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# A detrimental feedback loop: deleveraging and adverse selection

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# A detrimental feedback loop: deleveraging and adverse selection

Christoph Bertsch\*

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## Abstract

Market distress can be the catalyst of a deleveraging wave, as in the 2007/08 financial crisis. This paper demonstrates how market distress and deleveraging can fuel each other in the presence of adverse selection problems in asset markets. At the core of the detrimental feedback loop is agents' desire to reduce their reliance on distressed asset markets by decreasing their leverage which in turn amplifies the adverse selection problem in asset markets. In the extreme case, this leads to a market breakdown.

I find that adverse selection creates both an “ex-ante” inefficiency because it distorts agents' long-term leverage choices and an “interim” inefficiency because it distorts agents' short-term liquidity management. I derive important implications for central bank policy.

*Keywords:* Leverage, endogenous borrowing constraints, financial crisis, liquidity, asymmetric information, central bank policy.

*JEL classification:* D82, E58, G01, G20.

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# 1 Introduction

At the start of the financial crisis in July 2007, interbank market spreads jumped up and subprime asset markets experienced a large drop. Temporarily important market segments dried up completely.<sup>1</sup> At the same time, a pronounced deleveraging wave in the financial sector began. US investment banks drastically cut leverage immediately after the crisis erupted while data show that US commercial banks as well as EU and UK banks only began heavily reducing leverage from 2008 onwards. Financial market conditions were the main driver of deleveraging in 2007 and 2008. Later, the effect of financial market disorder on deleveraging was compounded by regulatory initiatives and a change in economic and policy conditions.<sup>2</sup>

In this paper, I propose a novel mechanism which draws a connection between financial market distress and deleveraging. The two key elements of the mechanism are *(i)* adverse selection problems in asset markets that cause market distress and *(ii)* the possibility of market participants to shield themselves against market distress in asset markets by adjusting their leverage. Empirically, the link between distress in subprime markets and adverse selection problems can be attributed to a substantial rise in counterparty risk and severe asymmetric information problems in subprime markets at the beginning of the crisis.<sup>3</sup> My model uses the same trigger for market distress. The model then links market distress in a novel way to the financial sector deleveraging wave witnessed during the crisis. In particular, I demonstrate that deleveraging and the intensity of an Akerlof (1970) type adverse selection problem in asset markets are interconnected in a potentially detrimental way through a novel feedback mechanism. This mechanism has, to my knowledge, yet to be studied in the existing literature.

The mechanism works as follows. If liquidity in a certain market is expected to contract because of adverse selection problems, investors try to access alternative markets for financing that are not prone to adverse selection problems, such as the prime market segment for collateralized credit. However, a better access to the prime market segment requires the provision of unencumbered high quality collateral that can be used for future trades. As a result, investors are forced to free high quality collateral by reducing their leverage. Interestingly, investors' attempts to improve their access to prime markets by reducing their leverage can amplify the

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<sup>1</sup>The spread between LIBOR and the overnight Federal Funds rate for 3-month loans jumped from sub 20 basis point levels before July 2007 to elevated levels between 40 and 100 basis points (see Cecchetti (2009a), p. 58). A similar picture holds for Europe where the spread between EURIBOR and the 3-month overnight index swap jumped from below 10 basis points to elevated levels fluctuating around 60 basis points during the year after August 2007. Then the spread shot up to over 180 basis points in November 2008 (see Heider et al. (2009), p. 8). US subprime markets for asset-backed securities and global high-yield corporate bonds were largely affected. In the year after August 2007, the US subprime index fell by over 80% and global high-yield corporate bond spreads climbed to over 60% (see Bank of England Financial Stability Report, April 2008).

<sup>2</sup>Feyen and González del Mazo (2013) provide a detailed account of the deleveraging wave. For US investment banks, leverage ratios (measured as weighted tangible assets over tangible common equity) dropped from around 40% in 2007 to under 30% in 2008, followed by a further drop to under 20% in 2009. Main factors contributing to the deleveraging wave in the initial crisis period till 2008 were the distress in interbank, subprime asset and high-yield corporate bonds market.

<sup>3</sup>See Gorton (2008) amongst others. Lax screening incentives under the existing securitization procedures may have contributed to the emergence of substantial asymmetric information problems, as argued by Keys et al. (2010).

adverse selection problem in subprime asset markets. Consequently, the benefits to an individual investor accruing from a reduction in her leverage in order to improve the access to prime markets increase even further, thereby laying the foundation for a detrimental feedback mechanism. Specifically, my model exhibits a simultaneous deleveraging and worsening of adverse selection problems in subprime asset markets which fuel each other. In this way, the model draws a compelling connection between the deleveraging wave in the financial crisis of 2007/08 and the adverse selection problems in sub-prime markets.

Importantly, the novel feedback mechanism presented in this paper does not rely on portfolio constraints and margin requirements.<sup>4</sup> Instead, the effect is solely generated by investors' desire to shield themselves from the negative implications of adverse selection problems in asset markets. Furthermore, I show that adverse selection problems distort both the leverage choice and the liquidity choice of financial market participants. This has important implications for central bank and regulatory policy and sheds new light on policy responses to the financial crisis.

From a more technical point of view, my paper proposes a model of liquidity provision with adverse selection that is enriched by an *ex-ante* leverage choice. Thus, the model combines short-term liquidity management with the more long-term problem of deciding about the optimal leverage of the business model. Agents are subject to idiosyncratic liquidity and solvency risk which they can share in two distinct spot markets. First, a subprime asset market that is prone to private information problems and second, a market for collateralized credit in the prime segment that is not prone to private information problems. However, the access to this alternative prime market is costly for individual agents. Access requires the provision of costly "spare borrowing capacity" (e.g. in the form of unencumbered assets of the highest quality) and thus demands a reduction in leverage. I show that agents may have an incentive to reduce their leverage and install a costly "spare borrowing capacity" in order to insulate themselves from future adverse selection problems in the asset market. Interestingly, there appears to be a strategic complementarity in borrowing capacity (or leverage) choices. This strategic complementarity is the building block of a detrimental feedback loop between financial market distress and deleveraging. In particular, I demonstrate analytically that there can exist detrimental deleveraging spirals induced by adverse selection which, in the extreme, can lead to a breakdown of the asset market.

Due to the strategic complementarity, the model exhibits an "ex-ante" inefficiency in borrowing capacity or leverage choices. Furthermore, for a given leverage choice, agents' liquidity choice is also distorted, leading to an "interim" inefficiency. The economy exhibits an under-investment in cash for a large parameter range. The private information is at the root of the inefficient liquidity choice.

The paper is organized as follows. Section 2 reviews the literature and discusses how my paper relates to it. The model is introduced in section 3. Section 4 contains the equilibrium

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<sup>4</sup>As it is, for instance, in Brunnermeier and Pedersen (2008).

analysis. Thereafter, I analyze efficiency in section 5. Important policy implications for liquidity regulation, leverage regulation and central bank interventions are discussed in section 6. Finally, section 7 concludes and discusses the robustness of the results. All proofs are in the Appendix.

## 2 Literature

Bolton et al. (2011) and Malherbe (2013)'s work is the most closely related to mine. They find that the anticipation of adverse selection in the future leads to excessive early asset trading and liquidity hoarding, respectively. In Bolton et al. (2011), the most efficient source of financing for banks with liquidity needs is outside liquidity. However, asymmetric information about the quality of bank assets constitutes a detrimental friction when market participants anticipate future liquidity shocks. The authors show that asymmetric information can lead to excessive early asset trading and excessive cash reserves. Moreover, banks reduce their origination of assets in anticipation of fire sale prices due to future liquidity shocks. In Malherbe (2013), the information about the quality of assets is also asymmetric. He finds that adverse selection and hoarding behavior can fuel each other.

Different from Bolton et al. (2011) and Malherbe (2013), investors in my model anticipate future adverse selection problems and, as a result, seek to reduce their leverage today. This, in turn, can intensify the adverse selection problem in the future and lead to stronger deleveraging today. Unlike Malherbe (2013), the supply of cash is endogenous in my model and, hence, cash holdings do not pose a negative externality. My model also differs from Malherbe (2013) in that it focuses on a novel interplay between two frictions, a private information problem and endogenous collateral constraints.

The interplay between the two frictions private information and collateral constraints is also analyzed by Martin and Taddei (2013) and Boissay (2011). However, their setting differs substantially from mine and they consider an environment in which both frictions affect the same asset. Furthermore, Boissay (2011) focuses on self-fulfilling pessimistic beliefs to generate a liquidity dry-up. While Martin and Taddei (2013) find that limited pledgability exacerbates adverse selection problems, the opposite result arises in my model with *endogenous* borrowing constraints. In earlier work Caballero and Krishnamurthy (2001) and Caballero and Krishnamurthy (2002) analyze the interplay between international and domestic collateral constraints. When the domestic credit market is underdeveloped domestic agents over-borrow and do not hold sufficient international collateral. Binding international collateral constraints, in turn, lead to fire sales in domestic markets with negative implications for financial intermediation. Caballero and Krishnamurthy's domestic and international credit market share some similarities with my paper's sub-prime asset market and its prime market for collateralized credit, respectively. Different to their work, the provision of high quality collateral plays in my model a rather negative role due to the adverse selection problem in sub-prime markets. Moreover, the provision of high quality collateral is in my model connected to leverage and investment.

There are several related papers that examine adverse selection problems in macro models

following the partial equilibrium model of Eisfeldt (2004), such as Kurlat (2009), Bigio (2011) and Taddei (2010). Kurlat and Bigio both extend the framework of Kiyotaki and Moore (2012) by introducing endogenous resaleability through asymmetric information. While Kurlat (2009) focuses on the relationship between liquidity and macroeconomic fluctuations as well as the amplification of shocks through learning, Bigio (2011) adds a labor market friction and analyzes how dispersion shocks to capital quality affect the liquidity of assets and the macroeconomy. Taddei (2010) rationalizes the positive relationship between aggregate economic activity and the cross-firm divergence of bond yields. Their models contrast with mine in that they abstract from the role of a liquid asset that co-exists with illiquid assets prone to adverse selection problems, which is a key ingredient of the mechanism presented in this paper.<sup>5</sup>

A separate strand of literature examines adverse selection problems and liquidity in asset markets (see Kirabaeva (2011)) or in interbank credit markets (see Freixas and Holthausen (2004), Heider et al. (2009) and Heider and Hoerova (2009)). Freixas and Holthausen (2004) is most closely related to this paper. They analyze a model with secured and unsecured credit which is similar to my model in which illiquid assets co-exist with high quality collateral. However, Freixas and Holthausen (2004) consider an exogenous change in the income structure that changes the composition between secured and unsecured credit thereby affecting the intensity of adverse selection in interbank credit markets.

The detrimental deleveraging loop developed in my paper also substantially differs from related mechanisms proposed by Brunnermeier and Pedersen (2008) and Geanakoplos (2009) among others, as it rests on a connection between deleveraging and adverse selection in asset markets. Brunnermeier and Pedersen (2008) develop a liquidity spiral consisting of an interplay between a loss spiral and a margin spiral. In Geanakoplos (2009), leverage cycles evolve through endogenous margin requirements. Similar to my paper, Geanakoplos (2009) predicts too little leverage and an asset price drop in a crisis. Yet the trigger in Geanakoplos's model is margin requirements, whereas in mine it is the anticipation of future adverse selection problems which leads to a desire to reduce leverage today with the purpose of having more unencumbered high quality assets available in the future. The emergence of a high demand for unencumbered high quality assets that can be used for future trades has some features of a "flight to quality". However, the mechanism does not rely on Knightian uncertainty as in Caballero and Krishnamurthy (2008) and other related papers that attempt to explain flight to quality.

From a theoretical point of view, my paper features a model with endogenous liquidity supply drawing from related work by Shleifer and Vishny (1992) and the subsequent literature on cash-in-the-market pricing pioneered by Allen and Gale.<sup>6</sup> The notion of liquidity used in this paper refers to the cost of converting expected future income into cash. The stronger the adverse selection problem in the asset market and the less cash is available in the economy, the higher the cost. As such, my model incorporates the two important reasons for market breakdowns

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<sup>5</sup>More recently, Cui and Radde (2013) developed a version of Kiyotaki and Moore (2012) with a liquid asset and search frictions in illiquid asset markets. However, they abstract from adverse selection and focus on the pro-cyclicality of asset liquidity.

<sup>6</sup>See, for instance, Allen and Gale (2004) and Allen and Gale (2007).



discussed by Tirole (2011), with adverse selection and insufficient financial muscle.

The efficiency analysis in this paper is similar to Lorenzoni (2008) and Plantin and Parlour (2008) but with the special feature of two distinct inefficiencies. First, this paper exhibits an “ex-ante” inefficiency in borrowing capacity or leverage choices and, second, an “interim” inefficiency in liquidity choices. My result that the economy exhibits an under-investment in cash for a large parameter range contradicts the prescription of models with adverse selection who predict over-investment in cash (compare Malherbe (2013)). However, my result is in line with the cash-in-the-market pricing literature and with Bhattacharya and Gale (1987) who find that moral hazard and adverse selection are associated with under-investment in reserves.

### 3 The model

The model has four dates indexed by  $t = 0, 1, 2, 3$ . It comprises a model of liquidity provision spanning over dates  $t = 1, 2, 3$  that is enriched by a leverage choice at date  $t = 0$ .

#### 3.1 Agents

There is a continuum of ex-ante identical agents with mass equal to one. Agents are born at  $t = 0$  and derive utility from consuming at dates  $t = 2$  and  $t = 3$ . Ex-ante agents are uncertain about whether they would prefer to consume at  $t = 2$  or  $t = 3$ . Similar to Diamond and Dybvig (1983), there are two types of agents, the *early type* and the *late type*. The likelihood of an individual agent being either *early type* or *late type* is given by  $\lambda$  and  $(1 - \lambda)$ , respectively. *Early types* prefer to consume at the intermediate date whereas *late types* prefer consumption at date  $t = 3$ . Formally, the preferences of *early types* (subscript  $E$ ) are represented by the utility function:

$$u(c_2, c_3) = \beta_E U(c_2) + (1 - \beta_E) U(c_3)$$

and the preferences of *late types* (subscript  $L$ ) by the utility function:

$$u(c_2, c_3) = \beta_L U(c_2) + (1 - \beta_L) U(c_3),$$

where  $c_t$  is the consumption of an agent at date  $t$  and  $U(c) = \log(c)$  for simplicity.<sup>7</sup>

A higher relative valuation of consumption at  $t = 2$  by *early types* is reflected in  $\beta_E > \beta_L$ . In the remainder, I concentrate on parameters satisfying  $1 > \beta_E \geq \beta_L = 0$ .<sup>8</sup>

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<sup>7</sup>I use log-utility to ensure tractability. However, most qualitative results should hold for neoclassical utility functions satisfying the Inada conditions.

<sup>8</sup>For the main mechanism of this paper to work, there has to be a “limited” liquidity need in the intermediate period. Hence,  $\beta_E$  must be strictly smaller than one as this allows *early types* to shift consumption to  $t = 3$  in case there are some benefits of doing so. Moreover, setting  $\beta_L = 0$  shows to give a large advantage in analytical tractability without affecting the main results qualitatively.

### 3.2 Outside financiers

Next to the continuum of ex-ante identical agents, there are also *outside financiers*. Their only role is to provide financing to agents at the initial date  $t = 0$  at a fixed rate  $q > 1$ . For that reason, outside financiers are not explicitly modeled. Agents can be thought of as banks or other specialized investors in financial markets. Outside financiers can be thought of as fixed income funds or insurers. They have a long-term investment horizon and only invest in the safest asset classes, demanding highest quality collateral.

