

Dahlhaus, Tatjana

**Working Paper**

## Monetary policy transmission during financial crises: An empirical analysis

Bank of Canada Working Paper, No. 2014-21

**Provided in Cooperation with:**

Bank of Canada, Ottawa

*Suggested Citation:* Dahlhaus, Tatjana (2014) : Monetary policy transmission during financial crises: An empirical analysis, Bank of Canada Working Paper, No. 2014-21, Bank of Canada, Ottawa, <https://doi.org/10.34989/swp-2014-21>

This Version is available at:

<https://hdl.handle.net/10419/103034>

**Standard-Nutzungsbedingungen:**

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

**Terms of use:**

*Documents in EconStor may be saved and copied for your personal and scholarly purposes.*

*You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.*

*If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.*



BANK OF CANADA  
BANQUE DU CANADA

Working Paper/Document de travail  
2014-21

# **Monetary Policy Transmission during Financial Crises: An Empirical Analysis**

by Tatjana Dahlhaus

Bank of Canada Working Paper 2014-21

June 2014

# **Monetary Policy Transmission during Financial Crises: An Empirical Analysis**

by

**Tatjana Dahlhaus**

International Economic Analysis Department  
Bank of Canada  
Ottawa, Ontario, Canada K1A 0G9  
dahl@bankofcanada.ca

Bank of Canada working papers are theoretical or empirical works-in-progress on subjects in economics and finance. The views expressed in this paper are those of the author. No responsibility for them should be attributed to the Bank of Canada.

## **Acknowledgements**

I am grateful to Luca Gambetti for his very helpful guidance. I also thank Frank Schorfheide, Fabio Canova, Juan Rubio-Ramirez, Juan Carlos Conesa, Evi Pappa, and Tatevik Sekhposyan as well as seminar participants at the UAB, Bank of Canada, Bank of Mexico, ECB, DIW, Banca d'Italia, and Norway Statistics for helpful comments and suggestions.

## Abstract

This paper studies the effects of a monetary policy expansion in the United States during times of high financial stress. The analysis is carried out by introducing a smooth transition factor model where the transition between states (“normal” and high financial stress) depends on a financial conditions index. Employing a quarterly data set over the period 1970Q1 to 2009Q2 containing 108 U.S. macroeconomic and financial time series, I find that a monetary policy shock during periods of high financial stress has stronger and more persistent effects on macroeconomic variables such as output, consumption, and investment than it has during “normal” times. Differences in effects among the regimes seem to originate from non-linearities in the credit channel.

*JEL classification: C11, C32, E32, E44, G01*

*Bank classification: Financial markets; Econometric and statistical methods;*

*Transmission of monetary policy*

## Résumé

L’auteure étudie les effets d’une politique monétaire expansionniste aux États-Unis en périodes de fortes tensions financières. Pour procéder à son analyse, elle élabore un modèle factoriel à transition lisse dans lequel le passage d’un état à l’autre (d’une situation « normale » à un régime de fortes tensions) dépend d’un indice des conditions financières. À partir de données trimestrielles allant du premier trimestre de 1970 au deuxième trimestre de 2009, soit 108 séries chronologiques de données macroéconomiques et financières américaines, l’auteure observe qu’un choc de politique monétaire a des répercussions plus importantes et persistantes sur des variables macroéconomiques comme la production, la consommation et l’investissement s’il survient en période de fortes tensions plutôt que lorsque la conjoncture est « normale ». L’incidence différente selon le régime (« normal » / fortes tensions) semble attribuable à la non-linéarité du canal du crédit par lequel se transmettent les mesures de politique monétaire.

*Classification JEL : C11, C32, E32, E44, G01*

*Classification de la Banque : Marchés financiers; Méthodes économétriques et statistiques; Transmission de la politique monétaire*

## Non-technical summary

The U.S. economy has experienced several financial crises in the past 40 years. Financial conditions have long been understood to play an important role in macroeconomic dynamics and the recent financial crisis has strengthened interest in exploring the interactions between monetary policy and financial market imperfections. Policy-makers reduced interest rates to historical lows in order to support demand and alleviate the financial stress and its contractionary effects on the economy. Some interesting and natural questions arise: are the effects on economic activity of conventional monetary policy, i.e., decreasing the federal funds rate, different during periods of high financial stress from what is usually observed in good or “normal” times? Have the transmission channels of U.S. monetary policy been different during the financial crises of the past 40 years?<sup>1</sup>

This paper aims to shed light on these questions and contributes to the literature in two ways: first, since the questions at hand demand a non-linear environment, I develop an empirical model, which allows monetary policy shocks to propagate differently in times of low and high financial stress; second, I use this model to study the transmission of a conventional monetary policy shock in the United States during times of low or normal and high financial stress.

The analysis is of interest because the literature has not yet reached a consensus on whether and how responses of macroeconomic variables to monetary policy shocks differ in times of financial stress from what is usually observed in normal times. On the one hand, it has been argued that monetary policy has not been effective during financial crises. This view is motivated by the fact that, during the recent financial crisis, credit standards tightened despite the Federal Reserve decreasing interest rates substantially. On the other hand, one might argue that monetary policy has been effective and actually more potent during financial crises, since it not only lowers interest rates on default-free securities, but also helps to lower credit spreads. This view can be linked to the credit channel of monetary policy or the so-called financial accelerator. The impact of the financial accelerator is assumed to be non-linear, meaning that its effects are stronger the lower the financial conditions of the borrower, i.e., lower net worth and higher cost of credit. Therefore, during times of high financial stress, firms and households would be more sensitive to any change

---

<sup>1</sup>Throughout the paper, I define financial crises as periods of high financial stress.

in their cost of credit induced, for example, by a monetary policy shock.

To study the effects of monetary policy during times of high financial stress in the United States, I employ a data set that contains 108 quarterly series over the period 1970 to 2009. Estimating the empirical model, I find the following main results: first, a monetary policy shock has stronger and more persistent effects on macroeconomic variables such as output, consumption, and investment during financial crises than it has during normal times; second, differences in effects among the two regimes seem to originate from non-linearities in the credit channel.

More specifically, I find that a negative monetary policy shock decreases the external finance premium (EFP) and increases stock prices, i.e., the entrepreneurs' wealth, in the financial crises as well as the normal regime. The effects on the EFP and stock prices are stronger during financial crises. This provides supportive evidence for the existence of a balance-sheet channel that is more pronounced during times of high financial stress. Moreover, an expansionary monetary policy shock increases loans during financial crises while it seems to have no significant effects on loans during times of low financial stress, indicating the potential existence of a bank-lending channel during periods with severe disruptions in financial markets. Summing up, a monetary expansion during financial crises increases loans and asset prices by more than in times of low financial stress, leading to a higher decrease of the EFP which, in turn, provokes the stronger effects of macroeconomic variables such as output, investment, consumption, and employment.

# 1 Introduction

The U.S. economy has experienced several financial crises in the past 40 years. Financial conditions have long been understood to play an important role in macroeconomic dynamics and the recent financial crisis has strengthened interest in exploring the interactions between monetary policy and financial market imperfections. Policy-makers reduced interest rates to historical lows in order to alleviate the financial stress and its contractionary effects on the economy. Some interesting and natural questions arise: are the effects on economic activity of conventional monetary policy, i.e., decreasing the federal funds rate, different during periods of high financial stress from what is usually observed in good or “normal” times? Have the transmission channels of U.S. monetary policy been different during the financial crises of the past 40 years?

This paper aims to shed light on these questions and introduces an empirical model which allows for non-linearities in the dynamic propagation of monetary policy shocks along financial conditions. In particular, I study the transmission of conventional monetary policy during times of low (normal) and high financial stress. So far, little empirical work has been devoted to the investigation of monetary policy transmission during financial crises. Among the very few papers addressing the issue that transmission of shocks may depend on the level of financial stress are Davig and Hakkio (2010) and Hubrich and Tetlow (2012), who both employ a Markov-switching vector autoregression (VAR). Davig and Hakkio (2010) focus on the effects of financial shocks and find that increases in financial stress have had a much stronger effect on the real economy when the economy is in a distressed state. Hubrich and Tetlow (2012) show that macroeconomic dynamics are highly dependent on financial conditions. Moreover, while not explicitly modelling monetary policy, their counterfactual experiments suggest that conventional monetary policy is not particularly effective in times of high financial stress. In this paper, I contribute to the literature in two ways: first, a conventional monetary policy shock is explicitly identified in the model in order to study its effects depending on the level of financial stress; second, the analysis not only establishes the existence of differences in monetary policy transmission during periods of high financial stress, but exploits where these differences stem from.

The economic profession has not yet reached a consensus on whether and how responses of macroeconomic variables to monetary policy shocks differ in times of high financial stress



from what is usually observed in normal times. On the one hand, it has been argued that monetary policy has not been effective during financial crises. This view is motivated by the fact that, during the recent financial crisis of 2008/09, credit standards tightened and the cost of credit increased further, despite the Federal Reserve reducing interest rates substantially (see Krugman (2008)). On the other hand, one might argue that monetary policy has been effective and actually more potent during financial crises because it not only lowered interest rates on default-free securities, but also helped to lower credit spreads. Since shocks from the recent financial crisis were unusually large versions of shocks previously experienced (Stock and Watson (2012)), one might argue that monetary policy was simply not able to offset these massive contractionary shocks (see Mishkin (2009)).

The latter view can be linked to the credit channel of monetary policy or the so-called financial accelerator mechanism (see the seminal contributions of Bernanke and Gertler (1989), Bernanke et al. (1996) and Bernanke et al. (1999)). The key premise of the financial accelerator is the inverse relationship between the premium that borrowers have to pay when they ask for external credit in the banking system (the so-called external finance premium (EFP)) and the financial condition of the borrower. The impact of the financial accelerator is assumed to be non-linear, i.e., its effects are stronger the lower the financial fundamentals such as the net worth of firms or the higher the cost of credit. During normal times, when financial stress is low, firms and households are less sensitive to changes in their cost of credit, whereas during times of high financial stress or crises, firms and households are more sensitive to any change in their cost of credit. Therefore, changes in the net worth induced by, for example, monetary policy may yield large changes in the cost of credit for low-net-worth borrowers (borrowers during financial crises), but should not much affect the cost of credit for borrowers with wide internal finance (borrowers in normal times).<sup>2</sup>

The possibility of a non-linear credit channel has been noted in the literature by, for example, Bernanke and Gertler (1989), Gertler and Gilchrist (1994), and Bernanke et al. (1999). Bernanke et al. (1999), using a two-sector model with the financial accelerator, find that firms with relatively poor access to external credit markets respond more strongly to an expansionary monetary policy shock than do firms with better access to credit. Moreover, Gertler and Gilchrist (1994) provide empirical evidence that the performance of small firms

---

<sup>2</sup>Note that, during financial crises, when asset prices are low firms are likely to be liquidity constrained and in need of external financing.

is more sensitive to interest rate changes the weaker the balance sheets of the firms, suggesting that financial accelerator effects should be stronger the higher the financial stress. Finally, Ravn (2014) introduces a model in which the strength of the financial accelerator is non-linear. The asymmetric financial accelerator is modelled by assuming different values of the elasticity of the EFP with respect to the net worth of firms. This implies that when the entrepreneurs' wealth is already low, such as during financial crises, the EFP reacts more strongly to small changes in net worth.

