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Working Paper

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Economics Discussion Papers, No. 2014-47

Provided in Cooperation with:

Kiel Institute for the World Economy – Leibniz Center for Research on Global Economic Challenges

Suggested Citation: Bossone, Biagio (2014) : Secular stagnation, Economics Discussion Papers, No. 2014-47, Kiel Institute for the World Economy (IfW), Kiel

This Version is available at:

<https://hdl.handle.net/10419/103958>

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Discussion Paper

No. 2014-47 | November 19, 2014 | <http://www.economics-ejournal.org/economics/discussionpapers/2014-47>

Secular Stagnation

Biagio Bossone

Abstract

This study analyzes the emergence of secular stagnation as the consequence of a rise in the preference for liquidity. Such a rise is caused by a persistent set of pessimistic expectations. This study also investigates the effectiveness of a broad range of demand-management policies in dealing with secular stagnation. To obtain these results, this study uses a model where agents derive utility from holding assets of different degrees of liquidity. In this environment, rational expectations interact with changes in market sentiment, to produce secular stagnation.

JEL E2 E3 E4 E5 E61 E62 E63 G11 G12

Keywords helicopter money, liquidity preference; market sentiment; quantitative easing; pessimistic (optimistic) expectations; utility analysis

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Citation Biagio Bossone (2014). Secular Stagnation. Economics Discussion Papers, No 2014-47, Kiel Institute for the World Economy. <http://www.economics-ejournal.org/economics/discussionpapers/2014-47>

I. Purpose and Plan of the Study

The purpose of this study is twofold. Firstly, it analyzes secular stagnation as caused by strong liquidity preference in an economy where rationality and emotionality are integral and mutually interacting factors within the agent's decision making processes (Pfister and Böhm, 2008). The study also investigates the effectiveness of alternative demand management tools available to policymakers to deal with secular stagnation. Secondly, this study seeks to show the analytical power and simplicity of the utility-based approach used to derive the demand for consumption and (real, monetary and financial) assets within a unifying intertemporal optimizing framework. The study complements my recent research work on liquidity preference and market sentiment and on endogenous instability in market financial economies (Bossone 2014, 2015).¹

Section II reviews the literature, section III describes the model, and section IV derives its results. The model specifications, based on my previous works cited above, are reported in the Appendix. Innovations in model specifications are duly noted and described. Concluding remarks follow in section V.

II. Review of the Literature

The secular stagnation hypothesis, originally introduced by Hansen (1939), was recently revived by Summers (2013) as he argued that an age of secular stagnation, in which the equilibrium interest rate is negative, might explain the lack of inflationary pressure the US economy experienced in the boom years of the previous decades and the slow recovery from the 2007 crisis. While much discussion has followed Summers' provocation,² only Gauti Eggertsson and Neil Mehrotra (EM) have so far attempted to model the hypothesis formally (Eggertsson and Mehrotra 2014).

¹ The model has proved very fruitful to study resource allocation under uncertainty and changes in market moods as well as to show that instability is endogenous to the working of market financial economies.

² See *Blogs review: The secular stagnation hypothesis*, by Jérémie Cohen-Setton, Bruegel, 25 November, 2013 (Bruegel <http://www.bruegel.org/nc/blog/detail/article/1201-blogs-review-the-secular-stagnation-hypothesis/>), and the recent collection of contributions edited by Teulings and Baldwin (2014).

EM use an overlapping generations New Keynesian model and show that a very persistent slump is possible, triggered by shocks that create an oversupply of savings (typically, population ageing, income inequality, a decline in the relative price of investment, and a debt deleveraging shock). No self-correcting force intervenes, and a permanently negative equilibrium real interest rate is required to restore full employment. EM also show that fiscal policy is effective in bringing demand back towards full employment, while central bank commitments to keep nominal rates low are ineffective if nominal rates are expected to remain low indefinitely.

Unlike EM, this study focuses on liquidity preference – caused by the upsurge of pessimistic expectations – as the source of a persistent drop in demand and output. This is made possible by using a model where agents derive utility from holding assets of different liquidity, the utility provided by each asset is explicitly formalized, and rational expectations interact with market sentiment changing in response to changes in economic observables.

The results of this study are complementary to those of EM, in that they relate to the relationship between expectations and aggregate demand – an aspect that is not specifically considered by EM – suggesting that secular stagnation may ensue from agents being induced to increase dramatically their demand for liquidity as a significant and persistent deterioration in the state of their expectations raises the utility they draw from being ultra liquid. In this sense, while this study – unlike EM – says nothing about secular stagnation already affecting the economy ahead of the 2007 crisis, it may contribute to explain the persistent forces that have been at work in the economy since after the crisis and the severe challenges encountered by policy makers in reversing them.

Similarly to EM, nominal wage rigidity in this study does not play any role in determining secular stagnation. Finally, this study uses utility analysis to investigate the effectiveness of a broader range of demand management policy tools than those evaluated by EM, including the unconventional monetary policies adopted in some countries, and concludes that while negative interest rate, quantitative easing and forward (interest rate) guidance are ineffective under strong and persistent liquidity preference, fiscal policy and – even more – helicopter money drops provide the demand boost needed to exit secular stagnation.

III. The Model

A. The Economy

The economy consists of a representative infinitely-lived, intertemporal utility-maximizing agent, a government, a central bank, and four asset types: money M , which is also used as money for executing payment transactions, a short-term government bond T , a long-term government bond B , and non-liquid asset K , which can be thought of as a claim on the economy's productive capital stock. Each asset trades at price, $P^{Q(=M,T,B,K)}$ where $P^M = 1$.

Capital K , owned by the agents, is used to produce composite output (consisting of consumption and investment goods, that is, $Y = C+k$, where $k = K - K_{-1}$ is the variation of capital from the previous period in the form of investment) selling at composite price P , with a stochastic technology that employs the agent's labor at a competitive nominal income salary y . In forming expectations on the future developments of y , the agent considers the value of her job as defined in the Appendix. In each period, the agent earns nominal returns $r^{Q(=M,T,B,K)} = (1 + \rho_t^Q)(1 + \pi_t^Q)$ from her asset holdings, where ρ is the nominal interest rate or the dividend paid on the assets and π^Q is rate of change of asset price P^Q . The real rate of return on asset Q is $R^{Q(=M,T,B,K)} = (1 + \rho_t^Q)(1 + \pi_t^Q)(1 - \pi)$, where π is the rate of inflation. As a simplifying assumption, the supply of output adjusts instantaneously to demand.

The model's building blocks are analytically described in the Appendix.

The agent orders her preferences across consumption C , M , T , B and K , according to a strictly quasi-concave, time-separable, and well-behaved utility function $u_t = u(C_t, M_t, T_t, B_t, K_t)$, with $u'''(\cdot) > 0$ (risk aversion).

The utility-based approach to asset allocation and pricing, originally developed in Bossone (2014), builds upon three fundamental concepts. These are discussed below, while their formalization is reported in the Appendix:

All assets deliver utility. Assets of all types are considered as "vehicles" to future consumption, each characterized by its own peculiar "speed" (that is, the immediacy and the cost of converting it into consumption) and "power" (that is, its capacity to accumulate and store wealth over time at some risk). Greater speed would come at some power cost, and vice versa. Also, at each instant the agent is faced with the likelihood of having to liquidate the asset to face a consumption shock: changes in likelihood affect differently the utility derived from assets with different speed and power load. The instantaneous utility of any asset (Eq.(1) below) is thus calculated across the agents' time-horizon as the expected

value of the discounted summation of stochastic (uncertain) consumption utility, to which the asset gives access, net of the (uncertain) consumption utility lost to asset liquidation cost (see next).

