

Mimra, Wanda; Wambach, Achim

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# A Note on Uniqueness in Game-Theoretic Foundations of the Reactive Equilibrium

Wanda Mimra\* and Achim Wambach†

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## Abstract

Riley (1979)'s reactive equilibrium concept addresses problems of equilibrium existence in competitive markets with adverse selection. The game-theoretic interpretation of the reactive equilibrium concept in Engers and Fernandez (1987) yields the Rothschild-Stiglitz (1976)/Riley (1979) allocation as an equilibrium allocation, however multiplicity of equilibrium emerges. In this note we imbed the reactive equilibrium's logic in a dynamic market context with active consumers. We show that the Riley/Rothschild-Stiglitz contracts constitute the unique equilibrium allocation in any pure strategy subgame perfect Nash equilibrium.

*JEL classification:* C72, D82, G22, L10.

*Keywords:* asymmetric information, competitive insurance market, contract addition, reactive equilibrium

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\*Center of Economic Research, ETH Zurich, Zuerichbergstrasse 18, 8032 Zuerich, Switzerland. Email: [wmimra@ethz.ch](mailto:wmimra@ethz.ch)

†University of Cologne, Department of Economics, Albertus Magnus Platz, 50923 Cologne, Germany. Email: [wambach@wiso.uni-koeln.de](mailto:wambach@wiso.uni-koeln.de)

# 1 Introduction

Following the seminal work by Rothschild-Stiglitz (1976, RS in the following) on insurance markets with asymmetric information, the equilibrium non-existence problem in competitive markets with adverse selection has given rise to an extensive literature.<sup>1</sup> Riley (1979) proposed an equilibrium concept where firms anticipate that if they offer contracts different from the equilibrium contracts, they expect that competitors will react by adding new contracts as well. “A set of offers is a reactive equilibrium if, for any additional offer which generates an expected gain to the agent making the offer, there is another which yields a gain to a second agent and losses to the first” (Riley, 1979). Engers and Fernandez (1987, EF in the following) generalize results for the reactive equilibrium and provide a game-theoretic interpretation for the equilibrium concept. In their game-theoretic modelling, EF assume that firms offer contracts repeatedly, and privately informed consumers choose contracts only after the final contract addition has been made (which might never occur if firms keep on adding contracts).<sup>2</sup> EF show that the Riley result, namely that the RS contracts constitute equilibrium contracts, holds in their model. However, many other allocations are also possible, as the possibility to offer contracts immediately following a deviation can be used as a punishment to sustain collusive outcomes. Thus the problem of non-existence of equilibrium has been replaced by the problem of multiplicity of equilibria.

We embed the Riley logic in a dynamic set-up where consumers arrive at the market in each period. So rather than waiting until all contracts have been added as in EF, consumers enter the market frequently and choose among those contracts which are on the table. By this we add a realistic component to the basic model and allow for repeated market interaction. Firms’ choices in any period, in this case to offer new contracts, are not made independent of the demand. It will be shown that with this addition to the EF model, the RS allocation is the unique equilibrium allocation of the game.

We use the insurance market à la RS as the canonical application to display our results. In our model, new consumers which are either high or low risks arrive in each period. Insurers live forever. In each period, insurers can add contracts to their existing portfolio of contracts. A contract which has been offered cannot be

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<sup>1</sup>For an overview see Mimra and Wambach (2014).

<sup>2</sup>Wilson (1977), Myazaki (1977) and Spence (1978) use an equilibrium concept where following a deviating offer the other firms can withdraw their contracts rather than adding new contracts. A game-theoretic modeling of this concept is provided in Mimra and Wambach, 2011.

withdrawn.<sup>3</sup>

To gain some intuition for the result, consider first the case where insurers offer cross subsidizing contracts, i.e. contracts are such that the low risks contract makes a profit while the high risks contract makes a loss. Then, another insurer could enter the market and, as the single crossing property holds, could cream skim the low risks, i.e. the deviating company offers a new contract which only attracts the low risks and which is profit making. This insurer is assured that the high risks remain with the other insurers offering the original set of contracts, as these contracts remain in the market. Now, the other insurers can start to cream skim the deviating insurer in the next period, but if the profit of a single period is sufficiently large, cream skimming low risks will still be attractive. Note that there is no way how the other insurers can impose a loss on the deviating insurer, as it is them who are stuck with the contract for the high risk type. As a result, any cross subsidizing pair of contracts cannot be an equilibrium. That is different to the outcome in EF. In their model, cross subsidizing contracts, as well as other profit making contracts, can be part of an equilibrium allocation, as any deviating insurer will be immediately cream-skimmed himself. As consumers only enter once all contracts have been added, cream skimming a deviator ensures that deviation is unprofitable.