### 3.3 The leveraged investment at date $t = 0$

#### 3.3.1 Endowments & technology

At the initial date  $t = 0$ , agents are each endowed with  $\kappa > 0$  units of capital. Agents can invest in a leveraged long-term investment project and a risk-less storage technology. Storage is available in every period and can be used to transform one unit of cash today into one unit of cash in the subsequent period. Leveraged long-term investment projects can only be initiated at  $t = 0$  and can only mature at  $t = 3$ . These projects yield a deterministic date  $t = 3$  return of  $\rho > 1$  per unit invested.

The long-term investment project can be scaled at date  $t = 0$  by raising funds from *outside financiers* at a rate  $q$ , where  $1 < q < \rho$ . However, only a fraction  $0 < \gamma < 1$  of the date  $t = 3$  income from the investment project is pledgeable and, hence, leverage is limited.<sup>9</sup> *Outside finance* is discussed in more detail in section 3.3.2.

Let  $\Upsilon$  denote the scale of the leveraged long-term investment project. The maximum investment scale, say  $\bar{\Upsilon}$ , can then be computed as the solution to:

$$\Upsilon \leq k + \frac{\gamma\Upsilon - \theta}{q}, \quad \text{which gives } \Upsilon \leq \bar{\Upsilon}(k, \theta) \equiv \frac{k - \theta/q}{1 - \gamma/q}, \quad (1)$$

where  $k \in [0, \kappa]$  is the actual amount of capital invested and  $\kappa - k \geq 0$  is the amount of capital stored. The variable  $\theta \geq 0$  denotes the amount of “*spare borrowing capacity*” available for future periods. It is positive if agents decide not to fully lever-up their long-term investment project.

The total return on the leveraged long-term investment project, say  $G(k, \theta)$ , can be derived as:

$$G(k, \theta) \equiv \rho\Upsilon(k, \theta) - \left[ \frac{\gamma\Upsilon(k, \theta)}{q} - \frac{\theta}{q} \right] q = \frac{\rho - \gamma}{1 - \gamma/q} \left( k - \frac{\theta}{q} \right) + \theta. \quad (2)$$

The function is increasing in  $k$  and decreasing in the spare borrowing capacity  $\theta$ .

#### 3.3.2 Outside financing

I assume that outside financiers have deep pockets and provide outside finance at the fixed rate  $q$ . Furthermore, I assume that agents have access to financing from outside financiers only at

<sup>9</sup>The assumption of limited pledgability could, for instance, be justified by a moral hazard problem (see for instance Holmström and Tirole (2010)) or by the inalienability of human capital (see Hart and Moore (1994)).

$t = 0$  and not thereafter. This implies that only agents participate in date  $t = 1$  and  $t = 2$  markets. Outside financiers, who lack expertise are excluded due to limited participation.<sup>10</sup> The assumption of limited participation by outsiders simplifies the analysis. However, I will argue that the main qualitative results are robust when the assumption is relaxed.

### 3.4 The model of liquidity provision spanning over dates $t = 1, 2, 3$

Agents enter date  $t = 1$  with predetermined “*spare borrowing capacity*”. In particular, agents have  $\theta \geq 0$  units of spare borrowing capacity and carry over  $\kappa - k$  units of cash into date  $t = 1$ . I assume that the leverage choices at  $t = 0$  summarized in  $(k, \theta)$  are *common knowledge* at  $t = 1$ .

#### 3.4.1 Endowments & technology

At  $t = 1$ , agents each receive an endowment of  $\vartheta > 0$  units of cash each and no additional endowment thereafter. Including the cash carried over into  $t = 1$ , their total available resources are given by  $\zeta(k) = \vartheta + \kappa - k$ . Furthermore, agents have access to a collateralized credit market at  $t = 1$  where they can borrow or lend at the endogenous market interest rate  $r_1$ .<sup>11</sup>

As stated earlier, agents also have access to a riskless storage technology at  $t = 1$ . Alternatively, agents can invest in a risky long-term asset which can be thought of as a risky portfolio of mortgage loans that mature at the terminal date. The decision problem at date  $t = 1$  for each agent is to decide whether she wants to become an *illiquid investor* who invests in risky long-term assets (i.e. to issue risky mortgage loans) or a *liquid investor* who stores cash. In other words, agents have a discrete liquidity choice at date  $t = 1$ . They can either store all their available resources or instead fully invest them in a risky long-term asset.<sup>12</sup> I denote the endogenous population fraction of *liquid investors* with  $0 < f < 1$  and the fraction of *illiquid investors* with  $(1 - f)$ .

The payoffs of risky long-term assets are given in table 1.

<sup>10</sup>The assumption is standard in the cash-in-the-market pricing literature. Limited participation can, for instance, be justified by fixed costs that cause outside finance to be only available for long-term financing and not to cover short-term liquidity needs. In reality, outside financiers like insurers do not have the expertise to value certain complex risky assets classes such as structured mortgage-backed securities. Moreover, outside finance is in practice often associated with substantial fixed costs because financiers have to learn “about the basic features of the market such as the distribution of asset returns (...) and how to monitor changes over time” (see Allen and Gale (2007), p. 101). Allen and Gale (2007) also review substantial empirical evidence in favor of limited participation in financial markets.

<sup>11</sup>The subscript 1 stands for date  $t = 1$ .

<sup>12</sup>Indivisibility of investments in risky long-term assets at  $t = 1$  is a strong assumption and I invoke it solely to simplify the analysis. It has the character of an occupational choice. In practice the indivisibility could for instance be the result of some sufficiently high fixed costs for investments in a portfolio of mortgage loans. Importantly, the main qualitative results of the paper can also be established in an economy where agents can select mixed portfolios at date 1, meaning that only part of the endowment is invested in risky long-term assets. The interested reader can refer to my supplementary paper “Supplementary note on detrimental deleveraging and adverse selection” (2013), where I solve a model without the assumption of indivisibility that has an otherwise identical setup.

date $t = 1$		date $t = 2$	date $t = 3$
-1	w.p. $1 - \alpha$	$r_G$	$R_G$
-1	w.p. $\alpha$	$r_B$	$R_B$

Table 1: Risky long-term assets

At  $t = 3$ , the risky long-term asset yields a stochastic return for each unit invested at  $t = 1$ . Hence, illiquid investors face idiosyncratic return risk. There are two states. The *good state* (subscript  $G$ ) with a return of  $R_G > 1$  occurs with probability  $(1 - \alpha)$  and the *bad state* (subscript  $B$ ) with a lower return of  $0 \leq R_B < R_G$  occurs with probability  $\alpha$ . Let the long-term asset returns of individual investors be independently distributed and also be independent of investors' preferences. I assume that illiquid investors do not have the possibility at date  $t = 1$  to hedge against their return risk from the risky long-term assets, for instance, with the help of a mutual fund.<sup>13</sup>

At  $t = 2$ , risky long-term assets can be partially or fully liquidated prematurely. *Physical* liquidation yields  $r_G < R_G$  in the *good state* and  $r_B < R_G$  in the *bad state*. Alternatively, illiquid investors can securitize their portfolio of risk long-term mortgage loans at  $t = 2$  and sell their long-term asset partially or fully at the endogenous market price  $p$ .

### 3.5 Market institutions & information structure

At date  $t = 2$ , there are two distinct spot markets in which trades take place simultaneously. First, an asset market where illiquid investors can securitize and sell shares in their risky long-term mortgage loans to liquid investors at the endogenous price  $p$ . Second, a collateralized credit market where agents can borrow or lend against the leveraged long-term investment project initiated at  $t = 0$  at the endogenous interest rate  $r_2$ .

At date  $t = 1$  only the collateralized credit market exists because long-term assets are not yet initiated. Agents can borrow or lend against the leveraged long-term investment project initiated at  $t = 0$  at the endogenous interest rate  $r_1$ .

#### 3.5.1 The asset market at date $t = 2$

The date  $t = 2$  market for long-term assets is an anonymous and competitive spot market prone to private information problems. In particular, there are two layers of private information. First, illiquid investors learn privately at the beginning of date  $t = 2$  whether they are of the *early type* or *late type*. Second, illiquid investors learn privately at the beginning of date  $t = 2$  whether their long-term asset is *good* or *bad*, i.e. if it has a fundamental date  $t = 3$  value of  $R_G$  or  $R_B$ , respectively.

Due to the private information problem, illiquid investors with long-term assets of bad quality can potentially gain *from trading on private information* by securitizing their long-term mortgage

<sup>13</sup>This assumption can be justified whenever there is an incentive problem that requires investors to keep risk on their books when the portfolio of risky mortgage loans is created at date  $t = 1$ . See for instance Plantin and Parlour (2008).

loans of bad quality and selling them “lemons” at date  $t = 2$  even if they do not have a liquidity need.

For simplicity, I assume that buyers of long-term assets at date  $t = 2$  do not face risk but instead purchase a portfolio of risky long-term assets at the endogenous market price  $p$  with a fundamental value that is determined by the average quality of assets traded in the market.<sup>14</sup>

### 3.5.2 The collateralized credit market at dates $t = 1$ and $t = 2$

In the second spot market, agents can obtain collateralized credit up to their spare borrowing capacity  $\theta$ . This market is available at  $t = 1$  and  $t = 2$ . Private information about the liquidity need (*early type* or *late type*) does not play a role because the collateralized credit is fully secured with safe collateral. The borrowing constraint of an agent with spare borrowing capacity  $\theta$  is given by  $\frac{\theta}{r_t}$ , where  $r_t$  is the endogenous interest rate on collateralized credit at date  $t$ .

## 3.6 Parameter assumptions

**Assumption 1:**  $\alpha R_B + (1 - \alpha)R_G > 1$ .

**Assumption 2:**  $R_B < r_G < 1$ .

**Assumption 3:**  $R_B = r_B$ .

Assumption 1 guarantees that investments in risky long-term assets are not dominated by cash. Otherwise, the problem is trivial. Assumption 2 allows for a *lemons problem* in the asset market to arise. Whenever the return of a long-term asset in the bad state is sufficiently low, then a *lemons problem* can occur where the average quality of assets traded in the market falls short of  $r_G$  and illiquid investors with good quality assets leave the market, because they prefer to physically liquidate their assets. Finally, Assumption 3 helps to streamline the exposition of the model, as it simplifies the analysis of non-pooling equilibria. The key results of the paper are unaffected when relaxing Assumption 3.

To generate a tension between the leverage choice of agents’ business models in the long-term, e.g. at date  $t = 0$ , and the liquidity management of agents in the short-term, e.g. at date  $t = 1$ , I invoke the following parameter assumption.

**Assumption 4:**  $\alpha R_B + (1 - \alpha)R_G < q$ .

Assumption 4 guarantees that agents only want to raise cash at  $t = 0$  in order to leverage their investment project at  $t = 0$  instead of transferring the borrowed funds into the future for investment at  $t = 1$ .

## 3.7 Timeline

The model can be summarized with the help of a timeline depicted in figure 1.

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<sup>14</sup>This assumption is used in most of the related literature and has to be maintained for analytical tractability. It is justified as long as buyers can buy from multiple sellers at the same time without incurring an additional cost, e.g. a positive fixed cost per transaction and could also be implemented with the help of an intermediary.

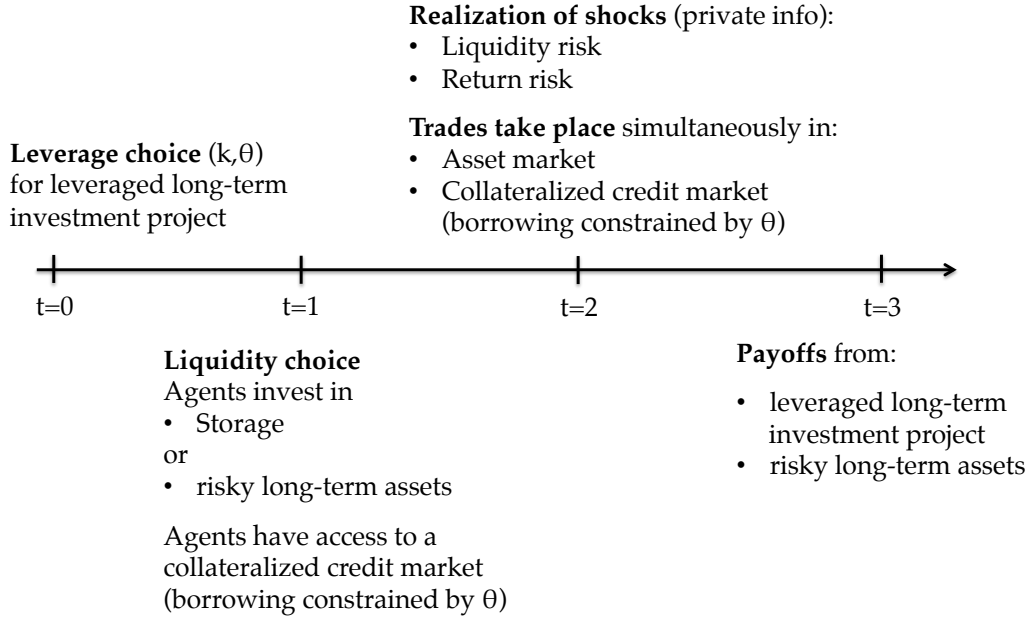


Figure 1: Timeline

## 4 Equilibrium analysis

Section 4 presents the equilibrium analysis. First, section 4.1 discusses how to solve the model and give a definition of equilibrium. In section 4.2, I analyze the model of liquidity provision spanning over dates  $t = 1, 2, 3$ . Then, in section 4.3, I examine the date  $t = 0$  leverage choice. Thereafter, I analyze efficiency in section 5.

The model must be solved backwards. At  $t = 2$ , illiquid and liquid investors face the realization of idiosyncratic liquidity risk. They can use two distinct competitive spot markets at  $t = 2$  to share their liquidity risk by trading long-term assets against cash and by borrowing or lending against safe collateral in the credit market. Because illiquid investors learn privately about the value of their risky long-term assets, they can also use the asset market to share their return risk. At  $t = 1$ , agents have to decide whether they want to become illiquid or liquid investors. Furthermore, agents have access to the collateralized credit market and decide on how much to borrow or lend. Finally, agents face a leverage choice at  $t = 0$ . It is summarized in the tuple  $(k, \theta)$ .

This paper focuses on symmetric equilibria, meaning that all agents of a certain type take identical decisions. An equilibrium is defined as follows.

### 4.1 Definition of equilibrium and classification of equilibria

Given the existence of two spot markets at  $t = 2$ , where trades takes place simultaneously, there exists a no-arbitrage condition in equilibrium.

**No-arbitrage.** The return from lending in the collateralized credit market at  $t = 2$  is  $r_2$ . Let  $a$  denote the average quality of assets traded at  $t = 2$ . For one unit of cash,  $\frac{1}{p}$  units of

long-term assets can be purchased. Hence, the return from purchasing assets in the market is  $\frac{a}{p}$ . No-arbitrage demands agents to be indifferent between purchasing assets or lending. As a result, in equilibrium:

$$\frac{a}{p} = r_2 \quad (3)$$

holds.

**A competitive equilibrium consists of** (i) a date  $t = 2$  asset price  $p^*$  and interest rates  $r_1^*$  and  $r_2^*$  at dates  $t = 1$  and  $t = 2$  at which markets clear, (ii) a no-arbitrage condition, (iii) type-dependent decision rules at date  $t = 2$  as functions of  $p^*$ ,  $r_1^*$ ,  $r_2^*$ , the leverage choice and  $f^*$ , (iv) investment decisions by agents at date  $t = 1$ , which map into equilibrium measures of liquid investors  $f^*$  and illiquid investors  $(1 - f^*)$ , and (v) a leverage choice at date  $t = 0$ .

**Classification of equilibria.** In the remainder, I refer to a *pooling equilibrium* where, given the equilibrium asset price, illiquid investors of the *early type* with a *good quality* long-term asset are willing to sell (at least partially) their long-term assets at date  $t = 2$ . If, instead, the equilibrium asset price is sufficiently low such that only illiquid investors with a *bad quality* long-term asset are willing to sell, then I refer to a *break down of the asset market*.

