and reinterpretation of classical utilitarianism. In particular, Harsanyi introduced the ideal of considering the choice of an ethical criterion in a hypothetical situation where individuals do not know who they are or what their interests will be (this approach was used independently and somewhat later by John Rawls, under the now widely-used term, 'original position'; it has also been introduced independently and somewhat earlier by William Vickrey). Hence, the social rule amounts to maximizing under uncertainty. (p. vii).

Arrow is apparently willing to let ordinal and cardinal theories co-exist, even though he had denied the possibility of the latter to exist in any meaningful way.

A convincing rationale for the adoption of ordinalism cannot be found in the work of Arrow, nor as far as I can see anywhere else. What was the actual motivation? In his survey of social choice theory, Sen (1986) has this to say:

No approach to welfare economics has received as much support over the years as utilitarianism. If  $U_i(\cdot)$  is the utility function of person  $i$  defined for each person  $i=1,\dots,n$ , over the set  $X$  of alternative social states, then on the utilitarian approach any state  $x$  is at least as good as another  $y$ , denoted  $x R y$ , if and only if  $\sum_{i=1}^n U_i(x) \geq \sum_{i=1}^n U_i(y)$ .

It is clear that utilitarianism uses cardinality and interpersonal comparability of utilities. Both these practices received severe reprimand in the 1930's, with the rebuke drawing sustenance from a single-minded concern with basing utility information on non-verbal behavior only, dealing with choices in the absence of risk. It thus appeared that social welfare must be based on just the  $n$ -tuple of ordinal, interpersonally non-comparable, individual utilities. This informational restriction would, of course, make the traditional utilitarian approach -and a great many other procedures -unworkable. (p.1073-4).

Sen is here referring to the influential book of Robbins (1932). Robbins pointed out that all social or economic policies, not based on the equivalent criteria of Pareto optimality or unanimous decision making, involve value judgments that compare the gains and losses of different individuals; interpersonal comparisons of utility in the parlance of economists. He stated further that there was no scientific foundation for value judgments. Therefore, economists cannot make any policy recommendations and claim scientific validity for them. The literature dealing with values in relation to social science and policy, as well as in relation to science generally is in fact vast. Robbins' position was directly challenged by the Nobel Prize winning economist Gunnar Myrdal (1958, 1970). According to Myrdal, the answer to the problem raised by Robbins is to make the value judgments that enter into social policies explicit. Policy decisions always involve assumptions about cause and effect. Therefore, there is a role for scientific analysis of policy decisions even if we accept Robbins' position that value judgments cannot be scientific. Further more, value judgments are certainly capable of being debated. Religious leaders and philosophers have done this for millennia. More recently, it has been a subject for psychologists and, as Layard (2005) points out, values are increasingly being studied by the methods of the natural sciences also.

For the purpose of the present paper it suffices to point out that Robbins' argument is not about voting. A vote aggregates the preferences of the voters. These may be influenced by the pronouncements of economists and others, but the voter is the ultimate arbiter of their validity. There is no logical link from Robbins' position to that of Arrow who excluded utilitarian voting on the ground that the interpersonal comparison of utilities is impossible.

One can also make a positive argument to the effect that the *avoidance of interpersonal utility comparisons is impossible*. Any non-unanimous social decision involves gainers and losers and thus implies at least an implicit judgment that the gains outweigh the losses. Scientific analysis can help in devising suitable rules for aggregating preferences. If this were not the case, the continued existence of departments of economics and of political science seems hard to justify.

## 6. THE PROBLEM WITH COLLECTIVE CHOICE AND VOTING THEORIES

The formal theory of voting has existed for about 200 years, the more abstract theory of collective choice for about half a century. Both traditions have produced large and highly formal literatures. If we define progress as agreement on successively superior methods of voting and other methods of collective choice, then it would appear that no progress has been made. Plurality voting is still the standard and, though theorists agree on its defects, no agreement on a superior method has been arrived at. In my view, this rather uniquely negative



record is a result of premature formalization and concomitant lack of attention to the conceptualization of the empirical problems that motivated the theories in the first place. In the case of collective choice theory the mistake was the fixation on an ordinal representation. The problem with voting theories is that the different voting methods that are analyzed are based on inferior generalizations from the case of two alternatives, i.e., majority rule (MR). If the voter chooses one alternative out of two, it seems plausible to let him choose one out of many, which is PV. An alternative generalization is the Borda count (BC), which has the voting scale  $VS = (0, 1, \dots, J-1)$ . MR is equivalent to BC in case of two alternatives. Finally, the Condorcet method of pair wise elimination by means of MR is an obvious generalization.