Since the questions at hand demand a non-linear environment, I introduce the smooth transition factor model (STFM), which is a regime-dependent factor model where the transition between regimes (of normal and high financial stress) is smooth. In contrast to estimating a model for each regime separately, a smooth transition model makes use of more information by exploiting variations in the probability of being in a particular regime so that estimation for each regime is based on a larger set of observations. Moreover, since I am especially interested in exploring where differences in effects of a monetary policy shock stem from, the STFM allows me to obtain conditional regime-dependent responses from a broad set of macroeconomic and financial variables. As a conditioning factor, I use a financial conditions index summarizing information from financial market variables. The STFM is estimated using Bayesian Markov chain Monte Carlo (MCMC) methods, i.e., a Metropolis-within-Gibbs sampler.

Estimating the model using 108 macroeconomic and financial variables over the period 1970Q1 to 2009Q2, I find the following main results:

1. A monetary policy shock has stronger and more persistent effects on macroeconomic variables such as output, consumption, and investment during financial crises than during normal times.
2. Differences in effects among the regimes seem to originate from non-linearities in the credit channel.

More specifically, I find that a monetary expansion decreases the EFP and increases stock prices, i.e., entrepreneurs' wealth, in the financial crises as well as the normal regime. The effects on the EFP and stock prices are stronger during financial crises. This provides supportive evidence for the existence of a balance-sheet channel that is more pronounced

during times of high financial stress. Moreover, an expansionary monetary policy shock increases loans during financial crises while it seems to have no significant effects on loans during times of low financial stress, indicating the potential existence of a bank-lending channel during periods with severe disruptions in financial markets. Summing up, a monetary expansion during financial crises increases loans and asset prices by more than in times of low financial stress, leading to a higher decrease of the EFP which, in turn, provokes the stronger effects of macroeconomic variables such as output, investment, and consumption.

The remainder of this paper is organized as follows: Section 2 introduces the STFM; the Metropolis-within-Gibbs sampler used to estimate the model is described in Section 3; Section 4 reports the empirical results of the analysis; Section 5 concludes.

## 2 The Smooth Transition Factor Model

This section introduces the smooth transition factor model (STFM). The model is based on the dynamic factor model (see, e.g., Forni et al. (2009)), into which I introduce a non-linear relation among the common factors. To allow for responses being different across normal times and times of high financial stress, i.e., financial crises, I employ a smooth transition vector autoregression (STVAR) for the factors (see, for example, Weise (1999) and Auerbach and Gorodnichenko (2012)). The STVAR is a multivariate extension of the smooth transition autoregressive models developed in Granger and Terasvirta (1993).

The key advantage of estimating an STVAR for the factors rather than estimating structural vector autoregressions for each regime separately is that with the latter I may have relatively few observations in a regime, especially for episodes of high financial stress. In contrast, by specifying the factors as an STVAR, the dynamics of the common factors may be determined by both regimes, with one regime having more impact in some times and the other regime having more impact in other times. Therefore, the model makes use of more information by exploiting variations in the probability of being in a particular regime, so that estimation for each regime is based on a larger set of observations.

Let  $x_t = (x_{1t}, x_{2t}, \dots, x_{Nt})'$  be an  $N$ -dimensional vector of macroeconomic variables at

time  $t$ ,  $t = 1, \dots, T$ .  $x_t$  has the following factor model representation:

$$\begin{aligned} x_t &= \chi_t + e_t \\ &= \Lambda' f_t + e_t, \end{aligned} \tag{1}$$

where  $f_t$  is an  $r \times 1$  vector of unobserved “common” or “static” factors and  $\Lambda$  is an  $r \times N$  matrix of factor loadings associated with  $f_t$ . Let  $\Lambda' = [\Lambda'_1 \ \Lambda'_2]'$ , where  $\Lambda_1$  is the first  $r \times r$  matrix of loadings and  $\Lambda_2$  is the remaining  $r \times N - r$  matrix of loadings. I assume that  $\Lambda'_1$  is a lower triangular matrix. The linear combination of the  $r$  factors, i.e.,  $\chi_t = \Lambda' f_t$ , is called the common component of  $x_t$  that is responsible for co-movements between macroeconomic variables.  $e_t$  is the idiosyncratic component of  $x_t$  and it is assumed to be normally distributed and cross-sectionally uncorrelated, i.e.,  $e_t \sim i.i.d.N(0, H)$ .

As mentioned earlier, the factors are assumed to follow an STVAR where the choice between the two regimes is operated by a non-linear transition function,  $F(z_{t-1}, \nu)$ , which takes a value of between zero and one. The dynamic behavior of the factors depends on the observable transition variable,  $z$ , and the transition function parameters,  $\nu$ :

$$\begin{aligned} f_t &= (1 - F(z_{t-1}, \nu)) \sum_{i=1}^p D_i^{(1)} f_{t-i} + F(z_{t-1}, \nu) \sum_{i=1}^p D_i^{(2)} f_{t-i} + u_t \\ \text{Var}(u_t) &= Q, \end{aligned} \tag{2}$$

with  $u_t$  being normally distributed and of constant variance over the two regimes. I assume that the covariance matrix of  $u_t$  is the identity matrix, i.e.,  $Q = I^3$ . Note that differences in the transmission of shocks come from differences in the lag polynomials, i.e.,  $D^{(1)}(L)$  and  $D^{(2)}(L)$ . For estimation purposes, it is useful to rewrite the factors in Equation (2) as

$$f_t = \sum_{i=1}^p D_i^{(1)} f_{t-i} + F(z_{t-1}, \nu) \sum_{i=1}^p \Pi_i f_{t-i} + u_t, \tag{3}$$

---

<sup>3</sup>The assumptions made on the first  $r$  loadings,  $\Lambda'_1$ , and the covariance matrix,  $Q$ , identify the factor model (see, for example, Del Negro and Schorfheide (2011), Otrok and Whiteman (1998) or Lopes and West (2004)). Additionally, I assume that the diagonal elements of  $\Lambda'_1$  are equal to 1 to identify the signs of the factors,  $f_t$ , and the loadings,  $\Lambda'$ . Note that this last restriction is over-identifying the model, since restricting the loadings on the diagonal of  $\Lambda'_1$  to be positive should be enough to identify the factors and loadings. However, in the empirical application of this paper, this set of restrictions worked best in guaranteeing convergence of the latent factors.

where  $\Pi = D^{(2)} - D^{(1)}$  is the contribution to the regression of considering a second regime. Equation (3) can be written as

$$\begin{aligned} D_t(L)f_t &= u_t \\ D_t(L) &= I - \left[ D_1^{(1)} + F(z_{t-1}, \nu)\Pi_1 \right] L - \dots - \left[ D_p^{(1)} + F(z_{t-1}, \nu)\Pi_p \right] L^p. \end{aligned} \tag{4}$$

Therefore, using Equations (1) and (4), macroeconomic variables can be described as

$$x_t = \Lambda'(D_t(L))^{-1}u_t + e_t, \tag{5}$$

where  $u_t$  are the economic shocks and  $B_t(L) = \Lambda'(D_t(L))^{-1}$  are the associated impulse-response functions which are varying over time. Note that, assuming  $F(z_{t-1}, \nu) = 1$  or  $F(z_{t-1}, \nu) = 0$ , I can obtain the impulse-response function of each extreme regime.

The regime changes are assumed to be captured by a logistic smooth transition function:

$$F(z_t, \nu) = \frac{1}{1 + \exp(-\gamma(z_t - c))}, \tag{6}$$

where  $\nu = (\gamma, c)$  is a vector containing the transition function parameters. The parameter,  $\gamma > 0$ , is responsible for the smoothness of the function,  $F$ . For high values of  $\gamma$ , the model switches sharply at a certain threshold. The location or threshold parameter,  $c$ , is the point of inflection of the function, and therefore it is the threshold around which the dynamics of the model change. The transition variable,  $z$ , is observable and normalized to unit variance so that  $\gamma$  is scale invariant. Moreover,  $z$  is dated by  $t-1$  to avoid contemporaneous feedback.

### 3 Estimation

The parameters and unobserved latent factors of the STFM are estimated using a Metropolis-within-Gibbs sampling procedure. The Gibbs sampler allows me to sample from conditional distributions for a subset of the parameters conditional on all the other parameters, instead of sampling from the joint posterior distribution, which would be a rather complex problem. Note that, conditional on  $\nu = (\gamma, c)$ , the STFM becomes a linear factor model. Bayesian estimation of linear factor models is discussed in, for example, Del Negro and Schorfheide (2011) and Otrok and Whiteman (1998). Therefore, my Gibbs sampling procedure re-

duces to three main blocks. First, I draw the factors,  $f_t$ , using the standard algorithm for state space models of Carter and Kohn (1994) given the model's parameters  $\gamma$ ,  $c$ ,  $H$ ,  $\Lambda$ ,  $D_1^{(1)}, \dots, D_p^{(1)}$ ,  $\Pi_1, \dots, \Pi_p$ . In the second block, I draw the linear parameters  $H$ ,  $\Lambda$ , and  $D_1^{(1)}, \dots, D_p^{(1)}$ ,  $\Pi_1, \dots, \Pi_p$ . Conditional on the factors, Equation (1) represents just  $N$  normal linear regression models. Given the factors, Equation (3) also becomes a normal multivariate regression. In the third block, the transition function parameters  $\gamma$  and  $c$  are drawn using a Metropolis-Hastings step.

### 3.1 Notation

Before reporting further estimation details, it is useful to rewrite the STFM. Define the  $1 \times rp$  vector  $w_t = (f'_{t-1}, \dots, f'_{t-p})$  and the  $1 \times 2rp$  vector  $w_t^{ST} = (w_t F(z_{t-1}, \nu) w_t)$ . Next, I stack the vectors over  $t$  to generate the  $T \times r$  matrix  $F = (f_1, f_2, \dots, f_T)'$ , the  $T \times 2rp$  matrix  $W^{ST} = (w_1^{ST}, \dots, w_T^{ST})'$ , the  $2rp \times r$  matrix  $D = (D_1^{(1)}, \dots, D_p^{(1)}, \Pi_1, \dots, \Pi_p)'$ , the  $T \times N$  matrix  $X = (x_1, \dots, x_T)'$ , the  $T \times N$  matrix  $E = (e_1, \dots, e_T)'$ , and the  $T \times r$  matrix  $U = (u_1, \dots, u_T)'$ :

$$X = F\Lambda + E \tag{7}$$

$$F = W^{ST}D + U, \tag{8}$$

where  $E \sim N(0, H)$  and  $U \sim N(0, Q)$ , for  $t = 1, \dots, T$ . Vectorizing Equations (1) and (3), the STFM is transformed into

$$x = (I_N \otimes F)\lambda + e \tag{9}$$

$$f = (I_r \otimes W^{ST})d + u, \tag{10}$$

where  $x = \text{vec}(X)$ ,  $\lambda = \text{vec}(\Lambda)$ ,  $d = \text{vec}(D)$ ,  $e = \text{vec}(E)$ ,  $f = \text{vec}(F)$ ,  $u = \text{vec}(U)$ ,  $e \sim N(0, H \otimes I_T)$  and  $u \sim N(0, Q \otimes I_T)$ . Therefore, the likelihood of the STFM can be shown to be of the standard normal Wishart form. See Appendix A for details.

