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"Relative Movements of Real Wages and Output": How does Keynes's 1939 essay relate to his Principle of effective demand?

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KOF Working Papers

“Relative Movements of Real Wages and Output” – How does Keynes’s 1939 essay relate to his Principle of Effective Demand?

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ABSTRACT

Keynes’s essay “Relative Movements of Real Wages and Output” is widely believed to be an important amendment to his *General Theory* because, in this essay, Keynes relaxed his core assumption of decreasing marginal returns to labour. Non-decreasing marginal returns, however, do not sit comfortably with the prime innovation of the *General Theory*: the Principle of Effective Demand. This will be demonstrated by performing – for the first time in the literature – numerical simulations with Keynes’s Aggregate-Demand-Aggregate-Supply (D/Z) model. The view that Keynes’s 1939 essay constitutes an important amendment to his *General Theory* thus has to be put into perspective.

I. INTRODUCTION

In *The Collected Writings of John Maynard Keynes*, the seminal *General Theory of Employment, Interest and Money* is volume 7. The difference between vol. 7 and the original edition is that three appendices accompany the main text. Appendix 1 lists the printing errors in the first edition of the *General Theory* corrected for in vol. 7. Appendix 2 also contains corrections to the *General Theory* which Keynes published in the September 1936 issue of the *Economic Journal* as “Fluctuations in Net Investment in the United States” after Simon Kuznets had pointed out some errors in Keynes’s use of statistics Kuznets had generated. Appendix 3 is Keynes’s March 1939 *Economic Journal* article “Relative Movements of Real Wages and Output”. Obviously, this article has been chosen for inclusion in vol. 7 because the editors of the *Collected Writings* believed that it also amended – if not corrected – the *General Theory* in a significant way.

In “Relative Movements of Real Wages and Output”, Keynes discusses various issues, one of which is of prime importance and will therefore be the sole focus of this essay. Keynes relaxes a core assumption he thoroughly relies on in the *General Theory*: the assumption of decreasing marginal returns to labour in the short period. Why is this important?

First of all, this assumption was very dear to Keynes. Only two years before “Relative Movements of Real Wages and Output” appeared in print, he had written to Ohlin: “I have always regarded decreasing physical returns in the short period as one of the very few incontrovertible propositions of our miserable subject!”.¹ In the *General Theory*, especially the beginning of the book heavily relies on the assumption of decreasing physical returns. In chapter 2, for instance, Keynes reconstructs what he calls ‘classical economics’ in terms of two postulates. The ‘first classical postulate’ states that “*The wage is equal to the marginal product of labour*” (Keynes 1936, p. 5). Keynes should have added: “...; and the marginal

¹ See *The Collected Writings of John Maynard Keynes*, vol. 14, p. 190.

product of labour is decreasing in the short period'. He makes this amendment on pp. 17-18 of the *General Theory*, where he states that he accepts the 'first classical postulate'. He explains what this means:

"It means that, with a given organisation, equipment and technique, real wages and the volume of output (and hence of employment) are uniquely correlated, so that, in general, an increase in employment can only occur to the accompaniment of a decline in the rate of real wages. ... This is simply the obverse of the familiar proposition that industry is normally working subject to decreasing returns in the short period during which equipment etc. is assumed to be constant; so that marginal product in the wage-good industries (which governs real wages) necessarily diminishes as employment is increased. So long, indeed, as this proposition holds, *any* means of increasing employment must lead at the same time to a diminution of the marginal product and hence of the rate of wages measured in terms of this product."

So the conclusion is that real wages and output move in opposite directions. In the *General Theory*, Keynes even conjectures that "a statistical enquiry" would confirm this (see Keynes 1936, pp. 9-10). Three years later, however, such statistical enquiries had been carried out (by Dunlop, 1938, and Tarshis, 1938, 1939), and their results failed to confirm Keynes's conjecture. This prompted Keynes to write the essay "Relative Movements of Real Wages and Output" where he suggests that – contrary to what is assumed in the *General Theory* – marginal returns may be non-decreasing over parts of the output range.

Keynes does not seem to have believed that this concession hurts his theory or his policy recommendations in any way. On the contrary, he writes: "If the falling tendency of real wages in periods of rising demand is denied, ... it would be possible to simplify considerably the more complicated version of my fundamental explanation which I have expounded in my

‘General Theory.’ My practical conclusions would have, in that case, *à fortiori* force” (Keynes 1939, pp. 40-41).²

What this essay wants to achieve is to show that Keynes is too optimistic in his assessment that replacing decreasing by non-decreasing returns is harmless for his theory. Chapter 2 is not the only part of the *General Theory* that would have to be revised if non-decreasing marginal returns were assumed. Also the theoretical core – the “Principle of Effective Demand” introduced in chapter 3 of the book – would be affected. Section 2 below reconstructs the Aggregate-Demand-Aggregate-Supply (D/Z) model which Keynes uses to determine the level of effective demand in chapter 3. This section builds on earlier literature. The first attempt to provide a formal structure for Keynes’s D/Z apparatus was Weintraub (1958), followed by Davidson and Smolensky (1964) and a number of publications by Paul Davidson (Davidson 1978, 1994, 2002).³ According to Weintraub and Davidson, *Z* stands for expected demand and *D* for actual demand. Hartwig (2007) criticizes this as not in line with chapter 3, where *D* stands for expected demand and *Z* stands for hypothetical proceeds that would maximize profits. Section 2 below briefly restates the interpretation of the D/Z model that was advertised in Hartwig (2007) and also in Hartwig (2000, 2004a, 2006, 2011a, 2011b). Chick (1983, chapter 4), Amadeo (1989, chapter 6), Darity (1989), Darity and Horn (1993, section V.), Galbraith and Darity (2005, chapter 13), and Lawlor (2008) interpret the D/Z model in a similar (although not identical) way.⁴

² Keynes attaches a footnote to the first sentence in the quote which states that especially chapter 2 of the *General Theory* could be simplified and needs to be revised in the light of the new evidence.

³ King (1994) has scrutinised the early discussion of Keynes’s aggregate-demand/aggregate-supply model. “To conclude that there was some confusion about aggregate supply and demand analysis in the early 1950s would be a grotesque understatement”, he sums up (p. 14).

⁴ It is not my aim here to pursue the details and subtleties of the differences in the interpretations. I feel that it is most constructive to lay out my own interpretation in section 2 and then to proceed swiftly to what is new in this essay.

What section 3 adds to this literature is that concrete functional forms are ascribed to the D- and Z-functions, so that they can be used for numerical simulations. It turns out that the model works well under the assumption of decreasing marginal returns and that it can produce interesting results, for instance, on the effects of different ‘shocks’ on price and wage dynamics. This is not the case, however, under the assumption of non-decreasing marginal returns, as section 4 shows. The model either fails to determine the level of effective demand or produces inconsistent results. Section 5 goes on to argue that the empirical evidence which led Keynes to suggest the possibility of non-decreasing returns – namely the finding of rising real wages at rising output – can be understood from the ‘traditional’ D/Z model with thoroughly decreasing returns if ‘supply shocks’ are admitted. The final section concludes.

II. KEYNES’S MODEL OF EFFECTIVE DEMAND AND ITS RELATION TO THE ‘FIRST CLASSICAL POSTULATE’

In chapter 3 of the *General Theory*, Keynes develops the Principle of Effective Demand in the context of a thought experiment by entrepreneurs, who aim at maximizing profit. To understand the Principle, it is important to visualize the economic process as a sequence of production periods. Entrepreneurs plan for a certain period of the future and are bound by their decisions until the end of the period. The Principle of Effective Demand is what guides their planning. To simplify the exposition, let us assume that the individual plans can be aggregated straightforwardly and that the planning period is the same for all entrepreneurs.

Keynes models the entrepreneurs’ planning task in terms of two functions, the Aggregate Supply Function (Z) and the Aggregate Demand Function (D). Z is “the aggregate supply price of the output from employing N men” (Keynes 1936, p. 25). The aggregate supply price is defined by Keynes as “the expectation of proceeds which will just make it worth the while of the entrepreneur to give that employment” (Keynes 1936, p. 24).