The original RS/Riley contracts, however, which provide zero profit with each type, do constitute an equilibrium allocation in our model. And this equilibrium exists even in those cases where in the static model of RS an equilibrium in pure strategies does not exist. The reason is as follows. If an insurer intends to deviate from the RS contracts, he has to attract both high and low risks, e.g. by offering a pooling contract. But now another insurer will find it profitable to cream skim the deviator, so that the deviator is left with the high risks only and makes a loss. So even if this deviator makes a profit for one period with the pooling contract, he will face losses for all future periods, making deviation unattractive.

## 2 The model

The set-up inserts the static model by RS in a repeated framework, and adds consumer arrival in each period to the dynamic model by EF.

In each period a continuum of individuals with mass 1 enter the insurance market

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<sup>3</sup>By only allowing for addition of contracts, we model the situation analogously to EF and capture the notion of the Riley concept. In parallel work we investigate repeated market interaction where insurers can also withdraw contracts with some probability.

and stay there for one period. Each individual faces two possible states of nature: In state 1, no loss occurs and the endowment is  $w$ . In state 2 a loss of size  $m$  occurs and the endowment is  $w - m$ . There are two types of individuals, an individual may be a high risk type ( $H$ ) with loss probability  $p^H$ , or a low-risk type ( $L$ ) with loss probability  $p^L$ , with  $0 < p^L < p^H < 1$ . The share of high risk types in each cohort is given by  $\gamma$  with  $0 < \gamma < 1$ .

Insurance is provided by insurers in the set  $I := \{1, \dots, i, \dots, n\}$ . Insurers live forever and discount their profit with discount rate  $\delta \in (0, 1)$ . In each period, insurers can offer a menu of contracts. An insurance contract  $\omega$  consists of a premium and an indemnity:  $\omega = (P, I)$ . The set of feasible contracts,  $\Omega$ , is given by  $\Omega := \{(P, I) \in \mathbb{R}^2\}$ . As usual, insurers do not know any individual's type.

Formally, the game proceeds as follows: There are  $t = 0, 1, 2, \dots$  periods. In each period  $t$ , the stage game is as follows:

*Stage 0:* Individuals of mass 1 enter the market. The risk type of each individual is chosen by nature. Each individual has a chance of  $\gamma$ ,  $0 < \gamma < 1$  to be a  $H$ -type, and of  $(1 - \gamma)$  to be a  $L$ -type.

*Stage 1:* Each insurer  $i \in I$  offers a compact set of contracts  $\Omega_i^t \subset \Omega$ , which contains the contracts from the previous period, i.e.  $\Omega_i^{t-1} \subseteq \Omega_i^t$ .  $\Omega_i^0 = \emptyset$ .

*Stage 2:* Individuals choose among the contracts  $\Delta^t := \bigcup_F \Omega_i^t$  or remain uninsured.

The game in each period with its three stages is equivalent to the RS game. The only difference is that contracts from the previous period remain in the market. The dynamic structure and stage 1 are similar to EF.<sup>4</sup> While in EF consumers make their choice only after all contracts have been offered, in our framework (new) consumers arrive and choose contracts in each period.

The preferences by the policyholders satisfy the von Neumann-Morgenstern axioms. The expected utility of a  $J$ -type individual,  $J \in \{H, L\}$  from choosing a contract  $\omega \in \Omega$  is abbreviated by  $u^J(\omega) := (1 - p^J)v(w - P) + p^Jv(w - m - P + I)$  where  $v$  is a strictly increasing, twice continuously differentiable and strictly concave utility index.

As policyholders only live for a single period, the game does not entail any strategic

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<sup>4</sup>In EF insurers choose contracts sequentially rather than simultaneously, the order of moves being exogenously given.

role for them. Thus, we will henceforth assume that consumer strategy does not depend on the history of play, and that consumers just choose the best contract available in the market. If consumers are indifferent between contracts, they choose the contract with the larger indemnity.

