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International Capital Markets, Oil Producers and the Green Paradox

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Abstract

A rapidly rising carbon tax leads to faster extraction of fossil fuels and accelerates global warming. We analyze how general equilibrium effects operating through the international capital market affect this Green Paradox. In a two-region, two-period world with identical homothetic preferences and without investment, the global interest rate falls and the Green Paradox weakens. With investment or a relatively more impatient oil-importing region, the Green Paradox may be strengthened because the future oil demand function shifts downward or because the interest rate rises. If the oil-importing region is very much more patient than the oil-exporting region, the Green Paradox may be reversed but in our calibrated model the effects are tiny. With exploration and endogenous initial oil reserves, a future carbon tax lowers cumulative oil extraction in partial equilibrium. If the boost to current oil extraction is weakened, strengthened or reversed in general equilibrium, so is the fall in cumulative extraction. A partial and general equilibrium welfare analysis of a future carbon tax, both for full and partial exhaustion, is given.

JEL-Code: D900, H200, Q310, Q380.

Keywords: global warming, Green Paradox, Hotelling rule, oil importers, oil producers, investment, capital markets, carbon tax, exploration investment, general equilibrium.

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1. Introduction

The idea that a rapidly rising carbon tax or a subsidy for renewable energy encourages oil producers to extract oil more quickly\(^1\) and accelerates global warming has gained traction and is known as the Green Paradox (Sinn, 2008, 2012; Gerlagh, 2011). The underlying mechanism is that a future carbon tax forces oil producers to supply less oil in the future due to lower future demand, which implies that current oil supply goes up. This pushes the current oil price down and therefore boosts today’s demand for oil. This is true when a given stock of oil is fully exhausted, but the effect is also present when stock-dependent extraction costs lead to partial exhaustion of reserves albeit that more of reserves are left abandoned.

It is well understood under which conditions the Green Paradox raises its head in partial equilibrium settings, where the level of investment and the interest rate are taken as given. However, front-loading of oil extraction also influences the global supply of savings and the demand for capital, so that the interest rate must adjust to clear bonds and capital markets. Although integrated assessment models of climate change like DICE and RICE typically allow for endogenous changes in investment and the interest rate (cf. Nordhaus, 1992; Nordhaus and Zhang, 1996; van der Ploeg and Withagen, 2014; Golosov et al., 2014; Rezai and van der Ploeg, 2014), they do not provide a decomposition analysis to assess the importance of effects operating through the international capital market for the Green Paradox.

Intuition suggests that a rapidly rising carbon tax increases current output relative to future output, which boosts global savings and depresses the global interest rate. This makes it less attractive to extract oil so that acceleration of the burning of oil and of global warming are mitigated. However, in general equilibrium there will be effects on the wealth positions of oil importers and oil exporters, so that changes in the interest rate and in oil extraction will affect investment and will impinge on future oil demand. Our objective is to gain a better understanding of these general equilibrium effects and to show that the benchmark general equilibrium result of mitigation of the Green Paradox is not robust when we allow for investment or when we allow for multiple jurisdictions. It is then possible that the Green Paradox is reinforced or reversed in general equilibrium.

We use a two-country, two-period model of oil importers and oil producers, where the interest rate and current and future real oil prices are determined from the conditions for clearing the markets for financial assets and oil, and the Hotelling rule governs optimal oil extraction (cf. Dixit, 1981; Marion and Svensson, 1984; van Wijnbergen, 1985; Djajić, 1988). We suppose that all markets operate under perfect competition and that only the oil-importing countries produce final goods which can be used for consumption and investment. We analyze the effects of changes in future carbon taxes on current oil extraction and the interest rate. In addition, we are interested in the amount of oil that is left unexploited.

\(^1\) We refer throughout to ‘oil’ as short-hand for ‘exhaustible fossil fuel resources’ such as oil, natural gas and coal.
in the crust of the earth as this affects the ultimate degree of global warming. We focus on changes in future taxes instead of optimal climate policy (typically requiring immediate action), because in reality gradual greening policies are observed and strong current action is lacking.

In general equilibrium the interest rate plays a role on both the production and consumption side of the economy. Demand for oil depends on oil prices, which are intertemporally related via the Hotelling rule and feature the interest rate as the opportunity cost of conserving oil. Furthermore, the level of investment depends on the interest rate, because it determines the marginal cost of renting a unit of capital. On the demand side the interest rate determines the relative price of current and future consumption. Moreover, each region’s wealth is affected by the interest rate. The interplay between these aspects drives the direction of the change in the interest rate induced by climate policy. The importance of investment for the Green Paradox arises from the imperfect substitutability of capital and oil, which implies that changes in investment shift the future oil demand function. The effects on the interest rate and investment together determine the general equilibrium repercussions for the Green Paradox.

Starting with the case of full exhaustion, we demonstrate that in general equilibrium the Green Paradox induced by a future carbon tax is mitigated if oil importers and oil exporters have identical homothetic preferences over current and future consumption and there are no investment possibilities. Although our results suggest that mitigation is still the most likely outcome under more general conditions with investment possibilities and asymmetric preferences between regions, we are also able to construct cases under which strengthening or reversal of the Green Paradox occurs. In particular, we show that a reversal can occur if oil producers are very much more impatient than oil importers but in our calibrated model the effects are tiny. Strengthening of the Green Paradox can occur if oil producers are relatively patient or if there is investment in physical capital. Accounting for partial exhaustion of the stock of oil reserves by imposing exploration costs, we show that in partial equilibrium a future carbon tax ensures that more oil will be locked in the crust of the earth. We demonstrate that general equilibrium effects have similar repercussions for cumulative extraction as they have for current extraction.

There are at least five other factors driving in the direction of a mitigated Green Paradox that we obtain in our case with identical homothetic preferences without investment. First, Hart and Spiro (2011, p. 7834) report that “scarcity rents seem to have been marginally or non-existent empirically”, so that Green Paradox effects will not be large. Potential explanations for this observation are a finite planning horizon of resource owners (Spiro, 2014) and endogenous field openings (Venables, 2014). Moreover, the emerging abundance of shale gas and other forms of unconventional fossil energy reserves might curb existing Hotelling rents even further. Second, a heavily polluting backstop alongside oil and clean renewable resources tends to mitigate the Green Paradox (van der Ploeg and Withagen, 2012b;
Michielsen, 2014). Third, if extraction costs of fossil fuel increase with subsoil reserves, climate policy potentially decreases cumulative extraction, thus mitigating the Green Paradox (e.g., Hoel, 2012; van der Ploeg and Withagen, 2012a; van der Ploeg, 2014). Fourth, learning by doing in the renewables sector can mitigate the Green Paradox (Nachtigall and Rübbelke, 2014). Finally, the Green Paradox that occurs after subsidizing renewables might be mitigated if fossil and renewable energy are used simultaneously due to increasing marginal production costs of renewables (cf. van der Ploeg and Withagen, 2012a; Grafton et al., 2013) or imperfect substitution between fossil and renewable resources (Michielsen, 2014).

Eichner and Pethig (2011; 2013) and Sen (2014) also study climate policy in a two-period, multi-country general equilibrium setting, but their focus is on carbon leakage as a result of unilateral action and they abstract from effects on cumulative extraction. Ritter and Schopf (2014) extend the model of Eichner and Pethig to allow for effects on cumulative extraction. Both Eichner and Pethig (2011; 2013) and Ritter and Schopf (2014) do not take investment in physical capital into account. Long and Stähler (2013) consider the general equilibrium effects of green technological progress on the speed of oil extraction but abstract from capital accumulation, and note that this may lead to a rise in the interest rate thereby offsetting and possibly reversing the initial increase in the rate of oil extraction.

Section 2 sets up the general two-country model with costless oil extraction. Section 3 derives the key partial equilibrium Green Paradox result. Section 4 shows that in general equilibrium Green Paradox effects are weakened if there is no investment and preferences are identical and homothetic, and demonstrates that amplification or reversal of the Green Paradox can occur if there is investment and or preferences differ very much between oil importers and exporters. Section 5 deals with exploration costs and partial exhaustion. Section 6 performs a welfare analysis. Section 7 concludes.

2. A two-country, general equilibrium model

We consider a two-period model with two blocks of countries (or regions), one block of homogeneous countries being the oil importers and the other block the oil exporters (denoted by an asterisk). Oil importers produce final goods, which are demanded by both blocks. They use capital and oil (as well as fixed factors such as land and labour) to produce final goods. Their assets consist of capital and bonds. We assume that oil extraction is costless, and discuss exploration costs in Section 5. Oil exporters have given initial oil reserves, which will be fully exhausted. Their other assets are capital and bonds. All markets operate under perfect competition and clear in each period. Oil producers have rational foresight and oil prices obey the Hotelling rule. The government of the oil-importing block might levy a specific carbon tax on the use of oil by final goods producers. Tax revenues are distributed in lump-sum fashion to
households in the oil-importing region. Our aim is to investigate the effects of the carbon tax on the real price of oil, the world interest rate, investment, and the intertemporal pattern of oil depletion.