Keynes defines Z as the product of an aggregate price and output component. The latter, the ‘output of N men’, we can identify as net value added⁵ (which is dependent on employment). I chose the symbol $Y(N)$ for this component. The price level implicit in Keynes’s aggregate supply function, P^s , must have the property that the proceeds it generates “will just make it worth the while of the entrepreneurs to give that employment” – in other words, P^s must be the profit-maximising price level. With respect to the micro-foundations of aggregate supply, the *General Theory* does not part company with the (neo)classical approach. Therefore, Keynes (1973 pp. 24-25) takes it for granted that the “entrepreneurs will endeavour to fix the amount of employment at the level which they expect to maximise the excess of proceeds over the factor cost”.

The mathematical approach to find out that level is standard. Simply differentiate the profit function with respect to employment to obtain the first-order condition. From this, the profit-maximising supply-price level P^s can be derived (see equations 1 and 2).⁶

$$\Pi = P^s \cdot Y(N) - w \cdot N \quad (1)$$

$$\frac{d\Pi}{dN} = 0 \Rightarrow P^s \cdot \frac{dY}{dN} - w = 0 \Rightarrow P^s = w \cdot \frac{dN}{dY} \quad (2)$$

Z , being the mathematical product of the output and the supply price levels, is thus given by (3):

$$Z = P^s \cdot Y(N) = w \cdot \frac{dN}{dY} \cdot Y(N) \quad (3)$$

Under decreasing marginal returns to labour, P^s grows progressively while $Y(N)$ grows with diminishing returns. Altogether, Z might be a linear function of N . At least, this seems to be hinted at in a somewhat opaque footnote on pp. 55-56 of the *General Theory* in which Keynes

⁵ Not gross value added because Keynes subtracts what he calls ‘user cost’ – the sum of intermediate consumption and depreciation allowances – from gross output in the aggregate (see Keynes 1936, pp. 23-24).

⁶ With Π = aggregate profit, P^s = aggregate supply price level, Y = net value added, N = employment, w = wage unit (= average nominal wage rate, see Keynes, 1936, p. 41).

suggests two arguably inconsistent things: first, that Z is linear with a slope of 1, and second, that the slope of Z is given by the reciprocal of the money wage. Ambrosi (2011) has recently shown that the second proposition could be made sense of if the word ‘share’ was added at the very end. In other words, Ambrosi argues that the slope of Z was given by the inverse of the money wage *share*. Hartwig (2011a) confirms this, showing in a general way that the slope of Z is given by the inverse of the output elasticity (which is identical to the money wage share for the standard neoclassical production function that Keynes subscribed to).⁷

Now let us turn to the Aggregate Demand Function D . According to Keynes, it gives “the proceeds which entrepreneurs expect to receive from the employment of N men” (Keynes 1936, p. 25). In a diagram with employment as abscissa and expected proceeds as ordinate, which Keynes describes verbally on page 25 of the *General Theory*, D lies above Z for small N . At a certain point – corresponding to a certain level of employment N – however, D and Z intersect. Keynes calls this point of intersection ‘effective demand’ and states that “it is at this point that the entrepreneurs’ expectation of profits will be maximised” (Keynes 1936, p. 25).

The interpretation of this passage of the *General Theory* is straightforward if we remember that Keynes adopted the (neo)classical micro-assumptions of profit-maximisation and price-taking.⁸ Because entrepreneurs cannot hope to dictate prices neither in their individual markets nor at the aggregate level they use the calculus of equations (1) and (2) to find out which price level would maximise profits. P^s , the price level implicit in Z , is in a way purely

⁷ Unfortunately, the exposition in Hartwig (2011a) is tainted with printing errors in the formulas. See the Working Paper version (KOF Working Paper No. 282) at www.kof.ethz.ch instead.

⁸ Keynes’s notion of price-taking departs from the strict microeconomic theory of the small firm operating under perfect competition. That theory would not allow for entrepreneurs forming *ex ante* expectations about demand. Keynes – who was concerned with the real world – did not have such firms in mind. In his theory, firms are not ‘atomistic’, but also not powerful enough to dictate the price. They have to form expectations about the price for their products the market will accept and about the market share that might be attributable to them (see Chick 1992).

hypothetical. *If*, for a certain N_1 , the entrepreneurs expected the price level given by (2) to rule in the market they would employ N_1 men because they knew that profits would thereby be maximised. But which price level do they *really* expect? This question is not answered by the supply function at all but by the demand function. The price level implicit in D , which we can call the demand price level P^d , is the price level the entrepreneurs really expect to rule in the market. Hence Keynes writes “let D be the proceeds which entrepreneurs expect to receive from the employment of N men”. If, for a certain N , $P^d > P^s$, “there will be an incentive to entrepreneurs to increase employment beyond N and, if necessary, to raise costs by competing with one another for the factors of production, up to the value of N for which Z has become equal to D ” (Keynes 1936, p. 25).

Now it is important to understand how Keynes’s model of effective demand, according to which the D and Z functions determine the volumes of output and employment as well as the price level at their point of intersection, relates to his acceptance of the ‘first postulate of classical economics’. Misunderstanding this relationship directly leads to watering down the importance of Keynes’s theoretical contribution. An example for what I would argue is a misunderstanding is provided by Meltzer (1983). He concludes that Keynes’s acceptance of the ‘first postulate’ implies that he must have believed that the market demand curve for labour was a theoretically viable concept for the determination of the employment level.

But this conclusion is not warranted. Keynes does not interpret the connection between real wages and employment the way Meltzer does: a lower real wage leads to a higher (labour demand and) level of employment. Keynes merely writes that the two magnitudes – real wages and employment – are ‘correlated’ (see the quote given above). The curve depicting the marginal product of labour is *not* a labour demand curve (see also Davidson 1983a, 1983b). The marginal product of labour schedule rather gives the real wage that will be associated *ex post* with a certain employment level – the latter depending on effective demand.

The left part of Figure 1 illustrates Keynes's analysis in chapter 3 of the *General Theory*. The D and Z curves intersect in the employment (N) / aggregate proceeds (PY) space at the point of effective demand. The total quantity of employment for the respective production period is thus given (N_{ed}). If we transfer N_{ed} to the employment / real wage (w/P) space, we can read the effective real wage level for the period (w/P_{ed}) off the Marginal-Product-of-Labour (MPL) curve. Note that this curve can no longer be interpreted as a labour demand curve since the demand for labour has already been determined. But if we are prepared to accept the concept of the labour supply curve (N^s), then w/P_{ed} can be used to deduce the level of involuntary unemployment ($N^s_{ed} - N_{ed}$).

<Insert Figure 1 around here>

But why is the real wage given by the MPL curve? – To answer this question we will have to distinguish two cases within the entrepreneurs' decision calculus for which the Principle of Effective Demand is a model. In the first case, the entrepreneurs anticipate next period's *de facto* aggregate demand correctly. Then, *ex post*, the *de facto* aggregate demand will coincide with the proceeds given by the point of intersection of the (expectation-dependent) D and Z curves. In this case, we have:

$$P^D \cdot Y(N) = P^S \cdot Y(N) \Leftrightarrow P^D \cdot Y(N) = w \frac{dN}{dY} \cdot Y(N) \Leftrightarrow \frac{w}{P^D} = \frac{dY}{dN} \quad (4)$$

Ex post, the real wage is equal to the marginal product of labour if the *ex ante* expectations have been correct. Now let us assume that the *ex ante* expectations have been incorrect. Let us assume that, at the beginning of a production period, the entrepreneurs underestimate the period's *de facto* demand. Since, according to the definition of the production period given above, the entrepreneurs are bound by their decision how many workers to employ until the end of the period, they cannot react to the higher-than-expected demand by increasing output. Their only two options are raising prices or depleting inventories. This result may contradict

the standard ‘Keynesian’ argument of quantity reactions at fixed prices; but it is exactly this way that Keynes describes the adjustment process between supply and demand in the *General Theory* (Keynes 1936, pp. 122-125). As a result of the rise in prices, the real wage will be reduced; and it will be reduced below the marginal productivity of the labour force employed in that period. If the entrepreneurs expect the higher demand to prevail in the next production period, then the point of effective demand for that period will entail a higher level of employment because the D curve shifts to the top. Consequently, the marginal product of labour will fall to the new real wage level. So Keynes’s acceptance of the ‘first postulate’ has to be qualified to the extent that, if entrepreneurs’ *ex ante* expectations are incorrect, it will take two production periods to establish an equality between the real wage and the marginal product of labour; and this statement only holds if the supply and demand conditions do not change in the transition from the first production period to the second.