Variable cost of asset liquidation. Every asset is characterized by an optimal “speed”, defined as the shorter time-interval possible for the asset to be sold at the minimum liquidation (or transaction) cost possible. Asset optimal speeds are structural parameters determined by the economy’s level of institutional and technological development: all else equal, a more efficient and safe financial infrastructure allows asset liquidation to be effected more rapidly and at lower costs. Some assets can be liquidated and converted into consumption immediately and at no cost. They can thus serve as monies in the exchange system. Other assets require longer time-intervals and involve positive liquidation costs. Having to liquidate an asset at a higher than optimal speed (owing, for instance, to immediate and unexpected consumption needs) results in higher liquidation costs or forces the agent to accept larger discounts on the asset sale price. Uncertainty affects asset utility also by influencing expected asset liquidation costs.

Asset price volatility. When holding an asset, the agent faces the risk that, at any time when the asset needs to be liquidated, it might sell at a loss due to the volatility of its market price. Asset price volatility affects risk-averse agent choices even if the actual price of the asset fluctuates symmetrically around its expected value: in translating the effects of future price dynamics in terms of asset utility gains/losses, a risk-averse agent weighs the contribution of negative deviations from the mean relatively more than the contribution of positive deviations of equal size and duration. The utility of the asset, therefore, responds inversely to changes in the expected volatility of the asset price.

Rational expectations and emotions. Expectations depend on the state of knowledge: better knowledge and information help the agents to form more precise expectations about future relevant variables, while lower-quality knowledge and information cause agent expectations to be less determinate. Expectations depend also on emotions: defining optimism (pessimism) as the state of mind that induces the agents to expect superior (inferior) outcomes of future events than would otherwise be reasonable for them to expect exclusively on the basis of the given knowledge and information, optimistic (pessimistic) expectations derive from a “distortive” process. This process introduces a deviation (i.e., a distortion) between purely rational expectations and expectations that are affected by emotional states of mind (e.g., “animal spirits”): optimism (pessimism) distorts the way agents process and interpret information.

The general formal expression for the utility of any asset Q at any time t , is

$$(1) \quad u(Q_t) = E_t[\sum_{T=t+1}^{\infty} u(\frac{Q_T P_t^Q}{P_T^C}) \prod_{i=1}^{T-1} \beta_i R_i^Q (1 - \theta_{t+i-1}) \vartheta_T (1 - \xi_T^Q) (1 - \sigma_T^Q) | \omega_t]$$

where ϑ_t is the probability of having to finance unexpected consumption at time t ; ξ^Q is the expected utility loss due to asset liquidation costs as a ratio of the expected utility from the consumption financed through sale of Q ; σ^Q is the expected utility loss due asset price volatility as a ratio of the

expected utility from the consumption financed through sale of Q , and where utility is conditional on current knowledge and information, and on the prevailing market sentiment.

From Eq.(1), and considering that for a fully liquid asset, like money, $\xi^M = \sigma^M = \rho^M = \pi^M = 0$, the utility-of-money function reduces to

$$(1b) \quad u(M_t) = E_t \left\{ \left[\sum_{T=t+1}^{\infty} u \left(\frac{M_t}{P_T^C} \right) \prod_{i=1}^{T-1} \beta_i (1 - \pi_i) (1 - \vartheta_{t+i-1}) \vartheta_T \right] \middle| \omega_t \right\},$$

that is, as a perfectly liquid asset, money trades against any goods or assets at a sure nominal price ($P^M = 1$) and at zero transaction cost, and does not bear interest.

In economies with advanced financial market infrastructures and a government with strong fiscal reputation, short-term (risk-free) bond T would trade in highly liquid monetary markets at low liquidation cost and interest rate, and would be considered as a close substitute to money. Longer term bonds and, even more so, corporate shares and obligations would be less liquid, trade at higher liquidation costs, and require adequate return premiums in order to attract demand. All else equal, a rise in pessimistic (optimistic) expectations would decrease (increase) the utility of more (less) liquid assets, consequently affecting their demand and relative price and return structures.

The supply of M is governed by the central bank (see Appendix), which adjusts it so as to keep the nominal interest rate on (riskless) asset T at the level of the agents' rate of time preference δ , assuming this can be derived from available data, where real-sector supply and demand are in intertemporal equilibrium and inflation is constant ($\pi = 0$).

The government produces a non-pecuniary public good G and finances production by issuing bonds T and B to match agent portfolio preferences of different maturity and liquidity services, under an intertemporal constraint whereby all debt and debt servicing costs must be repaid over time through taxation. The case will be examined where the government budget is partly financed through new money creation by the central bank.

Asset K is supplied by enterprises to support their investment plans and remunerate capital so as to attract investor demand.

B. Optimal allocations

The solution to plan (A10)-(A18) reported in the Appendix determines a path to optimal intertemporal consumption and to a sequence of optimal allocations $(C_r^*, M_r^*, T_r^*, B_r^*, K_r^*)$, equilibrium prices

$(P_\tau^{C^*}, 1, P_\tau^{T^*}, P_\tau^{B^*}, P_\tau^{K^*})$, and equilibrium gross returns $(R_\tau^{M^*}, R_\tau^{T^*}, R_\tau^{B^*}, R_\tau^{K^*})$, which at each future date $\tau \in (\tau + 1, \dots, \infty)$ satisfy the optimal intra-date rule of weighted marginal utility (w.m.u.) equality

$$(2) \quad E_t \left[\frac{w'(C_\tau^*)}{P_\tau^{C^*}} \middle| \omega_t \right] = E_t [u'(M_\tau^*) | \omega_t] = E_t \left[\frac{w'(T_\tau^*)}{P_\tau^{T^*}} \middle| \omega_t \right] = E_t \left[\frac{w'(B_\tau^*)}{P_\tau^{B^*}} \middle| \omega_t \right] = E_t \left[\frac{w'(K_\tau^*)}{P_\tau^{K^*}} \middle| \omega_t \right],$$

and clear the markets for the economy's goods and assets. Once the optimal intertemporal path for consumption is established, rule (2) requires that the agent equates at each moment of her future time-horizon the w.m.u that she draws from consumption and from monetary and financial asset holdings, conditional on the knowledge and information available and the prevailing market sentiment.

C. Secular Stagnation

From plan (A10)-(A20), a large and persistent deterioration of market sentiment $\omega_{t \in (t, \dots, t^+)} < \omega_{t-1}$ where t^+ is an indefinite future date, causes the agent's job value to fall (Eq.(A11) in the Appendix), leading to lower intertemporal optimal consumption, and raises her demand for liquidity (typically money and short-term bonds). Liquidity preference increases as a rational response to fears of future worsening of economic conditions, and triggers a change in equilibrium interest rates and asset rates of return.

Under the assumption that output adjusts instantaneously to demand, goods prices remain constant and $\pi = 0$. For a low enough level of $\omega_{t > t, \dots, t^+}$ and a far distant t^+ , restoring optimal equilibrium allocation $(C_\tau^*, M_\tau^*, T_\tau^*, B_\tau^*, K_\tau^*)$ may require $\rho^{M^*} < \rho^{T^*} < 0$, where the interest rate adjustment $|D\rho^{M^*}| = |\rho^{M^*} - 0| = \rho^{K^*} - 0$ equals the liquidity premium on M , which drives agent portfolio preferences, and where interval $(\rho^{M^*}, 0)$ defines the space where equilibrium nominal interest rates on liquid assets would have to fall for the economy to exit secular stagnation.