I propose a different conceptualization of collective choice as a problem of *measurement*. If preferences, or more generally judgments, are to be aggregated, they must first be measured. Measurement universally proceeds by means of *scales* that exist independently of the objects being measured. Given the scale, measurement proceeds without further restrictions. Suppose we wish to know of a pile of sticks which are longer than one meter and which are shorter. It makes no sense to say that we can ‘allow’ one stick to be longer, but then the rest must be shorter. But, this corresponds to the measurement of preference in PV where we can choose one alternative and are forced to reject the rest. Lifting this irrational restriction, while keeping the two-valued scale, produces AV.

Given the long history of voting theory, it is surprising that the first step towards UV, in the form of AV, was taken as late as the 1970s and the second step, the introduction of a scale with arbitrary divisions, was taken in 2000 with Smith’s paper on RV.

## 7. ARROW, CONDORCET AND UTILITARIAN VOTING

In his review of collective choice theory, Sen (1986, p. 1074) put the difference between ordinal and cardinal SWFs succinctly:

It appears that some conditions that look mild – and are indeed satisfied comfortably by utilitarianism when translated into its cardinal interpersonally comparable framework – cannot be fulfilled by *any* rule whatsoever that has to base the social ordering on n-tuples of individual orderings.

The purpose of this section is to make the translation, demonstrating at the same time that the conditions are trivially satisfied. My translation is based on Vickrey’s (1960) restatement of Arrow’s conditions.

1. *Unanimity*:  $s_{ik} \geq s_{ij} \quad \forall i, \forall j \Rightarrow s_k \geq s_j \quad \forall j$ .
2. *Nondictatorship*: There exists no individual  $i$ , such that  $s_{ih} > s_{ik} \Rightarrow s_h > s_k$ .
3. *Transitivity*: The ordering of the alternatives implied by the vector of the total vote  $(s_1, \dots, s_J)$  is transitive.
4. *Unrestricted Domain*: Each voter can score any alternative with any score permitted by the given voting scale.
5. *Independence of Irrelevant Alternatives*: The total score for any alternative does not depend on the scores given to other alternatives. A fortiori this is true of the relationship between the scores of two alternatives.

The conditions follow trivially from the definition of UV. This ‘possibility result’ is made possible by the fact that UV makes the gains and losses of different voters *commensurable* by measuring them on a voting scale. Arrow’s impossibility theorem is a consequence of the fact that he rejects commensurability and thereby any method of choice in the absence of unanimity.

Condition 5 also clarifies the relationship between UV and the Condorcet condition. If voters consistently vote in accordance with a given voting scale, the UV winner must also win any bilateral contest and hence would also be the Condorcet winner. This statement holds under sincere voting. In a bilateral contest, voters have a strong incentive to utilize the entire scale, in which case the result need not hold. Voters are more likely to vote sincerely in an election with multiple candidates. In that case the UV result would be superior to the Condorcet result.

## **8. THE COMMON SENSE ARGUMENT FOR UTILITARIAN VOTING**

In Section 2 I argued that voting is an instance of the aggregation of judgments. If we look at how judgments are aggregated in fields other than voting, we find some startling contrasts. There judgments are measured on a fixed scale, added and averaged. This was not done because some abstract theory demanded it. It was done because this is the way that measurement universally proceeds and no reasonable alternative suggests itself. In considering the examples below, I will only describe the scales used. That the numbers obtained are added and averaged is true in all cases.

Consider first the example that is closest to political voting: surveys of the popularity of politicians. Such surveys are conducted in most democratic countries. In the US they are conducted by the Survey Research Center of the University of Michigan. Respondents are asked to rate politicians on a 'temperature scale' ranging from 0 to 100. In Germany, Forschungsgruppe Wahlen uses the 'Politbarometer' scale ranging from -5 (very negative) to +5 (very positive).