III. SIMULATING THE D/Z MODEL WITH DECREASING MARGINAL RETURNS

This section goes beyond the existing literature in performing numerical simulations with the D/Z model. Hopefully, this will contribute to a better understanding of the modus operandi of the model.

First of all, it is necessary to specify a production function. I will use a simple neoclassical production function with diminishing marginal returns to labour. The capital stock is constant during the short period and will be normalised to one. The first line of Table 1 lists the assumed production function as well as the functions Z and D. The nominal wage rate which enters Z (see eq. 3 above) is normalised to 1. For D, I assume that the entrepreneurs expect a demand price level of 5.

<Insert Table 1 around here>

Figure 2 shows the D and Z curves for the first set of assumptions. The slope of Z equals the inverse of the output elasticity, which corroborates the theoretical analysis by Hartwig (2011a). Employment on the abscissa runs from 1 to 70. D and Z intersect at an employment level between 65 and 66. At this employment level, the marginal product of labour lies between 0.200 and 0.199, which is equal to the real wage for the assumed values $w=1$, $P^d=5$.

<Insert Figure 2 around here>

We can also add investment demand. In chapter 3 of the *General Theory*, Keynes distinguishes between two components of D, which he calls D_1 and D_2 . D_1 designates expected consumption demand and is, according to Keynes (1936, pp. 28-29) a function of employment $\chi(N)$. Although he does not say it directly, from what he writes on page 30 of the *General Theory* it is clear that Keynes regarded expected investment demand (D_2) *not* to be a function of employment (see also Chick 1983, p. 67). This means that if we draw D_2 in the PY/N space of Figure 1, it should be a horizontal line – with the concave D_1 curve set on top of it.⁹ In the first simulation it was implicitly assumed that the entrepreneurs expect zero investment. This implied zero savings and a propensity to consume of 1.

If we now assume positive investment (expectations) we must also relax the assumption that the propensity to consume equals 1. We can use the model to calculate which propensity to consume is consistent with a certain expected level of investment. For example, if the expected level of investment equals 10, the D curve becomes

$$D = c \cdot P^d \cdot Y(N) + 10 \quad (5)$$

If we still assume a demand price level of 5, we know that the real wage still equals 0.2. A marginal product of labour consistent with this real wage level will still be associated with an

⁹ It is the entrepreneurs in the consumption-goods sector who have to form expectations about the level of investment spending in order to calculate how much demand will be forthcoming to them through the multiplier mechanism (see Hartwig 2004b, 2006, 2008).

employment level around 65. So we are looking for a value of c that generates a value for D equal to 92.86 (the value of Z for an employment level of 65, see Figure 2). We can calculate c from equation 6:

$$92.86 = c \cdot 5 \cdot 65^{0.7} + 10 \Leftrightarrow c = 0.89 \quad (6)$$

Savings ($0.11 \cdot 5 \cdot 65^{0.7}$) are equal to the investment of 10 (apart from rounding errors).

Now let us return to the model without investment and assume a ‘supply shock’. For instance, due to a positive shock to productivity, let the output elasticity rise from 0.7 to 0.75. The second line of Table 1 lists the production function as well as Z and D for the second set of assumptions, and Figure 3 shows the result of the simulation. The Z curve moves to the right. Its slope is now the inverse of 0.75, so it is flatter than before. The D curve moves to the top because every unit of employment now produces more real income and expected demand. The new point of intersection lies at an employment level between 197 and 198. Again, the marginal product of labour is close to the real wage of 0.2.

<Insert Figure 3 around here>

We can also simulate a ‘demand shock’ as a rise or decrease in the demand price level. Keynes’s acceptance of the ‘first classical postulate’ implies that entrepreneurs expand output (and employment) only if they expect that the market will accept the price increase necessary to cover the rise in marginal cost (due to decreasing marginal returns). In other words – absent positive supply shocks –, entrepreneurs must be able to expect a higher demand price level in order to expand output. For this reason, Keynes wrote in an open letter to President Roosevelt:

“Rising prices are to be welcomed because they are usually a symptom of rising output and employment. When more purchasing power is spent, one expects rising output at rising prices. Since there cannot be rising output without rising prices, it is essential to

insure that the recovery shall not be held back by the insufficiency of the supply of money to support the increased monetary turnover” (Keynes 1933, p. 33).¹⁰

Figure 4 shows the consequences of raising the demand price level from 5 to 6 (without specifying yet which improvement in demand conditions prompted entrepreneurs to revise their demand price expectations upward). Table 1 gives the details for the third simulation. The D curve moves to the top, employment and output rise, and the marginal product of labour drops to the value of the new real wage rate (of 1/6).¹¹

<Insert Figure 4 around here>

Now assume that a positive shock to investment expectations is the reason for the improvement in demand conditions. This shock can raise employment only if it is associated with a higher expected demand price level (or a lower nominal wage rate). If it is not, something inconsistent occurs, as is shown in Figure 5. The simulation assumes a jump in expected investment from 10 to 20 without a change in the demand price level or the nominal wage rate. The figure shows that output and employment rise. If output and employment are

¹⁰ Note that Keynes does not indicate that rising prices raise output and employment because they lower the real wage rate. They are a ‘symptom’, not a cause of rising output and employment.

¹¹ It is apposite to revisit Keynes’s much debated definition of involuntary unemployment from chapter 2 of the *General Theory* at this juncture, according to which involuntary unemployment prevails when “*in the event of a small rise in the price of wage-goods relatively to the money wage, both the aggregate supply of labour willing to work for the current money-wage and the aggregate demand for it at that wage would be greater than the existing volume of employment*” (Keynes 1936, p. 15). A rise in the demand price level raises the demand for labour because the D curve moves to the top. Labour supply drops – a downward move along the labour supply curve in the right panel of Figure 1 –, but it remains higher than the existing volume of employment before the rise in the price level. Keynes’s definition of involuntary unemployment is thus perfectly in line with his model of effective demand and invokes in no way a neo-classical ‘labour market’ – at least no downward-sloping ‘labour demand curve’. Contrary to Darity and Young (1997), it would therefore not be necessary to rewrite this definition for a hypothetical ‘second edition’ of the *General Theory*.

higher than in simulation 1, the marginal product of labour must be lower. The table accompanying Figure 5 shows that the MPL drops from 0.2 to around 0.186. Still, the real wage is unchanged at $1/5$. So there is a discrepancy between the marginal product of labour and the real wage, which, according to Keynes, must be cured by a price reaction. Hence the market price – as opposed to the ‘demand price’, which is an *ex ante* expectation by the entrepreneurs – will rise to a value of around 5.38 in order to equalise the marginal product and the real wage.

<Insert Figure 5 around here>

If the entrepreneurs fully anticipate the situation, they know that their output expectation of around 22 is inconsistent with their demand price expectation of 5. So they will probably plan with a higher demand price in the first place. Of course, this would move the D curve further upward, leading to an equilibrium with a still lower marginal product of labour. On the other hand, the entrepreneurs can expect that the workers will resist ever lower real wages and will start asking for higher nominal wages. This moves the Z curve inwards and can lead to an equilibrium closer to the origin. In Hartwig (2006), I argued that the Principle of Effective Demand is a model that allows for determining a demand price level – and hence real wage level – that is consistent with a given expected output relation between the consumption-goods producing and the investment-goods producing departments of the economy. In the model, this determination in terms of expectation-building takes place *ex ante* – i.e. at the beginning of the production period before production is started.