## 4.2 Liquidity management at date $t = 1$ and liquidity provision at date $t = 2$

This section focuses on liquidity provision. Specifically, I analyze the liquidity choice at  $t = 1$  and market functioning at  $t = 2$ , taking the date  $t = 0$  leverage choice as given. In other words, I consider an agents' decision problem at  $t = 1$  and her trading decisions at  $t = 2$  for all  $k \in [0, \kappa]$  and  $\theta \in [0, \gamma k]$ . Most importantly, the leverage choice at  $t = 0$  not only pins down the total available resources at  $t = 1$ , which are given by  $\zeta(k) = \vartheta + \kappa - k$ , but also impacts market functioning.

Since the model must be solved backwards, I first analyze trading decisions at  $t = 2$  in section 4.2.1. Section 4.2.2 continues with a discussion of market equilibria for a given leverage and liquidity choice. The average quality of assets traded ( $a$ ) and market-clearing prices ( $p, r$ ) at  $t = 2$  are derived in sections 4.2.2.1 and 4.2.2.2, respectively. Thereafter, I move to the liquidity choice at  $t = 1$  in section 4.2.3. This section also contains the main results of equilibrium existence, uniqueness and efficiency.

Given the structure of the model, no agent wants to borrow in the collateralized credit market at  $t = 1$  in order to increase the available resources for investments above  $\zeta(k)$ . For sections 4.2.1 and 4.2.2, I postulate that this is in fact the case before providing a formal argument in section 4.2.3. Furthermore, it has to be true that the interest rates on collateralized credit are equalized across periods, i.e.  $r_1 = r_2 = r$ , because any other relationship between interest rates would create excessive demand or supply.<sup>15</sup>

<sup>15</sup>If  $r_1 < r_2$  ( $r_1 > r_2$ ), then liquid investors would not supply any cash at date  $t = 1$  (date  $t = 2$ ). Furthermore, if  $r_1 < r_2$  illiquid investors would want to borrow cash at  $t = 1$  in order to gain by supplying the cash at a higher rate in the subsequent period. This is impossible as it would lead to an excessive demand of cash at  $t = 1$ .

### 4.2.1 Trading decisions in financial markets at date $t = 2$

Agents enter the period with a predetermined leverage choice summarized in the tuple  $(k, \theta)$ . Depending on their liquidity choice at  $t = 1$ , agents are either *liquid investors* or *illiquid investors*. At the beginning of  $t = 2$ , agents learn privately if they are of the *early type* or *late type*. Moreover, illiquid investors learn privately if they have a *good* or *bad quality* long-term asset.

In the remainder, I use the following notation:

- *Liquid investors* (investment in storage) and *illiquid investors* (investment in risky long-term assets) are indexed with superscripts  $j = F$  (for fully liquid) and  $j = I$  (for illiquid).
- *Early* and *late types* are indexed with subscripts  $i = E$  and  $i = L$ , respectively.
- Long-term assets of *good* and *bad quality* have subscripts  $h = G$  and  $h = B$ , respectively.

Since the leverage choice at  $t = 0$  can potentially differ for agents who expect to become *liquid* and *illiquid investors* at  $t = 1$ , that choice is indexed with the superscript  $j$ , i.e.  $(k^j, \theta^j)$ .

#### 4.2.1.1 Decision problems at date $t = 2$ and supply & demand schedules

##### Liquid investors (superscript $F$ )

Let us start with the decision problem of a *liquid investor*. She enters the period with  $\zeta(k^F)$  units of cash and may be either of *early type* or *late type*. Her problem is to decide how much cash to consume at  $t = 2$  and how to invest the remainder. The remainder can either be stored, or invested in financial markets. The return from purchasing long-term assets in the market is given by  $\frac{a}{p}$  per unit of cash invested and the return from lending in the collateralized credit market is  $r$ . Due to no-arbitrage, liquid investors are indifferent between the two markets. However, they prefer investing in financial markets over storage whenever  $\frac{a}{p} = r > 1$ . Formally, the date  $t = 2$  problem of a *liquid investor* of type  $i$  writes:

$$\begin{aligned} & \max_{-\frac{\theta^F}{r\zeta(k^F)} \leq s_i^F \leq 1} \{ \beta_i \log(c_{2i}^F) + (1 - \beta_i) \log(c_{3i}^F) \} \\ \text{s.t.} \quad & c_{2i}^F = \zeta(k^F) (1 - s_i^F) \\ & c_{3i}^F = \zeta(k^F) s_i^F r + G(k^F, \theta^F), \end{aligned} \tag{4}$$

where the choice variable  $s_i^F$  is the supply of cash to the market. It takes on a negative value if liquid investors want to borrow in the collateralized credit market. Their collateral constraint is given by  $-\frac{\theta^F}{r}$ . As a result, the highest possible consumption level at  $t = 2$  is given by  $c_{2i}^F = \zeta(k^F) + \frac{\theta^F}{r}$  and the lower bound of  $s_i^F$  by  $-\frac{\theta^F}{r\zeta(k^F)}$ . Notice that the consumption level at  $t = 2$  is decreasing in  $s_i^F$ , while the consumption level at  $t = 3$  is increasing in  $s_i^F$ . The latter effect is larger, the higher the return  $r$  of investing in financial markets. Moreover, liquid investors receive a  $t = 3$  return on the leveraged long-term investment project given by  $G(k^F, \theta^F)$ .

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Similarly,  $r_1 > r_2$  is impossible as it would imply that  $r_2$  and, hence,  $r_1$  have to be infinite. As a result, in equilibrium, it has to be true that  $r_1 = r_2$ .



The first-order necessary condition is:

$$s_i^F : -\frac{\beta_i}{1-s_E^F} + \frac{(1-\beta_i)\zeta(k^F)r}{\zeta(k^F)s_i^F r + G(k^F, \theta^F)} + \mu_1 - \mu_2 = 0, \quad (5)$$

where  $\mu_1$  and  $\mu_2$  are the multipliers on the first and second inequality constraint, respectively. Hence, the supply schedule of *liquid investors* can be derived as:

$$s_i^F = \begin{cases} (1 - \beta_i) - \beta_i \frac{G(k^F, \theta^F)}{r\zeta(k^F)} & \text{if } (1 - \beta_i) - \beta_i \frac{G(k^F, \theta^F)}{r\zeta(k^F)} \geq -\frac{\theta^F}{r\zeta(k^F)} \\ -\frac{\theta^F}{r\zeta(k^F)} & \text{otherwise.} \end{cases} \quad (6)$$

The incentive for liquid investors to save part of their available resources increases in the return from investing  $r$  and decreases in  $\beta_i$  (which captures the relative utility derived from early consumption). Notice that *late types* select  $s_L^F = 1$  because  $\beta_L = 0$ .

### Illiquid investors (superscript $I$ )

The date  $t = 2$  problem of *illiquid investors* is more difficult. They must decide how many long-term assets to sell in the market (or to physically liquidate) and how much to borrow in the collateralized credit market. In case illiquid investors are active on both sides of the market, they must also decide how much cash to invest in financial markets through asset purchases or through lending. Illiquid investors enter the period with  $\zeta(k^I)$  units of long-term assets that are worth either  $R_G$  or  $R_B$ . Formally, the problem of an *illiquid investor* of type  $i$  with long-term assets of quality  $h$  writes:

$$\begin{aligned} & \max_{s_{ih}^I, d_{ih}^I} \{ \beta_i \log(c_{2ih}^I) + (1 - \beta_i) \log(c_{3ih}^I) \} \\ & -\frac{\theta^I}{r\zeta(k^I)} \leq s_{ih}^I \\ & s_{ih}^I \leq \max\{p, r_h\} \\ & 0 \leq d_{ih}^I \leq 1 \end{aligned} \quad (7)$$

$$s.t. \quad \begin{cases} c_{2ih}^I = \begin{cases} \zeta(k^I)(pd_{ih}^I - s_{ih}^I) & \text{if } p \geq r_G \\ \zeta(k^I)(r_h d_{ih}^I - s_{ih}^I) & \text{otherwise} \end{cases} \\ c_{3ih}^I = \zeta(k^I)(s_{ih}^I r + (1 - d_{ih}^I)R_h) + G(k^I, \theta^I), \end{cases}$$

where the choice variable  $s_{ih}^I$  captures the supply of cash to the market and the choice variable  $d_{ih}^I$  captures the fraction of long-term assets sold or physically liquidated by an investor with preferences  $i = E, L$  and long-term asset quality  $h = B, G$ .

Given that illiquid investors have the option to either sell or physically liquidate their assets, one has to distinguish between two cases. If  $p > r_G$ , then the *good types* with high quality long-term assets are willing to sell in order to raise  $pd_{ih}^I$  units of cash. Instead, if  $p \leq r_G$  they are indifferent or strictly prefer physical liquidation and raise  $r_G d_{ih}^I$  units of cash. In the latter case of  $p < r_G$ , the asset market breaks down as no asset price above  $r_B$  can be supported. Here the *bad types* with bad quality long-term assets are indifferent whether to sell or to physically liquidate their lemons in order to raise  $r_B d_{ih}^I$  units of cash.

Unlike to liquid investors, the supply of cash now potentially differs depending on whether illiquid investors are of good or bad type. For illiquid investors the collateral constraint is given by  $-\frac{\theta^I}{r}$ . The highest possible consumption level at  $t = 2$  is given by  $c_{2ih}^I = \zeta(k^I)p + \frac{\theta^I}{r}$  if  $p \geq r_G$ , while the upper and lower bound of  $s_{ih}^I$  are given by  $\max\{p, r_h\}$  and  $-\frac{\theta^I}{r\zeta(k^I)}$ , respectively. Notice that the consumption level at  $t = 2$  is decreasing in  $s_{ih}^I$  and increasing in  $d_{ih}^I$ , whereas the consumption level at  $t = 3$  is increasing in  $s_{ih}^I$  and decreasing in  $d_{ih}^I$ . Moreover, consumption at  $t = 3$  includes the return on the leveraged long-term investment project  $G(k^I, \theta^I)$ .

Suppose there exists a pooling equilibrium, i.e.  $p \geq r_G$ . The first-order necessary conditions thus write:

$$\begin{aligned} s_{ih}^I &: -\frac{\beta_i}{pd_{ih}^I - s_{ih}^I} + \frac{(1-\beta_i)\zeta(k^I)r}{\zeta(k^I)(s_{ih}^I r + (1-d_{ih}^I)R_h) + G(k^I, \theta^I)} + \mu_1 - \mu_2 = 0 \\ d_{ih}^I &: \frac{\beta_i p}{pd_{ih}^I - s_{ih}^I} - \frac{(1-\beta_i)\zeta(k^I)R_h}{\zeta(k^I)(s_{ih}^I r + (1-d_{ih}^I)R_h) + G(k^I, \theta^I)} + \mu_3 - \mu_4 = 0, \end{aligned}$$

where  $\mu_1, \mu_2, \mu_3, \mu_4$  are the multipliers on the first, second, third, and fourth inequality constraint, respectively. The first-order conditions together with the no-arbitrage condition show that  $s_{ih}^I$  and  $d_{ih}^I$  cannot simultaneously take on an interior solution. This is because  $R_G/p > r$  for all  $\alpha > 0$ , while  $R_B/p < r$  for all  $\alpha > 0$  and  $p \geq r_G$ .

The demand and supply schedules of *illiquid investors of bad type* can be derived as:

$$pd_{iB}^I = \check{p} \quad (8)$$

and:

$$s_{iB}^I = \begin{cases} \check{p} - \beta_i \left( \check{p} + \frac{G(k^I, \theta^I)}{\zeta(k^I)r} \right) & \text{if } \check{p} - \beta_i \left( \check{p} + \frac{G(k^I, \theta^I)}{\zeta(k^I)r} \right) \geq -\frac{\theta^I}{r\zeta(k^I)} \\ -\frac{\theta^I}{r\zeta(k^I)} & \text{otherwise,} \end{cases} \quad (9)$$

where  $\check{p} = \max\{p, r_B\}$ . Bad types optimally sell all their lemon assets whenever there are gains from trading, i.e. if  $p \geq r_B$ . Otherwise, they physically liquidate. Again, their supply of cash decreases in  $\beta_i$ . Recall from (7) that the lower bound of  $s_{iB}^I$  is given by  $-\frac{\theta^I}{r\zeta(k^I)}$ .

By analyzing the various corner solutions, the demand and supply schedules of *illiquid investors of good type* can be derived as:

$$pd_{iG}^I = \begin{cases} 0 & \text{if } -\beta_i \left( R_G + \frac{G(k^I, \theta^I)}{\zeta(k^I)} \right) \geq -\frac{\theta^I}{\zeta(k^I)} \\ \beta_i \left( 1 + \frac{G(k^I, \theta^I) - \theta^I}{R_G \zeta(k^I)} \right) \check{p} - \frac{1-\beta_i}{r\zeta(k^I)} \theta^I & \text{if } -\beta_i \left( R_G + \frac{G(k^I, \theta^I)}{\zeta(k^I)} \right) < -\frac{\theta^I}{\zeta(k^I)} \text{ and} \\ \beta_i \left( 1 + \frac{G(k^I, \theta^I) - \theta^I}{R_G \zeta(k^I)} \right) \check{p} - (1-\beta_i) \frac{\theta^I}{r\zeta(k^I)} < \check{p} \\ \check{p} & \text{otherwise} \end{cases} \quad (10)$$

where  $\check{p} = \max\{p, r_G\}$  and:

$$s_{iG}^I = \begin{cases} -\beta_i \left( R_G + \frac{G(k^I, \theta^I)}{\zeta(k^I)} \right) \frac{1}{r} & \text{if } -\beta_i \left( R_G + \frac{G(k^I, \theta^I)}{\zeta(k^I)} \right) \geq -\frac{\theta^I}{\zeta(k^I)} \\ -\frac{\theta^I}{r\zeta(k^I)} & \text{otherwise.} \end{cases} \quad (11)$$

Whenever the borrowing constraint is relatively relaxed because of high spare borrowing capacity (i.e. a high  $\theta^I$ ), illiquid investors of good type only borrow in the collateralized credit market and do not sell any of their high quality assets (i.e.  $s_{iG}^I = -\beta_i (R_G + G(k^I, \theta^I) / \zeta(k^I)) \frac{1}{r}$  and  $pd_{iG}^I = 0$ ). Instead, if the borrowing constraint is binding, then they start to sell some or all of their long-term assets. This happens if the desired borrowing exceeds  $\frac{\theta^I}{r\zeta(k^I)}$ , as can be seen in the first line of equation (11). Finally, notice that asset sales of good types are larger as  $\beta_i$  increases. As intuition suggests, it only happens that all high quality long-term assets are sold if there is a high preference to consume at  $t = 2$ , i.e. if:

$$\beta_i \left( 1 + \frac{G(k^I, \theta^I) - \theta^I}{R_G \zeta(k^I)} \right) p - (1 - \beta_i) \frac{\theta^I}{r\zeta(k^I)} \geq p.$$

This is the case in the last line of equation (10).

**Financing asymmetry.** The pecking order of *illiquid investors* with *good* and *bad quality* long-term assets is crucially different. Whereas *bad types* prefer to sell their lemons for all  $p \geq r_B$  and then borrow the remainder, *good types* prefer first to borrow as much as they can, i.e. up to  $\frac{\theta^I}{r\zeta(k^I)}$  units of cash, and then finance the remainder with asset sales.

The financing asymmetry arises because *good types* face a discount in the lemons asset market and therefore prefer borrowing in the prime market for collateralized credit, while *bad types* always gain from selling their lemons whenever  $p > r_B$ . Hence, a better access to the prime market (a higher  $\theta^I$ ) crowds out the supply of high quality long-term assets in the sub-prime market. Formally, the discount in the lemons asset market for *good types* arises because they face a financing cost of  $\frac{R_G}{p}$  when selling assets and a financing cost of  $r$  when borrowing in the credit market. Due to no-arbitrage,  $r = \frac{a}{p} < \frac{R_G}{p}$  as soon as a positive measure of *bad types* sell their lemons. This is because  $\alpha > 0$  implies that  $a < R_G$ .