Since for all  $i \in I$  and  $t$ ,  $\Omega_i^t$  is a compact subset of  $\Omega$ ,  $\Delta^t$  is compact. We denote by  $\bar{\omega}_J^t$  the contract such that

$$\bar{\omega}_J^t \in \arg \max_{\omega \in \Delta^t} u^J(\omega)$$

and

$$\bar{I}_J \geq \tilde{I}_J \quad \forall \tilde{\omega}_J \in \arg \max_{\omega \in \Delta^t} u_J(\omega)$$

Let  $\bar{k}_J^t$  denote the number of firms offering  $\bar{\omega}_J^t$  in  $t$ , i.e.  $\bar{K}_J^t := \{i \in I \mid \bar{\omega}_J^t \in \Omega_i^t\}$  and  $\bar{k}_J^t := |\bar{K}_J^t|$ . We assume throughout that the strategy of a consumer of type  $J$  is to choose  $\bar{\omega}_J^t$  at firm  $f \in \bar{K}_J^t$  with probability  $1/\bar{k}_J^t$ . This allows us to insert consumer choice directly into the firms' objective function such that the game reduces to a game of complete information between firms. With this we can reduce the stage game in every  $t$  to a single stage, stage 1'.

*Stage 1'*: Each insurer  $i \in I$  offers a compact set of contracts  $\Omega_i^t \subset \Omega$ , which contains the contracts from the previous period, i.e.  $\Omega_i^{t-1} \subset \Omega_i^t$ .  $\Omega_i^0 = \emptyset$ . Contract  $\bar{\omega}_H^t$  ( $\bar{\omega}_L^t$ ) will be chosen by the  $H(L)$ -type.

An insurer who offers contract  $\omega_J^t = (P_J^t, I_J^t)$  will make a profit of  $\pi_J^t = P_J^t - p_J I_J^t$  per customer of type  $J$  with  $J \in \{L, H\}$ . Call  $\pi_i^t$  the profit in period  $t$  of insurer  $i$ . Then the overall profit of insurer  $i$  is given by:

$$\pi_i = \sum_{t=0}^{\infty} \delta^t \pi_i^t \tag{1}$$

The reduced game is an infinite horizon game with observed actions that is continuous at infinity. The corresponding equilibrium concept we will use is that of subgame-perfect Nash equilibrium. The history of the game is given by the set of all contracts being offered in each period, i.e.  $h^t = (\Omega^1, \Omega^2, \dots, \Omega^{t-1})$ . The set of all histories in period  $t$  is denoted by  $H^t$ . A strategy for insurer  $i$  is a sequence of maps



$\{s_i^t\}_{t=0}^\infty$  where each  $s_i^t$  maps histories in sets of contracts, i.e.

$$\begin{aligned} s_i^t : H^t &\rightarrow \Omega \\ h^t &\mapsto \Omega_i^t \end{aligned}$$

We say that a contract pair  $(\omega_L, \omega_H)$  constitutes an equilibrium allocation if an equilibrium of the game exists where on the equilibrium path in each period the consumers of type  $L$  ( $H$ ) choose contract  $\omega_L$  ( $\omega_H$ ).

### 3 Equilibrium analysis

Define  $\Omega^{RS} = (\omega_L^{RS}, \omega_H^{RS})$ , as the RS contracts, where  $\omega_H^{RS}$  is the full insurance contract with fair premium for the high risks, and  $\omega_L^{RS}$  gives partial insurance for the low risks at their fair premium, and is such that high risks are indifferent between  $\omega_H^{RS}$  and  $\omega_L^{RS}$ .

Two assumptions are required for our equilibrium analysis. First, the discount rate  $\delta$  should be sufficiently large, so that deviating behavior from the RS contracts can be effectively punished in future periods. Second, the number of insurers should be sufficiently large, so that collusive agreements are not stable.

**Assumption 1.** (i)  $\delta > \frac{K}{K+1}$  with  $K$  being defined in Lemma 2 below.

(ii)  $n > \frac{1}{1-\delta}$ .

With these assumptions we can formulate our first proposition which shows that no pair of profit-making contracts and no pair of cross-subsidizing contracts can constitute an equilibrium allocation.

