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## The equitable top trading cycles mechanism for school choice

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## **The Equitable Top Trading Cycles Mechanism for School Choice**

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Rustamdjan Hakimov, Onur Kesten  
**The Equitable Top Trading Cycles Mechanism for School Choice**

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## Abstract

# **The Equitable Top Trading Cycles Mechanism for School Choice**

by Rustamdjan Hakimov and Onur Kesten<sup>\*</sup>

A particular adaptation of Gale's top trading cycles procedure to school choice, the so-called TTC mechanism, has attracted much attention both in theory and practice due to its superior efficiency and incentive features. We discuss and introduce alternative adaptations of Gale's original procedure that can offer improvements over TTC in terms of equity along with various other distributional considerations. Instead of giving all the trading power to those students with the highest priority for a school, we argue for the distribution of the trading rights of all slots of each school among those who are entitled to a slot at that school, allowing them to trade in a thick market where additional constraints can be accommodated. We propose a particular mechanism of this kind, the Equitable Top Trading Cycles (ETTC) mechanism, which is also Pareto efficient and strategy-proof just like TTC and eliminates justified envy due to pairwise exchanges. Both in simulations and in the lab, ETTC generates significantly fewer number of justified envy situations than TTC.

*Keywords:* School choice, stability, top trading cycles

*JEL classification:* C78, C79, D61, D78, I20

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# 1 Introduction

School choice programs that give families the flexibility to express preferences over schools have become increasingly popular both across the U.S. and worldwide. Despite such popularity, market designers are yet to reach a consensus on the “right” assignment mechanism. One major cause for the lack of agreement can be attributed to the existing trade-offs among the desirable features of mechanisms.

There are three most important criteria to evaluate an assignment mechanism: (1) equity in assignments (*i.e.*, the extent to which student priorities can be accommodated), (2) student welfare, and (3) immunity to strategic action. In this context, however, there are apparent tensions between the three requirements. Even equity and welfare are in conflict: there is no mechanism which is both stable<sup>1</sup> and Pareto efficient (Roth, 1982a). This tension automatically forces the designer to make a decision between the two properties. If one values stability over Pareto efficiency, then the choice is clear. The celebrated student-proposing *deferred acceptance* (DA) mechanism of Gale and Shapley (1962) is not only stable but it also produces the stable allocation that is most favorable to each student. Moreover, this mechanism is strategy-proof<sup>2</sup> (Dubins and Freedman, 1981; Roth, 1982a).<sup>3</sup>

On the other hand, an alternative mechanism is available should Pareto efficiency be the preferred feature between the two. Abdulkadiroğlu and Sönmez (2003) proposed an intuitive adaptation of Gale’s ‘top trading cycles’ procedure to school choice. The *top trading cycles* (TTC) mechanism is not only Pareto efficient but it is also strategy-proof. TTC has also been regarded as a viable assignment mechanism to be used in practice. The task force for the Boston school district recommended the use of TTC as opposed to the eventually adopted DA.<sup>4</sup> Similarly, the San Francisco school district announced plans to implement TTC in 2011. Most recently, two major school districts, Denver and New Orleans adopted TTC for student assignment.<sup>5</sup>

The top trading cycles idea was originally introduced in the context of a housing market (Shapley and Scarf, 1974). A housing market consists of a set of distinct objects, each with unit supply, and each owned by a distinct agent. The procedure works as follows: each agent points to the agent who owns his best choice object. Since the number of agents is finite, there is at least one cycle. In each cycle, the intended trades are performed and these agents and objects are removed. Then the same procedure is iteratively applied to the reduced market until no object remains in the market. In a housing market this procedure has been shown to possess quite appealing properties. It yields the unique *core* allocation of the housing market (Roth and Postlewaite, 1977) and it is strategy-proof (Roth, 1982b). Moreover, this procedure stands out as the unique strategy-proof, Pareto efficient, and individually rational mechanism (Ma, 1994). For the case of the unit supply of each object, large classes of assignment mechanisms

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<sup>1</sup>An allocation is stable if there is no unmatched student-school pair  $(i, s)$  such that student  $i$  prefers school  $s$  to his assignment, and school  $s$  either has not filled its quota or prefers student  $i$  to at least one student who is assigned to it.

<sup>2</sup>That is, no student ever gains by misrepresenting his preferences.

<sup>3</sup>Today two major school districts, Boston and NYC, have been implementing DA in student assignment for about a decade now upon abandoning their existing plans.

<sup>4</sup>See Kesten (2006) for a property-specific theoretical comparison of TTC and DA.

<sup>5</sup>In all these places the decision was made in collaboration with economists. The details of the subsequent stages of the San Francisco plan, however, are not publicly available.

based on this procedure have been characterized by (group-)strategy-proofness and efficiency in conjunction with various other auxiliary requirements (Pápai, 2001; Pycia and Ünver, 2009). The procedure has already proven useful for other applications such as an on-campus housing assignment (Abdulkadiroğlu and Sönmez, 1999), assignment problems with transfers (Miyagawa, 2001), and kidney exchange (Roth et al., 2005).

Nevertheless, Gale’s top trading cycles procedure cannot be directly applied to the school choice context. Since an object may now be in multiple supply (i.e., each school has several slots) and students have priorities over schools as opposed to the ownership rights in a housing market, an adaptation of the procedure calls for a suitable interpretation of student priorities. In TTC the trading market is generated by assigning each school to the student with the current highest priority for that school and keeping track of the remaining slots at each school. Then trades among these top priority students are carried out according to the top trading cycles procedure and the remaining slots are adjusted. Once a student has been placed to a school, the student is removed and the next highest priority student enters the market to be part of a trading cycle that is formed in a similar fashion. Despite its compelling welfare and incentive features, TTC may still introduce otherwise avoidable equity violations. We illustrate this point via a simple example.

Let the set of agents be  $I = \{i_1, i_2, i_3\}$  and the set of schools be  $S = \{s_1, s_2\}$  where school  $s_1$  has one slot and school  $s_2$  has two slots. The priorities for the schools and the preferences of the students are given as follows:

$\succ_{s_1}$	$\succ_{s_2}$	$P_{i_1}$	$P_{i_2}$	$P_{i_3}$
$i_1$	$i_3$	<u><math>s_2</math></u>	<u><math>s_1</math></u>	<u><math>s_1</math></u>
$i_2$	$i_2$	$s_1$	<u><math>s_2</math></u>	<u><math>s_2</math></u>
$i_3$	$i_1$			

When we apply TTC to this problem, student  $i_1$  who has the highest  $s_1$ –priority, exchanges one slot at school  $s_1$  in return for one slot at school  $s_2$  from student  $i_3$  who has the highest  $s_2$ –priority. The resulting allocation is indicated in boxes above. This allocation is Pareto efficient. However, the priority of student  $i_2$  for school  $s_1$  is violated by student  $i_3$  i.e.,  $i_2$  has justified envy over  $i_3$ . On the other hand, notice that a Pareto efficient and stable allocation is also available for this problem.

In this paper we argue that it may be possible to avoid the kind of justified envy situations observed in the above example under TTC at no cost in terms of welfare or incentives. Observe that TTC gives student  $i_3$  ownership over both slots of school  $s_2$  before student  $i_2$  enters the market. But then student  $i_1$  has no choice but to trade with student  $i_3$ , which in turn leads to the violation of the priority of student  $i_2$  for school  $s_1$ . However, had student  $i_1$  traded his right for one slot at school  $s_1$  with student  $i_2$  for his right for one slot at school  $s_2$ , there would not be any priority violations. Indeed, such a trade would have led to the Pareto efficient and stable allocation underlined in the above profile.<sup>6</sup>

We argue that alternative adaptations of Gale’s top trading cycles procedure can make it possible to integrate other additional considerations into the trading process relative to TTC. In

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<sup>6</sup>The introduction of avoidable justified envy situations in TTC may be further exacerbated when working with coarse priorities, which often require the use of random tie-breaking. Such randomization is known to lead to avoidable welfare losses under DA (Erdil and Ergin, 2008; Abdulkadiroğlu et al., 2009b).

fact, while expressing his view on TTC, the former superintendent of Boston Thomas Payzant underscored the need for such flexibility in trades in his memorandum to the school committee, dated May 25, 2005:

*“There may be advantages to this approach...It may be argued, however, certain priorities –e.g., sibling priority– apply only to students for particular schools and should not be traded away.”*

Since TTC allows only the current highest priority students to participate in the trading process, there are at most  $|S|$  participants in the market at any instant of this algorithm. Such a thin market may, however, entail justified envy for non-participants as shown above, or may not be able to accommodate the trading constraints concerning specific priorities such as those due to sibling status. Our alternative approach consists of two parts. First is the construction of a much thicker trading market than TTC by increasing the number of “active” participants at any given instant of the trading market. We advocate for assigning all slots of each school  $x$  to all the  $q_x$  students with the highest  $x$ –priority giving one slot to each student and endowing them with equal trading power subject to any additional constraints that may be desired. This would generate a sufficiently thick trading market leading to a multitude of trading opportunities in which various considerations including fairness, affirmative action (e.g., diversity), class composition, etc., may also be incorporated into the trading process while making placements. A key observation is that in a thick market with competing participants *being pointed to* is what leverages an agent to be part of a trading cycle, directly impacting the composition of the final allocation. Therefore, once market thickness is guaranteed, the second part of the approach is the choice of an appropriate “pointing rule” that specifies *who can point to whom*, which can be determined depending on the desideratum.

At any instant of the TTC algorithm, for example, each active participant of the market is endowed with a seat from a distinct school. Such thinness of the market automatically precludes the use of additional criteria for making assignments and already pins down all possible trades. To exemplify how the alternative approach works, consider the following pointing rule to address superintendent Payzant’s remark.<sup>7</sup> For simplicity, assume that the a priori priority assignments are independent of students’ sibling status (as opposed to the current practice in Boston public schools). Coupled with the construction of the thick market where each slot in the district is up for trade, first let each student point only to a student (or students) holding a slot from his favorite school—note that unlike in TTC there are now multiple such students—and second, point only to that student (or those students) whose favorite school at this instant is one where he himself has a sibling. Such a pointing rule, which can lexicographically use other criteria (e.g., other priorities) during cases of ties, indeed ensures that a student is favored in trading due to his sibling status only when he is to be assigned to his sibling’s school and is treated the same way as the rest of the students otherwise.

Whereas generalized trading mechanisms can readily attain efficiency by restricting to trades among the participants’ most desired choices, achieving strategy-proofness becomes challenging

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<sup>7</sup>Note that this issue cannot be resolved for TTC by simply giving higher priority to students, who have siblings, at some or at all schools. The reason is that, aside from the size of the market in which it operates, the pointing rule of TTC exclusively considers each student’s priority at his *endowed* school and is agnostic about his priority at his *intended* assignment.

due to the possibility of new participants joining the market, who may be endowed with a slot from a school that an existing participant may already be endowed with. Therefore, maintaining strategy-proofness requires particular attention to whether the pointing rule depends on the exogenous (e.g., priorities) or endogenous (e.g., preferences) specifications of a problem as well as on the organization of the entry of new participants into the market. As a specific illustration for fairness considerations that have been central in school choice, we propose and study a particular manifestation of the above ideas as a competitor to TTC. We call this mechanism the *equitable top trading cycles mechanism (ETTC)*. ETTC is also Pareto efficient and strategy-proof just like TTC (Propositions 1 & 2). Furthermore, we show that although ETTC considerably improves the equity aspects of TTC without paying any cost when attention is restricted to pairwise cycles (Propositions 3 & 4), there is in general no specific type of top trading cycles mechanism, strategy-proof or not, that always eliminates justified envy among students whenever efficiency and stability are compatible.

Given the growing popularity of TTC in practice, it is important to see how the two mechanisms perform in the controlled environment of the lab. Therefore we designed experiments to compare TTC with ETTC. Ours is not the first attempt to evaluate the performance of TTC in the lab. TTC has been experimentally compared to other mechanism in the context of on-campus housing and school choice. In the former context, Chen and Sönmez (2006) experimentally compare TTC with a popular real-life mechanism, the so-called random serial dictatorship with squatting rights and show that TTC leads to superior welfare gains.<sup>8</sup> In the latter context, Chen and Sönmez (2006) find that TTC and DA both provide higher performance in terms of efficiency relative to the much debated Boston mechanism primarily due to the significant differences in truth-telling rates. Our experiments also adopt much of their design idea. Calsamiglia et al. (2009) test TTC versus the DA and Boston mechanisms when participants' preferences may be constrained in length. They show that constraining the preference list leads to efficiency losses. Guillen and Hing (2014) show that the truth-telling under TTC can be distorted by third-party advice. Pais and Pintér (2008) study the role of information in student placement mechanisms and find that the three mechanisms achieve their best performances when participants do not have information about either the preferences of the other participants or the priorities of the schools. Additionally, they report that TTC outperforms Boston and DA when participants hold partial or full information about each other's preferences and priorities. Guillen and Hakimov (2014) show that in the presence of full information students tend to misrepresent their preferences even in a simple environment under TTC especially in the case of uncertainty about the behavior of other participants in the market. These findings, together with other experimental considerations such as the minimization of favorable default choices, led to our choice of a complete information environment for the experiment.