### 3.2 Block I: Factors

In the first block of the Gibbs sampler, I draw the factors conditional on all the parameters, i.e., the linear and the transition function parameters. Before I apply the algorithm developed by Carter and Kohn (1994), in order to draw from the posterior distribution of the factors the STFM has to be rewritten in state space form:

$$x_t = \underbrace{\begin{pmatrix} \Lambda & 0 & \cdots & 0 \end{pmatrix}}_{\bar{\Lambda}} \underbrace{\begin{pmatrix} f_t \\ f_{t-1} \\ \vdots \\ f_{t-p+1} \end{pmatrix}}_{\bar{f}_t} + e_t \quad e_t \sim N(0, H) \quad (11)$$

$$\underbrace{\begin{pmatrix} f_t \\ f_{t-1} \\ \vdots \\ f_{t-p+1} \end{pmatrix}}_{\bar{f}_t} = \underbrace{\begin{pmatrix} D_{1t} & D_{2t} & \cdots & D_{pt} \\ I & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & I & 0 \end{pmatrix}}_{\bar{D}_t} \underbrace{\begin{pmatrix} f_{t-1} \\ f_{t-2} \\ \vdots \\ f_{t-p+2} \end{pmatrix}}_{\bar{f}_{t-1}} + u_t \quad u_t \sim N \left( 0, \underbrace{\begin{pmatrix} Q & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{pmatrix}}_{\bar{Q}} \right),$$

where  $D_{it} = [D_i^{(1)} + F(z_{t-1}, \nu)\Pi_i]$  for  $i = 1, \dots, p$ . I am therefore ready to draw the factors using the Kalman filter and smoother algorithm developed by Carter and Kohn (1994). Details on the procedure are provided in Appendix A.

### 3.3 Block II: Linear Parameters

In the second block, I draw the linear parameters, i.e., the loadings, the idiosyncratic variances, and the regime-dependent coefficients. Given the factors and the transition function parameters, the STFM is a linear factor model. Therefore, drawing from the conditional distribution of the linear parameters requires that these parameters be split into the two parts that refer to the observation equation in (9) and to Equation (10), respectively. The blocks can be sampled independently from each other conditional on the extracted factors, the data, and the transition function parameters.

## Loadings and Idiosyncratic Variances

First, I draw the loadings,  $\Lambda$ , and the idiosyncratic variances,  $H$ . Conditional on the factors and the transition function parameters,  $\nu$ , Equation (9) represents simply  $N$  normal linear regression models, and therefore the conditional posterior distributions are of standard forms. Since the idiosyncratic errors,  $E$ , are independent across equations, the sampling can be implemented one equation at a time. Note that the subscript  $n$  refers to the  $n$ -th equation and that  $\sigma_n^2$  denotes the  $n$ -th diagonal element of  $H$ .

Using an improper prior for the idiosyncratic variances might result in a Bayesian analogue of the Heywood problem, which takes the form of multi-modality of the posterior of  $\sigma_n^2$  with one mode lying at 0 (Martin and McDonald (1975)). To avoid this potential problem, I assume natural conjugate priors:

$$\begin{aligned}\lambda_n &\sim N(\underline{\lambda}_n, \sigma_n^2 \underline{V}_n) \\ \sigma_n^2 &\sim IG(\underline{\nu}/2, \underline{\delta}/2).\end{aligned}$$

The variance of the normal prior for  $\lambda_n$  depends on  $\sigma_n$  because this allows joint drawing of  $\Lambda$  and  $\sigma_1, \dots, \sigma_N$ .

Combining the likelihood of the STFM, which is given in Appendix A, with the prior distributions, I obtain the following posterior distributions for  $\Lambda$  and  $\sigma_n^2$ :

$$\begin{aligned}\lambda_n | \sigma_n^2, X, F, \nu &\sim N(\bar{\lambda}_n, \sigma_n^2 \bar{V}_n) \\ \sigma_n^2 | X, F, \nu &\sim IG(\bar{\nu}/2, \bar{\delta}/2),\end{aligned}$$

where

$$\begin{aligned}\bar{\lambda}_n &= \bar{V}_n(\underline{V}_n^{-1} \underline{\lambda}_n + F' F \hat{\lambda}_n) \\ \bar{V}_n &= (\underline{V}_n^{-1} + F' F)^{-1} \\ \bar{\delta} &= \underline{\nu} \underline{\delta} + \hat{e}'_n \hat{e}_n + (\hat{\lambda}_n - \underline{\lambda}_n)' (\underline{V}_n + (F' F)^{-1})^{-1} (\hat{\lambda}_n - \underline{\lambda}_n) \\ \bar{\nu} &= T + \underline{\nu}.\end{aligned}$$

Variables with a hat refer to their respective ordinary least-squares estimates, i.e.,  $\hat{\lambda}_n =$



$(F'F)^{-1}F'x_n$  and  $\hat{e}_n = x_n - F\hat{\lambda}_n$ .

### Regime-Dependent Coefficients

The second part refers to Equation (10), which is the STVAR of the factors. As mentioned earlier, conditional on the factors and the transition function parameters, Equation (8) becomes a multivariate regression and, hence, conditional posterior distributions can be obtained in their standard forms. Since I assume the covariance matrix,  $Q$ , to be the identity matrix, I only need to draw the regime-dependent coefficients, i.e.,  $D_1^{(1)}, \dots, D_p^{(1)}$ ,  $\Pi_1, \dots, \Pi_p$ .

Since I do not have any strong a priori beliefs about the parameter values of  $D$ , I choose the uninformative Jeffreys prior. Combining the likelihood with the prior, I obtain a standard posterior distribution for  $vec(D) = d$ :

$$\begin{aligned} d|Q, X, F, \nu &\sim N(\bar{d}, \bar{V}_d) \\ \bar{d} &= vec\left((W^{ST'}W^{ST})^{-1}W^{ST'}F\right) \\ \bar{V}_d &= Q \otimes (W^{ST'}W^{ST})^{-1}. \end{aligned}$$

To ensure stationarity, I truncate the draws of  $D^{(1)}$  and  $D^{(2)} = \Pi + D^{(1)}$ . I discard the draws whenever not all of the roots of the characteristic polynomials  $D^{(1)}(L)$ ,  $D^{(2)}(L)$ , as well as  $D_t(L)$ , lie outside the unit circle.

### 3.4 Block III: Transition Function Parameters

In the last block of the Gibbs sampler, I draw the transition function parameters, i.e., the smoothness parameter,  $\gamma$ , and the threshold value,  $c$ . Since their full conditional distribution has no known form, I update them jointly using the Metropolis-Hastings algorithm following Lopes and Salazar (2006). I assume similar priors as proposed by Lopes and Salazar (2006):

$$\begin{aligned} \gamma &\sim G(a, b) \\ c &\sim N(m_c, \sigma_c^2). \end{aligned}$$

For given starting values  $(\gamma^{(0)}, c^{(0)})$  at iteration  $s$  of the Gibbs sampler, the Metropolis-Hastings algorithm consists of two steps.

Step 1:

I generate draws from the following proposal densities:

$$\begin{aligned}\gamma^* &\sim G\left((\gamma^{(s-1)})^2/\Delta_\gamma, \Delta_\gamma/\gamma^{(s-1)}\right) \\ c^* &\sim N\left(c^{(s-1)}, \Delta_c\right).\end{aligned}$$

The normal proposal density is truncated at the interval  $[c_A, c_B]$  with  $\hat{F}(c_A) = 0.05$  and  $\hat{F}(c_B) = 0.95$ , where  $\hat{F}$  is the empirical distribution of the transition variable,  $z$ .

Step 2:

I set  $(\gamma^{(s)}, c^{(s)}) = (\gamma^*, c^*)$  with probability  $\alpha(\gamma^*, c^*|\gamma^{(s-1)}, c^{(s-1)}) = \min\{1, A\}$ , where

$$\begin{aligned}A &= \frac{\prod_{t=1}^T p_N(x_t|f_t^*, \Lambda^*, D^{(1)}(L)^*, D^{(2)}(L)^*, Q^*)}{\prod_{t=1}^T p_N(x_t|f_t, z_{t-1}, \Lambda, D^{(1)}(L), D^{(2)}(L), Q)} \frac{p_G(\gamma^*|a, b)p_N(c^*|m_c, \sigma_c^2)}{p_G(\gamma^{(s-1)}|a, b)p_N(c^{(s-1)}|m_c, \sigma_c^2)} \\ &\times \frac{p_G(\gamma^{(s-1)}|(\gamma^*)^2/\Delta_\gamma, \Delta_\gamma/\gamma^*)}{p_G(\gamma^*|(\gamma^{(s-1)})^2/\Delta_\gamma, \Delta_\gamma/\gamma^{(s-1)})} \frac{\Phi\left(\frac{c_B - c^{(s-1)}}{\sqrt{\Delta_c}}\right) - \Phi\left(\frac{c_A - c^{(s-1)}}{\sqrt{\Delta_c}}\right)}{\Phi\left(\frac{c_B - c^*}{\sqrt{\Delta_c}}\right) - \Phi\left(\frac{c_A - c^*}{\sqrt{\Delta_c}}\right)}.\end{aligned}$$

Otherwise  $(\gamma^{(s)}, c^{(s)}) = (\gamma^{(s-1)}, c^{(s-1)})$ . Therefore,  $A$  is the product of the likelihood ratio, the prior ratio, and the proposal ratio.  $p_N$  and  $p_G$  denote the probability density function of the normal and gamma, respectively.  $\Phi(\cdot)$  denotes the cumulative distribution function of the standard normal distribution.  $\Delta_\gamma$  and  $\Delta_c$  are adjusted on the fly for the burn-in period to generate an acceptance probability, which lies between 10% and 50%.

## 4 Empirical Results

In this section, I report the results of the analysis. First, I discuss the choice of the transition variable. Then, the data, the model specification, the identification of the monetary policy shock, and the choice of the prior hyperparameters are described. Next, I analyze the convergence of the proposed Gibbs sampler algorithm, followed by estimation results of the transition function. Finally, I report results for the persistence of shocks among the different regimes and regime-dependent impulse responses to the monetary policy shock.

## 4.1 Transition Variable

The transition variable,  $z$ , is chosen to be an index that reflects tensions in various credit markets. A broad measure of financial stress is preferred, rather than a market-specific measure, since using only one measure of financial stress may ignore potential stresses in other financial markets. To this end, I use the financial conditions index (FCI), recently constructed by Hatzius et al. (2010). This FCI has several advantages. First, it covers a large span of history. Second, the FCI pools information across 45 financial indicators, whereas other indices are relatively narrow and might exclude potentially important indicators. Third, the FCI is purged of endogenous movements related to business cycle fluctuations or monetary policy. Figure 1 shows a time-series plot of the FCI. An increase in the FCI corresponds to an improvement in overall financial conditions, while a decrease reflects a worsening. Therefore,  $z_{t-1} < c$  indicates the high financial stress (financial crises) regime, with  $D^{(1)}(L)$  describing the behavior of the system in that regime, i.e.,  $F(z_{t-1}, \nu) \approx 0$ . For sufficiently high values of the FCI, i.e.,  $z_{t-1} > c$ ,  $D^{(2)}(L)$  describes the behavior of the system in the low financial stress (normal) regime, i.e.,  $F(z_{t-1}, \nu) \approx 1$ .

