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Over-aging: Are present human populations too old?

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Working Paper No. 137

Over-aging - Are present human populations too old?

by

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Over-aging - Are present human populations too old?*

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Abstract

This paper investigates the problem of an “optimum population” concerning age structures in a 3-period OLG-model with endogenous fertility and longevity. The first-best solution for a number-dampened total social welfare function, including Millian and Benthamite utilitarianism as two extreme cases, identifies the optimal age structure, generally failed in the laissez-faire economy. Individuals over-invest in health expenditures and choose a non-optimal number of offspring. A calibration exercise for 80 countries emphasizes that mean ages in the optimal solution with the highest feasible individual utility exceed the observed in all countries, especially due to a very low first-best number of children. Introducing a preference for the population stock in the social welfare function increases fertility, but reduces individual utility, in the first-best solution. Optimal mean age shrinks and an over-aging of the laissez-faire economy becomes more likely. To decentralize first-best solutions health expenditures are taxed, whereas children are either taxed or subsidized.

Keywords: endogenous fertility; adult mortality; optimal age structure; over-aging; optimal taxation

JEL classification: H20; I10; J18

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1 Introduction

The demographic transition, most countries underwent during the past 150 years, leads to a variety of age structures. In developed countries, low fertility and mortality rates imply relatively high dependency ratios and mean ages. The high fraction of old individuals arouses fear of an over-aged population. On the contrary, in developing countries, featured by many children and low survival probabilities, populations are seen to be too young. Since both suggest sub-optimal situations, this papers aims to highlight the differences between observed and optimal age structures.

The problem of sub-optimal age structures differs between developed and developing countries. The former are presumably too old and the latter too young, implying different social and political challenges, since demographic aging involves strong economic effects, due to varying intergenerational transfers (Weil, 2006). Developed economies, like in Europe, with a high fraction of old individuals are plotted on the upper left on the LHS of figure 1. The high dependency ratios have consequences on old age security, health care as well as labor markets and are seen as a risk on social and individual welfare quite often. On the contrary, in developing economies, mean ages are still low, but dependency ratios likewise high, as illustrated on the RHS of figure 1. The large number of births combined with a low life expectancy e.g. requires policies on birth control, nutrition or health care and is as challenging to societies and especially policy makers as presumably over-aged populations. Providing an instrument to evaluate the existence and extent of the sub-optimality of age structures plotted in figure 1 is the main objective of the paper. Each policy implemented to avoid a population that is either over-aged or too young requires such an instrument or point of reference: an optimal age structure. On the contrary, the paper does not aim to address specific issues raised by different sub-optimal age structures.

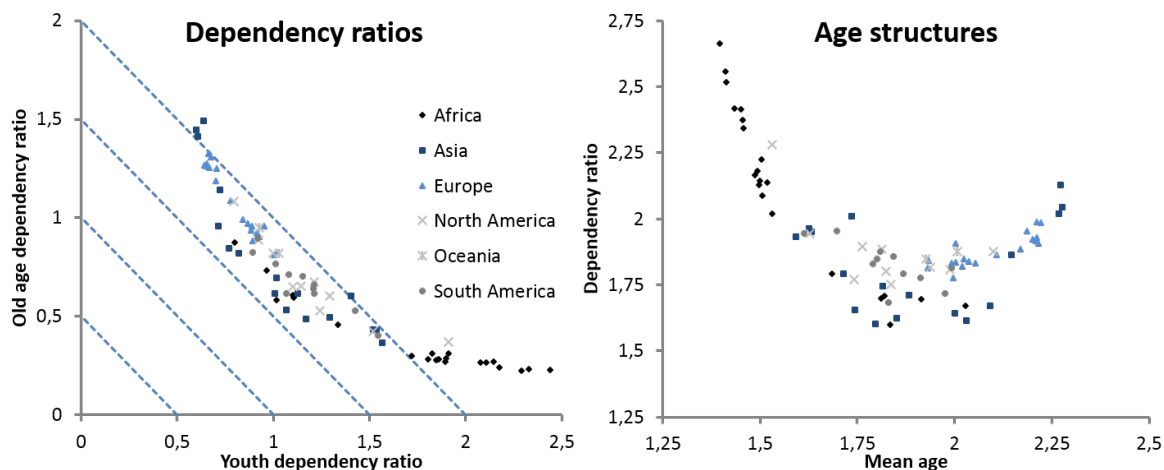


Figure 1: Steady state age structures across countries

The figure presents the stable populations arising from NRR (Net Reproduction Rate) and female life expectancies at age 60, according to section 3.1. The LHS illustrates dependency ratios across countries distinguished by youth and old age dependency ratio. Their overall value increases with distance to the point of origin, see Weil (2006). The U-shaped relation between mean ages and overall dependency ratios is plotted on the RHS. (Data source: United Nations (2011))

An overlapping-generations model (OLG-model) with three co-existing generations, children, adults and retirees, enables to evaluate discrepancies between observed and optimal age structures. Individual decisions in a laissez-faire (LF) economy endogenously determine observed age-

structures, illustrated in figure 1. Selfish representative individuals choose savings, the number of offspring and health expenditures to influence longevity.¹ These age structures are faced to the optimal, or social welfare maximizing, solutions of the first-best (FB) economy. Its social welfare function (SWF) follows number-dampened total utilitarianism, which allows both Millian and Benthamite utilitarianism as extreme as well as intermediate cases (Ng, 1986). Comparing both solutions and decentralizing optimal age structures points out, that observed and optimal age structures differ. To avoid over-investments in health, expenditures are always taxed. On the contrary, a dominated or dominating capital dilution effect determines whether children are taxed or subsidized and hence if parents c.p. have a too high or low fertility. Furthermore, lump-sum transfers in working and retirement age correct savings and balance government’s budget constraint.

Calibrating the LF economy reproduces age structures of 80 countries presented in figure 1. The resulting parameter set permits to compute optimal age structures. In the optimal solution with the highest feasible individual life-cycle utility among all first-best - the “Golden Age” - mean ages in all countries are considerably above observed, whereas findings on the dependency ratio notably depend on observed fertility. In developing countries with high fertility observed exceed optimal dependency ratios and vice versa in populations with low fertility. Even if “Golden Age” implies the highest expected life-cycle utility, results are striking from a demographic point of view. Fertility is substantially below replacement level. Introducing a preference for the population stock and/or a social discount factor (SDF) in the SWF increases number of offspring and alters age structures. A sufficient weight of the population in the SWF changes outcome and populations are over-aged. However, both the weight of population size and the SDF are fixed arbitrary. Finally, the findings lead to the following dilemma: “Golden Age” is favorable from an ethical and economic point of view but problematic from a demographic perspective. On the contrary, fixing the weight of the population size and the SDF arbitrarily facilitates plausible demographic results but is questionable from an ethical point of view.

The outline of the paper is as follows. Before introducing the theoretical framework in section 3, the related literature is reviewed. The first subsection (3.1) presents the demographic structure in the OLG-model. Afterwards the solution chosen by individuals in a LF economy (3.2) and the optimal decisions of a social planner (3.3) are evolved. A decentralization of the first-best solution in subsection (3.4) enables to discuss tendency concerning an over-aged population. Section 4 presents the calibration exercise. After describing the data set (4.1), the laissez-faire model is calibrated in subsection 4.2. Subsection 4.3 compares observed and optimal age structures and discusses the outcomes. Section 5 concludes.

2 Literature review

The idea of an optimal age structure contributes to the literature on “optimum population”, traditionally dealing with population size by applying the “economic principle” on populations (Wolf, 1908). As “an optimum population is the one that achieves a given aim in the most satisfactory way” (Sauvy, 1969), we define maximization of social welfare as objective. Using a number-dampened total utilitarianism (Ng, 1986) provides a flexible SWF that allows different preferences for the population stock between the two extreme cases of Millian and Benthamite utilitarianism. A Millian or Average SWF optimizes utility of an average or representative individuals. As population size is not valued, a very small number of individuals characterized by a high (average) utility presumably is optimal. On the contrary population size is fully internalized in Benthamite, Total or Classical

¹In contrast to Becker and Barro (1988), Soares (2005) and Baudin (2012), we abstract from investigating the role of child mortality and focus on longevity.

utilitarianism. The sum over welfare or utility of all individuals is maximized. Parfit [1976, 1982, 1984] criticizes that the repugnant conclusion may emerge: it is optimal to increase population size while per capita consumption goes to zero.²

Theories on optimal population emphasizes different aspects of populations, starting with the population size, see e.g. Dasgupta [1969, 1998, 2005] or Boucekkine *et al.* (2011). Investigating the growth rate adds the dynamic perspective. For an OLG-framework with two overlapping generations the famous “serendipity theorem” (Samuelson, 1975) claims that in the case of a unique and stable steady state the laissez-faire economy converges to the most advantageous golden rule steady state, the “Golden Age”, if fertility is fixed optimally.³ The “serendipity theorem” still holds if mortality, and hence an uncertain life time for the second period, is added (de la Croix *et al.*, 2012). Stressing the idea of an optimal age structure in a related OLG-framework introduces a third aspect to theories of “optimum population”, not done yet to the best of our knowledge. Additionally, some major differences arise: First of all, highlighting age structures adds another dimension. Individuals and populations evolve along the dimension time and age (Arthur and McNicoll, 1977). The latter is generally neglected in theories of optimal size. Secondly, in line with Boucekkine and Fabbri (2013) applying a number-dampened total utilitarianism offers a more flexible SWF than exclusively focusing on “Golden Age”. Thirdly, models on optimal population growth neglect child rearing costs and health expenditures. As fertility only balances the negative capital widening and positive intergenerational transfer effect, optimal population growth is probably too high in those models (Arthur and McNicoll, 1978). Considering costly aspects of fertility and longevity in the LF and FB solution links the paper to frameworks on optimal fertility and longevity, jointly determining population size, growth rate as well as the age structure.

The socially optimal level of fertility is strongly related to the choice of the SWF. In general, fertility and hence population size of a Benthamite SWF exceeds those of a Millian SWF. However, the number of children in the LF economy is either above or below the optimal level in both kinds of SWF (Nerlove *et al.*, 1985) and depends on individual preferences. Utility of offspring is an argument in their parents’ utility function if they are altruistic, like in Razin and Ben-Zion (1975), Becker and Barro (1988), Baudin (2011) and Nerlove *et al.* (1986). This kind of perfect altruism leads to dynastic preferences such that the time horizon of parents is infinite.⁴ In contrast to altruistic individuals, time horizon is limited by death and therefore finite for selfish parents. In line with Eckstein and Wolpin (1985), Galor and Weil (1996) or van Groezen and Mejidam (2008) we assume that children appear as a kind of consumption good in parents’ utility function and go along with costs.⁵ Related to the idea of an optimal fertility, missing property rights, causing an inefficiently low number of offspring, are an another possible explanation for the low fertility in developed countries (Schoonbroodt and Tertilt, 2013).

Endogenous health expenditures are one possibility to model an endogenous length of life. Individuals invest in their health capital stock, simultaneously depreciated age specifically in each period. They die, if the health capital stock is zero (Grossman (1972), Ehrlich and Chuma (1990),

²Boucekkine and Fabbri (2013) figure out that the repugnant conclusion never occur for realistic parameters in an AK-like growth model. Blackorby and Donaldson (1984) propose a critical-level generalized utilitarianism as an alternative to avoid a population that is too large. Individual utility only contributes positively to the SWF if it exceeds a critical level.

³Deardorff (1976) shows that the solution is a global minimum for a Cobb-Douglas utility and production function. Jäger and Kuhle (2009) add the exact general conditions for an interior optimal growth rate of populations.

⁴Optimal family policies to decentralize FB fertility in a framework with quality quantity trade off have to account for interaction between instruments on education and fertility, due to the non-linear budget constraint (Baudin, 2011).

⁵On the contrary, Neher (1989) or Bental (1989) consider children as an investment good. Another alternative are “warm glove” preferences or ad hoc altruism. Parents achieve utility by the number of children and an argument of their children’s utility function, see e.g. Benabou (1996).

Dalgaard and Strulik (2014)). Focusing on longevity, we assume that health expenditures determine the probability to survive retirement age. The optimal survival probability might be failed as several effects, like externalities in preventing contagious diseases (Kuhn *et al.*, 2011) or the negative impact of health expenditures on the life-cycle income, are not taken into account (Davies and Kuhn (1992), Philipson and Becker (1998)).

3 The model

3.1 Demographic structure

The model is an extension of the overlapping generations model introduced in the literature by Allais (1947), Diamond (1965) and Samuelson (1958) in discrete time t from 0 to infinity. For simplicity, a single sex population in a closed economy is assumed. The age structure is pooled in three age groups, children, working population and retirees. Hence, at any point in time three generations coexist. The cohort born in period t constitutes the children N_t^t .⁶ Childhood as first period of life is passive. Simultaneously the cohort N_t^{t-1} , born in $t-1$, is in middle age as working adults. During this active period of life, individuals solve their maximization problem, endowed with one unit of time, allocated on child-rearing and labor. As the old generation N_t^{t-2} is retired and does not participate in labor market, total labor supply is:

$$L_t = (1 - \tau n_t) N_t^{t-1}, \quad (1)$$

where τ represents the time required to bring up a child. The number of children born in t determine size of birth-cohort in t , hence $N_t^t = n_t N_t^{t-1}$. Whereas all individuals survive childhood and undergo middle age, only a fraction q_t becomes old: $N_t^{t-2} = q_t N_{t-1}^{t-2}$.⁷ Thus, the age structure is as simple as possible and composed of the three at least necessary age groups to describe a human life-cycle: the working period including parenthood surrounded by the two periods in dependency, childhood and retirement (Bommier and Lee, 2003).

The dependency ratio (DR_t) and the mean age (\bar{x}_t) are used to describe age structures. The former indicator links economic and demographic aspects. Individuals participating in the labor market are compared to non-working and thus dependent people. Due to interactions between generations, e.g. intergenerational transfers, this indicator is of particular interest for policy makers. Applied to the population of the model, the dependency ratio is:

$$DR_t = \frac{N_t^{t-2} + N_t^t}{N_t^{t-1}}.$$

Rearranging allows to express the dependency ratio in terms of the survival probability and number of children:

$$DR_t = \frac{q_t + n_{t-1} n_t}{n_{t-1}}. \quad (2)$$

It has to be mentioned that the term “dependent” is deceptive in the particular case of the presented framework. Indeed, the old generation does not participate in the labor market. However, retirees are not supported by the working generation. Nevertheless, it is not possible to conclude on the age of a population by means of the dependency ratio. Both children and retirees are elements of the numerator. A high value can imply either a very young or old population, as emphasized on

⁶The subscript indicates the period and the superscript the birth cohort.