The main purpose of our experiment lies in comparing TTC and ETTC from the point of view of incentives, welfare, and fairness. We work with three environments: (1) *a designed environment* which mimics that of Chen and Sönmez (2006); (2) *a random-correlated environment* which generates high degree of correlation among preferences; and (3) *a random-uncorrelated environment*. We observe similar rates of misrepresentation by subjects under both mechanisms. Nevertheless, ETTC produces significantly less justified envy in treatments with some correlation

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<sup>8</sup>For the same context, Guillen and Kesten (2012) compare TTC to another, much less popular real-life mechanism, which turns out to be equivalent to Gale-Shapley's (1962) deferred acceptance mechanism.



of the preferences. In addition to the usual comparison with respect to a number of justified envy situations, we employ the idea of Guillen and Kesten’s (2012) ordinal efficiency test to compare the allocations by the two mechanisms with respect to equity dominance, which we refer as the ordinal fairness test. Given the same input of stated preferences, we determine the more plausible allocation from the criterion of fairness. Then we compare the number of dominations of one mechanism versus the other. The test shows that ETTC is significantly more likely to generate less justified envy situations than TTC in all environments. The advantage of ETTC becomes more prominent given the full preference revelation and the ETTC allocation is more likely to equity dominate the TTC allocation even in the case of the uncorrelated preferences of students. We also couple our lab experiment with computational exercises, which confirm the equity domination of ETTC.

The rest of the paper is organized as follows. The next section introduces the formal model. Section 3 describes TTC and its properties. Section 4 delineates the generalization of the top trading cycles idea and provides theoretical comparisons of TTC and ETTC. Section 5 offers the experimental analysis, and section 6 concludes. All the proofs and the experimental instructions are relegated to the appendix.

## 2 School Choice Problem

In a school choice problem, a certain number of students are to be assigned to a certain number of schools. Each school has a certain number of available slots, and the total number of slots is no less than the number of students.<sup>9</sup> Let  $I = \{i_1, i_2, \dots, i_n\}$  denote the set of students. A generic element in  $I$  is denoted by  $i$ . Let  $S = \{s_1, s_2, \dots, s_m\}$  denote the set of schools. A generic element in  $S$  is denoted by  $s$ . Let  $q_s$  be the number of available slots at school  $s$ , or the *quota* of  $s$ . Each student has strict preferences over all schools. Let  $P_i$  denote the preferences of student  $i$ . Let  $R_i$  denote the at-least-as-good-as relation associated with  $P_i$ . Formally, we assume that  $R_i$  is a linear order, i.e., a complete, transitive, and anti-symmetric binary relation on  $S$ . That is, for any  $s, s' \in S$ ,  $s R_i s'$  if and only if  $s = s'$  or  $s P_i s'$ . A strict priority order of all students for each school is exogenously given. Let  $\succ_s$  denote the priority order for school  $s$ .

Since priorities are pre-specified, a **school choice problem**,<sup>10</sup> or simply a problem, is a preference profile  $P = (P_i)_{i \in I}$ . Let  $\mathcal{R}$  be the set of all problems. An **allocation**  $\mu$  is a list of assignments such that each student is assigned to one school and the number of students assigned to a particular school does not exceed the quota of that school. Formally, it is a function  $\mu : I \rightarrow S$  such that for each  $s \in S$ ,  $|\mu^{-1}(s)| \leq q_s$ . Given  $i \in I$ ,  $\mu(i)$  denotes the assignment of student  $i$  at  $\mu$  and given  $s \in S$ ,  $\mu^{-1}(s)$  denotes the set of students assigned to school  $s$  at  $\mu$ . Let  $\mathcal{M}$  be the set of all allocations. An allocation  $\mu$  is **non-wasteful** if no student prefers a school with unfilled quota to his assignment, i.e., for all  $i \in I$ ,  $s P_i \mu(i)$  implies  $|\mu^{-1}(s)| = q_s$ . An allocation

<sup>9</sup>This assumption is not needed for any of the results in this paper. In the absence of this assumption, the identical analysis can be carried out by introducing the “null school” that represents not being placed in any *real* school and that has unlimited seats.

<sup>10</sup>The student placement problem is also closely related to the “house allocation problem” in which there is a set of objects collectively owned by the society. In that problem, however, the quota for each house is one. See for example, Pápai (2000), Abdulkadiroğlu and Sönmez (1998, 1999), Ehlers et al. (2002), Ehlers (2002), Ehlers and Klaus (2003), Ehlers (2002), and Kesten (2009).

$\mu$  is **Pareto efficient** if there is no other allocation which makes all students at least as well off and at least one student better off, i.e., there is no  $\alpha \in \mathcal{M}$  such that  $\alpha(i) R_i \mu(i)$  for all  $i \in I$  and  $\alpha(j) P_j \mu(j)$  for some  $j \in I$ .

The central equity notion is “stability.” We say that **student  $i$  justifiably envies student  $j$  for school  $s$**  at a given allocation  $\mu$  (or alternatively, *priority of student  $i$  for school  $s$  is violated*) if  $i$  would rather be assigned to  $s$  to which some student  $j$  who has a lower  $s$ –priority than  $i$ , is assigned, i.e.,  $s P_i \mu(i)$  and  $i \succ_s j$  for some  $j \in I$ . An allocation is **stable** (or fair) if it is non-wasteful and no student’s priority for any school is violated.

A **school choice mechanism**, or simply a mechanism  $\varphi$ , is a systematic procedure that chooses an allocation for each problem. Formally, it is a function  $\varphi : \mathcal{R} \rightarrow \mathcal{M}$ . Let  $\varphi(P)$  denote the allocation chosen by  $\varphi$  for problem  $(P)$  and let  $\varphi_i(P)$  denote the assignment of student  $i$  at this allocation. A mechanism is Pareto efficient (stable) if it always selects Pareto efficient (stable) allocations. A mechanism  $\varphi$  is **strategy-proof** if it is a dominant strategy for each student to truthfully report her preferences. Formally, for every problem  $(P)$ , every student  $i \in I$ , and every report  $P'_i$ ,  $\varphi_i(P) R_i \varphi_i(P'_i, P_{-i})$ .

### 3 Top Trading Cycles Mechanism

Abdulkadiroğlu and Sönmez (2003) propose what they call the **top trading cycles mechanism (TTC)**. TTC is based on *Gale’s top trading cycles procedure* proposed in the context of “housing markets” (Shapley and Scarf, 1974). In a housing market, there is a set of “indivisible objects” (e.g., houses) each of which is initially *assigned* to a different agent among a set of “agents.” Gale’s top trading cycles procedure works as follows:<sup>11</sup> Each agent points to the agent who is assigned to his best choice object. Since the number of agents is finite, there is at least one cycle. Then in each cycle, the corresponding trades are performed (i.e., each agent in a cycle receives the object assigned to the agent he points to), and these agents and objects are removed. Some agents may not be able to participate in a cycle and therefore remain in the market. Then the same procedure is applied to the new market and so on. The algorithm terminates when there are no agents left. This procedure yields the unique *core* allocation of this housing market (Roth and Postlewaite, 1977). The core mechanism for the housing market context has been shown to be the unique strategy-proof and Pareto efficient mechanism that ensures that no agent receives a worse object than his initial assignment (Ma, 1994).

Gale’s top trading cycles procedure cannot be directly applied to the school choice context. Since now there may be multiple copies (slots) of a particular object (school), this difference in the models necessitates further modification of the procedure. TTC is one such adaptation of the procedure. For a given problem, the outcome of TTC can be found via the following algorithm:<sup>12</sup>

*Step 1: Each student who has the highest priority for a school is assigned to all slots of that school. (A student may be assigned to the slots of different schools.) Each student points to*

<sup>11</sup>The procedure we describe here is not the same as the one proposed by Gale. But, the two are equivalent. For reasons to be made clear shortly, we adopt this alternative procedure.

<sup>12</sup>The algorithm we give here is an equivalent algorithm to the one that was proposed by Abdulkadiroğlu and Sönmez (2003). Since this version of the algorithm will make it easier for us to compare it with the alternative adaptation of Gale’s top trading cycles procedure that we will propose later in the paper, we work with this equivalent algorithm. This alternative algorithm appears in Kesten (2006).

the student (possibly himself) who is assigned to (all slots of) his best choice. There is at least one cycle. Each student in a cycle is placed in the school that was assigned to the student he is pointing to. Since each student who is part of a cycle is already placed in a school, he is removed and the number of available slots at that school is decreased by one.

In general,

Step  $k$ ,  $k \geq 2$ : All the remaining slots of each school which were assigned to a student who was part of a cycle at step  $k-1$  are assigned to the student with the highest priority for that school among the remaining students. (A student may be assigned to the slots of different schools.) Each student points to the student (possibly himself) who is assigned to (all remaining slots of) his best choice among the remaining schools. There is at least one cycle. Each student in a cycle is placed in the school that was assigned to the student he is pointing to. Since each student who is part of a cycle is already placed in a school, he is removed and the number of available slots at that school is decreased by one.

The algorithm terminates when no student is left.

Being based on Gale's top trading cycles procedure, TTC inherits the desirable properties it has. The first such property is Pareto efficiency.

**Proposition** (Abdulkadiroğlu and Sönmez, 2003) *The top trading cycles mechanism is Pareto efficient.*

However, TTC is not stable.<sup>13</sup> (We will shortly return to this aspect of TTC.) The second important property TTC has is strategy-proofness.

**Proposition** (Abdulkadiroğlu and Sönmez, 2003) *The top trading cycles mechanism is strategy-proof.*

**Proposition** (Roth 1982) *A Pareto efficient and stable allocation may not always exist and if it exists, it is unique.*

## 4 Generalized Top Trading Cycles Mechanisms

Abdulkadiroğlu and Sönmez (2003) have adopted *Gale's top trading cycles procedure* to school choice and propose a Pareto efficient and strategy-proof mechanism based on this procedure. Despite its appealing properties, this mechanism leaves room for improvement as far as equity is concerned. To elaborate, in the TTC algorithm all slots of a given school are assigned to the

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<sup>13</sup>Kesten (2006) gives a sufficient and necessary condition (which is similar to the one given in Ergin (2002)) for the equivalence of SOSM and TTCM or, to restore the stability or, the resource monotonicity, or the population monotonicity of TTCM.

student with the highest priority for that school. Since all students who have this school as a best choice have to point to this student, this particular student is given all the trading power of the slots of this school. As we have illustrated in the introduction, such “excessive” power may, however, result in the justified envy of students who may have lower priority for that school but higher priority for the school this student is placed in. Of course, due to the incompatibility between fairness and Pareto inefficiency, it is not always possible to totally avoid cases of justified envy. Nonetheless, we shall show that it may still be possible to considerably reduce the number of these cases by considering alternative adaptations of Gale’s top trading cycles procedure.

The key idea in our alternative approach is the construction of a much thicker trading market by increasing the number of “active” participants at any given instant of the trading market. Since TTC allows only the current highest priority students to be part of the trading process, there are at most  $|S|$  participants at any instant of this algorithm. Such a thin market may, however, entail justified envy for non-participants. Instead of assigning all  $q_x$  slots of a given school  $x$  to the highest  $x$ –priority student, we propose assigning all slots to all the  $q_x$  students with the highest  $x$ –priority giving one slot to each and endowing them with equal trading power. This would, for example, lead to an initial market with  $\sum_x q_x$  active participants. And, in order to maintain the thickness of the market throughout the process, whenever a student is removed from the market, any unallocated endowments will be “inherited” by the next highest priority student remaining in the market.

An important subtlety that arises with a market containing multiple students who are assigned a slot from the same school is determining the terms of trade. A *pointing rule* specifies for each school-student pair which school-student pair(s) should be pointed to among those who contain the favorite school currently remaining in the market at any instant of the algorithm. The choice of the pointing rule will shape the ensuing incentive and equity properties of the resulting trading mechanism.

While there is a rich set of pointing rules one can conceive, by and large we classify them into two groups. Consider any step  $t$  of the trading process applied to a given problem. Let  $(i, x)$  be a pair whose<sup>14</sup> currently remaining favorite school in the market is some  $y \in S \setminus \{x\}$ . Let  $X_t^{\rightarrow y}$  be the set of such pairs, i.e., those who are endowed with a slot from  $x$  and whose remaining favorite school at step  $t$  is some  $y \in S \setminus \{x\}$ , and let  $Y_t$  be the set of pairs who are endowed with a slot from  $y$ , i.e.,  $Y_t = \cup_z Y_t^{\rightarrow z}$ .

- *Pointing rules that depend only on the exogenous parameters.* Formally, a rule of this kind is a mapping  $r : (X_t^{\rightarrow y}, Y_t, E) \rightarrow Y_t$  where  $E$  captures an exogenously given constraint such as the priority structure. These pointing rules in effect describe which pair(s) in  $Y_t$  will be pointed to by a member in  $X_t^{\rightarrow y}$  depending on the characteristics of students contained in the two sets. Specifically, rules of this kind are agnostic about the preferences of students in  $Y_t$ . For example, such a rule may be the one based on the lottery draw used to break ties in priorities in school districts such as Boston, where many students fall in the same priority class, e.g., each pair in  $X_t^{\rightarrow y}$  points to that pair in  $Y_t$  which contains the student

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<sup>14</sup>With a slight abuse of language, the favorite school of a student in a given school-student pair will also be referred as the favorite school of the pair.

with the best lottery draw. Alternatively, the pointing rule may be chosen to fulfill certain affirmative action considerations and to promote diversity in student composition as in San Francisco, e.g., each pair in  $X_t^{\rightarrow y}$  points to those pairs in  $Y_t$  which contain a minority student.