Clearly, in the (typical) case where the Zero Lower Bound (ZLB) holds, that is, $\rho^M \geq 0$, an equilibrium allocation $(C_\tau^{**}, M_\tau^{**}, T_\tau^{**}, B_\tau^{**}, K_\tau^{**})$ is forced upon the economy, where

$$(3) \quad C_\tau^{**} < C_\tau^*, \quad M_\tau^{**} > M_\tau^*, \quad T_\tau^{**} > T_\tau^*, \quad B_\tau^{**} < B_\tau^*, \quad K_\tau^{**} < K_\tau^*.$$

This result derives from the model discussed above and formalized in the Appendix, whereby the rise of pessimistic expectations:

- a) Worsens the agent's job value JV , thus lowering consumption

- b) Moves forward in time probability θ_T , thus raising the marginal utility of liquidity M and T ;
- c) Raises expected utility losses ξ^B and ξ^K as well as σ^B and σ^K , thus lowering the marginal utility of B and K ; and
- d) Lowers price P^K reflecting the expected decline in the marginal efficiency of productive capital asset K due to deteriorating economic prospects.³

This yields inequality

$$(4) \quad E_t \left[\left(\frac{u'(C_t)}{P_t^C} \right) \middle| \omega_t \right] = E_t [u'(M_t) | \omega_t] \geq E_t \left[\left(\frac{u'(T_t)}{P_t^T} \right) \middle| \omega_t \right] > E_t \left[\left(\frac{u'(B_t)}{P_t^B} \right) \middle| \omega_t \right] > E_t \left[\left(\frac{u'(K_t)}{P_t^K} \right) \middle| \omega_t \right],$$

which steers the economy toward the new allocation in (3) above.

As the model of expectations formation underpinning this study suggests, the absence of exogenous (market or policy driven) shocks or innovations, which could re-energize market sentiment via positive stimuli and feed back into expectations, leads the latter to adjust to the new equilibrium thereby making it persistent. As a result, the demand for liquidity would continue dominating agent preferences, and the economy would find itself entangled in secular stagnation indefinitely.

While the model used in this study does not incorporate the financial sector, and therefore cannot say anything on how financial institutions can affect secular stagnation, Bossone (2015) suggests that the financial sector may indeed play an important role in deepening or even causing secular stagnation by persistently constraining the provision of liquidity to the system. This result would be consistent with the findings of the EM paper, and would provide an important element to evaluate economic policies designed to revitalize aggregate demand by restoring the credit channel. The result, however, does not deflect from the fundamental conclusion of this study that strong and persistent liquidity preference driven by deeply pessimistic expectations can be a critical explaining factor of secular stagnation, especially since such expectations may similarly influence heterogeneous (private-sector) agents prompting them to withdraw stimulus from the economy either through reducing consumption, rationing lending, or cutting capital expenditure.

³ Note that while P^B adjust downwards as a result of the decrease in the w.m.u. of asset B caused by factors a)-c), price P^K is affected directly by expectations under d). Its drop thus compounds the effect of factors a)-c) on the w.m.u. of K , requiring as a consequence a larger adjustment in the equilibrium rate of return on the asset.

IV. Policy Evaluation

D. Negative Interest Rates

Would breaching the ZLB through negative interest rates (NIR) really solve the problem, as suggested above? Theoretically yes, practically it is much less certain. Even assuming that application of NIR to cash can be resolved (Buiter 2009, Kimball 2012) and, similarly, that the government can set $\rho^T < 0$ as necessary, dynamically the agents can search for, and eventually identify alternative assets earning non-zero returns (e.g., a reserve foreign currency or a government bond, including from a foreign country, considered to be safe enough). If such an asset, say A is found, inequality (4) would become

$$E_t \left[\left(\frac{u'(C_\tau)}{P_\tau^C} \right) \middle| \omega_t \right] = E_t [u'(M_\tau) | \omega_t] = E_t \left[\left(\frac{u'(T_\tau)}{P_\tau^T} \right) \middle| \omega_t \right] < E_t \left[\left(\frac{u'(A_\tau)}{P_\tau^A} \right) \middle| \omega_t \right] > E_t \left[\left(\frac{u'(B_\tau)}{P_\tau^B} \right) \middle| \omega_t \right] \\ > E_t \left[\left(\frac{u'(K_\tau)}{P_\tau^K} \right) \middle| \omega_t \right].$$

Asset A would then become the newly preferred liquid asset, and would tend supplant within the agent portfolios those liquid assets whose liquidity premium has been neutralized through NIR. With liquidity preference dominating agent attitudes, the NIR signals would fail to stimulate consumption and would not be transmitted across the whole spectrum of interest rates, since at the non-negative zero interest rate paid on the “new” asset the agents would absorb as much of it as they could – not to mention that the increasing demand for the new asset might even raise its price and deliver a positive return to holders. As a result, a new liquidity trap would emerge, the NIR stimulus would fail to incentivize agents’ decisions to go less liquid (that is, investment in productive capital), and the economy would still be in secular stagnation.

E. Quantitative Easing

For similar reasons, quantitative easing (QE) – whereby the central bank buys specified amounts of financial assets from commercial banks and other private institutions, thus raising their prices (lowering their yield) and increasing the monetary base – is bound to be ineffective to boost aggregate demand.

While QE may succeed in raising asset prices (contrary to anticipations of Wallace neutrality⁴), under liquidity preference dominance it would not be able to stimulate consumption and/or investment.

With inequality (4) holding, liquidity preference pushing toward $\rho^{T*}, \rho^{M*} < 0$, but hitting the ZLB, QE would amount to the central bank buying B in exchange for M so as to bring the w.m.u.'s of M , T and B down to equality by lowering r^B . In the limit, QE would pressurize asset prices until $\rho^{M*} = \rho^{T*} = \rho^{B*} = 0$ would yield. Yet, under liquidity preference dominance and a binding ZLB, agents would willingly absorb whatever amounts of M they could afford, and would hold on to them without changing their consumption and investment decisions since $Dr^B < r^{K*} - 0$; that is, the policy-induced reduction in the nominal interest rate on the safe long-term asset is not large enough to incentivize investment in (illiquid) asset K . The only way to ensure QE success, as Nick Rowe has recently put it, would be for the government-owned central bank to “move towards communism”, where the government purchases and owns all the assets in the economy...⁵

In fact, for extremely high level of pessimism, one could even conceive of the possibility that the agents holding asset B would be willing to get all the M they could in exchange for B at its going price, thus neutralizing any effect that the central bank seeks to exert on interest rates via QE: a sort of “super liquidity trap” would emerge as the agents strongly separated the assets that they consider to be liquid from all the others. In this case, too, obviously and a fortiori, QE would fail to affect demand.

F. Acting Irresponsibly

Assume that the central bank is willing to “commit to being irresponsible”, in Krugman’s (1998) words, and is ready “to do whatever it takes” to drive the economy out of secular stagnation by affecting inflation expectations. Ruling out “super liquidity trap” contingencies, and with the ZLB constraint binding, the central bank executes QE to achieve $\rho^B = 0$ and continues to purchase outstanding volumes of B so as to inject additional doses of M into the economy with a view to increasing the w.m.u. of consumption C and capital asset K by raising inflation expectations.

⁴ This issue of Wallace (1981) neutrality as relates to QE has recently become the subject of blogosphere debate. See, for instance, Richard H. Serlin, “The Intuition Behind Wallace Neutrality”, Economics, Finance, Personal Finance, Politics, and Other Subjects with a Focus on Intuition, Clarity, and Non-Misleading, August 10, 2014, available at <http://richardhserlin.blogspot.it/2014/08/the-intuition-behind-wallace-neutrality.html>.

⁵ See Nick Rowe, “Fractional reserves, capital, communism, and the optimum quantity of money”, Worthwhile Canadian Initiative, 8 September, 2014, available at http://worthwhile.typepad.com/worthwhile_canadian_initi/2014/09/fractional-reserves-and-the-optimum-quantity-of-money.html

The question is: what are the channels available to the central bank for its QE policy to influence expectations?

First, to the extent that the central bank purchases massive amounts of B , and commits to hold onto them perpetually (eventually rolling over all the maturing B 's), QE policy actually becomes “helicopter money” policy and can impact spending decisions through the fiscal lever, as will be discussed later on.

Second, short of this twist in policy, the central bank is left without effective channels.

Take consumption first: as $\rho^{M^*} = \rho^{T^*} = \rho^{B^*} = 0$, current consumption cannot be affected through the interest rates in the Euler equation. The only remaining strategic variable for the central bank to aim at would be the agent’s job value, but this could increase only if aggregate demand were expected to grow, which in turn requires that expected sales and sale prices would go up: a vicious circle. QE has no way to affect consumption.

