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Considering the extremely poor: Multidimensional poverty measurement for Germany

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Keywords: Poverty index, multivariate poverty, Alkire-Foster index, SOEP.

JEL Classification: I32, D63.

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Considering the Extremely Poor – Multidimensional Poverty Measurement for Germany

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October 6, 2014

Abstract¹

This paper applies the Alkire and Foster (2011) index of multidimensional poverty to German data. This is done with respect to the politically most important dimensions of poverty mentioned in the *German federal government's report on poverty and wealth*. Additionally, a modification of the identification step of the Alkire-Foster index is proposed to guarantee that individuals, who are extremely poor in only few dimensions, are not omitted by the index.

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1 Introduction

Starting with the Headcount measure much research on poverty measurement was done in the last century, see e.g. Atkinson (1987), Zheng (1997) and Chakravarty and Muliere (2004) for surveys on poverty measurement. One area is the development of more appropriate indices fulfilling reasonable postulates like monotonicity or transfer principle. Nevertheless, the indices have to be easily interpretable, since the recipients of poverty studies are more likely politicians than professional statisticians. This may

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be the reason why the statistically insufficient Headcount is still the most popular poverty index. In the last decades the focus on poverty measurement has changed more and more from unidimensional to multidimensional poverty measurement, see e.g. Bourguignon and Chakravarty (2003) or Wagle (2008). This increases the gap between statistically appropriate measures and measures that are interpretable for non-professional recipients even more. Due to this change three additional questions arise: What are the (additional) dimensions of poverty, who is considered to be poor in the multidimensional setting and how to aggregate the different dimensions?

To answer the first question, we rely on the *German federal government's report on poverty and wealth*. Concerning the second question we use the approach of Alkire and Foster (2011) in which an individual is considered to be poor, if he or she is deprived in at least a given number of dimensions. Since this approach may not be capable of considering individuals who are extremely poor in only a few dimensions, we propose a modification of the identification step that includes the extent of deprivation. To answer the third question we will again rely on the approach of Alkire and Foster (2011), which extends the unidimensional Foster et al. (1984) aggregation step to the multidimensional setting.

This paper is organised as follows: Section 2 shortly describes the Alkire and Foster (2011) poverty index (henceforth AF index). In Section 3, we motivate and define a modified version of this poverty index. Section 4 describes the data and the choice of parameters used in Section 5 to apply the original and modified AF index to German data. Section 6 concludes.

2 The Alkire and Foster poverty index

We start this section with some notations: Consider a population with n individuals and d dimensions like income, education or health. Let $\mathbf{X} = (x_{ij})_{nd}$ be a matrix with value x_{ij} for individual i in dimension j and $\mathbf{x}_i = (x_{i1}, \dots, x_{id})$ vectors of individual outcomes so that $\mathbf{X} = (\mathbf{x}_1, \dots, \mathbf{x}_n)'$. Let $\pi = (\pi_1, \dots, \pi_d)$ be a vector of univariate poverty lines and $\mathbf{w} = (w_1, \dots, w_d)$ be a vector of nonnegative dimensional weights, such that w.l.o.g. $\sum_{j=1}^d w_j = d$.

Alkire and Foster (2011) define a new index of multidimensional poverty using the common two step method of Sen (1976): On the first stage the (multidimensional) poor individuals are identified while on the second stage the deprivations of the (poor) individuals are aggregated. In the approach of Alkire and Foster (2011) an individual is identified as poor, if his or her values do not reach at least k out of d univariate poverty lines, i.e. if

$$\left(\sum_{j=1}^d \mathbf{1}_{x_{ij} < \pi_j} \geq k \right), \quad k \in \{1, 2, \dots, d\}.$$

Considering potentially different importance of the dimensions they use weights to obtain the multidimensional poverty identification function

$$\rho_i = \rho_{i,k}(\mathbf{x}_i, \boldsymbol{\pi}, \mathbf{w}) = \mathbf{1}\left(\sum_{j=1}^d w_j \mathbf{1}_{x_{ij} < \pi_j} \geq k\right), \quad k \in]0, d]. \quad (1)$$

For the second step Alkire and Foster (2011) use the Foster et al. (1984) aggeration procedure, i.e. they evaluate the deprivation of individual i in dimension j by

$$\left(\frac{\pi_j - x_{ij}}{\pi_j}\right)_+^\alpha := \mathbf{1}_{x_{ij} < \pi_j} \cdot \left(\frac{\pi_j - x_{ij}}{\pi_j}\right)_+^\alpha$$

and add first across dimensions and afterwards across individuals. Altogether, the Alkire and Foster (2011) poverty index is defined by

$$P_{\alpha,k}^{AF}(\mathbf{X}, \boldsymbol{\pi}, \mathbf{w}) = \frac{1}{n} \sum_{i=1}^n \rho_i \cdot \left(\frac{1}{d} \sum_{j=1}^d w_j \left(\frac{\pi_j - x_{ij}}{\pi_j}\right)_+^\alpha\right). \quad (2)$$

Similar to the univariate FGT indices usually three cases for α are of special interest, namely $\alpha = 0$, $\alpha = 1$ and $\alpha = 2$. For these cases Alkire and Foster (2011) provide special factorizations, which provide some additional possibilities of interpretation. Therefore let p be the number of poor individuals, $p = \sum_{i=1}^n \rho_i$, and define²

$$\begin{aligned} H &= \frac{p}{n}, \\ A &= \frac{\sum_{\{i|\rho_{i,k}=1\}} \sum_{\{j|x_{ij} < \pi_j\}} w_j}{p \cdot d}, \\ G &= \frac{\sum_{\{i|\rho_{i,k}=1\}} \sum_{\{j|x_{ij} < \pi_j\}} w_j \cdot \left(\frac{\pi_j - x_{ij}}{\pi_j}\right)}{\sum_{\{i|\rho_{i,k}=1\}} \sum_{\{j|x_{ij} < \pi_j\}} w_j} \quad \text{and} \\ S &= \frac{\sum_{\{i|\rho_{i,k}=1\}} \sum_{\{j|x_{ij} < \pi_j\}} w_j \cdot \left(\frac{\pi_j - x_{ij}}{\pi_j}\right)^2}{\sum_{\{i|\rho_{i,k}=1\}} \sum_{\{j|x_{ij} < \pi_j\}} w_j}. \end{aligned}$$

For $\alpha = 0$ the index is called the “Adjusted Headcount Ratio”. It holds that $P_{0,k}^{AF} = H \cdot A$, where H is the share of poor individuals among the population and A is the average percentage of deprived dimensions among the poor. The “Adjusted Headcount Ratio” can be applied even in situations where only ordinal data is available.

The magnitude of the deprivation is incorporated for $\alpha > 0$. In the special case $\alpha = 1$ the index is called the “Adjusted Poverty Gap”. It holds that $P_{1,k}^{AF} = H \cdot A \cdot G$. In addition to the share of poor individuals and the average deprivation among the poor, the index is sensitive to the average poverty gap. Here the average poverty

²The definitions and terms are according to section 4 of Alkire and Foster (2011).

gap means the average relative deprivation across all dimensions in which a poor individual is deprived.

If $\alpha = 2$ it holds that $P_{2,k}^{AF} = H \cdot A \cdot S$. In this case the measure accounts for the “severity” of the deprivation, making it sensitive to transfers among the poor.³

At the end of this section we want to refer to some extensions and applications of the AF index, without claiming the completeness of this list. Ravallion (2011) and Alkire et al. (2011) discuss the general use of single indicators of poverty. Bennett and Mitra (2013) generalize this measure, allowing a mixture of ordinal and metric variables. Alkire and Santos (2010) adapt the measure to developing countries and obtain the so called “Multidimensional Poverty Index”, which is part of the Human Development Report⁴ of the United Nations. For other applications see for example Whelan et al. (2014) for an application to Europe, Battiston et al. (2013) for an application to Latin American, Yu (2013) for an application to China or Mitra et al. (2013) for an analysis of the connection between disability and poverty in developing countries.