- *Pointing rules that depend on the priority structure and the preferences.* Formally, a rule of this kind is a mapping  $r : (X_t^{\rightarrow y}, Y_t, E, P) \rightarrow Y_t$ . These pointing rules in effect describe which pair(s) in  $Y_t$  will be pointed to by a member in  $X_t^{\rightarrow y}$  depending not only on the two sets, but possibly also on the preferences of the students in  $Y_t$ . For example, this kind of a rule may allow for considering the priority of students in  $Y_t$  for their favorite schools remaining in the market.

TTC is an example of the former type of pointing rules. Indeed, at any step  $t$  of TTC each pair  $(i, x) \in X_t$  points to the pair in  $Y_t$  containing the student with the highest  $y$ -priority.<sup>15</sup> In the next section we present yet another example of these rules that specifically aims to establish equity among pairwise exchanges. An important advantage of these pointing rules is that because such pointing rules depend only on exogenous specifications, the top trading cycles mechanisms they induce are readily strategy-proof in addition to being Pareto efficient.<sup>16</sup>

On the other hand, by allowing for dependence on the internal specifications of a problem, the latter type of pointing rules may render more ground in terms of stability possibly at the expense of strategy-proofness while also maintaining Pareto efficiency. In the sequel we also discuss the construction of a top trading cycles mechanism based on an intuitive example of this kind of pointing rule.

## 4.1 Equitable Top Trading Cycles

Recall that in TTC each student-school pair points to the student-school pair that contains the highest priority student for the school contained in the latter pair. We propose an alternative adaptation of Gale's top trading cycles procedure in which *each student-school pair points to the student-school pair that contains the highest priority student for the school contained in the initial pair*. This proposal is based on a “dual” pointing rule to that of TTC which ensures that whenever a cycle forms between two student-school pairs, then the students included in that cycle have the highest priority for their favorite schools among their competitors at that step of the trading market.

Here is a description of our proposal. At the first step, for each school slots are assigned to students one by one following their priority order to form student-school pairs in a thick market. A student can be contained in more than one student-school pair. We denote a student-school pair by  $(i, s)$  where  $i$  is a student and, with a slight abuse of notation,  $s$  denotes one slot from school  $s$ . Each student-school pair  $(i, s)$  points to the student-school pair  $(i', s')$  such that (i) school  $s'$  is the best choice of student  $i$  and, (ii) student  $i'$  is the student with the highest

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<sup>15</sup>Formally, the pointing rule underlying TTC is as follows. For any given sets  $X_t^{\rightarrow y}$  and  $Y_t$  of any step  $t$ ,  $r^{TTC}(X_t^{\rightarrow y}, Y_t, \succ_y) = \{(j, y) \in Y_t : j \succ_y j' \text{ for any } (j', y) \in Y_t\}$ .

<sup>16</sup>More precisely, this statement is true for pointing rules that are functions. In the case when the pointing rule is a correspondance, two or more cycles may be nested and an additional exogenous criterion also needs to be used for cycle selection. We shall discuss cycle selection rules subsequently.

priority for school  $s$  among the students who are assigned to a slot from school  $s'$ . If there is already a student-school pair at which student  $i$  is assigned to one slot from his best choice school, then all student-school pairs containing him point to that student-school pair. Since the number of student-school pairs is finite, there is at least one cycle. In each cycle, corresponding trades are performed, *i.e.*, if a student-school pair  $(i, s)$  is pointing to the pair  $(i', s')$  in a cycle, then student  $i$  is placed in school  $s'$  and he is removed as well as the slot student  $i'$  is assigned.

It is possible that some student-school pairs that contain the same student appear in the same cycle or in different cycles. In such a case, the extra slots of that school (to which different student-school pairs containing him are pointing in other cycles) remain to be “inherited” by the remaining students.<sup>17</sup> An important twist of our algorithm is that the slots that remain to be inherited at the end of a step  $t$ ,  $t \geq 1$ , are not necessarily inherited at the very next step by the remaining students. The inheritance of slots of a school  $s$  does not take place until all students who are contained in a student-school pair including school  $s$  at step  $t$  are removed.<sup>18</sup> Immediately after the last student who was contained in a student-school pair with a slot from school  $s$  at step  $t$  is removed, all slots of school  $s$  which thus far remain to be inherited, are inherited by the remaining students one by one following the priority order for school  $s$ , *i.e.*, these students are assigned those slots to again form student-school pairs. At each step, again student-school pairs point to each other in the way described above. Corresponding trades are carried out in each cycle and some slots remain to be inherited at the appropriate subsequent step. The procedure continues in a similar fashion. The following algorithm summarizes this procedure for a given problem:

*Step 1: For each school, all available slots are assigned to students one by one following their priority order to form student-school pairs. Each student-school pair  $(i, s)$  points to the student-school pair  $(i', s')$  such that (i) school  $s'$  is the best choice of student  $i$  and, (ii) student  $i'$  is the student with the highest priority for school  $s$  among the students who are assigned to a slot from school  $s'$ . If student  $i$  is already assigned to one slot from his best choice school, then all student-school pairs containing him point to that student-school pair. There is at least one cycle. In each cycle, corresponding trades are performed, and all student-school pairs which participate in a cycle are removed. It is possible that student-school pairs containing the same student, say student  $i$ , appear in the same cycle or in different cycles. In such a case, student  $i$  is placed in his best choice, and the other slots of that school (to which the student-school pairs containing him are pointing in those other cycles) remain to be inherited. For each student-school pair  $(i, s)$  which participates in a cycle, the slots assigned to student  $i$  in other student-school pairs which do not participate in a cycle also remain to be inherited.*

*In general,*

*Step  $k$ ,  $k \geq 2$ : **Inheritance:** For each school  $s$  such that (i) there are slots of school  $s$*

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<sup>17</sup>The idea of “inheritance of slots” we use here is in part inspired by Pápai (2000), who introduces and characterizes quite a large family of rules which she calls “endowment inheritance rules” that are also based on Gale’s top trading cycles algorithm.

<sup>18</sup>As will be clear shortly, this restriction ensures that our mechanism is strategy-proof.

which remained to be inherited from previous steps, and (ii) there are no existing pairs who were assigned to a slot of school  $s$  at a previous steps of the algorithm, the slots which remained to be inherited from previous steps are assigned to the remaining students one by one following the priority order for school  $s$  to form new student-school pairs.

**Pointing:** Each student-school pair  $(i, s)$  points to the student-school pair  $(i', s')$  such that (i) school  $s'$  is the best choice of student  $i$  and, (ii) student  $i'$  is the student with the highest priority for school  $s$  among the students who are assigned to a slot from school  $s'$ . If student  $i$  is already assigned to one slot from his best choice school, then all student-school pairs containing him point to that student-school pair.<sup>19</sup> There is at least one cycle. In each cycle, corresponding trades are performed, and all student-school pairs which participate in a cycle are removed. It is possible that student-school pairs containing the same student, say student  $i$ , appear in the same cycle or in different cycles. In such a case, student  $i$  is placed in his best choice and the other slots of that school (to which the student-school pairs containing him are pointing in those other cycles) remain to be inherited. For each student-school pair  $(i, s)$  which participates in a cycle, the slots assigned to student  $i$  in other student-school pairs which do not participate in a cycle also remain to be inherited.

**Remark 1:** Note that slots remain to be inherited in one of two ways: (1) More than one student-school pair containing the same student participate in a cycle. Then that student is given one slot from his best choice and the other slots of his best choice (to which he is pointing in any other cycle) remain to be inherited; (2) A student-school pair participates in a cycle and there are other student-school pairs containing the same student which do not participate in a cycle: Then the slots assigned to him in those other student-school pairs remain to be inherited.

We call the mechanism that associates the outcome of the above algorithm to each problem the **equitable top trading cycles mechanism (ETTC)**. Next we give an example to illustrate the dynamics of this algorithm:

**Example 1.** Let  $I = \{i_1, i_2, \dots, i_7\}$  and  $S = \{s_1, s_2, s_3, s_4\}$ , where schools  $s_1$  has one slot and the rest of the schools have two slots each. The priorities for the schools and the preferences of the students are given as follows:

$\succ_{s_1}$	$\succ_{s_2}$	$\succ_{s_3}$	$\succ_{s_4}$								
$i_1$	$i_2$	$i_3$	$i_7$	$P_{i_1}$	$P_{i_2}$	$P_{i_3}$	$P_{i_4}$	$P_{i_5}$	$P_{i_6}$	$P_{i_7}$	
$i_5$	$i_4$	$i_5$	$i_6$	<u><math>s_2</math></u>	$s_1$	<u><math>s_3</math></u>	<u><math>s_1</math></u>	<u><math>s_4</math></u>	$s_2$	$s_3$	
$i_4$	$i_5$	$i_1$	$i_5$	$s_3$	<u><math>s_2</math></u>	$s_4$	$s_3$	$s_3$	<u><math>s_3</math></u>	<u><math>s_4</math></u>	
$i_7$	$i_7$	$i_6$	$i_4$	$s_4$	$s_4$	$s_1$	<u><math>s_2</math></u>	$s_1$	$s_1$	$s_1$	
$i_6$	$i_6$	$i_2$	$i_3$	$s_1$	$s_2$	$s_2$	$s_4$	$s_2$	<u><math>s_4</math></u>	$s_2$	
$i_2$	$i_1$	$i_4$	$i_2$								
$i_3$	$i_3$	$i_7$	$i_1$								

*Step 1:* Since students  $i_1$  has the highest priority for school  $s_1$ , the only slot of this school is assigned to him to form the student-school pair  $(i_1, s_1)$ . Since students  $i_2$  and  $i_4$  have the top two

<sup>19</sup>We add this particular restriction to the description of the algorithm to facilitate the formation of self-cycles. This addition is immaterial for all of our results.

highest priorities for school  $s_2$ , the two slots of this school are assigned to these students to form the student-school pairs  $(i_2, s_2)$  and  $(i_4, s_2)$ . The other student-school pairs which form similarly are:  $(i_3, s_3)$ ,  $(i_5, s_3)$ ,  $(i_7, s_4)$ , and  $(i_6, s_4)$ .

We determine which student-school pair points to which pair. Consider, for example, the student-school pair  $(i_1, s_1)$ . The best choice of student  $i_1$  is school  $s_2$ , hence student  $i_1$  will point to “one” of the student-school pairs that contain school  $s_2$ , which are  $(i_2, s_2)$  and  $(i_4, s_2)$ . Since student  $i_4$  has a higher  $s_1$ –priority than  $i_2$ , student-school pair  $(i_1, s_1)$  points to  $(i_4, s_2)$ . The best choice of student  $i_3$  is school  $s_3$ . Hence, the student-school pair  $(i_3, s_3)$  points to itself and forms a self-cycle. Other student-school pairs point to each other in a similar fashion (see Figure 1(a)).

Next we identify the cycles of step 1. Two cycles form in this step: the pairwise cycle  $\{(i_1, s_1), (i_4, s_2)\}$  and the self-cycle  $\{(i_3, s_3)\}$ . Corresponding trades are performed: that is, student  $i_1$  is placed in school  $s_2$ , student  $i_4$  is placed in school  $s_1$ , and student  $i_3$  is placed in school  $s_3$ . All three student-school pairs that have participated in a cycle are removed.

*Step 2:* The remaining student-school pairs are  $(i_2, s_2)$ ,  $(i_5, s_3)$ ,  $(i_6, s_4)$ , and  $(i_7, s_4)$  (see Figure 1(b)). The only cycle of this step is the self-cycle  $\{(i_2, s_2)\}$ . The corresponding trade places student  $i_2$  in school  $s_2$ .

*Steps 3 & 4:* The remaining student-school pairs are  $(i_5, s_3)$ ,  $(i_6, s_4)$ , and  $(i_7, s_4)$  (see Figure 1(c)). The only cycle of this step is the pairwise cycle  $\{(i_5, s_3), (i_6, s_4)\}$ . The corresponding trade places  $i_5$  in  $s_4$  and  $i_6$  in  $s_3$ . In the next step, the self-cycle  $\{(i_7, s_4)\}$  forms and student  $i_7$  is placed in  $s_4$ .

The ETTC allocation is indicated in boxes. The underlined allocation is the outcome of TTC for the same problem. Whereas the former allocation is stable, students 4 and 6 face priority violations at the latter allocation. ♦

Just like TTC, ETTC too, is Pareto efficient.

