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**Working Paper**

## Efficiency versus Robustness of Markets - Why improving market efficiency should not be the only objective of market regulation

EWL Working Paper, No. 4 [01/09]

**Provided in Cooperation with:**

University of Duisburg-Essen, Chair for Management Science and Energy Economics

*Suggested Citation:* Weber, Christoph (2009) : Efficiency versus Robustness of Markets - Why improving market efficiency should not be the only objective of market regulation, EWL Working Paper, No. 4 [01/09], University of Duisburg-Essen, Chair for Management Science and Energy Economics, Essen

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Chair for Management Sciences and Energy Economics  
University of Duisburg-Essen

EWL Working Paper No. 4

**EFFICIENCY VERSUS ROBUSTNESS OF MARKETS**  
**– WHY IMPROVING MARKET EFFICIENCY SHOULD NOT BE**  
**THE ONLY OBJECTIVE OF MARKET REGULATION**

by

*Christoph Weber*

May 2009

# **Efficiency versus Robustness of Markets**

## **– Why improving market efficiency should not be the only objective of market regulation**

by Christoph Weber

### **Abstract**

The efficiency of capital markets has been questioned almost as long as the efficient market hypothesis had been worked out. Numerous critics have been formulated against this hypothesis, questioning notably the behavioural assumptions underlying the efficient market hypothesis. The present contribution does not focus on the behavioural assumptions but rather looks at the implications of focusing purely on the objective of market efficiency when considering market design questions. Hence it aims at discussing the following, possibly rather fundamental issue: Is the objective of efficiency, which has guided most of the market reforms in the last decades, sufficient? Or has it to be complemented by the objective of robustness?

Mathematical and engineering control theory has developed the concept of robust control (e.g. Zhou and Doyle, 1998) and it has been shown that there is always a trade-off between the efficiency of a control system and its robustness (cf. e.g. Safonov, 1981, Doyle et al., 1988). The efficiency of the system describes its reactions to disturbance signals. The lower the integral loss function over the so-called transfer or sensitivity function, the less a system is affected by disturbances such as demand fluctuations, and the more efficient is the control. The economic equivalent clearly is the maximisation of welfare, which results in an efficient economic system. Robustness by contrast is defined as stability of the control system in the presence of model uncertainty (deviations in the model parameters or misperceptions of the underlying system).

These concepts are applied to the financial markets in their interaction with the real economy. The financial markets being understood as the controllers of real world activity through investments, the implications of misperceptions in the financial sphere are analysed both theoretically and in an application example. From the theory it may readily be derived that financial markets providing efficient, i.e. welfare-optimal solutions, must have limitations with respect to robustness. Also in the application example it turns out that in the presence of potential misperception a reduction of irreversible cost shares in investments may lead to an increase in overall expected system costs. Hence improvements in (conventional) market efficiency may be counter-productive by facilitating misallocation of capital as a consequence of misperceptions in the financial markets. This leads to the conclusion that a sole focus on the efficiency objective in market design is problematic and some of the recent turmoil in financial markets may be explained by the lack of consideration given to robustness issues.

*Keywords : market efficiency, robustness, optimal control, stochastic dynamic growth*

*JEL-Classification : E20, G14, Q40*

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*The authors are solely responsible for the contents which do not necessarily represent the opinion of the Chair for Management Sciences and Energy Economics.*

## 1 Introduction

The efficiency of capital markets has been questioned almost as long as the efficient market hypothesis had been worked out and popularised in the beginning of the 1970s, cf. notably Fama (1970), Malkiel (1973). Criticisms have addressed the formalisation of the concept (notably LeRoy 1976) and the simplifying assumptions, particularly on the absence of information costs (Grossman and Stiglitz 1980). But the by far most numerous critiques focused on the idealising if not unrealistic behavioural assumptions underlying the theory and proposed alternative market models, leading to the establishment of an alternative school of thought in finance, namely behavioural finance (cf. e.g. Thaler 1993, Shleifer 2000). The proponents have identified several anomalies in stock markets, like overreaction to new information (De Bondt and Thaler 1985) or mean-reversion (Poterba, Summers 1988, Lo, MacKinlay 1988). These hypotheses and the empirical results used to underpin their validity have remained controversial (e.g. Fama 1991, Fama 1998, Malkiel 2003). Yet even proponents of the efficient market hypothesis have come to admit that observed market valuations may turn out to be erroneous in an ex post perspective (Malkiel 2003, p. 61).

However recent turmoil on world financial markets as well as previous crises may give lead to a more fundamental questioning: Is the objective of market efficiency itself sufficient – sufficient notably when dealing with issues of market design and market regulation? Obviously this question has a normative dimension. Yet for most economists the first-hand answer will probably be yes – the reasoning behind being that information efficiency will lead to efficient allocation of scarce (capital) resources and hence to allocative or Pareto efficiency, which is the best contribution of markets we can expect when aiming at welfare maximization<sup>1</sup>. For people outside economics things are often less clear-cut. They notably question whether the overall objective of economic welfare maximization fully describes societal objectives and they fear destabilization as a consequence of increased reliance on market mechanisms and notably advanced, global financial markets<sup>2</sup>. The first issue is related to the normative dimension mentioned above. It has been taken up by welfare economics making the distinction between (general) welfare and economic welfare (e.g. Pigou 1920) and stressing the separability of allocation and distribution issues in the fundamental welfare theorems (e.g. Mas-Colell et al. 1995). Taking these economic propositions as given, still the second concern remains. Within

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<sup>1</sup> This reasoning is for example touched upon in the introduction of Williams (2005). Also Beechey et al. (2000, p. 6) state “The link between an asset market that efficiently reflects available information (at least up to the point consistent with the cost of collecting the information) and its role in efficient resource allocation may seem natural enough”. However already Stiglitz (1980) provides evidence that information efficiency neither is a necessary nor a sufficient condition for Pareto efficiency of financial markets - his major arguments being based on the assumption of costly information.

<sup>2</sup> One prominent voice articulating these criticisms internationally is the ATTAC network ([www.attac.org](http://www.attac.org)), founded originally following a plea for an international Tobin tax in a journal editorial (cf. Ramonet 1997).

economics it has been taken up by Keynes (1936), in what later has been labelled the “castle-in-the-air” theory of investment, and later by various authors developing models of bubbles in stock and other financial markets. Theories and experiments on bubble formation have now been derived based on various formulations, describing and explaining bubbles notably based on endogenous formation of expectations (Smith et al. 1988), concepts of mimetic contagion (Topol 1991), intrinsic formation due to dividend policy (Froot and Obstfeld 1991), rational behavior of overlapping generations (Tirole 1985), agency problems for financial intermediaries (Allen and Gale 2000), interaction between different types of agents (e.g. Day and Huang 1990) or agents switching between different sets of predictors (e.g. Brock and Hommes 1997), herding behaviour (e.g. Brunnermeier 2001, Sornette 2002) and cognitive biases like overconfidence (e.g. Daniel et al. 1998, Daniel et al. 2001). These papers provide a broad variety of potential explanations for stock market bubbles. Closely related are also papers addressing the topic of “excess volatility” (e.g. Shiller 1981, LeRoy and Porter 1981) and the possibility of long-term deviation of stock prices from fundamental values (e. g. Summers 1986).