**Firms**

Output of final goods is given by $F(K, R_t)$, where $K_t$ denotes employed capital and $R_t$ the oil extraction rate in period $t$ ($t = 1, 2$). Taking account of other fixed factors such as land and labour, this production function has decreasing returns to scale and is strictly increasing for positive inputs and strictly concave in each input. With the net rate of return on capital indicated by $r_t$, the world market oil price by $q_t$, the specific carbon tax levied on the producers of final goods by $\tau_t$, and the constant rate of depreciation by $\mu$, profits by firms in the oil-importing region in each period are

$$\Pi_t = F(K, R_t) - (r_t + \mu)K_t - (q_t + \tau_t)R_t, \quad t = 1, 2.$$  

Profit maximization under perfect competition gives:

1. $F_K(K_t, R_t) = r_t + \mu$, $F_R(K_t, R_t) = q_t + \tau_t$,
2. $F_K(K_2, R_2) = r_2 + \mu$, $F_R(K_2, R_2) = q_2 + \tau_2$.

This yields conditional oil demand $R_1(r_1, q_1 + \tau_1)$, capital demand $K_1(r_1, q_1 + \tau_1)$, and the profit function $\Pi_1(r_1, q_1 + \tau_1)$. For period two this yields the factor demands $K_2(r_2, q_2 + \tau_2)$ and $R_2(r_2, q_2 + \tau_2)$, which gives the profit function $\Pi_2(r_2, q_2 + \tau_2)$. Factor demands decrease in own factor prices. If capital and oil are cooperant factors, $F_{KR} > 0$ which we assume, future oil demand decreases in the interest rate too.

Profit maximization by oil exporters, facing the real interest rate $r_2$, yields the Hotelling rule:

$$q_2 = (1 + r_2)q_1,$$

Hence, the after-tax return on keeping a marginal barrel of oil in the earth, $r_2q_1$, must equal the return on taking a marginal barrel of oil out of the earth, i.e., the expected capital gains, $q_2 - q_1$.

**Households**

Households in the oil-importing region derive utility from present and future consumption and disutility from carbon dioxide in the atmosphere. Preferences of the representative household in the oil-importing region can be represented by:

$$\Lambda(C_t, E_t) = U(C_t) - D(E_t),$$

where $C_t$ and $E_t$ denote consumption in the oil-importing region and the concentration of carbon dioxide in the atmosphere, respectively, in period $t$. Carbon emissions are proportional to oil use. By appropriate
We assume that the utility function $U(C_1, C_2)$ is continuously differentiable, increasing and strictly concave. The atmospheric carbon stock causes convex damages $D(E_1, E_2)$. Households ignore the consequences of their consumption decisions on carbon emissions. The representative household in the oil-exporting region is not affected by climate change or, equivalently, oil exporters do not conduct climate policy. Its preferences are represented by the continuously differentiable, increasing, strictly concave utility function $U^*(C_1^*, C_2^*)$, where $C_t^*$ denotes consumption in period $t$. The consolidated budget restrictions for both regions read:

$$C_1 + A_2 = (1 + r_1)A_t + \Pi_1 + \tau_1 R_1, \quad C_2 = (1 + r_2)A_2 + \Pi_2 + \tau_2 R_2,$$

$$C_1^* + A_2^* = (1 + r_1)A_t^* + q_1 R_1, \quad C_2^* = (1 + r_2)A_2^* + q_2 R_2,$$

where $A_t$ and $A_t^*$ denote asset holdings at the start of period $t$. The initial asset endowments $A_t$ and $A_t^*$ are given. It follows that the present-value budget constraints for both regions are

$$C_1 + \frac{C_2}{1 + r_2} = (1 + r_1)A_t + \Pi_1 + \tau_1 R_1 + \frac{\Pi_2 + \tau_2 R_2}{1 + r_2} \equiv M,$$

$$C_1^* + \frac{C_2^*}{1 + r_2} = (1 + r_1)A_t^* + q_1 R_1 + \frac{q_2 R_2}{1 + r_2} \equiv M^*,$$

where $M$ and $M^*$ denote wealth of the oil-importing and oil-exporting region, respectively. Wealth of the oil-importing region is the sum of the net return on assets and the present discounted value of profits and carbon tax refunds. Wealth of the oil-exporting block consists of the return on assets and the present discounted value of oil revenues. Ideally, the carbon taxes in the oil-importing region are optimal (from the perspective of this region). However, in the present paper we consider carbon taxes as exogenous. In Section 6 we perform a welfare analysis taking damages into account.

**Equilibrium conditions**

Equilibrium on the asset markets requires that capital must be held by one of the two regions:

$$K_1 = A_t + A_t^*, \quad K_2 = A_2 + A_2^*.$$

The initial oil stock is $S_1$, so that oil market equilibrium (OME) requires

$$S_1 = R_1 + R_2.$$

---

2 We abstract from carbon depreciation. For a more detailed modeling of the carbon cycle, see Golosov et al. (2014).
The goods market equilibrium conditions (GME) for periods one and two are:

\[(9) \quad F(K_1, R_1) + (1 - \mu)K_1 = C_1 + C_1^* + K_2,\]
\[(10) \quad F(K_2, R_2) + (1 - \mu)K_2 = C_2 + C_2^*.\]

This completes the description of the model with full exhaustion of oil reserves, which we analyze in Sections 3-4. In Section 5 we extend the model to allow for partial exhaustion by introducing exploration costs so that the initial oil stock becomes endogenous.

### 3. Partial equilibrium

To understand the Green Paradox in partial equilibrium, we study here the resource market in isolation thus taking the interest rate \(r_2\) and the capital stocks \(K_1\) and \(K_2\) as fixed before moving to a general equilibrium analysis in Sections 4-6. Consider an increase in the future carbon tax \(\tau_2\), keeping \(\tau_1\) constant. To see that current oil extraction increases, substitute oil demand from (1)-(2) and the Hotelling rule (3) in the OME (8):

\[(11) \quad R_1(q_1 + \tau_1) + R_2((1+r_2)q_1 + \tau_2) = S_1,\]

where we use that \(K_1\) and \(K_2\) are given and oil demand \(R_1\) and \(R_2\) are functions of the tax-inclusive oil price only. If the future carbon tax goes up, equilibrium on the oil market does not allow for a higher current world market oil price \(q_1\). From (11) it is immediately apparent that this would reduce oil demand in both periods so that there would be excess supply. Hence, a future carbon tax increases the future cost of oil and curbs future oil use, but depresses the current oil price and thus boosts current oil demand and current carbon emissions. Hence, a future carbon tax forces oil suppliers to supply less oil in the future and thus to supply more today. This is the essence of the Green Paradox for the partial equilibrium context with full exhaustion of reserves.

Next we change carbon taxes in proportion, i.e., \(\tau_2 = (1+\psi)r_1\). We can rewrite the OME condition (11) as

\[(12) \quad R_1(q_1 + \tau_1) + R_2 \left( (1+r_2) \left[ q_1 + \tau_1 + \left( \frac{\psi - r_2}{1 + r_2} \tau_1 \right) \right] \right) = S_1.\]

So, if the growth rate of the carbon tax, \(\psi\), equals the return on capital for the oil-exporting region, \(r_2\), an increase in the first-period carbon tax does not affect \(q_1 + \tau_1\) or the intertemporal pattern of oil extraction
rates. If the growth rate of the carbon tax is bigger than \( r_2 \), the future cost of oil rises while the current cost of oil falls so that there is a Green Paradox. However, if the growth rate of the carbon tax is lower than \( r_2 \), we deduce immediately from (12) that the future cost of oil falls so that more oil is extracted in the future and less today (no Green Paradox). We summarize the results so far as follows.

\textbf{Proposition 1 (Green Paradox in partial equilibrium):} With a given interest rate and a fixed resource demand function a higher future carbon tax accelerates oil extraction and global warming whilst a carbon tax that rises faster (slower) than the interest rate accelerates (decelerates) oil extraction and global warming.

We now extend our analysis towards general equilibrium and focus on a higher future carbon tax (setting the current carbon tax to zero), which captures a carbon tax that rises faster than the interest rate.

\section*{4. General equilibrium}

In general equilibrium, both the interest rate and the resource demand function are affected by climate policy. As a result, we show that the Green Paradox can be attenuated, reversed or amplified.

\textbf{Definition 1:} Assume the future carbon tax is increased.

(i) ‘Attenuation’ occurs if current oil extraction expands by less than in partial equilibrium.

(ii) ‘Reversal’ occurs if current oil extraction decreases.

(iii) ‘Amplification’ occurs if current oil extraction expands by more than in partial equilibrium.

In Section 4.1 we present a benchmark case where there is no investment and regions have identical, homothetic preferences. In Sections 4.2-4.4 we allow for capital accumulation and non-identical preferences.