There will be a w/P^d combination that leads to a D/Z equilibrium in which the marginal product of labour is equal to that real wage. For the sake of the argument, let us assume that the adjustment process does not take place instantaneously in the minds of the entrepreneurs at the beginning of the production period, but as a trial-and-error process over a sequence of periods. More concretely, let us assume that simulation 4 describes what happens in period 1.

For period 2 let us assume that the entrepreneurs raise their demand price expectation to 5.38 and their expectation for the nominal wage rate from 1 to 1.1.

Figure 6 shows the results of simulation 5. The curves intersect at a smaller volume of employment. Therefore, the MPL rises from 0.186 (simulation 4) to 0.19. Still, however, the MPL is lower than the *ex ante* expected real wage (of 1.1/5.38), so the nominal wage and price levels will rise further. Over the next periods, output and employment can be expected to decline further, while the real wage keeps rising, until the point is reached where the real wage and the marginal product of labour are equal. Declining output and employment at rising real wages was regarded as the typical case by Keynes – at least until the publication of “Relative Movements of Real Wages and Output”.

<Insert Figure 6 around here>

IV. SIMULATING THE D/Z MODEL WITH NON-DECREASING MARGINAL RETURNS

The upshot of the foregoing analysis is that the D/Z model works well under the assumption of decreasing marginal returns. This is not the case, however, when this assumption is dropped. The sixth line of Table 1 gives as an example for a production function with increasing returns: the function $Y=N^2$. Z remains a linear function of employment with the slope given by the inverse of the output elasticity; and D remains the product of the demand price level and output. Even without a simulation it is clear that D lies above Z for all N and that the two curves are driven apart exponentially. There is no point of intersection – in other words: no point of effective demand. Without such a point, however, the D/Z model fails to determine the level of output and employment. This means that for production functions with thoroughly increasing returns, the model of effective demand does not work.¹²

¹² This statement carries over to production functions with thoroughly constant returns.

However, in “Relative Movements of Real Wages and Output” Keynes does *not* suggest that marginal returns are thoroughly increasing. Rather, he suggests a nexus between marginal returns and capacity utilisation. He writes:

“We should all agree that if we start from a level of output very greatly below capacity, so that even the most efficient plant and labour are only partially employed, marginal real cost may be expected to decline with increasing output, or, at the worst, remain constant. But a point must surely come, long before plant and labour are fully employed, when less efficient plant and labour have to be brought into commission ... Even if one concedes that the course of the short-period marginal cost curve is downward sloping in its early reaches, Mr. Kahn’s assumption that it eventually turns upwards is, on general common-sense grounds, surely beyond reasonable question; and that this happens, moreover, on a part of the curve which is highly relevant for practical purposes” (Keynes 1939, pp. 44-45).

Keynes here describes an S-shaped production function. The marginal returns schedule of such a production function is an inverted U. Marginal returns rise when output and employment are low. From some point onwards, however, they begin to decline. Keynes believes that the part of the curve beyond the turning point is ‘highly relevant for practical purposes’. This may reflect his reluctance to give away entirely the ‘incontrovertible proposition’ of decreasing physical returns in the short period in the light of the empirical evidence collected by Dunlop and Tarshis.

How does the D/Z model of effective demand cope with an S-shaped production function? To investigate this, a concrete S-shaped production function will be specified, and the D and Z functions will be calculated. This is done in the seventh line of Table 1. Figure 7 shows the two curves – or, more precisely, the two curves up to the point where the marginal product is still positive. As Pindyck and Rubinfeld (1995, p. 172) point out, production past this point

“is not technically efficient and therefore is not part of the production function; technical efficiency rules out negative marginal products”.

<Insert Figure 7 around here>

As the figure shows, an S-shaped production function gives rise to a D curve which is also S-shaped, and to a Z curve which is convex. The curves intersect in the region which Keynes regarded as ‘highly relevant for practical purposes’, namely in the region of decreasing marginal returns. Most notably, a point of intersection in the region of the employment spectrum for which marginal returns are upward-sloping is *impossible* for the same reasons discussed in the context of production functions with thoroughly increasing marginal returns. This means, however, that the Principle of Effective Demand as a model of entrepreneurial decision making under uncertainty, leading up to a planned level of output and employment for the upcoming production period which entrepreneurs expect to be profit-maximising, cannot produce points of effective demand in the region ‘of output very greatly below capacity’. In other words, if one allows for an S-shaped production function as Keynes did in his 1939 essay (as opposed to the *General Theory*), the model of effective demand tells us that entrepreneurs never expect that an ‘output very greatly below capacity’ will be profit-maximising. Hence, remembering that entrepreneurs decide about their output and employment levels *ex ante* based on the model of effective demand, we have to conclude that a situation with output greatly below capacity will never occur. This conclusion, of course, would be inconsistent with the empirical evidence, especially during the Great Depression. The argument in this essay is that it is not the model of effective demand that is to blame for this inconsistency, but Keynes’s suggestion that marginal returns may be increasing over a certain range of output in his 1939 essay. If we deny this possibility and return to the *General Theory*’s assumption of thoroughly decreasing marginal returns, no such inconsistency emerges. A concave D curve can intersect a linear Z curve at very low levels of employment

(and capacity utilisation) if the demand price level is sufficiently low. For instance, if we replace the demand price level of 5 in simulation 1 by a demand price level of 2, D would intersect Z at an employment level, not between 65 and 66 as in Figure 2, but at an employment level between 3 and 4.

But what about the reason why Keynes dropped the assumption he previously thought to be ‘incontrovertible’ – the assumption of decreasing marginal returns in the short period – in the first place: the empirical evidence collected by Dunlop and Tarshis? The next section shows that a movement of real wages and output in the same direction – or even constant real wages in the face of changing output, which was Keynes’s preferred ‘statistical generalisation’ in 1939 (see below) – can be simulated based on a production function with thoroughly diminishing marginal returns to labour.

V. MUST REAL WAGES AND OUTPUT MOVE IN OPPOSITE DIRECTIONS UNDER DECREASING MARGINAL RETURNS?

In the *General Theory*, Keynes suggests that economics can be divided into “the theory of stationary equilibrium and the theory of shifting equilibrium – meaning by the latter the theory of a system in which changing views about the future are capable of influencing the present situation” (Keynes 1936, p. 293). Obviously, the ‘theory of shifting equilibrium’ refers to his own Principle of Effective Demand, which is a model for what has been called a ‘board room economy’. Keynes’s ‘equilibrium’ – the point of effective demand – is not some kind of ‘market equilibrium’ as in the ‘theory of stationary equilibrium’. Rather it is the result of a thought experiment of each entrepreneur aiming to estimate *ex ante* which output and employment level will realise maximum profit. This equilibrium ‘shifts’ (from each production period to the next) because entrepreneurs’ expectations are in a permanent flux.

Above it has been argued that ‘supply and demand shocks’ can shift the equilibrium. These shocks consist in changes in entrepreneurial expectations as to the conditions of supply and/or demand. More optimistic views about demand conditions either mean that entrepreneurs expect a higher investment volume or that they expect the market to accept a higher demand price level. Both types of expectations move the D curve upward. More optimistic views about supply conditions can also take two forms. Either nominal wages are expected to drop or productivity is expected to rise. In both cases, the Z curve moves outward.

An increase in productivity, for instance thanks to an improved organisation of production processes, can be expected to happen frequently; and they will move the Z curve outward. At the same time, the D curve moves upward because higher productivity means that the same number of workers will produce a higher level of output (income).