3 A modified AF index

To motivate our modified identification step we start with a small example: Consider $d = 5$ dimensions and set the univariate poverty lines to $\pi_j = 10$ for $j = 1, \dots, 5$. Identify individual i as poor, if he or she is poor in at least $k = 3$ dimensions and set $\alpha = 1$. The following table shows the data for four individuals:

individual	dimension $j =$					poor in ... dimensions	“contribution” to the AF index: $\mathbf{1}_{\dots} \cdot \frac{1}{d} \sum_{j=1}^d \left(\frac{\pi_j - x_{ij}}{\pi_j} \right)_+$
	1	2	3	4	5		
$i = 1$	12	13	12	14	15	0	$0 \cdot 0.00 = 0.00$
$i = 2$	9	11	9	11	9	3	$1 \cdot 0.06 = 0.06$
$i = 3$	0	11	0	11	11	2	$0 \cdot 0.40 = 0.00$
$i = 4$	0	0	0	0	0	5	$1 \cdot 1.00 = 1.00$

Given the relevant dimensions and poverty lines, it is incontestable that individual 1 is not poor and individual 4 is poor. However, classifying individuals 2 and 3 is more complicated. Using the AF index with $k = 3$, individual 2 is poor, because he or she is deprived in three dimensions, while individual 3 is not poor, since he or she is deprived in only two dimensions. So individual 2 will contribute to the index while individual 3 will not.

³For a discussion of the properties of the AF index see section 6 of Alkire and Foster (2011) and the discussion of the properties of the modified index at the end of section 3.

⁴For information on the use of the MPI in the Human Development Report see UNDP (2014a) and UNDP (2014b).

We think there are dimensions where this does not make sense, e.g. individual 2 is poor, because he or she is slightly ill-nourished, slightly not healthy enough and slightly not well-educated whereas individual 3 is not poor, since he or she is well-educated but has nothing to eat and is fatally ill. Combined with the calculated lack of 0.06 for $i = 2$ and 0.40 for $i = 3$, according to our opinion, the situation of individual 2 is much better than of individual 3. So we recommend to additionally identify individual 3 as poor.

Therefore we modify the identification step of the AF index. Instead of identifying individual i as poor, if he or she is deprived in k (weighted) dimensions (see formula (1)) now individual i is also identified as poor, if his or her “contribution”⁵ to the poverty index would be equal or above a threshold m ,

$$\rho_i^* = \rho_{i,k,m,\beta}^*(\mathbf{x}_i, \boldsymbol{\pi}, \mathbf{w}) = \mathbf{1}_{\sum_{j=1}^d w_j \mathbf{1}_{x_{ij} < \pi_j} \geq k} \quad \vee \quad \sum_{j=1}^d w_j \left(\frac{\pi_j - x_{ij}}{\pi_j} \right)_+^\beta \geq m, \quad k, m > 0. \quad (3)$$

With this modified identification function the resulting poverty measure is

$$P_{\alpha,k,m,\beta}^{AF*}(\mathbf{X}, \boldsymbol{\pi}, \mathbf{w}) = \frac{1}{n} \sum_{i=1}^n \rho_i^* \cdot \left(\frac{1}{d} \sum_{j=1}^d w_j \left(\frac{\pi_j - x_{ij}}{\pi_j} \right)_+^\alpha \right). \quad (4)$$

Using this identification function, there are two possibilities for an individual to be considered as poor. If an individual is deprived in many dimensions, the single deprivations can be minimal, nevertheless we would consider the individual to be poor. The reason we add the second condition is to guarantee that an individual, who is extremely deprived in only a few dimensions, will be identified as poor. Obviously this modified identification step c.p. (weakly) increases the number of poor individuals.

Note that due to the additional character of the modified identification function the new identification step nests the old one. It simplifies to the Alkire and Foster (2011) identification step if either $m > d$ or $\beta = 0$ and $k = m$: In the first case the second condition can never be fulfilled and in the second case the two conditions are equal. On the other hand, if $k > d$, the first condition can never be fulfilled and an individual is poor only if the second condition is fulfilled.⁶

Of course one can argue that this is an ad hoc method and it would be better to construct identification procedures, that somehow rely on the substitutability of the different dimensions of poverty. But we have to keep in mind who are the

⁵In general we allow $\beta \neq \alpha$, so strictly speaking it is not necessary the contribution to the index that is used for identification. For a discussion about the choice of the parameter β see section 4.3.

⁶ $k > d$ is not possible in the original index of Alkire and Foster (2011) since it would lead to $P_{\alpha,k}^{AF}(\mathbf{X}, \boldsymbol{\pi}, \mathbf{w}) = 0$ by definition. We allow for $k > d$ because our second condition can still be fulfilled and by setting $k > d$ we can analyse the sole influence of the new condition.

recipients of such poverty studies and therefore this simple identification procedure seems appropriate to us.

The new identification step is easy to interpret if $\beta = \alpha$: In this case the additional condition means that the sum of weighted deprivations of individual i has to exceed m . Combined with the original condition the identification function can be interpreted as follows: Individual i is considered to be poor if either he or she is deprived in at least k dimensions or if his or her (weighted) deprivations reach or exceed a threshold m .

For an analysis of the properties of the modified measure we can rely on the properties of the original measure. Alkire and Foster (2011) deduce the properties of their measure by analyzing the properties of a certain combination of an identification function with an aggregation function. The benefit of this approach is that some of the properties solely rely on the aggregation function used and do not depend on the identification function. Since we have only modified the identification function, we can directly deduce that our modified measure fulfills the properties *Decomposability*, *Replication Invariance*, *Symmetry*, *Deprivation focus*, *Weak Monotonicity*, *Nontriviality* and *Normalization* for $\alpha \geq 0$.⁷

A second group consists of properties that are defined for a given identification function, e.g. an individual receives an improvement in any of the dimensions given he or she is identified as poor. *Poverty Focus*, *Weak Rearrangement*, *Dimensional Monotonicity*, *Monotonicity* (for $\alpha > 0$) and *Weak Transfer* (for $\alpha \geq 1$) belong to this group. Since they are defined conditional on a given identification procedure, we can deduce that they are fulfilled for our modified identification procedure as well.

4 Data and parameters

Using poverty measures like (2) and (4) requires a number of choices for different parameters that imply value judgements about the importance and relation of the dimensions, the extent of poverty and many more topics. Although for some parameters there is only a number of meaningful possibilities and for some other parameters there have been established specific choices in the literature, in general these choices remain somehow arbitrary.

Since measuring poverty should be the foundation of political decisions, in our opinion these choices have to be made by the society itself, at best after a broad public discussion. Task of a researcher should therefore be to point out the consequences of different choices of parameters and to provide a detailed background for the discussion.

⁷For a definition and a discussion of these properties see Alkire and Foster (2011).

Nevertheless, if the researcher wants to exemplify the different measures or parameters, he or she has to make some choices in the first place. This is what we want to do in this section. For presenting first results for the modified measure, we choose a setting of parameters, that we think is meaningful in the context of poverty measurement in Germany. But again we want to emphasize, that these choices are not mandatory. All we can do is to make the process of choice transparent and thereby open for discussion.

4.1 Included dimensions

For the choice of the dimensions, we rely on Part C of the “German federal government’s 4th report on poverty and wealth” (Bundesministerium für Arbeit und Soziales, 2013).⁸ The report covers nine major topics, i.e. (i) distribution of material resources, (ii) employment market, (iii) child care and education, (iv) health, (v) home and rent, (vi) homelessness, (vii) prisoners and their chances on participation, (viii) social commitment and social contact and (ix) social responsibility of the rich and wealthy.