\subsection*{4.1. Benchmark case: identical homothetic preferences, no capital}

Without capital as factor of production the oil market equilibrium (OME) condition reads:

\begin{equation}
R_1(q_t) + R_2((1 + r_2)q_t + \tau_2) = S_1.
\end{equation}

For a given \( \tau_2 \), equation (13) gives a negative relationship between \( r_2 \) and \( q_t \) and, for given \( r_2 \), it gives a negative relationship between \( \tau_2 \) and \( q_t \). The reason is that a higher interest rate or future carbon tax curbs future oil demand, thus requiring a fall in the current oil price to clear the oil market. Hence, as shown in Figure 1 below, equation (13) corresponds to the downward-sloping OME locus in \((r_2,q_t)\)-space which shifts inwards as the future carbon tax is increased.
To characterize the goods market equilibrium (GME), we assume without loss of generality zero initial asset endowments. Then the present-value budget constraints (4) and (5) become

\[
C_1 + \frac{C_2}{1 + r_2} = F(R_t(q_1)) - q_1 R_t(q_1) + \frac{F(R_t((1 + r_2)q_1 + \tau_2)) - (1 + r_2)q_1 R_t((1 + r_2)q_1 + \tau_2)}{1 + r_2} \equiv M(q_1, r_2; \tau_2),
\]

\[
C_1^* + \frac{C_2^*}{1 + r_2} = q_1 R_t(q_1) + \frac{q_2 R_2}{1 + r_2} = q_1 S_1 \equiv M^*(q_1),
\]

where profit maximization, the Hotelling rule (3), and (13) have been used. We have explicitly indicated that oil importers’s wealth depends on the carbon tax. Let the tax rate be given. For any triplet of prices \((q_1, q_2, r_2)\) such that the Hotelling rule is satisfied, the oil-exporting region derives demand for the final good as functions of this triplet. The Hotelling rule makes sure that (perceived) discounted income of this region equals \(q_1 S_1\). For this set of prices (and the given tax rate) the oil-importing region determines profit-maximizing demand for oil, and thereby discounted total income \(M\). Then follows demand for final goods for the two periods. So, we can write \(C_t(r_2, M(r_2, q_1; \tau_2))\) and \(C_t^*(r_2, M^*(q_1)), t = 1, 2\). Using (9)-(10), the GME locus of points for which there is equilibrium on the final goods market is defined by

\[
\frac{C_2(r_2, M(r_2, q_1; \tau_2)) + C_2^*(r_2, M^*(q_1))}{C_1(r_2, M(r_2, q_1; \tau_2)) + C_1^*(r_2, M^*(q_1))} = \frac{F(R_2((1 + r_2)q_1 + \tau_2))}{F(R_t(q_1))}.
\]

A general equilibrium exists where the GME and OME locus intersect (due to Walras’s Law). In general little is known about the shape of GME and the shift that occurs in GME following a change in the tax. A higher carbon tax will affect the current oil price which will lead to a reallocation of wealth, as can be seen from (14) and (15), which in turn affects relative aggregate final goods demand.

One way to partly get around the problem with regard to the effect of a change in the tax rate is to assume that future oil use equals remaining supply \(S_1 - R_t(q_1)\), irrespective of the future price (instead of assuming that future oil use is determined by demand in (2) irrespective of remaining supply). The GME locus of the initial oil price and interest rate for which this is taken into account is given by the Modified GME or MGME locus:

\[
\frac{C_2(r_2, M(r_2, q_1)) + C_2^*(r_2, M^*(q_1))}{C_1(r_2, M(r_2, q_1)) + C_1^*(r_2, M^*(q_1))} = \frac{F(S_1 - R_t(q_1))}{F(R_t(q_1))}.
\]
The MGME does not feature the future carbon tax \( r_2 \). To determine the general equilibrium effect of a higher carbon tax, we still need to establish whether the MGME locus is upward-sloping or downward-sloping in \((r_2, q_1)\)-space.

Figure 1 shows the MGME and OME loci in \((r_2, q_1)\)-space, for three different cases. Because the OME line shifts down upon an increase in the future carbon tax whereas the MGME line remains unaffected, the comparative statics results depend on the relative slopes of the MGME and OME loci with a crucial role played by the interest rate. In partial equilibrium the interest rate is exogenous and the equilibrium oil price jumps down from point E to the level corresponding with point PE (partial equilibrium) in each panel, which implies shifting oil extraction from the future to the present. In general equilibrium, however, the interest rate adjusts to the level corresponding with point GE to clear the asset market.

Attenuation and reversal of the Green Paradox feature a fall in the interest rate. This induces oil exporters to pump more slowly, so that the partial equilibrium Green Paradox is attenuated (panel (a)) or reversed (panel (b)). Amplification of the Green Paradox (panel (c)) is associated with a higher interest rate, which further boosts current oil extraction.

Which of the cases prevails, depends on preferences, and in particular on the wealth effects of a future carbon tax. With homothetic preferences, relative aggregate consumption of future and current final goods in each country is independent of wealth. It follows from the optimality conditions for households in each region that demand for future relative to current goods depends solely on the interest rate, i.e. \( \frac{C_2}{C_1} = \Phi(r_2) \) and \( \frac{C_2^*}{C_1^*} = \Phi^*(r_2) \), with \( \Phi' > 0 \) and \( \Phi^{*'} > 0 \). Hence, with identical preferences the MGME locus reduces to

\[
\Phi(r_2) = \frac{F(S_1 - R_1(q_1))}{F(R_1(q_1))}.
\]

We thus get an upward-sloping MGME locus, which implies weakening of the Green Paradox.

**Proposition 2 (Attenuation of the Green Paradox):** Suppose the two regions have identical and homothetic preferences, that there is no investment, and that the future carbon tax is increased. Then, in general equilibrium acceleration of oil extraction occurs but is always less than in partial equilibrium.
Figure 1: Three different general equilibrium scenarios

Panel (a) - Attenuation

Panel (b) - Reversal

Panel (c) - Amplification

Notes: The intersection of the solid lines (point E) gives the initial equilibrium. The intersection at point GE gives the equilibrium after the increase in the future carbon tax. The movement from point E to point PE gives the partial equilibrium effect of an increase in the future carbon tax.
The intuition is as follows. The partial equilibrium effect of a future carbon tax lowers $q$, so that current oil extraction and current production of final goods increase and future oil extraction and final goods production decrease. Because of identical homothetic preferences, wealth effects do not affect the aggregate relative demand for future goods. As a result, an excess demand for future goods materializes at the prevailing interest rate. Hence, the interest rate must decrease to reduce relative demand for future goods, thus eliminating excess demand. The lower interest rate encourages oil-exporters to extract oil less quickly, thus attenuating the Green Paradox.

4.2. Asymmetric preferences

To see whether Proposition 2 also holds more generally, we allow for different, but homothetic, preferences across regions. The aggregate relative consumption of future and current goods is then

\[ \frac{C_2 + C_2^*}{C_1 + C_1^*} = \Phi(r_2) + \left[ \Phi^*(r_2) - \Phi(r_2) \right] \frac{C_1^*}{C_1 + C_1^*}. \]

The extra term in square brackets in (18) captures a wealth reallocation effect: Even with homothetic preferences, a reallocation of wealth between countries affects aggregate relative demand for future goods at an unchanged interest rate. If relative current consumption of the region with the highest equilibrium future-to-current consumption ratio increases, then aggregate future-to-current consumption increases and vice versa. To illustrate these effects for the Green Paradox, let us consider CES utility functions.3

Assumption 1: The utility functions are given by

\[ U(C_1, C_2) = \begin{cases} \frac{C_1^{1-\eta}}{1-\eta} + \frac{1}{1+\rho} \frac{C_2^{1-\eta}}{1-\eta} & \text{if } \eta \neq 1 \\ \ln C_1 + \frac{1}{1+\rho} \ln C_2 & \text{if } \eta = 1 \end{cases}, \]

\[ U^*(C_1^*, C_2^*) = \begin{cases} \frac{C_1^{1-\eta^*}}{1-\eta^*} + \frac{1}{1+\rho^*} \frac{C_2^{1-\eta^*}}{1-\eta^*} & \text{if } \eta^* \neq 1 \\ \ln C_1 + \frac{1}{1+\rho^*} \ln C_2^* & \text{if } \eta^* = 1 \end{cases}, \]

where $\eta > 0$ and $\eta^* > 0$ denote the elasticities of marginal utility (the inverses of the intertemporal elasticities of substitution), and $\rho \geq 0$ and $\rho^* \geq 0$ are the rates of pure time preference in both regions.

3 Bergson’s theorem states that preferences are time separable and homothetic if and only if they are of the CES type.
In this section we restrict ourselves to the case $\eta = \eta^* = 1$, in which the income and substitution effects of changes in the interest rate cancel out, resulting in the fixed expenditure shares

\[(19) \quad C_i = \alpha M, \quad \frac{C_i^*}{1 + r_2} = (1 - \alpha) M, \quad C_i^* = \alpha^* M, \quad \frac{C_i^*}{1 + r_2} = (1 - \alpha^*) M^*, \]

where $\alpha \equiv (1 + \rho) / (2 + \rho)$ and $\alpha^* \equiv (1 + \rho^*) / (2 + \rho^*)$. Note that $\alpha, \alpha^* \in [0.5, 1)$ due to non-negative discounting. The case of non-unitary elasticities of marginal utility is explored in Section 4.4.

**Proposition 3 (Logarithmic utility and general equilibrium):** Suppose that Assumption 1 holds with $\eta = \eta^* = 1$, and that there is no investment. With an increase in the future carbon tax, the Green Paradox is attenuated if oil importers are not more impatient than oil exporters ($\rho \leq \rho^*$), and may be amplified if oil importers are more impatient than oil exporters ($\rho > \rho^*$). Reversal of the Green Paradox cannot occur.