## 4.2 Data and Model Specification

The data set consists of 108 U.S. quarterly series over the period 1970Q1 to 2009Q2. The sample starts in 1970 since the FCI (the transition variable) is not available for earlier dates. Moreover, I stop the sample in 2009Q2 to exclude the zero lower bound period, since the analysis focuses on assessing the effects of conventional monetary policy. The series of the data set are listed in Table 1 and are mostly obtained from the Federal Reserve Economic Data. Some financial variables are obtained from International Financial Statistics and the flow of funds account provided by the Federal Reserve Board of Governors. The series include national accounts data such as GDP, consumption, investment and the GDP deflator; labor market variables such as hours worked, unemployment rate, employment, prices, industrial production; public sector variables; and financial variables such as stock market returns and realized volatilities,<sup>4</sup> spreads, and interest rates. All data series are transformed

---

<sup>4</sup>The realized or historical volatility is calculated as the standard deviation of the daily equity index, i.e.,  $\hat{\sigma}_t = \sqrt{\frac{1}{T_t-1} \sum_{\tau_t=1}^{T_t} (r_{\tau_t} - \mu_t)^2}$ , where  $r_{\tau_t}$  is the price index at time  $\tau_t$  in quarter  $t$  and  $\mu_t$  is the average of the index over quarter  $t$  (see Bloom (2009)).

to reach stationarity. A list of transformations applied to the variables is given in the fourth column of Table 1. Moreover, I set the number of factors to 4. Since the number of factors determines the number of structural shocks in the STFM framework described here, this choice corresponds to my preferred monetary policy identification scheme (see Section 4.3). However, I experimented with specifications containing 3, 5 or 6 factors and the results I obtained are very similar. Finally, I report results based on a lag length of one quarter, i.e.,  $p = 1$ . I tried several versions with different lag lengths, but the results compared to the ones reported here do not differ much.<sup>5</sup>

### 4.3 Identification of the Monetary Policy Shock

Following most of the literature, I assume a recursive or Cholesky scheme to identify a conventional monetary policy shock (e.g., Sims (1992), Bernanke et al. (2005), and Christiano et al. (1999)). Equation (5) in Section 2 becomes a representation of the macroeconomic variables,  $x_t$ , in terms of structural shocks assuming a Cholesky identification scheme. Therefore, the choice and order of the first  $r$  variables of my data set become relevant for identification. Ordering the federal funds rate fourth in my data set, I assume that a monetary policy shock has no contemporaneous effects on the first three variables of the data set. I order GDP, the GDP deflator, and the CPI before the federal funds rate. Consequently, under this ordering, GDP and prices react only with a lag to a monetary policy shock. The CPI is ordered before the federal funds rate, since its inclusion can mitigate the price puzzle (Eichenbaum (1992)).

### 4.4 Prior Choices

In choosing the hyperparameters for the priors of the loadings and idiosyncratic variances, first, following, for example, Lopes and West (2004) and Bernanke et al. (2005), I choose a vague prior for  $\lambda_n$ , i.e.,  $\underline{\lambda}_n = 0_{r \times 1}$ ,  $\underline{V}_n = I_r$ ; second, the hyperparameters for  $\sigma_n^2$  are defined as  $v = 0.001$  and  $\delta = 6$  (see, for example, Otrok and Whiteman (1998) and Bernanke et al. (2005)). These values produce a quite diffuse though proper prior.

Concerning the priors of the transition function parameters  $\gamma$  and  $c$ , the Gibbs sampling

---

<sup>5</sup>Note that, with increasing lag length, the number of degrees of freedom in each regime decreases. To avoid the curse of dimensionality problem (especially in the high financial stress regime), the model is estimated using a higher prior mean of the threshold parameter.

algorithm is sensitive to the choice of hyperparameters. In particular, if the priors are not sufficiently informative, the algorithm has difficulties converging. Therefore, I choose relatively informative hyperparameter values in order to obtain well-mixing Markov chains and an adequate acceptance rate. Typically, the data are not very informative on the shape parameter, since a wide range of values of  $\gamma$  will lead to similar shapes of the transition function (Deschamps (2008) and Teräsvirta (1994)). Here I choose the hyperparameters,  $a$  and  $b$ , of the prior for  $\gamma$  such that the mean of  $\gamma$  is equal to 3 and the variance is equal to 0.1. To set the values of the hyperparameters for  $c$ , consider Figure 1, which plots the FCI. Since I am interested in high financial stress episodes/financial crises, it seems intuitive that I choose the prior mean of  $c$  in such a way that mainly financial crises episodes are considered for the estimation of  $D^{(1)}$ . Therefore, I choose  $m_c = -1.35$  and  $\sigma_c^2 = 0.01$ . This allows me to partly exclude the low values of the FCI at the beginning of the 1980s that originated from monetary policy that was responsible for very high interest rates. Nevertheless, since the threshold parameter provides the point around which the dynamics of the model change, I also experiment with choosing different prior means of  $c$ , i.e.,  $m_c = \{-1.2, -1.1, -1.0, -0.9, -0.8\}$ . Results are robust to these prior choices.

## 4.5 Convergence

To make sure that the results are based on converged simulations, I follow various strategies. First, all my results are based on 20,000 draws of the Metropolis-within-Gibbs sampler, where the first 4,000 draws are discarded as burn-in. Second, I thin the draws by considering only every fourth draw to reduce possible autocorrelations of the sequence (since I have a Metropolis-Hastings step). Third, the Gibbs sampler is run several times to compare the results obtained each time assuring that the chain is converging to the same stationary distribution. Moreover, I assess convergence visually by checking the trace plots, which show the evolution of draws of the parameters and the log-likelihood. This is helpful for checking whether there are jumps in the level and variance of the respective parameter. Furthermore, to assure that the Gibbs sampler has moved to its target distribution, I plot the estimated factors obtained in the first half of simulations against the ones obtained in the second half of the sampler (see Figure 2). Small deviations show that the simulated chain has converged. Finally, to assess the precision of the Gibbs sampler, I plot the estimated

factors with their 95% confidence bands (Figure 3). Since the estimated factors of the first and second halves of the simulations are nearly identical and the bands of the estimated factors are tight, the chain seems to converge properly.

#### 4.6 Weight on Financial Crises Regime

The time profiles of the transition function,  $F(z_{t-1}, \nu)$ , give a first insight into the regime-changing behavior of the model. The transition function can be interpreted as the weight on the normal and financial crises regimes, respectively, where values close to one correspond to the normal regime and values close to 0 correspond to the high financial stress regime.

First, I report the estimates of the transition function parameters, i.e., the smoothness parameter,  $\gamma$ , and the threshold value,  $c$ , which are calculated as the medians of the respective parameter draws kept after thinning. As expected, the estimated values of these parameters are close to their prior values. That is, the estimate for  $\gamma$  is equal to 2.860 with the associated 95% confidence interval being [2.276, 3.566], and the estimate of the threshold value,  $c$ , is equal to -1.351 with the 95% confidence interval being [-1.370, -1.331].

Using these parameter simulations, Figure 4 plots the estimated transition function with its 95% confidence bands. Since the confidence intervals of  $\gamma$  and  $c$  are relatively small, the bands for the transition function are tight. Most of the time, the model is in the normal regime ( $F(z_{t-1}, \nu)$  is close to 1). This is not surprising, since financial crises are rather infrequent events. Four high financial stress/financial crises periods can be identified where the regime changes are rather abrupt. The first period in 1974 corresponds to the so-called Franklin National crisis, when the failure of the Franklin National Bank and the Herstatt Bank led to a crisis of confidence that brought the international banking system close to disaster. The second period of high financial stress in 1987 corresponds to Black Monday, when stock markets crashed worldwide. Another regime change from the normal to the financial crises regime occurs around 2001 and matches up well with the burst of the dot-com bubble. The last period where the transition function reaches values close to zero corresponds to the recent financial crisis of 2008/2009. It is worth mentioning that the first three high financial stress episodes were rather short-lived, while for the recent crisis the model stays longer in the high financial stress regime.

## 4.7 Persistence among Regimes

One may expect to find a difference in the persistence of effects between the normal and the financial crises regimes. The persistence of effects depends on the eigenvalues of the companion form of the regime-dependent coefficient matrices. More specifically, the closer the eigenvalues are to unity, the higher is the persistence of effects following shocks. In the case of greater-than-unity eigenvalues, the impulse responses would be explosive. Note that the latter case is automatically excluded, since I discard draws of the coefficient matrices that are associated with eigenvalues bigger than or equal to one.

First, I report the results for the maximum eigenvalues of the extreme regime lag polynomials,  $D^{(1)}(L)$  and  $D^{(2)}(L)$ , that correspond to  $F(z_{t-1}, \nu) = 0$  and  $F(z_{t-1}, \nu) = 1$ , respectively. The median of the maximum eigenvalue associated with the draws of  $D^{(1)}(L)$  kept after thinning is equal to 0.928, with the 68% confidence interval being [0.821,0.979]. Moreover, for the draws of  $D^{(2)}(L)$ , the median of the maximum eigenvalue is 0.644 and its confidence interval is given by [0.551,0.742]. It seems that the persistence of shocks indeed varies in the two regimes.

Next, the maximum eigenvalues associated with  $D_t(L)$  are obtained for each draw kept. This provides me with a plot of the median and 68% confidence interval of the maximum eigenvalues for each point in time  $t$  (see Figure 5). Whenever the model is in the financial crises regime (i.e., in 1974, 1987, 2001 and 2008/2009), the plot shows a high spike for the maximum eigenvalue. Therefore, it seems that shocks have more persistent effects during financial crises than in normal times.

## 4.8 Monetary Policy Shock

Impulse-response functions are the key statistics to shed light on whether monetary policy transmission is different during normal times and times of high financial stress. This section reports the results for the regime-dependent impulse responses of an expansionary monetary policy shock. Note that I construct impulse responses to monetary policy shocks conditional on a given regime. In other words, I assume that once the system is in a regime, it can stay in that regime for a long time.<sup>6</sup> Moreover, as in Auerbach and Gorodnichenko (2012),

---

<sup>6</sup>The advantage of this approach is that, once a regime is fixed, the model is linear and hence impulse responses are not functions of history (for details, see Koop et al. (1996)). Nevertheless, allowing for feedback from changes in  $z$  into the dynamics of macroeconomic variables, I consider an interesting issue, which is

impulse responses for the normal and financial crises regimes are constructed using the extreme regime lag polynomials  $D^{(2)}(L)$  and  $D^{(1)}(L)$ , respectively. Therefore, impulse responses correspond to weights equal to  $F(z_{t-1}, \nu) = 1$  in the case of the normal regime and  $F(z_{t-1}, \nu) = 0$  in the case of the financial crises regime.<sup>7</sup> Note that  $F(z_{t-1}, \nu) = 0$  corresponds to the estimated weights during the recent financial crisis (and during Black Monday). Hence, one could interpret the responses of the financial stress regime as responses to a monetary policy expansion during the financial crisis of 2008/2009.

Figures 6, 7, 8, 9, and 10 show the median impulse responses (of the draws kept after thinning) to a monetary policy shock decreasing the federal funds rate by one standard deviation. Solid lines indicate responses in the financial crises regime, while dashed lines indicate responses in the normal regime. The dotted lines are the respective 68% confidence bands.

Figure 6 shows the regime-dependent responses of the federal funds rate, GDP, prices and employment. One of the most striking results is that an expansionary monetary policy shock has a larger effect on GDP during financial crises than during normal times. In the financial stress regime, GDP significantly and persistently increases, reaching the maximum rise, of about 2.5%, after around a year. The shape of the response of GDP in times of low financial stress is very similar to that in the financial crises regime. However, there is a difference in the maximum increase that is reached at about 0.6% in the low financial stress (normal) regime. Moreover, prices increase significantly in the normal regime while they increase for only about two quarters in the financial crises regime. After that the effects on prices become insignificant in the high financial stress regime. Although I cannot observe the so-called “price puzzle” in times of low financial stress, there seems to be a version of it in times of high financial stress, since effects are non-significant after about two quarters. Concerning employment, a monetary expansion has positive and persistent effects on employment in the financial stress regime (after a small negative impact effect), while having no significant effects in the normal regime.

To further analyze the differences among the two regimes, Figure 7 shows the impulse responses of consumption, investment, industrial production, and new orders. Most notably,

---

left for further research.