It will become clear that the financing asymmetry interacts in a novel way with agents' leverage decisions. Before arriving at this result, it is necessary to consolidate the understanding of financial markets at  $t = 2$  in section 4.2.2 and to analyze the liquidity choice at  $t = 1$  in section 4.2.3.

## 4.2.2 Financial market equilibria at date $t = 2$

**4.2.2.1 Average quality of assets traded** The average quality of assets traded in the market at date  $t = 2$  is defined as:

$$a = \frac{\lambda [\alpha d_{EB}^I(p) R_B + (1 - \alpha) d_{EG}^I(p) R_G] + (1 - \lambda) \alpha d_{LB}^I(p) R_B}{\lambda [\alpha d_{EB}^I(p) + (1 - \alpha) d_{EG}^I(p)] + (1 - \lambda) \alpha d_{LB}^I(p)}. \quad (12)$$

Suppose there exists a pooling equilibrium (i.e.  $a \geq p \geq r_G$ ), then:<sup>16</sup>

$$a(k^I, \theta^I) = \frac{\lambda(1 - \alpha) \left[ \beta_E \left( 1 + \frac{G(k^I, \theta^I) - \theta^I}{R_G \zeta(k^I)} \right) - (1 - \beta_E) \frac{\theta^I}{a \zeta(k^I)} \right] R_G + \alpha R_B}{\lambda(1 - \alpha) \left[ \beta_E \left( 1 + \frac{G(k^I, \theta^I) - \theta^I}{R_G \zeta(k^I)} \right) - (1 - \beta_E) \frac{\theta^I}{a \zeta(k^I)} \right] + \alpha}. \quad (13)$$

Equation (13) implicitly defines  $a(k^I, \theta^I)$ . It has two solutions of which the one with the higher value is more relevant since it describes a scenario where  $a(k^I, \theta^I) > r_G$  and, hence, a pooling equilibrium can exist. Interestingly,  $a$  does not depend on  $p$  as long as  $a \geq p \geq r_G$  holds. Further, equation (13) reveals that the average quality of assets traded is independent of  $f$  (the level of liquidity available in the economy) if  $a \geq p \geq r_G$ . This contrasts with models in which short-term funding is fully elastic, such as in Malherbe (2013). In Lemma 1 of Malherbe (2013), the author employs a model with elastic supply of cash and demonstrates that investments in storage pose a negative externality. This is not the case if the demand for cash is inelastic for prices above  $r_G$ , which is a typical property of models with cash-in-the-market pricing and also a property of my model, as can be seen in equation (13). Intuitively, in models with cash-in-the-market pricing, higher cash holdings and a higher availability of cash in the economy are typically beneficial for market functioning instead of posing a negative externality.

In the following analysis, it is critical to understand how the average quality of assets traded depends on the borrowing constraint and key deep parameters of the model. Given the financing asymmetry discussed in section 4.2.1.1, it is of particular interest to understand the dependency of  $a$  on the tightness of the borrowing constraint of illiquid investors with high quality long-term assets in hand.

Equation (13) implicitly defines  $a$ . Using the implicit function theorem, one can show that  $a$  tends to decrease in  $\theta^I$  and to increase in  $R_B$ . This is intuitive: a better ability to borrow tends to reduce asset sales of illiquid investors with high quality long-term assets and thereby amplifies the adverse selection problem. If the average quality of traded assets is depressed by too much, then a pooling equilibrium cannot exit. The result is summarized formally in Lemma 1 below. Define  $\bar{\theta}^I(a)$  as the solution to:

$$\theta^I = \frac{\beta_E (R_G \zeta(k^I) + G(k^I, \theta^I)) + \frac{\alpha R_G \zeta(k^I)}{\lambda(1 - \alpha)}}{(1 - \beta_E) (R_G/a)^2 + \beta_E} \quad (14)$$

<sup>16</sup>Recall that there cannot exist a pooling equilibrium if  $a < r_G$ . It shows that this is the case if  $\alpha$  is too high and  $\lambda$  and/or  $R_B$  too low. Here, the adverse selection problem is too strong.

and observe that a permissible solution to equation (14), i.e. a  $\bar{\theta}^I \in [0, \gamma k^I]$ , can only exist if:

$$\gamma k^I \geq \frac{\beta_E (R_G \zeta(k^I) + q k^I) + \frac{\alpha R_G \zeta(k^I)}{\lambda(1-\alpha)}}{(1 - \beta_E) (R_G/r_G)^2 + \beta_E}. \quad (15)$$

**Lemma 1. Average quality of assets traded**

(a) If equation (15) is violated or if  $\theta^I < \bar{\theta}^I(r_G)$  then equilibria with pooling in the date  $t = 2$  asset market are characterized by  $\frac{da^*}{d\theta^I} < 0$  and  $\frac{da^*}{dR_B} > 0$ .

(b)  $a \geq r_G$  is a necessary condition for a pooling equilibrium to exist and holds if:

$$\frac{\lambda(1-\alpha) \left[ \beta_E \left( 1 + \frac{G(k^I, \theta^I) - \theta^I}{R_G \zeta(k^I)} \right) - (1 - \beta_E) \frac{\theta^I}{r_G \zeta(k^I)} \right] R_G + \alpha R_B}{\lambda(1-\alpha) \left[ \beta_E \left( 1 + \frac{G(k^I, \theta^I) - \theta^I}{R_G \zeta(k^I)} \right) - (1 - \beta_E) \frac{\theta^I}{r_G \zeta(k^I)} \right] + \alpha} \geq r_G. \quad (16)$$

*Proof.* See Appendix A.2.

If equation (15) is violated, then result (a) of Lemma 1 always holds. Otherwise, a sufficient condition is given by  $\theta^I < \bar{\theta}^I(r_G)$ . Intuitively, equation (15) is guaranteed to be violated whenever  $k^I$  is low. This occurs when the relative size of the leveraged long-term investment project is not too high. In that case, illiquid investors with high quality long-term assets in hand reduce their asset sales whenever their borrowing constraint is relaxed. It is only when (15) is unviolated, that illiquid investors with high quality long-term assets in hand may, for large values of  $\theta^I$ , be inclined to increase their asset sales when their borrowing constraint is further relaxed. This requires extreme parameter values with a very large relative size of the long-term investment project  $k^I$ . However, such extreme parameter values are not of interest in this paper.<sup>17</sup>

**4.2.2.2 Market-clearing at date  $t = 2$**  Taken together, the equations derived in section 4.2.1 give us a market-clearing condition. For markets to clear, the demand for long-term assets has to be weakly larger than the supply of long-term assets:  $S(p) \leq D(p)$ . Demand and supply depend on  $f$ , the fraction of liquid investors. Market-clearing demands that:

$$\underbrace{(1-f) * (\lambda (\alpha d_{EB}^I + (1-\alpha) d_{EG}^I) + (1-\lambda) \alpha d_{LB}^I)}_{\text{supply of long-term assets}} = \underbrace{\left( \frac{f * (\lambda s_E^F + (1-\lambda) s_L^F) + (1-f) * \alpha (\lambda s_{EB}^I + (1-\lambda) s_{LB}^I)}{p} \right)}_{\text{demand for long-term assets}}. \quad (17)$$

<sup>17</sup>A very large size of  $k^I$  results in investors with high quality long-term assets in hand who have no incentive to increase their  $t = 3$  income through a reduction in their asset sales at  $t = 2$ . The problem becomes trivial.

If a pooling equilibrium exists (i.e.  $a \geq p \geq r_G$ ), then the market-clearing price is:<sup>18</sup>

$$p(f, a, k^I, k^F, \theta^I, \theta^F) = \min \left\{ \frac{f}{Z(f, a, k^I, k^F, \theta^I, \theta^F)} \frac{1 - \lambda\beta_E}{\lambda\beta_E}, a \right\} \quad (18)$$

where:

$$Z(f, a, k^I, k^F, \theta^I, \theta^F) \equiv (1-f) + (1-f) * \left( (1-\alpha) \left[ \frac{G(k^I, \theta^I) - \theta^I}{R_G \zeta(k^I)} - \frac{1-\beta_E}{\beta_E} \frac{\theta^I}{a \zeta(k^I)} \right] + \alpha \frac{G(k^I, \theta^I)}{a \zeta(k^I)} \right) + f * \frac{G(k^F, \theta^F)}{a \zeta(k^F)}.$$

One can show that  $p$  increases in the level of liquidity available in the economy:

$$\frac{dp(f, a, k^I, k^F, \theta^I, \theta^F)}{df} > 0 \quad (19)$$

whenever the solution to equation (18) is interior, i.e.  $a > p \geq r_G$ . Instead, if  $p = a$ , then the average quality of assets traded and, hence,  $p$  is independent of  $f$ . This is the case if the level of liquidity available in the economy is very high. Finally, if the available liquidity is very low, then  $p < r_G$  and the asset market breaks down. In the latter case,  $p$  is independent of  $f$  as long as a small increase in the liquidity available does not restore market functioning at date  $t = 2$ . The results are formally summarized in Lemma 2 below. I define  $r_G^- \equiv \lim_{\varepsilon \rightarrow 0} (r_G - \varepsilon)$  where  $\varepsilon > 0$ .

**Lemma 2. Market-clearing asset price**

If  $\left( \frac{f}{Z(f, a, k^I, k^F, \theta^I, \theta^F)} \frac{1 - \lambda\beta_E}{\lambda\beta_E} \right) \in [r_G^-, a)$ , then  $\frac{dp(f, a, k^I, k^F, \theta^I, \theta^F)}{df} > 0$ . Otherwise the market-clearing asset price is independent of  $f$ .

**Proof.** The proof follows from the previous discussion.

Following Allen and Gale (2007), I refer to cash-in-the-market pricing when the equilibrium asset prices is below the “fundamental value” of assets (i.e.  $p < a$ ) due to a shortage of aggregate liquidity in the economy.<sup>19</sup> In my model, cash-in-the-market pricing arises because the cash available in the economy is endogenous and, therefore, the supply of short-term funding is limited (very low elasticity; bounded supply).

### 4.2.3 Liquidity choice at date $t = 1$

Agents have to decide whether they want to become *liquid investors*, who are the natural providers of liquidity in the markets at  $t = 2$ , or *illiquid investors*, who are the natural demanders of cash at  $t = 2$ . This paper focuses on rational expectations equilibria where agents form correct perceptions on future prices ( $p$  and  $r$ ) and the average quality of assets traded ( $a$ ) at  $t = 2$ .

<sup>18</sup>Recall that the market-clearing price cannot exceed the average quality of assets traded  $a$ .

<sup>19</sup>Notably, the use of the expression “fundamental value” is not uncontroversial in this context. It means that long-term assets trade at a discount in the intermediate period when compared to their expected return in the final period, which itself is determined by the average quality of assets traded in the market.

In section 4.2.3.1, I analyze the date  $t = 1$  decision problem. Section 4.2.3.2 discusses existence and characterization of equilibria. Finally, section 5.1 analyzes efficiency. I find that pooling equilibria are, for a large parameter range, characterized by a tendency for inefficient under-investment in cash due to the private information problem.

**4.2.3.1 The problem at date  $t = 1$**  Agents face a discrete choice at date  $t = 1$  described by  $x \in \{0, 1\}$ . Either they become *liquid investors* and store their entire endowment ( $x = 0$ ) or they become *illiquid investors* and fully invest their available resources at  $t = 1$  in risky long-term assets ( $x = 1$ ). Let the amount borrowed in the date  $t = 1$  collateralized credit market be denoted by  $b^j$ , where the superscript  $j = I, F$  stands for illiquid and fully liquid, respectively. The variable  $b^j$  takes a positive (negative) value whenever a given agent borrows (lends). Given that  $\beta_L = 0$ , the agents' maximization problem reads:

$$\begin{aligned} & \max_{\substack{x \in \{0, 1\} \\ b^I \leq \frac{\theta^I}{r}, b^F \leq \frac{\theta^F}{r} \\ c_{ih}^I, c_{ti}^F \geq 0 \forall t, i, h}} \left\{ \begin{aligned} & x * V(c_{2EB}^I, c_{3EB}^I, c_{2EG}^I, c_{3EG}^I, c_{3LB}^I, c_{3LG}^I) \\ & + (1 - x) * W(c_{2E}^F, c_{3E}^F, c_{3L}^F) \end{aligned} \right\} \quad (20) \end{aligned}$$

s.t.

$$\begin{aligned} V(\cdot) &= \left( \lambda \left[ \begin{aligned} & \alpha (\beta_E U(c_{2EB}^I) + (1 - \beta_E) U(c_{3EB}^I)) \\ & + (1 - \alpha) (\beta_E U(c_{2EG}^I) + (1 - \beta_E) U(c_{3EG}^I)) \\ & + (1 - \lambda) [\alpha U(c_{3LB}^I) + (1 - \alpha) U(c_{3LG}^I)] \end{aligned} \right] \right) \\ W(\cdot) &= \lambda [\beta_E U(c_{2E}^F) + (1 - \beta_E) U(c_{3E}^F)] + (1 - \lambda) U(c_{3L}^F) \\ c_{2ih}^I &= \begin{cases} [\zeta(k^I) + b^I] (pd_{ih}^I - s_{ih}^I) & \text{if } p > r_G \\ [\zeta(k^I) + b^I] (r_h d_{ih}^I - s_{ih}^I) & \text{otherwise} \end{cases} \\ c_{3ih}^I &= [\zeta(k^I) + b^I] (s_{ih}^I r + (1 - d_{ih}^I) R_h) + G(k^I, \theta^I) - b^I r \\ c_{2E}^F &= [\zeta(k^F) + b^F] (1 - s_E^F) \\ c_{3E}^F &= [\zeta(k^F) + b^F] s_E^F r + G(k^F, \theta^F) - b^F r \\ c_{3L}^F &= [\zeta(k^F) + b^F] r + G(k^F, \theta^F) - b^F r. \end{aligned}$$

Recall that the superscripts  $j = I, F$  correspond to *illiquid investors* and *liquid investors*, respectively. Preferences are captured by the subscripts  $i = E, L$  and the long-term asset quality is denoted by subscripts  $h = B, G$ . The supply and demand schedules  $(s_E^F, s_{ih}^j, d_{ih}^I)$  are derived in section 4.2.1.1.

At date  $t = 1$ ,  $k^j$  and  $\theta^j$  are predetermined and taken as given. They can potentially differ for agents who expect to become an *illiquid* or *liquid investor* at date  $t = 1$ . This is because agents can correctly anticipate at date  $t = 0$  what their desired liquidity choice will be at date  $t = 1$  and thus adjust their leverage choice accordingly depending on their type  $j$ .<sup>20</sup> Finally,

<sup>20</sup>The potential heterogeneity in  $\theta$ s does not arise in an alternative setup where the indivisibility assumption in the date  $t = 1$  liquidity choice is relaxed. However, the main qualitative result are unaffected.

notice that the resources available for investments at  $t = 1$  depend on  $b^j$ , the amount borrowed or lent.

**Solution to the date  $t = 1$  problem without borrowing (i.e.  $b^j = 0$ ).** Suppose again that  $a \geq p \geq r_G$  and that the condition in equation (16) of Lemma 1 holds.<sup>21</sup> Moreover, I continue to postulate that  $b^I = b^F = 0$  as in sections 4.2.1 and 4.2.2 before arguing in the subsequent paragraph that borrowing at  $t = 1$  never takes place.