**Proposition 1** *The equitable top trading cycles mechanism is Pareto efficient.*

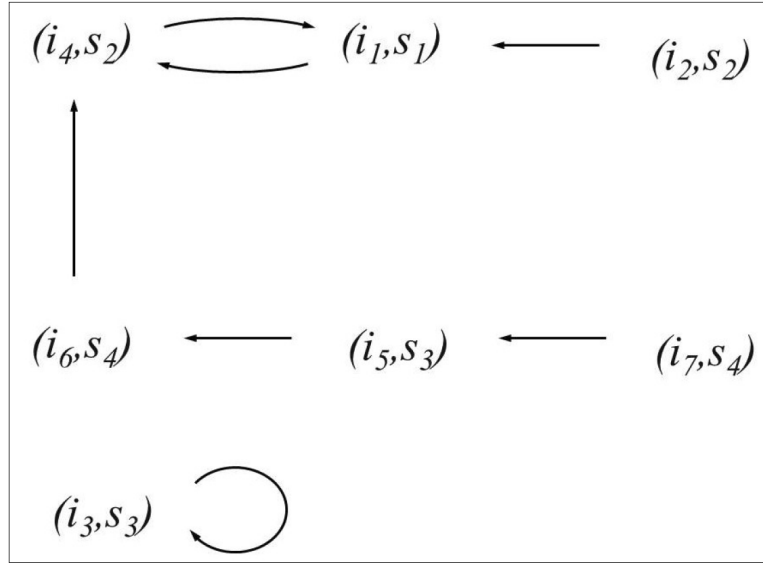
Just like TTC, ETTC too, is strategy-proof.

**Proposition 2** *The equitable top trading cycles mechanism is strategy-proof.*

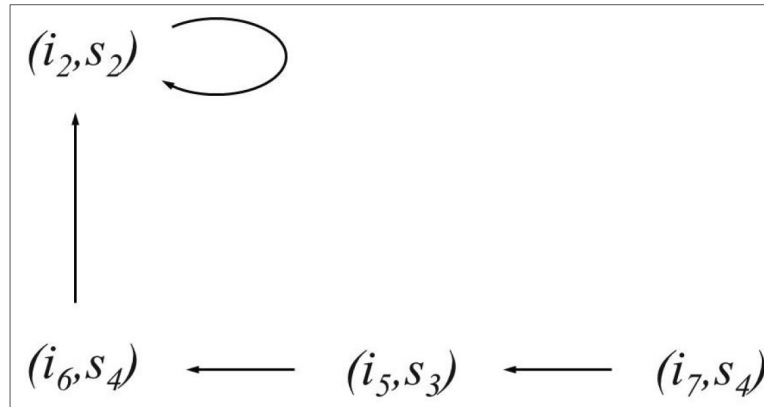
ETTC not only shares the same compelling properties with TTC, it also has more to offer in terms of equity. To make the argument more transparent, we give examples to compare ETTC with TTC in terms of equity aspects. The advantage of ETTC over TTC is most apparent when we consider cycles consisting of two student-school pairs.

**Example 2. ETTC vs. TTC when there are cycles consisting of two student-school pairs:** Let  $I = \{i_1, i_2, \dots, i_7\}$  and  $S = \{s_1, s_2\}$ , where school  $s_1$  has three slots, and school  $s_2$  has four slots. The priorities for the schools and the preferences of the students are given as follows:

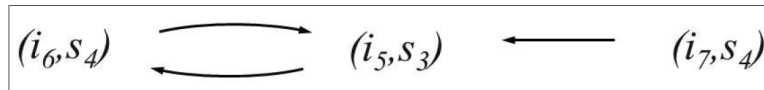




**(a)**



**(b)**



**(c)**

Figure 1: Steps of the ETTC for the Example 1.

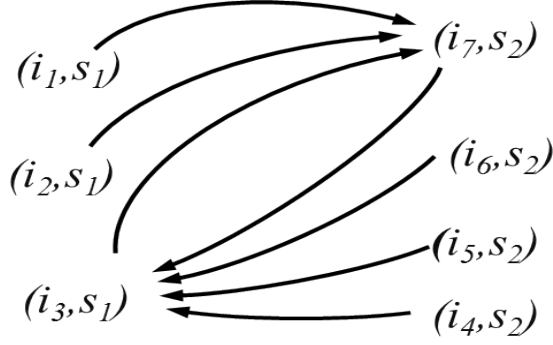


Figure 2: The first step of ETTC algorithm for Example 2.

$\succ_{s_1}$	$\succ_{s_2}$							
$i_1$	$i_4$							
$i_2$	$i_5$							
$i_3$	$i_6$							
$i_7$	$i_7$							
$i_5$	$i_3$							
$i_6$	$i_1$							
$i_4$	$i_2$							
		$P_{i_1}$	$P_{i_2}$	$P_{i_3}$	$P_{i_4}$	$P_{i_5}$	$P_{i_6}$	$P_{i_7}$
		$\boxed{s_2}$	$\boxed{s_2}$	$\boxed{s_2}$	$\boxed{s_1}$	$\boxed{s_1}$	$\boxed{s_1}$	$\boxed{s_1}$
		$s_1$	$s_1$	$s_1$	$\boxed{s_2}$	$s_2$	$s_2$	$\underline{s_2}$

Here, students  $i_4$ ,  $i_5$ ,  $i_6$ , and  $i_7$  have identical preferences, and are competing for one slot at school  $s_1$ . According to the priority order  $\succ_{s_1}$ , it is student  $i_7$  who “deserves” one slot at school  $s_1$  before any other student among the four students.

When we apply the TTC algorithm to this problem, three cycles form in the first three steps. In these cycles, each of students  $i_1$ ,  $i_2$ , and  $i_3$  trades one slot of school  $s_1$  for one slot of school  $s_2$  with students  $i_4$ ,  $i_5$ , and  $i_6$  respectively. Student  $i_7$  inherits the last slot of school  $s_2$  and forms a self-cycle. The TTC allocation is the underlined allocation above. Note that at this allocation, student  $i_7$  has justified envy for all three students who have been placed to school  $s_1$ . Let us now apply the ETTC algorithm to the same problem.

The first step of the ETTC algorithm is depicted in Figure 2. The only cycle is  $\{(i_3, s_1), (i_7, s_2)\}$ . Student  $i_7$  is placed in school  $s_1$  and student  $i_3$  in school  $s_2$ . Then these student-school pairs are removed. In the second step, all student-school pairs containing a slot of school  $s_1$  point to the student-school pair  $(i_5, s_2)$  because student  $i_5$  has the highest priority for school  $s_1$  among the remaining students who are assigned to a slot of school  $s_2$ . All student-school pairs containing a slot of school  $s_2$  point to the student-school pair  $(i_1, s_1)$  because student  $i_1$  has the highest priority for school  $s_2$  among the remaining students who are assigned to a slot of school  $s_1$ . Then the only cycle is  $\{(i_5, s_2), (i_1, s_1)\}$ . Student  $i_5$  is placed in school  $s_1$  and student  $i_1$  to school  $s_2$ . Continuing in a similar way, we obtain the allocation marked with rectangles. Note that this allocation is stable.<sup>20</sup> Now student  $i_7$  is better off whereas student  $i_4$  is worse off as compared to the outcome of TTC. Unlike ETTC, TTC severely violates the priorities of student  $i_7$  by giving

<sup>20</sup>Of course, this is not to say that it will be the case in general. In fact, there is no strategy-proof and Pareto efficient mechanism that selects the Pareto efficient and stable matching whenever such a matching exists (Kesten, 2010).

all the trading power for slots of school  $s_2$  to student  $i_4$ . ♦

One can also observe that the magnitude of the priority violations caused by TTC in the way described in the above example grows with the number of available slots. In the U.S. and abroad, many school district authorities put limitations on the number of schools a student can list in his preferences.<sup>21</sup> In Columbus for example, the Columbus city school district allows each student to apply to at most three schools. Such a restriction would considerably limit the number of participants in a cycle. The next two results establish the superiority of ETTC over TTC when this limitation comes naturally.

**Proposition 3** *Suppose there are two schools. Let  $i$  be any student who is entitled to one slot at a school  $s \in S$ , i.e., he is one of the students in the top  $q_s$  priority group. Then, under ETTC student  $i$  never has justified envy. This is not the case under TTC.*

**Proposition 4** *Suppose there are two schools. If ETTC selects an unstable allocation for a problem, then TTC also selects an unstable allocation for the same problem. However, the converse is not necessarily true.*

**Remark 2.** The two-school restriction in Propositions 3 and 4 renders a clear analytical comparison of the two mechanisms possible. Nevertheless, these results are not meant to say that ETTC improves upon TTC only when there are two schools. Indeed, the simulation results for more general cases, reported in the subsequent experimental analysis, also indicate a smaller number of priority violations under ETTC relative to TTC. In addition to the constraints imposed on the lengths of preference lists, another reason that may also contribute to the relative stability difference between the two mechanisms is that student preferences are often correlated in practice,<sup>22</sup> and this salient feature further restricts the scope of large exchanges. For example, in the extreme case of perfect correlation i.e., when preferences are identical across all students, only self-cycles form under either mechanism.

## 4.2 Cyclewise Equitable Top Trading Cycles Mechanisms

One plausible idea is to establish equity among arbitrary sized trades that possibly involve more than two student-school pairs. To fix ideas, consider a potential three-way cycle that will involve a first student-school pair chosen from those pairs containing school  $a$ , a second pair chosen from those containing school  $b$ , and a third chosen from those containing school  $c$ . In order to avoid possible priority violations in the resulting allocation, equity would require that the first pair we choose contain the highest  $b$ -priority student, the second pair contain the highest  $c$ -priority student, and the third contain the highest  $a$ -priority student. This reasoning motivates an interesting pointing rule that belongs to the second type of pointing rules.

<sup>21</sup>See Haeringer and Klijn (2009) for the implications of such a restriction on the equilibria of the resulting preference revelation games under TTC and DA. See also Pathak and Sönmez (2011) for its effects on students incentives.

<sup>22</sup>Motivated by this observation, Abdulkadiroğlu et al. (2009a) study a stylized model of school choice where students have identical preference rankings.

For simplicity, first suppose that (any student in) any student-school pair containing the same school has the same favorite school, e.g., every student-school pair of the kind  $(\cdot, x)$  will point to a pair containing school  $y$ . We require that *each student-school pair*, e.g., all pairs of the kind  $(\cdot, a)$  in the example, *point to the student-school pair holding his favorite school*, e.g., one of the kind  $(\cdot, b)$  in the example, *which contains the current highest priority student for the school which is the favorite school of the latter kind of pairs*, e.g., highest  $c$ -priority student contained in pairs of the kind  $(\cdot, b)$  in the example.

The rationale behind such a pointing rule is clear. In a top trading cycles procedure for a student-school pair, being pointed to is tantamount to gaining trading power. Hence when determining which pair needs to be pointed to among a set of pairs that are competing for a particular school, equity necessitates that the decision should be based on students' priority for that particular school.

One subtlety that is missing in the above discussion is that two pairs that contain the same school may have different favorite schools at a given instant of the algorithm. For example, suppose that while the favorite school of pair  $(j, y)$  is school  $z'$ , the favorite school of pair  $(k, y)$  is a different school  $z''$ . Meanwhile, suppose that  $y$  the favorite school of pair  $(i, x)$  at this instant. Then the question is: which of the two pairs will be pointed to by pair  $(i, x)$ ?

One solution to this question is to consider pointing *correspondances*, whereby a pair possibly points to multiple pairs. For instance, in the above example we can require  $(i, x)$  to point to the highest  $z'$ -priority pair among those pairs containing school  $y$  whose favorite school is  $z'$ , *and* point to the highest  $z''$ -priority pair among those pairs containing school  $y$  whose favorite school is  $z''$ . On the other hand, pointing correspondances give rise to cycle selection issues, as we may now have overlapping cycles once a pair is allowed to point to multiple pairs. Therefore, one can imagine a rich inventory of cycle selection methods including those based on cycle size, composition (e.g., affirmative action considerations), equity across cycles, etc.

Although pointing correspondances may allow for superior equity gains compared to the first type of pointing rules that depend only on the exogenous specifications, the preceding discussion leads to two observations about such correspondances. First, they involve critical choice decisions depending on the desideratum of the market designer.<sup>23</sup> And second, although Pareto efficiency of the resulting mechanisms is ensured, use of cycle selection rules makes these mechanisms vulnerable to strategic behavior.<sup>24</sup>

Given that pointing correspondances may be able to improve upon ETTC on the equity front, a curious question at this point is whether there can be any pointing rule/correspondance such that the associated mechanism selects the stable and efficient allocation when it exists? The following example leads to a negative answer.

**Example 3. (No top trading cycles mechanism, strategy-proof or not, selects the stable and efficient allocation)** Let  $I = \{i_1, i_2, i_3\}$  and  $S = \{s_1, s_2, s_3\}$ , where each school has one slot. The priorities for the schools and the preferences of the students are given as follows:

<sup>23</sup>Similar choice decisions are the subject of Roth et al. (2005) in the context of kidney exchange, where the authors propose and discuss a wide range of cycle and chain selection rules.

<sup>24</sup>In fact, it is possible to construct examples showing that strategy-proofness is lost regardless of the cycle selection rule. Of course, this is not to say that these mechanisms are easily manipulable, especially in a large market where participants lack complete information. A thorough investigation of pointing correspondances and the ensuing top trading cycles mechanisms is beyond the scope of the present paper.