Look next at investment: in order to stimulate investment demand, the central bank seeks to raise the w.m.u. of capital asset K , which from Eq.(2) is

$$(2c) \quad u'(K_t)/P_t^K = E_t[\sum_{\tau=t+1}^{\infty} \beta^{\tau} \prod_{i=1}^{\tau-t} R_i^K (1 - \theta_{t+i-1}) \vartheta_{\tau} (1 - \xi_{\tau}^K) (1 - \sigma_t^Q) u'(\frac{K_t P_t^K}{P_t^C}) | \omega_t] / P_t^K.$$

As Eq.(2c) is always increasing in P^K , the central bank aims at raising this price. However, P^K reflects the marginal efficiency of capital, which can increase only if future sales and sale prices are expected to grow. The central bank should therefore stimulate consumption. Yet, as just discussed, it is unable to do so under the circumstances.

The conclusion is that QE is not effective in revamping aggregate demand as a way to help the economy out of secular stagnation. On the other hand, and asymmetrically, expectations of a QE reversal (or “tapering”) might negatively affect P^K and instantly lower the w.m.u. of asset K as a result.

Finally, and importantly, in the circumstances where QE succeeds in driving ρ^B down to zero, thus ensuring $E_t[u'(M_{\tau}) | \omega_t] = E_t[\frac{u'(T_{\tau})}{P_{\tau}^T} | \omega_t] = E_t[(\frac{u'(B_{\tau})}{P_{\tau}^B}) | \omega_t]$, but where $D\rho^B < \rho^{Q^*} - 0$ and gives no incentive for agents to invest in K , some of them may search for, and eventually identify, assets that are in short supply, whose supply adjusts only slowly to demand, and whose prices show prospects of rapid gains. Assuming an asset S with such characteristics is found, inequality (4) becomes

$$(4b) \quad E_t \left[\left(\frac{u'(C_\tau)}{P_\tau^C} \right) \middle| \omega_t \right] = E_t [u'(M_\tau) | \omega_t] = E_t \left[\left(\frac{u'(T_\tau)}{P_\tau^T} \right) \middle| \omega_t \right] = E_t \left[\left(\frac{u'(B_\tau)}{P_\tau^B} \right) \middle| \omega_t \right] < E_t \left[\left(\frac{u'(S_\tau)}{P_\tau^S} \right) \middle| \omega_t \right] > E_t \left[\left(\frac{u'(K_\tau)}{P_\tau^K} \right) \middle| \omega_t \right].$$

If, as it would likely be the case, spread $r^S - r^K$ is sufficiently large and is expected to increase swiftly, agents might want to shift some of their portfolio liquidity into S holdings. As shown by Bossone (2014), this could lead to the formation of speculative bubbles. Such is a risk that QE policies do pose to the economy, and the willingness by the central bank to “act irresponsibly” via the interest rates channel might end up actually engendering irresponsible consequences.

G. Forward Guidance

From the above findings follows (trivially) the ineffectiveness of forward interest rate policy guidance, which the central bank may use to communicate its intention to keep interest rates at a low level for an indefinite period of time, even beyond the point when normalizing them would be in order and, thus, signaling its willingness to tolerate higher future inflation rates. In terms of the model above, forward guidance amounts to the central bank committing to keeping $E_t(\rho_\tau^{M*} = \rho_\tau^{T*} = \rho_\tau^{B*} = 0)$, for $\tau \in (t, \dots t^+)$ and with t^+ being an indefinite date in the future. As it has been seen already, this policy measure has no effect on aggregate demand under liquidity preference dominance.

H. Helicopter Money

The idea of helicopter money (HM) – originally evoked by Friedman (1969) – is a policy whereby new money is created by the central bank and provided (“dropped from helicopter”) directly to households and private businesses, without generating new (public or private) debt, to stimulate spending when the economy is in deep recession. Since central banks have generally no mandate to give money away (they can only exchange one asset for another), HM drops need to be backed by the budget-approval process and must essentially involve fiscal policymaking decision (Grenville 2013). The advantage of this *monetary cum fiscal* policy tool is to provide newly created purchasing power directly to (private and public sector) agents who can more immediately spend it.

Today, the concept of HM generally refers to money creation operations intended to support aggregate demand by financing state budget programs of public spending or tax reduction. On the wake of Bernanke (2002), various authors have supported HM as a most effective policy tool for demand

management purposes in economies undergoing deep recessions.⁶ Buiter (2014) evaluates HM analytically through a formal model, and identifies the conditions under which it boosts aggregate demand most effectively. One of these conditions is the irreversibility of the new money base stock creation, which constitutes a permanent addition to the total net wealth of the economy.⁷

The impact of HM and the importance of the irreversibility condition can both be appreciated by considering the intertemporal budget constraints of the agent and the government (see Eqs.(A11) and (A18) in the Appendix). As the government budget is partly and irreversibly funded through money creation, its budget constraint can be permanently relaxed by an equivalent amount, implying that taxation can in turn be equally reduced and disposable income increased (see Eq.(A17b) in the Appendix). Since irreversibility removes possible Ricardian equivalence effects, consumption spending increases permanently.

In fact, HM acts also through the expectations channel: a large and sustained stimulus combining monetary and fiscal levers increases both the agent's job value (Eq.(A11) in the Appendix), and hence consumption C through the intertemporal budget constraint (Eq.(A13) in the Appendix), and the marginal efficiency of capital asset K , as reflect in price P^K , thus raising the w.m.u. of both C and K . (Eq.(2c)).

The effect of HM can reverse expectations via Eq.(A8) in the Appendix and facilitate the economy's exit from secular stagnation.

⁶ See Bossone et al (2014) for references.

⁷ This is made possible by the ('fiat') money base constituting an asset for the holder but not a liability for the issuer (Buiter 2004). Operationally, irreversibility can be attained if HM drops are executed either by:

(i) having the government issue interest bearing debt, which the central bank would buy and hold in perpetuity, rolling over into new government debt when the existing debt on its balance sheet reaches maturity. In this case, the government would face a debt interest servicing cost, but the central bank would make an exactly matching profit from the difference between the interest rate it receives on its debt and the zero cost of its money liabilities, and would return this profit to the government;

or by

(ii) having the central bank purchase special government securities that are explicitly non-interest bearing and never redeemable.

In terms of the fundamentals of money creation and government finance, the choice of these two routes would make no difference (Turner 2013).

A comment on irreversibility. Notice that, unlike one could be led to believe,⁸ the irreversibility condition has nothing to do with the fact that, at any future date, the central bank might decide to withdraw part or all of the liquidity injected in the system by selling bonds held in its portfolio. In this case, the holders of liquidity would exchange money for the bonds sold by the central bank. Yet the total net worth of the economy would not change, only its composition would (shifting from more to less liquid assets): the addition to the economy's net worth originally operated through HM would not (and could not) be undone by any new open market operation

Finally, it should be observed that the way HM affects aggregate demand in the model used in this study is exclusively through taxation. In real world cases, however, HM can as well (and importantly) operate through public spending. Its overall effectiveness, therefore, depends on how the newly created money can be funneled through channels that can more readily facilitate its circulation via spending acts.

I. Fiscal Policy

By comparison with the HM policy just discussed, the fundamental factor characterizing fiscal policy is that the government can use only debt and taxation to finance its budget: relaxing the budget constraint now in order to allow for current lower taxation (or higher public spending) requires larger government indebtedness, which in turn implies higher taxation and/or lower public spending at some future dates. Whether, and to what extent, this is going to weigh negatively on current consumption decisions from rational agents in real (as opposed to theoretical) circumstances depends on various factors, such as, inter alia, the agents' relevant time-horizon and factors binding their rationality, the state and sustainability of public finances, and the credibility of the fiscal and monetary authorities.