The effects of such a positive supply shock have already been shown in Figure 3.¹³ The D and Z curves intersect at higher levels of output and employment. Concomitantly, the Marginal-Product-of-Labour (MPL) curve shifts to the right (see Figure 8). Now it becomes clear that accepting the ‘first classical postulate’ that the real wage equals the marginal product of labour need not imply that real wages and output move in opposite directions even under decreasing marginal returns. The outward shift of the MPL curve breaks this nexus. This can be verified by scrutinising the tables accompanying Figures 2 and 3. For the lower output elasticity of 0.7, the D and Z curves intersect at an employment level between 65 and 66. The mean marginal product of labour (real wage) over these two employment levels equals 0.1996. For the higher output elasticity of 0.75 on the other hand, the D and Z curves intersect at an employment level between 197 and 198. The mean marginal product of labour

¹³ In this example, the shock raises the output elasticity. It is also possible to model a rise in total factor productivity, which has been set to unity in the simulations so far. For example, the production function $Y=N^{0.7}$ could be changed to $Y=1.1*N^{0.7}$. In this case, the Z-function remains unchanged. Only the D curve moves upward, similar to the case of simulation 3.

(real wage) over these two employment levels equals 0.2001, which is a little bit higher, not lower. However, for practical purposes these two values are equal. This is in line with what Keynes believed to be the general case. In “Relative Movements of Real Wages and Output”, he writes:

“If, at the present stage of the inquiry, we are to make any single statistical generalisation, I should prefer one to the effect that, for fluctuations within the range which has been usual in the periods investigated which seldom approach conditions of full employment, short-period changes in real wages are usually so small compared with the changes in other factors that we shall not often go far wrong if we treat real wages as substantially constant in the short period (a very helpful simplification if it is justified). The conclusion, that changes in real wages are not usually an important factor in short-period fluctuations until the point of full employment is approaching, is one which has been already reached by Dr. Kalecki on the basis of his own investigations” (Keynes 1939, pp. 42-43).

The upshot of this section is that constant real wages – or even real wages moving in the same direction as output and employment – are perfectly compatible with decreasing marginal returns to labour in the short period in which the capital stock is constant.¹⁴ There was no need for Keynes to give away the ‘incontrovertible proposition’ of decreasing physical returns.

<Insert Figure 8 around here>

¹⁴ Darity (1989, pp. 21-22) discusses a number of additional reasons why real wages might move pro-cyclically in spite of decreasing marginal returns, such as expectational errors, time lags and shifts in the output composition between consumption goods and investment goods.

VI. CONCLUSION

In this essay, I specify exact functional forms for the ‘aggregate demand function’ (D) and the ‘aggregate supply function’ (Z) which together make up Keynes’s model of effective demand and use them to simulate equilibria of the model and its behavior in the face of various shocks. In addition to gaining a deeper understanding of Keynes’s model, these simulations also highlight the importance of the assumption of diminishing marginal returns to labour for Keynes’s model.

Keynes does not seem to have believed that ceding diminishing marginal returns to Dunlop and Tarshis in his 1939 essay “Relative Movements of Real Wages and Output” hurt his theory or his policy recommendations in any way. The essay argues that Keynes was wrong in this assessment. Rather, his model of effective demand described in chapter 3 of the *General Theory* only works under the assumption of decreasing marginal returns; it fails to determine the level of effective demand or produces inconsistent results if this assumption is relaxed or dropped.

Some may be tempted to ask: “So what?”. The chapter 3 model is widely believed to be unconnected with the rest of the *General Theory* and hence to be unimportant. In chapter 18 of that book for instance, titled ‘The General Theory of Employment Re-stated’, the D/Z model does not resurface. So what does it matter if non-decreasing returns are in conflict with the chapter 3 model? They may nevertheless be in line with the rest of the *General Theory*.

Amadeo (1989, p. 90) notes that there is a striking difference between chapters 3 and 18: “In the former, the whole argument is based on producers’ decision to produce and employ based on the expected demand for their product and the cost structure of their firms; in the latter, the theory of employment and output is based essentially on the determinants of actual aggregate consumption and investment expenditure”. Amadeo calls the chapter 3 model the ‘supply version of the principle of effective demand’ and the chapter 18 version the ‘expenditure dimension version of the principle of effective demand’. He argues that “in the

book, there is a gradual substitution of actual demand for expected demand. Or, to put it differently, one is led to assume that demand creates its own supply, or that supply gradually adapts to demand in such a way that the role of supply can be totally neglected” (Amadeo 1989, p. 91).

Amadeo supports this move he perceives. In his view, Keynes was right to drop expectations because the “emphasis on expectations tends to obscure equilibrium analysis” (Amadeo 1989, p. 106). But to move from the ‘supply version of the principle of effective demand’ to the ‘expenditure dimension version’ would not only imply a de-emphasis on expectations, it would also imply to drop the aggregate production function. There seem to be two versions of Keynes’s own theory and Keynesian theory more generally: one that works with an aggregate production function and one that works without it. Mainstream Keynesianism has – until recently – opted for the second variant.¹⁵ There is no aggregate production function in the Keynesian cross model, the Keynesian multiplier model and the IS/LM model.¹⁶ Darity and Young (1997, p. 26) explicitly advise Keynesian economists to focus “on the theory of investment and the rehabilitation of the multiplier” instead of on chapter 2 of the *General Theory* in which Keynes accepts the ‘classical’ production function with decreasing returns to labour. Lawlor et al. (1987, p. 517) characterize Keynes’s acceptance of the first classical postulate in chapter 2 as a rather unnecessary attempt by

¹⁵ There is a production function, however, in the now fashionable New Keynesian model, see Blanchard and Galí (2007) for instance.

¹⁶ Although there is no production function in the standard textbook version of the IS/LM model, Hicks (1937) incorporates production functions with decreasing marginal returns. Cottrell and Darity (1991) replace decreasing with increasing returns and show that under certain assumptions about the relative dynamics of nominal wages and prices, this model can produce realistic (i.e. pro-cyclical) real wage and price dynamics. Therefore, unlike Keynes’s D/Z model, Hicks’s model can at least be reconciled with increasing returns. More research along these lines would be welcome.

Keynes to offer his classical colleagues common ground and as “peripheral to the main features of his analytical system”.

In the ‘expenditure dimension version of the principle of effective demand’, the question whether marginal returns to labour are decreasing, constant or increasing is of course irrelevant because there is no production function. This, however, is the version of Keynesianism that has come under attack by competing schools of thought. This attack, which has been quite successful in sweeping Keynesianism out of academia, has been launched on two grounds. The first contention was that Keynesianism had no micro-foundations (see for instance Thurow 1983, pp. 3-4) and the second was that it ignored expectations in general and rational expectations in particular (see Lucas and Sargent 1978). The ‘supply version of the principle of effective demand’ is much less prone to such criticism than the ‘expenditure dimension version’. First of all, the ‘supply version’ is micro-founded, and its micro-foundations (profit maximisation, price-taking, decreasing returns) are the same as those of the ‘classical’ school. And furthermore, the ‘supply version’ features expectations very prominently.¹⁷ The ‘supply version of the principle of effective demand’ therefore is the paradigm to build on. Given this – and remembering that chapter 3 lays the groundwork for the ‘supply version’ (see Amadeo 1989) – it can be assessed how damaging Keynes’s admittance of non-decreasing returns in “Relative Movements of Real Wages and Output” for his overall project of building a case for ‘equilibrium’ unemployment really was.

But what about Amadeo’s claim that Keynes himself dropped the ‘supply version’ in favour of the ‘expenditure dimension version’ as he progressed through the *General Theory*? I do not think that this claim holds water. Neither did Keynes drop expectations, which is clear from his re-statement of the essence of the *General Theory* one year after the book went to press (Keynes 1937). Nor did he drop the production function. The latter re-emerges in chapters 20 and 21 of the *General Theory* – that is, further down in the book than chapter 18.

¹⁷ Darity and Horn (1993) even argue that those expectations encompass rational expectations as a special case.

Of course, in these two chapters, Keynes emphasises repeatedly that marginal returns are decreasing (see Keynes 1936, p. 289 and pp. 302-306).