**Proof:** See Appendix A1. □

If the oil-importing region is more patient or not too much more impatient than the oil-exporting region, Green Paradox effects are attenuated. However, Figure 2 shows that if the oil-importing region is much more impatient than the oil-exporting region, the Green Paradox may be amplified.\(^4\) We have Assumption 1 with $\eta = \eta^* = 1$ and $S_i = 2$. The carbon tax $\tau_2$ is increased from zero to 0.01. The CES production function underlying the results in the figure is given by

\[(20) \quad F(R) = \left[ \beta R^{(\sigma - 1)/\sigma} + (1 - \beta) L^{(\sigma - 1)/\sigma} \right]^{\sigma/(\sigma - 1)} \quad \text{with} \quad \beta = 0.1 \quad \text{and} \quad L = 1.\]

The intuition for amplification of the Green Paradox is as follows. At a given interest rate, the future carbon tax induces excess supply of current goods (as in Proposition 1), but also increases the relative wealth of the oil-importing region due to the decline in the oil price. If the oil-importing region has a relatively high current-to-future consumption rate (which is the case if $\rho > \rho^*$), the wealth reallocation effect positively affects aggregate current-to-future consumption, which diminishes the excess supply of current final goods that is caused by the partial equilibrium Green Paradox effect. If strong enough, this wealth reallocation effect can even lead to excess demand for current goods. The interest rate then needs to rise instead of fall to restore general equilibrium so that the Green Paradox is amplified.

\(^4\) In this example and all examples to come, existence of an equilibrium poses no problem. Moreover, local stability of the usual Walrasian tâtonnement process is easily verified (i.e., Assumption A1 in Online Appendix A2 is satisfied in all the reported cases), so that there is no need to introduce a sophisticated auctioneer to steer the economy from a pre-tax equilibrium to an equilibrium with a tax.
**Figure 2: Attenuation versus amplification of the Green Paradox**

Notes: Parameters are set to $\beta = 0.1$, $\eta = \eta^* = 1$, $L = 1$, and $S_i = 2$. The carbon tax $\tau_1$ is increased from 0 to 0.01. The horizontal axis features a mean-preserving spread with $\alpha = 1/2 + x$, $\alpha^* = 1 - x$, $x \in (0, 1/2)$; the shaded area indicates amplification of the Green Paradox.

The strength of the wealth reallocation effect depends on the difference between current expenditures shares, $\alpha - \alpha^*$ (or, equivalently on how much more impatient the oil-importing region is relative to the oil-exporting region, as measured by $\rho - \rho^*$). The strength of the supply effect (the change in current output relative to future output due to the lower current world price of oil) depends crucially on the elasticity of factor substitution, $\sigma$, through its effect on the price elasticity of oil demand $\varepsilon_1$:

$$dF[R_1(q_1)] = F'(R_1(q_1)) \left( \frac{\partial R_1}{\partial q_1} \right) dq_1 = -\varepsilon_1 R_1 dq_1 \quad \text{with} \quad \varepsilon_1 \equiv -\left( \frac{q_1 \partial R_1}{R_1 \partial q_1} \right) = \sigma \left[ 1 + \left( \frac{\beta}{1 - \beta} \right) \left( \frac{R_1}{L} \right)^{\sigma - 1} \right] > 0. $$

The grey area in Figure 2 gives combinations of $\alpha - \alpha^*$ and $\sigma$ for which the Green Paradox is amplified.

**4.3. Physical capital**

Changes in investment affect the future capital stock and will thus shift future resource demand. We show that, as a consequence, amplification of the Green Paradox no longer requires an increase in the interest rate. With investment, the OME condition reads

$$R_1(q_1) + R_2(1 + r_2)q_1 + \tau_2 = S_i. \quad (21)$$
Note that second period oil demand decreases if both input prices $r_2$ and $(1 + r_2)q_i + r_2$ increase. With the use of (9)-(10), the GME locus is now:

$$
\frac{C_v(r_2, M(r_2, q_i; r_2)) + C'_v(r_2, M'(q_i))}{C_i(r_2, M(r_2, q_i; r_2)) + C'_i(r_2, M'(q_i))} = \frac{(1-\mu)K_v(r_2, (1 + r_2)q_i + r_2) + F(K_v(r_2, (1 + r_2)q_i + r_2), R_v(r_2, (1 + r_2)q_i + r_2))}{(1-\mu)K_i + F(R_i(q_i)) - K_i(r_2, (1 + r_2)q_i + r_2)},
$$

where we have used that the first-period interest rate can be written as a function of $q_i$ only as it follows from $F_k(K_1, R_1(q_i)) = r_1 + \mu$ with $K_1 = A_4 + A_4^*$ given. As in the previous section the GME locus can be turned into an MGME locus without the carbon tax appearing, but it is a bit more complicated now. First, we replace $R_v(r_2, (1 + r_2)q_i + r_2)$ by $S_i - R_i(q_i)$. Second, for every $q_i$ we know $S_i - R_i(q_i)$, so that (with slight abuse of notation), future demand for capital is $K_2(r_2, S_i - R_i(q_i))$. So, we arrive at

$$
\frac{C_v(r_2, M(r_2, q_i)) + C'_v(r_2, M'(q_i))}{C_i(r_2, M(r_2, q_i)) + C'_i(r_2, M'(q_i))} = \frac{(1-\mu)K_v(r_2, S_i - R_i(q_i)) + F(K_v(r_2, S_i - R_i(q_i)), S_i - R_i(q_i))}{(1-\mu)K_i + F(R_i(q_i)) - K_i(r_2, S_i - R_i(q_i))}.
$$

In terms of the direction of changes in the equilibrium prices, the different possibilities are still described by the three panels of Figure 1. Still, there is a key difference with Sections 4.1-4.2: a change in the future capital stock shifts future oil demand, so the movement from E to PE no longer corresponds with the partial equilibrium effect. Hence, the Green Paradox can be amplified even without a higher interest rate.

We illustrate this using Assumption 1 and identical preferences, so $(C_v + C_v^*)/(C_i + C_i^*) = \Phi(r_2)$.

Figure 3: Attenuation versus amplification under identical homothetic preferences

Panel (a): attenuation vs. amplification

Panel (b): strength of general equilibrium effect

Notes: $A_4 = 1$, $A_4^* = 0$, $\beta = 1/3$, $\eta = \eta^* = 1$, $\lambda = 0.1$, $\mu = 0.1$, $L = 1$, and $S_i = 2$. In panel (a), the shaded area indicates amplification and the white area attenuation of the Green Paradox. Panel (b) shows the difference in the change in current extraction between the general and the partial equilibrium model, as a share of the change in the partial equilibrium model, for the specification with physical capital (solid line) and without physical capital (dashed line). In panel (b), $\rho = \rho^* = 0.05$, implying $\alpha = \alpha^* = 1.05 / 2.05$. The carbon tax $r_2$ is raised from zero to 0.01.
It can be seen from (22) that the interest rate then always goes down after an increase in the future carbon tax. Figure 3 shows the outcome when \( \tau_2 \) is increased from zero to 0.01 with the CES production function:

\[
F(R) = \left[ \beta R^{\sigma_1} + \lambda K^{\sigma_1} + (1 - \beta - \lambda) L^{\sigma_1} \right]^{\frac{\sigma}{\sigma_1}}, \text{ with } \beta = 1/3, \ \lambda = 0.1, \text{ and } L = 1.
\]

We use Assumption 1 with \( \eta = \eta^* = 1 \) and \( A_1 = 1, \ A_2^* = 0, \ \mu = 0.1, \text{ and } S_1 = 2 \). The shaded area in panel (a) indicates the combinations of the elasticity of factor substitution \( \sigma \) and the common current expenditure share \( \alpha \) for which the Green Paradox is amplified despite the decrease in the interest rate (resulting from a downward shift in future oil demand). Panel (b) shows the strength of the general equilibrium effect as a function of the elasticity of factor substitution, with \( \rho = \rho^* = 0.05 \) imposed (so \( \alpha = \alpha^* = 1.05 / 2.05 \)). More specifically, the graph in panel (b) shows the difference between the change in first-period extraction between general equilibrium and partial equilibrium as a share of the change in first-period extraction in partial equilibrium when there is investment (solid line) and when there is no investment (dashed line).\(^5\) For the plotted range of elasticities of factor substitution (from 0.05 to 0.95), the general equilibrium effect on first-period extraction varies from an amplification of 7.5 percent in the model with investment to an attenuation of 6.5 percent without investment. Allowing for investment does not change the result of Proposition 3 that reversal of the Green Paradox is impossible under Assumption 1 with \( \eta = \eta^* = 1 \). We summarize the results of this section as follows.

**Proposition 4 (Investment and Green Paradox):** Assumption 1 holds with \( \eta = \eta^* = 1 \) and investment in physical capital is possible. With an increase in the future carbon tax, amplification of the Green Paradox can occur even if preferences are identical but reversal of the Green Paradox cannot occur.