**Proposition 1.** *No pair of contracts  $(\omega_L, \omega_H) \neq \Omega^{RS}$  can constitute an equilibrium allocation.*

*Proof.* First we will consider the symmetric case, where all firms offer the same contracts in equilibrium. Suppose that for all firms  $i \in I$  and for all  $t$ ,  $(\omega_L, \omega_H) \in s_i^t$  where  $(\omega_L, \omega_H) \neq \Omega^{RS}$  and  $\omega_J = \bar{\omega}_J^t$ ,  $J \in \{H, L\}$  in equilibrium. There are four relevant cases, either both  $\omega_L$ , and  $\omega_H$  are individually profit-making, or  $\omega_H$  is profit-making, or  $\omega_L$  is profit-making, or both  $\omega_L$  and  $\omega_H$  are individually break even, but different from RS contracts. Suppose that the contract menu is profit making with profit  $\pi$  for the industry. Consider first the case where both contracts are profit-making. Then

a deviating insurer could undercut both contracts slightly in some  $t$ , and attract all types for one period. As both contracts are profit making, the deviating insurer will not face losses in the future. Thus, the deviating insurer makes a profit no less than  $\pi$ , while staying with the original contract gives a continuation profit of  $\frac{\pi}{(1-\delta)^n}$ . Given assumption (ii), it is then profitable to deviate in some  $t$ . If only  $\omega_H$  is profit-making with  $\omega_L$  zero profit or loss-making, i.e. cross-subsidization from high to low risks, a deviating insurer can slightly undercut  $\omega_H$  in  $t = 0$  without offering  $\omega_L$ . Thus the deviator only attracts the high risks on which profits of no less than  $\pi$  are made. As  $\omega_H$  is profit making, the deviating insurer will not face losses in the future. Again given assumption (ii), it is then profitable to deviate in  $t = 0$ .

Next consider the case where only the low risk type contract is profit making, while the high risk contract makes zero profit or a loss, i.e.  $(\omega_L, \omega_H)$  are cross subsidizing from low to high risks. Then a deviator could cream skim the low risks in  $t = 0$ , and would make a profit close to the profit of the low risks. As the contract of the high risks is zero profit or loss making, the profit accruing to the deviator will be larger than or equal to  $\pi$ . Given that the (loss-making) high risk contract remains on offer for all subsequent  $t$ , the lower bound on deviator profits for all subsequent periods is 0. With the same reasoning as above it then holds that deviation is profitable.

Furthermore, if  $\omega_L$  and  $\omega_H$  do individually break even, but are different from RS contracts, a deviating insurer could undercut both contracts slightly in some  $t$  such that the high risk type contract remains break-even, the low risk type contract makes a positive profit, and all types are attracted for one period. Again the lower bound on deviator profits for all subsequent periods is 0 and with the same reasoning as above it then holds that deviation is profitable.

Next consider asymmetric equilibria. Suppose that in equilibrium for all  $t$ ,  $(\omega_L, \omega_H) \in s_i^t$  for some firm  $i \in I$  where  $\omega_J = \bar{\omega}_J^t$ ,  $J \in \{H, L\}$  and  $(\omega_L, \omega_H) \neq \Omega^{RS}$ . The set of firms that offer the contracts which are bought by the consumers might well differ between the periods. Then for some firm  $i \in I$  and some  $\tilde{t}$ , firm  $i$ 's continuation payoff in  $\tilde{t}$  is lower or equal to the continuation payoff when all firms  $i \in I$  set  $(\omega_L, \omega_H) \in s_i^t$  for  $t > \tilde{t}$ . But then, by the reasoning as above, there always exists a profitable deviation.  $\square$

Thus any profit making pair of contracts cannot be equilibrium outcomes, as would be expected from a competitive market. This result is different from the analysis in EF, where profit making contracts could be sustained. In their model, insurers prevent undercutting by threatening to undercut the deviator. As consumers

only enter once all contract additions have been made, this makes deviation from profitable contracts unattractive.

Proposition 1 also shows that cross-subsidizing contracts, and in particular the second best efficient pair of contracts (the MWS contracts, Miyazaki, 1977, Wilson, 1977, Spence, 1978) cannot be equilibrium outcomes. Here the dynamics as already proposed by RS sets in: With cross subsidizing contracts, some insurers might find it profitable to offer contracts which only attract the profit making part of the policyholders, i.e. to cream skim the low risks.