Facing these criticisms the efficient market proponents on the one hand tend to emphasize that even if these anomalies are empirically observable, they are not durably exploitable by investors<sup>3</sup>. Hence the assumption of information efficiency in the weaker understanding that all information that is exploitable is exploited by market participants is still intact. Another line of argumentation stresses that even under such alternative settings regulatory interference into the market, e.g. through the introduction of a Tobin or transaction tax, is far from improving market outcomes with certainty – consequently efficiency of the market mechanism in the sense of being the best of all envisageable coordination mechanisms is not refuted. E.g. Dow and Rahi (2000) and Subrahmanyam (1998) describe models with different types of traders allowing for the analysis of welfare effects due to the introduction of a transaction cost and they find mixed evidence on the overall effects. Empirically, Hau (2006) shows that an increase of transaction costs at the Paris stock exchange has led to an increase in volatility and thus not contributed to the expected market stabilisation.

Yet obviously the first refutation primarily focuses on informational efficiency and not on allocative efficiency and the second only shows that government intervention may provide more harm than good, at least when taking short-term volatility as stability indicator. But both arguments do not consider dynamic allocative efficiency in the long-run and they neither consider the risk of instability.

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<sup>3</sup> Fama (1998) for example emphasizes that different behavioural models predict alternatively overreaction and underreaction to new information. If markets are efficient, both will occur with similar probabilities, so that the investors cannot make any ex-ante predictions on which phenomenon will prevail.

Both issues have been also been repeatedly discussed in the past<sup>4</sup>, however a combined comprehensive treatment is so far missing. The present paper proposes to use concepts from dynamic control theory to provide an integrated assessment of both aspects. Thereby the stochastic and robust control theory (e.g. Zhou and Doyle 1998) is used as framework for analysis, given that it allows dealing with issues of efficiency and stability in the presence of noise and misspecification<sup>5</sup>. The concepts of robust control theory have been exploited increasingly during the last years in macroeconomics and finance following the pioneering work of Hansen and Sargent (e.g. Hansen, Sargent 1995, Hansen, Sargent 2001, Hansen, Sargent 2003, Hansen, Sargent 2007). While the present analysis has many formal similarities with the works of Hansen and Sargent, the intention of the analysis and the implications derived are considerably different. Hansen and Sargent on the one hand are interested in deriving equilibrium conditions in the presence of fears of misspecification (e.g. Cagetti et al. 2002, Hansen, Sargent 2005, Hansen, Sargent 2007a). On the other hand they use the robust control approach to derive recommendations for individual agents, notably in the field of monetary policy (e.g. Hansen, Sargent 2003, Hansen, Sargent 2007), i.e. they see the central bank in the role of controller facing considerable parameter uncertainty.

In the analysis at hand the financial markets and the agents on these markets, namely the financial intermediaries, are viewed as the controller because they allocate capital to different sectors of the economy and thus control investment in these sectors. In this view, the secondary asset markets like the stock market are included in the controller, where as the system consists of the real economy. The intervention of the controller in the system in this perspective corresponds to the allocation of savings of the representative consumer to those sectors in the economy where they are most beneficial.

The approach chosen allows descriptive insights in the behaviour of the overall system in the presence of (potential) misperception. I.e. the trade-off between robustness and efficiency, which exists in any approach to market design for financial markets is demonstrated both theoretically and through an illustrative example.

Correspondingly the present paper is complementary to earlier research on the formation of bubbles and crashes. In the stream of literature cited above, the focus is on identifying mechanisms explaining the emergence of bubbles – be it through expectation formation, herding behaviour, cognitive biases etc. All these mechanisms may contribute to the formation of distorted valuations or misperceptions of the real economy in the fi-

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<sup>4</sup> E.g. Abel et al. (1997) investigate theoretically and empirically the dynamic efficiency of the U.S. and other industrialized economies, taking up earlier analyses by Phelps (1961) and Diamond (1965). Cagetti et al. (2002) address the issue of dynamic efficiency under uncertainty yet they do not consider issues of stability.

<sup>5</sup> A profound discussion of the role of noise for finance and economics is provided already by Black (1985). The potential effect of noise trading on financial market stability is investigated by De Long et al. (1990)

nancial sphere aiming at controlling it. In the following we do not focus on the emergence of these misperceptions but rather on their consequences in the dynamic interaction between system (real economy) and controller (financial markets and agents).

The remaining of the paper is organised as follows: first a stochastic dynamic growth model is specified as the macroeconomic, welfare theoretical framework for analysis in section I. Then optimal control and robustness are introduced in this context in section II. Section III provides an application to a small multi-sector model. In Section IV the implications for market design are discussed whereas section V concludes and provides indications for further research.

## 2 Stochastic dynamic growth

We consider an economy with consumer preferences represented by the maximization of the utility function of a representative consumer

$$U = E \left[ \sum_{\beta=1}^{+\infty} \beta^t u_t(s_t, b_t, l_t) \right] \quad (1)$$

under the following restrictions:

$$s_t = S(c_t, h_t) \quad (2)$$

$$h_t = H(h_{t-1}, c_t) \quad (3)$$

$$\Phi_c c_t + \Phi_g g_t + \Phi_i i_t = G(k_{t-1}, d_t) \quad (4)$$

$$k_t = K(k_{t-1}, i_t) \quad (5)$$

$$l_t = V(g_t) \quad (6)$$

$$\begin{pmatrix} b_t \\ d_t \end{pmatrix} = W(z_t) \quad (7)$$

$$z_t = Z(z_{t-1}, \varepsilon_t) \quad (8)$$

$s_t$  thereby describes household services, produced by the representative household using consumption goods  $c_t$  and household production stock  $h_t$  combined through the household production function  $S$ <sup>6</sup>.  $s_t$ ,  $c_t$  and  $h_t$  as well as the following variables are thereby vectors of flexible size. The household production stock follows a capital depreciation and replacement relation  $H(h_{t-1}, c_t)$  analogous to the equation governing capital stock  $k_t$  evolution using the function  $K(k_{t-1}, i_t)$ . Market clearing is described through equation (4). Thereby besides consumption goods  $c_t$  and investment goods  $i_t$  also intermediate goods  $g_t$  appear, which are notably used to model adaptation costs through

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<sup>6</sup> Examples of household services include mileage produced by driving one's own car, or living in a self-owned home. In an energy economics context these correspond to the concept of energy services. Another motivation to their introduction may also be a desire to cope with shifting, recursive household preferences (cf. Hansen, Sargent 2005).



equation (6). The corresponding cost term then enters the utility function as indicated in equation (1). Furthermore, stochastic preference shocks  $b_t$  are included in equation (1) as well as stochastic shocks  $d_t$  in endowments respectively production appear in equation (4). These shocks are related to i.i.d. errors  $\xi_t$  through equations (7) and (8).