<sup>7</sup>I also calculated impulse responses assuming weights between 0.8 and 1, i.e.,  $0.8 \leq F(z_{t-1}, \nu) \leq 1$ , for the normal regime and between 0 and 0.2, i.e.,  $0 \leq F(z_{t-1}, \nu) \leq 0.2$  for the financial crises regime. The results obtained are similar.



for all four variables the responses to an expansionary monetary policy shock are stronger in high financial stress periods than in periods of low financial stress. As for consumption, during financial crises a decrease in the federal funds rate increases consumption permanently by 1.5%, but by only about 0.6% in periods of low financial stress. Investment reacts with a maximum effect of about 6% in the financial crises state and 1.5% in the normal state. The responses of industrial production and new orders follow similar patterns. Summing up, variables from the business cycle seem to respond more extremely to a monetary policy shock during financial crises than during normal times.

So far, it is not clear where these differences in impulse responses come from. To gain insight into what might cause the differences in the transmission of the monetary policy shock, I plot impulse responses for credit spreads, stock market indices and their corresponding realized volatilities in Figures 8 and 9, respectively. Additionally, I consider the responses of bank loans to a monetary expansion (Figure 10).

The two upper rows of Figure 8 show the regime-dependent impulse responses of the spread between the bank prime loan rate and the 3-month T-bill rate (95), and the spread between the bank prime loan rate and the 6-month T-bill rate (96). These spreads measure the premium that entrepreneurs have to pay when they ask for credit in the banking system (i.e., when they raise funds externally), and have been used in the literature as a proxy for the external finance premium (EFP) (see Bernanke et al. (1999)). An expansionary monetary policy shock has no significant impact effect on the EFP and then significantly decreases the EFP during times of normal as well as high financial stress. The maximum decrease is reached at about three quarters. Strikingly, the EFP reacts more extremely to a monetary expansion in the financial crises regime than in the normal regime, i.e., during financial crises the EFP decreases by twice as much as it does during low financial stress periods. Moreover, the response of the EFP in the financial crises regime reverts back to the pre-shock level about two quarters later than the response of the normal regime. Therefore, a monetary expansion during financial crises decreases the cost of external funding by more than in times of low financial stress.

Additionally, Figure 8 shows the responses of the TED spread (103), which is a measure of the risk of default on interbank loans (counterparty risk) and therefore proxies the premium that banks have to pay when lending to each other. After an expansionary mon-

etary policy, the TED spread decreases on impact and reverts back to the pre-shock level after about a year in both regimes. The responses of the normal and the financial crises regimes are nearly identical in the case of the TED spread. This suggests that even though a monetary expansion decreases the counterparty risk in both regimes, its effects are not asymmetric. Finally, the last row of Figure 8 shows the regime-dependent impulse response of the Baa-Aaa spread (94), i.e., the corporate bond spread. A monetary policy shock increases the spread slightly on impact in both regimes. While in the normal regime the response of the corporate bond spread stabilizes at the pre-shock level after about a year, the response of the financial stress regime turns negative after about two quarters and reverts back to the pre-shock level after about two years. Therefore, the monetary expansion decreases the cost of bond financing via lower-rated bonds during times of financial stress.

Figure 9 shows the regime-dependent impulse responses of the S&P 500 and NYSE price index, and of their respective realized volatilities. An expansionary monetary policy shock significantly and permanently increases both stock price indices. The S&P 500 index increases by about 2% on impact in both regimes. While the response of the financial crises state reaches its maximum increase of about 7.5% after a year and then stabilizes at a little bit less than 7%, the response of the normal state stabilizes at around 5% after two quarters. A very similar pattern arises for the regime-dependent responses of the NYSE index. Since the stock market indices can be seen as a proxy for entrepreneurs' wealth, a monetary expansion during financial crises increases the worth of a firm by more than it does during periods of low financial stress. Moreover, a monetary policy shock seems to have no effects on the volatility of stocks in the normal regime, since the responses are not significantly different from zero (except for a slightly positive effect on impact). Nevertheless, in the financial crises regime, the response of the realized volatility significantly decreases after a monetary expansion, reverting to the pre-shock level after about 1.5 years. This indicates that a monetary expansion decreases the uncertainty regarding the value of assets during financial crises, while it has no effect on this uncertainty in normal times.

Finally, I consider the effects of a monetary expansion on several loans at commercial banks (see Figure 10). A similar pattern arises for all the different types of loans. During financial crises, commercial and industrial loans, consumer loans, total loans, and real estate loans at commercial banks increase significantly after an expansionary monetary

policy shock. Conversely, I cannot observe any clear-cut effects on the different types of loans in the normal regime. Responses of loans during times of low financial stress are insignificant, except that one might argue that the response of consumer loans is slightly increasing. Therefore, a monetary expansion has asymmetric effects on loans, i.e., while a monetary expansion during normal times does not have any significant effects on loans, it raises loans during financial crises.

These findings for the financial market variables have various implications. First, the negative response of the credit risk spreads, i.e., EFP, in the financial crises as well as in the normal regime, suggests that monetary policy and the premium that entrepreneurs have to pay when they ask for external credit may be linked. Therefore, this finding provides supportive evidence for the existence of a credit channel of monetary policy or the so-called financial accelerator (see the seminal contributions of Bernanke and Gertler (1989), Bernanke et al. (1996) and Bernanke et al. (1999)). The credit channel can be broken down into two components: the balance-sheet channel and the bank-lending channel (Bernanke and Gertler (1995)). Consequently, the positive response of entrepreneurs' wealth, i.e., the S&P 500 and NYSE index, in both regimes supports the potential existence of a balance-sheet transmission channel. More specifically, the balance-sheet channel suggests that an expansionary monetary policy shock increases the net worth of borrowers via the increase in asset prices, forcing down the EFP, which decreases the effective cost of credit, and therefore further stimulates investment and output. I observe these balance-sheet effects for the high financial stress as well as for the low financial stress regime, with effects being stronger during times of high financial stress. Moreover, a monetary policy expansion may also affect the EFP by shifting the supply of intermediated credit, in particular, loans by commercial banks. Consequently, the positive response of loans in the financial crises regime may suggest the existence of a bank-lending channel at least during financial crises, when financial market imperfections are high.

Second, and more important for the question at hand, is the fact that effects on the EFP are stronger in the financial crises regime than in the normal regime. Moreover, stock prices, volatilities, and loans are affected more in the case of high financial stress. This may indicate that the financial accelerator is stronger when the financial fundamentals, e.g., the entrepreneurs' wealth, are low, that is, during financial crises. This possibility of non-linear

balance-sheet effects was discussed for example in Bernanke and Gertler (1989), Gertler and Gilchrist (1994), and Bernanke et al. (1996). More specifically, in a high financial stress regime, an expansionary monetary policy shock increases asset prices and decreases the EFP by more than in the low financial stress regime. This translates into a stronger increase of investment, output, and other variables describing the state of the economy such as consumption, industrial production, employment, and new orders.

## 5 Conclusion

This paper investigates whether monetary policy transmission during financial crises differs from what is usually observed in low financial stress or normal times. In order to do so, I introduce the STFM, which is a regime-dependent factor model where the transition between states (normal and high financial stress times) depends on a financial conditions index.

My analysis shows substantial evidence that the transmission of a conventional monetary policy shock is different in times of financial crises. More specifically, I find that a monetary expansion during financial crises has stronger and more persistent effects on macroeconomic variables such as output, consumption, and investment than during normal times. These differences in effects among the regimes seem to originate from non-linearities in the credit channel. That is, a monetary expansion increases asset prices and loans by more during financial crises than in normal times, leading to a higher decrease in the EFP which, in turn, provokes the stronger effects on macroeconomic variables such as output, investment, and consumption.

## References

- Auerbach, A. J. and Gorodnichenko, Y. (2012). Measuring the output responses to fiscal policy, *American Economic Journal of Economic Policy* **4**(2): 1–27.
- Bernanke, B., Boivin, J. and Eliasziw, P. S. (2005). Measuring the effects of monetary policy: A factor-augmented vector autoregressive (FAVAR) approach, *The Quarterly Journal of Economics* **120**(1): 387–422.
- Bernanke, B. S. and Gertler, M. (1989). Agency costs, collateral, and business fluctuations, *American Economic Review* (79): 14–31.
- Bernanke, B. S. and Gertler, M. (1995). Inside the black box: The credit channel of monetary policy transmission, *Journal of Economic Perspectives* **9**(4): 27–48.
- Bernanke, B. S., Gertler, M. and Gilchrist, S. (1996). The financial accelerator and the flight to quality, *Review of Economics and Statistics* **78**(1): 1–15.
- Bernanke, B. S., Gertler, M. and Gilchrist, S. (1999). The financial accelerator in a quantitative business cycle framework, in J. Taylor and M. Woodford (eds), *The Handbook of Macroeconomics*, Elsevier Science B. V., Amsterdam, pp. 1341–1393.
- Bloom, N. (2009). The impact of uncertainty shocks, *Econometrica* **77**(3): 623–685.
- Carter, C. and Kohn, R. (1994). On Gibbs sampling for state space models, *Biometrika* **81**: 541–553.
- Christiano, L. J., Eichenbaum, M. and Evans, C. L. (1999). Monetary policy shocks: What have we learned and to what end?, in J. B. Taylor and M. Woodford (eds), *Handbook of Macroeconomics*, Vol. 1 of *Handbook of Macroeconomics*, Elsevier, chapter 2, pp. 65–148.
- Davig, T. and Hakkio, C. (2010). What is the effect of financial stress on economic activity, *Federal Reserve Bank of Kansas City Economic Review 2nd quarter*.
- Del Negro, M. and Schorfheide, F. (2011). Bayesian macroeconometrics, in J. Geweke, G. Koop and H. van Dijk (eds), *The Oxford Handbook of Bayesian Econometrics*, Oxford University Press.

- Deschamps, P. J. (2008). Comparing smooth transition and Markov switching autoregressive models of US unemployment, *Journal of Applied Econometrics* **23**(4): 435–462.
- Eichenbaum, M. (1992). Comment on “Interpreting the macroeconomic time series facts: The effects of monetary policy”: by Christopher Sims, *European Economic Review* **36**(5): 1001–1011.
- Forni, M., Domenico, G., Lippi, M. and Reichlin, L. (2009). Opening the black box: Structural factor models with large cross sections, *Econometric Theory* **25**(05): 1319–1347.
- Gertler, M. and Gilchrist, S. (1994). Monetary policy, business cycles, and the behavior of small manufacturing firms, *Quarterly Journal of Economics* **109**(2): 309–40.
- Granger, C. W. and Terasvirta, T. (1993). *Modelling Nonlinear Economic Relationships*, Oxford University Press, New York.
- Hatzius, J., Hooper, P., Mishkin, F., Schoenholtz, K. and Watson, M. W. (2010). Financial conditions indexes: A fresh look after the financial crisis, *NBER Working Paper No. 16150*.
- Hubrich, K. and Tetlow, R. J. (2012). Financial stress and economic dynamics: The transmission of crises, *Finance and Economics Discussion Series No. 2012-82*, Federal Reserve Board.
- Kim, C. J. and Nelson, C. R. (1999). *State-Space Models with Regime Switching: Classical and Gibbs-Sampling Approaches with Applications*, Vol. 1 of *MIT Press Books*, The MIT Press.
- Koop, G., Pesaran, M. H. and Potter, S. M. (1996). Impulse response analysis in nonlinear multivariate models, *Journal of Econometrics* **74**(1): 119–147.
- Krugman, P. (2008). Depression economics returns, *The New York Times*.
- Lopes, H. F. and Salazar, E. (2006). Bayesian model uncertainty in smooth transition autoregressions, *Journal of Time Series Analysis* **27**(1): 99–117.
- Lopes, H. F. and West, M. (2004). Bayesian model assessment in factor analysis, *Statistica Sinica* **14**: 41–67.