It should be noted that, at times when $\rho_\tau^{T*} \approx \rho_\tau^{B*} \approx 0$, and interest rates are even negative in real terms, running a front-loaded expansionary budget with a view to back-loading the fiscal adjustment needed to some future date may seem to be a winning strategy to help the economy out of stagnation. However, the question is whether the issuance of new public debt obligations would affect the equilibrium interest rate, and the answer ultimately depends on the market assessment of debt sustainability looking forward. In the case of a largely indebted country, for instance, the success of the fiscal stimulus would depend on the markets trusting that the stimulus were to actually succeed in triggering output growth, thus improving debt sustainability. This would be enough to allow interest rates on debt to remain low while the government stretches its budget through larger indebtedness to

⁸ See, for, instance White (2013).

finance the stimulus. However, multiple equilibria would be possible and would depend on market beliefs, meaning that an element of uncertainty is inherent in the exclusive use of fiscal policy as a way out of secular stagnation. For this reason, fiscal policy ranks second to HM, which can boost demand without creating debt and adds to the economy's net wealth.

V. Concluding Remarks

This study has analyzed the emergence of secular stagnation as the consequence of a rise of strong liquidity preference caused by the upsurge and persistence of pessimistic expectations. The study has used a model where agents derive utility from holding assets of different liquidity, the utility provided by each asset is explicitly formalized, and rational expectations interact with changes in market sentiment driven by variations in economic observables. The study has also investigated the effectiveness of a broad range of demand management policies in dealing with secular stagnation, concluding that “helicopter money” and fiscal policy (in that order) can effectively boost aggregate demand and output in an economy in secular stagnation, whereas other unconventional forms of monetary policy are very likely to prove ineffective under pessimistic expectations and when liquidity preference strongly dominates agent attitudes, or they might even provoke undesirable consequences (as it could be the case with quantitative easing).

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Appendix

A. The Utility of Assets

The utility of asset Q at date t can be obtained by summing over two terms: (i) the utility derived from converting the asset into consumption at the next date $t+1$ with probability ϑ_{t+1} , and (ii) the utility from holding the asset available to access consumption at some later date with residual probability $(1 - \vartheta_{t+1})$. In the following, for convenience of exposition, the agent's time discount factor $\beta = 1/(1+\delta)$, where δ is the agent's rate of time preference, and return R^Q are assumed to be constant, and the price of consumption is set at $P^C = 1$. (These assumptions will be removed later on.) Substituting iteratively for $u(Q)$ at each forward date yields

$$\begin{aligned}
 \text{(A1)} \quad u(Q_t) &= \beta R^Q [E_t u(P_{t+1}^Q Q_t) \mathcal{G}_{t+1} + u(Q_t)(1 - \mathcal{G}_{t+1})] \\
 &= \beta R^Q E_t [u(P_{t+1}^Q Q_t)] \mathcal{G}_{t+1} + (\beta R^Q)^2 (1 - \mathcal{G}_{t+1}) \{E_t [u(P_{t+2}^Q Q_t)] \mathcal{G}_{t+2} + u(Q_t)(1 - \mathcal{G}_{t+2})\} \\
 &= \beta R^Q E_t [u(P_{t+1}^Q Q_t)] \mathcal{G}_{t+1} + (\beta R^Q)^2 (1 - \mathcal{G}_{t+1}) E_t [u(P_{t+2}^Q Q_t) \mathcal{G}_{t+2}] \\
 &\quad [\dots] + (\beta R^Q)^3 (1 - \mathcal{G}_{t+1})(1 - \mathcal{G}_{t+2}) \{E_t [u(P_{t+3}^Q Q_t)] \mathcal{G}_{t+3} + u(Q_t)(1 - \mathcal{G}_{t+3})\} \\
 &= \beta R^Q E_t [u(P_{t+1}^Q Q_t)] \mathcal{G}_{t+1} + (\beta R^Q)^2 (1 - \mathcal{G}_{t+1}) E_t [u(P_{t+2}^Q Q_t)] \mathcal{G}_{t+2} + \dots \\
 &\quad \dots + (\beta R^Q)^i (1 - \mathcal{G}_{t+1}) \dots (1 - \mathcal{G}_{t+i-1}) \{E_t [u(P_{t+i}^Q Q_t)] \mathcal{G}_{t+i} + u(Q_t)(1 - \mathcal{G}_{t+i})\}
 \end{aligned}$$

and so on for each subsequent substitution of $u(Q)$. Assuming that the agent consumes all her wealth throughout the time horizon, so that holding of Q vanishes in the limit since $\lim_{i \rightarrow \infty} u(Q_t)(1 - \mathcal{G}_{t+i-1}) = 0$, and summing over the agent's infinite time horizon, the utility of asset Q at date t is

$$\text{(A2)} \quad u(Q_t) = E_t \left[\sum_{T=t+1}^{\infty} u(P_T^Q Q_t / P_T^C) \prod_{i=1}^{T-t} (\beta R^Q)^i (1 - \mathcal{G}_{t+i-1}) \mathcal{G}_T (1 - \xi_T^Q) \right]$$

Releasing the assumption of constant δ , R^Q , and $P^C = 1$ allows us to write equation (2) in an extended form

$$(A2a) \quad u(Q_t) = E_t \left[\sum_{T=t+1}^{\infty} u(P_T^Q Q_t / P_T^C) \prod_{i=1}^{T-1} \beta_i R_T^Q (1 - g_{t+i-1}) g_T (1 - \xi_T^Q) \right]$$

Every financial asset Q can therefore be regarded as a vehicle for transferring purchasing power across time. Each asset has its own capacity to store and to accumulate purchasing power over time through its real return profile. Two additional features qualify each asset's performance as a vehicle of purchasing power: the costs involved in the process of trading the asset or of transforming it into cash (i.e., its liquidity), and the volatility of the purchasing power granted by the asset grants to its holder (i.e., the risk profile of the asset's real return).

B. The Cost of Asset Liquidation

Liquidating assets may involve resource costs such as for information acquisition, search, evaluation and verification, legal and administrative requirements, bargaining and negotiations, etc. Depending on the efficiency of the financial system where asset trading takes place, as well as on the state of market mood, each asset Q requires its own minimum amount of time τ_Q^* (to be defined more precisely below) for its holder to be able to sell it at the ongoing market price P^Q , net of unit liquidation cost $q^* \in (0,1)$. If the agent is compelled to realize the asset within a time interval $\tau < \tau_Q^*$, then she must be willing to accept a sale price lower than $(1 - q^*)P^Q$, that is, the asset must sell at a price discount larger than the unit liquidation transaction cost under optimal timing ($q > q^*$). The liquidity of asset Q is therefore variable and endogenously determined, and can be modeled in terms of the following structure for asset liquidation cost

$$q = q(\tau_Q^* / \tau),$$

where

- if $0 < \tau_Q^* < \tau$ then $q = q^* > 0$: the seller has enough time to liquidate Q and pays only q^* for the transaction
- if $\tau_Q^* > \tau \geq 0$ then $q > q^*$: the seller has not enough time and must sell Q at a discount larger than the optimal unit transaction cost

- $\lim_{\substack{\tau^* > \tau \\ \tau \rightarrow 0}} q = 1$: the discount increases with the time pressure on the seller to sell Q , and
- if $\tau_Q^* = 0$ then $q^* = 0$: Q is perfectly liquid (cash),

and where

$$\tau_Q^* = \tau(\psi^Q, \omega), \quad \tau'_\psi < 0, \tau'_\omega < 0, \quad \text{and } 0 \leq \omega \leq 1,$$

that is, the minimum time interval required to sell Q optimally decreases with structural variable ψ^Q , which reflects the level of financial system efficiency in the trading of asset Q (including such features as technology; market platform, legal, regulatory and supervisory infrastructure; etc.), and increases with ω , which captures the prevailing market mood for trading Q , with a high (low) ω indicating the state of exuberance (pessimism) in the market for Q as perceived by the agents. Thus, greater (lower) efficiency of the financial infrastructure where Q is traded and a “seller” (“buyer”) market would shorten (lengthen) τ_Q^* and lower (raise) q . Variable ω will be defined below.