To sum up, non-decreasing marginal returns to labour are incompatible with key aspects of Keynes's theory; and it is not necessary to assume non-decreasing returns in order to explain pro-cyclical movements of real wages and output. Therefore, Keynes should not have given away the assumption of decreasing marginal returns in "Relative Movements of Real Wages and Output". The view that that essay constitutes an important amendment to his *General Theory* thus has to be put into perspective.

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A discussion with Till van Treeck about the significance of the assumption of decreasing marginal returns for Keynes's theoretical edifice prompted me to write this essay. I would like to thank William Darity, Jr. for his excellent comments on earlier versions of this paper. All remaining errors are mine.

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Figure 1: Determination of employment and unemployment according to the model of effective demand

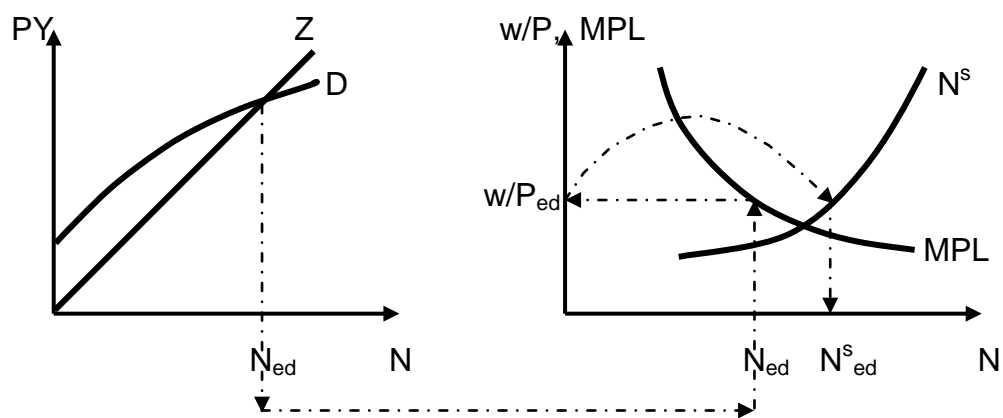
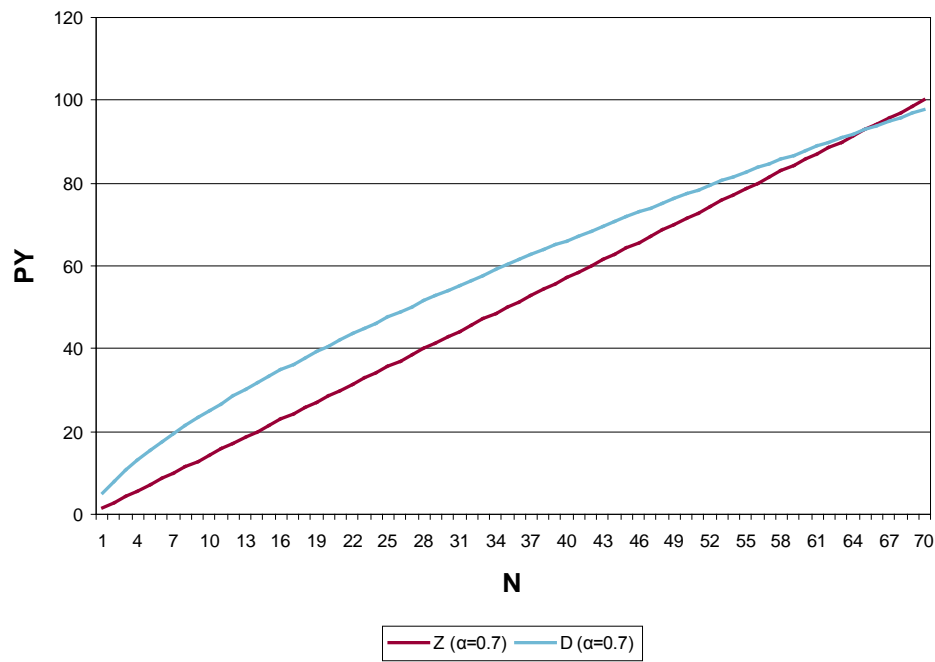
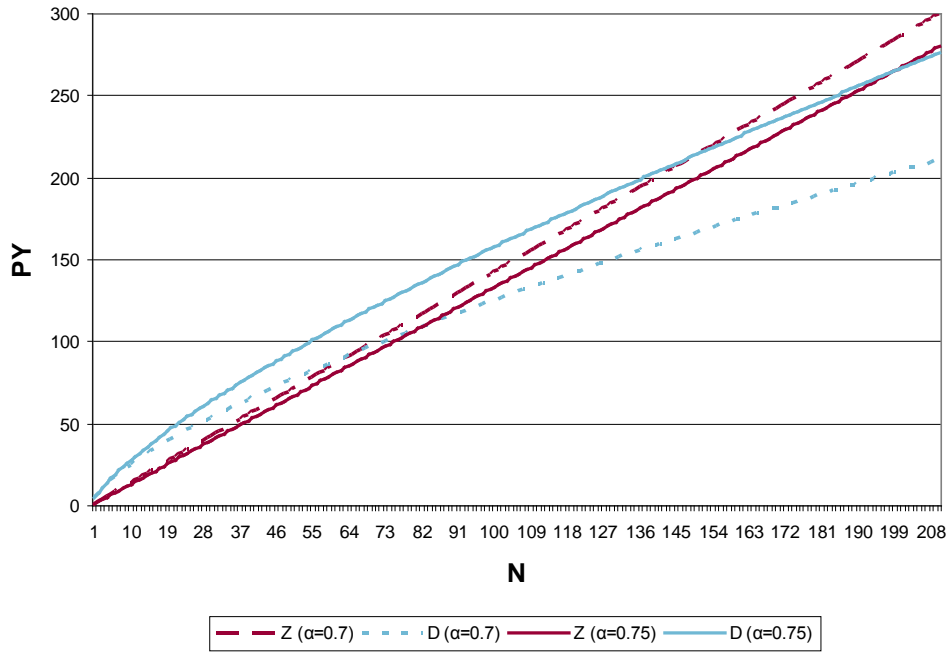


Figure 2: Simulated D and Z functions for $Y(N)=N^{0.7}$



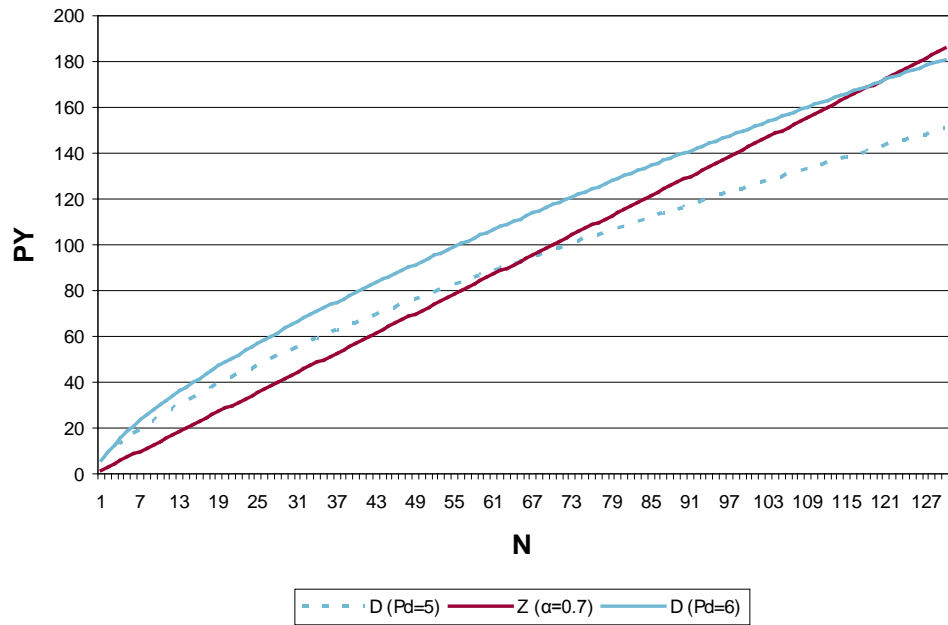
N	Y(N)	MPL	Ps (=1/MPL)	Z ($\alpha=0.7$)	D ($\alpha=0.7$)
61	17.7717782	0.20393844	4.90344051	87.1428571	88.858891
62	17.9752187	0.20294602	4.92741869	88.5714286	89.8760934
63	18.1776771	0.20197419	4.95112767	90	90.8883855
64	18.3791737	0.20102221	4.97457465	91.4285714	91.8958684
65	18.5797279	0.20008938	4.99776656	92.8571429	92.8986395
66	18.7793586	0.19917502	5.02071004	94.2857143	93.896793
67	18.9780839	0.19827849	5.04341146	95.7142857	94.8904194
68	19.1759213	0.19739919	5.06587692	97.1428571	95.8796066