Observe that the problem formulated in (20) can be simplified by plugging in the respective consumption, supply, and demand terms for  $p \geq r_G$ . In a pooling equilibrium, it has to be true that agents are indifferent between becoming an *illiquid* or *liquid investor* at date  $t = 1$ . The fraction of *liquid investors*  $f$  has to solve:

$$V(p(f)) = W(p(f)). \quad (21)$$

From Lemma 2,  $\frac{dp(\cdot)}{df} > 0$  if  $a \geq p \geq r_G$ . Hence, for given tuples  $(k^I, \theta^I)$  and  $(k^F, \theta^F)$ , it follows that  $\frac{dV}{df} = \frac{dV}{dp} \frac{dp(\cdot)}{df} > 0$  and:

$$\frac{dW}{df} = \frac{\lambda \left[ \beta_E G(k^F, \theta^F) - (1 - \beta_E) \frac{a(\cdot)}{p(\cdot)} \zeta(k^F) \right] - (1 - \lambda) \frac{a(\cdot)}{p(\cdot)} \zeta(k^F) \frac{dp(\cdot)}{df}}{a(\cdot) \zeta(k^F) + p(\cdot) G(k^F, \theta^F)} < 0 \text{ if } s_i^F > 0.$$

From equation (6) one can derive a sufficient condition for  $s_i^F > 0$  to hold. It is given by:

$$1 > \beta_E \left( 1 + \frac{G(k^F, \theta^F)}{\zeta(k^F)} \right). \quad (22)$$

Suppose equation (22) holds. As a result, if there does exist a value of  $f$  that solves equation (21), say  $\hat{f}$ , then that value must be unique.

Using the pricing kernel from equation (18), it follows that if  $r_G \leq p(\hat{f}) \leq a$ , then we found a date  $t = 1$  equilibrium characterized by  $f^* = \hat{f}$  and  $p^* = p(\hat{f})$ . Instead if  $p(\hat{f}) > a$ , then  $f^* = 1$  and there is a date  $t = 1$  equilibrium with no trade at date  $t = 2$  where all agents decide to become *liquid investors* because  $V$  is strictly smaller than  $W$  when evaluated at some price  $p' = a$ , where  $a < p(\hat{f})$ . Finally, if  $p(\hat{f}) < r_G$  then the asset market breaks down. The equilibrium with a breakdown of the asset market is discussed in Appendix A.1.

**Solution to the date  $t = 1$  problem with borrowing.** Suppose again that  $a \geq p \geq r_G$  and that the condition in equation (16) of Lemma 1 holds. I argued earlier that  $r_1 = r_2$  has to hold in equilibrium. At  $r_1 = r_2$ , liquid investors are indifferent between storing cash at  $t = 1$  or lending in the collateralized credit market. Instead, illiquid investors may have an incentive to borrow at  $t = 1$  and thereby increase their available resources for investments in risky long-term

<sup>21</sup>Notice that the condition in equation (16) of Lemma 1 implies that the borrowing constraint is binding. Otherwise, good types would not be willing to sell their high-quality assets leading to  $a < r_G$ .



assets to  $\zeta(k^I) + b^I$ . However, it can be shown that, in equilibrium,  $b^{I*} = b^{F*} = 0$  due to the concave curvature of the utility function.<sup>22</sup> As a result, the solution is identical to the one of the date  $t = 1$  problem without borrowing.

**4.2.3.2 Equilibrium: existence and characterization** One can distinguish between *three types of equilibria*. In “*pooling equilibria*” both *good* and *bad types* are selling. Pooling equilibria are characterized by cash-in-the-market pricing, i.e.  $r_G \leq p \leq a$  and exist if  $r_G \leq p(\hat{f}) \leq a$ . Furthermore, there *always* exists either an equilibria characterized by a “*market breakdown*” where the good types prefer to physically sell their high quality long-term assets instead of selling them in the market, i.e.  $p < r_G$ , or an equilibria characterized by “*liquidity hoarding*” where investors only invest in storage at date  $t = 1$ , i.e.  $f^* = 1$ . The results are formally summarized in Proposition 3.

**Proposition 3. Existence and characterization**

For a given  $k^I, k^F$  and  $\theta^I, \theta^F$ , there exists:

1. a pooling equilibrium if and only if  $r_G \leq p(\hat{f}, a, k^I, k^F, \theta^I, \theta^F) \leq a$ ; it is characterized by  $f^* = \hat{f}$  and  $p^* = p(\hat{f}, a, k^I, k^F, \theta^I, \theta^F) \leq 1$ , which holds with strict inequality if  $\alpha > 0$
2. a liquidity hoarding equilibrium if  $p(\hat{f}, a, k^I, k^F, \theta^I, \theta^F) > a \geq r_G$ , characterized by  $f^* = 1$
3. an equilibrium where the asset market breaks down, characterized by  $f^* \in [0, 1]$  and  $p^* \in [0, r_B]$ ,

where  $a$  is implicitly defined by equation (13).

**Proof.** The proofs of the first two results are developed in section 4.2.3.1 using the result of Lemma 2. It remains to be argued that any  $p \geq 1$  cannot be a pooling equilibrium price because nobody would be willing to invest in storage. The proof of the third result is in Appendix A.4.

**4.3 Leverage choice at date  $t = 0$**

While section 4.2 focused on liquidity management, I now move to the date  $t = 0$  leverage choice with the intent of describing the leverage choice and understanding the interplay between the intensity of adverse selection problems in the date  $t = 2$  asset market. I start with a discussion of the special case  $\alpha = 0$  in section 4.3.1. In section 4.3.2 I move to the general case with  $\alpha > 0$  and show that only in the latter case does a tension arise between the leverage choice at  $t = 0$  and the future asset market at  $t = 2$ . Adverse selection problems in the asset market at  $t = 2$  are responsible for this interaction. Section 4.3.2 also establishes the main results of the paper on the existence of a detrimental feedback loop between deleveraging and adverse selection in

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<sup>22</sup>Please refer to Appendix section A.3 for a formal argument.

asset markets. Finally, section 4.3.3 concludes with a numerical example that illustrates the feedback loop. Moreover, I relate the analytical result to the financial crisis.

#### 4.3.1 Special case: $\alpha = 0$

In the special case when  $\alpha = 0$ , long-term assets are riskless and thus there is no adverse selection in the asset market. Hence, the previous analysis of dates  $t = 2$  and  $t = 1$  is greatly simplified. The main results derived above are summarized in Proposition 4. To reduce notation, I define  $R \equiv R_G$ .

**Proposition 4. Equilibrium existence, uniqueness and efficiency if  $\alpha = 0$**

If:

$$1 \geq \beta_E \left( 1 + \frac{G(k^I, \theta^I)}{R\zeta(k^I)} \right) \quad (23)$$

is satisfied when evaluated at  $k^I = \kappa$  and  $\theta^I = 0$ , then there exists a unique equilibrium characterized by  $k^* = k^{I*} = k^{F*} = \kappa$ ,  $\theta^* = \theta^{I*} = \theta^{F*} = 0$ ,  $f^* = \lambda\beta_E \left( 1 + \frac{G(k^*, \theta^*)}{R} \right)$  and  $p^* = 1$ . The equilibrium is efficient.

**Proof.** See Appendix A.5.

Interestingly, the results of Proposition 4 on uniqueness and efficiency change in section 4.3.2 when introducing risky long-term assets and private information.

#### 4.3.2 General case: $\alpha > 0$

Section 4.2 contains the analysis of dates  $t = 1, 2, 3$  for the general case with  $\alpha > 0$ . What remains is the analysis of the leverage choice at  $t = 0$ . In section 4.3.2.1, I solve the  $t = 0$  problem. The results on leverage are summarized in section 4.3.2.2. Section 4.3.2.3 establishes the detrimental feedback loop between deleveraging and adverse selection. Thereafter, section 4.3.3 relates the findings to the financial crisis of 2007/08. To illustrate the main analytical results, I present a numerical example.

**4.3.2.1 Solution to the problem at date  $t = 0$**  Recall that the return on the leveraged long-term investment project is increasing in the amount of capital invested and decreasing in the spare borrowing capacity. Formally:

$$G_k \equiv \frac{dG(k, \theta)}{dk} = q \frac{\rho - \gamma}{q - \gamma} > 0 \quad (24)$$

and:

$$G_\theta \equiv \frac{dG(k, \theta)}{d\theta} = \frac{q - \rho}{q - \gamma} \leq 0 \quad (25)$$

with  $-G_\theta < G_k$ .

In a rational expectations equilibrium, agents correctly anticipate the future measure  $f$  of illiquid investors at  $t = 0$ , as well as the asset price  $p$  and the average quality of assets traded

$a$  at  $t = 2$ . Furthermore, they anticipate their individual liquidity choice at  $t = 1$ . As a result, agents who expect to become *liquid investors* at  $t = 1$  select  $\theta^{F*} = 0$  and  $k^{F*} = \kappa$  as long as  $G(\kappa, 0)$  is not too large. The necessary and sufficient condition is given by:

$$\beta_E \left( 1 + \frac{G(\kappa, 0)}{\zeta(\kappa)} \right) \leq 1, \quad (26)$$

which guarantees that  $s_i^F \geq 0$ . It is similar to the necessary and sufficient condition used in Proposition 4 (compare equation (37)). I assume for the remainder of the analysis that equation (26) holds.<sup>23</sup>

*Illiquid investors* may reduce leverage at  $t = 0$  even if equation (26) holds because they may incur future benefits from installing a spare borrowing capacity. To see this, consider their problem at  $t = 0$ :<sup>24</sup>

$$\begin{array}{l} \max \\ 0 \leq \theta^I \leq \gamma\kappa \\ 0 \leq k^I \leq \kappa \end{array} \left( \begin{array}{l} \lambda \left[ \begin{array}{l} \alpha \left( \begin{array}{l} \beta_E U \left( \beta_E (a(k^I, \theta^I) \zeta(k^I) + G(k^I, \theta^I)) \frac{p(f)}{a(k^I, \theta^I)} \right) + \\ (1 - \beta_E) U \left( (1 - \beta_E) (a(k^I, \theta^I) \zeta(k^I) + G(k^I, \theta^I)) \right) \end{array} \right) + \\ \\ (1 - \alpha) \left( \begin{array}{l} \beta_E U \left( \beta_E \left( \zeta(k^I) + \frac{G(k^I, \theta^I) - \theta^I}{R_G} + \frac{\theta^I}{a(k^I, \theta^I)} \right) p(f) \right) + \\ (1 - \beta_E) U \left( (1 - \beta_E) \left( \zeta(k^I) + \frac{G(k^I, \theta^I) - \theta^I}{R_G} + \frac{\theta^I}{a(k^I, \theta^I)} \right) R_G \right) \end{array} \right) \end{array} \right] \\ + (1 - \lambda) [\alpha U (a(k^I, \theta^I) \zeta(k^I) + G(k^I, \theta^I)) + (1 - \alpha) U (R_G \zeta(k^I) + G(k^I, \theta^I))] \end{array} \right) \quad (27)$$

The payoffs in the different contingencies are the same as in section 4.2.3.1.

**4.3.2.2 Results on leverage** An analysis of (27) reveals that illiquid investors are willing to install a spare borrowing capacity at date  $t = 0$  as long as a reduction of leverage is not too expensive. The formal conditions are stated in Lemma 5.

**Lemma 5. Leverage**

*Given that a pooling equilibrium exists at date  $t = 2$ , investors optimally select at date  $t = 0$ :*

(a)  $\theta^I > 0$  whenever the cost of reducing leverage is not too high, i.e. if for a given  $k^I \in [0, \kappa]$  the conditions in Lemma 1(a) hold and:

$$-G_{\theta^I} < \frac{\lambda(1 - \alpha) \left( \frac{R_G}{a(0,0)} - 1 \right)}{1 + \alpha \left( \frac{R_G}{a(0,0)} - 1 \right)}, \quad (28)$$

<sup>23</sup>This paper focuses on the interplay between the leverage choice and the adverse selection problem in asset markets at  $t = 2$ . For this reason, I restrict myself to parameter values such that liquid investors are willing to fully leverage their long-term investment project, while illiquid investors may decide not to fully leverage their long-term investment project, i.e. select  $\theta^I > 0$ , if the adverse selection problem is too strong.

<sup>24</sup>Recall that leverage is positive for all  $\theta^I < \gamma\kappa$ . The upper bound of  $\theta^I$  follows.

where  $a(k^I, \theta^I)$  is implicitly defined by equation (13).

(b)  $k^{I*} = \kappa$  whenever the cost of reducing capital is sufficiently high, i.e. if:

$$G_k > R_G. \quad (29)$$

**Proof.** See Appendix A.6.

Both inequalities in equations (28) and (29) are sufficient but not necessary for the results stated in Lemma 5. Intuitively, the first result arises because investors have a benefit from selecting  $\theta^I > 0$  whenever they turn out to be of the good type at  $t = 2$ . In this scenario, the spare borrowing capacity helps investors to reduce costly asset sales at a discount, which arises due to the adverse selection problem. It is optimal to select  $\theta^I > 0$  whenever the cost of reducing leverage (captured by  $G_{\theta^I}$ ) is not too high. Further, investors prefer to shield themselves against the adverse selection problem by reducing their leverage instead of by increasing their capital investment at  $t = 0$  whenever  $G_k > R_G$ . This happens despite a reduction in  $k$  allows investors to have more cash available in the future.

Interestingly, it turns out that the incentive for an individual illiquid investor  $m$  to select a positive value of  $\theta^I$  is increasing in the leverage choice of other investors  $\theta_{-m}^I$ . In other words, I find a strategic complementarity in leverage choices which is stated formally in Proposition 6. The optimal  $\theta^I$  chosen by investor  $m$  is a function of  $\theta_{-m}^I$  and I denote it with  $\hat{\theta}^I(\theta_{-m}^I)$ .

**Proposition 6. Strategic complementarity in leverage choices**

*Given that a pooling equilibrium exists at date  $t = 2$ , there is a strategic complementarity in leverage choices if the conditions in Lemma 1(a) hold and the cost of reducing leverage is not too high. Formally, for any  $k_{-m}^I$  and  $\theta_{-m}^I$  that are part of a pooling equilibrium:*

$$\frac{d\hat{\theta}^I(\theta_{-m}^I, k_{-m}^I)}{d\theta_{-m}^I} > 0 \quad (30)$$

*if, first, the conditions in Lemma 1(a) hold when evaluated at  $k^I = k_{-m}^I$  and  $\theta^I = \theta_{-m}^I$ , second, the condition (28) in Lemma 5 holds and, third, the following condition is satisfied:*

$$\left( -\frac{\alpha}{\lambda(1-\alpha)} \zeta(k_{-m}^I) + \frac{k_m^I q}{R_G} \right) G_{\theta^I} < \zeta(k_{-m}^I) + \frac{k_m^I q}{R_G} \text{ for all } k_m^I \in [0, \kappa]. \quad (31)$$

**Proof.** See Appendix A.7.

The proof uses the fact established in Lemma 5(a) that investors have an incentive to select a positive  $\theta^I$  whenever the cost of keeping a spare borrowing capacity is low. Notice that the conditions of Lemma 5(a) and Lemma 1(a) are necessary for the above result to hold but not sufficient. In particular a third sufficient condition is needed and is given by the inequality in equation (31). Notably, equation (31) is guaranteed to hold independently of  $k_m^I$  for a sufficiently

low cost  $-G_{\theta^I}$  of reducing leverage as well as for small values of  $\alpha$ . Intuitively, this is the case because a smaller value of  $\alpha$  implies that agents face a higher probability of being a *good type* and, hence, are more likely to gain from selecting a higher  $\theta^I$ . The strategic complementarity established in Proposition 7 is at the root of the detrimental feedback mechanism described in section 4.3.2.3.

**4.3.2.3 Deleveraging and the severity of adverse selection in the asset market** The main result of this paper is the existence of a detrimental feedback loop between deleveraging and the intensity of adverse selection in the asset market. I find that a more severe adverse selection causes an reduction in leverage, as long as the equilibrium level of  $\theta^I$  is not too large. The result is formally stated in Proposition 7 below and the focus is on symmetric equilibria, i.e. equilibria where  $\theta_m^I = \theta_{-m}^I = \theta^{I*}$  for all  $m$ .