Since the expected utility lost to Q 's liquidation is a fraction ξ^Q of the expected utility from the consumption financed through the proceeds of Q ,

$$\xi_T^Q = E_t[u(q_T P_T^Q Q_t / P_T^C)] / E_t[u(P_T^Q Q_t / P_T^C)]$$

where $\xi^Q = \xi(q)$, $\xi^Q > 0$, $\xi(0) = 0$, and $\xi(1) = 1$. Note that ξ^Q decreases with the improvement in market sentiment (and viceversa). Equation (A2a) can then be rewritten as

$$(A2b) \quad u(Q_t) = E_t \left\{ \left[\sum_{T=t+1}^{\infty} u(P_T^Q Q_t / P_T^C) \prod_{i=1}^{T-1} \beta_i R_i^Q (1 - \mathcal{G}_{t+i}) \mathcal{G}_T (1 - \xi_T^Q) \right] \mid \omega_t \right\}.$$

C. Asset price volatility

When holding an asset, the agent faces the risk that, at any time when the asset is to be liquidated, it might sell at a loss due to the volatility of its market price. Asset price volatility affects risk-averse agent choices even if the actual price of the asset fluctuates symmetrically around its expected real value since, in translating the effects of future price dynamics in terms of asset utility gains/losses, a

risk-averse agent weighs the contribution of negative deviations from the mean relatively more than the contribution of positive deviations of equal size and duration, so that

$$E_t[u(P_T^Q Q_t / P_T^C)] < u[E_t(P_T^Q Q_t / P_T^C)]$$

where prices P^Q and P^C are governed by dynamics to be discussed in Section IV. Call σ the ratio of the expected utility loss caused by price volatilities to the expected utility from the consumption financed by Q 's liquidation,

$$(A3) \quad \sigma_T = \{u[E_t(P_T^Q Q_t / P_T^C)] - E_t[u(P_T^Q Q_t / P_T^C)]\} / E_t[u(P_T^Q Q_t / P_T^C)]$$

Eq.(A3) varies directly with the dispersion of the distribution functions of P^Q and P^C , which both increase with the degree of economic uncertainty perceived by the agent and indicated by state variable ω (see below). Also, the ratio increases if P^Q and P^C are negatively correlated. The expression for equation (A2b) can therefore be further extended as

$$(A2c) \quad u(Q_t) = E_t \left\{ \left[\sum_{T=t+1}^{\infty} u(P_T^Q Q_t / P_T^C) \prod_{i=1}^{T-1} \beta_i R_Y^Q (1 - g_{t+i-1}) g_T (1 - \xi_T^Q) (1 - \sigma_T) \right] \mid \omega_t \right\}$$

where $E_t[\cdot \mid \omega_T]$ stands for the expectations conditional on ω_T . Equation (A2c) determines the utility of liquid and less liquid financial assets in an economy with uncertainty, variable (endogenous) asset liquidation costs, and financial market volatility.

D. Agent Expectations and Market Sentiment

*Expectations formation*⁹

Let $X_t = [Y_t, P_t]$, with $\tau = t + 1, \infty$, where Y refers to future income levels and vector P denotes future prices of goods and assets. At each date, the representative agent uses current information set $i_t \in I$ to form and update expectations of the future values of X , as a solution of the economy's model. The agent operates transformation $\mathbf{T}: i\mathbf{T} \in (I \times \mathbf{R}) \rightarrow \iota \in (0,1)$, which converts every information set i into a

⁹ The approach here used generalizes the one proposed by Giovannini (1989).

real number \mathbf{z} drawn from the interval (0,1), reflecting the state of knowledge and information of the agents: as \mathbf{z} moves up (down) along the interval, it reflects a higher (lower) level of knowledge and information about the economy, and more (less) precise expectations about of the future values of X , which the agent extracts from the information set available and processes through her accumulated knowledge of the economy. In every period t , the evolution of the agent's expectations of the future values of X is governed by the following distribution function

$$(A4) \quad F_t(\mathbf{X}_T | \mathbf{z}_t) = \sum_{n=1}^{N(\mathbf{z}_t)} (\mathbf{w}_{nT} | \mathbf{z}_t) F_{tn}(\mathbf{X}_T), \quad T > t$$

where $\mathbf{w}_T : 0 \leq \mathbf{w}_{nT} \leq 1, \sum_n \mathbf{w}_{nT} = 1$, and where the weights w 's and the order N of the set of selected distribution functions are conditional on state of knowledge and information \mathbf{z}_t . Equation (A4) associates a specific structure of weights to N different probability distribution functions of X . A "high" value of state variable \mathbf{z} , reflecting strong knowledge and good information, implies a small set (i.e., a small N) of narrowly dispersed distribution functions of X , each with a large \mathbf{w}_T attached, leading to more precise predictions of future X 's: in the limit case of $\mathbf{z} = 1$ (that is, perfect knowledge and full information), Eq.(A4) reduces to $F_t(\mathbf{X}_T | 1) = F_t(\mathbf{X}_T)$. On the other hand, a low value of \mathbf{z} , reflecting poor knowledge and limited information, implies a broad set (i.e., a large N) of dispersed distribution functions, each with a small \mathbf{w}_T attached: no single distribution dominates the others in the agent's expectations, and each has a very low probability of being the "true" one.¹⁰ The effect of more or less dispersed expectations is of great relevance for discussing the impact of asset price volatility on resource allocation.

Market sentiment

Agent expectations are affected by (changes in) market mood, that is, the sentiment about the state of the world, which prevails in the environment where the agents operate. We define optimism (pessimism) as the state of mind inducing an agent to expect superior (inferior) outcomes of future

¹⁰ In a fan chart, the band would be very large and all predicted values within the band would have the same shadings since none would have a higher likelihood than the others. In the extreme case of a value of \mathbf{z} very proximate to 0, then $\mathbf{w}_{nT} \cong 0, \forall n$ and the distribution $F_t(\mathbf{X}_T)$ would be indeterminate (the width of the fan-chart band would be infinite).

events than would be reasonable for the agent to anticipate on the exclusive basis of the knowledge and the information available to her. Formally, expectations are said to be “optimistic” when

$$E_t(X_T | t'_t) > E_t(X_T | t_t)$$

or

$$W_t[E_t(X_T | t'_t)] > W_t[E_t(X_T | t_t)] \text{ for } T > t, \text{ and } t'_t > t_t$$

where W the welfare associated with the expected value of X and where the reverse sign holds in both expressions for “pessimistic” expectations, and where the *perceived* state of the world t' is the result of information “distortive” process $\mathbf{D}: iD \in (I \times R) \rightarrow t'/t \equiv \omega \neq 1$ and $t', t \in (0,1)$, where deviations of ω from 1 signal the effect of perceptive “distortions” on agent expectations, and where $\omega > 1$ ($\omega < 1$) indicates optimistic (pessimistic) expectations: under optimism or pessimism, the agent’s *perceived* state of the world is “distorted” relative to the state of the world which would be reasonable to expect to prevail based on the agent’s knowledge and information available. Process \mathbf{D} reflects the agent emotions, and distorts the expected value of the variables that are relevant to the agent to an extent that depends on the intensity of the prevailing emotions: much as individuals experience in real life situations when they see the very same contingency under a different light depending on the state of their mind, often under the influence of the prevailing social context. This process can be understood as the product of (changing) individual “animal spirits” as these may be socially influenced by the individuals’ mutual observation of others’ moods and behaviors, as well as by individual psychological attitudes. As knowledge progresses, better information becomes available, and new feedbacks are gained from observed reality, agents adjust their perceptions possibly causing their “distorted” (optimistic or pessimistic) expectations to gravitate toward their correct (“undistorted”) equivalent. However, it is also possible that, at least for some time, those “distorted” expectations are confirmed ex post by real events.