Figure 3: Simulated D and Z functions for $Y(N)=N^{0.7}$ and $Y(N)=N^{0.75}$



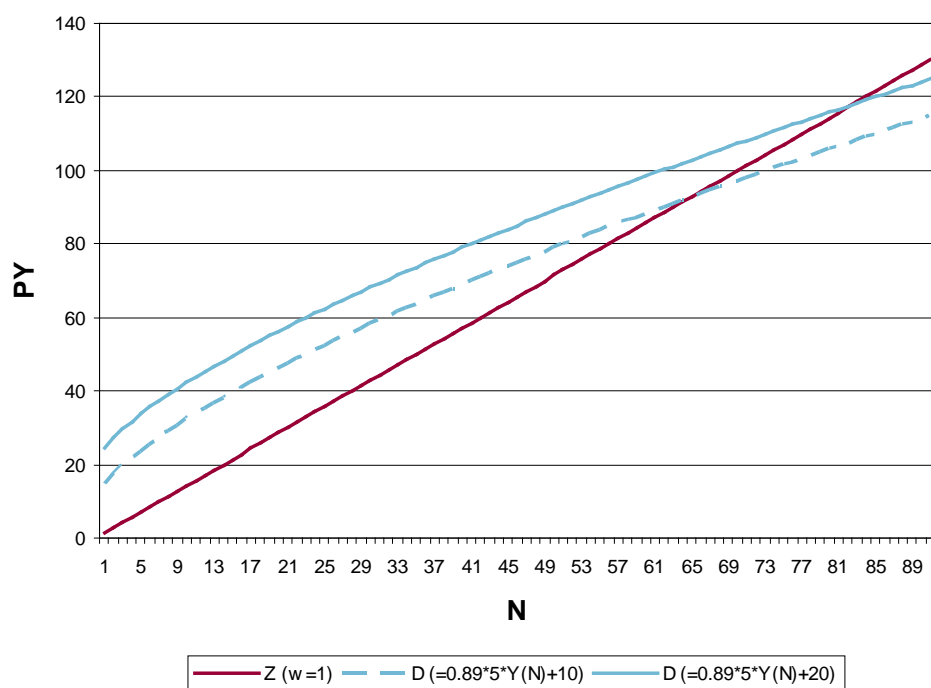
N	Y(N)	MPL	Ps (=1/MPL)	Z (α=0.75)	D (α=0.75)
193	51.7807077	0.20122037	4.96967587	257.333333	258.903539
194	51.981798	0.20096056	4.9761008	258.666667	259.90899
195	52.1826294	0.20070242	4.98250094	260	260.913147
196	52.3832034	0.20044593	4.98887652	261.333333	261.916017
197	52.5835218	0.20019107	4.99522774	262.666667	262.917609
198	52.7835861	0.19993783	5.00155483	264	263.91793
199	52.983398	0.19968617	5.007858	265.333333	264.91699
200	53.182959	0.1994361	5.01413746	266.666667	265.914795

Figure 4: Simulated D and Z functions for $P^d=5$ and $P^d=6$



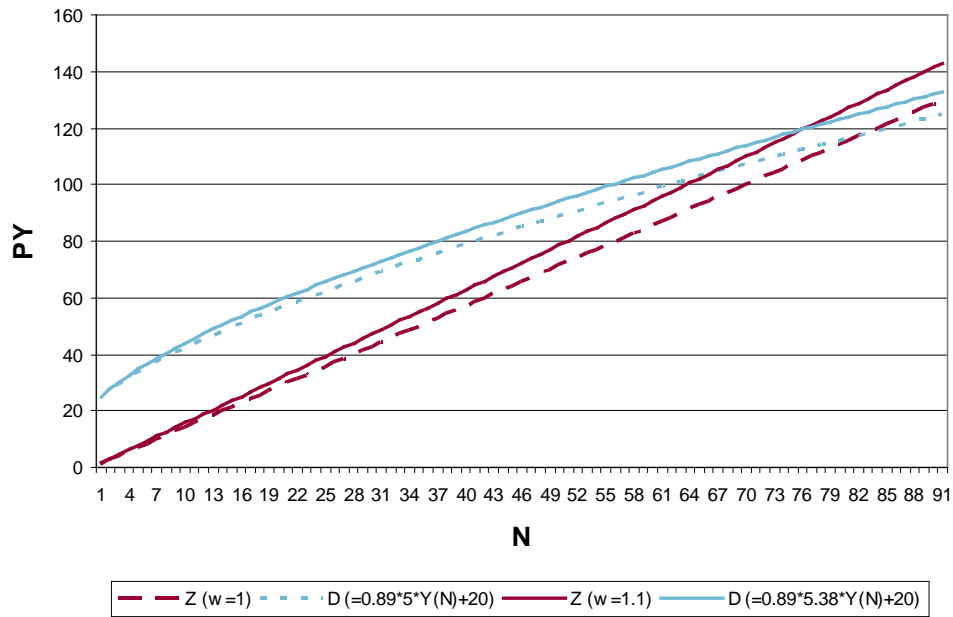
N	Y(N)	MPL	Ps (=1/MPL)	Z ($\alpha=0.7$)	D ($P^d=6$)
115	27.7005561	0.16861208	5.93077314	164.285714	166.2033368
116	27.8689491	0.16817469	5.94619787	165.714286	167.2136946
117	28.0369071	0.16774218	5.96152979	167.142857	168.2214428
118	28.2044351	0.16731445	5.97677026	168.571429	169.2266103
119	28.3715376	0.1668914	5.99192058	170	170.2292256
120	28.5382194	0.16647295	6.00698204	171.428571	171.2293163
121	28.704485	0.166059	6.0219559	172.857143	172.2269099
122	28.8703389	0.16564949	6.03684339	174.285714	173.2220332

Figure 5: Simulated D and Z functions for $D=0.89*5*Y(N)+10$ and $D=0.89*5*Y(N)+20$



Y(N)	MPL	Ps (=1/MPL)	Z (w=1)	D (=0.89*5*Y(N)+10)	D (=0.89*5*Y(N)+20)
17.7717782	0.20393844	4.90344051	87.14285714	89.26213079	99.26213079
17.9752187	0.20294602	4.92741869	88.57142857	90.16947533	100.1694753
18.1776771	0.20197419	4.95112767	90	91.07243988	101.0724399
18.3791737	0.20102221	4.97457465	91.42857143	91.97111461	101.9711146
18.5797279	0.20008938	4.99776656	92.85714286	92.86558647	102.8655865
18.7793586	0.19917502	5.02071004	94.28571429	93.75593932	103.7559393
18.9780839	0.19827849	5.04341146	95.71428571	94.64225412	104.6422541
19.1759213	0.19739919	5.06587692	97.14285714	95.52460905	105.524609
19.3728878	0.19653654	5.08811229	98.57142857	96.40307965	106.4030796
19.5689998	0.19569	5.11012322	100	97.27773895	107.2777389
19.764273	0.19485903	5.13191512	101.4285714	98.14865758	108.1486576
19.9587228	0.19404314	5.15349322	102.8571429	99.01590391	109.0159039
20.1523641	0.19324185	5.17486254	104.2857143	99.87954409	109.8795441
20.3452113	0.1924547	5.19602792	105.7142857	100.7396422	110.7396422
20.5372781	0.19168126	5.21699402	107.1428571	101.5962604	111.5962604
20.7285782	0.19092112	5.23776534	108.5714286	102.4494588	112.4494588
20.9191246	0.19017386	5.25834622	110	103.2992959	113.2992959
21.1089301	0.18943912	5.27874084	111.4285714	104.1458283	114.1458283
21.2980069	0.18871652	5.29895324	112.8571429	104.9891109	114.9891109
21.4863671	0.18800571	5.31898733	114.2857143	105.8291971	115.8291971
21.6740222	0.18730636	5.33884688	115.7142857	106.6661389	116.6661389
21.8609835	0.18661815	5.35853554	117.1428571	107.4999865	117.4999865
22.0472621	0.18594076	5.37805684	118.5714286	108.3307889	118.3307889
22.2328685	0.1852739	5.39741418	120	109.1585937	119.1585937
22.4178133	0.18461729	5.41661088	121.4285714	109.9834473	119.9834473