Much more common than political surveys are those conducted by firms to measure the satisfaction of their customers. The typical scale is discrete and has the values: excellent (+2), good (+1), satisfactory (0), poor (-1), bad (-2), or some equivalent descriptive terms, or numbers. The scale may be presented directly in numbers, or verbally and then converted to numbers.

A similar scale is used to measure performance either in scholastic or vocational settings. Here a typical scale is: excellent (1), good (2); average (3), poor (4), failing (5). In the German system a grade of 0.5 is also sometimes given to denote an exceptional performance. These tests may involve multiple levels of aggregation, for example by combining examination notes to obtain course notes and then combining these to obtain a grade for the entire course of study. Performance ratings illustrate particularly well why I prefer to use the term 'judgment' rather than 'preference', since the examiner is expected to be guided by his objective knowledge of the subject matter, not by any kind of preference.

In all of the examples cited, there is no restriction as to how the values allowed by a given scale may be applied. For example, no one has suggested that in a class of, say, 30 students should be rated by the BC, 30 points for the best and 1 for the worst student. The only exception is political voting. I have never seen any explicit justification for violating voter sovereignty in this manner and believe that non exists.

## **9. THE UTILITARIAN TRADITION AND VOTING**

Beginning with the early utilitarians, particularly Bentham and J. S. Mill, utilitarianism provided the generally accepted ethical foundation of economics and much of social thought. The basic utilitarian position is that the aim of social policy should be the maximization of the population's total utility, defined as the sum of individual utilities. Utilitarians were convinced of the measurability of utility in principle, but did not progress in operationalizing this view.

It is not my purpose here to review the vast literature pro and con utilitarianism. Instead I limit myself to describing the connection between voting and utilitarian social choice theory. It depends on

*Condition Cardinal Utility: The scores by means of which voters express their preferences on an appropriate voting scale can be interpreted as their cardinal utilities regarding the relevant alternatives.*

Given this condition, it follows that UV is the utilitarian solution to the voting problem. Let  $s_{ij}$  be the score of the  $i$ th voter for the  $j$ th alternative. If  $a_h$  is the winning alternative, then from the definition of UV,  $\sum_i s_{ih} = \max(j) \sum_i \sum_j s_{ij}$ .

Accepting utilitarianism as the relevant ethical postulate is, in connection with this condition sufficient to establish UV.

## 10. DERIVING THE VOTING RULE FROM BEHIND THE VEIL OF IGNORANCE

A persistent theme of both religion and ethics throughout the ages has been that ethical behavior is not narrowly self-centered, but involves an emphatic identification with others. This idea was formalized by both Rawls (1958, 1971) and Harsanyi (1955, 1976). Rawls has been justifiably criticized for employing the minimax rule, while Harsanyi uses the standard assumption of expected utility maximization. While adopting Harsanyi's formal argument, I interpret it differently.

Harsanyi postulates that when an individual decides behind the veil of ignorance he assumes that in a future situation of collective decision making he could, with equal probability, be one of the other members of society involved in that decision. The following quotation (Harsanyi, 1976, Ch. II, p.22, footnote 16) makes this clear:

Or rather, if he had an equal chance of being 'put in place of' any individual member of the society, with regard not only to his objective social (and economic) conditions, but also to his subjective attitudes and tastes. In other words, he ought to judge the utility of another individual's position not in terms of his own attitudes and tastes but rather in terms of the attitudes and tastes of the individual actually holding this position.

When Harsanyi goes on to postulate that an individual maximizes his expected utility under the assumption just described, he is not using the conventional assumption of maximizing *ones own* expected utility. Harsanyi here squeezes the concept of empathy into a narrow mathematical corset taken from individual decision making under uncertainty and not clearly relevant for social decisions.

My position is that voting theory is distinct from ethics in the sense that there cannot be any control over the votes that are cast to determine if they are ethical or not. The aggregation of votes may well be considered from an ethical viewpoint, but the evaluation of individual preferences is beyond the scope of voting theory.