Market sentiment and the external environment

While changes in market moods necessarily reflect the internal (psychological) processes of individual human beings and the mechanisms governing their social interactions, they are nonetheless

influenced by the observation of reality as filtered by knowledge. Market moods, thus, respond to evolving features of the economic environment, among which critical are: (a) the perceptions of general economic (in)stability; (b) the performance of relevant markets; and (c) significant news, innovations, and forecasts. Assumptions follow as to how these three classes of features influence market moods. While such assumptions are here given the form of specific algorithms, so as to facilitate the formal analysis below, they should be considered more broadly as suggestive that agent emotions are not necessarily unexplained “sunspots”: they do interact with information, knowledge, and rationality in defining the agents’ responses to real circumstances.

The assumptions are:

- a. *General economic stability* is here evaluated in terms of social loss function $L = L(\bar{x})$, where x is the set of arguments representing the key macroeconomic policy objectives pursued by the policymakers, and each argument is expressed as a deviation between actual and target values.

Agents know that policymakers pursue socially optimal target $L^* = \min L(\bar{x})$. They thus observe the actual values of L and compare them to L^* . Looking backward, say τ periods from current date set at $t_0 = 0$, the agents evaluate economic stability using function

$$(A5) \quad S = -(1/t) \sum_{t=t_0-\tau}^{t_0-1} t \ln(L_t - L^*) = \mathfrak{S}(\Lambda, t), \text{ where } t_0 = 0 \text{ and } L_t - L^* \equiv \Lambda_t \quad \text{with } \mathfrak{S}_\Lambda < 0; \mathfrak{S}_t > 0$$

where recent deviations Λ weigh more heavily than distant ones. Note that S increases with lower Λ and with higher scalar factor t , which determines the duration of (in)stability: all else equal, a value of S calculated over a longer time interval is more informative than if calculated over a shorter interval. In words: protracted stability (instability) feeds optimism (pessimism).

- b. *Market performances* can be measured as observed changes in past realizations over a relevant time period

$$(A6) \quad PPerf_t = -(1/t) \sum_{t=t_0-\tau}^{t_0-1} t(X_t - X_{t-1}),$$

where, too, observations far distant in time weigh less than those closer to the present and persistent same-directional changes increase the value of the indicator.

c. *News, innovations, and forecasts* work their effects on market mood by improving expectations of future discounted market performances

$$(A7) \quad FPerf_t = \sum_{t=t_0-\tau}^{t_0-1} \beta^t E_{t_0} (X_t - X_{t-1}),$$

Notice that, in the light of relation (A7), there would be mutually reinforcing retro-feedback effects from market sentiment to expectations and back to market sentiment.

From Eqs.(A5)-(A7):

$$(A8) \quad \omega = \omega(S, PPerf, FPerf) \quad \text{with} \quad \omega'_S, \omega'_{PPerf}, \omega'_{FPerf} > 0,$$

that is: increasing (decreasing) perceptions of economic stability, improving (worsening) past market performances, and improving (worsening) expected future market performance, generate greater optimism (pessimism).

Notice from Eq.(A8) that repeated signals of policy ineffectiveness in minimizing the social loss function, a protracted flat market performance, and the lack of news and innovations suggesting improvements in future market performance would cause market sentiment to remain persistently pessimistic, thus continuing to affect economic variables negatively.

E. The Economy

The Representative Agent

As from the main text, the representative agent orders her preferences across C , M , T , B , and K , according to a strictly quasi-concave, time-separable, and well-behaved utility function $u_t = u(C_t, M_t, T_t, B_t, K_t)$, with $u''(\cdot) > 0$ (risk aversion). The general formal expression for the utility of any asset Q at any time t is

$$(A9) \quad u(Q_t) = E_t \left\{ \left[\sum_{T=t+1}^{\infty} u(P_T^Q Q_t / P_T^C) \prod_{i=1}^{T-1} \beta_i R_i^Q (1 - \vartheta_{t+i-1}) \vartheta_T (1 - \xi_T^Q) (1 - \sigma_T) \right] \mid \omega_t \right\}$$

where

- ϑ_{t+1} : probability of having to finance unexpected consumption at time $t+1$;
- ξ^Q : expected utility loss due to asset liquidation costs as a ratio of the expected utility from the consumption financed through sale of Q ; and
- σ^Q : expected utility loss due asset price volatility as a ratio of the expected utility from the consumption financed through sale of Q ,

and where utility is conditional on current knowledge, information, and the prevailing market mood. From Eq.(A9), and considering that in the case of a fully liquid asset, like money, $\xi^M = \sigma^M = \rho^M = \pi^M = 0$,¹¹ the utility-of-deposit function reduces to

$$(A9b) \quad u(M_t) = E_t \left\{ \left[\sum_{\tau=t+1}^{\infty} u(M_\tau / P_\tau^c) \prod_{i=1}^{T-1} \beta_i R_i^Q (1 - \vartheta_{t+i-1}) \vartheta_T \right] \middle| \omega_t \right\}.$$

A rise in optimism, or a decline in the perceived state of economic uncertainty, increases (decreases) the utility of less (more) liquid assets, consequently affecting their demand and relative prices (returns).

The agent is assumed to weigh her future income prospects by considering how the value of her job is going to be affected by economic developments. Following Hall (2013), the job value is the present discounted value of the future difference between a worker's productivity and the worker's pay, which is here written as:

$$(A10) \quad \mathcal{J}_t = E_t \left[\sum_{\tau=t+1}^{\infty} \prod_{i=t}^{\tau} \beta^i (p_\tau x_\tau - y_\tau) \middle| \omega_t \right]$$

where x and w are, respectively, the agent's output and remuneration, and expectations reflect market moods: the job value declines (rises) with pessimistic (optimistic) prospects about future levels of

¹¹ As a perfectly liquid asset, money trades against any goods or assets at a certain nominal price and zero transaction cost, and does not bear interest.

remuneration and risk of unemployment. As the representative agent determines her intertemporal budget constraint, she uses her expected job value to “modulate” her future expected income levels:

$$(A11) \quad E_t[y_\tau | \omega_t] = E_t(y_\tau^{(J V_t / \bar{\Pi})})$$

where $\bar{\Pi}$ is the normal rate of profit, $\frac{J V_t}{\bar{\Pi}} = 1$ holds under normal economic conditions (where all factors earn the same return), and $\frac{J V_t}{\bar{\Pi}} > 1$ (< 1) under optimistic (pessimistic) expectations. In other words, the agent expects a future lower (higher) income depending on her prevailing state of optimism (pessimism) looking forward.

At any date t , the agent plans to maximize

$$(A12) \quad U = \underset{\{C_\tau, M_\tau, T_\tau, B_\tau, K_\tau\}}{\text{Max}} E_t \left\{ \left[\sum_{\tau=t}^{\infty} u(C_\tau, M_\tau, T_\tau, B_\tau, K_\tau) \right] \left[\prod_{i=t}^{\tau-1} \mathbf{b}'_i \mathbf{r}_i^{\mathbf{Q}=\mathbf{M}, \mathbf{T}, \mathbf{B}, \mathbf{K}} \right] \mid \omega_t \right\}$$

where $\mathbf{b}'_i \mathbf{r}_i^{\mathbf{Q}=\mathbf{M}, \mathbf{T}, \mathbf{B}, \mathbf{K}}$ is the vector of asset rates of return, subject to budget constraint

$$(A13) \quad P^C_\tau C_\tau + P^T_\tau T_\tau + P^B_\tau B_\tau + P^K_\tau K_\tau + M_\tau = y_\tau - \text{tax}_\tau + M_{\tau-1} + r^T_{\tau-1} P^T_{\tau-1} T_{\tau-1} + r^B_{\tau-1} P^B_{\tau-1} B_{\tau-1} + r^K_{\tau-1} P^K_{\tau-1} K_{\tau-1}$$

$$(A14) \quad y_\tau = E_t[y_\tau | \omega_t] = E_t\{y_\tau^{(J V_t / \bar{\Pi})}\}$$

$$(A15) \quad C_\tau, M_\tau, T_\tau, B_\tau, K_\tau \geq 0$$

and transversality condition

$$(A16) \quad \sum_{\tau=0}^{\infty} P^C_\tau C_\tau = \sum_{\tau=0}^{\infty} (y_\tau).$$