Under appropriate convexity conditions a unique competitive equilibrium exists for each information state  $J_t$ . In order to facilitate the further treatment, the constraints are linearized around the equilibrium, whereas for the utility function a quadratic approximation is chosen<sup>7</sup>. This yields a linear-quadratic problem, in which the losses compared to the equilibrium stage are minimized

$$\min E \left[ \sum_{t=1}^{+\infty} \beta^t \left( (s_t - b_t)'(s_t - b_t) + l_t^2 \right) \middle| J_0 \right] \quad (9)$$

$$\text{s.t.} \quad s_t = \Lambda h_t + \Pi c_t \quad (10)$$

$$h_t = \Delta_h h_{t-1} + \Theta_h c_t \quad (11)$$

$$\Phi_c c_t + \Phi_g g_t + \Phi_i i_t = \Gamma k_{t-1} + d_t \quad (12)$$

$$k_t = \Delta_k k_{t-1} + \Theta_k i_t \quad (13)$$

$$l_t^2 = g_t' g_t \quad (14)$$

$$\begin{pmatrix} b_t \\ d_t \end{pmatrix} = \begin{pmatrix} U_b \\ U_d \end{pmatrix} z_t \quad (15)$$

$$z_t = A_{22} z_{t-1} + \varepsilon_t \quad (16)$$

This model formulation matches the one used by Hansen, Sargent (2005, p. 250ss.). Accordingly, this model can be formulated as a standard discounted stochastic optimization problem:

$$\min E \left[ \sum_{t=1}^{+\infty} \beta^t (x_t' u_t)' \begin{pmatrix} Q & W' \\ W & R \end{pmatrix} \begin{pmatrix} x_t \\ u_t \end{pmatrix} \right] \quad (17)$$

$$\text{s.t.} \quad x_{t+1} = Ax_t + Bu_t + \Xi \xi_t \quad (18)$$

Thereby the state vector  $x_t$  contains all variables with dynamic equations, namely household capital stock  $h_t$ , other capital stock  $k_t$  and auto-correlated shocks  $z_t$ :

$$x_t = \begin{pmatrix} h_{t-1} \\ k_{t-1} \\ z_t \end{pmatrix} \quad (19)$$

From an economic perspective, control of the system is mainly possible through the adjustment of investments. Correspondingly the control vector  $u_t$  has to be set to:

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<sup>7</sup> A similar approach is e.g. also chosen by Kydland, Prescott (1982).

$$u_t = i_t \quad (20)$$

When aiming at an optimal control of the system, usually not all the state variables are directly observable. Moreover even for the observable variables, measurement error may occur. E.g. the capital stock of firms is observed through their balance sheets, yet those may be influenced by valuation idiosyncrasies or reporting date effects. Therefore the optimal control problem as stated in equations (17) and (18) has to be supplemented by an observation equation:

$$y_t = Cx_t + Du_t + \Omega\omega_t \quad (21)$$

This stochastic discounted control problem may be transformed into a standard stochastic control problem – a so-called Linear Quadratic Gaussian (LQG) problem – using the transformations  $\tilde{x}_t = \beta^{1/2}(x_t - R^{-1}Wu_t)$  and  $\tilde{u}_t = \beta^{1/2}u_t$ <sup>8</sup>. The standard stochastic control problem has the form:

$$\min E \left[ \sum_{\beta=1}^{+\infty} \tilde{x}_t' \tilde{Q} \tilde{x}_t + \tilde{u}_t' R \tilde{u}_t \right] \quad (22)$$

$$\text{s.t.} \quad \tilde{x}_{t+1} = \tilde{A} \tilde{x}_t + \tilde{B} \tilde{u}_t + \tilde{\Xi} \xi_t \quad (23)$$

$$\tilde{y}_t = \tilde{C} \tilde{x}_t + \tilde{D} \tilde{u}_t + \Omega \omega_t \quad (24)$$

Standard conditions for the existence of a well-defined solution require notably the matrix  $\tilde{Q}$  to be positive semi-definite and the matrix  $R$  to be positive definite. Moreover conditions of observability and controllability are required, which will be discussed in the following where relevant.

### 3 Optimal control and robustness

In the presence of a control problem as summarized through equations (22) to (24), conventional control theory aims at determining the most efficient regulator  $K$ , i.e. the feedback law from outputs  $\tilde{y}_t$  to controls  $\tilde{u}_t$ , which minimizes the quadratic loss function stated. This corresponds to seeking for the market design ensuring maximum efficiency. A graphical representation of the system and the regulator is given in Figure 1.

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<sup>8</sup> Cf. Hansen, Sargent (2005), pp. 60 ss.

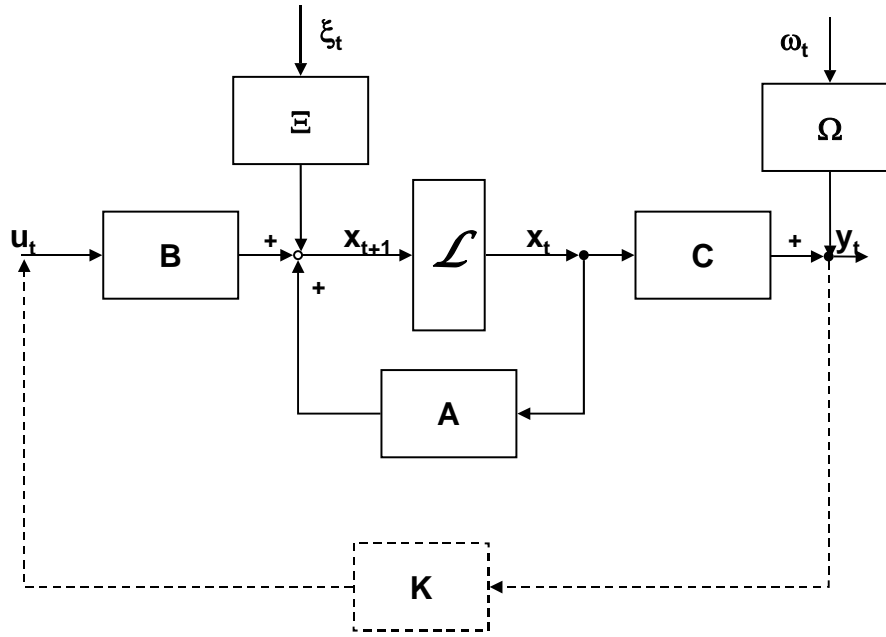


Figure 1: Control system in state space representation (A, B, C) with regulator K

In control theory it is well-established that the optimal regulator consists of the combination of the optimal Kalman-Bucy filter and the optimal linear regulator applied to the standard LQG-problem with observable state-vector  $\tilde{x}_t$  (i.e. consisting of equations (22) and (23) only)<sup>9</sup>. The Kalman-Bucy filter enables the (as good as possible) identification of the actual system state in a so-called observer system, which replicates the original system. The observer thus represents the model, which the controlling instance (the “invisible hand” in the economic framing of Adam Smith) has of the reality it is to control. From the observer state then the optimal feedback is generated using the same feedback law, which would also be applied if the states  $\tilde{x}_t$  were directly observable (cf. Figure 2).