With Proposition 1, we can proceed to show that we can concentrate on equilibria of the game where on the equilibrium path in each period the same menu of contracts is chosen by consumers.

**Lemma 1.** No sequence of contract menus  $\{(\omega_L^t, \omega_H^t)\}_{t=0}^\infty$  where for some  $t > 0$ ,  $(\omega_L^t, \omega_H^t) \neq (\omega_L^{t-1}, \omega_H^{t-1})$ , can constitute an equilibrium allocation.

*Proof.* In every period  $t$ , industry payoff is smaller or equal than the monopoly profit. Thus, there exists a supremum for per period profit. For any sequence of contract menus there exists a period  $\hat{t}$  where per period profit is within  $\epsilon$  of the supremum. With a similar argument as in proof of Proposition 1, there exists a profitable deviation in  $\hat{t}$ .  $\square$

While so far our results agree with the analysis by RS, our next proposition shows that in those cases where RS do not obtain an equilibrium, in our extended model an equilibrium with the RS allocation always exists.

**Proposition 2.** *The following strategies constitute a subgame perfect equilibrium:*

$t = 0$ : Each insurer  $i \in I$  offers the RS-contracts:  $s_i^0 = \Omega^{RS}$ .

$t > 0$ : If in  $t - 1$  only the RS contracts are offered, offer the RS-contracts again:  $s_i^t = \Omega^{RS}$ .

If in  $t - 1$  any contract  $\omega' \neq \omega_L^{RS}$  was offered which attracted low risk customers, offer in addition to  $\Omega^{RS}$  a contract  $\tilde{\omega}$  where  $\tilde{\omega}$  maximizes  $u^L(\tilde{\omega})$  subject to  $u^H(\omega') \geq u^H(\tilde{\omega})$  and  $\pi^L(\tilde{\omega}) \geq 0$ :  $s_i^t = \Omega^{RS} \cup \{\tilde{\omega}\}$ .

In equilibrium all insurers offer the RS contracts and make zero profits. If someone deviates, the other insurers will cream skim the deviator. Cream skimming is such that the low risks obtain the best possible contract which is not attractive for the high risks and which does not lead to a loss for the insurer. Thus by construction, further cream skimming will not be possible.

*Proof.* As the game is continuous at infinity, we can resort to the one-stage-deviation-principle to prove the proposition (Fudenberg and Tirole, 1991, p. 109 ff.). Therefore it is sufficient to analyse two cases: First, on the equilibrium path, where an insurer contemplates to make a different offer than the RS contracts. Second, off the equilibrium path, where an insurer contemplates to make an offer different from the RS contracts jointly with  $\tilde{\omega}$ .

Deviation on the equilibrium path is only possible by attracting both high and low risks. The equilibrium strategies are such that low risks will then be cream skimmed, and the deviator will be stuck with the high risks. Thus the optimal deviation will be such that the high risks obtain full insurance, which minimizes the costs the deviator has with the high risks for any given level of high risk type utility. Furthermore, the deviation pair of contracts will be such that low risks do not obtain more utility than they receive with the RS contracts, in order to maximize the profit with the low risk types. We can thus classify the set of potential deviation contract menus by  $\{(\omega_H(\Delta), \omega_L(\Delta))\}$ . For each  $\Delta$  the high risk contract specifies full insurance and a premium below the fair premium.

$$\omega_H(\Delta) = (p_h m - \Delta, m)$$

Thus the (negative) profit the insurer makes with this contract is given by  $-\Delta$ . The low risks contract  $\omega_L(\Delta)$  is such that high risk types are indifferent between their deviating contract and the contract designed for the low risks, i.e.  $u^H(\omega_H(\Delta)) = u^H(\omega_L(\Delta))$  (IC constraint), and the low risks are indifferent between their contract and the RS contract, i.e.  $u^L(\omega_L(\Delta)) = u^L(\omega_L^{RS})$  (PC constraint). A natural upper limit for  $\Delta$  is when the contract (pair) provides full insurance for both types at the fair pooling premium. Thus  $0 \leq \Delta \leq \Delta^*$  with  $\Delta^* = (p_H - (\gamma p_H + (1 - \gamma)p_L)m$ .