The central bank

The central bank governs the supply of M so as to minimize deviations over time between the interest rate on the riskless asset T and the agent's rate of time preference δ (assuming this can be derived from available data). The central bank thus minimizes loss function L :

$$(A17) \quad L = \underset{\{\bar{M}\}}{\text{Min}} E_t \left[\sum_{\tau=t}^{\infty} \beta^{\tau} L(r_{\tau}^T - \delta_{\tau}) \right]$$

subject to

$$(A18) \quad r_{\tau+1}^T - \delta_{\tau+1} = a(r_{\tau}^T - \delta_{\tau}) - b(\bar{M}_{\tau} - M_{\tau}) + e_{\tau+1}, \quad e_{\tau+1} \sim N(0, \sigma_e^2), \text{ i.i.d.}$$

The government

The government produces a non-pecuniary public good G and finances production by issuing bonds T and B (which match agent portfolio preferences of different maturity and liquidity services)

$$(A19) \quad G_{\tau} + r_{\tau-1}^T P_{\tau-1}^T T_{\tau-1} + r_{\tau-1}^B P_{\tau-1}^B B_{\tau-1} = P_{\tau}^T T_{\tau} + P_{\tau}^B B_{\tau} + \text{tax}_{\tau}$$

under an intertemporal constraint whereby all state expenditures must be paid over time through tax revenues

$$(A20) \quad \sum_{t=\tau}^{\infty} [G_{\tau} + P_{\tau}^T T_{\tau} \prod_{\tau} r_{\tau}^T + P_{\tau}^B B_{\tau} \prod_{\tau} r_{\tau}^B] = \sum_{t=\tau}^{\infty} \text{tax}_{\tau}$$

The case is examined in the main text where the government budget is partly financed through new money creation by the central bank. In terms of Eq.(A19), this would become

$$(A19b) \quad G_{\tau} + r_{\tau-1}^T P_{\tau-1}^T T_{\tau-1} + r_{\tau-1}^B P_{\tau-1}^B B_{\tau-1} = P_{\tau}^T T_{\tau} + P_{\tau}^T T_{\tau} + \text{tax}_{\tau} - D\text{tax}_{\tau}$$

where $DM_\tau = |Dtax_\tau|$.

F. Equilibrium

Plan (A12)-(A20) is the agent's program to allocate resources optimally between current consumption, money and non-money assets so as to maximize, both within each date and over time, the streams of utility derived from each, subject to the given resource constraint. The Bellman's equation to solve the plan is

$$V(y_t, C_t, M_t, T_t, B_t, K_t | \omega_t) = \underset{\{C, M, T, B, K\}}{\text{Max}} \{u(C_t, M_t, T_t, B_t, K_t) | \omega_t\} \\ + V(y_{t+1}, M_{t+1}, T_{t+1}, B_{t+1}, K_{t+1} | \omega_{t+1}) \mathbf{b}' \mathbf{R}' \}$$

The Euler equation is

$$u'(C_t, M_t, T_t, B_t, K_t | \omega_t) = E_t \{ [u'(C_{t+1}, M_{t+1}, T_{t+1}, B_{t+1}, K_{t+1})] \mathbf{b}'_{t+1} \mathbf{R}'_{t+1}^{\mathbf{Q}=\mathbf{M}, \mathbf{T}, \mathbf{B}, \mathbf{K}} | \omega_t \},$$

which determines the optimal inter-temporal path for consumption and asset holdings, conditional on state variable ω_t . The optimal intra-date allocation across consumption and assets, conditional on ω_t , is derived as a solution to

$$U = \underset{\{C, M, T, B, K\}}{\text{Max}} E_t \left\{ \sum_{\tau=t}^{\infty} u(C_\tau, M_\tau, T_\tau, B_\tau, K_\tau) \prod_{i=t}^{\tau-1} \mathbf{b}'_i \mathbf{r}_i^{\mathbf{Q}=\mathbf{M}, \mathbf{T}, \mathbf{B}, \mathbf{K}} \right\} | \omega_t \}$$

subject to constraints (A11)-(A14), which yields the following first order conditions:

$$U'(C_t) = E_t \left\{ [u'(C_t) / P_t^C] \prod_{i=0}^t \beta_i \right\} | \omega_t \} - \lambda_t = 0$$

$$U'(M_t) = E_t \left\{ [u'(M_t)] \prod_{i=0}^t \beta_i r_M^i \right\} | \omega_t \} - \lambda_t = 0$$

$$U'(T_t) = E_t \left\{ [u'(T_\tau) / P_t^T \prod_{i=0}^{\tau-t} \beta_i r_\tau^i] \mid \omega_t \right\} - \lambda_\tau = 0$$

$$U'(B_t) = E_t \left\{ [u'(B_\tau) / P_t^B \prod_{i=0}^{\tau-t} \beta_i r_\tau^i] \mid \omega_t \right\} - \lambda_\tau = 0$$

$$U'(K_t) = E_t \left\{ [u'(K_\tau) / P_t^K \prod_{i=0}^{\tau-t} \beta_i r_\tau^i] \mid \omega_t \right\} - \lambda_\tau = 0$$

$$\lambda_\tau E_t \{ [P_\tau^C C_\tau + M_\tau - M_{\tau-1} + P_\tau^T T_\tau - P_{\tau-1}^T T_{\tau-1} + P_\tau^B B_\tau - P_{\tau-1}^B B_{\tau-1} + P_\tau^K K_\tau - P_{\tau-1}^K K_{\tau-1} - y_\tau] \mid \omega_t \} = 0$$

$$\lambda_\tau \geq 0.$$

These conditions imply that, at planning date t , the agent selects the allocation $(C_\tau^*, M_\tau^*, T_\tau^*, B_\tau^*, K_\tau^*)$ and the economy determines prices $(P_\tau^{C^*}, 1_\tau, P_\tau^{T^*}, P_\tau^{B^*}, P_\tau^{K^*})$, which at each future date τ satisfy the optimal intra-date rule

$$(A21) \quad E_t [u'(C_\tau^*) / P_t^{C^*} \mid \omega_t] = E_t [u'(M_\tau^*) \mid \omega_t] = E_t [u'(T_\tau^*) / P_t^{T^*} \mid \omega_t] = E_t [u'(B_\tau^*) / P_t^{B^*} \mid \omega_t] = E_t [u'(K_\tau^*) / P_t^{K^*} \mid \omega_t]$$

and clear the markets for goods and assets. Rule (A21) equates at each date the w.m.u. of consumption and of monetary and financial assets. The consistency of optimal rule (A21) with the solution to plan (A12)-(A20) can be seen by showing that no allocation $(C_\tau^0, M_\tau^0, T_\tau^0, B_\tau^0, K_\tau^0)$ exists at equilibrium prices $(P_\tau^{C^*}, 1_\tau, P_\tau^{T^*}, P_\tau^{B^*}, P_\tau^{K^*})$, which solves (A12) while not violating (A21): with $(C_\tau^0, M_\tau^0, T_\tau^0, B_\tau^0, K_\tau^0)$ violating (A19), an allocation $(C_\tau^*, M_\tau^*, T_\tau^*, B_\tau^*, K_\tau^*) \succ (C_\tau^0, M_\tau^0, T_\tau^0, B_\tau^0, K_\tau^0)$ can always be attained at no extra cost, which is consistent with (A12)-(A20), yields a higher value for (A12), and solves (A21).

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