We try to build a multidimensional poverty index that includes most of these topics. Certainly (ix) is relevant for a general discussion on poverty reduction but not relevant for status quo poverty measurement. Furthermore we want to analyse poverty on an individual basis, therefore in our study (vi) is included in (v) and (vii) is assumed to be less important than the other dimensions. The rest of the topics should be included in our analysis, so our poverty measure considers up to six dimensions of poverty.

For our analysis we use data of the German Socio-economic Panel (SOEP).⁹ Bringing together the dimensions and the data, we operationalize the dimensions and define univariate poverty lines as follows:¹⁰

- I. **Income:** *Equivalized monthly household net income*¹¹, poor if the income is less than 60% of the median income,

⁸For an English summary see Federal Ministry of Labour and Social Affairs (2013). For the 1st to 3rd report see Bundesregierung (2001), Bundesregierung (2005) and Bundesregierung (2008), respectively.

⁹Socio-economic Panel (SOEP), Data for years 1984-2011, Version 28, SOEP, 2012, doi:10.5684/soep.v28. For more information see Wagner et al. (2007) and Wagner et al. (2008).

¹⁰The names of the variables are those from the English version of “SOEPinfo”, <http://panel.gsoep.de/soepinfo2011/>.

¹¹We use the modified OECD-equivalence scale, 1 to head of household, 0.3 for each child younger than approximately 14 (since only the year of birth is available), and 0.5 to all other household members.

- II. **Employment:** *Number of months registered employed within the previous year*, poor if less than 10 months employed (up to three months we consider unemployment to be frictional),
- III. **Education:** *Amount of education or training in years*, poor if less than 10.5 years of education (10.5 years correspond to a basic school degree and additional vocational training),
- IV. **Health:** *Satisfaction with health*¹², poor if less than 4,
- V. **Living:** *Size of housing unit in m^2* , poor if less than $45m^2$ (the maximum living space paid by the job center / social welfare office)¹³,
- VI. **Social participation:** *Number of activities per week*¹⁴, poor if less than one activity per week.

4.2 Dimensional weights

The choice of weights for the dimensions reflects the relative importance of the different dimensions of poverty. There are a number of different approaches to set or derive weights, which are roughly classified as either data-driven, normative or hybrid approaches by Decancq and Lugo (2013).

In our application we want to apply two different methods to set the weights. The first is to set the weights for the different dimensions to an equal level. Certainly not an elegant approach but for a number of reasons it is the only viable way in a broad number of situations.¹⁵

¹²Self-Evaluation, measured on a scale from 0 to 10. We assume this question to be a Likert-type item, so we are able to treat it like a metric variable, see for example Traylor (1983).

¹³We construct an equivalence scale similar to Frick (1995), i.e. 1 for the first individual, $\frac{1}{3}$ for the second and third individual and $\frac{2}{9}$ for all other household members. This equivalence scale was constructed to reflect the guidelines for appropriate living space of the social welfare offices in Germany, which are approximately $45m^2$ for one individual, $60m^2$ for two individuals, $75m^2$ for three individuals and $10m^2$ for each additional household member.

¹⁴We construct a variable by adding up the frequencies of nine different activity variables, namely *attend cultural events*, *attend cinema*, *pop*, *jazz concerts*, *participate in sports*, *artistic activities*, *attend social gatherings*, *helping relatives*, *friends*, *perform volunteer work*, *participate in local politics* and *attend church or other religious events*. We set at least one time per week equal to 1, at least one time per month equal to 0.5 (since less than one time per week and at least one time per month is 1,2 or 3 times per month and the mean is approximately 2 times per month, i.e. 0.5 times per week). Similarly we set less than one time per month and more than never equal to 0.125 times per week.

¹⁵Often it is argued that the researcher does not want to impose a value judgement and therefore uses equal weights. But this is no proper reasoning since equal weights mean that the dimensions are of equal importance, clearly a value judgement. We would rather argue that due to data availability many of the approaches to obtain weights do not work. So we use equal weights as a arbitrary starting point which clearly reveals that setting weights is a topic that has to be analysed in more detail.

The second approach is to use self stated weights, in our opinion the most favorable way to set the weights. Optimally the weights would come from the same survey as the data but since we have no such weights in the SOEP data, we use weights from the OECD BETTER Life Index¹⁶ as a proxy. The index consists of eleven dimensions that are supposed to influence the quality of life and we use these dimensions to cover our six dimensions of poverty. The dimensions of the index are attached to the dimensions of poverty according to the following list:

i. Civic Engagement and Government (CG), ii. Social connections (SC) \Rightarrow Social participation, iii. Education and Skills (ES) \Rightarrow Education, iv. Environment quality (EQ), v. Health status (HS) \Rightarrow Health, vi. Housing (HO) \Rightarrow Living, vii. Income and wealth (IW) \Rightarrow Income, viii. Jobs and earnings (JE) \Rightarrow Employment, ix. Subjective well-being (SW), x. Personal security (PS), xi. Work-Life Balance (WL).

The website allows users to set the weighting scheme according to their individual preferences. As of September 10, 2014, 6289 Germans have set their individual weights for these dimensions. We use the average weights of the dimensions to construct two weighting schemes, w_1 for all six dimensions of our poverty measure and w_2 for a subset that excludes the variable *social participation*. Table 1 gives the average weight for each of the dimensions and the two weighting schemes for our application.

Table 1

4.3 Other Parameters

The choices for α , k , β and m are in principal not less arbitrary than the choices for the weights, but especially for α there are some well-known choices.

The parameter α determines how the measure reacts to changes of a poor individual in a deprived dimension. There have been established three common choices for α in the literature, which have been mentioned before and correspond to the adjusted Headcount ($\alpha = 0$), the Adjusted Poverty Gap ($\alpha = 1$) and a transfersensitive measure ($\alpha = 2$). We will focus on these choices for α throughout the paper.

For β we will focus on the case $\beta = 1$. In general one could argue that setting $\beta = \alpha$ would be the most natural way, since in this case the identification step would mirror the contribution to the poverty measure. But for $\beta = 0$ the identification method would reduce to the original method and for $\beta = 2$ it would be difficult to interpret. In contrast, $\beta = 1$ is easy to interpret because in this case the second identification method is to compare the sum of the relative shortfalls in the different dimensions with a given threshold.

¹⁶See <http://www.oecdbetterlifeindex.org/> for information about the OECD BETTER LIFE Index.

With k and m we can determine the extent of deprivation an individual must suffer to be considered as poor. As for α we will provide results for different choices of these parameters and show how they influence our measure of multidimensional poverty.

5 Results of the empirical application

At first, we discuss the results of the well-known indices for the German data. In a second step, we illustrate the modified indices with the German data to see whether the constructed example from the beginning of Section 3 is relevant for real data or not.

In order to analyse multidimensional poverty over the complete time period, we will restrict the analysis in this section to five dimensions of poverty and leave out the dimension “social participation”. Some results for all six dimensions of poverty will be presented in the Appendix.

5.1 Results for the well-known measures

First of all we take a look at the univariate Headcounts for the above mentioned dimensions of poverty in Figure 1.

Figure 1

Until 1991 Figure 1 shows Headcounts for West Germany and since 1992 for the reunited Germany. This is probably the reason for the dramatic change in education and living poverty from 1991 to 1992. It also shows an increase of employment poverty, i.e. the increasing unemployment after German reunification until chancellor Schröder’s labour market reforms. Also you can see the decrease of employment poverty and the increase of income poverty in the last few years after these labour market reforms, whose different steps became law between 2003 and 2005.

Figure 2

Figure 2 shows the univariate FGT indices for the different dimensions of poverty for $\alpha = 1$ and $\alpha = 2$, which in contrast to the univariate Headcounts incorporate the magnitude of deprivations.