In classical utilitarianism the utilitarian decision rule was advanced as a fundamental ethical principle, for which no further justification was either needed, or available. In my view, if the argument involving the veil of ignorance is to carry conviction beyond postulating utilitarianism directly, it must involve a straight forward application of expected utility maximization based on the deciding individual's own utility function. A similar position was taken by Rae (1969) and Taylor (1969) who also used the veil of ignorance argument. They considered the choice between two alternatives on the basis of a 2-valued scale and arrived at majority rule. Had they considered an arbitrary number of alternatives, while retaining the 2-valued scale, they would have discovered AV. The general case, with an arbitrary number of alternatives is considered below.

Assume that the members of a constitutional convention, hereafter called *electors*, have to decide, possibly among other issues, on a voting rule. Each elector is perfectly selfish, solely interested in how he himself will fare in future elections, as measured by her expected utility.

The utility experienced by the  $i$ th voter in a future election given that the  $j$ th alternative is chosen is the score  $s_{ij}$  on her ballot, taken from the relevant voting scale.

Let  $N$  be the number of voters in a future election. The elector assumes that any of the ballots cast in that election could, with equal probability  $1/N$ , reflect her preferences in that election. If the  $j$ th alternative wins, the expected utility of this outcome to the elector is  $\frac{1}{N} \sum_i s_{ij}$ , the average utility of the  $j$ th outcome. Let  $h$  be the alternative that maximizes this expression:

$$\frac{1}{N} \sum_i s_{ih} = \max(j) \frac{1}{N} \sum_i s_{ij}.$$

Alternative  $h$ , that maximizes the electors expected utility is by definition the UV winner. This argument holds with regard to *any* election in which the elector may participate at some future time. UV is the election method that a rational utility maximizing elector would prefer. If all electors are expected utility maximizers and are able to understand the consequences of their choice, *they will unanimously choose UV*.

Implicit in this argumentation is not Harsanyi's assumption of an elector's serial *identity* with all voters of a future election, but rather an assumption of *similarity of tastes, or values* among the members of a society. If in a future election, alternative  $a$  gets twice as many votes as alternative  $b$ , then by assumption, the elector will be twice as likely to vote for  $a$  than for  $b$ . A similarity of tastes and values may be regarded as a defining characteristic of a *society*, as distinct from a random collection of individuals.

This derivation, based on self interest only, clearly differs from the previous section where the utilitarian rule was simply postulated as a fundamental ethical principle. The argument involving the veil of ignorance is also distinct from Harsanyi's axiomatic derivation of a utilitarian SWF. That derivation gave rise to a literature that focuses on the intermingling of differences in tastes and differences in beliefs in shaping the differences in individual choices.<sup>1</sup> It would be interesting to have an analysis along these lines of the choice from behind the veil of ignorance.

The connection between choice under uncertainty and voting that is provided by conception of voting behind a curtain of ignorance can be exploited to analyses other voting procedures as well. The minimax rule of decision theory corresponds to anti-plurality voting in which the alternative with the fewest negative votes is chosen. PV corresponds to a maximax rule of maximizing the probability of the largest gain, regardless of risk. This would appear to be another major flaw of PV.

## 11. AXIOMATIC DERIVATIONS OF UV

In this section I discuss two axiomatic systems that can be easily applied to UV. May (1952) postulated four conditions to derive MR for the case of two alternatives.<sup>2</sup> I reformulate these slightly so that they can be related to UV, which satisfied them easily. The following subsection considers a set of conditions due to d'Aspremont and Gevers (1977) that is necessary and sufficient to establish a utilitarian SWF.

### 11.1 May's conditions

*Decisiveness: The voting rule produces a definite outcome for any pattern of individual preferences.*

*Anonymity: The outcome depends only on the votes cast, and not on which voter cast which vote.*

<sup>1</sup> A recent contribution is Gilboa, Samet and Schmeidler (2004).

<sup>2</sup> An exposition of May's result can also be found in Mueller (2003, 133-136).