⁷Likewise $N_t^{t-2} = q_t N_{t-2}^{t-2}$ holds, as the survival probability between childhood and middle age is 1.

the RHS of figure 1. In other words, an age structure with the shape of a pyramid as well as an inverted pyramid can have the same dependency ratio. Moreover, this demographic indicator is inappropriate to describe the process of aging. Therefore the mean age is additionally considered.⁸ Applied to the model the mean age is calculated as:

$$\bar{x}_t = \frac{\sum_{j=1,2,3} j N_t^{t-j+1}}{\sum_{j=1,2,3} N_t^{t-j+1}},$$

whereby young individuals are defined to be at age $j = 1$. Accordingly, middle age is 2 and old age 3. Rearranging leads to:

$$\bar{x}_t = \frac{3q_t + 2n_{t-1} + n_{t-1}n_t}{q_t + n_{t-1} + n_{t-1}n_t}.$$

Hence, mean age and dependency ratio are expressed in terms of the survival probability and the individual number of children. As both are endogenous variables, the age structure is endogenous as well.

3.2 The laissez-faire economy

3.2.1 Production and capital market

Production follows the common neoclassical OLG framework (see e.g. de la Croix and Michel (2002)). At each point in time homogeneous firms produce a single commodity Y_t^{LF} with capital K_t^{LF} and labor L_t^{LF} by means of a neoclassical production function with constant returns to scale. Since capital is fully depreciated every period, $Y_t^{\text{LF}} = F(K_t^{\text{LF}}, L_t^{\text{LF}})$ is net production. Dividing by labor yields production in intensive terms $f(k_t^{\text{LF}}) = F\left(\frac{K_t^{\text{LF}}}{L_t^{\text{LF}}}, 1\right)$, where k_t^{LF} is the capital-labor ratio. The representative firm maximizes its profits in fully competitive markets. Labor is provided by the generation in middle age and remunerated by the wage rate w_t^{LF} . The interest factor R_t^{LF} is paid on capital. The maximization problem of firms in intensive terms is:

$$\max_{k_t^{\text{LF}}} = f(k_t^{\text{LF}}) - R_t^{\text{LF}} k_t^{\text{LF}} - w_t^{\text{LF}}.$$

Production factors are compensated by their marginal products:

$$R_t^{\text{LF}} = f'(k_t^{\text{LF}}), \tag{3}$$

$$w_t^{\text{LF}} = f(k_t^{\text{LF}}) - k_t^{\text{LF}} f'(k_t^{\text{LF}}). \tag{4}$$

Savings of the working generation are invested and constitute productive capital in the next period $S_t^{\text{LF}} = K_{t+1}^{\text{LF}}$. Therefore, capital of firms is owned by the current old generation. Since labor is determined by equation 1, the capital-labor ratio is:

$$k_t^{\text{LF}} = \frac{K_t^{\text{LF}}}{L_t^{\text{LF}}} = \frac{K_t^{\text{LF}}}{(1 - \tau n_t^{\text{LF}}) N_t^{t-1}}.$$

⁸Alternatively, a decomposition in youth and old age dependency ratio, as illustrated in figure 1, allows to conclude on the age structure (see e.g. Weil (2006)). Nevertheless, the mean age is preferred, as this single indicator measures the full age structure.

Hence, the labor decreasing effect, caused by the time required to rear a child, is considered. The equilibrium of the capital market in terms of individuals in working age is:

$$s_t^{\text{LF}} = (1 - \tau n_{t+1}^{\text{LF}}) n_t^{\text{LF}} k_{t+1}^{\text{LF}}. \quad (5)$$

Savings per capita of a middle-aged individual in t balance capital per individual in working age in $t + 1$.

3.2.2 Individuals

The representative individual lives at most three periods, childhood, working age, including parenthood, and retirement. She maximizes her expected life-cycle utility at the beginning of middle age and obtains utility from consumption in middle c_t^{LF} and old age d_{t+1}^{LF} . Furthermore, parents receive a “consumption” utility from rearing their children n_t^{LF} , weighted in relation to those of material consumption $u(c_t^{\text{LF}})$ by $\gamma > 0$. For simplicity, only the pure number of own children generates utility. Hence, no quality quantity trade off is considered.

With all individuals undergoing child- and parenthood, to experience retirement is uncertain. If she is alive, she consumes d_{t+1}^{LF} and receives a utility discounted by $0 < \beta \leq 1$. Otherwise if she dies, her utility is implicitly assumed to be zero. The survival probability q is a strictly increasing function of health expenditures (h_t^{LF}) during middle age and upper bounded by one:

$$q'(h) > 0, \quad q(h \rightarrow \infty) = 1.$$

Like in Leung and Wang (2010) individuals themselves choose health expenditures, which are a pure private good. As she is risk neutral with respect to longevity her expected life-cycle utility function is additive and time separable:⁹

$$EU_t = u(c_t^{\text{LF}}) + \beta q(h_t^{\text{LF}}) u(d_{t+1}^{\text{LF}}) + \gamma v(n_t^{\text{LF}}) \quad (6)$$

and is maximized with respect to the budget constraints in working age and retirement:

$$(1 - \tau n_t^{\text{LF}}) w_t^{\text{LF}} = c_t^{\text{LF}} + a n_t^{\text{LF}} + s_t^{\text{LF}} + h_t^{\text{LF}}, \quad (7)$$

$$d_{t+1}^{\text{LF}} = \tilde{R}_{t+1}^{\text{LF}} s_t^{\text{LF}}. \quad (8)$$

In midlife individuals are endowed with one unit of time allocated on labor and child rearing. To bring up a child causes two kinds of costs, exogenous goods cost a , like in van Groezen and Mejidam (2008), and time cost τ , see for example de la Croix and Doepke (2003). Time cost fixes an upper bound on the individual number of children $n_t^{\text{LF}} < \frac{1}{\tau}$. Labor income is spent on consumption in middle age c_t^{LF} , direct child cost in terms of goods $a n_t^{\text{LF}}$, health expenditures h_t^{LF} and savings for old age consumption s_t^{LF} . Supposing perfect annuity markets with an annuity for life (Yaari, 1965), the return factor on savings \tilde{R}_t^{LF} depends on the average survival probability of individuals:

$$\tilde{R}_t^{\text{LF}} = \frac{R_t^{\text{LF}}}{q(h_{t-1}^{\text{LF}})}. \quad (9)$$

⁹Risk neutral individuals with respect to longevity are a common assumption in literature. Supposing risk neutrality facilitates a time separable and additive life-cycle utility function in contrast to risk averse individuals (Bommier, 2006). In the more general case of risk aversion the shift of certain utilities is concave. Kuhn *et al.* (2010) show that in the case of constant health expenditures the optimal savings are lower in the case of risk aversion. The effect on health expenditures is ambiguous, if savings are assumed to be constant.

Taking prices as given, maximizing expected life-cycle utility with respect to health expenditures, the number of children, consumption in working and retirement age yields to the following set of first-order conditions (FOC):

$$u'(c_t^{\text{LF}}) = \beta \tilde{R}_{t+1}^{\text{LF}} q(h_t^{\text{LF}}) u'(d_{t+1}^{\text{LF}}), \quad (10)$$

$$u'(c_t^{\text{LF}}) = \frac{\gamma}{a + \tau w_t^{\text{LF}}} v'(n_t^{\text{LF}}), \quad (11)$$

$$u'(c_t^{\text{LF}}) = \beta q'(h_t^{\text{LF}}) u(d_{t+1}^{\text{LF}}). \quad (12)$$

The strictly positive set of variables $\{c_t^{\text{LF}}, d_t^{\text{LF}}, h_t^{\text{LF}}, k_t^{\text{LF}}, n_t^{\text{LF}}, s_t^{\text{LF}}, w_t^{\text{LF}}, R_t^{\text{LF}}\}_{t=0}^{t=+\infty}$, solving the system of FOCs (eq. 10-12) as well as the equations on wages (eq. 4), return factor on savings (eq. 9), capital market equilibrium (eq. 5) and budget constraints (eq. 7, 8), defines the interior solution of the laissez-faire economy.¹⁰ The first FOC (eq. 10) is the familiar Euler equation describing the distribution of consumption between midlife and old age. The individual decision on the number of children is illustrated by the second FOC (eq. 11). The marginal costs of children, goods ($au'(c_t^{\text{LF}})$) as well as time cost ($\tau w_t^{\text{LF}} u'(c_t^{\text{LF}})$) balance the marginal utility of a child ($\gamma v'(n_t^{\text{LF}})$). Deciding on health expenditures (eq. 12), individuals compare the additional utility, due to a higher survival probability in old age, with the loss of consumption possibilities in middle age. The gain achieved from the last marginal unit of health expenditures ($\beta q'(h_t^{\text{LF}}) u(d_{t+1}^{\text{LF}})$) equalizes its opportunity cost ($u'(c_t^{\text{LF}})$).¹¹ Thus, individuals do not take into account the influence of the changing survival probability on life-cycle income. The rising survival probability shrinks returns on savings \tilde{R}^{LF} . This reduces consumption possibilities during retirement and thus life-cycle income. Hence, individuals do not internalize the so-called Philipson-Becker-effect (Philipson and Becker, 1998).

3.3 The first-best solution

A point of reference is required to evaluate the age structures reflecting individual decisions. This optimal or first-best age structure results from a SWF following the idea of number-dampened total utilitarianism (Ng, 1986). Expected life-cycle utility of the current and all future generations, weighted by the size of the birth cohorts and the social discount factor ρ , is maximized:

$$W = \sum_{t=0}^{\infty} \rho^t (N_t^{t-1})^x EU(c_t^{\text{FB}}, d_t^{\text{FB}}, n_t^{\text{FB}}, h_t^{\text{FB}}), \quad \text{with} \quad 0 \leq x \leq 1, 0 < \rho \leq 1$$

The characteristic feature of number-dampened total utilitarianism, a concave weight of population size in the welfare function, holds for $0 < x < 1$. Furthermore, the two extreme cases of Average and Total utilitarianism are included. Population size doesn't matter at all in the SWF, if the planner is Millian ($x = 0$). Only the discounted life-cycle utility of a representative individual of each birth cohort is taken into account. On the contrary, a Benthamite Planner ($x = 1$) considers the discounted life-cycle utility of each born individual. Thus number-dampened total utilitarianism has the advantage to imply the two extreme and the whole spectrum of intermediate cases.

As the time horizon of the planner's dynamic maximization problem is infinite, an upper bound requires $\rho < n^{-x}$. Certainly, a discounting of future generation's utility $\rho < 1$ is problematic

¹⁰The Hessian-matrix, required to control for the second-order conditions (SOC), is presented appendix A.

¹¹As $u'(c_t) > 0$, a positive level of utility in old age $u(d_{t+1}) > 0$ is required. Intuitively, rational individuals only invest in health, if they enjoy old age.

from an ethical point of view. Blackorby *et al.* (2005) e.g. claim that “for the purpose of social evaluation, the well-being of future generations should not be discounted.” A different weight of present and (unborn) future individuals in the SWF is hardly defensible. Additionally, the choice of the social discount factor (SDF) is arbitrary. One way to avoid any kind of social discounting is to convert the dynamic maximization problem of the Millian planner into a static. This “Golden Age” situation is treated as point of reference.¹²

The planner includes preferences of the selfish individuals of each generation, implying that both, individuals and planner, have different time horizons. An individual only achieves utility in periods, she is alive. In contrast, the planner has a dynastic function. Due to the varying preferences, solutions differ, even if there is no externality or imperfection in the economy. Considering the preferences of the representative individual and rearranging from a longitudinal in a cross-section point of view results in:

$$W = \sum_{t=0}^{\infty} \rho^t \left[(N_t^{t-1})^x [u(c_t^{\text{FB}}) + \gamma v(n_t^{\text{FB}})] + (N_{t-1}^{t-2})^x \frac{\beta}{\rho} q(h_{t-1}^{\text{FB}}) u(d_t^{\text{FB}}) \right],$$

where consumption in working age of the first old generation (c_{-1}^{FB}) is not taken into account and the initial health expenditures (h_{-1}^{FB}) are preexisting. The social planner’s resource constraint at time t is:

$$Y_t = N_t^{t-1} (c_t^{\text{FB}} + an_t^{\text{FB}} + h_t^{\text{FB}}) + K_{t+1}^{\text{FB}} + N_t^{t-2} d_t^{\text{FB}}.$$

The initial capital stock K_0 and the two cohorts N_0^{-1}, N_0^{-2} are as well historically given. Aggregated production Y_t is allocated on consumption c_t^{FB} , health expenditures h_t^{FB} and exogenous goods cost to bring up children an_t^{FB} for all adults N_t^{t-1} as well as for consumption of the retirees alive $d_t^{\text{FB}} N_t^{t-2}$. Additionally, output is used to install capital for production in the next period K_{t+1} . Dividing by the size of the cohort N_t^{t-1} leads to the resource constraint in terms of individuals in working age. In doing so, time cost to raise a child has to be taken into account. Hence, output in terms of working individuals is $y_t^{\text{FB}} = (1 - \tau n_{t+1}^{\text{FB}}) f(k_{t+1}^{\text{FB}})$ and the resource constraint at t :

$$y_t^{\text{FB}} = c_t^{\text{FB}} + an_t^{\text{FB}} + (1 - \tau n_{t+1}^{\text{FB}}) n_t^{\text{FB}} k_{t+1}^{\text{FB}} + q(h_{t-1}^{\text{FB}}) \frac{d_t^{\text{FB}}}{n_{t-1}^{\text{FB}}} + h_t^{\text{FB}}. \quad (13)$$

In order to maximize the social welfare function, the benevolent planner chooses consumption in working and retirement age, health expenditures, the number of children and the capital-labor ratio for the next point in time. the following system of FOCs defines the optimal decisions:

$$u'(c_t^{\text{FB}}) = \frac{\beta}{\rho} (n_{t-1}^{\text{FB}})^{1-x} u'(d_t^{\text{FB}}), \quad (14)$$

$$u'(c_t^{\text{FB}}) = \frac{\rho}{(n_{t-1}^{\text{FB}})^{1-x}} f'(k_{t+1}^{\text{FB}}) u'(c_{t+1}^{\text{FB}}), \quad (15)$$

¹²The “Golden Age” situation additionally enables to relate the results to the literature on optimal population growth, e.g. Samuelson (1975), Deardorff (1976) and de la Croix *et al.* (2012). Different solutions to ensure an upper bound of the problem, required to determine the optimal path, are discussed in the literature. Already Ramsey (1928) discussed this problem in his paper “A Mathematical Theory of Saving”. For an overview on social discounting see e.g. Heal (2005).

$$u'(c_t^{\text{FB}}) = \frac{\gamma v'(n_t^{\text{FB}}) + \rho x (n_t^{\text{FB}})^{x-1} (u(c_{t+1}^{\text{FB}}) + \gamma v(n_{t+1}^{\text{FB}}) + \beta q(h_{t+1}^{\text{FB}}) u(d_{t+2}^{\text{FB}}))}{\tau \omega + a + (1 - \tau n_{t+1}^{\text{FB}}) k_{t+1}^{\text{FB}} - \frac{q(h_t^{\text{FB}}) d_{t+1}^{\text{FB}}}{n_t^{\text{FB}} f'(k_{t+1}^{\text{FB}})}}, \quad (16)$$

$$u'(c_t^{\text{FB}}) = \beta q'(h_t^{\text{FB}}) (u(d_{t+1}^{\text{FB}}) - u'(d_{t+1}^{\text{FB}}) d_{t+1}^{\text{FB}}), \quad (17)$$

with $\omega = f(k_t^{\text{FB}}) - f'(k_t^{\text{FB}}) k_t^{\text{FB}}$. The set of variables $\{c_t^{\text{FB}}, d_t^{\text{FB}}, h_t^{\text{FB}}, k_t^{\text{FB}}, n_t^{\text{FB}}\}_{t=0}^{t=+\infty}$ satisfying the 4 FOCs (eq. 14-17) and the resource constraint (eq. 13) defines the interior first-best solution.¹³ Consumption is allocated optimally between the working and retired generation according to eq. 14. By means of eq. 15 the FOC is converted in the familiar Euler equation, already known from laissez-faire economy.

$$u'(c_t^{\text{FB}}) = \beta f'(k_{t+1}^{\text{FB}}) u'(d_{t+1}^{\text{FB}}) \quad (18)$$

The modified golden rule of capital accumulation¹⁴ (eq. 15) describes the optimal investment in the capital stock. In steady state, a capital-labor ratio is installed that balances its marginal product and the inverse of the SDF in the extreme case of total utilitarianism ($x = 1$). Additionally the number of children and thus implicitly population growth is considered if the weight of population size is concave or even absent in the social welfare function ($0 \leq x < 1$).