Going on with the multidimensional results, Figures 3 and 4 show the results for the AF index for different choices of k .

Figures 3 & 4

For $\alpha = 0$, the AF index does not report the proportion of (multidimensional) poor individuals, but the "Adjusted Headcount Ratio" described in Section 2. Nevertheless, the different developments of the univariate Headcounts are reflected in the AF index: For $k = 1$ an individual is poor, if he or she is deprived in at least one dimension. The large percentage of individuals, who are poor in living and housing, decreased dramatically during the 1990s and 2000s. On the other hand poverty caused by income and unemployment increases during the early 2000s. Both developments are shown by the AF index for $k = 1$ in Figure 3. Because of the different range of values we standardize the values of the indices using 1992 as base year in Figure 4. After the German reunification the AF indices, where an individual is poor in at least one ($k = 1$) and at least two ($k = 2$) dimensions, decreased as described above. But if you focus only on those, who are poor in at least three ($k = 3$) and four ($k = 4$) dimension, the AF indices increased during the last decade.

Roughly speaking, if we define all individuals as poor, who are deprived in any of the dimensions ($k = 1$), poverty decreases. But the intense of poverty of those who are in a very bad situation, i.e. poor in at least three or four dimensions, rises. For similar results for $\alpha = 1$ and $\alpha = 2$ see Figure 5.

Figure 5

Since the Figures of the AF indices for different α look very similar at a first glance, we now take a look on how c.p. the choice of α affects the index. Do changes of α make any difference for real data?

Table 2

Of course in many situations the changes of the indices have the same sign for different choices of α , but there are also situations where you can see different behaviors. E.g. if you compare 1992 with 2005 the index decreased for $\alpha = 0$, was approximately constant for $\alpha = 1$ and increased for $\alpha = 2$, see table 2. This means that the magnitude of suffered deprivations increased and compensated for the decrease in the number of poor individuals. This is in line with the former results, also suggesting that the situation of the extremely poor has worsened.

Now we illustrate how c.p. the choice of the weighting factors w_j affects the index. Therefore we compare the weighting scheme w_2 with an weighting scheme with equal weights for all dimensions for $\alpha = 1$ and $k = 2$ or $k = 3$ respectively, see Figure 6.

Figure 6

We can see in Figure 6 that this special weighting scheme does not seem to affect the general development of the AF index. This is probably due to the fact that the

weights we have obtained are not that different from equal weights. Therefore – and because this should not be the main topic of this note – we will restrict the rest of the analysis to a weighting scheme with equal weights and leave the topic of setting weights to future research.

5.2 Results for the modified measures

Now we want to illustrate the modified AF index for several parameter specifications. In the first step, we start with various m and $k = 6$, i.e. an individual is identified as poor, if his or her illfare exceeds m and no one is identified as poor by the AF identification step, see Figure 7.

Figure 7

If an individual is identified as poor even for small illfare m , Figure 7 shows a decrease of the index, e.g. for $m = 0.25$ the index decreases from 0.0376 in 1984 to 0.0332 in 2011. If the illfare m has to be large to count someone as poor, the index increases, for $m = 1.5$ from 0.0037 in 1984 to 0.0052 in 2011. Again we interpret this as a worsening of the situation of the extremely poor, which is in line with the results of the original AF index. Moreover, the results of the modified measure seem to be more explicit since already the identification step focusses on “extreme” poverty.

The basic idea of the modified identification step is that the original measure omits individuals, who are extremely poor in only a few dimensions. Therefore the following figure illustrates how many individuals are additionally poor, if we modify the identification step.

Figure 8

Figure 8 shows an enormous difference between the number of poor defined by the AF index for $k = 3$ and the same index with the modified identification step. Of course the more restrictive the additional condition to identify the poor is, i.e. the larger m is, the less additional poor are identified. E.g. for $m = 1$ the new identification step means that either you are deprived in three out of five dimensions or the sum of the relative deprivations in at most two dimensions is one.¹⁷ The results show that even for this quite restrictive choice of m the number of poor individuals nearly doubles.

Figure 9

¹⁷This means that either the individual has nothing in one dimension or is on average 50% below the poverty line in two dimensions.

The last figure should demonstrate how the modified identification step changes the values of the index and not only the number of poor individuals. Therefore Figure 9 shows the results for the modified AF index for $\alpha = 1, k = 3$ and different choices of m . Most striking is the level shift corresponding to the different numbers of poor individuals. However, the general pattern of the development of poverty does not seem to depend on the identification step.

6 Conclusions and remarks

We have applied the Alkire Foster index of multidimensional poverty and defined a modified identification step. This new identification step allowed us to classify individuals with extreme deprivation in only few dimensions as poor with a very simple procedure, i.e. using the individual deprivations that are calculated for the aggregation step anyway. We have illustrated this modification with German data. This generalization came at the cost of an additional parameter m , who determines a “deprivation line”, but since the choice of the parameter depends on individual deprivations it should be relatively easy to interpret.

Our results for the AF index show that in general multidimensional poverty in Germany seems to decrease but the situation of the extremely poor individuals seems to worsen. This is in line with the results of our modified measure, which suggest that even for restrictive choices of the new “deprivation line” the number of poor individuals significantly increases compared to the original identification procedure. But clearly the situation of extremely poor individuals has to be analysed in greater detail in future applications.

Another problem – for our modification as well as for other multidimensional poverty measures – is the choice of dimensional weights. For our first application we have used equally weighted and additionally self stated dimension weights. Other approaches to determine the dimension weights seem to be a challenging but interesting topic for further research.

It would also be interesting to use the modified identification approach for data of developing countries, since our motivating example at the beginning of Section 3 probably fits even better for poverty measurement for developing countries: In some situations being poor in two dimensions (slightly ill-nourished and slightly not well-educated) is better than being poor in one dimensions (being well educated but having nothing to eat), isn't it?!

Acknowledgement *We would like to thank Karl Mosler for his very helpful contributions.*

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Tables and figures for the main text

Dim.	CG	SC	ES	EQ	HS	HO
Avg. weight	0.0675	0.0814	0.1002	0.0929	0.1060	0.0889
w_1	–	0.8912	1.0971	–	1.1606	0.9734
w_2	–	–	1.0737	–	1.1359	0.9526

Dim.	IW	JE	SW	PS	WL	
Avg. weight	0.0837	0.0878	0.1069	0.0908	0.0934	Σ
w_1	0.9164	0.9613	–	–	–	6
w_2	0.8969	0.9408	–	–	–	5

Table 1: Average weights from the OECD BETTER Life Index and resulting weighting schemes.

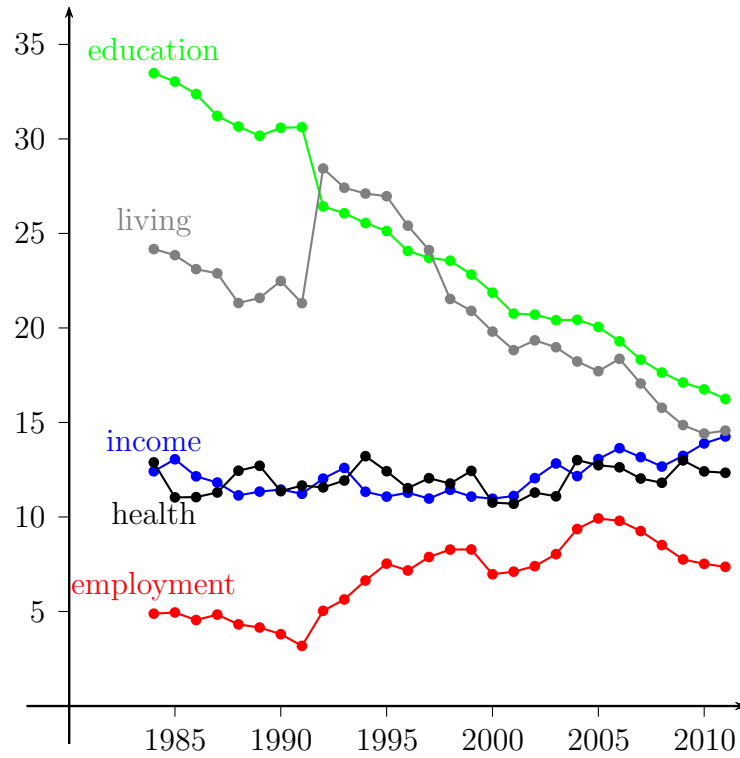


Figure 1: Univariate Headcounts (in %) for income, employment, education, health, and living.