$\succ_{s_1}$	$\succ_{s_2}$	$\succ_{s_3}$	$P_{i_1}$	$P_{i_2}$	$P_{i_3}$
$i_1$	$i_2$	$i_1$	<u><u>S2</u></u>	<u><u>S1</u></u>	<u><u>S1</u></u>
$i_3$	$i_3$	$i_2$	$s_3$	<u><u>S3</u></u>	<u><u>S3</u></u>
$i_2$	$i_1$	$i_3$	$s_1$	$s_1$	$s_2$

The school-student pairs that form are  $(i_1, s_1)$ ,  $(i_1, s_3)$ , and  $(i_2, s_2)$ . Regardless of the pointing rule, a single cycle forms: the pairwise cycle  $\{(i_1, s_1), (i_2, s_2)\}$ . Therefore, any top trading cycles mechanism selects the underlined allocation above. However, the unique Pareto efficient and stable allocation is the one marked with boxes. ♦

## 5 Experimental design

We designed an experiment to compare the performances of the TTC and ETTC mechanisms, with particular attention to the question of the elimination of justified envy situations. We implemented two treatments which differ only in the mechanism which is used to allocate slots in schools. We ran three environments in each treatment. In each environment there are 10 students and 10 school slots.

### 5.1 Designed environment

There are three schools in this environment. Schools A and B have three slots, school C has four slots. The preferences are generated by the algorithm which is similar to the one used in Chen and Sönmez (2006) and approximates the preference formation in the real market. The payoff of each student depends on the quality of the school,<sup>25</sup> the proximity of the school and the presence of siblings in the school at the time of application.

Thus each student's utility ranking of the schools is generated by the following utility function:

$$u^i(s) = u_q^i(s) + u_p^i(s) + u_s^i(s) + u_r^i(s)$$

$u_q^i(s)$  represents the quality utility for student  $i$  at school  $s$ . It equals 40 for school A, and 20 for schools B and C for all students.

$u_p^i(s)$  represents the proximity utility for student  $i$  at school  $s$ . It equals 10 for school  $s$ , if the student lives in the district of school  $s$ . Students 1, 4 and 7 live in the district of school A. Students 2, 5 and 8 live in the district of school B. Students 3, 6, 9 and 10 live in the district of school C.

$u_s^i(s)$  represents the utility from having a sibling at school  $s$ . It equals 20 if a student has a sibling in the school. Student 1 has a sibling at school B, students 4 and 7 have siblings at school C.<sup>26</sup>

<sup>25</sup>There is only one good school in this environment, namely school A. Schools B and C are of equal low quality.

<sup>26</sup>We introduced this utility in order to create preferences where the top choice of a student does not correspond to the school of her district, as this situation is not interesting from an experimental perspective.

$u_r^i(s)$  represents a random utility which captures diversity of tastes. It is drawn from the uniform distribution from the interval  $[0,20]$ . It reflects a variety of preferences depending on abilities and interests.

The monetary payoffs for subjects were determined based on the resulting ranking. They received €15 for getting a slot in the school of their top choice, €10 for the second choice and €5 for the last choice.

Table 1 presents the preferences of students in this designed environment.

ID	1	2	3	4	5	6	7	8	9	10
Top choice	B	C	B	C	A	A	C	A	A	B
2nd choice	A	A	A	B	C	C	A	B	B	A
3rd choice	C	B	C	A	B	B	B	C	C	C

Table 1: Preferences of students in the designed environment

In all environments each school has a strict priority order of students of other districts. In the designed environment these priority orders are generated so as to insure that an allocation with no justified envy is possible under mechanisms of interest. As for students who live in the school district, they have the highest and equal priority for that school. The weak priority of students of the district is transformed into a strict priority by a random draw in the experiment. Students' draws are transformed into a queue used for tie-breaking when the allocation is calculated in the experiment.

Table 2 presents the priority orders of schools in the designed environment.

School	A	B	C
<b>The highest priority</b>	<b>1,4,7</b>	<b>2,5,8</b>	<b>3,6,9,10</b>
The second highest	10	10	7
.	8	9	2
.	9	1	1
.	3	7	8
.	2	4	5
.	5	6	4
No priority	6	3	

Table 2: Priority orders of schools in the designed environment

## 5.2 Random-correlated environment

It is often the case in real life that different students want different schools depending on their abilities.<sup>27</sup> Given the abilities, students' preferences are strongly correlated, at least for the top

<sup>27</sup>Chen and Sönmez (2006) differentiate between two schools: one is stronger in the arts, another one in sciences. Here we employ a similar idea.

choices. The random-correlated treatment is constructed to approximate this structure, assuming there is no utility of proximity to school.<sup>28</sup>

There are five schools in this environment - A, B, C, D and E. Each school has only two slots. Students 1 and 2 live in the district of school A, students 3 and 4 live in the district of school B, students 5 and 6 live in the district of school C, students 7 and 8 live in the district of school D, students 9 and 10 live in the district of school E.

Six students (1–6) prefer either school D or E, with the other one coming second in preferences, and the other four students (7–10) prefer either school A or school C. Note that it is insured that students 1–6 do not live in the district of schools D and E, as well as that students 7–10 do not live in the districts of schools A and C. The rankings from the third to the last (5th) are generated randomly. In the experiment subjects received €15 for getting a slot in the school of their top choice, €12.50 for the second choice, €10 for the third choice, €7.50 for the fourth choice and €5 for the last choice.<sup>29</sup>

Table 3 presents the preferences of students in the random-correlated environment.

ID	1	2	3	4	5	6	7	8	9	10
Top choice	D	D	E	E	D	E	C	C	C	A
2nd choice	E	E	D	D	E	D	A	A	A	C
3rd choice	B	A	B	C	A	A	E	E	E	E
4th choice	C	B	A	B	C	C	B	B	B	B
Last choice	A	C	C	A	B	B	D	D	D	D

Table 3: Preferences of students in the random-correlated environment

Unlike in the designed environment, the schools' priority orders of non-district students are generated in such a way that a fair allocation is not feasible under both mechanisms. Table 4 presents the priority orders of schools in the random-correlated environment.

### 5.3 Random-uncorrelated environment

In order to check the robustness of the results we create a random-uncorrelated environment. There are four schools in this environment – A, B, C and D. Schools A and B have two slots each, and schools C and D have three slots each. Students 1 and 2 live in the district of school A, students 3 and 4 live in the district of school B, students 5, 6 and 7 live in the district of school C, students 8, 9 and 10 live in the district of school D.

The preferences of each student are generated randomly. In situations when the district school has the highest payoff, the preferences of the students were regenerated. The resulting preferences are presented in Table 5. In the experiment subjects received €15 for getting a slot in the school of their top choice, €11.50 for the second choice, €8 for the third choice, €4.50 for the fourth choice.

<sup>28</sup>One could think of this environment as a game, where only the most ambitious students from districts with relatively bad schools are competing to get slots in the best schools of the city.

<sup>29</sup>The payoffs for the schools are different in all environments, but we keep the payoff for the first choice the same – 15 euro. The payoff for the least preferred choice is also almost the same (5 or 4.5 euro). Payoffs for the other schools are generated in order to keep the same difference in payoffs between first and second schools, second and third schools and so on.

School	A	B	C	D	E
<b>The highest priority</b>	<b>1, 2</b>	<b>3, 4</b>	<b>5, 6</b>	<b>7, 8</b>	<b>9, 10</b>
The second highest .	10	6	10	9	3
.	8	5	4	10	2
.	5	8	1	3	5
.	6	7	7	2	1
.	9	10	8	1	6
.	3	2	9	4	8
.	4	1	2	6	4
No priority	7	9	3	5	7

Table 4: Priority orders of schools in the random-correlated environment

ID	1	2	3	4	5	6	7	8	9	10
Top choice	D	C	C	A	D	A	B	C	C	B
2nd choice	A	A	B	B	B	D	C	A	D	D
3rd choice	B	D	D	C	A	C	A	D	A	C
Last choice	C	B	A	D	C	B	D	B	B	A

Table 5: Preferences of students in the random-uncorrelated environment

The priority orders for students from other districts were generated randomly and presented in Table 6

School	A	B	C	D
<b>The highest priority</b>	<b>1,2</b>	<b>3,4</b>	<b>5,6,7</b>	<b>8,9,10</b>
The second highest	7	8	9	7
.	9	6	2	4
.	10	5	8	1
.	4	10	4	3
.	8	9	3	2
.	6	7	1	6
.	5	2	10	5
No priority	3	1	.	.

Table 6: Priority orders of schools in the random-uncorrelated environment

Both mechanisms are implemented as one-shot games of complete information. Each subject knows her own payoff table, the preference tables of other participants and the priority orders of schools.

The experimental design allows us to test three hypotheses based on the theoretical properties of TTC and ETTC:

**Hypothesis 1:** Participants of the experiment choose to state their true preferences for allocations under both TTC and ETTC as both mechanisms are strategy proof.

**Hypothesis 2:** TTC and ETTC should not differ with respect to the efficiency criteria, as



both mechanisms are Pareto-efficient.

**Hypothesis 3:** On average, the number of justified envy situations generated by ETTC should be lower than those of TTC.

## 5.4 Experimental procedures

The experiment was run at the experimental economics lab at the Technical University Berlin. We recruited subjects from our pool with the help of ORSEE by Greiner (2004). In total, 14 sessions were conducted, that is, seven sessions per treatment. In total, 140 subjects participated in the experiment. Each session included three environments which were played in random order. No feedback was provided between environments, thus we assume no learning effects. Only one of the three environments was payoff relevant for subjects and it was determined at the end of the experiment by a random draw. All subjects played in a perfect information environment, thus they were aware of the preferences of other players, as well as of the priorities of the schools. The experiment was paper-based. The sessions lasted approximately 70 minutes, with 40–45 minutes used for instructions and the public solution of an example. The average payoff, including a participation fee, was €15.32.

In the experiment, subjects were randomly assigned a seat in the lab which corresponds to a subject ID. The experimenter readed the instructions aloud. Subjects were allowed to ask questions, which were answered in public. Then subjects had an additional 20 minutes to go through the explanation of the mechanism and the solution to the example. After all subjects had completed the reading, an experimenter went through the solution of an example in public. Subjects then had 15 minutes to solve an allocation problem. Before the decision sheets were distributed, the subjects drew a number from the bag to determine the initial queue, which was used to tie-break the priority orders of students in the school of their district (one for each of the environments). Subjects were not made aware of the number they had drawn, neither were they aware of the procedure which mapped the drawn numbers into the queue. Then decision sheets were distributed for one of the three environments. After subjects made their decisions, the sheets were collected, and the decision sheets for the next environment are distributed. After all the decisions for all the environments were completed, subjects filled out the questionnaire, while the experimenter typed in the data to calculate the outcome. The feedback on the allocations was given, together with the payment, to every subject in private. Instructions which were used in the experiment can be found in the Appendix.

## 6 Experimental Results

We compare the outcomes generated by TTC and ETTC to evaluate the mechanisms. There are three important criteria of comparison: truthful preference revelation, efficiency of allocation, and fairness of allocation.

We first look at the proportions of truthful preference revelations. As under both TTC and ETTC, slots in the district schools are guaranteed for students, the truthful preference revelation requires that reported rankings coincide with truthful preferences from the top choice up to the district school of the participant.

	Designed environment	Random- correlated environment	Random- uncorrelated environment
TTC	57%	30%	37%
ETTC	56%	27%	40%
Test of propor- tions, p value	0.865	0.708	0.728

Table 7: Truth-telling rates

**Result 1 (Truthful Preference Revelation):** *In all environments, the differences in proportions of truthful preference revelation under TTC and under ETTC are not statistically significant.*

Table 7 presents the proportions of truthful preference revelation for each mechanism in each of the environments. The test of equality of proportions shows that proportions of truthful preference revelation under ETTC are not significantly different from that under TTC in all environments:  $z=0.170$  ( $p=0.805$ ) for the designed environment,  $z=0.374$  ( $p=0.708$ ) for the random-correlated environment and  $z=0.3473$  ( $p=0.728$ ) for the random-uncorrelated environment. The highest proportion of truthful preference revelation is in the designed environment, 57% under TTC and 56% under ETTC, and the lowest in the random-correlated environment: 30% and 27% correspondingly. Thus, neither TTC nor ETTC induced full truthful preference revelation for all participants and therefore we reject Hypothesis 1. The proportions of truthful preference revelation in designed and random-uncorrelated environments for TTC are similar to the ones in Chen and Sönmez (2006) (50% and 43% respectively). Low truthful-telling rates in the random-correlated environment can be explained by high correlation of preferences within two top choices, high number of schools and relatively low loss in payoffs by missing one rank reached. The variation shows that participants in the experiment react to the environment in the expected way.