Considering all effects of a newborn, fertility is optimal if marginal utility is equal to marginal costs. According to FOC 16 both consist of three elements. First of all, parents enjoy marginal utility from rearing an additional child ($\gamma v'(n_t^{\text{FB}})$). Furthermore, the intergenerational or (old age) dependency ratio effect $\left(\frac{q(h_t^{\text{FB}}) d_{t+1}^{\text{FB}}}{n_t^{\text{FB}} f'(k_{t+1}^{\text{FB}})} u'(c_t^{\text{FB}}) \right)$, see e.g. van Groezen and Mejidam (2008), captures the effect of an additional child on the future relation between working and retired generation. Children increase population growth and hence the number of working (and consuming) individuals related to the old, only consuming, in $t+1$.¹⁵ A third effect arises if the planner is characterized by a preference for the population stock, see e.g. Baudin (2011). The discounted marginal life-cycle utility of an additional child $\left(\rho x n_t^{\text{FB} x-1} EU_{t+1}^{\text{FB}} \right)$ is considered in the SWF and boosts fertility if $EU_{t+1} > 0$. On the contrary, fertility is reduced if output does not ensure a positive expected life-cycle utility. A sufficient output level $y_{t+1} > \bar{y}_{t+1}$ s.t. $EU_{t+1} > 0$ is assumed below. Obviously this effect vanishes if the planner is Millian ($x = 0$). The first two costs in eq. 16 are the marginal goods ($au'(c_t^{\text{FB}})$) and time costs $\tau f'(k_t^{\text{FB}}) u'(c_t^{\text{FB}})$ to bring up children. Due to the time cost for child rearing, the capital widening effect of children, as third costly aspect, is composed of two elements. Current fertility increases the present capital-labor ratio $\tau f'(k_t^{\text{FB}}) k_t^{\text{FB}} u'(c_t^{\text{FB}})$, as time cost lowers labor supply. Nevertheless, the capital-labor ratio in the next period decreases, because labor supply tomorrow is higher $(1 - \tau n_{t+1}^{\text{FB}}) k_{t+1}^{\text{FB}} u'(c_t^{\text{FB}})$. Consequently, the time cost of children slows down the capital widening effect. Hence, two effects influencing fertility in opposite directions are not internalized by individuals if the optimal solution follows the idea of Millian utilitarianism. Taking into account the dependency ratio effect increases c.p. the number of children, whereas a consideration of the capital dilution effect leads to a reduction. An existing preference for the population stock ($x > 0$) additionally boosts fertility.

¹³The Hessian-matrix to control for the SOC in “Golden Age” as well as in the general FB solution are in appendices B and C.

¹⁴Phelps (1961) introduced the term “golden rule”.

¹⁵Opposing working and retired generation, fertility of the previous period influences the current dependency ratio, but has no impact in the future. Current fertility determines the future relation, as present old are deceased. This result would change in an OLG-model with additional age groups.

Optimally fixed health expenditures follow the last FOC (eq. 17). Compared to de la Croix *et al.* (2012), a survival probability determined by health investments helps to obtain an interior solution. Rearranging eq. 17 emphasizes that

$$\varepsilon_{u(d_t^{FB}), d_t^{FB}} = \frac{u'(d_t^{FB})}{u(d_t^{FB})} d_t^{FB} < 1$$

is a necessary condition for an interior solution of the social planner problem. The instantaneous utility function $u(d_t^{FB})$ has to be inelastic with respect to changes in old age consumption d_t^{FB} ; implying a sufficient consumption level. In other words, the absolute level of consumption of retirees matters. An interior solution is only feasible, if consumption and hence utility during retirement exceeds a critical level. The gain achieved from an additional unit of health expenditures ($\beta q'(h_t^{FB})u(d_{t+1}^{FB})$) has to be as high as the arising costs. These opportunity costs consist of two elements, less consumption in middle age ($u'(c_t^{FB})$) and decreasing consumption possibilities in old age ($\beta q'(h_t^{FB})u'(d_{t+1}^{FB})d_{t+1}^{FB}$). Hence, the Philipson-Becker effect is internalized.

All effects are taken into account by the benevolent planner. Choices on fertility and mortality are optimal and hence implicitly the resulting age structure. Thus, first-best age structures are considered as optimal.

3.4 Decentralization of the optimal age structure

The interior FB and LF solution imply the optimal as well as the age structure, reflecting individual decisions. As well known, nothing guarantees that the “modified golden rule of capital accumulation” holds in laissez-faire economies. In contrast, in general FB and LF solutions differ, because of the double infinity in goods and agents (Shell, 1971). Fixing fertility optimally is one possibility to achieve the golden rule level in the standard OLG framework, claimed by the “serendipity theorem”. Anyway, this intervention is excluded, since individuals decide themselves on the number of offspring.

As individuals don’t consider all effects, fertility and mortality c.p. deviate from their optimal values. However, a conclusion on the relation between LF and FB age structure isn’t possible. Due to the double infinity in goods and agents, LF and FB capital-labor ratio vary. Thus, beside the sub-optimal decisions on demographic variables, production levels differ. Only a discussion of c.p. discrepancies is possible either by comparing the FOCs or by decentralizing optimal age structures. The latter is preferred, because of the advantage to display the required policy instruments.

Four instruments implement the optimal solution in a decentralized economy. Two pigouvian taxes ensure, that individuals internalize all effects in fertility as well as mortality. A child allowance or tax η_t corrects the desired number of children and a health subsidy or tax ϕ_t the health expenditures. In addition a lump-sum transfer in working age z_t^m guarantees that the capital-labor ratio in the laissez-faire economy satisfies the modified golden rule level. Finally, excluding debt, a second one in retirement age z_t^o balances the budget constraint of the government in each point in time:

$$(\eta_t n_t + \phi_t h_t + z_t^m) N_t^{t-1} + z_t^o N_t^{t-2} = 0,$$

or, in terms per individual in middle age:

$$\eta_t n_t + \phi_t h_t + z_t^m + \frac{q(h_{t-1})z_t^o}{n_{t-1}} = 0. \tag{19}$$

Individuals maximize their expected life-cycle utility function (eq. 6) with respect to the adjusted budget constraints, due to the interventions of the government:

$$c_t = (1 - \tau n_t)w_t - (a - \eta_t)n_t - s_t - (1 - \phi_t)h_t + z_t^m, \quad (20)$$

$$d_{t+1} = \tilde{R}_{t+1}s_t + z_{t+1}^o.$$

The corresponding system of FOCs is:

$$u'(c_t) = \tilde{R}_{t+1}\beta q(h_t)u'(d_{t+1}), \quad (21)$$

$$u'(c_t) = \frac{\gamma v'(n_t)}{\tau w_t + a - \eta_t}, \quad (22)$$

$$u'(c_t) = \frac{\beta q'(h_t)u(d_{t+1})}{(1 - \phi_t)}. \quad (23)$$

The FOCs eq. 16, 17, 22 and 23, the middle age budget and resource constraint (eq. 13 and 20), as well as the constraint of the government (eq. 19) determine the necessary levels for the four policy measures such that $\{c_t^{\text{FB}}, d_t^{\text{FB}}, h_t^{\text{FB}}, k_t^{\text{FB}}, n_t^{\text{FB}}\}_{t=0}^{t=+\infty} = \{c_t, d_t, h_t, k_t, n_t\}_{t=0}^{t=+\infty}$. The lump-sum transfer in working age corrects savings or capital accumulation in the LF economy (eq. 24) and those in retirement age (eq. 25) balances the budget constrain:

$$z_t^m = (1 - \tau n_t) f'(k_t)k_t - \eta n_t - \phi_t h_t - \frac{q(h_{t-1})d_t}{n_{t-1}}, \quad (24)$$

$$z_t^o = -\frac{n_{t-1}(\eta_t n_t + \phi_t h_t + z_t^m)}{q(h_{t-1})}. \quad (25)$$

Proposition 1 *Taxes on health expenditures*

A tax on individual health expenditures according to

$$\phi_t = \frac{d_{t+1}u'(d_{t+1})}{d_{t+1}u'(d_{t+1}) - u(d_{t+1})} \quad (26)$$

implements the Philipson-Becker-Effect.

Proof Resulting from equalizing 17 to 23 and rearranging, equation 26 describes the optimal policy on health expenditures at each point in time. Due to the non-negativity in consumption and $u'(\cdot) > 0$, the numerator is always positive. On the contrary, only a negative denominator allows a solution of the FOC (eq. 17). The negative result for ϕ_t confirms that health expenditures are taxed. ■

The tax on health expenditures indicates that the price of health investments is too low and individuals c.p. over-invest in their longevity. In contrast to the unambiguous findings for mortality, results concerning fertility are less obvious. The effects, not considered by individuals, operate in different directions.

Proposition 2 *Child allowances*

The optimal path on child policies to decentralize first-best fertility follows:

$$\eta_t = \frac{\gamma v'(n_t) \left[\frac{q(h_t)d_{t+1}}{n_t f'(k_t)} - (1 - \tau n_{t+1}) k_{t+1} \right] + \rho x n_t^{x-1} E U_{t+1} (a + \tau w_t)}{\gamma v'(n_t) + \rho x n_t^{x-1} E U_{t+1}}. \quad (27)$$

1. *Average utilitarianism*

The child allowance is positive if the dependency ratio effect $\left(\frac{q(h_t)d_{t+1}}{n_t f'(k_t)} \right)$ dominates the capital dilution effect $((1 - \tau n_{t+1}) k_{t+1})$ and negative in a vice versa situation.

2. *Number-dampened total and total utilitarianism*

The child allowance is positive if the dependency ratio effect $\left(\frac{q(h_t)d_{t+1}}{n_t f'(k_t)} \right)$ and the preference for the population stock, weighted by the ratio of marginal time and good costs to the marginal consumption utility of child rearing $\left(\rho x n_t^{x-1} E U_{t+1} \frac{a + \tau w_t}{\gamma v'(n_t)} \right)$, dominate the capital dilution effect $((1 - \tau n_{t+1}) k_{t+1})$. Children are taxed in a vice versa situation.

Proof Equalizing the FOC on fertility (eq. 16 and 22) and transposing for η_t leads to 27. If $x = 0$, the extreme case of Millian utilitarianism, the optimal policy on fertility simplifies to:

$$\eta_t|_{x=0} = \frac{q(h_t)d_{t+1}}{n_t f'(k_t)} - (1 - \tau n_{t+1}) k_{t+1} \quad \text{with} \quad \begin{cases} \eta < 0 & \text{if } LHS < RHS \\ \eta = 0 & \text{if } LHS = RHS \\ \eta > 0 & \text{if } LHS > RHS \end{cases}$$

In the case of number-dampened total or total utilitarianism $0 < x \leq 1$ the sign of the policy is determined by the numerator of eq. 27. Rearranging leads to:

$$\eta_t|_{0 < x \leq 1} \begin{cases} < 0 & \text{if } \frac{q(h_t)d_{t+1}}{n_t f'(k_t)} + \rho x n_t^{x-1} E U_{t+1} \frac{a + \tau w_t}{\gamma v'(n_t)} < (1 - \tau n_{t+1}) k_{t+1} \\ = 0 & \text{if } \frac{q(h_t)d_{t+1}}{n_t f'(k_t)} + \rho x n_t^{x-1} E U_{t+1} \frac{a + \tau w_t}{\gamma v'(n_t)} = (1 - \tau n_{t+1}) k_{t+1} \\ > 0 & \text{if } \frac{q(h_t)d_{t+1}}{n_t f'(k_t)} + \rho x n_t^{x-1} E U_{t+1} \frac{a + \tau w_t}{\gamma v'(n_t)} > (1 - \tau n_{t+1}) k_{t+1} \end{cases}$$

■

Hence, the dominating effect (or effects) determine(s) the sign of the child allowances. A positive value indicates costs of offspring that are too high in the laissez-faire economy. A situation more likely if the planner takes into account the population size in the SWF, as this adds an additional positive effect not considered by individuals. The allowance reduces costs and fertility increases, since children are a normal good.¹⁶ The opposite is true, if the capital dilution effect dominates. The costs of a child are too low to consider the dominating negative influence of additional children on the capital-labor ratio. The tax increases costs and lowers fertility.

Supposed both children and health expenditures are normal goods, taxing health expenditures implies an over-investment in health and a survival probability that exceeds c.p. the optimal level. The lower mortality raises the mean age. This tendency to an over-aged population is strengthened by fertility, if the dependency ratio effect, supported by the preference for the population stock, dominates. The costs of children are too high in the LF economy. Hence, the number of offspring is c.p. below its optimal level and thus upwards biases the mean age, too. On the contrary, a

¹⁶If the quantity of children is a Giffen good, fertility decreases in child allowance. This has to be checked if a quantity quality trade off is considered.

dominating capital dilution effect goes along with a LF fertility that tends to exceeds the FB level, reducing the LF mean age.