**Proposition 7. Deleveraging**

*A more severe adverse selection problem in the asset market, reflected in a lower value of  $R_B$  causes a:*

- (a) *reduction in leverage, i.e.  $\frac{d\theta^{I*}}{dR_B} < 0$ , which further amplifies the adverse selection problem if the conditions in equations (28), (29), (31) hold and if the conditions in Lemma 1(a) are satisfied when evaluated at  $(\kappa, \theta^{I*})$ .*
- (b) *breakdown of the asset market if the amplification of the adverse selection problem is sufficiently strong. Formally, if condition (16) in Lemma 1(b) is violated when evaluated at  $(\kappa, \theta^{I*})$ .*

**Proof.** See Appendix A.8.

Proposition 7 establishes the existence of a detrimental feedback loop by demonstrating that a stronger adverse selection problem in the asset market due to a decrease in  $R_B$  is causing deleveraging. However, the deleveraging itself triggers a further reduction in the average quality of assets traded as shown in Lemma 1(a). This in turn amplifies deleveraging. In the extreme, this detrimental feedback mechanism can lead to a breakdown of the asset market due to drastic deleveraging, causing a substantial welfare loss. Here, a pooling equilibrium ceases to exist.

Formally, the conditions in equations (28), (29), (31) and Lemma 1 are sufficient but not necessary for the result in Proposition 7 to hold.

**4.3.3 The detrimental feedback loop: Relation to the financial crisis of 2007/08 with the help of a numerical example**

The above mechanism explains how a deterioration in the quality of subprime mortgage-backed securities triggered both a breakdown of subprime markets and a deleveraging wave in the financial crisis of 2007/08. After observing distress in subprime markets, banks started to change their business models. In search of unencumbered high quality collateral, banks began

deleveraging. This enabled them to reduce their dependency on refinancing through subprime markets. However, the simultaneous escape from subprime exposure amplified the distress in those markets even further.

To illustrate the previous analytical results, consider a numerical example using the parameters given in table 2. The parameters are selected such that there exists a pooling equilibrium.

Variable	$\vartheta$	$\beta_E$	$\lambda$	$\alpha$	$R_G$	$r_G$	$R_B$	$\kappa$	$\rho$	$q$	$\gamma$
Value	1	0.7	0.5	0.2	1.5	0.7	0.5	0.1	1.6	1.5	0.8

Table 2: Model parameters

The existence of a pooling equilibrium is facilitated by a relatively mild adverse selection problem (a low  $\alpha$ ) and by relatively high returns from leverage due to a good pledegability of the long-term investment project, i.e. a high  $\gamma$  leading to  $G_k \approx 1.71 > R_G$  and  $G_{\theta^I} \approx -0.14$ .

Given the result from Lemma 5(b), the optimal capital investment at  $t = 0$  is  $k^* = k^{I*} = k^{F*} = \kappa$ . There exists a pooling equilibrium characterized by  $\theta^* = \theta^{I*} = \theta^{F*} = 0$  and  $f^* \approx 0.34$ . The corresponding average quality of assets traded and the market-clearing asset price are given by  $a^*(k^*, \theta^*) \approx 0.91$  and  $p^*(f^*, a^*, k^*, \theta^*) \approx 0.78$ , respectively. It shows that with a relatively mild adverse selection problem, agents do not have an incentive to install spare borrowing capacity at  $t = 0$ , i.e. to select a positive  $\theta$ . This changes when the adverse selection problem is stronger, as suggested by Proposition 7.

Let us examine what happens in a crisis scenario triggered by a deterioration in the quality of subprime assets. Consider an increase in the intensity of adverse selection caused by a drop in the value of lemons from  $R_B = 0.5$  to  $\widetilde{R}_B = 0$ . In this case, there no longer exists a pooling equilibrium because the incentives to deleverage by selecting a strictly positive  $\theta^I$  are too large and, hence,  $\theta^I = 0$  cannot be optimal. As a result, the asset market breaks down in equilibrium. Intuitively, the more intense adverse selection problem incentivizes agents to increase  $\theta^I$  which in turn amplifies adverse selection, eventually pushing  $a$  below  $r_G$ . In summary, a detrimental feedback loop evolves, leading to a breakdown of the asset market.

## 5 Efficiency

Now I analyze the efficiency of the date  $t = 1$  liquidity choice and the date  $t = 0$  leverage choice in sections 5.1 and 5.2, respectively. In section 5.1, the benchmark is the competitive equilibrium derived in section 4.2. In section 5.2, I examine when a constrained planner wants to prevent the excessive deleveraging found in section 4.3.

### 5.1 Efficiency of the equilibrium at date $t = 1$

To analyze “interim” efficiency, that is the efficiency of the date  $t = 1$  liquidity choice for a given date  $t = 0$  leverage choice, I consider the problem of a constrained planner who can select  $f$  at

$t = 1$  but who cannot infer with markets at date  $t = 2$ . Recall the pricing formula in equation (18) and from Lemma 2 that  $\frac{dp(\cdot)}{df} > 0$ . Using an envelope-type argument, I examine whether a constrained planner would select a level of  $f$  different to the one found in the competitive equilibrium. The result is summarized in Proposition 8.

**Proposition 8. Efficiency**

Pooling equilibria are efficient if  $p(\hat{f}) = r_G$  or  $p(\hat{f}) = a$ . Instead, for  $r_G < p(\hat{f}) < a$  pooling equilibria are generically inefficient if:

$$\left\{ \lambda\beta_E - \frac{f\zeta(k^F)}{\zeta(k^F) + \frac{p(f)}{a}G(k^F, \theta^F)} \right\} \Big|_{f=\hat{f}} \neq 0.$$

Sufficient conditions for inefficient under-invest in cash are given by  $Z' < 0$  or by:

$$G(k^F, \theta^F) > \frac{1 - \lambda\beta_E}{\lambda\beta_E} \zeta(k^F) \frac{a}{r_G} \quad (32)$$

where  $a$  solves equation (13).

**Proof.** See Appendix A.9.

**Intuition.** Equilibria are “typically” not efficient when the solution for the price is interior, i.e if  $r_G \leq p(\hat{f}) \leq a$ . The private information problem is at the root of the inefficiency. Due to a trading-on-private-information motive, late investors with bad quality assets can gain from private information, whereas investors with good quality assets lose. When making their date  $t = 1$  choice, agents do not take into account that their choice affects future market prices and the average quality of assets traded. The direction of the inefficiency depends crucially on the comparison between these gains and losses.

When abstracting from leveraged investment and inspecting the liquidity choice problem in isolation, i.e. if  $k^I = k^F = 0$ , then  $Z' < 0$  is always satisfied. Here, the gains from trading-on-private-information outweigh the losses, making investments in risky long-term assets too attractive from a constraint planner’s point of view. The pooling equilibrium is characterized by an *over*-investment in risky long-term assets or equivalently by an *under*-investment in cash.<sup>25</sup>

With leveraged investment  $Z' < 0$  is not guaranteed to hold. However, Proposition 8 offers an alternative sufficient condition with equation (32). Equation (32) shows that a lower  $R_B$  (which implies a lower  $a$ ) is more likely to be associated with inefficient *under*-investment in cash. Intuitively, the gains from trading-on-private-information increase.<sup>26</sup>

<sup>25</sup>To see this, notice that the benefits of bad types outweigh the costs of good types at an asset price of  $p = 1$  (the equilibrium price with  $\alpha = 0$ , as well as in the model without private information), Consequently agents have a relatively higher incentive to invest into risky long-term assets ex-ante, which causes a reduction in the equilibrium asset price. In particular, the equilibrium asset price shows to be strictly below 1 for all  $\alpha > 0$ .

<sup>26</sup>The tendency for inefficient *under*-investment in cash prevails in the model where the portfolio choice at date  $t = 1$  is divisible. Again, the interested reader can refer to my supplementary paper “Supplementary note on detrimental deleveraging and adverse selection” (2013) for details.

## 5.2 Efficiency of the equilibrium at date $t = 0$

How does the equilibrium at  $t = 0$  compare to the choice of a constrained social planner? A general analysis is difficult and limit cases are easier to analyze. Using an envelope-type argument, one finds that the equilibrium is characterized by an inefficiently low level of leverage, i.e. by a too high  $\theta^I$ , when condition (28) in Lemma 5(a) holds and  $\lambda \rightarrow 0$ . Hence the lemons problem not only causes an “interim” inefficiency due to a distortion of the liquidity choice at  $t = 1$ , but also an “ex-ante” inefficiency due to a distortion of leverage decisions at  $t = 0$ .

Given the results established in section 4.3.2.3, a situation can arise in which investors select a high value of  $\theta^I$  which negatively affects the average quality of assets traded in the economy and thereby endangers the existence of pooling equilibria. Here, a constrained social planner has a clear scope to improve social welfare by preventing “excessive” deleveraging that causes a breakdown of the asset market.

Intuitively, the reason for the inefficient leverage choice at  $t = 0$  lies in a combination of the strategic complementarity in leverage choices established in Proposition 6 and the negative impact of higher values in  $\theta$  on the average quality of assets traded established in Proposition 1(a). Neither effect is taken into account at  $t = 0$  by individual agents. A numerical example of such a situation was presented earlier in section 4.3.3.

## 6 Policy

The previous analysis reveals several immediate policy implications. Firstly, a policy maker can counteract inefficient under- or over-investment in cash by either taxing investments in risky long-term assets or by regulating  $f$  directly. Secondly, a policy maker can prevent a detrimental deleveraging spiral that arises due to a deterioration in asset quality (compare Proposition 7) by making deleveraging less attractive at date  $t = 0$ . Thirdly, a policy maker can intervene at date  $t = 2$  and provide liquidity to markets by purchasing assets under distress.

While the computation of a Pigovian tax to counteract inefficient under- or over-investment in cash is standard, the relevant policies to influence the leverage choice at  $t = 0$  and to provide liquidity at  $t = 2$  demand further discussion. In the remainder, I examine liquidity provision in section 6.1 and then explore policies to manipulate the leverage choice in section 6.2.

### 6.1 Liquidity provision at date $t = 2$

This section analyzes a central bank intervention at  $t = 2$  intended to support asset prices and market functioning. Such policies were used during the financial crisis of 2007/08 when leading central banks provided liquidity to markets under distress such as subprime asset markets and interbank credit markets.<sup>27</sup>

**Setting:** Consider a scenario in which a central bank anticipates an asset market breakdown at  $t = 2$  and wants to restore market functioning by providing liquidity through collateralized

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<sup>27</sup>See Cecchetti (2009b) and Borio and Nelson (2008) for excellent reviews.



credit in the prime market as well as through an asset purchase program. Suppose that central banks can only generate cash at no cost in nominal terms but not in real terms as in Allen et al. (2009). This implies that central banks have to either store cash before  $t = 2$  and then make it available at  $t = 2$  or they must accept future inflation that devalues claims at  $t = 3$ .

**Question:** Under what conditions can such an intervention be successful?

To answer this question, one has to analyze the equilibrium at  $t = 2$  with intervention. Arguably, such an intervention in date  $t = 2$  markets should be anticipated by market participants at date 1. I consider both anticipated and unanticipated interventions. This requires to consider three cases.

**1. Suppose that, absent intervention,  $p(\hat{f}, a, \hat{k}^I, \hat{k}^F, \hat{\theta}^I, \hat{\theta}^F) < r_G$  while  $a(\hat{k}^I, \hat{\theta}^I) > r_G$ .**<sup>28</sup>  
**Consider an unanticipated intervention.** If the reason for the asset market breakdown at  $t = 2$  is a depressed asset price due to a lack of liquidity, then a central bank liquidity provision at  $t = 2$  can successfully restore market functioning by raising the level of liquidity available in the economy. This is because  $dp(f, \cdot)/df > 0$ .

**2. Suppose that, absent intervention,  $p(\hat{f}, a, \hat{k}^I, \hat{k}^F, \hat{\theta}^I, \hat{\theta}^F) < r_G$  while  $a(\hat{k}^I, \hat{\theta}^I) > r_G$ .**  
**Consider an anticipated intervention.** Unlike the first case an anticipated intervention affects the  $t = 1$  liquidity choice of agents. A date  $t = 1$  equilibrium with a strictly positive measure of liquid investors demands that  $V(f) = W(f)$  as demonstrated in section 4.2.3.1. Since  $dV(f)/df > 0$  and  $dW(f)/df < 0$ , any liquidity provision at  $t = 2$  resulting in an increase in  $f$  must trigger an increase in the number of illiquid investors that completely off-sets the intervention. Consequently, a central bank can only increase  $f$  and restore market functioning if it crowds-out the private liquidity supply. Interestingly, the same outcome can be achieved if a central bank or regulator induces more agents at  $t = 1$  to become illiquid investors.

**3. Suppose that, absent intervention,  $a(\hat{k}^I, \hat{\theta}^I) < r_G$ .** Here a liquidity provision at  $t = 2$  has no effect on the asset market. This is because  $a$  is independent of  $f$  in a scenario where the average quality of assets traded and not cash-in-the-market pricing is at the root of the market breakdown.

## 6.2 Leverage regulation at date $t = 0$

In section 4.3, I show under which conditions a detrimental feedback loop exists between adverse selection at  $t = 2$  and deleveraging at  $t = 0$ . How can a policymaker who faces a deleveraging wave as in the financial crisis of 2007/08 prevent the development of a detrimental feedback loop?

Increasing the costs of reducing leverage is a successful policy to prevent excessive deleveraging (as a reaction to anticipated future market distress). In other words, the policymaker must

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<sup>28</sup>Where  $\hat{k}^I, \hat{k}^F, \hat{\theta}^I, \hat{\theta}^F$  are solutions to the problem in (27).

prevent market participants from starting a run into unencumbered high quality assets, which in turn amplifies market distress. The policymaker can achieve this outcome via several options.

First, the policymaker can reduce refinancing costs for agents at  $t = 0$ . Lowering  $q$  causes a reduction in  $-G_{\theta I}$ . If the reduction in  $-G_{\theta I}$  is sufficiently large, then agents do not deleverage (see Lemma 5(a)). The numerical example from section 4.3.3 shows that a decrease of the refinancing cost at date  $t = 0$  from  $q = 1.5$  to  $\tilde{q} = 1.4$  is sufficient to give agents an incentive to increase their leverage, thereby alleviating the adverse selection problem and preventing a market breakdown at date  $t = 2$ .<sup>29</sup>

Second, the policymaker can use an alternative policy and achieve the same outcome. By widening the collateral requirements for refinancing at  $t = 0$ , i.e. by increasing  $\gamma$ ,  $-G_{\theta I}$  is also reduced, discouraging deleveraging.

Third, if the central bank has any means with which to reduce either the asymmetric information problem at  $t = 2$  or the number of lemons in the market, then a credible commitment to achieving these goals positively influences the leverage choice at  $t = 0$  and can prevent a deleveraging wave. Equation (40) illustrates that the benefit from installing a positive spare borrowing capacity is smaller, the higher  $a$ .

## 7 Discussion and robustness of results

**Conclusion** This paper presents a novel feedback mechanism which explains how deleveraging and the intensity of adverse selection in asset markets can fuel each other as in the financial crisis of 2007/08. The interplay of adverse selection in asset markets and a financing asymmetry, which itself is a function of leverage, is at the core of the mechanism.

Adverse selection creates both an “interim” inefficiency in the liquidity choice (short-term) and an “ex-ante” inefficiency in the leverage choice (long-term). The first inefficiency is caused by a distortion of the liquidity choice which leads to a tendency for under-investment in cash over a large parameter range. The second inefficiency occurs because of a negative externality in borrowing capacity choices which generates a detrimental feedback loop.

I analyze several central bank policies that have been used during the crisis and discuss under what conditions they can be successful based on the nature of the underlying problem. All in all, the main results of the paper are fairly robust to variations of the model.

For future research a detailed welfare analysis which uncovers the ambiguous role played by transparency is of interest. On one hand, a higher degree of transparency in the asset market has direct negative implications for risk-sharing and, hence, social welfare. On the other hand, an increase in transparency can also be beneficial because it mitigates the incentives for market participants to reduce their leverage.