*Neutrality: If two alternatives receive the same total score, they have the same outcome.*

*Positive responsiveness: If a voter increases his score for some alternative, then the outcome for that alternative cannot be worse.*

That these conditions are satisfied by UV is trivial; I have not been able to extend May's proof of sufficiency from MR to the case of UV.

### 11.2 The conditions of d'Aspremont and Gevers

d'Aspremont and Gevers (1977) presented a set of conditions that are necessary and sufficient for a cardinal SWF.<sup>1</sup> I reinterpret these to fit the assumption that preferences are expressed as scores on a ballot. There is some overlap between these conditions and those of May.

*Universal domain (UD): This is the same as May's decisiveness.*

*Independence of irrelevant alternatives (IIA): The choice from any subset of alternatives is independent of the preferences over alternatives outside that subset.*

*Unanimity (U): If each voter gives a higher score to alternative a, than to alternative b, then a must be socially preferred to b.*

*Anonymity (A): Any permutation of ballots among voters leaves the result unchanged.*

*Positive linear transformation (PLT): A transformation of individual utilities of the form  $a_i + bu_i$ ,  $b > 0$ , where  $b$  is identical over all voters, does not change the outcome.*

d'Aspremont and Gevers demonstrate that in their formulation these assumptions imply a utilitarian SWF of the form  $w = \sum u_i$ .

Instead of PLT, I assume Condition VS. This assumption is stronger than PLT, since now  $a_i = a$ ,  $\forall i$ .

Given the five assumptions, the utilitarian SWF is simply UV, since the individual utilities are in this case the scores assigned by voters to the alternatives. That UV satisfies all conditions is easily checked. I omit the proof of necessity, given by d'Aspremont and Gevers for the more general case based on PLT.

## 12. PREFERENCE DISTORTION AND STRATEGIC VOTING IN TRADITIONAL VOTING SYSTEMS

The choice of one out of several alternatives may depend not only on how good the chosen alternative is, but also on how bad the others are. The alternatives to UV are the traditional voting methods. Their flaws have been much discussed in the literature and the search for superior alternatives has been a principal motivation of voting theory. They are of three kinds: The first occurs when voters cast sincere votes, expressing their true preferences as well as they can be expressed under a given voting system. The second involves strategic voting when voters deliberately cast votes that do not reflect their true preferences. The third has to do with the fact that a given voting rule may prevent voters from expressing their true preferences.

How can a possible outcome of a voting system be evaluated? The theory of this paper suggests that the best outcome is the candidate who would win a utilitarian vote (utilitarian winner), the worst is the candidate getting the worst score in a utilitarian vote (utilitarian loser). The Condorcet criterion (CC) is the one most often used in the literature. The best candidate here is the Condorcet winner, who wins all binary contests; the worst is the Condorcet loser, who loses all binary contests. Neither a Condorcet winner, nor a Condorcet loser, always exist. In the following example, the Condorcet winner exists and is identical

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<sup>1</sup> Sen (1986, p.1125) also discusses the result.

with the utilitarian winner; the Condorcet loser also exists and is identical with the utilitarian loser. For the purpose of evaluation I use the following definition:

*A ‘best’ candidate is one who wins under both UV and CC and a ‘worst’ candidate is one who loses under both UV and CC.*

*The Flaw of PV:*

*Under PV the best candidate may receive the fewest votes and the worst candidate the most votes.*

The statement is illustrated by the following example:

Table 5.1: The Worst Candidate Wins, the Best Loses

Preference	<i>abc</i>	<i>cba</i>	<i>bca</i>
No. of Voters	35	30	20
EV Score	1, 0, -1	-1, 0, 1	-1, 1, 0

The first row of the table shows the preference orderings of three groups of voters, the second their number. The third row shows the utilitarian vote of each group on the three-valued EV scale. These scores always relate to the *abc* ordering.

The results of pair wise contests are: *ba*(50/40), *ca*(50/40), *bc*(60/30), so that *b* always wins, *a* always loses. The aggregate score for the EV vote is (-15, 20, -5) so that again *b* is the winner, *a* the loser. In a PV election, the total scores are (35, 20, 30) so that *a*, the worst candidate wins.