The case of the dependency ratio is similar and depends positively on the survival probability, too. As the Philipson-Becker effect is not internalized by individuals, the fraction of the non-working population exceeds c.p. the optimal level. The effect of fertility is less obvious, based on different reasons. First of all, according to the relationship between capital dilution and the sum of dependency ratio effect and the preference for the population stock, costs of children are either above or below the LF solution. Additionally, fertility either rises or lowers the dependency ratio. A higher fertility boosts the dependency ratio in t but reduces its value in $t + 1$. According to

$$\frac{\partial DR}{\partial n} = \begin{cases} > 0 & \text{if } n^2 > q \\ < 0 & \text{if } n^2 < q \end{cases}$$

the steady state effect is ambiguous. Developing countries, commonly characterized by a higher fertility, are featured by a positive steady state relation between the number of children and the dependency ratio. India with a Net reproduction rate¹⁷ (NRR) of 1.169 and a survival probability of 0.57, used in the calibration exercise, belongs to this group. On the contrary, developed countries with a small number of offspring per women, like Germany with a NRR of 0.654 and high survival probability (0.83), are featured by a shrinking dependency ratio if fertility rises. Hence, four steady state scenarios are possible. Developing countries with high youth dependency ratios c.p. have a DR that is too low, if the intergenerational effect and the preference for the population stock dominate. In contrast, a tendency to exceed the optimal level features developed countries. If the capital dilution effect dominates, the situation is vice versa. However, as the effect of the capital-labor ratios is not considered, only a discussion on tendencies is possible. To get a clear picture, the model is calibrated in the next section.

4 Calibration exercise

4.1 Data

Laissez-faire age structures always differ from optimal. In order to analyze the deviation, the LF economy is calibrated to fit data of 80 countries ($i = 1, 2, \dots, 80$). The resulting parameters enable to replicate the age structures illustrated in figure 1. Afterwards, a calculation of the FB solutions is possible and allows a discussion on the direction and extent of difference between observed and optimal age structures.

A period and implicitly each age, childhood, parenthood and retirement, is fixed to 30 years. A ten-year average of GDP between 2000 and 2009 times the length of a period approximates steady state values of output Y_i . Like all economic variables, GDP is expressed in PPP (constant 2005 international \$). As well as for labor, health expenditures and gross fixed capital formation ratio in percentage of GDP, the world bank database is the source. The estimation of gross domestic fixed capital stock for 93 countries until 1990, offered by Nehru and Dhareshwar (1993), approximates real capital stocks. Converting their values in PPP and adjusted to the future according to:

$$K_{i,t+1} = (1 - \Delta) K_{i,t} + I_{i,t} \times GDP_{i,t},$$

¹⁷The NRR is “the average number of daughters a hypothetical cohort of women would have at the end of their reproductive period if they were subject during their whole lives to the fertility and mortality rates of a given period (United Nations, 2011).”

leads to the capital stock $K_{i,t}$. Gross fixed capital formation ratio in percentages of GDP ($I_{i,t}$) times GDP in PPP involves the required yearly investments. The capital depreciation rate Δ is fixed to 5% p.a. Furthermore, health expenditures between age 30 and 59 are required. As health expenditures per capita are fundamentally driven by age structures and no adequate data by age group is available, we approximate them by controlling for the effects of age structures. Total health expenditures in percent of GDP enable to calculate the average health expenditures per capita in PPP. In a second step, we correct the values by supposing the distribution of health expenditures over age groups and sexes in Germany to obtain the health expenditures between age 30 and 59 $h_{i,t}$.¹⁸ Table 1 presents the summary statistics of economic as well as demographic variables.

Table 1:
SUMMARY STATISTICS OF ECONOMIC AND DEMOGRAPHIC VARIABLES

| Variable | Mean | Std. dev. | Min | Max |
|-------------------------|--------|-----------|-------|---------|
| GDP in bn PPP | 18491 | 47499 | 59 | 366699 |
| Health exp. In K PPP | 37.945 | 41.896 | 1.452 | 183.492 |
| Total labor force in K | 31040 | 99124 | 165 | 761269 |
| Capital stock in bn PPP | 1666 | 4101 | 5 | 29953 |
| Survival prob | 0.721 | 0.123 | 0.510 | 0.950 |
| NRR | 1.204 | 0.478 | 0.600 | 2.437 |

Calibrating the model requires the demographic variables in addition to the economic ones. Ten-year averages of total labor forces approximate the population in working age $N_{i,t}^{t-1}$. Both fertility as well as mortality emanate from the United Nations (2011) world prospects 2010 and focus on female populations. The survival probability q_i is calculated from female life tables in 2009. The average life-expectancy at age 60 is related to the length of a period. The NRR 2005-2010 reflects fertility $n_{i,t}$ and implicitly captures mortality within childhood and reproductive age.¹⁹

4.2 Calibration of the laissez-faire economy

A calibration requires specific functional forms. We suppose logarithmic instantaneous utility functions for consumption in middle and old age, as well as for the utility generated by children $u(\cdot) = v(\cdot) = \ln(\cdot)$. Production technology follows a common Cobb-Douglas function with A_i as total factor productivity (TFP):

$$Y_{i,t} = F(K_{i,t}, L_{i,t}) = A_i K_{i,t}^\alpha L_{i,t}^{1-\alpha}.$$

Its partial production elasticity of capital (α) is fixed to 0.3. The survival probability is a monotonically increasing function of health expenditures $q' > 0$:

$$q(h_{i,t-1}) = \frac{1}{1 + \exp^{h_{0,i} - h_{i,t-1}^\delta}}, \quad (28)$$

upper bounded by 1. Determined by the parameters $h_{0,i}$, δ_i the function is either s-shaped or concave. The country specific parameter $h_{0,i}$ reflects general climate, medical and hygienic envi-

¹⁸In general the distribution of health expenditures over age groups is similar across countries (Dalgaard and Strulik, 2014). Appendix D offers details of the calculation.

¹⁹Figure 1 illustrates the implied age structures.

ronmental situations of economies and is independent from individual health expenditures. Their influence is captured by the country specific parameter $\delta_i > 0$.²⁰

The first step of the calibration fixes the country non-specific weight of children in the expected life-cycle utility function and the time to bring them up. Within the model, parents allocate their time between child rearing and paid work. Relating time for care to the sum of time for care and paid work leads to an average of 15.3% for both men and women over age 15 in 23 OECD countries. However, this value is sensitive to age structures and the average number of children within countries. Focusing alternatively on the time used for childcare as main or second activity compared to the total time for work and child rearing in all families with at least one child in preschool age results in a much higher average value of 30.5% in 14 OECD countries. As the time to bring up a child varies over age and potentially more than one is reared in the household, this value is likely to be an overestimation. Following de la Croix and Doepke (2003), assuming that only half of the time is necessary over a period of 30 years, results once again in a value around 15%; supposed for all countries in the calibration (OECD, 2012). Implicitly this fixes the average maximal fertility to around 13 children per women. This outcome seems to be rather realistic as Gagnon *et al.* (2008) state: “fertility was particularly high among the settlers of New France ... an average of 9.97 children for each fertile woman who survived married to the age of 50.” Thus, even this very high fertility is below the fixed natural limit.²¹ As part of the preferences, the country non-specific weight of utility generated by child rearing is fixed to fulfill the FOC on the number of children (eq. 11) in the US. The required good costs are 207.43 K and defined as total expenditures of a child for housing, food, health care etc. converted in constant international PPP \$.²²

A second step determines the eight remaining parameters or variables, supposing γ , the weighting of utility generated by child rearing, and the share of time required to do so (τ) are constant across countries. As the laissez-faire solution comprises of eight equations, the system is identified and the model perfectly replicates observed data. Following the definition of labor in eq. 1 allows to calculate L_i , considering the NRR, the total labor force and the time to bring up children. Afterwards, the TFP remains as residual in the Cobb-Douglas function and is ascertained. Additionally, eq. 3, 4 and the capital market equilibrium (eq. 5) permit to calculate interest factors, wages and savings. By means of savings, required to fit the capital-labor ratio, we choose the discount factor β_i corresponding to the Euler equation (eq. 10). The good cost in each country balances the individual choice on the number of children (eq. 11). Finally, the two parameters of the survival probability function are jointly determined by the decision on health expenditures (eq. 12) and the survival probability itself (eq. 28).

²⁰A common assumption in the literature, see e.g. Chakraborty (2004), Leung and Wang (2010) and Leroux *et al.* (2011), is a concave survival probability function with: $q'(h) > 0, q''(h) < 0, q(h \rightarrow \infty) = 1$ and $0 \leq q \leq 1$. The specification $q(h_{i,t-1}) = \frac{h_{0,i} + h_{i,t-1}^{\delta_i}}{1 + h_{0,i} + h_{i,t-1}^{\delta_i}}$ satisfies an upper bound of 1 and concavity ($q' > 0, q'' < 0$) for the set of parameters $(\delta_i, h_{0,i})$ fulfilling $h_{i,t-1}^{\delta_i} + h_{0,1} > 0$ and $h_{i,t-1}^{\delta_i} + \frac{(1+h_{0,i})(1-\delta)}{1+\delta} > 0$ for steady state solutions. As this specification inserts additional restrictions and qualitative results are not altered, we prefer the more flexible functional form in eq. 28.

²¹Fixing the time to bring up a child to 0.15 % goes along with the values detectable in OLG models with endogenous fertility. For a period of 25 years Doepke (2004) chooses a time costs of 0.155. Half of the time cost of 0.15 %, estimated by Knowles (1999), is used in the calibration of de la Croix and Doepke (2003).

²²The good costs are the sum over total expenditures at ages 0 to 17 for the younger child in middle-income, husband-wife households with two children. Child care and education expenses are only considered for families with expense (Lino, 2012).

Table 2:

| SUMMARY STATISTICS OF THE PARAMETERS | | | | |
|--------------------------------------|--------|-----------|-------|--------|
| Parameter | Mean | Std. dev. | Min | Max |
| α | 0.30 | | | |
| τ | 0.15 | | | |
| γ | 0.46 | | | |
| A_i (in K) | 31.23 | 21.05 | 4.58 | 103.19 |
| a_i (in K) | 118.05 | 150.13 | -4.16 | 714.76 |
| β | 0.29 | 0.13 | 0.14 | 0.87 |
| $h_{0,i}$ | 1.10 | 0.35 | 0.40 | 2.07 |
| δ | 0.07 | 0.03 | 0.02 | 0.14 |

Table 2 summarizes country non-specific and specific parameters, determined according to the defined calibration strategy. The elasticity of production and the time required to bring up a child are fixed by means of the literature. Utility generated by child rearing is weighted slightly below half of those originated by consumption during working age.

On average the TFP (A_i) is 31,229, with a range between 4,583 in Mozambique and 103,187 in Luxembourg. The latter is in addition characterized by the highest total good costs per child of 714.76 K PPP. In contrast, in many very poor, especially African countries, children still have to contribute to household income. To fit the data, their contribution is highest in Mali (-4.16 K PPP). On average the expenditures to bring up a child are 118.05 K PPP. The average individual discount factor (0.29) is very close to the text book value (0.3) for 30 years, see e.g. de la Croix and Michel (2002). Commonly, values are below mean in developed countries, e.g. Germany, UK or the US, whereas developing countries like Guyana, Malawi or Zambia have rather high values, implying a lower discounting. The two parameters in the survival probability function have an average of $\delta = 0.07$ and $h_0 = 1.1$. To fit both, mortality and health expenditures, the United States of America have the lowest survival probability in absence of health investments, due to $h_0 = 2.07$. A very low survival probability features Japan in this theoretical situation as well ($h_0 = 1.86$), even if the observed survival probability is highest. However, health expenditures have c.p. the highest influence on longevity ($\delta = 0.14$). On the contrary, the situation is vice versa in Israel with the highest survival probability in a hypothetical situation without health investments ($h_0 = 0.40$ and $\delta = 0.07$). Pakistan is featured by the lowest impact of health expenditures on longevity ($\delta = 0.02$).

4.3 Numerical results

The set of parameters perfectly replicates the observed age structures in the laissez-faire scenario (LF) illustrated in figure 1. Additionally, it enables to compute the corresponding optimal solutions in first-best economies. Table 3 presents the demographic variables of three first-best as well as the LF scenario.²³

The ‘‘Golden Age’’ scenario (GA) constitutes the point of reference, because of two reasons. First of all, this optimal solution for a Millian SWF with a SDF of $\rho = 1$ avoids the arbitrary and hardly defensible discounting of future generations. Secondly, the ‘‘Golden Age’’ characterizes the most favorable solution among all optimal from an individual point of view, as it goes along with the highest expected life-cycle utility. However, the neglected population size in the SWF and the

²³The values are non-weighted averages across continents. Appendix E offers parameters, variables and SOC for selected countries.

dominating capital dilution effect lead to a very low “Golden Age” fertility. Optimality of such a very fast shrinking population is highly questionable from a demographic point of view. On average populations halve every generation and reduce actually to 30 % in the European area. Africa, the continent with the highest observable number of offspring, coincidences with the highest “Golden Age” fertility. Still, on average the NRR of 0.67 is below replacement level. Only in Mali and Zambia optimal populations grow. Thus, adding children to the SWF and considering their goods and time costs clearly avoid an overestimation of fertility, detectable in theories on optimal growth rates (Arthur and McNicoll, 1978). Nevertheless, the low “Golden Age” fertility is rather intuitive. The planner only cares about utility of the representative individual, not about population size. Furthermore, accounting for the “modified golden rule” (eq. 15) a low fertility goes along with a lower marginal product of k^{FB} and hence c.p. a higher output per work-force. Finally, this implies more consumption possibilities and thereby utility for the representative individual. In addition, the higher output enables to invest more in health. Thus, even if taking into account the Philipson-Becker effect would suggest a lower survival probability than in the LF economy, the “Golden Age” exceeds the observed survival probability in all 80 countries.

Table 3:
AVERAGE OBSERVED AND OPTIMAL DEMOGRAPHIC VARIABLES

| Area | Fertility | | | | Survival probability | | | |
|---------------|-----------|------|------|------|----------------------|------|------|------|
| | LF | GA | FB1 | FB2 | LF | GA | FB1 | FB2 |
| Africa | 1.74 | 0.67 | 1.40 | 2.83 | 0.57 | 0.59 | 0.58 | 0.57 |
| Asia | 1.03 | 0.41 | 0.87 | 1.66 | 0.71 | 0.73 | 0.72 | 0.71 |
| Europe | 0.81 | 0.30 | 0.70 | 1.37 | 0.84 | 0.88 | 0.86 | 0.85 |
| North America | 1.21 | 0.44 | 0.99 | 1.95 | 0.76 | 0.79 | 0.77 | 0.76 |
| Oceania | 0.98 | 0.39 | 0.84 | 1.62 | 0.86 | 0.89 | 0.88 | 0.86 |
| South America | 1.16 | 0.47 | 0.97 | 1.85 | 0.75 | 0.78 | 0.76 | 0.75 |
| World | 1.20 | 0.46 | 1.00 | 1.96 | 0.72 | 0.74 | 0.73 | 0.72 |

As the very low “Golden Age” fertility is striking from a demographic point of view, we present two additional FB scenarios. Introducing a SDF and considering the population size in the welfare function ($x > 0$) changes the optimal number of children. The SDF is fixed to 0.5 % p.a in both scenarios FB1 and FB2. To achieve an optimal average fertility across countries that coincidences with the replacement level, the weighting of population size in the SWF (x) is fixed to 0.75 % in scenario FB1. In general this scenario implies an optimal NRR smaller than the observed. The number of offspring in less developed countries exceeds the replacement level but still remains below in developed economies. As output persists above the observed level, most populations in FB1 have similar or even higher survival probabilities than observed.