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<sup>29</sup>With  $\widetilde{R}_B = 0$  and  $\tilde{q} = 1.4$ , there does exist a pooling equilibrium characterized by  $\theta^* = 0$ ,  $k^* = \kappa$  and  $f^* \approx 0.35$ . The average quality of assets traded and the market-clearing asset price are given by  $a^*(k^*, \theta^*) \approx 0.91$  and  $p^*(f^*, a^*, k^*, \theta^*) \approx 0.77$ , respectively.

**Robustness of results** For tractability, I assumed throughout this paper that the date  $t = 1$  liquidity choice is binary, meaning that the investment decision at  $t = 1$  is indivisible. When solving a model without the assumption of indivisibility and an otherwise identical setup, I find that the most important qualitative results are unaltered. However, it comes at the cost of reduced tractability. For that reason, the results of Propositions 6 and 7 can only be demonstrated numerically under divisibility since the analytical analysis can only be done partially.

Another assumption is that agents only have access to financing from outside financiers at  $t = 0$  and not thereafter. All results derived in this paper are unaffected by this assumption whenever  $\frac{a}{p(\hat{f})} < q$  where  $a$  solves equation (13). This is because no agent would borrow from outsiders at date  $t = 2$  if the price for borrowing from outsiders exceeded the price for borrowing from insiders. Moreover, no agent would borrow at  $t = 1$  for any parameters satisfying Assumption 4. In addition, it is worth noting that  $\frac{a}{p(\hat{f})} < q$  is unlikely to hold given Assumption 4 though a formal proof is difficult. Nevertheless,  $\frac{a}{p(\hat{f})} < q$  is guaranteed to hold for a sufficiently big exogenous cost  $q$  of outside finance. Moreover, it can be argued that the main qualitative results of the paper are unaffected even in the case where  $\frac{a}{p(\hat{f})} < q$  is violated. To see this, observe that the ability to borrow from outsiders at  $t = 1$  preserves the result of Proposition 1(a) although cash-in-the-market pricing may vanish due to an increase in the demand for long-term assets.

Furthermore, I assumed throughout this paper that the inequality in equation (26) holds. Intuitively, equation (26) demands that agents do not have a strong incentive to transfer income from the long-term project to date  $t = 1$  by reducing their capital investment at date  $t = 0$ . This assumption is used to simplify the analysis, as it prevents that the investment decision at date  $t = 0$  competes with the consumption needs at date  $t = 2$ . Still, the assumption does not prevent that the investments at dates  $t = 0$  and  $t = 1$  compete between each other (see Lemma 5(b)). Only denying the latter may have limited the robustness of the results in this paper. This is because I allow agents to have two methods of shielding themselves against adverse selection problems. They can increase their spare borrowing capacity and they can carry cash from date  $t = 0$  to date  $t = 1$ .

Another concern is the assumption that an adverse selection problem only exists in the market for long-term assets and not with the leveraged long-term investment project started at date  $t = 0$ . This modeling choice is justified by the desire to model a prime market for collateralized credit and a subprime asset market in co-existence. However, the reader may wonder whether the results still hold if there is also a private information problem in the collateralized credit market. The answer to this question is yes. In particular, what matters is that there exist two spot markets at date  $t = 2$  with a varying degree of adverse selection problems. This assures that the financing asymmetry described in section 4.2.1.1 exists, which is crucial for the qualitative results thereafter.

Finally, the modeling feature of an asset market breakdown hinges on Assumption 2. A richer economy with more than two possible payoffs of the risky long-term asset (for instance a continuum approximation) would require a more complicated parameter assumption to generate

a market breakdown and preserve the existence of the detrimental feedback loop derived in Proposition 7. In particular, there has to be a relatively low probability mass for intermediate return realizations together with a sufficiently high cost for physical liquidation of high quality assets. For the application to the financial crisis of 2007/08 the presumption that low quality assets are so bad such that nobody wants to hold them in equilibrium (which is guaranteed by Assumptions 2 and 3) is arguably realistic. Prior to the crisis financial market participants with superior information like US investment banks were more than happy to off-load bad quality subprime assets at high prices to less informed German Landesbanken. However, one could think of variations in the model where bad assets are not so bad such that nobody wants to hold them in equilibrium. Here a reduction in the asset price may cause an increase in the average quality of assets traded and, thereby, potentially overturn some of the results obtained in this paper.

## A Appendix

### A.1 Breakdown of the asset market at $t = 2$

When the asset market breaks down, only the collateralized credit market is left and the market-clearing condition is given by:

$$\begin{aligned} & (1 - f) * \zeta(k^I) (\lambda (\alpha s_{EB}^I(r) + (1 - \alpha) s_{EG}^I(r)) + (1 - \lambda) \alpha r_B) \\ = & f * \zeta(k^F) (\lambda s_L^F(r) + (1 - \lambda)). \end{aligned} \quad (33)$$

Let  $\hat{r}(f)$  be the solution to equation (33). Then the market clearing interest rate is given by  $r^* = \max\{1, \hat{r}(f)\}$ . Given that  $s_L^F(r)$ ,  $s_{EB}^I(r)$  and  $s_{EG}^I(r)$  are increasing in  $r$ , it has to be true that  $\hat{r}(f)$  is decreasing in  $f$ . Intuitively, the interest rate is lower, the more liquidity is available in the economy. Thereby resembling equation (19). Moreover,  $\hat{r}(f)|_{f \rightarrow 0} = \infty$ .

The liquidity choice problem at  $t = 1$  if agents anticipate a breakdown of the asset market is constructed similar to before. If an interior solution exists, then  $\tilde{V}(f) = \tilde{W}(f)$ , where:

$$\begin{aligned} \tilde{V}(f) & \equiv \left( \begin{array}{c} \lambda \left[ \alpha (\beta_E U (\zeta(k^I) (r_B - s_{EB}^I(\cdot)))) + (1 - \beta_E) U (\zeta(k^I) s_{EB}^I(\cdot) r(f) + G(k^I, \theta^I)) \right] \\ + (1 - \alpha) \left( \begin{array}{c} \beta_E U (\zeta(k^I) (pd_{EG}^I(\cdot) - s_{EG}^I(\cdot))) + \\ (1 - \beta_E) U (\zeta(k^I) (s_{EG}^I(\cdot) r + (1 - d_{EG}^I(\cdot)) R_G) + G(k^I, \theta^I)) \end{array} \right) \end{array} \right) \\ & + (1 - \lambda) [\alpha U (\zeta(k^I) r_B r(f) + G(k^I, \theta^I)) + (1 - \alpha) U (\zeta(k^I) R_G + G(k^I, \theta^I))] \\ \tilde{W}(f) & \equiv \left( \begin{array}{c} \lambda [\beta_E U (\zeta(k^F) (1 - s_E^F(\cdot))) + (1 - \beta_E) U (\zeta(k^F) s_E^F(\cdot) r(f) + G(k^F, \theta^F))] \\ + (1 - \lambda) U (\zeta(k^F) r(f) + G(k^F, \theta^F)) \end{array} \right), \end{aligned}$$

with  $s_E^F(\cdot) = s_E^F(r(f))$ ,  $s_{Eh}^I(\cdot) = s_{Eh}^I(r(f))$  and  $d_{EG}^I(\cdot) = d_{EG}^I(r(f))$ . If:

$$\tilde{W}(f) \Big|_{f \rightarrow 1} \geq \tilde{V}(f) \Big|_{f \rightarrow 1} \quad (34)$$

then all agents prefer to become liquid investors and the equilibrium exhibits liquidity hoarding with  $r^* = 1$  and any  $p^* \in [0, r_B]$ .

### A.2 Proof of Lemma 1

Both results of Lemma 1 are proven in turn.

(a) The average quality of assets traded at  $t = 2$  is implicitly defined by:

$$W \equiv \lambda (1 - \alpha) \left[ \beta_E \left( 1 + \frac{G(k^I, \theta^I) - \theta^I}{R_G \zeta(k^I)} \right) - (1 - \beta_E) \frac{\theta^I}{a \zeta(k^I)} \right] (R_G - a) + \alpha (R_B - a) = 0.$$

By application of the implicit function theorem  $\frac{da}{d\theta^I} = -\frac{dW/d\theta^I}{dW/da}$  and  $\frac{da}{dR_B} = -\frac{dW/dR_B}{dW/da}$ , where  $\frac{dW}{d\theta^I} < 0$  and  $\frac{dW}{dR_B} > 0$ . Moreover,

$$\frac{dW}{da} = \frac{\lambda(1-\alpha)(1-\beta_E)R_G\theta^I}{a^2\zeta(k^I)} - \lambda(1-\alpha)\beta_E \left( 1 + \frac{G(k^I, \theta^I) - \theta^I}{R_G\zeta(k^I)} \right) - \alpha. \quad (35)$$

Given that  $\left. \frac{dW(\theta^I)}{da} \right|_{\theta^I=0} < 0$  and  $\frac{d}{d\theta^I} \left( \frac{dW}{da} \right) > 0$ , equation (35) is negative if and only if:

$$\theta^I < \frac{\beta_E (R_G\zeta(k^I) + G(k^I, \theta^I)) + \frac{\alpha R_G\zeta(k^I)}{\lambda(1-\alpha)}}{(1-\beta_E)\frac{R_G^2}{a^2} + \beta_E}. \quad (36)$$

Equation (36) holds for all  $\theta^I \in [0, \gamma k^I]$  if equation (15) is violated. Otherwise, a necessary condition for equation (36) to hold is that  $\theta^I < \bar{\theta}^I(r_G)$  because the right-hand side of equation (15) is increasing in  $a$  and  $r_G$  is the smallest possible value of  $a$  such that a pooling equilibrium can exist. The results of Lemma 1 (a) follow.

(b) Both, the left-hand side and the right-hand side of equation (13) are increasing in  $a$ . As a result, the larger root of equation (13) is only larger than  $r_G$  if the condition in Lemma 1 (b) holds. (*q.e.d.*)

### A.3 No borrowing at date $t = 1$

The argument is constructed in four steps. First, consider an economy with  $x \in [0, 1]$  (i.e. where the date  $t = 1$  choice is not discrete) and suppose there exists a symmetric equilibrium characterized by  $\tilde{x} \in (0, 1)$ ,  $\tilde{p} = p(\tilde{x})$  and  $\tilde{a} = a(\tilde{x})$  where every agent selects an identical level of portfolio liquidity. Second, consider a perturbation where a fraction  $f$  of agents select a level of liquidity given by  $x' = \tilde{x} - \epsilon'$  and a fraction  $(1 - f)$  of agents select a level of liquidity given by  $x'' = \tilde{x} - \epsilon''$  with  $\epsilon', \epsilon'' > 0$  being small. Suppose  $f, x'$  and  $x''$  are chosen such that both groups of agents with lower and higher portfolio liquidity have identical expected utility. Third, consider an increase in  $\epsilon'$  and  $\epsilon''$  that preserves the property that both groups have identical expected utility. Due to the concave curvature of the utility function, it must be that at least one group suffers a utility reduction independent of the adjustment in equilibrium prices because of the change in exposure to return risk. As a result, the increase in  $\epsilon'$  and  $\epsilon''$  must cause a reduction of the expected utility for both groups. Fourth, the above argument can be applied repeatedly. Hence, if there exists an equilibrium with  $x' = 0, x'' = 1$  and  $f = \hat{f}$ , than illiquid investors who have the possibility to borrow at  $t = 1$  will not do so. Because of the concave utility, they desire a reduction in portfolio liquidity and not an increase in portfolio liquidity given the equilibrium prices  $r_1 = r_2 = \frac{a}{p(\hat{f})}$ . Hence, in any pooling equilibrium  $b^{I*} = b^{F*} = 0$ . For the equilibrium with a breakdown in the asset market a similar argument can be made to show that  $b^{I*} = b^{F*} = 0$ .

#### A.4 Proof of Proposition 3

It remains to be shown that there always exists an equilibrium where the market breaks down characterized by  $f^* \in [0, 1]$  and  $p^* \in [0, r_B]$ . The equilibrium can be constructed as follows. Suppose investors believe at date  $t = 1$  that there will be an asset market breakdown at date  $t = 2$ , i.e. they believe that  $p < r_G$ . Notice that at date  $t = 2$  such an equilibrium can always be sustained since good types are not willing to sell for all  $p < r_G$  and, hence, buyers of long-term assets will not be willing to pay any price above  $r_B (< r_G)$ . The equilibrium where the asset market breaks down can be supported by any  $p \in [0, r_B]$ . If equation (34) in Appendix A.1 holds, then the equilibrium is characterized by liquidity hoarding, i.e.  $f^* = 1$ . Otherwise  $f^* \in [0, 1)$ . (*q.e.d.*)

#### A.5 Proof of Proposition 5

The proof proceeds as follows. I discuss first market-clearing at  $t = 2$  in section A.5.1 and, second, the solution to the date  $t = 1$  problem in section A.5.2. Finally, section A.5.3 analyzes the leverage choice at  $t = 0$  and concludes with the main results.

Notice that I can abstract from physical liquidation of long-term assets which does not play a role as long as assets are riskless. At  $t = 1$  investors take  $k, \theta \geq 0$  and  $G = G(k, \theta)$  as given and optimally choose whether to become a *liquid* or *illiquid investor*. Uncertainty realizes at the beginning of  $t = 2$  when investors learn whether they are of *early type* or *late type*. As before, there are two distinct spot markets at date  $t = 2$  where agents can share their liquidity risk. In the asset market agents buy or sell long-term assets at an endogenous price  $p$  and in the credit market investors agents lend or borrow at an endogenous interest rate  $r$ . Borrowing is collateralized with the safe proceeds of the leveraged long-term investment project and the borrowing capacity is  $\frac{\theta}{r} \geq 0$ , where  $r = \frac{R}{p}$  due to no arbitrage.

##### A.5.1 Market-clearing at date $t = 2$

The supply and demand schedules derived in section 4.2.1 simplifies for the special case with  $\alpha = 0$ . Given that  $p = R$  cannot be an equilibrium (because here nobody would have an incentive to purchase long-term assets at  $t = 1$ ) the focus is on  $p < R$ . For markets to clear, the demand for long-run assets has to be weakly larger than the supply of long-run assets:  $S(p) \leq D(p)$ .

Observe from equation (10) that illiquid investors can fully finance their consumption needs by selling long-term assets as long as:<sup>30</sup>

$$1 \geq \beta_E \left( 1 + \frac{G(k^I, \theta^I)}{R\zeta(k^I)} \right). \quad (37)$$

Let's assume the inequality in equation (37) holds and examine later under what conditions this is indeed the case. Then the market clearing price can be computed as:

<sup>30</sup>Notice that the condition in equation (22) implies that equation (37) holds for all  $1 + \kappa < R$  if  $k^F \geq k^I$  and  $\theta^I \geq \theta^F$ .

$$p(f) = \min \left\{ \frac{1 - \lambda\beta_E}{\lambda\beta_E} \frac{f}{(1-f) * \left(1 + \frac{G(k^I, \theta^I)}{R\zeta(k^I)}\right) + f \frac{G(k^F, \theta^F)}{R\zeta(k^F)}}, R \right\}. \quad (38)$$

### A.5.2 Solution to the problem at date $t = 1$

As in section 4.2.3 agents have to be indifferent whether to become *liquid investors* or *illiquid investors*.  $V(f)$  and  $W(f)$  simplify for the special case with  $\alpha = 0$ . Under symmetry of leverage choices, i.e. if  $k = k^I = k^F$  and  $\theta = \theta^I = \theta^F$ , it shows that investors are indifferent for  $p = 1$ . Suppose that leverage choices are in fact symmetric, then:

$$f^* = \lambda\beta_E \left(1 + \frac{G(k, \theta)}{R}\right). \quad (39)$$

Moreover, notice that for  $p = 1$  agents are indifferent whether to borrow or lend in the collateralized credit market at date  $t = 1$ .