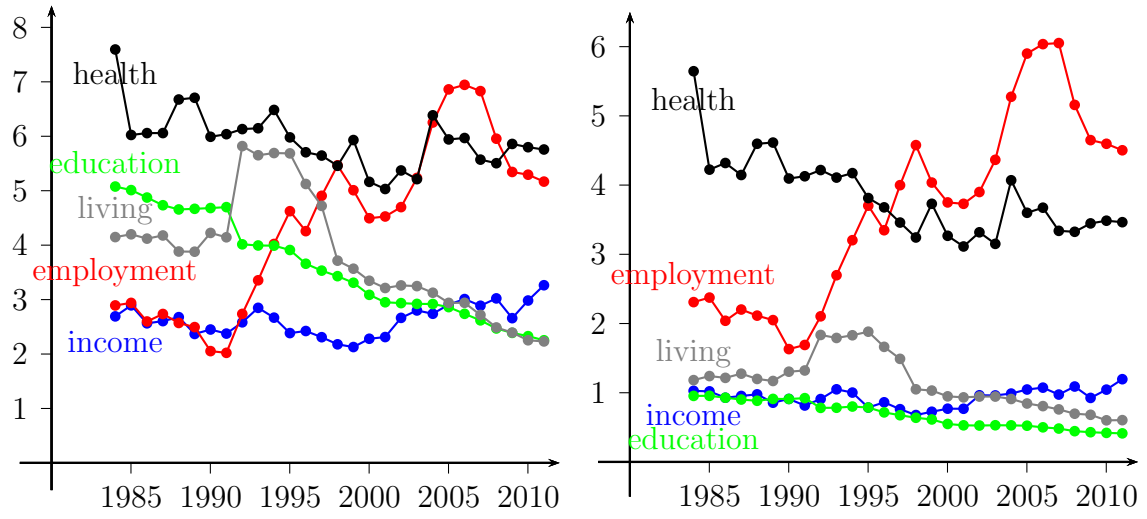


Figure 2: Univariate FGT indices (in %) for $\alpha = 1$ (left hand side) and $\alpha = 2$ for income, employment, education, health, and living.

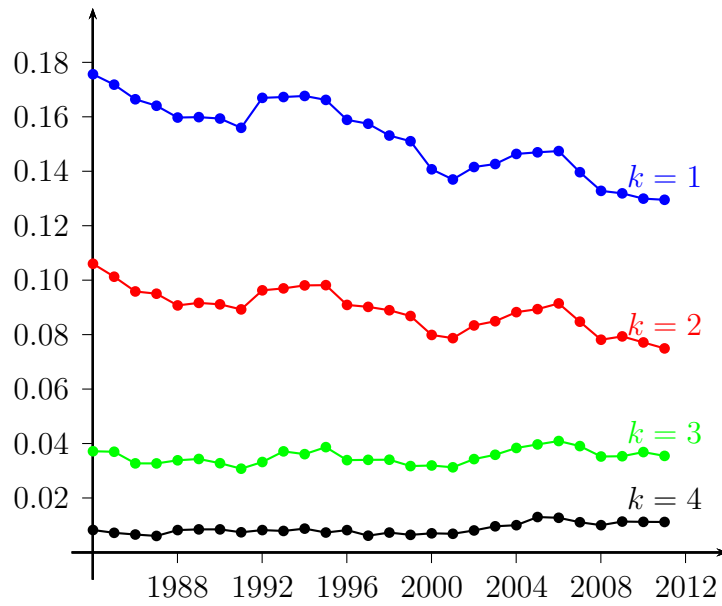


Figure 3: Alkire-Foster indices for $\alpha = 0$ and $w_j = 1$.

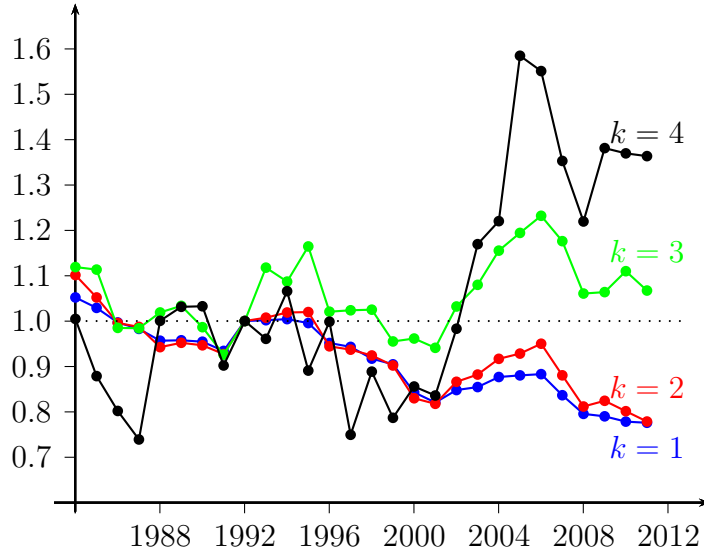


Figure 4: Alkire-Foster indices for $\alpha = 0$ and $w_j = 1$, base year 1992.

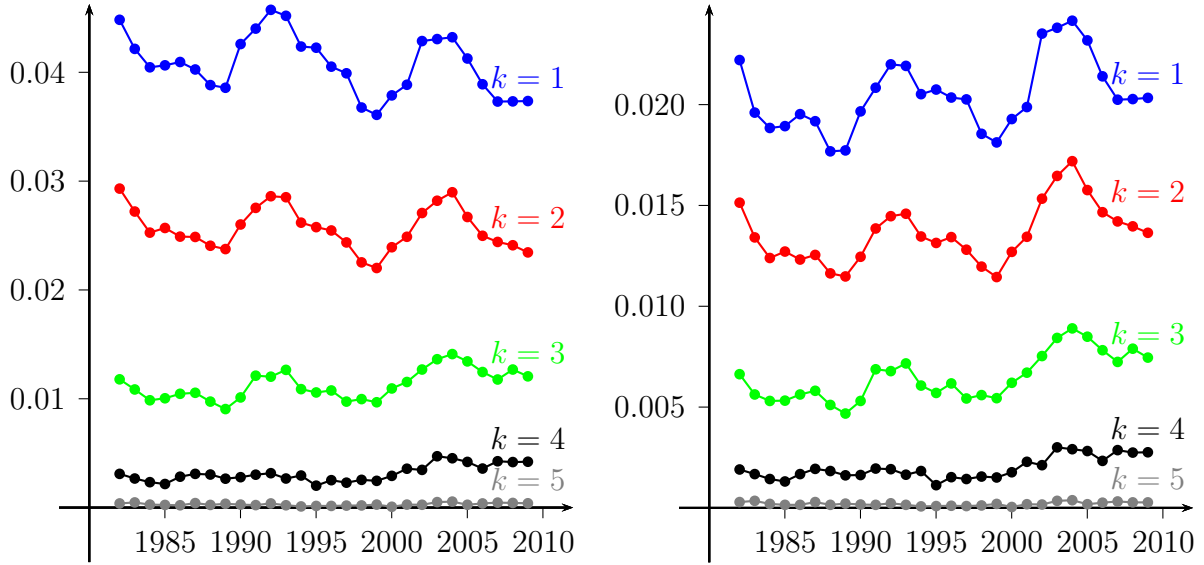


Figure 5: Alkire-Foster indices for $\alpha = 1$ (left) and $\alpha = 2$ and $w_j = 1$.