Scenario FB2 implies optimum populations featured by a high fertility, exceeding the replacement level everywhere. Supposing a weight of population size of $x = 5\%$ still guarantees a upper bounded maximization ($\rho n^x < 1$) in all economies. On average the cohort size almost doubles every period. Due to the lower output per capita, the survival probability further decreases compared to “Golden Age” and FB1. An observed longevity exceeding the optimal features a lot of economies, like Australia, France or Mali. Further increasing the weight of the population size to the boundary of maximization drops longevity below the optimal everywhere.

Table 4:
AVERAGE OBSERVED AND OPTIMAL AGE STRUCTURES

| Area | Mean age | | | | Dependency ratio | | | |
|---------------|----------|------|------|------|------------------|------|------|------|
| | LF | GA | FB1 | FB2 | LF | GA | FB1 | FB2 |
| Africa | 1.58 | 2.11 | 1.69 | 1.39 | 2.12 | 1.70 | 1.87 | 3.06 |
| Asia | 1.91 | 2.44 | 2.01 | 1.64 | 1.82 | 2.49 | 1.80 | 2.14 |
| Europe | 2.09 | 2.62 | 2.19 | 1.76 | 1.88 | 3.30 | 1.96 | 2.00 |
| North America | 1.82 | 2.42 | 1.95 | 1.55 | 1.88 | 2.45 | 1.82 | 2.36 |
| Oceania | 1.97 | 2.51 | 2.07 | 1.66 | 1.86 | 2.67 | 1.89 | 2.15 |
| South America | 1.83 | 2.39 | 1.94 | 1.57 | 1.83 | 2.27 | 1.79 | 2.27 |
| World | 1.84 | 2.39 | 1.95 | 1.58 | 1.92 | 2.44 | 1.86 | 2.39 |

The optimal age structures, involved by fertility and mortality, are presented in table 4 as non-weighted averages across continents. Additionally, figure 2 adds the optimal dependency ratios and mean ages to the observed, illustrated on the RHS of figure 1. Figure 3 supplements the LHS of figure 1 by the optimal dependency ratios distinguished by youth and old age dependency ratio.

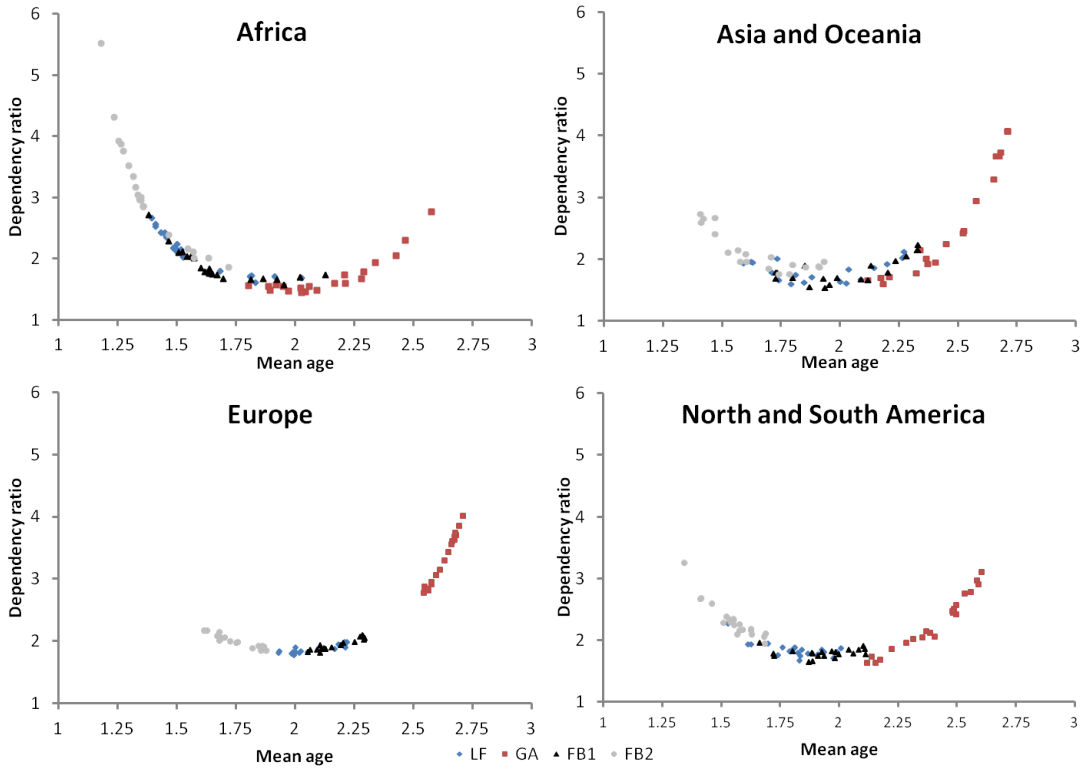


Figure 2: Laissez-faire and first-best age structures across countries

Both, the very low NRR and high survival probability, lead to a “Golden Age” mean age above the observed, plotted to the right of the LF scenario in figure 2. Hence, observed populations are far too young compared to the “Golden Age” situation, as point of reference. The increasing optimal

number of children combined with a shrinking survival probability decreases mean ages, if future generation’s utility is discounted and/or the SWF considers population size. As soon as optimal solutions are plotted to the left of the LF economy, populations are over-aged, like illustrated by scenario FB2.

Whether the “Golden Age” exceeds the laissez-faire share of non-working individuals or not, in particular depends on the observed number of children. All countries go along with a higher old-age and a lower youth dependency ratio than observed (figure 3). Even if the lower “Golden Age” fertility reduces the youth dependency ratio, the higher old age dependency ratio dominates in most countries. These economies are featured by a “Golden Age” dependency ratio that exceeds the observed level. Only economies with high observed NRR and low survival probabilities plotted on the lower left in fig. 1, like Cameroon, Egypt, Kenya, Mali, Uganda and Zambia, are featured by a lower distance to the point of origin and thus smaller “Golden Age” dependency ratios.

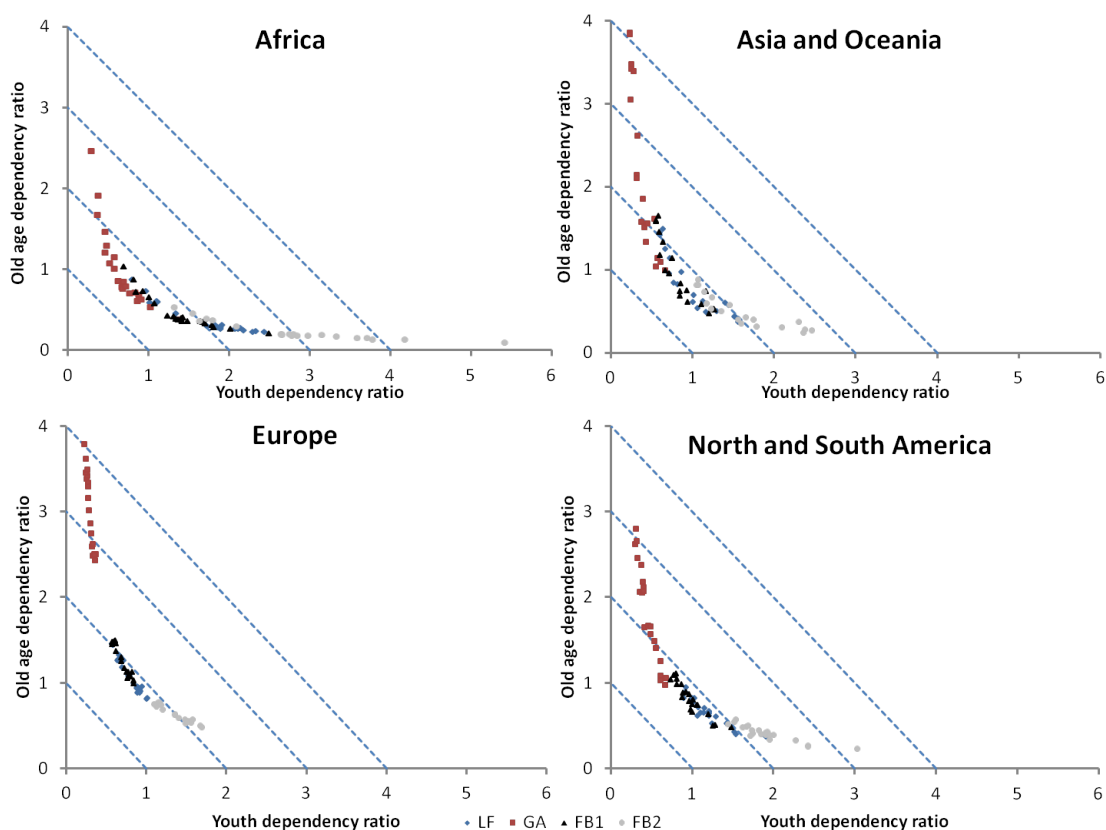


Figure 3: Laissez-faire and first-best dependency ratios across countries

Introducing a SDF and a preference for the population stock ($x > 0$) boosts optimal fertility and reduces the survival probability. The optimal mean age and old age dependency shrink, whereas the youth dependency ratio increases. First-best age structures move to the lower right, following a convex curve, accompanied by an initially shrinking share of non-working individuals, visualized in figure 3. The decreasing old-age dependency ratio initially dominates the growing youth dependency ratio. As soon as they balance each other, the minimal overall dependency ratio is obtained. A further increasing SDF and/or weight of population size in the SWF leads to a growing overall share of non-active individuals. Thus, the optimal dependency ratio follows the same U-shaped pattern

in the decreasing mean age detectable in the observed data of figure 1, if the SDF and/or weight of the population size increases, illustrated by the three FB scenarios in figure 2. Starting from the reference point of the “Golden Age”, the share of non-working individuals in most economies shrinks and converges to the observed in scenario FB1. Populations with a “Golden Age” dependency ratio below the observed are an exception. In general their “Golden Age” is below their FB1 dependency ratio. Stronger preferences for the population stock (FB2) imply even younger optimal populations (a lower mean age) and higher dependency ratios. The larger number of fertility further boost the youth dependency ratio, now dominating the shrinking old age dependency ratio.

All optimal populations plotted in figure 2 and 3 differ from the observed situations in figure 1. Whereby the SWF function determines whether optimal or observed age structures are characterized by a higher or lower mean age and share of non-working individuals. In “Golden Age” populations that are too young feature all economies and most developed countries have a higher share of working individuals than optimal. In scenario FB1 the dependency ratio is similar to the observed. However, except of Zambia, all populations are still too young. On the contrary the strong preference for the population stock leads to an over-aging in all countries, accompanied by an overshooting of the working individuals in most laissez-faire economies.²⁴ Table 4 illustrates the varying relation between optimal and observed age structures on average across continents. The policy instruments to decentralize the different optimal solutions summarizes table 5.

Table 5:
AVERAGE POLICY MEASURES TO IMPLEMENT THE “GOLDEN AGE”

| Area | Health Tax | | | Child | | | Lump-sum transfer in | | | | | |
|------------|------------|------|------|----------------|------|-----|----------------------|------|-----|--------------|------|------|
| | in % | | | allowance in K | | | middle age in K | | | old age in K | | |
| | GA | FB1 | FB2 | GA | FB1 | FB2 | GA | FB1 | FB2 | GA | FB1 | FB2 |
| Africa | -10.2 | -9.8 | -9.7 | -276 | -39 | 21 | 237 | 101 | 4 | -90 | -105 | -133 |
| Asia | -9.8 | -9.4 | -9.4 | -1553 | -228 | 91 | 938 | 427 | 72 | -201 | -256 | -361 |
| Europe | -9.0 | -8.6 | -8.6 | -4317 | -594 | 245 | 2542 | 1144 | 209 | -429 | -588 | -868 |
| N. America | -9.6 | -9.1 | -9.1 | -1547 | -194 | 83 | 1034 | 446 | 61 | -220 | -295 | -420 |
| Oceania | -8.9 | -8.5 | -8.5 | -2075 | -297 | 176 | 1627 | 723 | 74 | -352 | -443 | -649 |
| S. America | -9.7 | -9.3 | -9.3 | -693 | -95 | 47 | 537 | 235 | 27 | -136 | -171 | -243 |
| World | -9.7 | -9.3 | -9.2 | -1768 | -244 | 104 | 1109 | 495 | 79 | -223 | -293 | -420 |

The “Golden Age” goes along with the highest feasible expected life-cycle utility for the represented individuals in all points in time. To achieve this most favorable optimal age structure from an individual point of view, which is beyond doubt problematic from a demographic perspective, requires extensive policy measures. Their signs are identical across countries, only magnitudes differ. As expected, health expenditures are taxed by around 9 to 10 % to account for the Philipson-Becker effect. Children are taxed too, due to a capital dilution effect that dominates the intergenerational transfer effect in all economies. Additionally to the good cost a , a child costs 1,768 K PPP on average, certainly with a large variation between 7.22 K PPP in Zambia and 11.055 million PPP in Luxembourg. The working generation obtains a lump-sum transfer to increase the capital-labor ratio and the retired generations pays a transfer to close the budget constraint in all 80 countries. Generally the extents of instruments decrease in scenario FB1 and signs are unchanged for the majority of countries. Only in Zambia children are subsidized combined with a positive lump-sum transfer of the working age generation. The higher preference for the population stocks in FB2 leads

²⁴It has to be mentioned, that the expected first-best life-cycle utility is below laissez-faire in certain circumstances.

to a dominated capital dilution effect and a child allowance across countries, between 2.7 K PPP in Mozambique and 591.7 K PPP in Luxembourg. The sign of health expenditures isn't altered. The lump-sum transfers are either both negative, e.g. Israel, Guyana or Uganda, only positive in working age to increase the capital-labor ratio, e.g. China, Germany or Japan, or during retirement age as in Zambia to implement the optimal capital-labor ratio and close the government's budget constraint. Hence, assumptions concerning the SWF are important for the direction and magnitudes of the policy instruments necessary to implement the varying optimal age structures.

5 Conclusion

A simple OLG-model with endogenous fertility and longevity replicates observed age structures to evaluate a possible over-aging. Defining number-dampened total utilitarianism as objective for optimality and adding optimal to observed age structures in figure 2 and 3 offers an instrument to discuss sub-optimality of observed situations. Three first-best scenarios point out the sensitivity of results concerning the definition of social welfare functions.