If the system is observable, i.e. when the state vector  $\tilde{x}_t$  can be recovered (approximately) from observing the output, the optimal Kalman filter  $K_f$  can be obtained by solving the algebraic Riccati equation:

<sup>9</sup> Cf. e.g. Stengel (1994), pp. 451 ss.

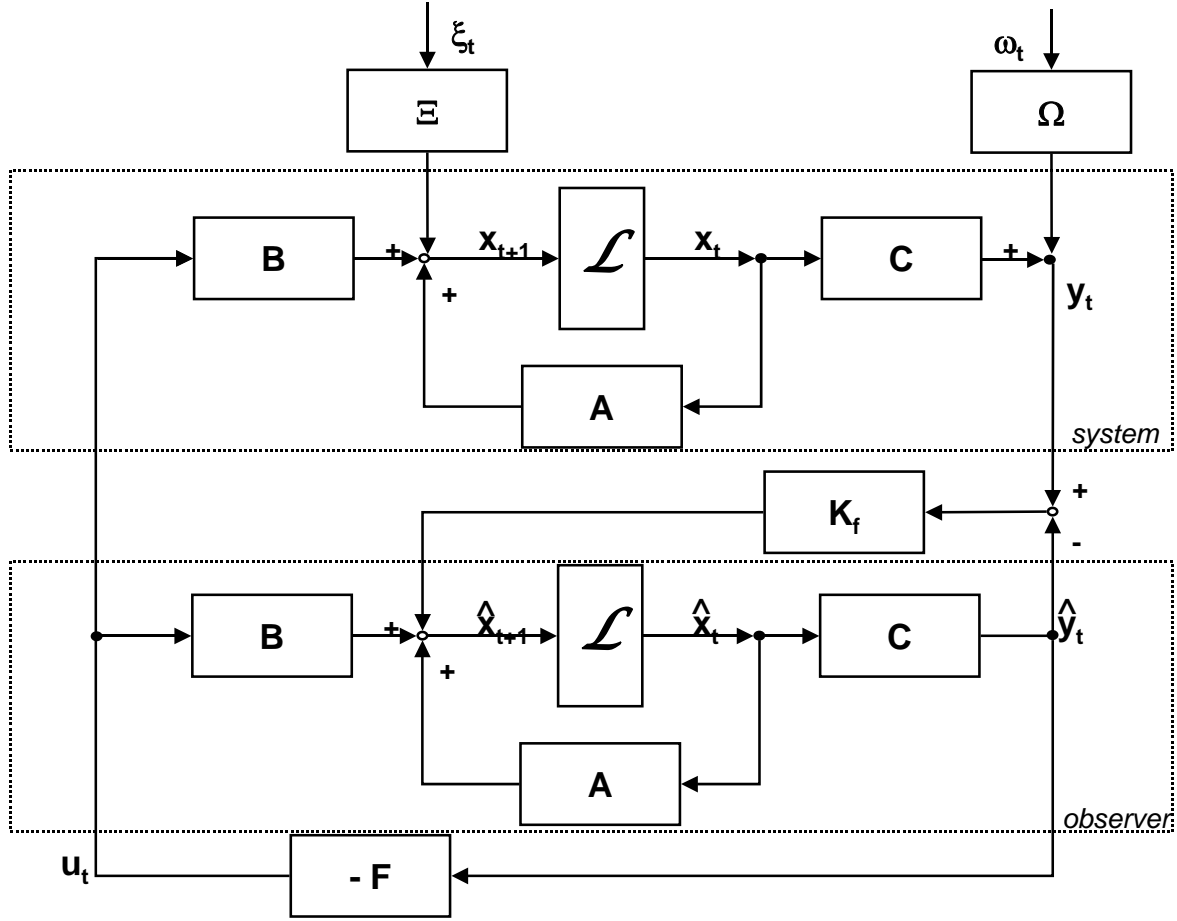


Figure 2: Control system with Kalman filter, observer and optimal feedback

$$P_f = AP_f A' + BB' - AP_f C' (CP_f C' + \Omega \Omega')^{-1} CP_f A' \quad (25)$$

for the unknown matrix  $P_f$  and then using the relationship

$$K_f = -AP_f C' (CP_f C' + \Omega \Omega')^{-1} \quad (26)$$

to determine the Kalman filter, which is found to be a constant matrix, if all the other matrices involved are also time-independent.

The optimal regulator using state information  $\tilde{x}_t$  is also found to be a constant feedback matrix  $F$ :

$$\tilde{u}_t = -F \tilde{x}_t \quad (27)$$

The matrix  $F$  can be determined again by solving a Riccati equation, if the system under study is controllable<sup>10</sup>. This means that all components of the state vector  $\tilde{x}_t$  can attain any arbitrary value starting from an initialization of 0 in a finite number of time periods through appropriately chosen controls. This condition is clearly not fulfilled here, given that the state vector  $\tilde{x}_t$  contains the exogenous, not controllable disturbances  $z_t$  (cf.

<sup>10</sup> Cf. Stengel (1994), pp. 139 ss.

equations (16) and (19)). For this kind of control problems optimal, stabilizing solutions can be established following the approach discussed by Hansen and Sargent (2005, pp. 65ss.). The result is a unique optimal feedback matrix  $F$ , which minimizes the optimization criterion (24).

Hence an overall optimal regulator for the LQG is established as a combination of a Kalman filter, an observer and a linear feedback law. This optimal regulator may be characterized by its (matrix) transfer function  $G_R(s)$  in the frequency domain<sup>11</sup>. The transfer function notably indicates the amplification and phase shifting for any sinusoidal input signal  $u(j\omega)$ . Similarly the original system is characterized by its transfer function  $G_S(s)$ , which is equal to  $C(sI - A)^{-1}B$ . The transfer function for the closed loop system is then the so-called sensitivity function  $S(s)$ , which can be shown to equal

$$S(s) = (I + G_S G_R)^{-1} \quad (28)$$

This function describes the response of the closed-loop system to disturbances in the frequency domain and a minimization of the so-called  $H_2$ -Norm of  $S(s)$  can be shown to be equivalent to the solution of the LQG-problem defined above (e.g. Weber (1989)).