- Martin, J. and McDonald, R. (1975). Bayesian estimation in unrestricted factor analysis: A treatment for Heywood cases, *Psychometrika* **40**(4): 505–517.
- Mishkin, F. S. (2009). Is monetary policy effective during financial crises?, *American Economic Review* **99**(2): 573–77.
- Otrok, C. and Whiteman, C. H. (1998). Bayesian leading indicators: Measuring and predicting economic conditions in Iowa, *International Economic Review* **39**(4): 997–1014.
- Ravn, S. H. (2014). Asymmetric monetary policy towards the stock market: A DSGE approach, *Journal of Macroeconomics* **39**, Part A(0): 24–41.
- Sims, C. A. (1992). Interpreting the macroeconomic time series facts: The effects of monetary policy, *European Economic Review* **36**(5): 975–1000.
- Stock, J. and Watson, M. W. (2012). Disentangling the channels of the 2007-2009 recession, *NBER Working Paper No. 18094*.
- Teräsvirta, T. (1994). Specification, estimation, and evaluation of smooth transition autoregressive models, *Journal of the American Statistical Association* **89**: 208–218.
- Weise, C. L. (1999). The asymmetric effects of monetary policy: A nonlinear vector autoregression approach, *Journal of Money, Credit and Banking* **31**(1): 85–108.

Series	Mnemonic	Description	Transformation
1	GDP1	Real Gross Domestic Product, 1 Decimal	5
2	GDPDEF	Gross Domestic Product: Implicit Price Deflator	6
3	CPIAUCSL	Consumer Price Index For All Urban Consumers: All Items	6
4	FEDFUNDS	Effective Federal Funds Rate	2
5	GNPC96	Real Gross National Product	5
6	NICUR/GDPDEF	National Income/GDPDEF	5
7	DPIC96	Real Disposable Personal Income	5
8	OUTNFB	Nonfarm Business Sector: Output	5
9	FINSLC1	Real Final Sales of Domestic Product	5
10	FPIC1	Real Private Fixed Investment	5
11	PRFIC1	Real Private Residential Fixed Investment	5
12	PNFIC1	Real Private Nonresidential Fixed Investment	5
13	GPDI1	Real Gross Private Domestic Investment	5
14	PCECC96	Real Personal Consumption Expenditures	5
15	PCNDGC96	Real Personal Consumption Expenditures: Nondurable Goods	5
16	PCDGC96	Real Personal Consumption Expenditures: Durable Goods	5
17	PCEVC96	Real Personal Consumption Expenditures: Services	5
18	GPSAVE/GDPDEF	Gross Private Saving/GDP Deflator	5
19	FGCEC1	Real Federal Consumption Expenditures & Gross Investment	5
20	FGEXPND/GDPDEF	Federal Government: Current Expenditures/GDP deflator	5
21	FGRECPT/GDPDEF	Federal Government Current Receipts/GDP deflator	5
22	FGDEF	Federal Real Expend-Real Receipts	2
23	CBIC1	Real Change in Private Inventories	1
24	EXPGSC1	Real Exports of Goods & Services	5
25	IMPGSC1	Real Imports of Goods & Services	5
26	CP/GDPDEF	Corporate Profits After Tax/GDP deflator	5
27	NFCPATAX/GDPDEF	Nonfinancial Corporate Business: Profits After Tax/GDP deflator	5
28	CNCF/GDPDEF	Corporate Net Cash Flow/GDP deflator	5
29	DIVIDEND/GDPDEF	Net Corporate Dividends/GDP deflator	5
30	HOANBS	Nonfarm Business Sector: Hours of All Persons	5
31	OPHNFB	Nonfarm Business Sector: Output Per Hour of All Persons	5
32	UNLPNBS	Nonfarm Business Sector: Unit Nonlabor Payments	5
33	ULCNFB	Nonfarm Business Sector: Unit Labor Cost	5
34	WASCUR/CPI	Compensation of Employees: Wages & Salary Accruals/CPI	5
35	COMPNFB	Nonfarm Business Sector: Compensation Per Hour	6
36	COMPRNFB	Nonfarm Business Sector: Real Compensation Per Hour	5
37	GDPCTPI	Gross Domestic Product: Chain-type Price Index	6
38	GNPCTPI	Gross National Product: Chain-type Price Index	6
39	GNPDEF	Gross National Product: Implicit Price Deflator	6
40	INDPRO	Industrial Production Index	5
41	IPBUSEQ	Industrial Production: Business Equipment	5
42	IPCONGD	Industrial Production: Consumer Goods	5
43	IPDCONGD	Industrial Production: Durable Consumer Goods	5
44	IPFINAL	Industrial Production: Final Products (Market Group)	5
45	IPMAT	Industrial Production: Materials	5
46	IPNCONGD	Industrial Production: Nondurable Consumer Goods	5
47	AWHMAN	Average Weekly Hours: Manufacturing	2
48	AWOTMAN	Average Weekly Hours: Overtime: Manufacturing	2
49	CIVPART	Civilian Participation Rate	2
50	CLF16OV	Civilian Labor Force	5
51	CE16OV	Civilian Employment	5
52	USPRIV	All Employees: Total Private Industries	5
53	USGOOD	All Employees: Goods-Producing Industries	5
54	SRVPRD	All Employees: Service-Providing Industries	5

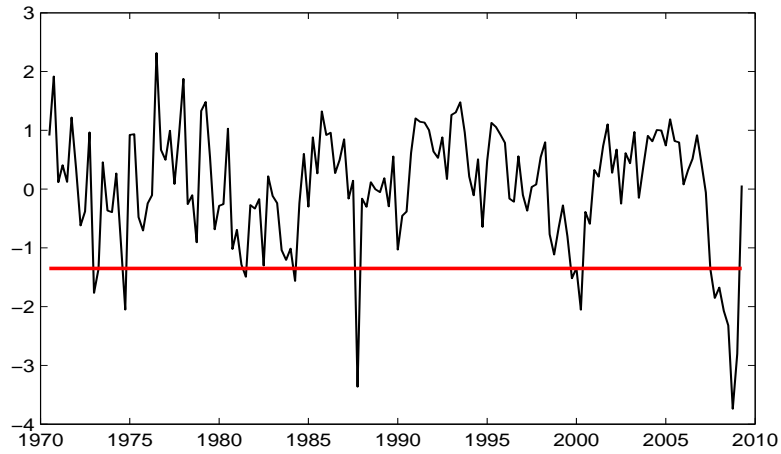
**Table 1:** Transformations of  $x_t$ :  $1 = x_t$ ,  $2 = \Delta x_t$ ,  $5 = \Delta \log x_t$ ,  $6 = \Delta^2 \log x_t$

Continued...

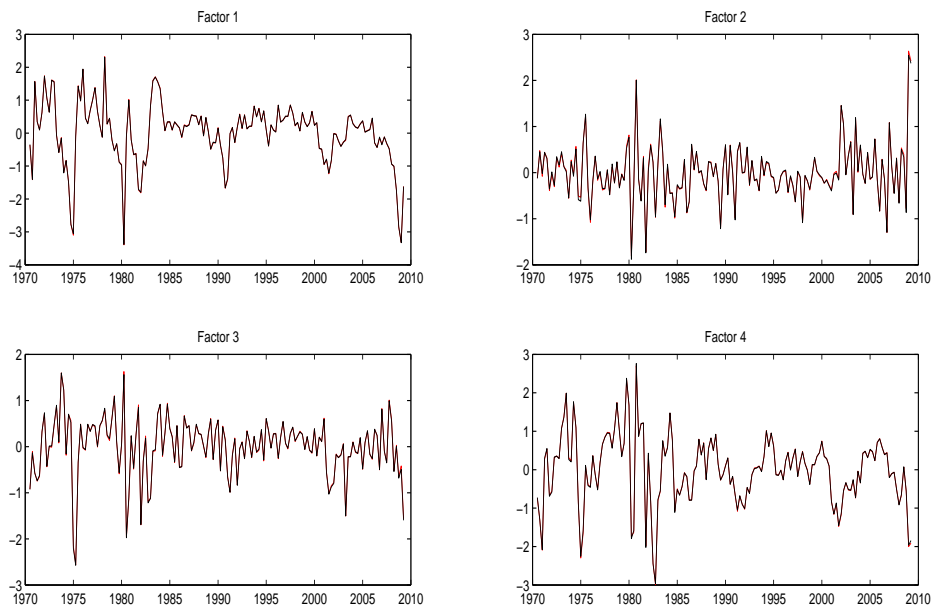


Series	Mnemonic	Description	Transformation
55	UNEMPLOY	Unemployed	5
56	UEMPMEAN	Average (Mean) Duration of Unemployment	5
57	UNRATE	Civilian Unemployment Rate	2
58	HOUST	Housing Starts: Total: New Privately Owned Housing Units Started	5
59	M1SL	M1 Money Stock	6
60	M2MSL	M2 Minus	6
61	M2SL	M2 Money Stock	6
62	BUSLOANS	Commercial and Industrial Loans at All Commercial Banks	6
63	CONSUMER	Consumer (Individual) Loans at All Commercial Banks	6
64	LOANINV	Total Loans and Investments at All Commercial Banks	6
65	REALLN	Real Estate Loans at All Commercial Banks	6
66	TOTALSL	Total Consumer Credit Outstanding	6
67	CPIULFSL	Consumer Price Index for All Urban Consumers: All Items Less Food	6
68	CPILEGSL	Consumer Price Index for All Urban Consumers: All Items Less Energy	6
69	CPILFESL	Consumer Price Index for All Urban Consumers: All Items Less Food & Energy	6
70	CPIENGSL	Consumer Price Index for All Urban Consumers: Energy	6
71	CPIUFDSL	Consumer Price Index for All Urban Consumers: Food	6
72	PPICPE	Producer Price Index Finished Goods: Capital Equipment	6
73	PPICRM	Producer Price Index: Crude Materials for Further Processing	6
74	PPIFCG	Producer Price Index: Finished Consumer Goods	6
75	PPIFGS	Producer Price Index: Finished Goods	6
76	OILPRICE	Spot Oil Price: West Texas Intermediate	6
77	USNEDG	US Manufacturers new Orders of durable Goods	5
78	USNOCG	US New Orders of Consumer Goods & Materials	5
79	NAPMNOI	US ISM Manufacturer Survey: New Orders Index	1
80	USCYLEAD	US the Conference Board Leading Economic Indicators Index	5
81	GEXPND/GDPDEF	Government Current Expenditures/ GDP deflator	5
82	GRECPT/GDPDEF	Government Current Receipts/ GDP deflator	5
83	GDEF	Government Real Expend-Real Receipts	2
84	GCEC1	Real Government Consumption Expenditures & Gross Investment	5
85	TB3MS	3-Month Treasury Bill: Secondary Market Rate	2
86	TB6MS	6-Month Treasury Bill: Secondary Market Rate	2
87	GS1	1-Year Treasury Constant Maturity Rate	2
88	GS10	10-Year Treasury Constant Maturity Rate	2
89	AAA	Moody's Seasoned Aaa Corporate Bond Yield	2
90	BAA	Moody's Seasoned Baa Corporate Bond Yield	2
91	MPRIME	Bank Prime Loan Rate	2
92	GS10-FEDFUNDS		1
93	GS1-FEDFUNDS		1
94	BAA-AAA	Default Rate Spread	1
95	MPRIME-TB3MS	External Finance Premium	1
96	MPRIME-TB6MS	Bank Prime Loan Rate minus 6-Month Treasury Bill	1
97	US500STK	US Standard & Poor's Index of 500 common Stocks	5
98	USSHRPRCF	US Dow Jones Industrial Share Price Index	5
99		Realized Volatility of S&P 500 Index	1
100	LIBOR3M	3-Month US Deposit London Offer	2
101	LIBOR6M	6-Month US Deposit London Offer	2
102	LIBOR3M-FEDFUNDS		1
103	LIBOR3M-TB3M	TED spread	1
104	NYSE	US NYSE Composite Index	5
105		Realized Volatility of NYSE Index	1
106	EXCRESNS	Excess Reserves of Depository Institutions	5
107	ADJRESSL	Adjusted Monetary Base	5
108		Real Net Taxes	5