“Golden Age” populations are always older than observed ones and featured by a shrinking number of very happy individuals in almost every economy. A successive raise in the preference for the population stock increases optimal fertility and lowers the survival probability as well as expected life-cycle utility. Optimal mean ages and dependency ratios trace the U-shaped pattern detectable across observed age structures. A sufficient weighting of the population size in SWF, as in scenario FB2, leads to an over-aging of observed populations. Four policies implement the varying optimal age structures. Health expenditures are always taxed, whereas the sign of the child allowances and the lump-sum transfers in working as well as retirement age are sensitive to the SDF and the preference for the population stock. A positive child allowance is more likely the higher the latter. However, the discounting of future generation's utility is problematic from an ethical point of view. Furthermore, both parameters, the SDF (ρ) and the magnitude of the preference for the population size (x), are fixed arbitrary and enable first-best solutions with lower life-cycle utilities than in the laissez-faire economy. Finally, the following dilemma arises:

At least one aspect of the different optimal solutions is problematic. The shrinking, very old populations in “Golden Age” are striking from a demographic point of view but favorable from an ethical and economic perspective. In contrast, first-best solutions with younger age structures are demographically less problematic, but are less advantageous for individuals and imply ethical problems, due to an arbitrary discounting of future generation's utility and preference for the population stock.

However, due to assumptions and simplifications, findings should be interpreted carefully. First of all, age structures are aggregated to three generations. The child survival probability and age-dependent heterogeneity within age groups are not considered, e.g. fast growing health expenditures in old age. Furthermore, the calibration exercise focuses on steady state and hence stable populations. Aging as a dynamic phenomenon is not investigated. Indeed, non of the 80 calibrated economies is in a steady state situation. Beside the demographic aspects, economic assumptions limit results. Estimated parameters might be biased, in consequence of country-specific policies and social security systems, neglected in the calibration. Diversity of social security systems across countries would require totally different models to consider them adequately.

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A Hessian-matrix of the laissez-faire economy

The FOCs describe a maximum if the following 3x3 Hessian-matrix is negative definite:

$$H \equiv \begin{pmatrix} \frac{\partial^2 EU_t}{\partial s_t^2} & \frac{\partial^2 EU_t}{\partial s_t \partial n_t} & \frac{\partial^2 EU_t}{\partial s_t \partial h_t} \\ \frac{\partial^2 EU_t}{\partial n_t \partial s_t} & \frac{\partial^2 EU_t}{\partial n_t^2} & \frac{\partial^2 EU_t}{\partial n_t \partial h_t} \\ \frac{\partial^2 EU_t}{\partial h_t \partial s_t} & \frac{\partial^2 EU_t}{\partial h_t \partial n_t} & \frac{\partial^2 EU_t}{\partial h_t^2} \end{pmatrix}$$

Following Young's theorem, the Hessian is symmetric. The 6 necessary partial derivations of the matrix H are:

$$\frac{\partial^2 EU_t}{\partial s_t^2} = u''(c_t) + \beta \tilde{R}_{t+1}^2 q(h_t) u''(d_{t+1})$$

$$\frac{\partial^2 EU_t}{\partial s_t \partial n_t} = (\tau w_t + a) u''(c_t)$$

$$\frac{\partial^2 EU_t}{\partial s_t \partial h_t} = u''(c_t) + \beta q'(h_t) \tilde{R}_{t+1} u'(d_{t+1})$$

$$\frac{\partial^2 EU_t}{\partial n_t^2} = \gamma v''(n_t) + (\tau w_t + a)^2 u''(c_t)$$

$$\frac{\partial^2 EU_t}{\partial n_t \partial h_t} = (a + \tau w_t) u''(c_t)$$

$$\frac{\partial^2 EU_t}{\partial h_t^2} = u''(c_t) + \beta q''(h_t) u(d_{t+1})$$

To determine the SOC the Eigenvalues and Sylvester's Criterion are computed by the matrix H in the calibration exercise.

B Hessian-matrix of the “Golden Age” solution

To verify that the optimality conditions describe a maximum in the Golden age situation, the following Hessian-matrix is computed for the static case of $\rho = 1$:

$$H \equiv \begin{pmatrix} \frac{\partial^2 W}{\partial d^2} & \frac{\partial^2 W}{\partial d \partial k} & \frac{\partial^2 W}{\partial d \partial n} & \frac{\partial^2 W}{\partial d \partial h} \\ \frac{\partial^2 W}{\partial k \partial d} & \frac{\partial^2 W}{\partial k^2} & \frac{\partial^2 W}{\partial k \partial n} & \frac{\partial^2 W}{\partial k \partial h} \\ \frac{\partial^2 W}{\partial n \partial d} & \frac{\partial^2 W}{\partial n \partial k} & \frac{\partial^2 W}{\partial n^2} & \frac{\partial^2 W}{\partial n \partial h} \\ \frac{\partial^2 W}{\partial h \partial d} & \frac{\partial^2 W}{\partial h \partial k} & \frac{\partial^2 W}{\partial h \partial n} & \frac{\partial^2 W}{\partial h^2} \end{pmatrix}$$

The partial derivations of the matrix H are:

$$\frac{\partial^2 W}{\partial d^2} = \left(\frac{q(h)}{n} \right)^2 u''(c) + \beta q(h) u''(d)$$

$$\frac{\partial^2 W}{\partial d \partial k} = -(f'(k) - n) \frac{q(h)}{n} (1 - \tau n) u''(c) = 0$$

$$\frac{\partial^2 W}{\partial d \partial n} = \frac{q(h)}{n} \left(\tau (f(k) - nk) + a + (1 - \tau n) k - \frac{q(h) d}{n^2} \right) u''(c_t) + \frac{q(h)}{n^2} u''(c)$$

$$\frac{\partial^2 W}{\partial d \partial h} = \frac{q(h)}{n} \left(1 + \frac{q'(h) d}{n} \right) u''(c)$$

$$\frac{\partial^2 W}{\partial k^2} = (1 - \tau n) f''(k) u'(c)$$

$$\frac{\partial^2 W}{\partial k \partial n} = -(1 - \tau n) u'(c)$$

$$\frac{\partial^2 W}{\partial k \partial h} = (1 - \tau n) (n - f'(k)) \left(1 + \frac{q'(h)}{n} \right) u''(c) = 0$$

$$\frac{\partial^2 W}{\partial n^2} = \left(a + \tau (f(k) - nk) + (1 - \tau n) k - \frac{q(h) d}{n^2} \right)^2 u''(c) + \gamma u''(n) + 2 \left(\tau k - \frac{q(h) d}{n^3} \right)^2 u'(c)$$

$$\frac{\partial^2 W}{\partial n \partial h} = \frac{q'(h) d}{n^2} u'(c) + \left(a + \tau (f(k) - nk) + (1 - \tau n) k - \frac{q(h) d}{n^2} \right) \left(1 + \frac{q'(h) d}{n} \right) u''(c_t)$$

$$\frac{\partial^2 W}{\partial h^2} = q''(h) \left(\beta u(d) - \frac{d}{n} u'(c) \right) + \left(1 + \frac{q'(h) d}{n} \right)^2 u''(c)$$

C Hessian-matrix of the first-best solution

The Hessian matrix, to verify that the optimality conditions describe a maximum, is:

$$H \equiv \begin{pmatrix} \frac{\partial^2 W}{\partial d_t^2} & \frac{\partial^2 W}{\partial d_t \partial k_{t+1}} & \frac{\partial^2 W}{\partial d_t \partial n_t} & \frac{\partial^2 W}{\partial d_t \partial h_t} \\ \frac{\partial^2 W}{\partial k_{t+1} \partial d_t} & \frac{\partial^2 W}{\partial k_{t+1}^2} & \frac{\partial^2 W}{\partial k_{t+1} \partial n_t} & \frac{\partial^2 W}{\partial k_{t+1} \partial h_t} \\ \frac{\partial^2 W}{\partial n_t \partial d_t} & \frac{\partial^2 W}{\partial n_t \partial k_t} & \frac{\partial^2 W}{\partial n_t^2} & \frac{\partial^2 W}{\partial n_t \partial h_t} \\ \frac{\partial^2 W}{\partial h_t \partial d_t} & \frac{\partial^2 W}{\partial h_t \partial k_t} & \frac{\partial^2 W}{\partial h_t \partial n_t} & \frac{\partial^2 W}{\partial h_t^2} \end{pmatrix}$$

The partial derivations of the matrix H are:

$$\frac{\partial^2 W}{\partial d_t^2} = \left(\frac{q(h_t)}{n_{t-1}} \right)^2 u''(c_t) + \frac{\beta}{\rho n_{t-1}^x} q(h_t) u''(d_t)$$

$$\frac{\partial^2 W}{\partial d_t \partial k_{t+1}} = \frac{q(h_t)}{n_{t-1}} ((1 - \tau n_{t+1}) n_t) u''(c)$$

$$\frac{\partial^2 W}{\partial d_t \partial n_t} = \frac{q(h_{t-1})}{n_{t-1}} (\tau f(k_t) + a + (1 - \tau n_{t+1}) k_{t+1}) u''(c_t)$$

$$\frac{\partial^2 W}{\partial d_t \partial h_t} = \frac{q(h_{t-1})}{n_{t-1}} u''(c_t)$$

$$\frac{\partial^2 W}{\partial k_{t+1}^2} = (1 - \tau n_{t+1}) \left(\rho n_t^x f''(k_{t+1}) u'(c_{t+1}) + (1 - \tau n_{t+1}) \left(f'(k_{t+1})^2 u''(c_{t+1}) \rho n_t^x + n_t^2 u''(c_t) \right) \right)$$

$$\begin{aligned} \frac{\partial^2 W}{\partial k_{t+1} \partial n_t} &= (1 - \tau n_{t+1}) \left(\rho n_t^x f'(k_{t+1}) \left(\frac{x}{n_t} u'(c_{t+1}) + \frac{q(h_t) d_{t+1}}{n_t^2} u''(c_{t+1}) \right) - u'(c_t) \right. \\ &\quad \left. + (\tau f(k_t) + a + (1 - \tau n_{t+1}) k_{t+1}) n_t u''(c_t) \right) \end{aligned}$$

$$\frac{\partial^2 W}{\partial k_{t+1} \partial h_t} = (1 - \tau n_{t+1}) \left(n_t u''(c_t) - \rho \frac{q'(h_t) d_{t+1}}{n_t^{1-x}} f'(k_{t+1}) u''(c_{t+1}) \right)$$

$$\begin{aligned} \frac{\partial^2 W}{\partial n_t^2} &= (\tau f(k_t) + a + (1 - \tau n_{t+1}) k_{t+1})^2 u''(c_t) + \gamma u''(n_t) + \rho x(x-1) n_t^{x-2} E U_t \\ &\quad + \rho x n_t^{x-1} \frac{q(h_t) d_{t+1}}{n_t^3} u'(c_{t+1}) - \rho(2-x) \frac{q(h_t) d_{t+1}}{n_t^{3-x}} u'(c_{t+1}) + \\ &\quad \rho \left(\frac{q(h_t) d_{t+1}}{n_t^3} \right)^2 n_t^x u''(c_{t+1}) + \frac{1}{\rho} \tau^2 k_t^2 n_t^{2-x} u''(c_{t-1}) \end{aligned}$$

$$\begin{aligned} \frac{\partial^2 W}{\partial n_t \partial h_t} &= \rho(1-x) \frac{q'(h_t) d_{t+1}}{n_t^{2-x}} u'(c_{t+1}) - \rho \frac{q'(h_t) q(h_t) d_{t+1}^2}{n_t^{3-x}} u''(c_{t+1}) \\ &\quad + (\tau (f(k_t) + a + (1 - \tau n_{t+1}) k_{t+1})) u''(c_t) \end{aligned}$$

$$\frac{\partial^2 W}{\partial h_t^2} = \beta q''(h_t) u(d_{t+1}) - \rho \frac{d_{t+1}}{n_t^{1-x}} \left(q''(h_t) u'(c_{t+1}) - \frac{q'(h_t)^2 d_{t+1}}{n_t} u''(c_{t+1}) \right) + u''(c_t)$$

D Calculation of the Health expenditures

The total health expenditures per capita in PPP are the weighted sum over sexes and age groups:

$$H_\Sigma = \sum_{\text{age group}} \sum_{\text{sex}} x_{\text{age group, sex}} h_{\text{age group, sex}}$$

with $h_{\text{age group, sex}}$, the age and sex specific health expenditures and $x_{\text{age group, sex}}$, the share on total population. Supposed $\frac{h_{\text{Germany}}^{\text{age group}+j}}{h_{\text{Germany}}^{0-14,\text{female}}}$, the relation of health expenditures between age groups and sexes in Germany, holds across countries, rearranging allows to express the female health expenditure in the youngest age group for each country:

$$h_{i,0-14,\text{female}} = \frac{H_\Sigma}{\sum_{j=0}^{J=12} x_{\text{age group, sex}} \frac{h_{\text{Germany}}^{\text{age group}+j}}{h_{\text{Germany}}^{0-14,\text{female}}}}$$

with J as the age-groups of both sexes. By means of $H_{j,\text{age group},\text{female}}$ and the age structures given by the United Nations (2011), it is possible to approximate the total health expenditures of females between age 30 and 59 for the 84 countries:

$$h_i = 15 * h_{i,0-14} * \left(\frac{h_{30-44}^{Germany}}{h_{0-14}^{Germany}} + \frac{h_{30-64}^{Germany}}{h_{0-14}^{Germany}} \right).$$

E Results for selected countries

Table 6:

EXPECTED LC-UTILITIES IN SELECTED COUNTRIES

| Country | Life-cycle utility | | | |
|------------------|--------------------|-----------|------------|------------|
| | <i>LF</i> | <i>GA</i> | <i>FB1</i> | <i>FB2</i> |
| Argentina | 15.18 | 15.80 | 15.72 | 15.43 |
| Austria | 15.64 | 16.39 | 16.31 | 16.02 |
| Brazil | 14.30 | 15.04 | 14.96 | 14.68 |
| Cameroon | 14.32 | 14.71 | 14.64 | 14.38 |
| Canada | 15.51 | 16.31 | 16.22 | 15.92 |
| Chile | 14.80 | 15.55 | 15.47 | 15.19 |
| China | 12.70 | 13.59 | 13.51 | 13.26 |
| Egypt, Arab Rep. | 14.66 | 15.24 | 15.17 | 14.89 |
| France | 16.22 | 16.84 | 16.76 | 16.47 |
| Germany | 15.44 | 16.23 | 16.14 | 15.85 |
| Guyana | 15.77 | 16.02 | 15.96 | 15.72 |
| India | 13.39 | 14.00 | 13.93 | 13.68 |
| Israel | 16.76 | 17.25 | 17.17 | 16.86 |
| Italy | 15.49 | 16.25 | 16.18 | 15.90 |
| Japan | 15.72 | 16.36 | 16.29 | 16.02 |
| Kenya | 14.73 | 14.97 | 14.91 | 14.65 |
| Luxembourg | 16.51 | 17.28 | 17.20 | 16.89 |
| Malawi | 13.61 | 13.88 | 13.81 | 13.52 |
| Mali | 14.83 | 15.02 | 14.96 | 14.66 |
| Mozambique | 13.20 | 13.51 | 13.45 | 13.21 |
| New Zealand | 16.18 | 16.74 | 16.67 | 16.37 |
| Pakistan | 13.85 | 14.40 | 14.33 | 14.07 |
| Philippines | 14.86 | 15.22 | 15.16 | 14.90 |
| Singapore | 15.36 | 16.21 | 16.13 | 15.85 |
| South Africa | 15.08 | 15.71 | 15.63 | 15.35 |
| Sweden | 15.84 | 16.53 | 16.45 | 16.16 |
| Uganda | 13.51 | 13.89 | 13.81 | 13.51 |
| United Kingdom | 15.57 | 16.34 | 16.26 | 15.96 |
| United States | 16.35 | 17.07 | 16.98 | 16.65 |
| Zambia | 16.25 | 16.31 | 16.25 | 15.90 |

Table 7:

OVERVIEW ON PARAMETERS, SAVINGS, CAPITAL-LABOR RATIO AND HEALTH EXPENDITURES IN SELECTED COUNTRIES

| Country | Parameters | | | | Capital-labor ratio | | | | Health expenditures | | | | | |
|------------------|------------|--------|---------|----------|---------------------|----------|--------|----------|---------------------|---------|--------|--------|--------|--------|
| | A_i | a_i | β | δ | $h_{0,i}$ | s_{LF} | LF | GA | $FB1$ | $FB2$ | LF | GA | $FB1$ | $FB2$ |
| Argentina | 30706 | 58072 | 0.24 | 0.08 | 1.04 | 66409 | 73322 | 1773040 | 440339 | 175311 | 33770 | 91146 | 58157 | 33828 |
| Austria | 54492 | 392278 | 0.19 | 0.10 | 1.47 | 146387 | 243869 | 7488750 | 1774080 | 701492 | 95820 | 256881 | 169858 | 103847 |
| Brazil | 23114 | 60736 | 0.22 | 0.09 | 1.46 | 38861 | 50248 | 1721940 | 385307 | 150484 | 27480 | 81379 | 51269 | 30266 |
| Cameroon | 11082 | 348 | 0.46 | 0.03 | 1.23 | 18543 | 14540 | 189101 | 55629 | 23032 | 4355 | 9952 | 6425 | 3651 |
| Canada | 56436 | 278102 | 0.17 | 0.11 | 1.86 | 128373 | 183338 | 6955000 | 1517920 | 585188 | 98373 | 258841 | 173225 | 108131 |
| Chile | 35465 | 108631 | 0.18 | 0.09 | 0.98 | 64293 | 81146 | 2843220 | 656640 | 258907 | 32186 | 93317 | 59847 | 35392 |
| China | 12482 | 39952 | 0.17 | 0.09 | 1.38 | 12110 | 18994 | 977563 | 209097 | 82672 | 7830 | 26825 | 16678 | 9883 |
| Egypt, Arab Rep. | 23450 | 21151 | 0.28 | 0.04 | 0.95 | 39871 | 37302 | 889497 | 229928 | 92838 | 9991 | 28162 | 17642 | 9898 |
| France | 56882 | 198666 | 0.21 | 0.11 | 1.38 | 171449 | 210279 | 4538550 | 1191240 | 480691 | 87176 | 193653 | 135437 | 86205 |
| Germany | 52379 | 380419 | 0.18 | 0.10 | 1.55 | 129895 | 220219 | 7497880 | 1722760 | 676646 | 89521 | 245406 | 161720 | 99017 |
| Guyana | 10459 | 12819 | 0.57 | 0.04 | 0.76 | 33948 | 37850 | 199240 | 79751 | 38246 | 7341 | 12336 | 8779 | 5484 |
| India | 11262 | 11828 | 0.30 | 0.04 | 1.11 | 14407 | 14945 | 358634 | 96739 | 40326 | 3940 | 11037 | 7070 | 4117 |
| Israel | 60393 | 60588 | 0.25 | 0.07 | 0.40 | 191494 | 172779 | 2934490 | 786538 | 314095 | 57888 | 139954 | 89990 | 50827 |
| Italy | 55022 | 415738 | 0.17 | 0.09 | 0.87 | 139222 | 233490 | 7092890 | 1765320 | 712083 | 59944 | 161124 | 107457 | 65551 |
| Japan | 45678 | 355027 | 0.20 | 0.14 | 1.86 | 143160 | 248483 | 4862910 | 1411060 | 594280 | 62258 | 119062 | 90574 | 63463 |
| Kenya | 8865 | -1802 | 0.50 | 0.03 | 0.84 | 17582 | 12893 | 93071.3 | 31077 | 13278 | 2687 | 4978 | 3319 | 1903 |
| Luxembourg | 103187 | 714755 | 0.17 | 0.09 | 1.30 | 310598 | 452476 | 16163500 | 3570600 | 1380300 | 183492 | 525422 | 336317 | 198059 |
| Malawi | 5419 | -1933 | 0.53 | 0.04 | 1.30 | 7377 | 4907 | 47429.4 | 13453 | 5126 | 2206 | 4497 | 2845 | 1545 |
| Mali | 8976 | -4158 | 0.55 | 0.02 | 1.02 | 17038 | 11248 | 76581.5 | 23728 | 9213 | 2359 | 4282 | 2761 | 1476 |
| Mozambique | 4583 | -533 | 0.52 | 0.04 | 1.23 | 6049 | 4512 | 44690.2 | 13988 | 5933 | 1629 | 3339 | 2202 | 1287 |
| New Zealand | 45568 | 116377 | 0.25 | 0.09 | 1.07 | 137633 | 158167 | 2996910 | 794260 | 320316 | 70599 | 168651 | 111961 | 67144 |
| Pakistan | 14206 | 4928 | 0.30 | 0.02 | 0.89 | 18611 | 15540 | 353020 | 94400 | 38552 | 2418 | 6593 | 4183 | 2366 |
| Philippines | 13831 | 4543 | 0.38 | 0.03 | 0.64 | 29038 | 24780 | 261487 | 84435 | 36590 | 4680 | 9940 | 6622 | 3855 |
| Singapore | 64428 | 607417 | 0.14 | 0.09 | 0.75 | 143971 | 263683 | 10634300 | 2455930 | 974574 | 58268 | 174324 | 113084 | 67542 |
| South Africa | 29085 | 64852 | 0.31 | 0.05 | 1.35 | 61726 | 71736 | 1766760 | 439357 | 175928 | 29421 | 84904 | 52766 | 29920 |
| Sweden | 55970 | 212170 | 0.20 | 0.09 | 1.10 | 144143 | 182762 | 5204430 | 1245250 | 491380 | 76176 | 204725 | 133160 | 79157 |
| Uganda | 7988 | -3188 | 0.38 | 0.05 | 1.24 | 9215 | 5960 | 106878 | 25535 | 9142 | 3144 | 7906 | 4768 | 2499 |
| United Kingdom | 58816 | 240770 | 0.17 | 0.09 | 1.23 | 129558 | 169125 | 6434620 | 1406050 | 543184 | 76245 | 223668 | 142240 | 83650 |
| United States | 69526 | 207432 | 0.22 | 0.10 | 2.07 | 195705 | 230812 | 7888160 | 1608610 | 595529 | 183491 | 491983 | 315429 | 189125 |
| Zambia | 6970 | -3723 | 0.87 | 0.03 | 1.14 | 19133 | 13060 | 34784.4 | 12238 | 4509 | 3492 | 4570 | 3010 | 1531 |

Table 8:

OVERVIEW ON DEMOGRAPHIC VARIABLES AND AGE STRUCTURES IN SELECTED COUNTRIES

| Country | Net Reproduction Rate | | | Survival probability | | | Mean age | | | Dependency ratio | | |
|------------------|-----------------------|-------|-------|----------------------|-------|-------|----------|-------|-------|------------------|-------|-------|
| | LF | GA | FB1 | FB2 | LF | GA | FB1 | FB2 | LF | GA | FB1 | FB2 |
| Argentina | 1.081 | 0.389 | 0.887 | 1.739 | 0.770 | 0.800 | 0.787 | 0.770 | 1.868 | 2.484 | 2.000 | 1.593 |
| Austria | 0.667 | 0.252 | 0.592 | 1.145 | 0.837 | 0.876 | 0.860 | 0.840 | 2.201 | 2.682 | 2.283 | 1.857 |
| Brazil | 0.893 | 0.299 | 0.732 | 1.443 | 0.737 | 0.783 | 0.763 | 0.741 | 1.975 | 2.592 | 2.112 | 1.685 |
| Cameroon | 1.718 | 0.673 | 1.367 | 2.655 | 0.517 | 0.525 | 0.521 | 0.515 | 1.531 | 2.044 | 1.641 | 1.361 |
| Canada | 0.795 | 0.275 | 0.684 | 1.358 | 0.860 | 0.904 | 0.887 | 0.865 | 2.100 | 2.660 | 2.205 | 1.759 |
| Chile | 0.919 | 0.323 | 0.774 | 1.519 | 0.823 | 0.857 | 0.843 | 0.826 | 1.992 | 2.586 | 2.110 | 1.682 |
| China | 0.714 | 0.240 | 0.606 | 1.173 | 0.683 | 0.733 | 0.714 | 0.693 | 2.091 | 2.655 | 2.206 | 1.789 |
| Egypt, Arab Rep. | 1.337 | 0.482 | 1.069 | 2.092 | 0.610 | 0.623 | 0.617 | 0.610 | 1.685 | 2.292 | 1.814 | 1.468 |
| France | 0.951 | 0.373 | 0.818 | 1.583 | 0.910 | 0.935 | 0.925 | 0.910 | 2.002 | 2.549 | 2.106 | 1.681 |
| Germany | 0.654 | 0.242 | 0.581 | 1.128 | 0.833 | 0.875 | 0.859 | 0.838 | 2.212 | 2.695 | 2.294 | 1.866 |
| Guyana | 1.068 | 0.612 | 1.000 | 1.719 | 0.657 | 0.663 | 0.659 | 0.653 | 1.831 | 2.174 | 1.872 | 1.568 |
| India | 1.169 | 0.437 | 0.940 | 1.787 | 0.570 | 0.584 | 0.578 | 0.571 | 1.744 | 2.324 | 1.873 | 1.528 |
| Israel | 1.404 | 0.538 | 1.165 | 2.307 | 0.850 | 0.866 | 0.858 | 0.848 | 1.735 | 2.341 | 1.852 | 1.472 |
| Italy | 0.662 | 0.264 | 0.600 | 1.144 | 0.877 | 0.904 | 0.893 | 0.879 | 2.222 | 2.674 | 2.288 | 1.871 |
| Japan | 0.637 | 0.286 | 0.582 | 1.075 | 0.950 | 0.968 | 0.961 | 0.951 | 2.273 | 2.664 | 2.331 | 1.936 |
| Kenya | 1.912 | 0.884 | 1.646 | 3.149 | 0.597 | 0.602 | 0.598 | 0.594 | 1.504 | 1.920 | 1.574 | 1.317 |
| Luxembourg | 0.777 | 0.278 | 0.688 | 1.362 | 0.843 | 0.879 | 0.864 | 0.846 | 2.108 | 2.649 | 2.193 | 1.752 |
| Malawi | 2.290 | 0.867 | 1.809 | 3.783 | 0.517 | 0.527 | 0.520 | 0.512 | 1.413 | 1.895 | 1.509 | 1.258 |
| Mali | 2.327 | 1.026 | 2.016 | 4.177 | 0.540 | 0.544 | 0.541 | 0.537 | 1.411 | 1.806 | 1.468 | 1.237 |
| Mozambique | 1.859 | 0.764 | 1.487 | 2.847 | 0.527 | 0.536 | 0.531 | 0.524 | 1.499 | 1.975 | 1.603 | 1.339 |
| New Zealand | 1.029 | 0.400 | 0.871 | 1.690 | 0.843 | 0.871 | 0.858 | 0.842 | 1.926 | 2.497 | 2.040 | 1.626 |
| Pakistan | 1.565 | 0.557 | 1.209 | 2.359 | 0.573 | 0.580 | 0.577 | 0.573 | 1.591 | 2.186 | 1.728 | 1.413 |
| Philippines | 1.517 | 0.669 | 1.273 | 2.383 | 0.657 | 0.663 | 0.660 | 0.655 | 1.632 | 2.121 | 1.730 | 1.424 |
| Singapore | 0.600 | 0.233 | 0.557 | 1.072 | 0.867 | 0.894 | 0.884 | 0.871 | 2.277 | 2.711 | 2.328 | 1.910 |
| South Africa | 1.015 | 0.370 | 0.841 | 1.639 | 0.593 | 0.617 | 0.606 | 0.594 | 1.834 | 2.428 | 1.953 | 1.575 |
| Sweden | 0.914 | 0.334 | 0.780 | 1.531 | 0.843 | 0.875 | 0.862 | 0.845 | 2.003 | 2.578 | 2.112 | 1.682 |
| Uganda | 2.437 | 0.723 | 1.702 | 3.716 | 0.553 | 0.569 | 0.560 | 0.550 | 1.397 | 2.025 | 1.547 | 1.266 |
| United Kingdom | 0.883 | 0.302 | 0.753 | 1.498 | 0.827 | 0.864 | 0.849 | 0.830 | 2.019 | 2.614 | 2.130 | 1.691 |
| United States | 0.997 | 0.310 | 0.811 | 1.670 | 0.817 | 0.868 | 0.846 | 0.818 | 1.937 | 2.606 | 2.082 | 1.627 |
| Zambia | 2.174 | 1.385 | 2.492 | 5.418 | 0.527 | 0.529 | 0.525 | 0.520 | 1.435 | 1.638 | 1.384 | 1.183 |