Unfortunately, modern control theory shows that this optimization comes at some expense. The robustness of the closed-loop system with respect to variations in the parameters of the original system can be shown to be equivalent to a limitation of the so-called complementary sensitivity function  $T(s)$ <sup>12</sup>:

$$T(s) = G_S G_R (I + G_S G_R)^{-1} \quad (29)$$

Given that the following identity holds

$$T(s) + S(s) = I \quad (30)$$

a minimization of the sensitivity function  $S$  – which corresponds in economic terms to a maximization of efficiency – will always have to be traded-off against a reduction in robustness, i.e. a loss in performance in the presence of modelling errors. That this trade-off is not purely theoretical is illustrated in the following sections using a specific application example for the model defined by equations (9) to (16).

## 4 Application

### 4.1 Model example

In the application example, we focus on issues of robust control in the energy sector (or more precisely the power sector) of an economy. Several reasons may be invoked for

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<sup>11</sup> Cf. Stengel (1994), pp. 151 ss.

<sup>12</sup> cf. e.g. Safonov (1981)

choosing this example: Primarily energy supply is a vital function for any modern society and failures in capital allocation in this field may induce through blackouts or other supply interruptions serious consequences for almost any other production and consumption process. Secondly, typical assets in the energy field like power plants, refineries or oil production platforms are long-lived and their “second-best-use” value is rather low. Consequently, investments in the field have to be considered mostly as being irreversible, leading to sunk cost which give rise to adaptation costs in the case of changing framework conditions. Thirdly, adaptation is further hampered by the fact that “time-to-build” for new production facilities, be it power plants, refineries or new oil platforms, is substantial. Exploration and development of new oil fields takes around ten years and also planning, approval procedures and construction of coal-fired power plants require around five years. Lastly, the aspect of household capital stocks and household production processes is of particular importance in the energy field, since many energy efficiency improvements like improved insulation, energy-saving light bulbs or higher efficiency cars involve in fact household investment into new, more efficient capital stock.

Hence we consider an economy, where utility to households is provided on the one hand through one (aggregate) energy service and on the other hand through one aggregate numeraire consumption good. Consequently the household service vector  $s_t$  comprises two elements<sup>13</sup>. The vector of consumption goods  $c_t$  encompasses the (aggregate) household appliance necessary to produce the energy service, the numeraire good and energy (taken to be peak electricity). Correspondingly the household production stock  $h_t$  includes only one component, the aggregate household appliance. Four types of capital goods are distinguished, one for the production of each consumption good plus one variable for power plants under construction. Hence the time to build is modelled in the case of power plants, similarly to the approach chosen by Kydland and Prescott (1982). Preferences shocks on energy services may occur in different forms – as momentary shocks (e. g. one year weather outliers), as permanent shocks (shift in preferences towards energy services) or as mean-reverting shocks (a sudden rush towards energy services, which is gradually absorbed over time). For the aggregate consumption good, only mean-reverting shocks are considered, as is done for shocks in production endowments (e.g. dry year for hydro power plants). The full description of the matrices characterising the system are available from the author upon request. The numerical values have been chosen to provide a rough approximation to the German situation, measuring energy in MWh, power plant and household production capacity in MW. The remaining quantities are measured in Mill. €, also the household services (and thus utility) are expressed in money-metric terms. Moreover a duration of 100 h is assumed for the considered (super-)peak period. Since the very peaking period is analysed, a (relatively) large disutility is associated to deviations between supply and demand in the case of the

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<sup>13</sup> The full detail of the matrices describing the model is given in the Annex.

energy service, given that this would mean supply interruption<sup>14</sup>. The size of the shocks has been chosen to represent around 1 % of the actual consumption / production.

For the state observation (matrix  $C$ ) all states are assumed to be observable (subject to some measurement noise), except the different preference shocks for energy services, where only the aggregate sum is observable.

A point of interest is on the choice of the quadratic matrix  $R$  in the objective function. It corresponds to the loss in utility associated with activating controls, i.e. investments. If investments were perfectly reversible - e.g. through liquid asset markets -, investments would reduce utility at time  $t$  only in as far as they reduce the consumption of the numeraire good in this period. In this case shifts between investment categories such as from power plants to numeraire good production would come at no utility loss. However this is unrealistic and moreover leads to mathematical problems. It is unrealistic since new information which e.g. reveals that power production (and consequently power plants) is less valuable will lead to a drop in asset values and somebody, namely the asset owner at the moment of information revelation, will have to bear the costs of this devaluation. Mathematically, the absence of irreversible disutility associated to investment would imply a matrix  $R$  which is singular. Yet in the standard formulation of the LQG problem,  $R$  is always assumed to be positive definite. Therefore a quadratic adaptation cost approach is implemented here, following e.g. Kydland and Prescott (1982), Ljungqvist and Sargent (2004) or Hansen and Sargent (2005). In the economic formulation chosen in equations (9) to (16), this corresponds to the introduction of intermediate goods  $g_t$ , which are multiples of the investment vector  $i_t$  and enter the objective function through the constraint (14).

## 4.2 Results

In order to understand the system behaviour and the impact of misperceptions in the controller, first the response to exogenous shocks is investigated for the system. In a first step this so-called disturbance response is considered for the original system controlled using an adequate Kalman-Filter and observer system. Then the control through a biased observer system is looked at and finally the exemplary results are generalised.

### 4.2.1 Disturbance response for the optimally controlled original system

The response of the optimally controlled system to external shocks can be depicted in several ways. In the following, the response to a disturbance impulse is depicted in the time domain. As an example the response to an autoregressive preference shock is taken, i.e. the preference for household energy services is suddenly increased at time 1,

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<sup>14</sup> Large disutility could possibly be avoided, if flexible customer response or interruptible load programs were implemented – Yet this is not the case at large scale in the current German electricity system.

afterwards this shock declines geometrically. The response for the different components of the components of the utility function and of the state vector is depicted in Figure 3. The other disturbances are thereby set to zero throughout.

The system as described above has one unit root, due to the permanent preference shock, which has been taken into account for energy services. Otherwise both the uncontrolled and the controlled system are stable, i.e. the responses to external shocks fade over time, unless new disturbances appear.