**Proof:** See Appendix A1. □

**4.4. Non-unitary elasticity of marginal utility**

Propositions 3 and 4 establish that reversal of the Green Paradox cannot occur with Assumption 1 and \( \eta = \eta^* = 1 \). Unitary elasticities of marginal utility give rise to constant current expenditure shares as income and substitution effects of changes in the interest rate cancel out against each other. However, if the income effect dominates the substitution effect and current expenditure shares depend positively on

\(^5\) In panel (b), at \( \sigma = 0.5 \) the oil income share \( q_i R_i / Y \) equals 8.8 percent, which matches the average US energy expenditure share in GDP over the period 1970-2009 (U.S. Energy Information Administration, 2012).
the interest rate, reversal of the Green Paradox may occur. The intuition behind this result is as follows. Recall that if the oil-importing region has a relatively low current-to-future consumption ratio, the wealth reallocation effect amplifies the excess supply of current goods resulting from an increase in the future carbon tax. Hence, the interest rate must fall to equilibrate relative demand for and relative supply of current goods. However, if current expenditure shares fall together with the interest rate, this constitutes a counteracting effect on the excess supply of current goods. Therefore, the required decrease in the interest rate to clear the intertemporal goods market is larger than in the case of constant expenditure shares. If the negative effect on current expenditure shares is strong enough, the Green Paradox can thus be reversed in general equilibrium.

As an example, consider the case of Assumption 1 with \( \eta = \eta^* > 1 \). The current expenditure shares resulting from utility maximization are given by

\[
\frac{C_1}{M} = \left[ 1 + (1 + r_2) \frac{1-\eta}{\eta} (1 + \rho) \right]^{-1}, \quad \frac{C_1^*}{M} = \left[ 1 + (1 + r_2) \frac{1-\eta^*}{\eta^*} (1 + \rho) \right]^{-1},
\]

which depend positively on the interest rate if \( \eta > 1 \) and \( \eta^* > 1 \). Figure 4 shows simulation results for different combinations of \( \sigma, \eta = \eta^* \), and \( \alpha - \alpha^* \). The two upper (lower) panels give the outcome for the model without (with) investment. The underlying CES production functions for the specification with and without capital are again given by (20) and (23), respectively. We set \( A_1 = 0, \ A^*_1 = 1, \ \mu = 0.1 \) and \( S_1 = 1 \). The carbon tax \( \tau_2 \) is increased from zero to 0.01. Attenuation occurs in the shaded areas of the four panels, whereas reversal of the Green Paradox occurs in the white areas. The figure illustrates that a combination of relatively patient oil importers, a sufficiently high elasticity of marginal utility, together with a sufficiently low elasticity of factor substitution can give rise to reversal of the Green Paradox. However, it is clear from Figure 4 that we need to impose rather extreme conditions to obtain this outcome.\(^6\)

A comparison of panels (a)-(b) and (c)-(d) suggest that with investment it is even more difficult to find conditions under which strong reversal occurs. To understand this, consider a strong reversal in the model without capital, so that \( R_2 \) goes up and \( r_2 \) goes down. In the model with capital, this would induce an increase in investment. The resulting increase in the future capital stock has two opposing effects on the

---

\(^6\) The reversal obtained in our numerical example is small in magnitude: the largest decrease in first-period extraction that we find is a factor 10 (10\(^3\)) smaller than the increase in extraction in the example with identical preferences underlying Figure 3 with \( \rho = \rho^* = 0.05 \) and \( \sigma = 0.5 \) in the model without (with) investment.
Green Paradox. First, relative supply of future consumption goods goes up, which increases the equilibrium rate of interest and thus boosts current extraction through the Hotelling rule, thereby enhancing the Green Paradox. Second, the increase in the future capital stock induces an upward shift in future oil demand, working against the Green Paradox. The first effect dominates the second one, so that on balance, strong reversal of the Green Paradox is less likely if the possibility of investment in physical capital is taken into account.

**Figure 4: Attenuation versus reversal of the Green Paradox with a future carbon tax**

Panel (a): without capital, $\alpha = 0.5, \alpha^* = 0.995$  
Panel (b): without capital, $\eta = \eta^* = 10$

Panel (c): with capital, $\alpha = 0.5, \alpha^* = 0.995$  
Panel (d): with capital, $\eta = \eta^* = 10$

**Notes:** $A_t = 0, A_t^* = 1, \beta = 0.1, \lambda = 1/3, L = 1,$ and $S_t = 1$. The carbon tax $\tau_c$ is increased from zero to 0.01. The white (shaded) area gives the region in which the Green Paradox is reversed (attenuated).

Proposition 5 summarizes the result of this section.

**Proposition 5:** Assumption 1 holds with $\eta = \eta^* > 1$ and there is no investment. An increase in the future carbon tax can reverse the Green Paradox in general equilibrium.
4.5 Summing up

The results so far have shown that the Green Paradox effect associated with the announcement of a future carbon tax is weakened in general equilibrium if the oil-importing and the oil-exporting region have identical, homothetic preferences and if the analysis abstracts from investment. However, if oil-importers are relatively impatient or if the possibility of investment in capital is taken into account, the Green Paradox may be amplified in general equilibrium. Finally, we have shown that under rather extreme conditions with very patient oil importers, a low elasticity of factor substitution, and an elasticity of marginal utility larger than unity, the Green Paradox can in theory be reversed but in our calibrated model the effects are tiny.

5. Exploration costs and partial exhaustion

For climate policy it matters not only to look at speed of extraction but also how much reserves to lock in the crust of the earth forever (e.g., Hoel, 2012; van der Ploeg and Withagen, 2012a; van der Ploeg, 2013). We thus assume that the total recoverable stock of oil $S_1$ depends on initial exploration investment

$$S_1 = G(I),$$

with $G' > 0$, $G'' < 0$, where $I$ denotes exploration investment.\(^7\) Hence, the return on oil exploration falls as less accessible fields have to be explored. Profit maximization gives the Hotelling rule (3) and exploration investment and initial reserves as increasing function of the initial oil price:

$$q_1 G'(I) = 1 \Rightarrow I = G^{-1}(1/q_1) = I(q_1), \quad I' > 0 \text{ and } S_1 = S_1(q_1) \text{ with } S_1'(q_1) > 0.$$ 

With exploration costs, the OME condition (21) becomes

$$R_1(q_1) + R_2(r_2, (1 + r_2)q_1 + \tau_2) = S_1(q_1).$$

In partial equilibrium with given interest rate $r_2$ and capital stocks $K_1$ and $K_2$ an increase in the future carbon tax $\tau_2$ boosts current oil extraction, since the return to conserving oil drops (as in the case without exploration costs). In addition, however, cumulative extraction goes down as the current oil price falls so that the return to exploration becomes lower. We have thus the following proposition.

**Proposition 6 (Abandoning oil reserves – partial equilibrium):** In partial equilibrium a higher future carbon tax $\tau_2$ boosts current oil extraction and curbs cumulative oil extraction.

\(^7\) Assuming that total oil reserves in the crust of the earth are given by $S_0 \geq S_1$, it follows that $S_0 - S_1$ units of oil remain untapped in the market equilibrium, because they are not worthwhile to exploit.
Proof: Given $d\tau > 0$, suppose that $dq > 0$. This would imply that the left-hand-side of (26) goes down, whereas the right-hand-side goes up. Therefore, we need $dq < 0$ so that $dR > 0$ and $dS < 0$. □

Hence, the effect of a future carbon tax on current oil extraction and cumulative oil extraction works in opposite direction. As a result, moving our focus to global warming, there is a trade-off between the speed of emissions and cumulative emissions (cf. van der Ploeg and Withagen, 2012a; van der Ploeg, 2013).

Using the Hotelling rule (3), the OME condition (26), and taking investment in exploration and the dependence of initial oil reserves on the oil price into account, the MGME condition is

$$
\frac{C(r_2, M(r_2, q_1)) + C(r_2, M^*(q_1))}{C_2(r_2, M(r_2, q_1)) + C_2(r_2, M^*(q_1))} = \frac{(1 - \mu)K_1 + F(R(q_1)) - K_2(r_2, S(q_1) - R(q_1)) - I(q_1)}{(1 - \mu)K_2(r_2, S(q_1) - R(q_1)) + F(K_2(r_2, S(q_1) - R(q_1)), S(q_1) - R(q_1))},
$$

where the wealth levels for the oil-importing and exporting-region are, respectively,

$$
M = (1 + r_1)A_1 + F(K_1, R_1(q_1)) - (r_1 + \mu)K_1 + \frac{F(K_2(r_2, q_1), S_1(q_1) - R_2(r_2, q_1)) + \nu_2A^*_2}{1 + r_2} - q_1S_1(q_1),
$$

$$
M^* = (1 + r_1)A^*_1 + q_1S_1(q_1) - I(q_1).
$$

Depending on the relative slopes of the OME and MGME loci, we get as before attenuation, reversal or amplification of the Green Paradox in general equilibrium. The various possible directions of changes in the equilibrium prices are once more described by the panels of Figure 1. Nevertheless, there is an important difference with the analysis in Section 4 and 5: the general equilibrium consequences for the equilibrium price of oil also affect cumulative resource extraction, so that we get the following result.

**Proposition 7 (Abandoning oil reserves – general equilibrium):** If the partial equilibrium effect on current oil extraction of a higher future carbon tax attenuated, reversed or amplified in general equilibrium, then so will be the effect on cumulative oil extraction.