α	1992	2005	Rel. diff.
0	0.1670	0.1470	-12%
1	0.0426	0.0430	o1%
2	0.0197	0.0238	o21%

Table 2: Alkire-Foster indices for $k = 1$ and $w_j = 1$.

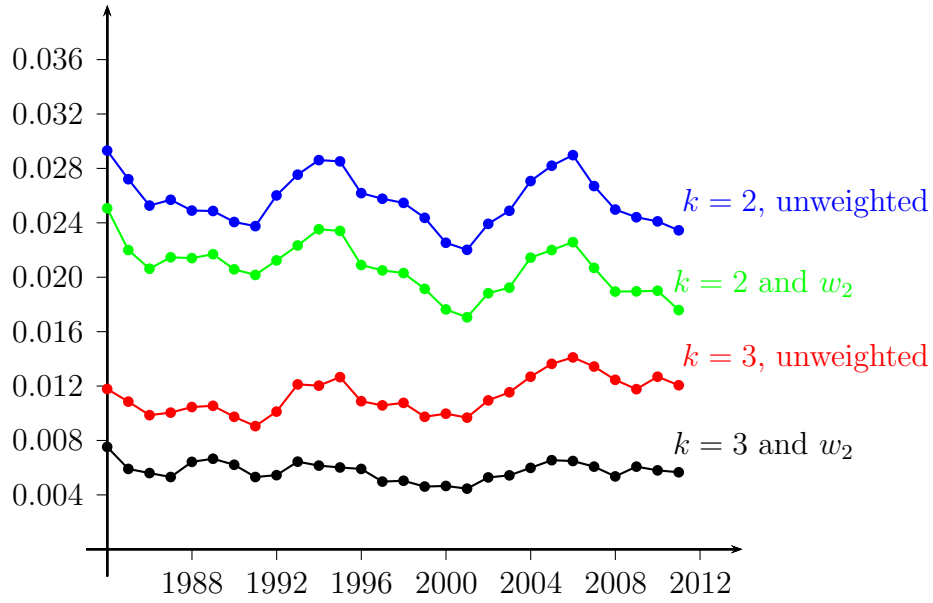


Figure 6: Alkire-Foster indices for $\alpha = 1$ and different weighting schemes.

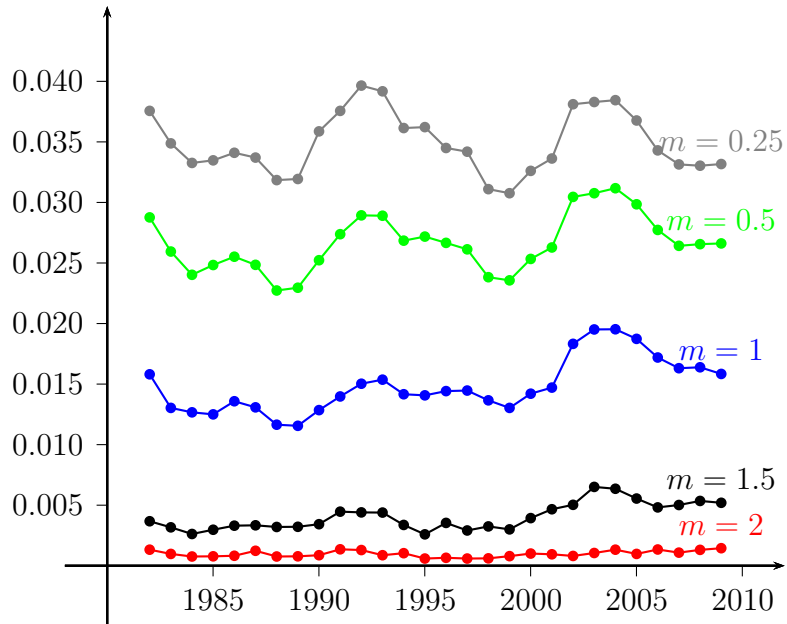


Figure 7: Modified Alkire-Foster indices for $\alpha = 1$, $k = 6$ and various m .

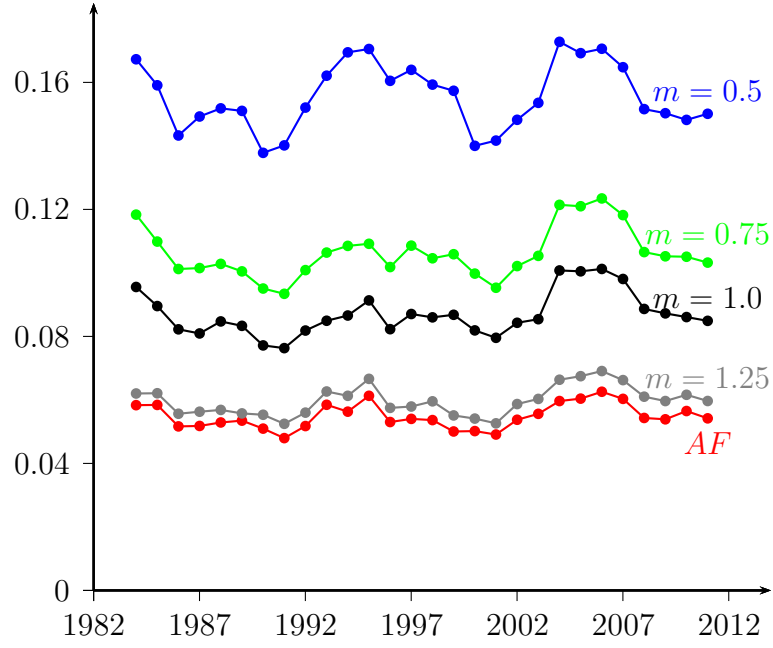


Figure 8: Proportion of the poor for the Alkire-Foster index, $k = 3$, $\alpha = 1$ and several m for the modified index.

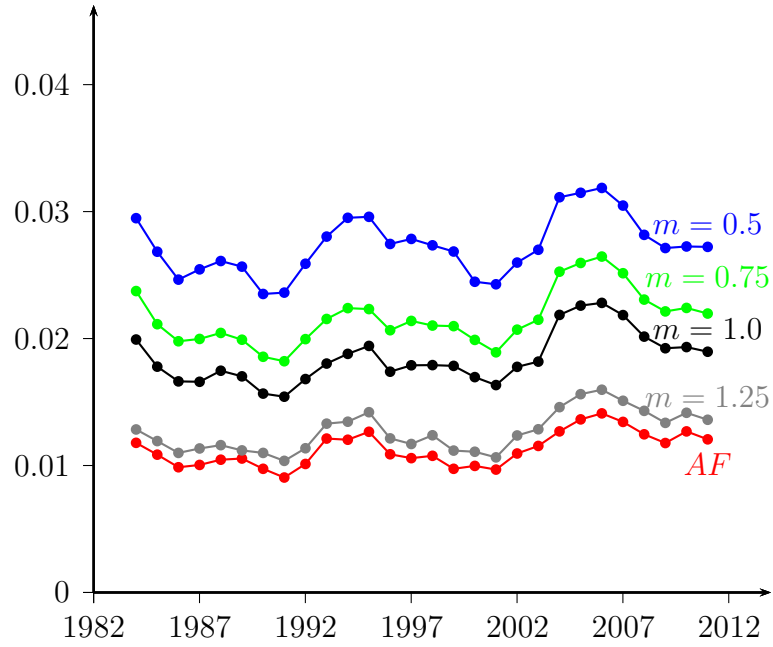


Figure 9: Values of the Alkire-Foster $k = 3$, $\alpha = 1$ and several m for the modified index.