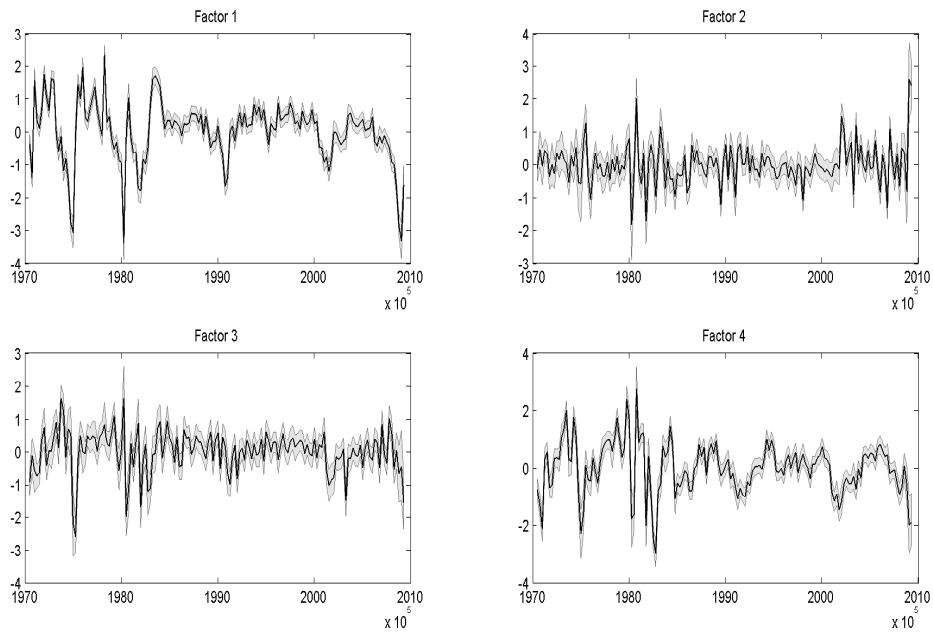
**Table 1 (Continued):** *Transformations of  $x_t$ : 1 =  $x_t$ , 2 =  $\Delta x_t$ , 5 =  $\Delta \log x_t$ , 6 =  $\Delta^2 \log x_t$*



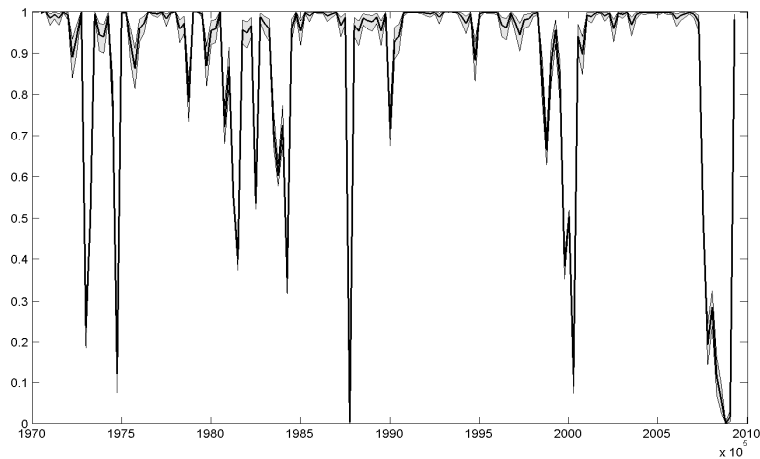
**Figure 1:** *The black line shows the FCI of Hatzius et al. (2010). The red line is the estimated threshold value,  $c$ . The estimate has been calculated as the median of the threshold value draws of the Metropolis-within-Gibbs sampler kept after thinning.*



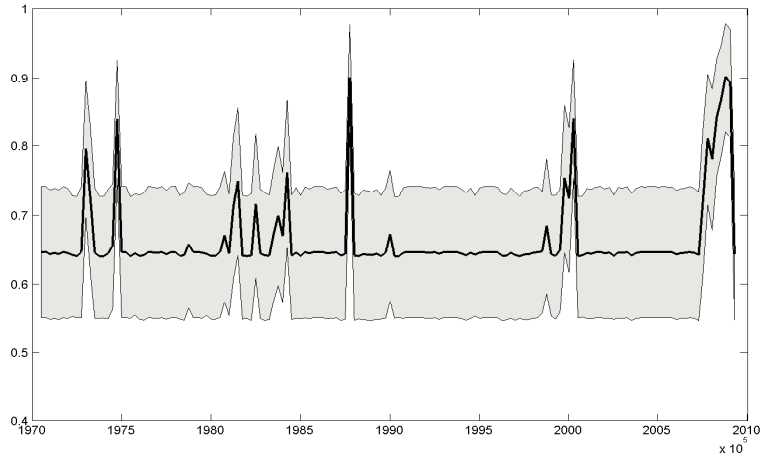
**Figure 2:** *This figure plots the estimated factors obtained in the first half of the Gibbs sampler against the ones obtained in the second half. The estimates are calculated as the median of the factor draws of the Gibbs sampler kept after thinning.*



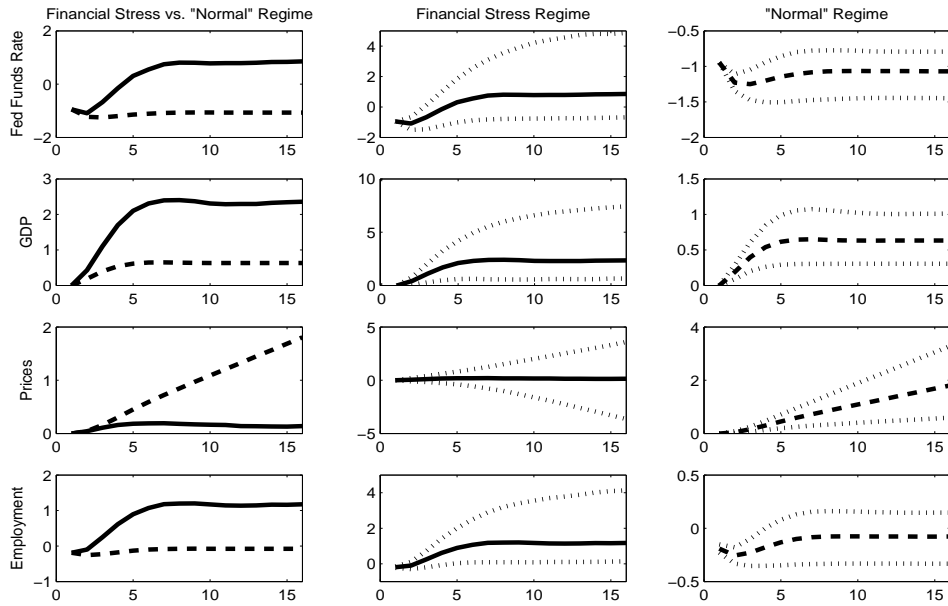
**Figure 3:** *This figure plots the estimated factors and their 95% confidence bands. The estimates have been calculated as the median of the factor draws of the Gibbs sampler kept after thinning.*



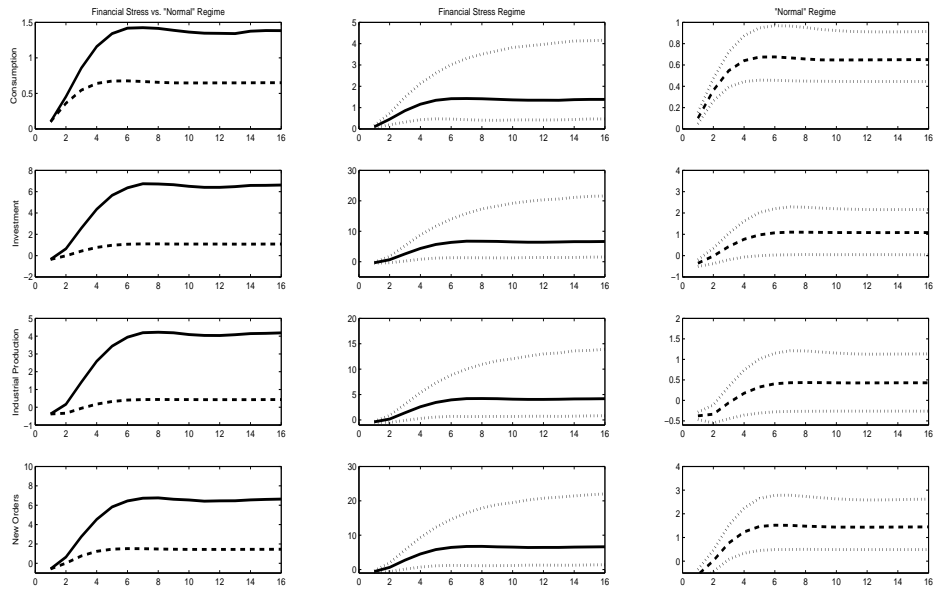
**Figure 4:** *This figure provides a time plot of the median of the weight on the financial crises regime together with its 95% confidence interval.*



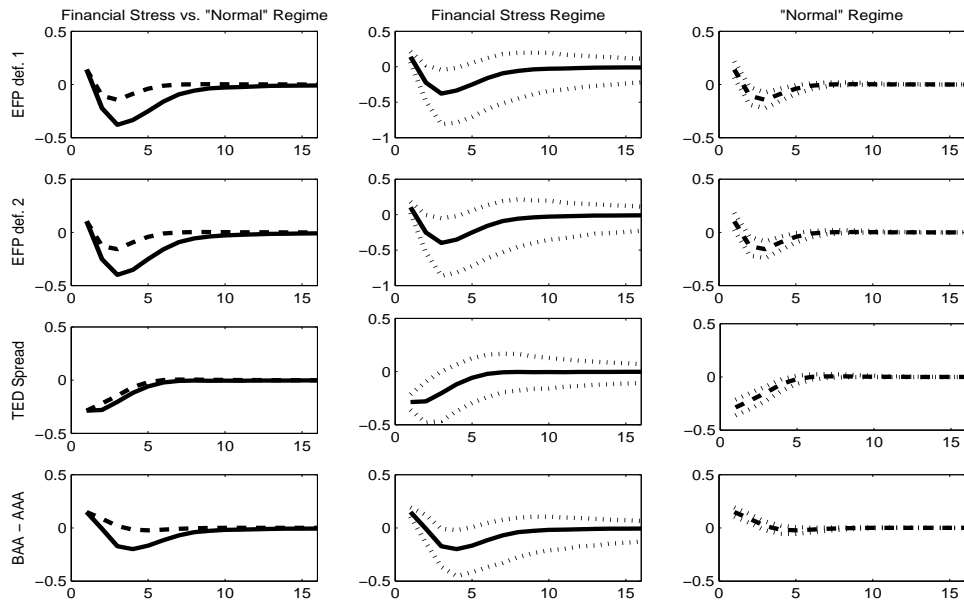
**Figure 5:** *This figure provides a plot of the median of the maximum eigenvalues of  $\bar{D}_t$  together with its 68% confidence intervals over time.*