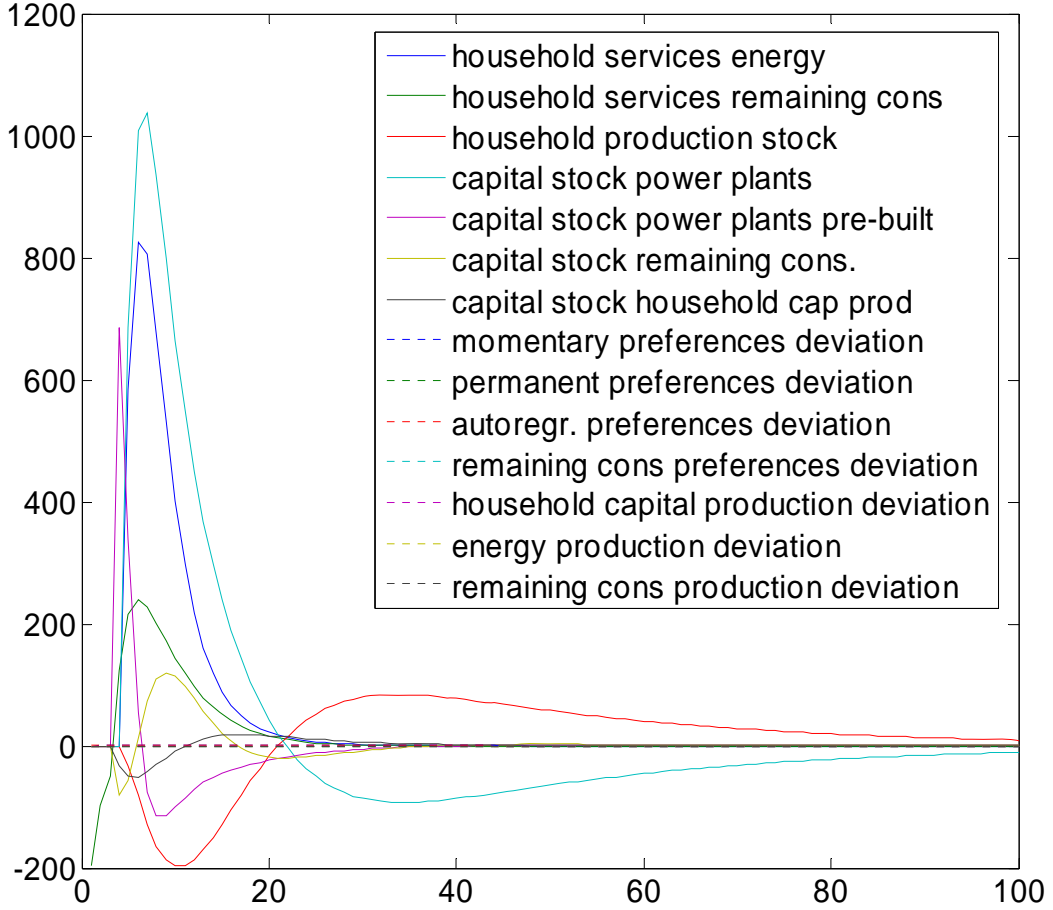


Figure 3: Response of the original system to an autoregressive energy service preference shock

#### 4.2.2 Disturbance response in case of optimal control combined with system misperception

In order to make explicit the role of robustness, the changes in the system response to external shocks may be investigated for a setup, where the optimal regulator as determined before is regulating a system which does not exactly match the assumptions underlying the design of the controller. In this case there is misperception of the system to



be controlled, or put in the language of the classics, the “invisible hand” is guided by a “squinting eye”.

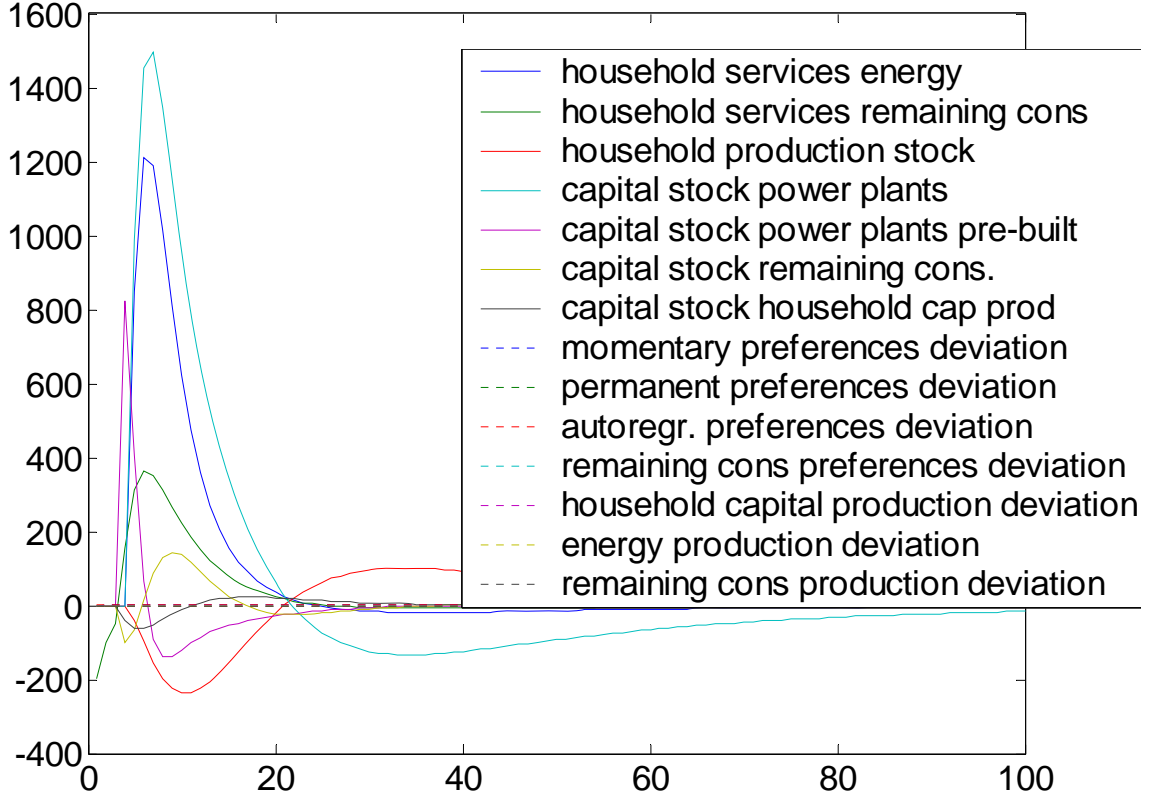


Figure 4: Response of the misperceived system with observable prices to an autoregressive energy service preference shock

For illustrative purposes one specific example is shown here: in the actual system the investment cost for power plants are 20 % higher than perceived and the proportion of autoregressive to permanent shocks is not 1 to 1 but 1.2 to 0.3. The consequences are shown in Figure 4. The peak in the power plant capital stock is increased by about 40 % (in MW, not in monetary units), and also the other responses of system components are more brusque. This is not entirely surprising since the controller is now not the optimal one which would fit best to the actual system. Similar results can be established for other disturbances or other misperceptions in the system. But in general system stability (with exception of the unit root for permanent shocks) remains preserved. Thus one might argue at first sight that robustness in the sense of perseverance of system stability in the case of misperception of system characteristics is not a crucial issue.

#### 4.2.3 Impact of adaptation costs on the system robustness

Yet this is only a part of the story. The trade-off between efficiency and robustness becomes more apparent when the value of the parameter  $b$  in the matrix  $\phi_i$  is modified.