The second criterion we consider is efficiency. We follow the cardinal concept of utility. Therefore we use the following formula to calculate the efficiency of allocation:

$$Efficiency = \frac{Actual\ sum\ of\ payments\ to\ participants}{Sum\ of\ payments\ to\ participants\ if\ they\ all\ state\ the\ truth}$$

Thus, we refer to the allocation with full preference revelation as being efficient. As we implemented sessions as one-shot games, we follow Chen and Sönmez (2002) in using the recombinant estimation technique proposed by Mullin and Reiley (2006) and modified by Abrevaya (2008)). Due to a relatively high number of sessions for each treatment, the full recombination is too demanding as it includes  $(7)^{10}$  recombinations. That is why we take 20,000 random recombinations of subjects from different sessions, keeping the ID fixed, and calculating the outcomes of interest. Thus, for every ID of a combination we just randomly determine a natural number from 1 to 7 from the uniform distribution which corresponds to the number of the session. We repeat this procedure 20,000 times, and control for the absence of repetitions afterwards. Based on these 20,000 combinations, we calculate a recombinant estimator for the expected value of the allocation outcome of interest. For the allocation of TTC the initial queue of subjects which is used to

break ties in the priority order of local students is crucial. To address this issue we calculate 10 different allocations for each combination for TTC based on 10 random initial queues. Note that ETTC does not require a tie-breaking of the priority order for local students. Thus we produce 200,000 allocations for each environment for TTC, and 20,000 allocations for each environment for ETTC. Following the results of Abrevaya (2008) we estimate asymptotic standard errors and asymptotic variance for these outcomes. Based on asymptotic variance we construct a Z-test for the significance of the difference between treatments.

**Result 2 (Efficiency):** *In all environments, the difference in efficiency under TTC and ETTC is not statistically significant.*

Table 8 presents the results of the recombinant estimation of expected efficiency in different environments. A two-sided Z test based on recombinant estimation of asymptotic variance shows that the efficiency of the allocation under ETTC is not significantly different from that under TTC in all environments:  $z=0.380$  ( $p=0.70$ ) for the designed environment,  $z=0.248$  ( $p=0.39$ ) for the random-correlated environment and  $z=0.064$  ( $p=0.95$ ) for the random-uncorrelated environment. Thus we can support Hypothesis 2.

Mechanism		TTC	ETTC	p-value
Designed environment	Mean efficiency	86.38%	82.37%	0.70
	Asymptotic standard error	0.026	0.028	
Random-correlated environment	Mean efficiency	91.37%	89.30%	0.80
	Asymptotic standard error	0.015	0.019	
Random-uncorrelated environment	Mean efficiency	84.51%	84.65%	0.95
	Asymptotic standard error	0.001	0.001	

Table 8: Mean efficiency

The main interest of comparison of mechanisms lies in the equity comparison. Using allocations from recombination we calculate two equity parameters:

1. *The number of justified envy situations* depicts the total number of student pairs where one student has justified envy to another second student in any school. If one student has justified envy to several students in one or more schools, each of the cases is counted for the sum of justified envy situations.

2. *The number of students with justified envy* depicts the number of students who have justified envy with regard to at least one other student.

**Result 3 (Number of justified envy situations):** *In the designed and random-correlated environments ETTC produces significantly less justified envy situations than TTC does. In the random-uncorrelated environment, the difference in the number of justified envy situations produced by ETTC and TTC is not statistically significant.*

Table 9 represents the results of the recombinant estimation of the mean number of justified envy situations under both mechanisms in all environments. In the designed environment,

Mechanism		TTC	ETTC	p-value
Designed environment	Mean number of justified envy outcomes	4.76	2.89	0.03
	Asymptotic standard error	0.66	0.58	
Random-correlated environment	Mean number of justified envy situations	9.62	8.43	0.00
	Asymptotic standard error	0.61	0.16	
Random-uncorrelated environment	Mean number of justified envy situations	3.80	3.42	0.25
	Asymptotic standard error	0.42	0.49	

Table 9: Mean number of justified envy situations

the mean number of justified envy situations under ETTC is 2.89 which is significantly less than 4.76 under TTC. The two-sided Z test based on recombinant estimation of asymptotic variance yields  $z=3.723$  ( $p=0.00$ ). In the random-correlated environment the relation is 8.43 to 9.62 correspondingly, and the difference is statistically significant:  $z=3.01$  ( $p=0.00$ ). As for the random-uncorrelated environment, the expected number of justified envy situations under ETTC is 3.42 and under TTC is 3.80, but the difference is not statistically significant:  $z=1.23$  ( $p=0.22$ ).

**Result 4 (Number of students with justified envy):** *In the designed and random-correlated environments under ETTC a significantly lower number of students have justified envy to other students than under TTC. In the random-uncorrelated environment, the difference in the number of students with justified envy under ETTC and TTC is not statistically significant.*

Mechanism		TTC	ETTC	p-value
Designed environment	Mean number of students with justified envy outcomes	3.85	2.25	0.00
	Asymptotic standard error	0.48	0.42	
Random-correlated environment	Mean number of students with justified envy situations	7.04	5.61	0.00
	Asymptotic standard error	0.29	0.36	
Random-uncorrelated environment	Mean number of students with justified envy situations	2.97	2.98	0.98
	Asymptotic standard error	0.29	0.45	

Table 10: Mean number of students with justified envy situations

Table 10 represents the results of the recombinant estimation of the number of students with justified envy under both mechanisms in all environments. In the designed environment, on average 2.25 out of 10 students have justified envy to at least one student under ETTC,

and 3.85 have it under TTC. The Z-test for equal means yields  $z=3.733$  ( $p=0.00$ ). For the random-correlated environment the number of students with justified envy is 5.61 and 7.04 correspondingly. And the difference is statistically significant:  $z=3.924$  ( $p=0.00$ ). In the random-uncorrelated environment the expected number of students with justified envy is 2.98 under ETTC and 2.97 under TTC, and the difference is not statistically significant:  $z=0.021$  ( $p=0.98$ ).

Combining result 3 and result 4 we can conclude that our Hypothesis 3 is only partially supported by the experimental results. It holds for the case of the designed and the random-correlated environments, but is rejected in the case of the random-uncorrelated environment.

In spite of the fact that in the random-uncorrelated environment ETTC produces a result similar to TTC from the perspective of the equity criterion, one should not underestimate the benefits of ETTC, as random-uncorrelated preferences are unlikely to appear in reality. Moreover, the result in the experiment is driven by the high rate of misrepresentations of preferences in the form of district school bias in the random-uncorrelated environment in both mechanisms. It can be explained by the fact that, by design, stating the district school as a top choice in the random-uncorrelated environment leads to a much lower average loss in pay-offs than in other environments. Note that seven out of 10 subjects have the district school as the second-best school in their preferences in the random-uncorrelated environment, while only three out of 10 in the designed environment, and none in the random-correlated environment. Thus in the random-uncorrelated environment it is likely that preferences are reported in a way that at most one local student in each school will apply to another school as the top choice. This fact diminishes the advantage of ETTC relative to TTC because it is identical to the case when each school has only one slot to allocate,<sup>30</sup> where TTC and ETTC are equivalent.

Next, we address the following question: if one cares about equity, given the stated preferences, which mechanism (TTC or ETTC) is fairer? Using the recombinations of stated preferences from both treatments, we create 20,000 different preference profiles in each of the three environments.<sup>31</sup> We use both mechanisms to calculate the allocation for each preference profile and 10 random initial queues and evaluate the allocation with respect to the *stated* preferences. Thus, in total we calculate 200,000 allocations for each mechanism in each environment. We run the procedure, similar to the ordinal efficiency test by Guillen and Kesten (2012). We calculate the number of times TTC dominates ETTC with respect to the equity criterion (TTC dominations)<sup>32</sup> and the number of times ETTC dominates TTC (ETTC dominations). We calculate it for both equity criteria: the number of justified envy situations and the number of students with justified envy. Finally, we use the Wilcoxon signed-rank test for the equality of dominations.

**Result 5 Equity dominance** *ETTC is more likely to generate less justified envy situations and less students with justified envy than TTC in all environments, given the students reveal their true preferences.*

For the number of justified envy situations, the Wilcoxon signed-rank test rejects the hypothesis of equality in the number of TTC and ETTC dominations in all environments. In the

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<sup>30</sup>As all other local students who applied to their home school can be excluded from the procedure, their tentative assignments are finalized.

<sup>31</sup>Alternatively, one can interpret the subsequent analysis as simulations.

<sup>32</sup>TTC creates less justified envy situations than ETTC.

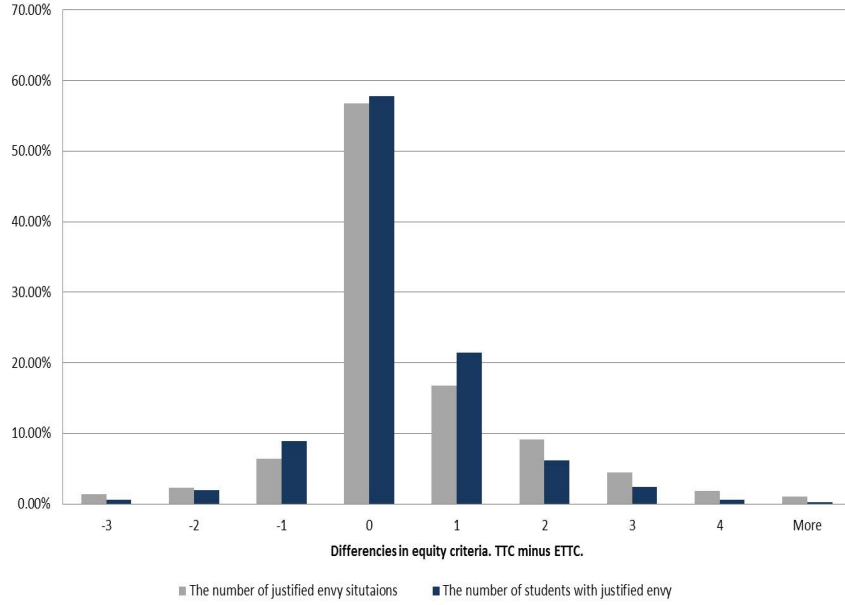


Figure 3: Difference in equity criteria in random environment

designed environment the number of TTC dominations is 16,990 out of 200,000 and the number of ETTC dominations is 111,924 leading to  $z=272.405$  ( $p=0.00$ ). In the random-correlated environment the number of TTC dominations is 33,815 and the number of ETTC dominations is 84,562 leading to  $z=152.338$  ( $p=0.00$ ). In the random-uncorrelated environment the number of TTC dominations is 20,231 out of 200,000 and the number of ETTC dominations is 66,294 leading to  $z=159.079$  ( $p=0.00$ ).

For the number of students with justified envy, the Wilcoxon signed-rank tests of equality in the number of TTC and ETTC dominations yield  $p$ -values  $< 0.01$  in all environments as well.

The histograms for the differences of equity criteria in a random environment are presented in Figure 3.

Result 5 is one of our main results from the perspective of policy. We showed that given Pareto efficiency and strategy proofness of the allocation procedure, caring about the frequency of justified envy situations while choosing between competing mechanisms, one should prefer ETTC to TTC.

Next we address the question of fairness from the individual's perspective. We construct a dataset on an individual level in the following way: we take 100 allocations for each of the participants in the experiment, based on a different recombination of the rest of the group, and calculate the individual result for the participant in both mechanisms. As a result, we have a dataset containing the following variables: a dummy for stating the truth; a dummy for having justified envy situations; and the rank reached by each participant. We assume that the revelation of true preferences is an exogenous decision, independent of the mechanism. Thus we interpret it as individual-based. This assumption is in line with the results of the experiment as there is no significant difference in the proportion of truth revelation across mechanisms. Thus we focus on the question: if one could choose between TTC and ETTC, which mechanism would she prefer

on average when controlling for the rank reached and truthful revelation of preferences?<sup>33</sup>

**Result 6 (Individual equity).** *The probability of a student having justified envy to any other student, controlling for preferences revelation and rank reached, is lower under ETTC than under TTC.*

Table 11 shows the results of the estimation of the probit model for the probability of having justified envy based on clustered robust standard errors at the level of individuals. The coefficients for the ETTC dummy are negative and statistically significant at the 5% level.

Table 11: Probit estimation of the dummy for justified envy

Variable	Coefficient (Std. Err.)
ETTC dummy	-0.229** (0.105)
Truth dummy	-0.239** (0.094)
Rank reached	1.052*** (0.069)
Intercept	-2.469*** (0.160)
N	84000
Log-likelihood	-34788.2
$\chi^2_{(3)}$	288.969

## 7 Conclusion

The contribution of this paper has been twofold. First, we have argued that although TTC has been a very attractive assignment tool in the literature mainly due to its efficiency and incentive properties; whenever the indivisible goods are in multiple supply such as in the school choice context, the same method can also be used in a much more flexible way to meet other distributional goals, thanks to the additional freedom the planner has emanating from the multitude of trading opportunities in general TTC mechanisms. To explore this potential one needs to first generate a thick market where all supplies of the goods are concurrently up for trade and then specify a pointing rule suitable for the planner's objectives. Second, we have proposed and studied a particular application of this idea to school choice as a potentially more equitable mechanism than TTC when dealing with pairwise exchanges which often arise under correlated preference

<sup>33</sup>We assume that from individual perspective the fairness is less important than the rank reached and we control for it explicitly in the regression. Thus, we assume that any subject would prefer an allocation to the second choice, with justified envy to an allocation to the third choice without justified envy.

profiles. We have shown that establishing full equity may not be possible via any mechanism employing the top trading cycles idea.

The findings from our experiment are consistent with the theory, as ETTC generally provides better equity results. In the most realistic settings (designed and random-correlated environments) ETTC generates a significantly lower number of students with justified envy as well as significantly less cases of justified envy. In the simulated settings, when allocations are evaluated with respect to stated choices, ETTC dominates TTC with respect to the equity criteria in all environments, including a random-uncorrelated environment. We have shown that from an individual point of view, ETTC is less likely to lead to justified envy than TTC given the rank reached in the allocation.