### A.5.3 Solution to the problem at date $t = 0$ and results on equilibrium existence, uniqueness and efficiency

Given the date  $t = 1$  equilibrium described in equations (37) and (39) there is neither a benefit from selecting  $\theta > 0$ , nor from selecting a  $k < \kappa$  as long as  $G(\kappa, 0)$  is not too large, i.e. as long as the inequality in equation (37) holds when evaluated at  $k = \kappa$  and  $\theta = 0$ . Therefore both, *liquid* and *illiquid investors* have no incentive to leave a positive spare borrowing capacity at  $t = 0$  whenever it is costly to do so, i.e. they choose  $\theta^* = 0$ . Moreover, both types of investors optimally select  $k^* = \kappa$  at  $t = 0$  because the leveraged long-term investment project yields a superior return given Assumption 4. Hence, the symmetry assumption underlying equation (39) can be validated if equation (37) holds.

Furthermore, the equilibrium is efficient. A constrained planner who can only choose  $k$ ,  $\theta$  and  $y$  is not able to improve upon the allocation since the economy is frictionless. A decrease in  $k$  or an increase in  $\theta$  is costly but would not bring any benefit.

It remains to show uniqueness. Recall that in equilibrium agents have to be indifferent whether to select a illiquid portfolio ( $I$ ) or a fully liquid portfolio ( $F$ ), i.e.:

$$\begin{aligned} X &\equiv \lambda \left[ \begin{array}{l} \beta_E U \left( \beta_E (R\zeta(k) + G(k^I, \theta^I)) \frac{p}{R} \right) + \\ (1 - \beta_E) U \left( (1 - \beta_E) (R\zeta(k) + G(k^I, \theta^I)) \right) \end{array} \right] + (1 - \lambda) U (R\zeta(k) + G(k^I, \theta^I)) \\ &= Y \equiv \lambda \left[ \begin{array}{l} \beta_E U \left( \beta_E \left( \frac{R}{p} \zeta(k) + G(k^F, \theta^F) \frac{p}{R} \right) \right) + \\ (1 - \beta_E) U \left( (1 - \beta_E) \left( \frac{R}{p} \zeta(k) + G(k^F, \theta^F) \right) \right) \end{array} \right] + (1 - \lambda) U \left( \frac{R}{p} \zeta(k) + G(k^F, \theta^F) \right). \end{aligned}$$

Suppose (37) holds when evaluated at  $k^I = \kappa$  and  $\theta^I = 0$ . Then the unique price solving the above system is  $p = 1$  since  $\frac{d}{dp}X(p) = \lambda\beta_E > \frac{d}{dp}Y(p)$ ,  $X(r_G) < Y(r_G)$  and  $X(1) = Y(1)$ . (*q.e.d.*)



## A.6 Proof of Lemma 5

The first-order necessary conditions to the problem in (27) read:

$$\theta^I : \left( \begin{array}{l} \left[ \frac{\alpha}{a\zeta(k^I)+G(k,\theta^I)} + \frac{(1-\lambda)(1-\alpha)}{\zeta(k^I)R_G+G(k^I,\theta^I)} \right] G_{\theta^I} \\ +\lambda(1-\alpha) \frac{G_{\theta^I}-1+\frac{R_G}{a(k^I,\theta^I)}}{R_G\zeta(k^I)+G(k^I,\theta^I)-\theta^I+\frac{R_G}{a(k^I,\theta^I)}\theta^I} \end{array} \right) + \mu_1 - \mu_2 = 0, \quad (40)$$

$$k : \left( \begin{array}{l} \alpha \frac{-a+G_k}{a\zeta(k^I)+G(k^I,\theta^I)} + (1-\lambda)(1-\alpha) \frac{-R_G+G_k}{R_G\zeta(k^I)+G(k^I,\theta^I)} \\ +\lambda(1-\alpha) \frac{-R_G+G_k}{R_G\zeta(k^I)+G(k^I,\theta^I)-\theta^I+\frac{R_G}{a(k^I,\theta^I)}\theta^I} \end{array} \right) + \mu_3 - \mu_4 = 0. \quad (41)$$

where  $\mu_1, \mu_2, \mu_3, \mu_4$  are the multipliers on the first, second, third and fourth inequality constraint, respectively. The two results of Lemma 5 are proven in turn using the two first-order necessary conditions in equations (40) and (41).

From equation (40) investors optimally select a positive  $\theta^I$  if:

$$\left( \frac{\alpha G_{\theta^I}}{a(k^I,\theta^I)\zeta(k^I)+G(k^I,\theta^I)} + \frac{(1-\lambda)(1-\alpha)G_{\theta^I}}{\zeta(k^I)R_G+G(k^I,\theta^I)} + \frac{\lambda(1-\alpha)\left(G_{\theta^I}-1+\frac{R_G}{a(k^I,\theta^I)}\right)}{R_G\zeta(k^I)+G(k^I,\theta^I)-\theta^I+\frac{R_G}{a}\theta^I} \right) \Big|_{\theta^I=0} > 0,$$

which can be reformulated to:

$$-G_{\theta^I} < \frac{\lambda(1-\alpha)\frac{R_G-a}{a}}{\alpha\frac{R_G\zeta(k^I)+G(k^I,0)}{a\zeta(k^I)+G(k^I,0)}+1-\alpha} \quad \text{for all } k^I \in [0, \kappa].$$

A sufficient condition for the above inequality to hold is given by equation (28) of Lemma 5, where I used the fact that  $\frac{da}{dk^I} > 0$  if the conditions in Lemma 1 hold (i.e. if  $G_k > R_G$ ).

From equation (41) investors optimally select  $k^* = \kappa$  if:

$$\left( \begin{array}{l} \alpha \frac{-a(k^I,\theta^I)+G_k}{a(k^I,\theta^I)\zeta(k^I)+G(k^I,\theta^I)} + (1-\lambda)(1-\alpha) \frac{-R_G+G_k}{R_G\zeta(k^I)+G(k^I,\theta^I)} \\ +\lambda(1-\alpha) \frac{-R_G+G_k}{R_G\zeta(k^I)+G(k^I,\theta^I)-\theta^I+\frac{R_G}{a(k^I,\theta^I)}\theta^I} \end{array} \right) \Big|_{k=\kappa} > 0.$$

A sufficient condition for the above inequality to hold is given by equation (29) of Lemma 5. (*q.e.d.*)

## A.7 Proof of Proposition 6

Given a pooling equilibrium exists and the condition in equation (28) of Lemma 5 holds, the optimal  $\hat{\theta}^I$  of speculator  $m$  solves (for a given  $k_{-m}^I$  and  $\theta_{-m}^I$ ):

$$H \equiv \left[ \frac{\alpha}{a(k_{-m}^I, \theta_{-m}^I) \zeta(k_{-m}^I) + G(k_m^I, \theta_m^I)} + \frac{(1-\lambda)(1-\alpha)}{R_G \zeta(k_{-m}^I) + G(k_m^I, \theta_m^I)} \right] G_{\theta^I} \\ + \frac{\lambda(1-\alpha) \left( G_{\theta^I} - 1 + \frac{R_G}{a(k_{-m}^I, \theta_{-m}^I)} \right)}{R_G \zeta(k_{-m}^I) + G(k_m^I, \theta_m^I) - \theta_m^I + \frac{R_G}{a(k_{-m}^I, \theta_{-m}^I)} \theta_m^I} = 0.$$

By application of the implicit function theorem  $\frac{d\hat{\theta}^I(\theta_{-m}^I, k_{-m}^I)}{d\theta_{-m}^I} > 0$  if:

$$\left( -\frac{\alpha}{\lambda(1-\alpha)} \zeta(k_{-m}^I) + \frac{k_m^I q}{R_G} \right) G_{\theta^I} < \zeta(k_{-m}^I) + \frac{k_m^I q}{R_G} \text{ for all } k_m^I \in [0, \kappa] \quad (42)$$

and the conditions in Lemma 1 hold when evaluated at  $(k_{-m}^I, \theta_{-m}^I)$ . To see this notice that  $\frac{dH}{d\theta_{-m}^I} < 0$  if the inequality in equation (42) holds. Furthermore,

$$\frac{dH}{d\theta_{-m}^I} = \frac{dH}{a} \frac{da}{d\theta_{-m}^I} > 0$$

if the inequality in equation (42) and the conditions in Lemma 1 hold when evaluated at  $(k_{-m}^I, \theta_{-m}^I)$ . This is because:

$$\frac{dH}{a} = \left( -\frac{\alpha G_{\theta^I}}{[a\zeta(k_{-m}^I) + G(k_m^I, \theta_m^I)]^2} + \lambda(1-\alpha) \frac{-\frac{R_G}{a^2} [R_G \zeta(k_{-m}^I) + G(k_m^I, \theta_m^I)] + G_{\theta^I} \frac{R_G}{a^2} \theta_m^I}{[R_G \zeta(k_{-m}^I) + G(k_m^I, \theta_m^I) - \theta_m^I + \frac{R_G}{a} \theta_m^I]^2} \right)$$

is strictly increasing in  $\theta_m^I$ . When evaluated at the highest possible value  $\bar{\theta}(k_m^I) = k_m^I q$  it shows that  $\frac{dH}{a}|_{\theta=\bar{\theta}(k_m^I)} < 0$  if the condition in equation (42) holds and, hence,  $\frac{dH}{a} < 0$ . (*q.e.d.*)

## A.8 Proof of Proposition 7

The two results of Proposition 7 are proven in turn.

(a) From equation (13) together with Lemma 1 it shows that for a given  $(k_{-m}^I, \theta_{-m}^I)$ :

$$\frac{da(k_{-m}^I, \theta_{-m}^I)}{dR_B} > 0 \quad (43)$$

if the conditions in Lemma 1 hold when evaluated at  $(k_{-m}^I, \theta_{-m}^I)$ .

Recall from the proof of Proposition 6 that given a pooling equilibrium exists and the condition in equation (28) of Lemma 5 holds, the optimal  $\hat{\theta}_m^I$  of investor  $m$  solves  $H = 0$ . In a symmetric equilibrium  $\theta_m^I = \theta_{-m}^I = \theta^I$  and  $k_m^I = k_{-m}^I = k^I$ . From the proof of Proposition 6  $\frac{dH}{d\theta_m^I} < 0$ ,  $\frac{dH}{a} < 0$  and, hence,  $\frac{d\hat{\theta}_m^I}{da} < 0$  if the inequality in (31) holds. Furthermore,  $\frac{da(\hat{\theta}_{-m}^I)}{d\theta_{-m}^I} < 0$  if the conditions in Lemma 1 hold when evaluated at  $(k_{-m}^I, \theta_{-m}^I)$ .

Now consider an increase in  $R_B$  and suppose that equation (29) holds, i.e.  $k^{I*} = \kappa$ . Due to (43) the decrease in  $R_B$  leads to an decrease in  $a(\hat{\theta}_{-m}^I)$ , which in turn increases  $\hat{\theta}_m^I$ . Given that in equilibrium  $\theta_m^I = \theta_{-m}^I$ , this increase in  $\hat{\theta}_m^I$  causes  $a(\hat{\theta}_{-m}^I)$  to decrease. Thereby causing

a further increase in  $\theta_m^I$  and completing the detrimental feedback loop. In conclusion  $\frac{d\theta^{I*}}{dR_B} < 0$  if the inequalities in equations (28), (29) and (31) hold, as well as the conditions of Lemma 1 when evaluated at  $(\kappa, \theta^{I*})$ .

(b) A necessary condition for the existence of a pooling equilibrium is given by equation (16) in Lemma 1(b). It is violated if the decrease in  $R_B$  leads to a sufficiently large increase in  $\theta^{I*}$  such that equation (16) does not hold when evaluated at  $(\kappa, \theta^{I*})$ . (*q.e.d.*)

## A.9 Proof of Proposition 8

In this proof I discuss in turn efficiency if the solution is interior ( $r_G \leq p(f) \leq a$ ) and if the solution is in one of the corners.

(a) Assuming an interior solution, the problem of the constraint social planner reads:

$$\max_{0 \leq f \leq 1} \left\{ (1-f) \left( \lambda \left[ \begin{aligned} & \alpha \left( \begin{aligned} & \beta_E U \left( \beta_E (a\zeta(k^I) + G(k^I, \theta^I)) \frac{p(f)}{a} \right) + \\ & (1-\beta_E) U \left( (1-\beta_E) (a\zeta(k^I) + G(k^I, \theta^I)) \right) \end{aligned} \right) + \\ & (1-\alpha) \left( \begin{aligned} & \beta_E U \left( \beta_E \left( \zeta(k^I) + \frac{G(k^I, \theta^I) - \theta^I}{R_G} + \frac{\theta^I}{a} \right) p(f) \right) + \\ & (1-\beta_E) U \left( (1-\beta_E) \left( \zeta(k^I) + \frac{G(k^I, \theta^I) - \theta^I}{R_G} + \frac{\theta^I}{a} \right) R_G \right) \end{aligned} \right) \right] \right. \\ & \left. + (1-\lambda) [\alpha U(\zeta(k^I) a + G(k^I, \theta^I)) + (1-\alpha) U(\zeta(k^I) R_G + G(k^I, \theta^I))] \right) \\ + f \left( \lambda \left[ \begin{aligned} & \beta_E U \left( \beta_E \left( \zeta(k^F) + \frac{p(f)}{a} G(k^F, \theta^F) \right) \right) \\ & + (1-\beta_E) U \left( (1-\beta_E) \left( \frac{a}{p(f)} \zeta(k^F) + G(k^F, \theta^F) \right) \right) \end{aligned} \right] \right) \\ \left. + (1-\lambda) U \left( \frac{a}{p(f)} \zeta(k^F) + G(k^F, \theta^F) \right) \right) \end{aligned} \right\}$$

With log-utility, the derivative with respect to  $f$  writes:

$$-V(f) + W(f) + \left\{ \lambda \beta_E - f \frac{\zeta(k^F)}{\frac{p(f)}{a} G(k^F, \theta^F) + \zeta(k^F)} \right\} \frac{p'(f)}{p(f)} \quad (44)$$

Using an envelope-type argument equation (44) simplifies. Given a pooling equilibrium exists:

$$\left\{ \lambda \beta_E - \frac{f \zeta(k^F)}{\frac{p(f)}{a} G(k^F, \theta^F) + \zeta(k^F)} \right\} \frac{p'(f)}{p(f)} \Big|_{f=\hat{f}} \quad (45)$$

The equilibrium is inefficient if equation (45) is non-zero. Equation (45) shows to take on a strictly positive value if (32) holds. In addition equation (45) also shows to take on a strictly

positive value if  $Z' < 0$ . To see this observe that (45) is positive if:

$$\frac{f}{Z(f)} \frac{G(k^F, \theta^F)}{a} > \frac{f - \lambda\beta_E}{1 - \lambda\beta_E} \zeta(k^F). \quad (46)$$

Both, the left-hand side and the right-hand side of equation (46) are continuous functions of  $f$ . For  $f \rightarrow 1$  the left-hand side and the right-hand side take on a value of  $\zeta(k^F)$ . Moreover, for  $f \rightarrow 0$  the left-hand side takes on a value of 0, while the right-hand side takes on a strictly negative value. If  $Z' < 0$  the right-hand side is increasing in  $f$  at an increasing rate (positive first and second derivative). As a result equation (45) takes on a strictly positive value if  $Z' < 0$  meaning that investors inefficiently under-invest in cash in equilibrium, i.e.  $f_{SP}^* > f^* = \hat{f}$ .

(b) Next, assume a corner solution with  $p(\hat{f}) > a$ . Here  $f^* = f_{SP}^* = 1$ . If instead assuming a corner solution with  $p(\hat{f}) < r_G$ , then  $f^* = f_{SP}^* \in [0, 1]$ . Hence, the equilibrium is efficient. (q.e.d.)

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