This parameter describes the link between investments  $i_t$  and the internal goods vector  $g_t$ . Since internal goods enter quadratically the utility function, they are a measure of the adaptation costs in the system. A reduction of the adaptation costs corresponds generally to an increase of the efficiency in the system. This is also illustrated through Figure 5, where the loss in utility is shown as a function of the adaptation costs. The results are normalized to the costs of disturbances at a parameter value of  $b^2 = 0.5$ , in order to handle the different sizes of the original vectors. Obviously, averaged over all disturbances, the loss in utility tends to decrease with decreasing adaptation costs.

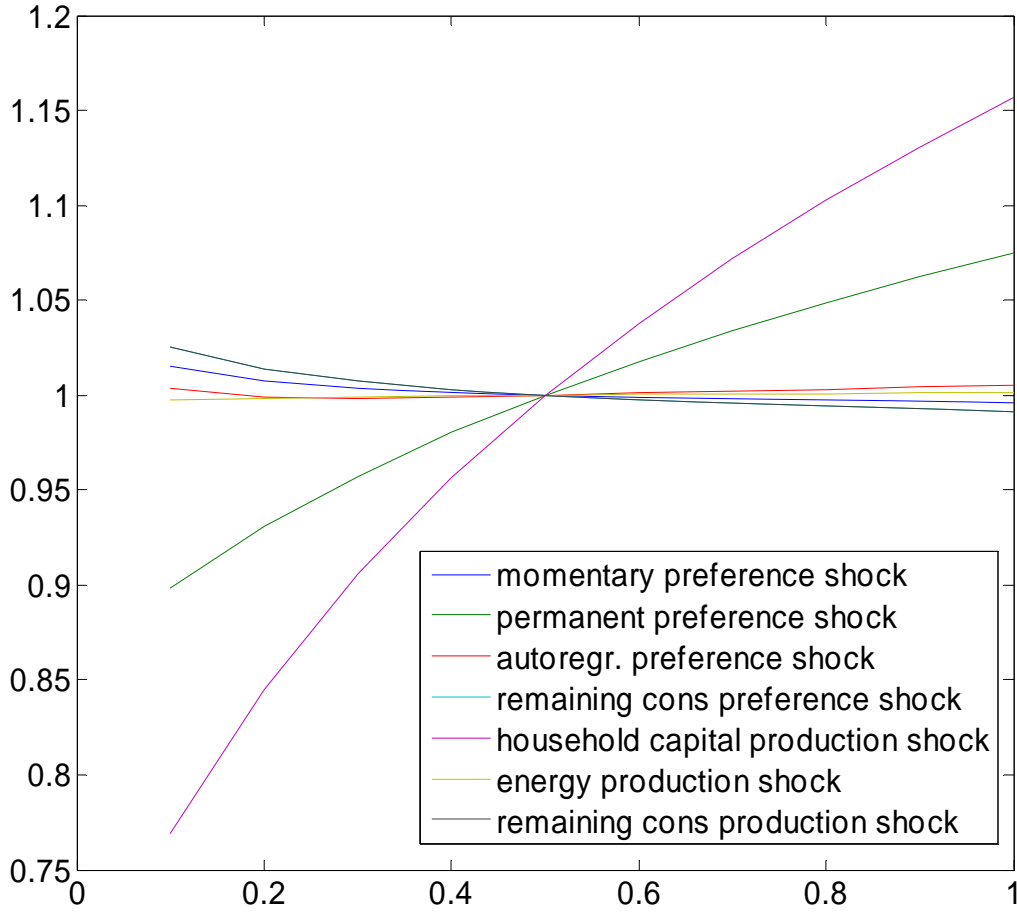


Figure 5 Variation of loss in utility as a function of adaptation costs (parameter  $b^2$ ) for the original system with optimal control

Precisely this feature does not hold, if there is misperception in the system. As shown in Figure 6, a reduction of adaptation costs does not lead to a reduction in utility losses. Rather the loss in utility increases, when adaptation costs are reduced. I.e., increased efficiency may come at the cost of higher utility losses in the case of system misperception. Or put the other way round: if misperception may arise in the markets, which are designed for regulating the original system, the strive for most efficient regulator design (i.e. market design) may lead to contraproductive results.

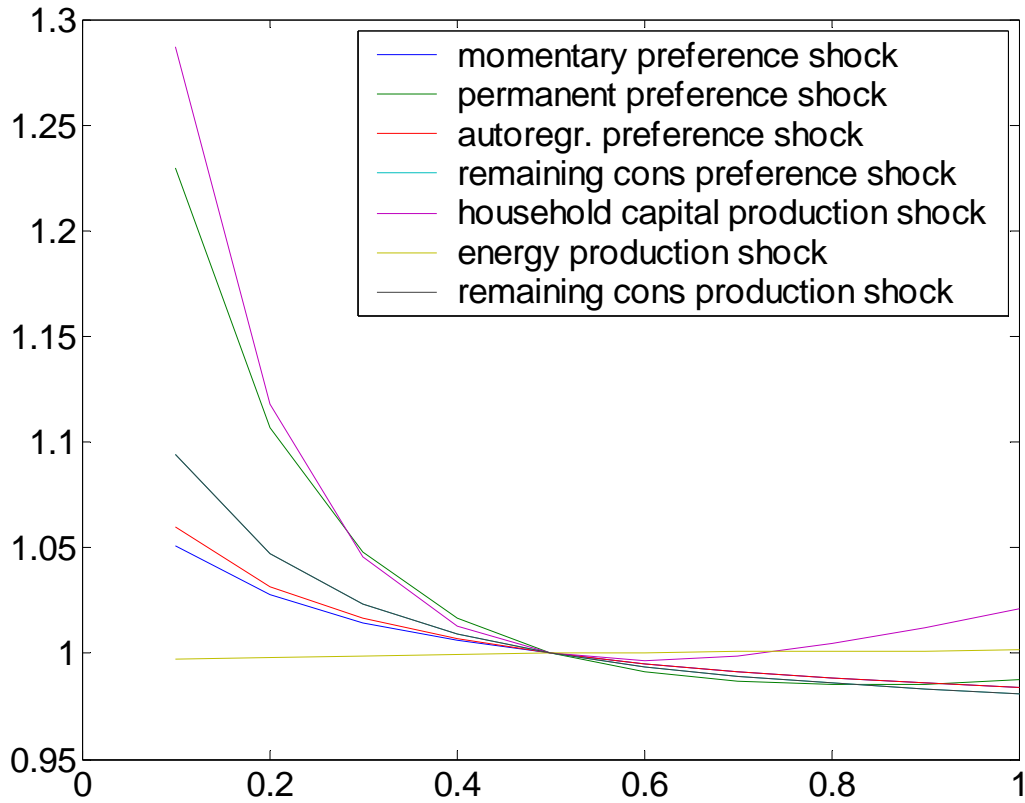


Figure 6 Variation of loss in utility as a function of adaptation costs (parameter  $b^2$ ) for the misperceived system

This result derived here for one specific combination of controller and misperceived system may also be generalised. For any stochastic distribution of misperceptions, the expected total system cost will decrease first with a decreasing share of irreversible costs (i.e. increased market efficiency). Yet beyond a certain point, the reduction in adaptation cost on the contrary leads again to an increase in overall expected system costs (cf. Figure 7).