ETTC has an additional advantage over TTC: it does not depend on the random tie-breaking of the top (up-to-quota) priority students. Notwithstanding our modeling assumption that priorities of the schools are strict and exogenously given, ETTC can easily be adopted to the coarse priority structures which are commonly observed in reality.<sup>34</sup> In cases when the number of students in the top priority class is more than the number of slots in the school, the slots are guaranteed for these students under both TTC and ETTC. However, only ETTC treats all students with the guaranteed slots *equally*, whereas TTC would require the use of tie-breaking among the students within the same class, thus unnecessarily favoring some students over others. In other words, although random tie-breaking does not harm student welfare under TTC as opposed to the case of the well-known DA (cf. Erdil and Ergin, 2008, Abdulkadiroğlu et al. 2009), it may nonetheless introduce an artificial loss of fairness. On the other hand, ETTC guarantees equal treatment of all students within the highest priority group, without requiring random-tie breaking and can eliminate avoidable justified envy situations.

## 8 The Appendix

### PROOF OF PROPOSITION 1

**Lemma 1** *Given a problem and a step of the ETTC algorithm, if the best choice of a student  $i$  among the remaining slots is a slot at school  $s$ , then student  $i$  does not ever point to a pair which contains a slot from a different school until no vacant slots of school  $s$  remain.*

**Proof:**

This simply follows from the fact that for each school  $s$  whose slots remain to be inherited from earlier steps, the to-be-inherited slots are assigned to the remaining students, immediately after the step at the end of which no student who was assigned to a slot of school  $s$  at an earlier step is left.

**Q.E.D.**

A critical observation about the ETTC algorithm is that by Lemma 1, if a school still has vacant slots at a step of the ETTC algorithm, then there is a student-school pair containing a slot from that school at that step. Then, the idea behind the Pareto efficiency of ETTC is the

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<sup>34</sup>For example, in Boston the top priority group of students consists of those students who live in the walk zone of the school and have a sibling. This priority group is unlikely to fill all the quotas in any given school.



same as that of TTC. Given a problem, each student who leaves at the first step is placed in his best choice, hence he can not be made better off. Each student who leaves at the second step is placed in his best choice among the remaining schools, hence he cannot be made better off without making someone who left at the first step worse off. Continuing in this way, no student can be made better off without making someone who left at an earlier step worse off.

**Q.E.D.**

## PROOF OF PROPOSITION 2

**Lemma 2** *Given a problem  $((\succ_s)_{s \in S}, (P_i)_{i \in I})$ , suppose a student  $i$  is removed at a step  $t$  of the ETTC algorithm and, if he submits preferences  $R'_i$  instead of  $R_i$ , then he is removed at a step  $t'$ . Then the remaining students, available slots of schools, and to-be-inherited slots of schools at the beginning of step  $\min\{t, t'\}$  are the same.*

**Proof:**

Given a problem  $((\succ_s)_{s \in S}, (P_i)_{i \in I})$ , because no student-school pair containing student  $i$  participates in a cycle before step  $\min\{t, t'\}$ , the same cycles form until step  $\min\{t, t'\}$  and the same students are placed in the same schools until step  $\min\{t, t'\}$ .

**Q.E.D.**

**Lemma 3** *Given a problem, if a student-school pair  $(i, s)$  is pointing to another student-school pair  $(i', s')$  at some step of the ETTC algorithm, then student-school pair  $(i, s)$  keeps pointing to the student-school pair  $(i', s')$  as long as student  $i'$  is not removed.*

**Proof:**

Given a problem, if a student-school pair  $(i, s)$  is pointing to another student-school pair  $(i', s')$  at a step  $t$ , then school  $s'$  is the best choice of student  $i$  among the remaining schools. Furthermore, student  $i'$  is the student among those who are assigned to a slot from school  $s'$  who has the highest priority for school  $s$ . By Lemma 1, student  $i$  will keep pointing to a pair containing a slot from school  $s'$  until no vacant slots at school  $s$  remain. Then the only case student-school pair  $(i, s)$  can point to another student-school pair  $(i'', s'')$  before student  $i'$  is removed is when (i)  $s'' = s'$  and (ii) student  $i''$  has higher priority than student  $i'$  for school  $s$ . But this is only possible if student  $i''$  is assigned to a slot of school  $s$  through inheritance at some step  $t', t' > t$ . But no inheritance of slots of school  $s'$  takes place before student  $i$  is removed.

**Q.E.D.**

Given a problem  $((\succ_s)_{s \in S}, (P_i)_{i \in I})$  and a student  $i$ , let  $t$  be the step at which student  $i$  is removed and  $s$  be the school he is placed in. We will show that if student  $i$  submits fake preferences  $R'_i$ , he can not be placed in a school which is better for him than school  $s$ . Let  $t'$  be the step at which student  $i$  is removed when he submits  $R'_i$  and  $s''$  be the school he is placed in at this step. We consider two cases:

**Case 1.**  $t \geq t'$  : Consider step  $t'$ . By Lemma 2, at the beginning of this step, the remaining students and slots are the same. Note that by Lemma 3, each student-school pair  $(i', s')$  that is pointing to a pair containing student  $i$  keeps pointing to that pair as long as student  $i$  stays. Similarly, each student-school pair  $(i'', s'')$  that is pointing to the pair  $(i', s')$  keeps pointing to

that pair as long as student  $i'$  stays which is the case as long as student  $i$  stays, and so on. Consequently, at step  $t$ , student  $i$  has the opportunity to participate in any of the cycles he participates under fake preferences  $R'_i$ . Since under his true preferences, he is pointing to his best choice at step  $t$ , school  $s$  cannot be worse than school  $s'$  for student  $i$ .

**Case 2.**  $t < t'$  : By Lemma 2, at the beginning of step  $t$ , the remaining students and slots are the same. Since student  $i$  is placed in his best choice at this step, he cannot be placed in a better school at a later step.

**Q.E.D.**

### PROOF OF PROPOSITION 3

Example 2 establishes the latter statement in the proposition. Let  $((\succ_s)_{s \in S}, (P_i)_{i \in I})$  be a problem,  $i$  a student and  $s$  a school such that  $i$  is entitled to one slot at school  $s$ . Suppose by contradiction that student  $i$  has justified envy for a student  $j$  who is placed in a school  $s'$  under ETTC. Since  $i$  is entitled to one slot at school  $s$ , we have  $s \neq s'$ . Note that student  $i$  has higher priority than student  $j$  for school  $s'$ . This means student  $j$  does not inherit any slot of school  $s'$  but in fact there is a step  $t$  such that a student-school pair  $(j, s)$  forms a cycle with another student-school pair  $(k, s')$  where  $k \neq i, j$ . Note that since student  $i$  has higher priority than student  $j$  for school  $s'$ , student-school pair  $(j, s)$  must be removed before step  $t$ . But this is possible only if student  $j$  is placed in school  $s'$ . A contradiction.

**Q.E.D.**

### PROOF OF PROPOSITION 4

Example 2 establishes the latter statement in the proposition. To prove the former statement, we shall show that if TTC selects a fair allocation for a problem, then ETTC also selects a fair allocation for the same problem. Let  $((\succ_s)_{s \in S}, (P_i)_{i \in I})$  be a problem,  $\alpha$  and  $\beta$  respectively be the TTC and the ETTC allocations. Suppose  $\alpha$  is fair. We show that  $\alpha = \beta$ . Let  $S = \{s, s'\}$ . Let  $I^* = I_s^* \cup I_{s'}^*$  be the set of students who form a student-school pair at the first step of the ETTC algorithm where  $I_s^*$  (resp.  $I_{s'}^*$ ) is the set of students who are contained in a student-school pair with school  $s$  (resp. school  $s'$ ). Also let  $I_1$  be the set of students who are contained in a student-school pair of the first step of the ETTC algorithm that form a self-cycle. Note that  $I_1 \subseteq I^*$ . Clearly,  $\alpha|_{I_1} = \beta|_{I_1}$ . If there is no cycle that forms after the student-school pairs containing the students in  $I_1$  are removed, then we proceed to the next paragraph. If there is such a cycle, call it cycle  $C_1$ . Let  $(i_1, s)$  and  $(j_1, s')$  be the two student-school pairs contained in cycle  $C_1$ . Note that student  $i_1$  (resp. student  $j_1$ ) has the highest priority for school  $s'$  (resp. for school  $s$ ) among those students in  $I^* \setminus I_1$  whose first choice is school  $s'$  (resp. school  $s$ ). The existence of cycle  $C_1$  implies that there also exists a cycle  $C'_1$  that forms at some step of the TTC algorithm such that two agents in  $I^* \setminus I_1$  point to one another. Then, since  $\alpha$  is fair and student  $i_1$  (resp. student  $j_1$ ) has the highest priority for school  $s'$  (resp. for school  $s$ ) among those students in  $I^* \setminus I_1$ , this means  $\alpha|_{\{i_1, j_1\}} = \beta|_{\{i_1, j_1\}}$ . If there is no cycle that forms after cycle  $C_1$ , then we proceed to the next paragraph. If there is such a cycle, call it cycle  $C_2$ . Let  $(i_2, s)$  and  $(j_2, s')$  with  $s \neq s'$ , be the two student-school pairs contained in cycle  $C_2$ . Note that student  $i_2$  (resp. student  $j_2$ ) has the highest priority for school  $s'$  (resp. for school  $s$ ) among those students in  $I^* \setminus I_1$  whose first choice is school  $s'$  (resp. school  $s$ ). Again, the existence of cycle  $C_1$  implies that there also exists a cycle  $C'_2$  that forms at some step of the TTC algorithm such that two agents in  $I^* \setminus (I_1 \cup \{i_1, j_1\})$  point to one another. As before, since  $\alpha$  is fair,  $\alpha|_{\{i_2, j_2\}} = \beta|_{\{i_2, j_2\}}$ . Continuing in this way, we conclude

that both ETTC and TTC agree on the placement of those students who are contained in a cycle until the inheritance of slots of some of the schools takes place for the first time.

Without loss of generality, let  $s$  be a school with  $n_s$  slots that remains to be inherited by the remaining students. (If there is no such school, then the result is trivial.) Note that at this step no student in  $I_s^*$  is left. From the preceding paragraph, the students who get the first  $q_s - n_s$  slots are the same under ETTC and TTC. This implies there is a step of the TTC algorithm in which the student with the highest priority for school  $s$  among the remaining students inherits all the  $n_s$  slots of school  $s$ . Note that under the ETTC algorithm, these  $n_s$  slots are assigned to the remaining students (starting with the student with the highest priority for school  $s$  among the remaining students) one by one following their priority order to form new student-school pairs. This means we are back where we started and can again use the same argument we used in the previous paragraph. Applying this argument iteratively, we conclude that  $\alpha = \beta$ .

**Q.E.D.**

## **INSTRUCTIONS- Mechanism TTC**

This is an experiment in the economics of decision making. The instructions are simple, and if you follow them carefully and make good decisions, you might earn a considerable amount of money. In this experiment, we simulate a procedure to allocate students to schools. The procedure, payment rules, and student allocation method are described below. Do not communicate with each other during the experiment. If you have questions at any point during the experiment, raise your hand and the experimenter will help you.

### ***Procedure***

There are 10 participants in this experiment. You are participant 1. (This number is drawn by each participant before entering the room.) Each participant represents a student, who wants to get a slot at a school. You will have to make three decisions in three different school markets. In all markets allocation will be determined by the same algorithm. This algorithm works as follows:

### ***Allocation Method***

Each participant is first tentatively assigned to the school within her respective district. Students of the school district have the highest priority in the school. Next, Decision Sheet rankings are used to determine mutually beneficial exchanges between two or more participants. The order in which these exchanges are considered is determined by a fair lottery. This means each participant has an equal chance of being the first in the line, the second in the line,..., as well as the last in the line. To determine this fair lottery, a participant will be asked to draw 10 numbers from an urn, one at a time. Each number corresponds to a participant ID number. The sequence of the draw determines the order in the lottery.

The specific allocation process is explained below.

1. Initially all 10 slots are available for allocation.
2. All participants are ordered in a queue based on the order in the lottery.

3. Next, an application to the highest-ranked school in the Decision Sheet is submitted for the participant at the top of the queue.

- If the application is submitted to her district school, then her tentative assignment is finalized (thus, she is assigned to a slot at her district school). The participant and her assignment are removed from subsequent allocations. The process continues with the next participant in line.
- If the application is submitted to another school, the procedure moves as follows: Say applicant Claudia's home district school is school A and she is applying to school B. Then Claudia's application is submitted to school B. After that, one of the students who tentatively hold the slot at school B has to be chosen. In particular, among all students, who tentatively hold a slot at school B, procedure chooses the one who is closest to the top of the queue. (So procedure follows the queue ordering while choosing among students of school B). Then this student is moved to the top of the queue directly in front of the requester (Claudia).

In general, whenever the queue is modified, the process continues similarly with the next student in the queue: An application of this student is submitted to her highest-ranked school with available slots.