Appendix

Appendix A: Univariate distributions and descriptive statistics

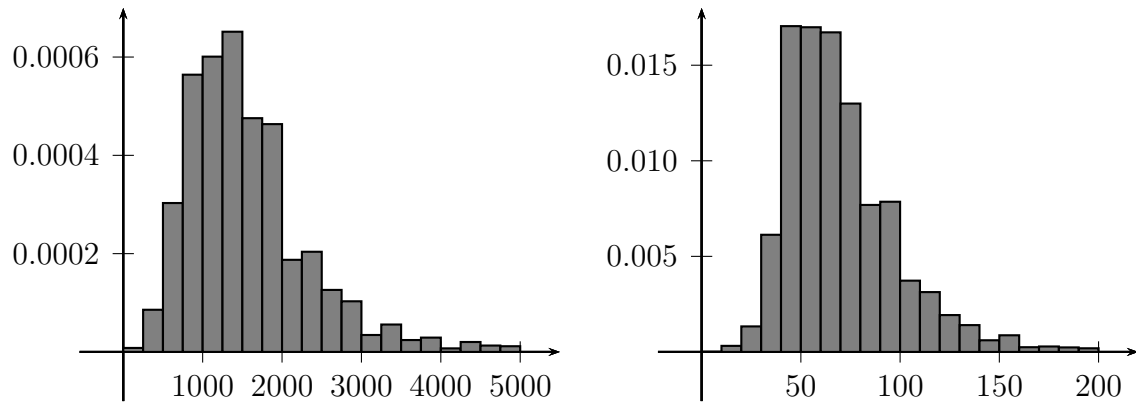


Figure 10: Histograms for income (left hand side) and living, year 2011.

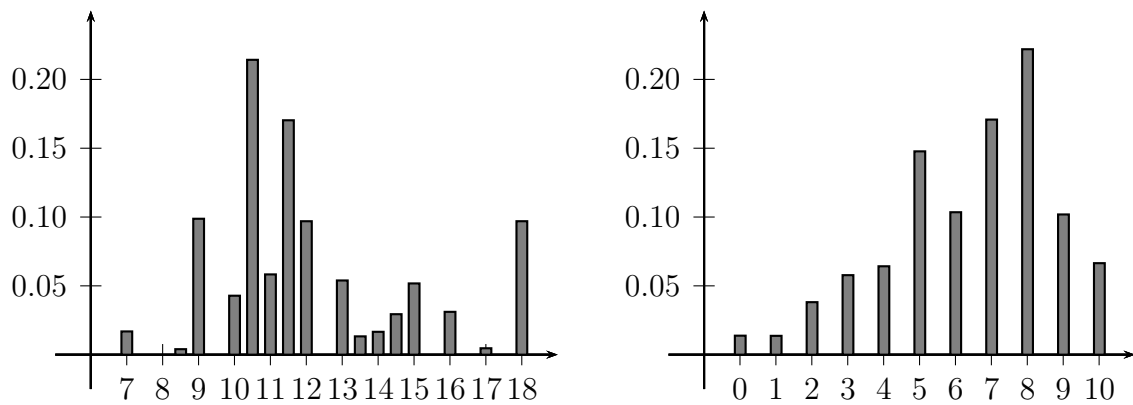


Figure 11: Bar charts for education (left hand side) and health, year 2011.

year	observations		income			employment		education		health		living	
	n	\tilde{n} (in m)	$\tilde{\mu}$	\tilde{m}	$\tilde{\sigma}$	$\tilde{\mu}$	$\tilde{\sigma}$	$\tilde{\mu}$	$\tilde{\sigma}$	$\tilde{\mu}$	$\tilde{\sigma}$	$\tilde{\mu}$	$\tilde{\sigma}$
1984	11518	46.9	803	716	701	11.6	1.9	10.8	2.2	6.8	2.7	59.3	21.7
1985	10359	47.2	815	750	446	11.6	1.9	10.9	2.2	6.8	2.5	59.6	22.6
1986	10020	47.7	863	777	704	11.6	1.8	10.9	2.2	6.7	2.5	60.0	23.0
1987	9955	48.0	894	818	626	11.6	1.8	11.0	2.3	6.7	2.4	60.3	22.0
1988	9500	48.1	919	844	560	11.6	1.8	11.0	2.3	6.5	2.4	61.1	22.6
1989	9205	48.4	965	869	592	11.7	1.8	11.0	2.3	6.5	2.5	61.9	23.3
1990	9020	49.1	1025	923	590	11.7	1.6	11.0	2.3	6.5	2.4	61.7	23.5
1991	8921	48.6	1063	972	503	11.7	1.6	11.1	2.4	6.5	2.4	62.4	24.4
1992	12714	62.1	1026	920	527	11.6	1.8	11.2	2.4	6.6	2.4	59.4	24.5
1993	12455	62.4	1101	1004	584	11.5	2.0	11.3	2.4	6.5	2.4	60.1	25.0
1994	12710	62.5	1137	1023	599	11.4	2.2	11.3	2.4	6.4	2.4	60.5	25.0
1995	12868	61.5	1194	1051	747	11.4	2.3	11.4	2.5	6.4	2.3	61.2	26.1
1996	12534	61.3	1206	1091	659	11.4	2.2	11.4	2.4	6.4	2.3	62.0	26.0
1997	12339	62.0	1215	1108	615	11.3	2.4	11.5	2.5	6.4	2.3	62.4	25.5
1998	13033	58.8	1229	1119	595	11.3	2.6	11.5	2.5	6.5	2.3	64.0	25.7
1999	12873	61.1	1270	1151	618	11.3	2.4	11.5	2.5	6.4	2.3	64.6	26.0
2000	22115	60.5	1302	1176	679	11.4	2.3	11.6	2.5	6.7	2.3	65.5	26.3
2001	20042	59.8	1334	1207	682	11.4	2.3	11.7	2.5	6.7	2.3	66.1	26.5
2002	21420	59.3	1365	1200	913	11.4	2.4	11.6	2.4	6.5	2.3	65.9	26.6
2003	20155	59.5	1399	1250	873	11.3	2.5	11.7	2.5	6.5	2.2	65.8	26.3
2004	19604	59.9	1406	1250	967	11.2	2.7	11.7	2.5	6.4	2.3	66.7	27.2
2005	18743	60.1	1413	1261	815	11.1	2.9	11.7	2.5	6.4	2.3	66.7	26.9
2006	19765	59.0	1442	1267	903	11.1	2.9	11.8	2.6	6.4	2.3	67.1	27.6
2007	18789	60.1	1466	1286	963	11.1	2.9	11.9	2.6	6.4	2.3	68.3	28.7
2008	17557	60.0	1507	1333	984	11.2	2.7	11.9	2.6	6.4	2.2	69.2	28.5
2009	18400	60.3	1547	1340	915	11.3	2.6	12.0	2.6	6.3	2.3	69.8	28.6
2010	16688	59.8	1597	1400	977	11.3	2.6	12.1	2.6	6.4	2.3	70.9	29.9
2011	16694	52.6	1619	1426	938	11.3	2.5	12.1	2.6	6.4	2.3	70.9	29.2

Table 3: Number of observations n and, if raising factors are used, observations \tilde{n} , mean $\tilde{\mu}$, median \tilde{m} and standard deviation $\tilde{\sigma}$.