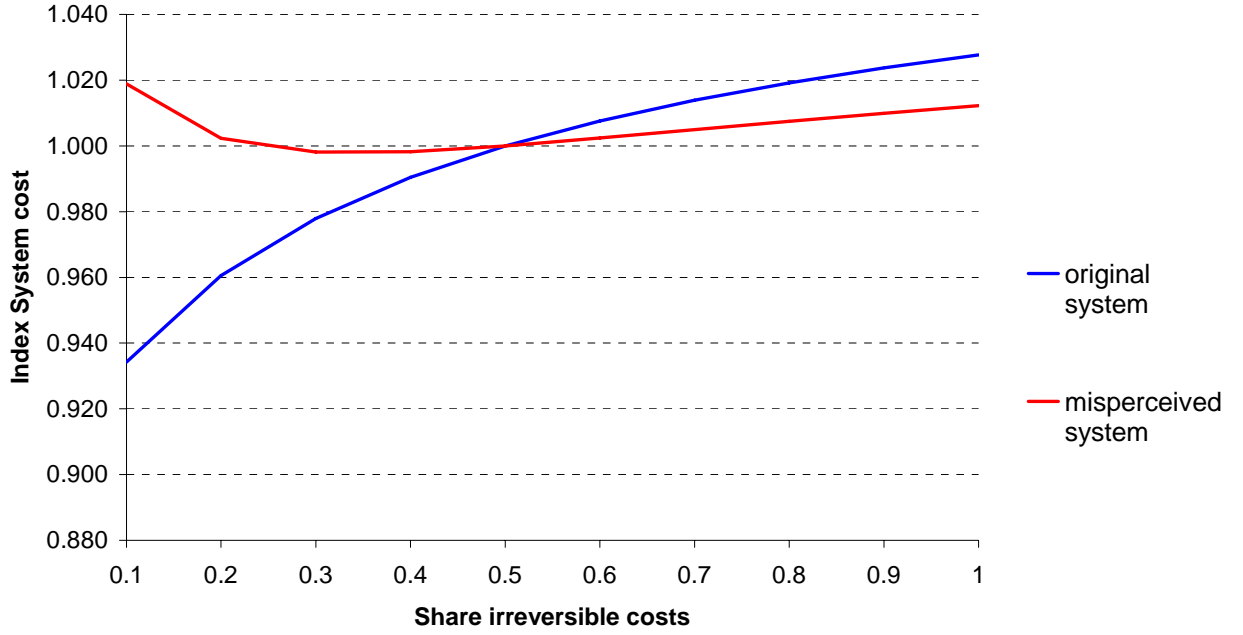


Figure 7 Expected total system cost depending on share of irreversible costs in investments

This at first sight counter-intuitive effect is related to the misperceptions occurring in the controller. Since the controller (i.e. the derivative financial markets) has a partly erroneous image of the real economy, the investments undertaken are not always efficient. If there are irreversible costs associated with these investments, they will be more carefully selected and lower in amplitude, but this avoids at the same time exaggerations which require costly ex-post corrections afterwards. Hence improving the efficiency of the capital markets by reducing the corresponding transaction costs or by lowering the capital requirements may in the end not increase overall welfare.

## 5 Concluding comments

The above analysis has shown both that there is a theoretical trade-off between efficiency and robustness of systems and that such trade-offs can be observed in stylized macro-economic models with financial markets as controllers of the real-world economy. Consequently issues of possible misperceptions among market participants and their implications should be taken into account when designing market mechanisms for financial markets.

This does not necessarily mean that markets are not suitable for coping with misperception issues. Hansen and Sargent (2005) show that market equilibria in the presence of model uncertainty exist and demonstrate how they can be computed. Yet they base their demonstration on the assumption that the market participants share a common approximating model and the knowledge about possible misperceptions - certainly an idealizing assumption.

The paper at hand does not rely on this idealizing assumption and provides first insights on the implications of misperceptions on market outcomes. Yet further research is needed for systematically identifying the factors in market design and structure, which have a crucial impact on market robustness. Also more detailed models are required to analyse the interplay between market misperceptions and real economic activity and how market designs may help in avoiding misperceptions with disastrous consequences.

## Annex

Matrices describing the original discounted stochastic control problem studied.

$A_{22} =$

0	1.0000	1.0000	0	0	0	0
0	1.0000	0	0	0	0	0
0	0	0.5000	0	0	0	0
0	0	0	0.5000	0	0	0
0	0	0	0	0.5000	0	0
0	0	0	0	0	0.5000	0
0	0	0	0	0	0	0.5000

$\beta =$

0.9700

$\Delta_h =$

0.9500

$\Delta_k =$

0.9750	1.0000	0	0
0	0	0	0
0	0	0.9000	0
0	0	0	0.9000

$\Gamma =$

0	0	0	1.0000
0	0	0.5000	0
90.0000	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0

$\Lambda =$

1  
0

O =

1.0e+004 \*

Columns 1 through 7

0.0002	0.0002	0.0002	0.0000	0.0002	0	-0.4876
0	0	0	-0.0001	0	0	0
-0.0000	-0.0000	0	0	0	0.0025	0
0.0002	0.0002	0.0002	0.0000	0.0002	0	-0.4391

Columns 8 through 12

-0.4859	-0.0062	0.0489	0.2429	0.0124
0	1.0000	0	0	-2.0000
0	0	0	-0.0025	0
-0.4373	-0.0062	0.0440	0.2187	0.0124

Phi =

1	0	0	0	0	0
0	1	0	0	0	0
0	0	1	0	0	0
0	0	0	-1	0	0
0	0	0	0	-1	0
0	0	0	0	0	-1

Phi\_i =

0	0	0
1	1	1
0	0	0
b	0	0
0	b	0
0	0	b

Pi =

0	0	0.0100
0	1.0000	0

Theta\_h =

1	0	0
---	---	---

Theta\_k =

0	0	0
0.8000	0	0
0	1.0000	0
0	0	0.1000

U\_b =

Columns 1 through 6

2500	0	0	0	0	0
------	---	---	---	---	---

0	0	0	10000	0	0
---	---	---	-------	---	---

Column 7

0
0

U\_d =

Columns 1 through 6

0	0	0	0	250	0
0	0	0	0	0	0
0	0	0	0	0	250000

Column 7

0
20000
0

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