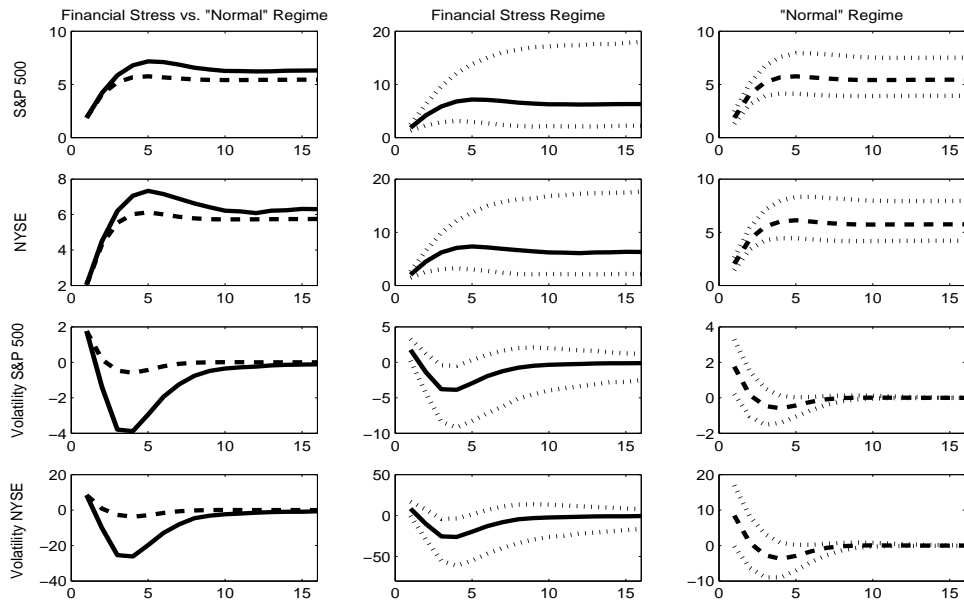
**Figure 6:** *This figure plots the impulse responses to an expansionary monetary policy shock of the federal funds rate (4), GDP (1), prices (2), and employment (51). Solid lines indicate the responses for the high financial stress regime and dashed lines indicate the responses for the normal regime. Dotted lines are the respective 68% confidence intervals.*



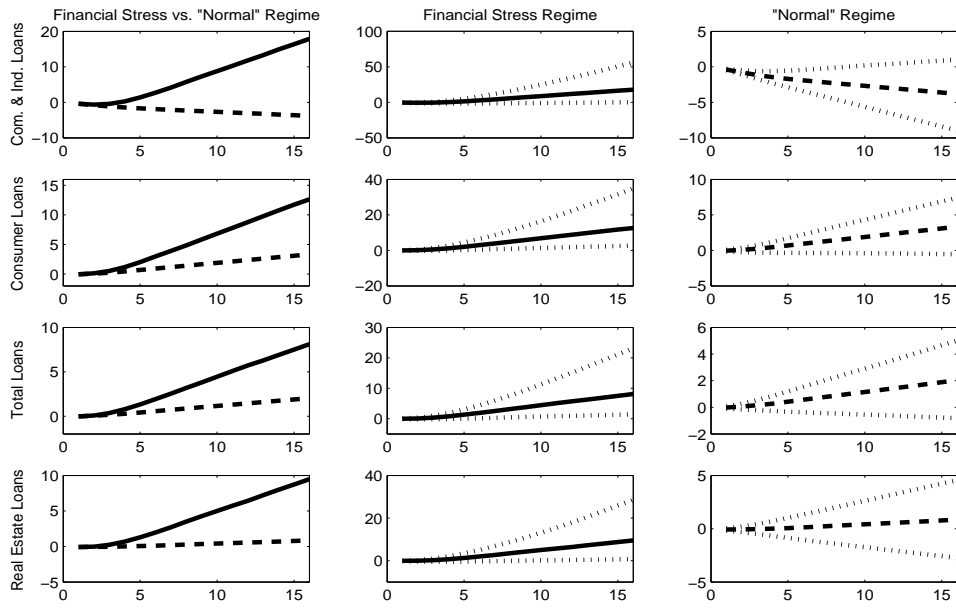
**Figure 7:** *This figure plots the impulse responses to an expansionary monetary policy shock of consumption (14), investment (10), industrial production (40), and new orders (78). Solid lines indicate the responses for the financial crises regime and dashed lines indicate the responses for the normal regime. Dotted lines are the respective 68% confidence intervals.*



**Figure 8:** *This figure plots the impulse responses to an expansionary monetary policy shock of the first measure of the external finance premium (95), the second measure of the external finance premium (96), the TED spread (103) and the corporate bond spread (94). Solid lines indicate the responses for the financial crises regime and dashed lines indicate the responses for the normal regime. Dotted lines are the respective 68% confidence intervals.*



**Figure 9:** *This figure plots the impulse responses to an expansionary monetary policy shock of the S&P 500 index (97), the NYSE index (104), the realized volatility of the S&P 500 (99), and the realized volatility of the NYSE index (105). Solid lines indicate the responses for the financial crises regime and dashed lines indicate the responses for the normal regime. Dotted lines are the respective 68% confidence intervals.*



**Figure 10:** *This figure plots the impulse responses to an expansionary monetary policy shock of commercial and industrial loans (62), consumer loans (63), total loans (64), and real estate loans (65). Solid lines indicate the responses for the financial crises regime and dashed lines indicate the responses for the normal regime. Dotted lines are the respective 68% confidence intervals.*



## A Details of Gibbs Sampler

### A.1 Likelihood

Under the normality assumptions, the likelihood of the model can be expressed as

$$L(\Lambda, f, D, H, Q, \nu) \propto |H|^{T/2} \exp \left\{ -\frac{1}{2} \text{tr} [(X - F\Lambda)' H^{-1} (X - F\Lambda)] \right\} \times \quad (12)$$

$$|Q|^{T/2} \exp \left\{ -\frac{1}{2} \text{tr} [(F - W^{ST}D)' Q^{-1} (F - W^{ST}D)] \right\},$$

where the first factor of the product is the likelihood of the observation equation and the second factor is the likelihood of the measurement equation. After some manipulations, I can rewrite the likelihood in the standard way as

$$L(\Lambda, f, D, H, Q, \nu) \propto |H|^{T/2} \exp \left\{ -\frac{1}{2} (\lambda - \hat{\lambda})' (H^{-1} \otimes F'F) (\lambda - \hat{\lambda}) \right\} \times \quad (13)$$

$$\exp \left\{ -\frac{1}{2} \text{tr} [(X - F\hat{\Lambda})' H^{-1} (X - F\hat{\Lambda})] \right\} \times$$

$$|Q|^{T/2} \exp \left\{ -\frac{1}{2} (d - \hat{d})' (Q^{-1} \otimes W^{ST'}W^{ST}) (d - \hat{d}) \right\} \times$$

$$\exp \left\{ -\frac{1}{2} \text{tr} [(F - W^{ST}\hat{D})' Q^{-1} (F - W^{ST}\hat{D})] \right\}.$$

Now the likelihood of each equation in the model given in (6) and (7) can be seen to be the product of an inverse Wishart density for  $H$  and  $Q$ , respectively, and a normal density for  $\lambda$  and  $d$ , respectively.

### A.2 The Kalman Filter and Smoother Algorithm

Let  $X^t = (x_1, x_2, \dots, x_t)$ ,  $\bar{F}^t = (\bar{f}_1, \bar{f}_2, \dots, \bar{f}_t)$  and  $\bar{D}^t = (\bar{D}_1, \bar{D}_2, \dots, \bar{D}_t)$  be the history from period 1 to  $t$  of  $x_t$ ,  $\bar{f}_t$  and  $\bar{D}_t$ , which are from the state space representation given by Equation (10). As in Carter and Kohn (1994) the conditional distribution of the whole history of the factors at time  $t$  is

$$p(\bar{F}^T | X^T, \bar{D}^T, \bar{\Lambda}, \bar{Q}, H, \nu) = p(\bar{f}_T | X^T, \bar{D}^T, \bar{\Lambda}, \bar{Q}, H, \nu) \prod_{t=1}^{T-1} p(\bar{f}^t | \bar{f}_{t+1}, X^t, \bar{D}^t, \bar{\Lambda}, \bar{Q}, H, \nu). \quad (14)$$

Since the state space model in (10) is linear and Gaussian, the distribution of the factors is given by

$$\bar{f}_T | X^T, \bar{D}^T, \bar{\Lambda}, \bar{Q}, H, \nu \sim N(\bar{f}_{T|T}, P_{T|T}) \quad (15)$$

$$\bar{f}_t | \bar{f}_{t+1}, X^t, \bar{D}^t, \bar{\Lambda}, \bar{Q}, H, \nu \sim N(\bar{f}_{t|t+1}, P_{t|t+1}) \quad t = T-1, \dots, 1. \quad (16)$$

First, I run the Kalman filter to obtain  $\bar{f}_{T|T}$  and  $P_{T|T}$ . Starting with  $\bar{f}_{0|0} = \mathbf{0}_{r \times p}$  and  $P_{0|0} = I_{r \times p}$ , the Kalman filter recursion over  $t = 1, \dots, T$  is given by

$$\bar{f}_{t|t-1} = \bar{D}_t \bar{f}_{t-1|t-1} \quad (17)$$

$$P_{t|t-1} = \bar{D}_t P_{t-1|t-1} \bar{D}_t' + \bar{Q}$$

$$\bar{f}_{t|t} = \hat{f}_{t|t-1} + P_{t|t-1} \bar{\Lambda}' (\bar{\Lambda} P_{t|t-1} \bar{\Lambda}' + H)^{-1} (x_t - \bar{\Lambda} \bar{f}_{t|t-1})$$

$$P_{t|t} = P_{t|t-1} - P_{t|t-1} \bar{\Lambda}' (\bar{\Lambda} P_{t|t-1} \bar{\Lambda}' + H)^{-1} \bar{\Lambda} P_{t|t-1}.$$

Then, having the draw of  $\bar{f}^T$  and the results of the filter, I run the Kalman smoother to obtain  $\bar{f}_{T-1|T}$  and  $P_{T-1|T}$ . This backward updating provides me with a draw of  $\bar{f}_{T-1}$ , and in the next updating step with a draw of  $\bar{f}_{T-2}$ , and so on until I arrive at  $\bar{f}_1$ . More specifically, the Kalman smoother steps for  $t = T-1, \dots, 1$  are as follows:

$$\bar{f}_{t|t+1} = \bar{f}_{t|t} + P_{t|t} \bar{D}_t' (\bar{D}_t P_{t|t} \bar{D}_t' + \bar{Q})^{-1} (\bar{f}_{t+1} - \bar{D}_t \bar{f}_{t|t}) \quad (18)$$

$$P_{t|t+1} = P_{t|t} + P_{t|t} \bar{D}_t' (\bar{D}_t P_{t|t} \bar{D}_t' + \bar{Q})^{-1} \bar{D}_t P_{t|t}.$$

If the lag order  $p$  exceeds one, then lags of the factors appear in  $\bar{f}_t$  and  $\bar{Q}$  is singular. In this case in the Kalman smoother steps, rather than conditioning on the full vector  $\bar{f}_{t+1}$  when drawing  $\bar{f}_t$ , I only can use the first  $p$  elements of  $\bar{f}_{t+1}$ . See Kim and Nelson (1999) for more details.