1986	inc	empl	edu	health	living	1991	inc	empl	edu	health	living
inc	1	.07	.21	.04	.25	inc	1	.11	.35	.06	.38
empl	.07	1	.04	.03	.08	empl	.11	1	.05	.06	.08
edu	.21	.04	1	.09	.26	edu	.35	.05	1	.06	.25
health	.04	.03	.09	1	.00	health	.06	.06	.06	1	-.01
living	.25	.08	.26	.00	1	living	.38	.08	.25	-.01	1
1996	inc	empl	edu	health	living	2001	inc	empl	edu	health	living
inc	1	.14	.26	.06	.34	inc	1	.17	.35	.09	.37
empl	.14	1	.07	.04	.09	empl	.17	1	.07	.07	.10
edu	.26	.07	1	.09	.13	edu	.35	.07	1	.12	.15
health	.06	.04	.09	1	-.01	health	.09	.07	.12	1	.01
living	.34	.09	.13	-.01	1	living	.37	.10	.15	.01	1
2006	inc	empl	edu	health	living	2011	inc	empl	edu	health	living
inc	1	.19	.39	.11	.46	inc	1	.19	.41	.13	.42
empl	.19	1	.10	.03	.15	empl	.19	1	.11	.05	.15
edu	.39	.10	1	.14	.23	edu	.41	.11	1	.15	.20
health	.11	.03	.14	1	.04	health	.13	.05	.15	1	.04
living	.46	.15	.23	.04	1	living	.42	.15	.20	.04	1

Table 4: Pairwise correlations for several years.

Appendix B: Results for six dimensions

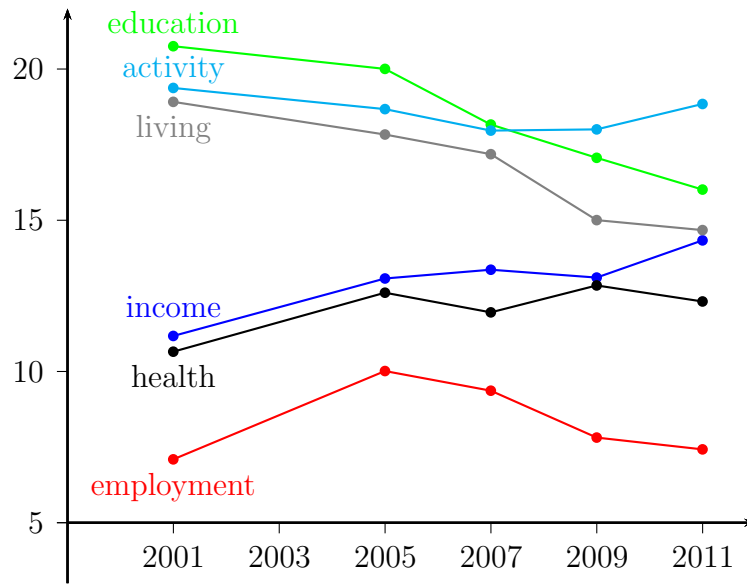


Figure 12: Univariate Headcounts (in %).

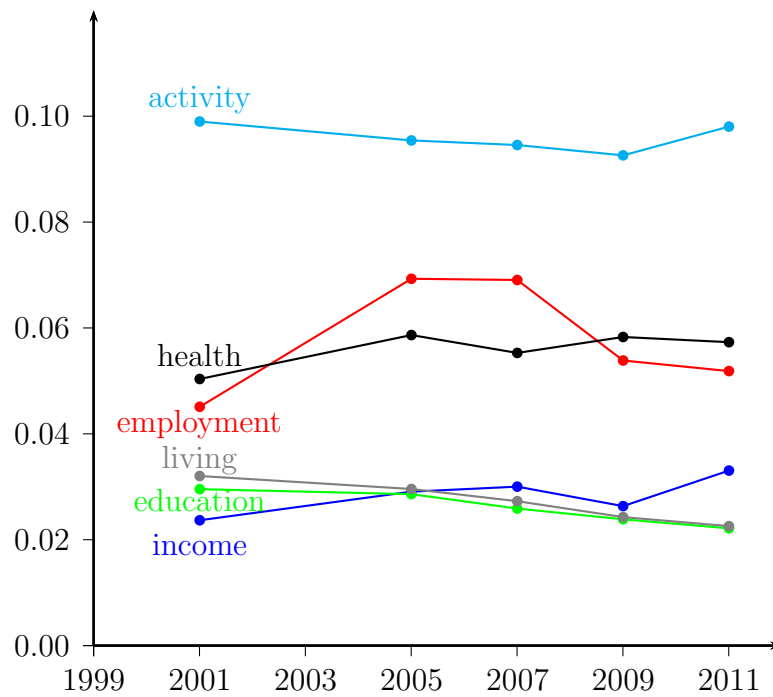


Figure 13: Univariate FGT indices for $\alpha = 1$.

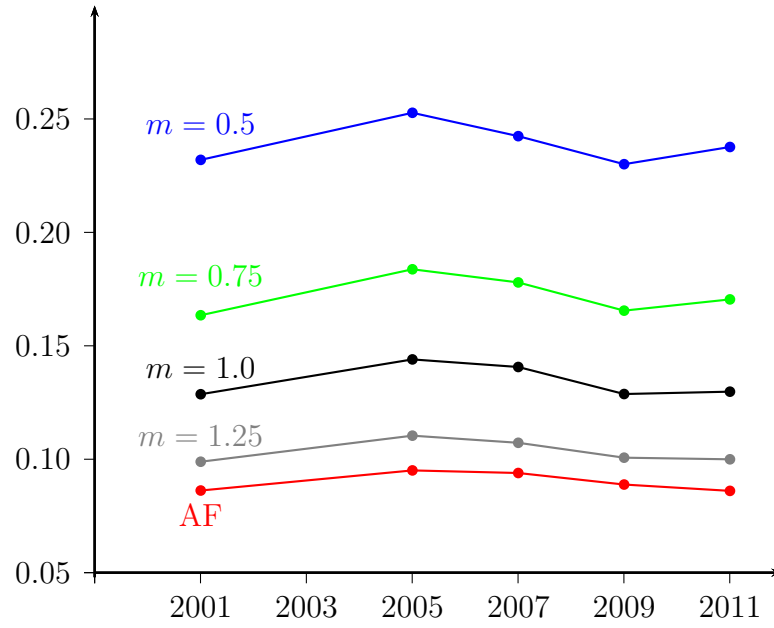


Figure 14: Proportion of the poor for the AF and the modified index with $k = 3$, $\alpha = 1$ and for several m .

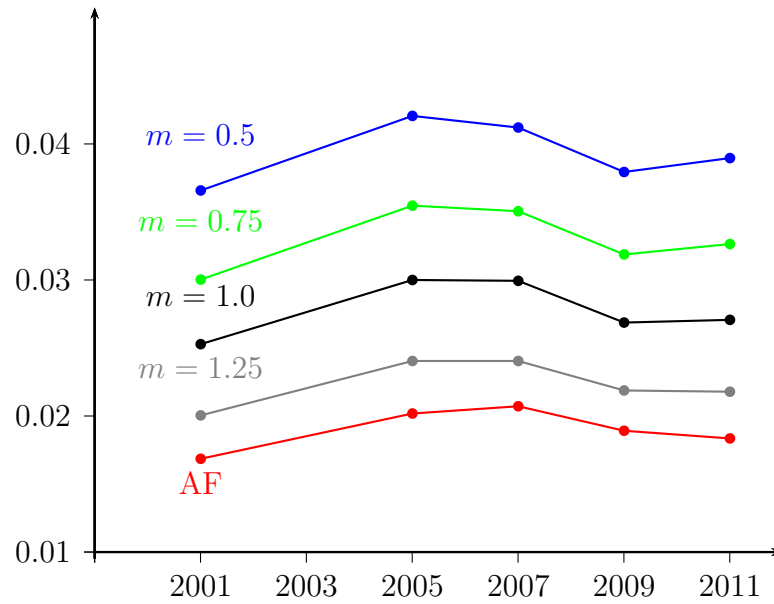


Figure 15: Values of the AF and the modified index with $k = 3$, $\alpha = 1$ and for several m .