The most prominent attempts at avoiding, or at least ameliorating, the defects of single stage voting systems are multiple stage voting systems. The most popular is plurality voting with a runoff (PVR) in which the two candidates with the most votes face each other. Another method, widely practiced in the English speaking world is the Hare system, or single transferable vote (STV). Originally it was proposed for elections in which several candidates from a single district are to be elected to an assembly; only the case where a single candidate is to be elected will be discussed here. Each round of STV is a PV leading either to the election of a majority candidate, or the elimination of the candidate with the fewest votes. The election stops when there is a majority winner. A computerized version of STV, called instant transferable vote (ITV), has been gaining in the U.S. Here voters state the order of their first three preferences, with successive rounds of voting being simulated on the computer.

The following statement shows that neither PVR nor STV remedy the basic flaw of PV

*The Flaw of PVSR and STV:*

*The worst candidate may win under either PVR or STV.*

Continuing the above example, we see that the best candidate, *b*, is eliminated in the first round under either PVR or STV. Under both voting rules, the worst candidate, *a*, faces *c* in the second round and wins by 35/20.

A second defect of traditional voting methods is the incentive they provide for strategic voting, involving a deliberate misrepresentation of voter preferences. It is a common observation that voters do not ‘waste’ their vote on a candidate with little or no chance of being elected. Under PV this means that they will vote for the candidate they prefer among those having a chance to win. This leads to an extreme preference distortion since the best liked and the worst liked candidate get the same 0 score. With a BC, preferred candidates are moved down the preference ranking to make room for less favored candidates with better

winning chances. Strategic voting under traditional voting rules is seen to lead to *preference reversal*, meaning that less preferred candidates may receive higher scores than more preferred candidates.

Preference reversal is a consequence of the fact that traditional voting methods restrict voters ability to express their preferences on a given scale. Under PV, a voter who for strategic reasons votes for his second choice, can no longer vote for his first choice. Under UV, there is no restriction on how a voter may vote under a given scale. If, for strategic reasons, a voter elevates his score for his second best candidate, he need not lower the score of the first best. There is no incentive for a preference reversal under UV.

The final source of preference distortion has to do with the fact that a given voting scale may not allow the voter to express as differentiated a judgment as he wishes. If, for example, a voter wishes to differentiate between those candidates about whom he feels positive, neutral, or negative, then a two valued voting scale does not allow him to express these judgments.

### 13. WHICH UV RULE?

A decision for UV leaves open the precise scale to be used. I believe that this choice should be regarded as a pragmatic issue to be decided by experimenting with the principal scales that have been proposed. Nevertheless, I offer my thoughts on this subject here by reviewing the principal choices.

AV: Extensive justifications for AV were given by Brams and Fishburn (1983) and more recently in by Weber (1995), as well as by Brams and Sanver (2005). Brams and Fishburn (2003) discuss their efforts at getting AV adopted, as well as attempting an evaluation of AV in those instances where it was used.

At the core of the arguments presented is the assumption that preferences are *dichotomous*. This means that the voter puts each of the alternatives facing him into one of two classes: those of whom he approves and those of whom he disapproves. If this assumption is correct, the AV is the simplest and best voting rule.

I am skeptical regarding the assumption of dichotomous preferences. It is a common experience that in addition to feeling positive or negative about candidates or issues, we may also feel neutral. The examples of common scales for aggregating judgments given in Section 8 typically use a scale with 5 or 6 values. These examples may involve more information than a voter has in a general election; but they do suggest that a two-valued scale may be too narrow. A psychological shortcoming of AV is that it does not allow voters the satisfaction of explicitly voting against a disliked candidate, or issue.

RV: Here the intent is clearly to allow voters a wide choice of possible scores in a given range. Smith (2000), the principal theoretical paper advocating RV, assumes that the score can be any number in the interval (0, 1). However, in an empirical study of RV, Smith, Quintal and Greene (2005) used the interval (0, 100). No explanation of the change was given, but it does seem more practicable to deal with whole numbers than with decimals. Following the 2004 US presidential election, the authors' conducted an exit poll by asking 122 respondents for the RV scores of 7 major and minor party candidates. From their analysis of the responses, they tried to determine among other things an optimal scale. From the fact that some voters used numbers that were multiples of 5, but not of ten, the authors' conclude that a scale with 20 values is desired by voters. This is not convincing, since these numbers may have been 25, 50, 75, suggesting a 4-valued scale. Also, I feel that experiments with different scales are needed, rather than trying to infer the optimal scale from votes on the 100 point scale.