Proof: With attenuation, $q_1$ changes in the same direction, but by less than in partial equilibrium. With reversal, $q_1$ changes in the opposite direction, compared to the partial equilibrium outcome. With amplification, $q_1$ changes in the same direction, but by more than in partial equilibrium. Because $S_1$ depends positively on $q_1$, the same qualifications hold for cumulative extraction. □

For example, consider the case of weakening of the Green Paradox in panel (a) of Figure 1. The current oil price drops by less than in partial equilibrium. As a result, cumulative oil extraction drops by less than in partial equilibrium, so general equilibrium effects weaken both the increase in current oil extraction and the decrease in cumulative oil extraction. Similarly, if the Green Paradox is strongly reversed, as in panel (b) of Figure 1, cumulative oil extraction goes up upon the announcement of a future carbon tax.
Since cumulative extraction directly affects the stock of atmospheric carbon, Proposition 7 implies that the general equilibrium consequences of future carbon taxes on global warming are (partly) offset by the general equilibrium consequences of future carbon taxes on cumulative extraction.

6. Welfare analysis

Here we quantify the general equilibrium welfare effects of a future carbon tax for the oil-importing region, both for full and partial exhaustion. We also compare the effects in general equilibrium with those in partial equilibrium. We present results for the case of Assumption 1 with identical homothetic preferences, \( \rho = \rho^* = 0.05 \) and \( \eta = \eta^* = 1 \). We vary the elasticity of factor substitution from 0.05 to 0.95. In the middle of this range, at \( \sigma = 0.5 \), the oil income share is 8.8 percent, which is in line with the average US energy expenditure share in GDP over the period 1970-2009 (U.S. Energy Information Administration, 2012).\(^8\) Starting from no taxation at all, we consider the introduction of a future carbon tax of 0.1, which roughly corresponds to a tax of 100 US dollars per ton carbon.\(^9\) Damages from carbon emissions in period 1 are \( \psi E_i \) and in period 2 are \( \psi(E_i + E_2) \), where we set \( \psi = 0.1 \) to get damages of about 100 US dollars per ton carbon (the implied marginal utility of consumption is close to 1 in the pre-tax equilibrium). Initial asset endowments are \( A_i = 1 \) and \( A^*_i = 0 \). The exploration function is \( G(I) = \chi \sqrt{I} \), where \( \chi \) is chosen so that \( S_i = 2 \) and we endow the oil-exporting region with an amount equal to the exploration cost that they incur. As a result, we start from an identical pre-tax equilibrium irrespective of whether we have full or partial exhaustion. Figure 5 shows the welfare effects in percentage changes of current consumption.

The total welfare effect (solid line) is decomposed in a ‘green’ effect (dashed line) and ‘grey’ effect (dotted line). Panels (a) and (b) show the results for full exhaustion. The welfare effects for partial exhaustion are given in panels (c) and (d). The left panels show the welfare changes in general equilibrium. With full exhaustion, grey welfare increases as a result of the fall in the oil price. Green welfare, however, falls because of the Green Paradox: extraction is brought forward so that discounted damages rise. The effect on total welfare varies from an increase of over 3 percent of current consumption to a decrease of about 1 percent for low and high values of the elasticity of factor substitution, respectively.

---

\(^8\) Van der Werf (2008) reports estimates for the elasticity of factor substitution varying from 0.17 to 0.61. Our implied resource income share varies from 6.6 to 9 percent over this range.

\(^9\) Take a world output of 75 trillion US2013\$ (World Bank, 2014), oil reserves amounting to 150 billion ton carbon (OPEC, 2013). If \( S_i = 2 \) and the implied value of output \( Y_i = 1 \), we get a tax of 100\$/ton carbon.
Figure 5: Welfare effects of a future carbon tax

Panel (a): Full exhaustion - GE

Panel (b): Full exhaustion - GE vs. PE

Panel (c): Partial exhaustion - GE

Panel (d): Partial exhaustion - GE vs. PE

Notes: $A^* = 1$, $A_x^* = 0$, $\beta = 1/3$, $\eta = \eta^* = 1$, $\lambda = 0.1$, $\mu = 0.1$, $\psi = 0.1$, $\rho = \rho^* = 0.05$, $L = 1$, and $\chi$ is set to get $S_1 = 2$ in the model with partial exhaustion. $\tau_z$ is raised from zero to 0.1. The solid black line gives the total welfare effect, the dashed and dotted lines represent the ‘green’ and ‘grey’ welfare effects, respectively. Panels (a) and (c) show the welfare effect under general equilibrium (GE). Panels (b) and (d) report the difference in welfare changes between general and partial equilibrium (PE).

With partial exhaustion green welfare goes up instead of down due to the decrease in cumulative extraction and locking up more fossil fuel in the crust of the earth: the adverse green welfare effect of more rapid extraction of a given stock of reserves (the Green Paradox) is swamped by the positive welfare effect of locking up more fossil fuel in the ground. The fall in cumulative extraction at the same time dampens the positive effect on grey welfare. Interestingly, the effect on total welfare now remains positive and varies from a 0.8 to a 1.8 percent increase in current consumption.

The two panels on the right of Figure 5 show the difference in welfare changes between general and partial equilibrium. Recall from Figure 3 that the Green Paradox is mitigated (amplified) in general equilibrium if the elasticity of factor substitution is high (low) enough. Accordingly, panel (b) shows that with full exhaustion, climate damages goes down by less in general equilibrium if $\sigma$ is high (where the
dashed line is positive) and by more if $\sigma$ is low (where the dashed line is negative). Panel (d) shows that with partial exhaustion the effect on cumulative oil extraction dominates: at high values, the effect on cumulative oil extraction is mitigated, leading to a smaller green welfare gain in general equilibrium. At low values of $\sigma$ the decrease in cumulative oil extraction is amplified, so that the increase in green welfare is larger in general equilibrium. Both for the specifications with full and partial exhaustion, the difference in green welfare effects between partial and general equilibrium ranges from about minus 0.05 percent to plus 0.1 percent of current consumption. Total welfare is not much affected either. For example, with $\sigma = 0.5$, consumption rises by approximately 1 percent in general equilibrium and by 0.9 percent in partial equilibrium (with full exhaustion). Hence, the ‘mistake’ made by assuming that the interest rate is unaffected upon an increase in the carbon tax is not detrimental to welfare.

7. Conclusion

The Green Paradox implies that a climate policy that becomes more ambitious during the next decades can be regarded as an announced expropriation of oil reserves. Decreased future demand for oil forces oil producers to accelerate oil extraction and carbon emissions and thus to exacerbate the problem of global warming. Since a gradual tightening of climate policy exerts a stronger downward pressure on future prices than on current ones, it curbs the expected rate of capital gains on oil reserves. Oil producers will try to avert this by extracting more quickly and putting the sales revenue into investments in the capital markets, which offer higher yields. Hence, a climate policy which becomes more aggressive over time accelerates carbon emissions. Most of the discussion of the Green Paradox has been cast in a partial equilibrium framework and has taken the interest rate as given. Since the interest rate is the key intertemporal price driving saving and investment decisions as well as oil depletion decisions and since the future oil demand function shifts upon changes in the future capital stock, this seems a serious shortcoming.

We show that the Green Paradox is mitigated in general equilibrium if oil exporters and oil importers have identical, homothetic preferences and if there are no investment possibilities. The mechanism behind this result is simple: the increase in current extraction induces a rise in relative current output. Consequently, there will be an excess supply of current output at the going interest rate. To restore equilibrium, the interest rate needs to fall, which implies that it becomes less attractive to pump oil out of the ground and invest the revenues on the capital market. As a result, oil exporters will slow down their extraction so that the Green Paradox is mitigated.
However, under less restrictive conditions, the Green Paradox might be strengthened instead of weakened in general equilibrium. First, if investment in physical capital is taken into account, the decrease of future extraction may cause a fall in investment given that oil and capital are cooperative in production. As a result, the future resource demand function shifts inwards, so that oil extraction is brought forward, thereby amplifying the partial equilibrium Green Paradox outcome. Second, the Green Paradox may be strengthened if the oil-importing region is relatively more impatient than the oil-exporting region. The reason is that environmental policy induces a wealth reallocation effect between oil producers and oil importers. If oil importers are relatively impatient, this wealth reallocation effect diminishes the excess supply of savings resulting from the increase in the future carbon tax. If this effect is strong enough, excess supply may turn into excess demand for current goods at the prevailing interest rate. In that case, the interest rate needs to rise to restore equilibrium, so that pumping oil becomes more attractive and the partial equilibrium Green Paradox outcome is amplified.