Figure 6: Simulated D and Z functions for $P^d=5.38$ and $P^s=1.1/0.7*N^{-0.3}$



Y(N)	MPL	Ps (=1.1/MPL)	Z (w=1.1)	D (=0.89*5.38*Y(N)+20)
19.9587228	0.19404314	5.668842541	113.142857	115.7811126
20.1523641	0.19324185	5.692348793	114.714286	116.7103894
20.3452113	0.1924547	5.715630711	116.285714	117.635855
20.5372781	0.19168126	5.738693423	117.857143	118.5575762
20.7285782	0.19092112	5.761541877	119.428571	119.4756177
20.9191246	0.19017386	5.784180842	121	120.3900424
21.1089301	0.18943912	5.806614924	122.571429	121.3009112
21.2980069	0.18871652	5.828848567	124.142857	122.2082833

Figure 7: Simulated D and Z functions for an S-shaped production function

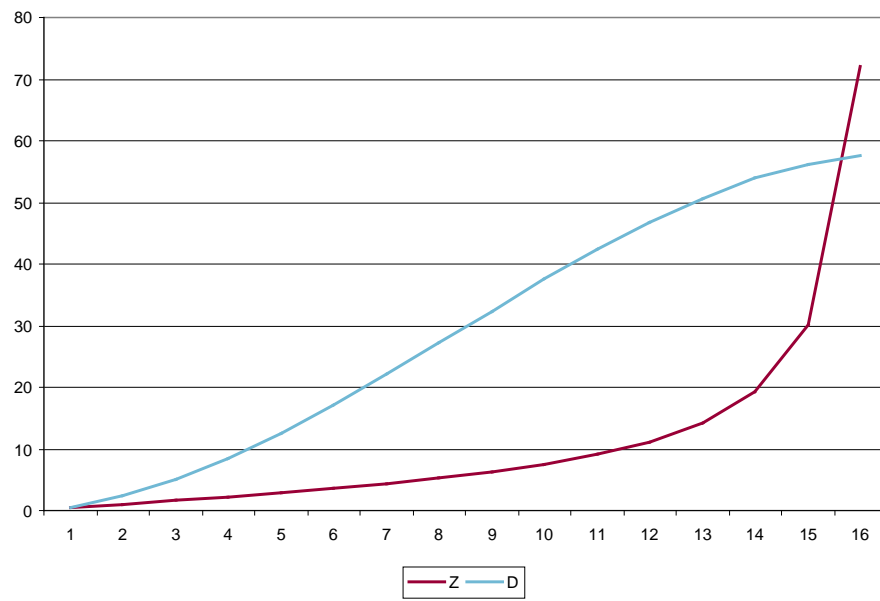


Figure 8: Simulated MPL functions for $Y(N)=N^{0.7}$ and $Y(N)=N^{0.75}$

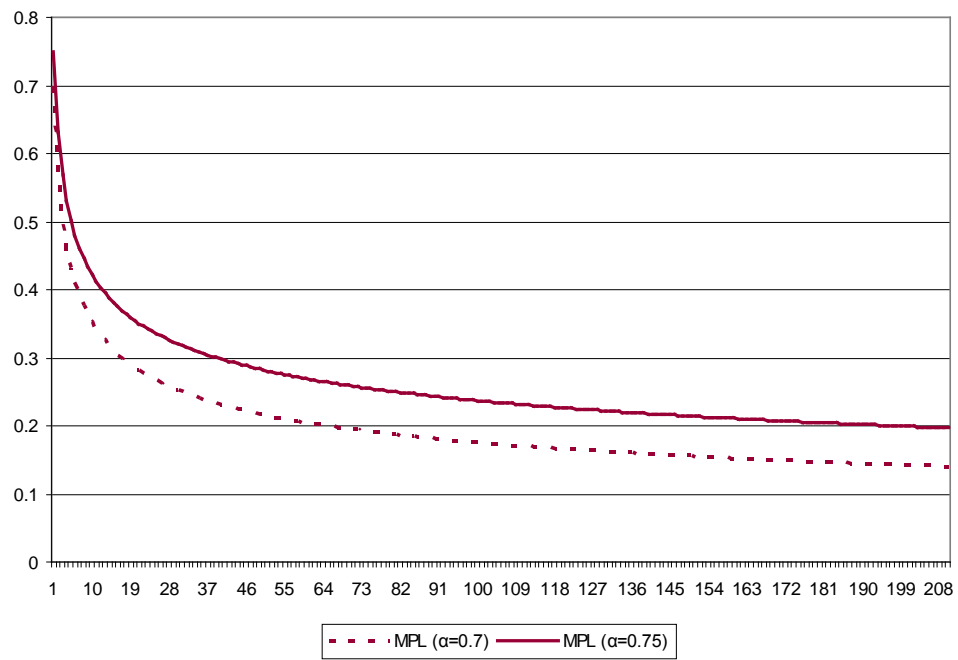


Table 1: Assumptions for numerical simulations

	Y(N)	MPL	P^s (=w/MPL)	Z (=P ^s *Y(N))	D (=P ^d *Y(N))
1	$Y=N^{0.7}$	$dY/dN=0.7*N^{-0.3}$	$P^s=1/0.7*N^{0.3}$	$Z=1/0.7*N^{0.3}*N^{0.7}$ $=1/0.7*N$	$D=5*N^{0.7}$
2	$Y=N^{0.75}$	$dY/dN=0.75*N^{-0.25}$	$P^s=1/0.75*N^{0.25}$	$Z=1/0.75*N^{0.25}*N^{0.75}$ $=1/0.75*N$	$D=5*N^{0.75}$
3	$Y=N^{0.7}$	$dY/dN=0.7*N^{-0.3}$	$P^s=1/0.7*N^{0.3}$	$Z=1/0.7*N^{0.3}*N^{0.7}$ $=1/0.7*N$	$D=6*N^{0.7}$
4	$Y=N^{0.7}$	$dY/dN=0.7*N^{-0.3}$	$P^s=1/0.7*N^{0.3}$	$Z=1/0.7*N^{0.3}*N^{0.7}$ $=1/0.7*N$	$D=0.89*5*N^{0.7}+10$ $D=0.89*5*N^{0.7}+20$
5	$Y=N^{0.7}$	$dY/dN=0.7*N^{-0.3}$	$P^s=1.1/0.7*N^{0.3}$	$Z=1.1/0.7*N^{0.3}*N^{0.7}$ $=1.1/0.7*N$	$D=0.89*5.38*N^{0.7}+20$
6	$Y=N^2$	$dY/dN=2*N$	$P^s=1/(2*N)$	$Z=1/(2*N)*N^2$ $=0.5*N$	$D=5*N^2$
7	$Y=25*N^2-N^3$	$dY/dN=50*N-3*N^2$	$P^s=1/(50*N-3*N^2)$	$Z=(25*N^2-N^3)/$ $(50*N-3*N^2)$	$D=(25*N^2-N^3)/40$