Denote by  $\pi^{HL}(\Delta)$  the profit an insurer makes if all high risks buy contract  $\omega_H(\Delta)$  and all low risks buy contract  $\omega_L(\Delta)$ . All low risks are cream skimmed after one period by the other insurers such that the deviator is stuck with the high risks, which will lead to a loss of  $\gamma\Delta$ . Thus the overall profit of a deviator, who offers the contract pair  $(\omega_L(\Delta), \omega_H(\Delta))$  is given by:

$$\pi^D(\Delta) = \pi^{HL}(\Delta) - \frac{\delta}{1 - \delta} \gamma \Delta$$

The following lemma is necessary to show that with  $\delta$  being large enough (Assumption i), deviation is not profitable.

**Lemma 2.** There exists a  $K$  such that for all  $\Delta \in [0, \Delta^*]$  it holds:

$$\frac{\pi^{HL}(\Delta)}{\gamma\Delta} < K.$$

Obviously, for any finite  $\Delta$  we can find a  $K$  large enough such that the inequality is satisfied. So we only have to consider the case  $\Delta \rightarrow 0$ . In that case, both numerator and denominator go to zero. Applying L'Hospital's rule gives a finite value for the expression, as the derivative of the denominator is equal to  $\gamma$ , while the derivative of the numerator is finite. Thus we can find a  $K$  large enough such that the inequality holds.

Finally we need to show that if contracts different from the RS contracts are offered (and bought) in period  $t - 1$ , then it is indeed optimal for any insurer to offer the RS contracts  $\Omega^{RS}$  and to cream skim the low risks (with contract  $\tilde{\omega}$ ) in period  $t$ . There are two cases to consider. First, suppose that after insurers made their offers, low risks buy the contract  $\tilde{\omega}$ . By construction of  $\tilde{\omega}$ , this contract is zero profit making, high risks are just indifferent between their contract and  $\tilde{\omega}$ . Thus as all other insurers offer this contract, a single insurer can offer no different contract which would make a profit and which not be bought by the high risks. Second, suppose that all policyholders buy the RS contracts, which would be the case if  $\tilde{\omega}$  is not attractive for the low risks compared to the RS contract. Then we are back to the equilibrium analysis above and again, deviation is unprofitable.  $\square$

Propositions 1, 2 and Lemma 1 lead to our main result:

**Corollary 1.** *The Rothschild-Stiglitz contracts are the unique equilibrium allocation in any pure-strategy equilibrium.*

## 4 Conclusion

Riley (1979) proposed the concept of a reactive equilibrium to tackle the equilibrium non-existence problem in competitive markets with adverse selection (Rothschild and Stiglitz, 1976). Engers and Fernandez (1987) gave a game-theoretic underpinning for the Riley concept, assuming that insurers can add contracts repeatedly in the market. However in their analysis many allocations can be sustained as equilibrium outcome thus replacing the problem of equilibrium non-existence with that of multiplicity. We include active consumers to the model of Engers and Fernandez by assuming that after each round of contract additions, a group of privately informed consumers

enters the market and chooses among the set of contracts. It is shown that then the Rothschild-Stiglitz contracts are the unique equilibrium allocation in any pure-strategy equilibrium, as originally proposed by Riley (1979).

## References

- Engers, M., Fernandez, L., (1987): Market equilibrium with hidden knowledge and self-selection. *Econometrica* 55 (2), 425–439.
- Fudenberg, D. and J. Tirole (1991): Game Theory, Cambridge (Mass.): MIT-Press
- Mimra, W. and A. Wambach (2011): A game-theoretic foundation for the wilson equilibrium in competitive insurance markets with adverse selection, CESifo Working Paper no. 3412.
- Mimra, W. and A. Wambach (2014): New developments in the theory of adverse selection in competitive insurance, *The Geneva Risk and Insurance Review*, 39 (2), 136-152.
- Miyazaki, H., (1977): The rat race and internal labor markets, *The Bell Journal of Economics* 8 (2), 394–418.
- Riley, J. G., (1979): Informational equilibrium, *Econometrica* 47 (2), 331–359.
- Rothschild, M., Stiglitz, J., (1976): Equilibrium in competitive insurance markets: An essay on the economics of imperfect information, *The Quarterly Journal of Economics* 90 (4), 629–649.
- Spence, M., (1978): Product differentiation and performance in insurance markets, *Journal of Public Economics* 10 (3), 427–447.
- Wilson, C., (1977): A model of insurance markets with incomplete information, *Journal of Economic Theory* 16 (2), 167–207.

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