If oil importers are very much more patient than oil exporters, the Green Paradox can even be reversed. The wealth reallocation effect then amplifies the excess supply of current goods resulting from the increase in the future carbon tax. Hence, the interest rate must decrease further than under identical preferences to restore equilibrium. We have shown that under certain conditions (an elasticity of factor substitution close to zero together with an extreme difference in the pure rate of time preference between the two regions, and an elasticity of marginal utility exceeding unity) the decline in the interest rate can be large enough to reverse the partial equilibrium Green Paradox outcome but effects will be tiny. Our numerical examples with factor income shares and elasticities of factor substitution in line with the empirics and with only moderate differences in patience, suggest that mitigation of the Green Paradox is the most likely outcome in a world with CES utility and CES production.\(^\text{10}\)

We have also studied the case of partial exhaustion in the presence of exploration costs. In practice, exploration costs rise as less accessible fields are explored and then it is seldom optimal to fully exhaust all oil reserves. An effective climate policy must thus focus on the supply side of the carbon market as well as on the demand side because it is crucial that not all oil that is in the crust of the earth is burned. We show that in partial equilibrium, a future carbon tax will reduce cumulative emissions. In general equilibrium, however, this effect may be mitigated, amplified or even reversed. In particular, we show that if the change in current extraction is mitigated, amplified or reversed in general equilibrium, so will be the change in cumulative extraction. We find that the adverse welfare effects of the Green Paradox can easily be swamped by the beneficial welfare effects of locking up more fossil fuel in the crust of the earth.

\(^\text{10}\) All our results are valid for non-marginal changes in carbon taxes. Online Appendix A2 uses Cramer’s rule to derive explicit expressions for the marginal effects around an outcome with zero carbon and asset holding taxes. These are in line with the results of our paper.
Our analysis has demonstrated that general equilibrium effects may have important consequences for the Green Paradox. In future research it would be interesting to assess these effects empirically, to evaluate how asset taxes as suggested by Sinn (2008, 2012) fare in general equilibrium and might avoid the Green Paradox altogether, and to explore how these affect the strategic analysis of climate policies.

References


Appendix A1: Proofs

For purposes of the proofs in this appendix, we distinguish between weak and strong reversal.

**Definition A1:** Assume the future carbon tax is increased.

(i) Weak reversal occurs if current oil extraction is unaffected.

(ii) Strong reversal occurs if current oil extraction decreases.

**Proof of Proposition 3:** Strong reversal implies a higher current world market oil price \( q_1 \), to have less demand for current oil, and a lower future world market oil price \((1 + r_2)q_1\), to have more demand for future oil (note that the oil price for the final good producer is \( (1 + r_2)q_1 + r_2 \), which has to fall in spite of a higher tax). Hence, it is immediate from (15) and (19) that demand for current final goods by oil exporters goes up and its demand for future final goods goes down. With lower present production and higher future production, the demand response of oil importers must be the other way around. But this is excluded by (19).

Weak reversal occurs if \( q_1 \) and \( (1 + r_2)q_1 + r_2 \) remain unaffected. Hence the wealth of oil exporters is unaffected as well as its first-period consumption. Its second-period consumption goes down because \( (1 + r_2)q_1 \) goes down. So, current consumption in the oil-importing region stays unaffected and future consumption increases. This is incompatible with (19).

Equality between first-period consumption and production requires \( \alpha M + \alpha^* M^* = F(R_i) \). Using \( M^* = q_1 S \) and \( M = F(R_i) + F(R_2) / (1 + r_2) - q_1 S_1 \), we get

\[
(\alpha^* - \alpha)q_1 S_1 = (1 - \alpha)F(R_i(q_1)) - \alpha F(R_2((1 + r_2)q_1 + r_2)) / (1 + r_2).
\]

From this expression, we see that \( \alpha > \alpha^* \) is a necessary condition for amplification: Occurrence of the Green Paradox implies an increase in \( R_i \) and a decrease in \( R_2 \). Hence, \( q_1 \) must go down. Furthermore, amplification of the Green Paradox requires an increase in \( r_2 \). Therefore, the right hand side of the expression increases. Given that \( q_1 \) goes down, the left hand side can only increase if \( \alpha > \alpha^* \). Finally, given existence, the conditions given in (iii) are sufficient. □

**Proof of Proposition 4:** The existence of the amplification region in panel (a) of Figure 4 proves the first part of the proposition. To prove the second part, we first show that weak reversal cannot occur. Weak reversal of the Green Paradox requires \( dR_i = 0 \) and \( dR_2 = 0 \) (from (21)), therefore \( dr_1 = 0 \) and \( dq_1 = 0 \) (from (1)), and \( dC_1^* = \alpha^* dM^* = \alpha^* dq_1 S_1 = 0 \). Furthermore, note that \( R_2(r_2, q_1(1 + r_2) + r_2) \) decreases if \( r_2 \) and \( q_1(1 + r_2) + r_2 \) increase. Hence, the constancy of \( R_2 \) and \( q_1 \) imply that \( dr_2 < 0 \), so that \( dK_2 > 0 \) (from
Consequently, (9) implies that $d(C_1 + C_1^*) < 0$. Given that $dC_1^* = 0$, this requires $dC_1 < 0$. However, we also have $C_1 = \alpha M = \alpha ((1 + r_1)A_1 + F(K_1, R_1) - (r_1 + \mu)K_1 + [F(K_2, R_2) - (r_2 + \mu)K_2] / (1 + r_2 - q_i S)$. The change in the bracketed term is given by $[F(K_2, R_2) - (r_2 + \mu)] dK_2 - K_2 d\tau_2 = -K_2 d\tau_2$, where the equality uses (2). Therefore, $dC_1 > 0$, so that we get a contradiction.

A consequence of the absence of weak reversal is that the use of first-period oil is monotonic in the second-period tax rate. To exclude strong reversal we thus need to show that for the tax large enough there exists an equilibrium with second-period oil use close to zero, implying first-period oil use close to the total available stock of oil. Hence, first-period oil use is increasing in the tax rate, and no strong reversal occurs. Define $\bar{q}_1$ by $F(K_2, R_2) - (r_2 + \mu) = \bar{q}_1$. Hence, it is the first-period world market oil price such that the entire stock of oil is demanded in the first period, with full employment of all capital. Define $\bar{r}_1$ by $F(K_1, S_1) - (\bar{r}_1 + \mu) = \bar{r}_1 + \mu$. Suppose $r_2 + \mu = 0$. Then, with $K_1 = A_1 + A_1^*$, the equilibrium in this economy can be characterized as follows:

$$C_1 + C_1^* = (1 - \mu)K_1 + F(K_1, S_1) - K_2,$$
$$C_2 + C_2^* = (1 - \mu)K_2.$$

Moreover

$$M = \{(1 + \bar{r}_1)A_1 + F(K_1, S_1) - (\bar{r}_1 + \mu)K_1 - \bar{q}_1 S_1\},$$
$$M^* = \{(1 + \bar{r}_1)A_1^* + \bar{q}_1 S_1\}.$$

We also have

$$C_2 + C_2^* = (1 - \mu)K_2 = (1 - \mu)[(1 - \alpha)M + (1 - \alpha^*)M^*] =
(1 - \alpha)(1 - \mu)[(1 + \bar{r}_1)A_1 + F(K_1, S_1) - (\bar{r}_1 + \mu)K_1 - \bar{q}_1 S_1] + (1 - \alpha^*)(1 + \bar{r}_1)A_1 + \bar{q}_1 S_1].$$

From this final condition we can solve for $K_2$. We now have a set of prices and allocations. It is claimed that this set constitutes the limit of an equilibrium of an economy for the second-period carbon tax going to infinity. Given these prices consumers maximize their utility, subject to their budget constraints. First-period profits are maximized. Demand for final goods as well as for oil equal supply in both periods. The difficulty lies in second-period profit maximization and demand for second-period capital in production. Profits can be written as $F(K_2, R_2) - (r_2 + \mu)K_2 - ((1 + r_2)\bar{q}_1 + \tau_2) R_2$. Given the solution $K_2$ of (29) the optimal oil input goes to zero as $\tau_2$ goes to infinity. Moreover, given zero oil input, the capital stock $K_2$ maximizes profits. This establishes the absence of a strong reversal. □
Online Appendix A2: Tâtonnement and comparative statics around an equilibrium with zero taxes

This appendix discusses the local Walrasian stability of the different equilibria and derives the comparative statics around an equilibrium outcome with zero taxes. The tâtonnement versions of the OME and GME conditions, respectively,

\[ \dot{q}_1 = \dot{\lambda}_1 \left[ R_1(q_1) + R_2 \left( r_2, (1 + r_2)q_1 + \tau_2 \right) - S_1 \right], \]
\[ \dot{r}_2 = \dot{\lambda}_2 \left[ C_1(r_2, M(r_2, q_2)) + C_1(r_2, M'(q_2)) + K_2(r_2, R_2 \left( r_2, (1 + r_2)q_1 + \tau_2 \right)) - F(K_1, R_1(q_1)) - (1 - \mu)K_1 \right], \]

where dots above variables denote changes and the wealth levels for both regions are given by

\[ M = (1 + r_1[R_1(q_1)])A_1 + F[R_1(q_1)] - (r_1[R_1(q_1)] + \mu)K_1 - q_1S_1, \]
\[ F(K_2, R_2 \left( r_2, (1 + r_2)q_1 + \tau_2 \right)) + R_2 \left( r_2, (1 + r_2)q_1 + \tau_2 \right) \]
\[ (r_2 + \mu)K_2(r_2, R_2 \left( r_2, (1 + r_2)q_1 + \tau_2 \right)) - \tau_2 \left( A_1'(r_2, q_1) + q_1R_2 \left( r_2, (1 + r_2)q_1 + \tau_2 \right) \right) \]
\[ 1 + r_2 \]

\[ M' = (1 + r_1[R_1(q_1)])A_1' + q_1R_1 + \frac{q_2}{1 + r_2}R_2 = (1 + r_1[R_1(q_1)])A_1 + q_1S_1. \]

Standard Walrasian tâtonnement implies that the auctioneer would raise oil prices if demand for oil exceeds the supply of oil, hence \( \dot{\lambda}_1 > 0 \). It also implies that the auctioneer would raise the current final goods price which corresponds to a fall in the future final goods price and a rise in \( r_2 \) if current demand for final goods exceeds current supply of final goods, hence \( \dot{\lambda}_2 > 0 \).