Table 9:
OVERVIEW ON POLICY INSTRUMENTS IN SELECTED COUNTRIES

| Country | Tax on health expenditures | | | Child allowance | | | Lump-sum transfer middle age | | | Lump-sum transfer old age | | |
|------------------|----------------------------|--------|--------|-----------------|------------|--------|------------------------------|---------|--------|---------------------------|----------|----------|
| | GA | FB1 | FB2 | GA | FB1 | FB2 | GA | FB1 | FB2 | GA | FB1 | FB2 |
| Argentina | -0.094 | -0.090 | -0.090 | -1084170 | -148753.0 | 72255 | 852631 | 374182 | 46938 | -205246 | -267214 | -383048 |
| Austria | -0.091 | -0.087 | -0.086 | -4996050 | -714570.0 | 263671 | 2541210 | 1173540 | 265123 | -362323 | -506460 | -760843 |
| Brazil | -0.100 | -0.095 | -0.094 | -1130070 | -153374.0 | 51271 | 684101 | 306963 | 66737 | -129159 | -182055 | -268685 |
| Cameroon | -0.099 | -0.096 | -0.095 | -96221 | -12521.3 | 11098 | 130518 | 51896 | -7872 | -83000 | -89699 | -109522 |
| Canada | -0.091 | -0.087 | -0.086 | -4746770 | -622261.0 | 222343 | 2632680 | 1177630 | 243951 | -396692 | -568547 | -842696 |
| Chile | -0.096 | -0.091 | -0.091 | -1923720 | -264638.0 | 97662 | 1251800 | 562370 | 107676 | -234278 | -323160 | -464582 |
| China | -0.113 | -0.107 | -0.106 | -710065 | -101154.0 | 20623 | 343944 | 160322 | 48126 | -55853 | -82567 | -120752 |
| Egypt, Arab Rep. | -0.095 | -0.091 | -0.090 | -548239 | -75861.3 | 40340 | 530950 | 229185 | 15460 | -204352 | -253834 | -339458 |
| France | -0.088 | -0.085 | -0.084 | -2767420 | -394511.0 | 211469 | 2084180 | 932329 | 115697 | -412769 | -529302 | -771170 |
| Germany | -0.092 | -0.088 | -0.087 | -5102020 | -718340.0 | 246744 | 2491990 | 1149020 | 271343 | -341447 | -485359 | -728532 |
| Guyana | -0.100 | -0.097 | -0.097 | -73833 | -9952.8 | 18665 | 91667 | 36797 | -8364 | -41763 | -39453 | -61038 |
| India | -0.105 | -0.101 | -0.100 | -223834 | -34926.0 | 14400 | 196784 | 89690 | 14673 | -73169 | -91332 | -125251 |
| Israel | -0.085 | -0.082 | -0.082 | -1641760 | -201413.0 | 166714 | 1778640 | 721921 | -60211 | -548673 | -651420 | -871531 |
| Italy | -0.091 | -0.088 | -0.087 | -4843080 | -729784.0 | 272482 | 2574680 | 1207970 | 271017 | -374335 | -510962 | -750978 |
| Japan | -0.092 | -0.089 | -0.089 | -3047840 | -516835.0 | 232232 | 1752910 | 847481 | 183730 | -257126 | -326212 | -483522 |
| Kenya | -0.101 | -0.097 | -0.097 | -38025 | -3786.1 | 7409 | 67760 | 22889 | -14290 | -49440 | -44933 | -46975 |
| Luxembourg | -0.084 | -0.081 | -0.080 | -11055000 | -1420420.0 | 591690 | 6201100 | 2748690 | 485152 | -975645 | -1388270 | -2052730 |
| Malawi | -0.108 | -0.103 | -0.102 | -20833 | -1915.8 | 2851 | 36594 | 11793 | -7872 | -29702 | -27936 | -20364 |
| Mali | -0.099 | -0.095 | -0.095 | -30015 | -1872.7 | 5976 | 62034 | 16824 | -22181 | -58139 | -47641 | -20524 |
| Mozambique | -0.112 | -0.107 | -0.106 | -20342 | -2591.2 | 2747 | 31455 | 11984 | -3023 | -22138 | -22118 | -25340 |
| New Zealand | -0.089 | -0.086 | -0.085 | -1735250 | -240856.0 | 143239 | 1403410 | 615241 | 55846 | -318865 | -401746 | -586868 |
| Pakistan | -0.100 | -0.096 | -0.096 | -214787 | -30250.3 | 16630 | 240089 | 102522 | 3200 | -115117 | -137401 | -173663 |
| Philippines | -0.098 | -0.095 | -0.094 | -127619 | -17740.8 | 17900 | 171857 | 70232 | -9963 | -86228 | -90733 | -117653 |
| Singapore | -0.091 | -0.087 | -0.087 | -7715390 | -1111970.0 | 361484 | 3612510 | 1686970 | 409618 | -468592 | -666568 | -974464 |
| South Africa | -0.093 | -0.089 | -0.088 | -1092880 | -152751.0 | 70789 | 815810 | 361960 | 52642 | -241931 | -317375 | -458127 |
| Sweden | -0.089 | -0.086 | -0.085 | -3867880 | -458024.0 | 206030 | 2267320 | 1008900 | 157224 | -429127 | -579725 | -844732 |
| Uganda | -0.106 | -0.101 | -0.100 | -56942 | -5692.0 | 4845 | 83208 | 29116 | -10482 | -52370 | -57562 | -49148 |
| United Kingdom | -0.090 | -0.086 | -0.085 | -4402780 | -566674.0 | 218136 | 2683340 | 1183990 | 213040 | -466037 | -660912 | -961750 |
| United States | -0.087 | -0.083 | -0.082 | -5008810 | -571897.0 | 251531 | 3148270 | 1324220 | 181433 | -554695 | -799735 | -1195490 |
| Zambia | -0.098 | -0.095 | -0.095 | -7216 | 1284.8 | 3676 | 20433 | -1767 | -24744 | -26162 | -5450 | 51851 |

Table 10:
OVERVIEW ON EIGENVALUES (EV) IN SELECTED COUNTRIES

| Country | Laissez-faire economy | | "Golden Age" | | | | FB1 | | | FB2 | | | | |
|--------------|-----------------------|----------|--------------|----------|----------|----------|--------|----------|----------|----------|-------|----------|----------|----------|
| | EV1 | EV2 | EV1 | EV2 | EV3 | EV4 | EV1 | EV2 | EV3 | EV4 | EV1 | EV2 | EV3 | EV4 |
| Argentina | -0.57 | -4.8E-11 | -3.1E-11 | -1.9E-11 | -3.5E-12 | -1.7E-14 | -2.14 | -9.7E-12 | -7.2E-12 | -1.3E-12 | -1.31 | -2.6E-11 | -1.5E-11 | -6.1E-12 |
| Austria | -1.50 | -8.3E-12 | -3.5E-12 | -7.4E-12 | -3.8E-13 | -8.1E-16 | -4.39 | -3.1E-12 | -9.1E-13 | -8.1E-14 | -2.73 | -2.7E-12 | -2.2E-12 | -1.0E-12 |
| Brazil | -0.84 | -1.2E-10 | -4.4E-11 | -5.7E-11 | -4.0E-12 | -1.3E-14 | -2.89 | -2.3E-11 | -1.0E-11 | -1.7E-12 | -1.69 | -2.9E-11 | -2.1E-11 | -1.6E-11 |
| Cameroon | -0.23 | -1.1E-09 | -7.9E-10 | -1.8E-10 | -6.6E-11 | -1.8E-12 | -1.05 | -4.2E-10 | -8.5E-11 | -3.5E-11 | -0.65 | -1.3E-09 | -7.7E-10 | -2.8E-11 |
| Canada | -1.06 | -9.8E-12 | -3.2E-12 | -6.8E-12 | -3.4E-13 | -8.0E-16 | -3.20 | -2.7E-12 | -8.0E-13 | -1.1E-13 | -1.96 | -2.3E-12 | -1.9E-12 | -1.4E-12 |
| Chile | -0.79 | -3.9E-11 | -2.8E-11 | -1.9E-11 | -2.6E-12 | -5.9E-15 | -2.49 | -8.2E-12 | -6.4E-12 | -5.6E-13 | -1.54 | -1.9E-11 | -6.9E-12 | -6.1E-12 |
| China | -1.31 | -8.8E-10 | -3.5E-10 | -4.2E-10 | -2.4E-11 | -3.6E-14 | -3.64 | -1.6E-10 | -6.3E-11 | -5.2E-12 | -2.12 | -1.8E-10 | -1.2E-10 | -6.4E-11 |
| Egypt | -0.37 | -1.7E-10 | -1.2E-10 | -2.2E-11 | -1.6E-11 | -7.2E-14 | -1.43 | -4.9E-11 | -8.6E-12 | -4.6E-12 | -0.90 | -1.5E-10 | -5.0E-11 | -7.0E-12 |
| France | -0.74 | -7.4E-12 | -5.1E-12 | -4.5E-12 | -8.1E-13 | -2.9E-15 | -2.54 | -2.2E-12 | -1.7E-12 | -1.9E-13 | -1.63 | -4.4E-12 | -2.2E-12 | -1.6E-12 |
| Germany | -1.56 | -9.9E-12 | -3.8E-12 | -8.9E-12 | -4.0E-13 | -7.5E-16 | -4.44 | -3.7E-12 | -9.5E-13 | -8.5E-14 | -2.75 | -3.0E-12 | -2.4E-12 | -1.1E-12 |
| Guyana | -0.59 | -7.5E-10 | -4.0E-10 | -2.6E-10 | -1.2E-10 | -2.8E-12 | -2.60 | -4.6E-10 | -8.3E-11 | -5.1E-11 | -1.77 | -1.1E-09 | -3.7E-10 | -6.9E-11 |
| India | -0.49 | -1.1E-09 | -9.2E-10 | -1.6E-10 | -1.1E-10 | -4.4E-13 | -1.80 | -2.9E-10 | -7.0E-11 | -2.6E-11 | -1.11 | -8.5E-10 | -2.6E-10 | -5.5E-11 |
| Israel | -0.34 | -1.3E-11 | -6.5E-12 | -2.3E-12 | -1.7E-12 | -7.8E-15 | -1.37 | -4.7E-12 | -9.6E-13 | -4.3E-13 | -0.89 | -1.5E-11 | -4.7E-12 | -7.9E-13 |
| Italy | -1.53 | -8.6E-12 | -8.2E-12 | -7.4E-12 | -9.3E-13 | -1.1E-15 | -4.11 | -3.3E-12 | -2.2E-12 | -7.9E-14 | -2.66 | -5.9E-12 | -2.6E-12 | -9.7E-13 |
| Japan | -1.65 | -1.0E-11 | -8.8E-12 | -1.2E-11 | -1.9E-12 | -2.9E-15 | -4.81 | -6.1E-12 | -3.4E-12 | -1.3E-13 | -3.19 | -7.3E-12 | -4.8E-12 | -1.5E-12 |
| Kenya | -0.18 | -3.0E-09 | -1.2E-09 | -7.7E-10 | -1.2E-10 | -8.9E-12 | -0.87 | -1.7E-09 | -2.9E-10 | -6.7E-11 | -0.55 | -5.1E-09 | -2.2E-09 | -5.5E-11 |
| Luxembourg | -1.11 | -1.7E-12 | -8.5E-13 | -1.1E-12 | -8.4E-14 | -1.7E-16 | -3.19 | -4.6E-13 | -2.1E-13 | -2.0E-14 | -2.03 | -6.1E-13 | -3.6E-13 | -2.6E-13 |
| Malawi | -0.13 | -6.5E-09 | -5.5E-09 | -1.3E-09 | -3.7E-10 | -2.5E-11 | -0.69 | -3.1E-09 | -1.5E-09 | -2.0E-10 | -0.39 | -1.2E-08 | -9.8E-09 | -1.5E-10 |
| Mali | -0.12 | -3.1E-09 | -1.2E-09 | -8.3E-10 | -8.2E-11 | -1.1E-11 | -1.01 | -2.2E-09 | -4.8E-10 | -4.4E-11 | -0.37 | -6.8E-09 | -3.4E-09 | -3.5E-11 |
| Mozambique | -0.19 | -1.0E-08 | -8.5E-09 | -2.2E-09 | -7.1E-10 | -3.3E-11 | -1.84 | -4.7E-09 | -1.4E-09 | -4.0E-10 | -0.58 | -1.3E-08 | -1.1E-08 | -3.1E-10 |
| New Zealand | -0.63 | -1.3E-11 | -8.7E-12 | -6.6E-12 | -1.2E-12 | -6.6E-15 | -2.38 | -3.4E-12 | -2.6E-12 | -4.3E-13 | -1.49 | -8.1E-12 | -4.9E-12 | -2.3E-12 |
| Pakistan | -0.27 | -1.7E-09 | -5.5E-10 | -1.9E-10 | -5.7E-11 | -4.7E-13 | -1.12 | -4.9E-10 | -2.8E-11 | -2.6E-11 | -0.70 | -1.5E-09 | -2.7E-10 | -2.2E-11 |
| Philippines | -0.29 | -1.0E-09 | -3.5E-10 | -2.0E-10 | -5.7E-11 | -1.1E-12 | -1.25 | -4.4E-10 | -3.8E-11 | -3.1E-11 | -0.82 | -1.3E-09 | -3.3E-10 | -2.6E-11 |
| Singapore | -1.86 | -7.2E-12 | -6.4E-12 | -5.8E-12 | -6.5E-13 | -4.4E-16 | -4.42 | -2.5E-12 | -1.6E-12 | -3.9E-14 | -2.88 | -4.6E-12 | -2.0E-12 | -5.1E-13 |
| South Africa | -0.65 | -5.4E-11 | -3.1E-11 | -1.4E-11 | -3.1E-12 | -1.5E-14 | -2.35 | -8.7E-12 | -5.5E-12 | -1.3E-12 | -1.44 | -2.5E-11 | -1.5E-11 | -4.5E-12 |
| Sweden | -0.80 | -8.9E-12 | -5.7E-12 | -4.9E-12 | -6.4E-13 | -1.9E-15 | -2.61 | -2.2E-12 | -1.5E-12 | -1.6E-13 | -1.64 | -4.5E-12 | -2.0E-12 | -1.6E-12 |
| Uganda | -0.11 | -2.9E-09 | -2.5E-09 | -3.7E-10 | -1.8E-10 | -4.2E-12 | -0.66 | -9.4E-10 | -3.8E-10 | -8.7E-11 | -0.36 | -3.7E-09 | -3.2E-09 | -6.5E-11 |
| UK | -0.86 | -9.4E-12 | -4.9E-12 | -5.0E-12 | -4.5E-13 | -1.0E-15 | -10.04 | -2.9E-12 | -1.1E-12 | -1.2E-13 | -1.63 | -3.4E-12 | -1.6E-12 | -1.5E-12 |
| US | -0.67 | -5.2E-12 | -1.3E-12 | -2.9E-12 | -1.3E-13 | -4.4E-16 | -10.33 | -2.9E-12 | -1.3E-13 | -1.0E-13 | -1.48 | -1.5E-12 | -9.8E-13 | -7.1E-13 |
| Zambia | -0.14 | -2.8E-09 | -1.6E-09 | -1.5E-09 | -1.7E-10 | -2.6E-11 | -0.66 | -1.5E-09 | -2.1E-09 | -6.6E-11 | -0.58 | -1.2E-08 | -8.8E-09 | -5.2E-11 |