The paper has some interesting findings that should be explored further. The mean scores of the major candidates were rather close to the scores of the same candidates on an AV

scale that was also tested. This raises the issue of how much difference a more differentiated scale actually makes to the outcome. Only a minority utilized the full range of the scale, suggesting that they voted sincerely, rather than strategically.

*EV*: Feldenthal (1989) analyzed this method as a combination of approval and disapproval voting. He concluded that with strategic voting in a small group *EV* reduces to *AV*. I proposed *EV* in Hillinger (2004a, 2004b) *EV* is intermediate between *AV* and *RV*. My motivation for advocating *EV* has been twofold. I feel that the 2-valued range of *AV* voting is not discriminating enough and that voters desire the emotional satisfaction of being able to directly vote against as well as for a candidate. I also feel that compared to the examples in Section 8 that use a 5, or 6 point scale, voters are generally less informed and hence less able to discriminate.

Direct evidence on this issue is provided by voting in committees. Here the voting generally proceeds issue by issue, or amendment by amendment. Members are asked to vote for, against, or abstain. An abstention in a committee is not comparable to staying home in a general election because of lack of interest, or because the voter thinks that he will have no effect on the outcome. The committee member who abstains is physically present and knows that his vote can make a difference. He abstains because he is neutral, rather than positive or negative about the issue being voted on. The fact that a committee vote typically includes some abstentions indicates a willingness to utilize a 3-point scale.

#### **14. UTILITARIAN VOTING IN THE REPUBLIC OF VENICE**

This section is based on the interesting article by Lines (1986), but has a somewhat different interpretation of the facts that she describes. Lines describes the electoral procedures used by the Venetian oligarchy to elect their dogi. They were in effect from 1268 to 1789, the last election before the conquest by Napoleon and thus in one of the longest lasting and successful republics of history. The total voting process was extremely complicated, involving many rounds of elimination, either by vote or by lot. The interpretation that suggests itself is that this was a design to make it impossible to predict the outcome and thus to avoid the usual wheeling and dealing by which parties usually try to influence the outcome. In the following I will discuss solely the voting procedures employed.

Lines, as reflected in her title, regards the voting procedures as *AV*. This is not quite correct. For one, she cites the historian Maranini to the effect that in some election three different balls could be cast into the urn, signifying favor, contrariness and doubt. In some elections ‘doubt’ meant a need for more discussion and a postponement of the election. In other cases the ‘doubt’ vote was simply not counted as the election proceeded. This is clearly *EV* rather than *AV*.

Even the probably more usual case, in which only two different balls, signifying ‘yes’, or ‘no’, were available is not equivalent to the contemporary definition of *AV*. Under *AV*, the voter actively selects the candidates of whom she approves and ignores the rest. Under the Venetian rules, a negative vote must actually be cast, just as a positive vote. More importantly, a negative vote had more weight than a positive vote. Two negative votes out of a total of nine constituted a veto.

The Venetian system was utilitarian in the sense that all available scores could always be used in relation to all candidates. It deviated from pure utilitarianism through the introduction of an element of veto power. In contrast to the risk maximizing *PV*, Venetians were risk averse. The proud 500 year history of their republic confirms this as having been the right choice.



## 15. CONCLUSION

The paper advances a number of positive arguments in favor of utilitarian voting. The most fundamental argument is the criterion of unrestricted consumer sovereignty. There is no justification for restricting voters' ability to evaluate alternatives by means of a given voting scale. I also argued that the fact that UV does not satisfy Arrow's criterion of independence of irrelevant alternatives is not a valid criticism.

The criticism of traditional voting methods is sharpened by showing that the most widely used methods may elect the worst candidate under both the utilitarian and Condorcet criteria.