Linearizing this system around a steady state with \( \tau_2 = 0 \) we get the following system:

\[
\begin{pmatrix}
\dot{q}_1 \\
\dot{r}_2
\end{pmatrix} = \begin{pmatrix}
-\lambda_1 \omega_q & -\lambda_1 \omega_r \\
\lambda_2 \gamma_q & -\lambda_2 \gamma_r
\end{pmatrix}
\begin{pmatrix}
dq_1 \\
dr_2
\end{pmatrix}
+ \begin{pmatrix}
-\lambda_1 \omega_q \\
-\lambda_2 \gamma_r
\end{pmatrix} d \tau_2,
\]

with the stationary state of this system giving the following steady-state solution

\[ dq_1 = -\frac{\Gamma_{q q}}{|\Delta|} d \tau_2, \quad dr_2 = -\frac{\Gamma_{r r}}{|\Delta|} d \tau_2, \]

where \( |\Delta| = \omega_q \gamma_q + \omega_r \gamma_r \), and we have defined the following coefficients and elasticities:

\[
\gamma_q = \frac{\partial C_1}{\partial M} \left( \frac{Y_2 - (r \cdot \mu)K_2 + K_2(1 + r_2)}{1 + r_2} \right) + \frac{\partial C_1}{\partial M} \left( \frac{\varepsilon_1}{1 + r_2} \right) + \frac{\partial C_1}{\partial M} \left( \frac{\varepsilon_2}{1 + r_2} \right),
\]

\[
\gamma_r = \frac{\partial C_2}{\partial M} \left( \frac{Y_1 - (r \cdot \mu)K_1 + K_1(1 + r_1)}{1 + r_1} \right) + \frac{\partial C_2}{\partial M} \left( \frac{\varepsilon_1}{1 + r_1} \right) + \frac{\partial C_2}{\partial M} \left( \frac{\varepsilon_2}{1 + r_1} \right),
\]

\[
\Gamma_{qq} = \gamma_q \omega_q \gamma_q + \gamma_r \omega_r > 0, \quad \omega_q = \varepsilon_1 R_1 + \varepsilon_2 R_2 > 0, \quad \omega_r = \left( \frac{\varepsilon_1}{1 + r_1} + \frac{\varepsilon_2}{1 + r_2} \right) q R_2 > 0, \quad \omega_q = \frac{\varepsilon_1 R_1}{1 + r_2} > 0,
\]

\[
\Gamma_{qq} = \gamma_q \omega_q - \gamma_q \omega_q = \frac{\varepsilon_1 R_2}{1 + r_2} \left( \frac{\partial C_1}{\partial M} \left( \frac{Y_2 - (r \cdot \mu)K_2 + K_2(1 + r_2)}{1 + r_2} \right) + \frac{\partial C_1}{\partial M} \left( \frac{\varepsilon_1}{1 + r_2} \right) + \frac{\partial C_1}{\partial M} \left( \frac{\varepsilon_2}{1 + r_2} \right) \right),
\]

\[
\Gamma_{qq} = \gamma_q \omega_q + \gamma_q \omega_r \left( \frac{\partial C_1}{\partial M} \left( \frac{\varepsilon_1}{1 + r_2} \right) + \frac{\partial C_1}{\partial M} \left( \frac{\varepsilon_2}{1 + r_2} \right) \right),
\]

\[
\Gamma_{qq} = \gamma_q \omega_q \left( \frac{\partial C_1}{\partial M} \left( \frac{\varepsilon_1}{1 + r_2} \right) + \frac{\partial C_1}{\partial M} \left( \frac{\varepsilon_2}{1 + r_2} \right) \right).
\[ |\Delta| = \omega_q \gamma_q + \omega_{q'}, \quad \theta_k = \frac{(r_2 + \mu)K_2}{Y}(0,1), \quad \theta_q = \frac{q_R R_2}{Y}(0,1), \quad \epsilon_1 = -\frac{\partial R_i(q_i)}{\partial q_i} R_1 > 0, \quad \epsilon_2 = -\frac{\partial R_i(r_i, q_i)}{\partial q_i} R_2 > 0, \]

\[ \epsilon^{i} = -\frac{\partial R_i(r_i, q_i)}{\partial r_i} r_2 + \mu > 0, \quad \eta_q^1 = \frac{\partial r_i(K_i, R_i)}{\partial r_i} R_i > 0, \quad \eta_2 = -\frac{\partial K_i(r_i, R_i)}{\partial r_i} r_2 + \mu > 0, \quad \eta^{i} = \frac{\partial K_i(r_i, R_i) R_i}{\partial r_i} R_2 > 0. \]

The terms in curly brackets drop out if there is no investment, the terms in square brackets drop out if expenditure shares are constant (i.e., if the elasticity of intertemporal substitution (EIS) equals unity in both regions), and the terms in double square brackets drop out if preferences are identical and homothetic. We discuss comparative statics induced by a future carbon tax in four different cases. Stability of the tâtonnement mechanism requires that the Jacobian matrix of the dynamic system for the tâtonnement process, i.e.,

\[ J = \begin{pmatrix} -\lambda_q \omega_q & -\lambda_2 \omega_r \\ \lambda_2 \gamma_q & -\lambda_2 \gamma_r \end{pmatrix}, \]

has two eigenvalues with negative real parts. This requires \( \det(J) = \lambda_q \lambda_2 \left[ \gamma_q \omega_q + \gamma_r \omega_r \right] > 0 \) or \( |\Delta| > 0 \) and \( \text{trace}(J) = -\lambda_q \omega_q - \lambda_2 \gamma_r < 0 \).

**Assumption A1:** \(|\Delta| > 0 \) and \( \lambda_q \omega_q + \lambda_2 \gamma_r > 0, \forall \lambda_q > 0, \lambda_2 > 0 \).

In the comparative statics below, we assume that Assumption A1 is satisfied.

**Identical homothetic preferences, EIS=1 and no investment:** Terms in curly, square and double square brackets drop out, so \( \Gamma_{qr} = \frac{\epsilon^2 R_2}{1 + r_2} \frac{\partial C_1}{\partial r_2} \left( Y_2 - (r_2 + \mu)K_2 \right) > 0, \quad \Gamma_{rr} = \frac{\epsilon^2 R_2}{1 + r_2} > 0. \) Therefore, a future carbon tax pushes down the interest rate and the Green Paradox is attenuated.

**Heterogeneous homothetic preferences, EIS=1 and no investment:** Terms in curly and square brackets drop out, so

\[ \Gamma_{qr} = \frac{\epsilon^2 R_2}{1 + r_2} \frac{\partial C_1}{\partial r_2} \left( Y_2 - (r_2 + \mu)K_2 \right) > 0, \quad \Gamma_{rr} = \frac{\epsilon^2 R_2}{1 + r_2} \left( \frac{\partial C^*_1}{\partial M^*} - \frac{\partial C_1}{\partial M} \right) \left( S_1 + \epsilon_1 R_1 \right). \]

We thus see that \( \Gamma_{rr} \) can become negative if oil exporters are much more patient than oil importers.\(^{11}\) In that case, a future carbon tax pushes up the interest rate and the Green Paradox is amplified.

**Heterogeneous homothetic preferences and no investment:** Terms in curly brackets drop out, so

\[ \Gamma_{qr} = \frac{\epsilon^2 R_2}{1 + r_2} \left( \frac{\partial C_1}{\partial r_2} \left( Y_2 - (r_2 + \mu)K_2 \right) + \frac{K_2}{1 + r_2} \right) \left( \frac{\partial C^*_1}{\partial r_2} + \frac{\partial C^*_1}{\partial r_2} \right), \quad \Gamma_{rr} = \frac{\epsilon^2 R_2}{1 + r_2} \left( \frac{\partial C^*_1}{\partial M^*} - \frac{\partial C^*_1}{\partial M} \right) \left( S_1 + \epsilon_1 R_1 \right). \]

Hence, \( \Gamma_{qr} \) can become negative if the IES < 1 and importers are not too impatient. In that case, the Green Paradox is strongly reversed.\(^{12}\)

**Investment:** The terms in curly brackets appear. Investment makes it more likely for \( \Gamma_{qr} \) to be positive. Hence, strong reversal is less likely with investment.

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\(^{11}\) As in the main text, we use the first-period expenditure share, \( \frac{\partial C_1}{\partial r_2} \), as a measure of impatience.

\(^{12}\) We interpret IES here as a weighted average of the intertemporal elasticities of substitution in both regions, such that IES < 1 if \( \frac{\partial C_1}{\partial r_2} + \frac{\partial C^*_1}{\partial r_2} > 0 \).