- If the application is submitted to her district school, then her tentative assignment is finalized. The process continues with the next participant in line.
- If the application is submitted to another school, say school S, then we follow the following procedure (explained above for Claudia): One of the students who tentatively hold a slot at School S needs to be chosen. In particular, among all the students who tentatively hold a slot at school S, the procedure chooses the student who is closer to the top of the queue. Then this student is moved to the top of the queue directly in front of the requester.

4. A mutually-beneficial exchange is obtained when a cycle of applications are made in sequence, which benefits all affected participants, e.g., I apply to Stefan's district school, Stefan applies to your district school, and you apply to my district school. In this case, the exchange is completed and the participants as well as their assignments are removed from subsequent allocations.

5. The process continues until all participants are assigned a school slot.

### **Example:**

In order to understand the mechanism better, let us go through an example together.

If you have any questions about any step of the allocation procedure please feel free to ask at any point. There are six students (ID numbers from 1 to 6) on the market, and three schools (school A, school B, and school C) with two free slots each. Students 1 and 2 live in the district of school A, students 3 and 4 live in school district B, and, finally, student 5 and 6 live in school district C. It means that tentative assignments look as follows:

Tentative assignments of students ( IDs)	School A	School B	School C
slot 1	1	3	5
slot 2	2	4	6

The lottery determined the following order (student IDs): 1-2-3-4-5-6

Students submitted the following schools' rankings in their Decision Sheets:

Student ID	1	2	3	4	5	6
Top choice	B	C	A	C	C	A
Middle choice	A	A	C	B	A	B
Last choice	C	B	B	A	B	C

This allocation method consists of the following steps:

**Step 1.** The queue looks as follows: 1-2-3-4-5-6 (the initial queue order is always determined by the lottery.) Thus student 1 (the first in the queue) applies to school B. It is not her district school. Students 3 and 4 are tentatively assigned to school B. One of the two students needs to be chosen. Between two students, student 3 is the closest to the top of the queue, that is why she moves to the top of the queue. And thus the queue is modified.

**Step 2.** The queue looks as follows: 3-1-2-4-5-6. Thus student 3 (the first in the queue) applies to school A. This school is not her district school, but the cycle of beneficial exchange appears. Student 3 wants to attend student 1's school, and student 1 wants to attend student 3's school. The beneficial exchange is obtained. Allocations of students 1 and 3 are finalized and they are excluded from the queue, and also 1 slot in each of the schools A and B is excluded from the allocation process.

Finalized assignments	School A	School B	School C
slot 1	3	1	-
slot 2	-	-	-

**Step 3.** The queue looks as follows 2-4-5-6. Student 2 (the first in the queue) applies to school C. It is not her district school. Students 5 and 6 are tentatively assigned to the school C. One of the two students needs to be chosen. Between two students, student 5 is closer to the top of the queue, that is why she moves to the top of the queue. And thus the queue is modified.

**Step 4.** The queue looks as follows: 5-2-4-6. Thus student 5 (the first in the queue) applies to school C. This school is her district school: Thus student 5 is assigned to school C. Her allocation is finalized and she is excluded from the queue as well as the slot in school C.

Finalized assignments	School A	School B	School C
slot 1	3	1	5
slot 2	-	-	-

**Step 5.** The queue looks as follows: 2-4-6. Student 2 (the first in the queue) applies to school C again. It is not her district school. Student 6 is the only student left, who is tentatively assigned to school C. Thus student 6 moves to the top of the queue. And thus the queue is modified.

**Step 6.** The queue looks as follows 6-2-4. Thus student 6 (the first in the queue) applies to school A. This school is not her district school, but the cycle of beneficial exchange appears. Student 6 wants to attend student 2's school, and student 2 wants to attend student 6's school. The beneficial exchange is obtained. Allocations of students 2 and 6 are finalized and they are excluded from the queue, and also 1 slot in each of the schools A and C is excluded from the allocation process.

Finalized assignments	School A	School B	School C
slot 1	3	1	5
slot 2	6	-	2

Step 7. There is only one student in the queue – student 4. She wants to apply to school C, but there are no more free slots there, so she applies to the second choice – school B. It is her district school and she is assigned to the slot in school B.

Thus final allocation of students looks as follows:

Finalized assignments	School A	School B	School C
slot 1	3	1	5
slot 2	6	4	2

## INSTRUCTIONS - Mechanism ETTC

This is an experiment in the economics of decision making. The instructions are simple, and if you follow them carefully and make good decisions, you might earn a considerable amount of money. In this experiment, we simulate a procedure to allocate students to schools. The procedure, payment rules, and student allocation method are described below. Do not communicate with each other during the experiment. If you have questions at any point during the experiment, raise your hand and the experimenter will help you.

### *Procedure*

There are 10 participants in this experiment. You are participant 1. (This number is drawn by each participant before entering the room.) Each participant represents a student, who wants to get a slot at a school. You will have to make 3 decisions in 3 different school markets. In all markets allocation will be determined by the same algorithm. This algorithm works as follows:

### *Allocation Method*

Each participant is first tentatively assigned to the school within her respective district. Each school has a priority ordering of all other students. Students of the school district have the highest priority in the school. Next, Decision Sheet rankings are used to determine mutually beneficial exchanges between two or more participants. The order in which these exchanges are considered is determined by a fair lottery. This means each participant has an equal chance of being the first in the line, the second in the line, , as well as the last in the line. To determine this fair lottery,

a participant will be asked to draw 10 numbers from an urn, one at a time. Each number corresponds to a participant ID number. The sequence of the draw determines the order in the lottery.

The specific allocation process is explained below.

1. Initially all 10 slots are available for allocation.
2. All participants are ordered in a queue based on the order in the lottery.
3. Next, an application to the highest-ranked school in the Decision Sheet is submitted for the participant at the top of the queue.
  - If the application is submitted to her district school, then her tentative assignment is finalized (thus, she is assigned to a slot at her district school). The participant and her assignment are removed from subsequent allocations. The process continues with the next participant in line.
  - If the application is submitted to another school, the procedure moves as follows: Say applicant Claudia's home district school is school A and she is applying to school B. Then Claudia's application is submitted to school B. After that, one of the students who tentatively hold the slot at school B has to be chosen. In particular, among all students, who tentatively holds a slot at school B, we choose the student with the highest priority at Claudia's district school, i.e., school A. ( So we follow the priority ordering of Claudia's school – school A) Then this student is moved to the top of the queue directly in front of the requester (Claudia).

Whenever the queue is modified, the process continues similarly: An application is submitted to the highest-ranked school with available slots for the participant at the top of the queue.

- If the application is submitted to her district school, then her tentative assignment is finalized. The process continues with the next participant in line.
  - If the application is submitted to another school, say school S, then we follow the procedure, explained for Claudia: a participant with the highest priority in the district school of the requester among those who tentatively hold a slot at School S is moved to the top of the queue directly in front of the requester. This way, each participant is guaranteed an assignment which is at least as good as her district school based on the preferences indicated in her Decision Sheet.
4. A mutually-beneficial exchange is obtained when a cycle of applications are made in sequence, which benefits all affected participants, e.g., I apply to Stefan's district school, Stefan applies to your district school, and you apply to my district school. In this case, the exchange is completed and the participants as well as their assignments are removed from subsequent allocations.

5. The process continues until all participants are assigned a school slot.

**Example:**

In order to understand the mechanism better, let us go through an example together.

If you have any questions about any step of the allocation procedure please feel free to ask at any point. There are six students (ID numbers from 1 to 6) on the market, and three schools (school A, school B, and school C) with two free slots each. Students 1 and 2 live in the district of school A, students 3 and 4 live in school district B, and, finally, student 5 and 6 live in school district C. It means that tentative assignments look as follows:

Tentative assignments of students ( IDs)	School A	School B	School C
slot 1	1	3	5
slot 2	2	4	6

The lottery determined the following order (student IDs): 1-2-3-4-5-6

Students submitted the following schools' rankings in their Decision Sheets:

Student ID	1	2	3	4	5	6
Top choice	B	C	A	C	C	A
Middle choice	A	A	C	B	A	B
Last choice	C	B	B	A	B	C

This allocation method consists of the following steps:

**Step 1.** The queue looks as follows: 1-2-3-4-5-6 (the initial queue order is always determined by the lottery.) Thus student 1 (the first in the order) applies to school B. It is not her district school. Students 3 and 4 are tentatively assigned to school B. Student 1 has to choose between them. Student 4 has higher priority in school A (school of student 1), that is why student 1 chooses student 4, and she moves to the top of the queue. And thus the queue is modified.

**Step 2.** The queue looks as follows: 4-1-2-3-5-6. Thus student 4 applies to school C. This school is not her district school. Students 5 and 6 are tentatively assigned to school C. Student 4 has to choose between them. Student 5 has higher priority in school B (school of student 4), that is why student 4 chooses student 5, and she moves to the top of the queue. And thus the queue is modified.

**Step 3.** The queue looks as follows 5-4-1-2-3-6. Student 5 applies to the school C. It is her district school. Thus student 5 is assigned to the school C. Her allocation is finalized, and she is excluded from the queue as well as the slot in school C.

Finalized assignments	School A	School B	School C
slot 1	-	-	5
slot 2	-	-	-

**Step 4.** The queue looks as follows: 4-1-2-3-6. Thus student 4 applies to school C again. This school is not her district school. Student 6 is the only student left, who is tentatively assigned to school C, and she moves to the top of the queue. And thus the queue is modified.



**Step 5.** The queue looks as follows: 6-4-1-2-3. Thus student 6 applies to school A. This school is not her district school. Students 1 and 2 are tentatively assigned to school A. Student 6 has to choose between them. Student 1 has higher priority in school B (school of student 6), that is why student 6 chooses student 1, and she moves to the top of the queue. And thus the queue is modified.

**Step 6.** The queue looks as follows: 1-6-4-2-3. Thus student 1 applies to the school B again. It is not her district school, but the cycle of beneficial exchange appears. Student 1 wants to attend student 4's district school again, and at the same time student 4 wants to attend student 6's district school, and student 6 wants to attend student 1's district school. The beneficial exchange is obtained. Allocations of students 1, 4 and 6 are finalized and they are excluded from the queue, and also 1 slot in each of the schools A, B and C is excluded from the allocation process.

Finalized assignments	School A	School B	School C
slot 1	6	1	5
slot 2	-	-	4

**Step 7.** The queue looks as follows 2-3. Student 2 wants to apply to school C, but there are no free slots any more. Thus student 2 applies to her second choice – school A. It is her district school. Thus student 2 is assigned to the school A. Her allocation is finalized, and she is excluded from the queue, and also the slot in school A is excluded from the allocation process.

Finalized assignments	School A	School B	School C
slot 1	6	1	5
slot 2	2	-	4

**Step 8.** There is only one student in the queue – student 3. She wants to apply to school A, but there are no more free slots there, so she applies to the second choice – school C, but there are no more free slots there, either. So she applies to school B. It is her district school and she is assigned to the slot in school B.

Thus the final allocation of students looks as follows:

Finalized assignments	School A	School B	School C
slot 1	6	1	5
slot 2	2	3	4

### ***Questionnaire in Instructions(common for both mechanisms)***

In order to check the level of understanding of the allocation procedure we ask you to find out the allocation of the student for the following market: There are six students (ID numbers from 1 to 6) on the market, and three schools (school A, school B and school C) with two free slots each. Students 1 and 2 live in the district of school A, students 3 and 4 live in the district of school B and, finally, students 5 and 6 live in the district of school C. It means that the tentative assignments look as follows:

Tentative assignments of students ( IDs)	School A	School B	School C
slot 1	1	3	5
slot 2	2	4	6

The lottery determined the following order (student IDs): 1-2-3-4-5-6  
Students submitted the following schools' rankings in their decision sheets:

Student ID	1	2	3	4	5	6
Top choice	B	C	A	C	B	B
Middle choice	A	A	C	B	C	A
Last choice	C	B	B	A	A	C

You have 10 minutes to perform the task. If you have any questions, raise your hand and we will come to you. After 10 minutes you must submit your answer sheet and then the experimenter will go through the solution on the board.

***Decision Sheets (common for both mechanisms)***<sup>35</sup>

**Decision sheet. Market TP1. Student ID1.**

There are three schools in the market: school A, school B and school C. School A and B have three slots each. School C has four slots.

Students 1, 4 and 7 live in the district of school A.

Students 2, 5 and 8 live in the district of school B.

Students 3, 6, 9 and 10 live in the district of school C.

Recall: You are student 1.

Your payoff amount depends on the school slot you hold at the end of the experiment. Payoff amounts for market 1 are outlined in the following table.

School	A	B	C
Payoff, EUR	10	15	5

You will be paid €10 if you hold a slot at School A in the market 1 at the end of the experiment.

You will be paid €15 if you hold a slot at School B in the market 1 at the end of the experiment.

You will be paid €5 if you hold a slot at School C in the market 1 at the end of the experiment.

Preferences of other students are the following:

ID	1	2	3	4	5	6	7	8	9	10
top	B	C	B	C	A	A	C	A	A	B
middle	A	A	A	B	C	C	A	B	B	A
bottom	C	B	C	A	B	B	B	C	C	C

Note that the preferences above correspond to payoffs. They are not necessarily coincide with stated preferences, which are used for the ballocation.

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<sup>35</sup>We use preferences of student 1 and the fixed order of the markets, for example.

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