I believe that further progress on voting rules, particularly the selection of an appropriate scale, calls above all for empirical and experimental studies of voter behavior under different rules. In this connection it is interesting to note that the past several decades have seen the evolution of a substantial engineering literature on 'voting' by information processing machines and software. In contrast to the political theory of voting, this literature is largely empirical, examining the performance of different voting rules in practice. Contributors to this literature recognize that the problems that they deal with are similar to those encountered in political voting.<sup>1</sup> An influence from the empirical approach of the engineers on the political theory of voting would be salutary.

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<sup>1</sup> For a review see Parhani (2005)

## REFERENCES

- Arrow, K. J. (1951), *Social Choice and Individual Values*, Wiley, New York. Second edition, 1963, Yale University Press.
- Brams, S. J. and Fishburn, P. C. (1983), *Approval Voting*, Birkhauser, Boston.
- (2003), Going from theory to practice: The mixed success of approval voting. Paper presented at the Annual Meeting of the American Political Science Association, forthcoming in *Social Choice and Welfare*.
- Brams, S. J. and Sanver, M. R. (2005); Critical Strategies Under Approval Voting, Who Gets Ruled In and Ruled Out, Working Paper, forthcoming in electoral studies.
- Felsenthal, D. S. (1989), On combining approval with disapproval voting, *Behavioral Science*, 34, 53-60.
- Harsanyi, J., C. (1953), Cardinal utility in welfare economics and in the theory of risk taking, *Journal of Political Economy*, 61, 434-435. Reprinted in Harsanyi (1976).
- (1955), Cardinal welfare, individualistic ethics, and the interpersonal comparisons of utility, *Journal of Political Economy*, LXIII, 309-321. Reprinted in: Harsanyi (1976).
- (1976), *Essays on Ethics, Social Behavior, and Scientific Explanation*, D. Reidel, Dordrecht.
- Hillinger, C. (2004a), On the possibility of democracy and rational collective choice, *Discussion Paper 2004-21, University of Munich*. The paper can be downloaded at the SSRN website under: <http://ssrn.com/abstract=608821>.
- (2004b), Utilitarian collective choice and voting. The paper can be downloaded at the SSRN website under: <http://ssrn.com/abstract=637521>
- Hook, S. (ed.), (1967), *Human Values and Economic Policy: A Symposium*, New York University Press.
- Layard, R. (2005), *Happiness: Lessons from a New Science*, London, Penguin Books.
- Levin, J. and Nalebuff, B. (1995), An introduction to vote-counting schemes, *Journal of Economic Perspectives*, 9, 1, 3-26.
- Lines, M. (1986), Approval voting and strategic analysis: A Venetian example, *Theory and Decision*, 20, 155-172.
- McLean, I. and Urken, A. B. (eds.), (1995), *Classics of Social Choice*, University of Michigan Press.
- Mueller, D. C. (2003), *Public Choice III*, Cambridge University Press.
- Rawls, J. (1958), Justice as fairness, *Journal of Philosophy*, 67, 164-194.
- (1971), *A Theory of Justice*, Cambridge, MA, Harvard University Press.
- Robbins, L. (1932), *An Essay on the Nature and Significance of Economic Science*, London, Macmillan. Third ed., 1984, New York University Press.
- (1981), Economics and political economy, Richard T. Ely Lecture, *American Economic Review*, 71,2. Reprinted in Robbins (1984).
- Parhami, B. (2005), Voting: A Paradigm for Adjudication and Data Fusion in Dependable Systems. Ch. 4, in Diab, H. B. and Zomaya, A. Y. (eds.), *Dependable Computing Systems*, John Wiley & Sons.
- Samuelson, P. (1967), Arrow's mathematical politics. In: Hook (1967).
- Sen, A. (1986), Social choice theory, Ch. 22 in: Arrow, K., J. and Intriligator, M., D. (eds.), *Handbook of Mathematical Economics*, vol. III., North-Holland, Amsterdam.
- Vickrey, W. (1960), Utility, strategy, and social decision rules, *Quarterly Journal of Economics*, 74, 507-35.
- Weber, R. J. (1995), Approval voting, *Journal of Economic Perspectives*, 9, 1, 39-49.