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# Post-Malthusian Dynamics in Pre-Industrial Scandinavia\*

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## Abstract

Theories of economic growth hypothesize that the transition from pre-industrial stagnation to sustained growth is associated with a post-Malthusian phase in which technological progress raises income and spurs population growth while offsetting diminishing returns to labour. Evidence suggests that England was characterized by post-Malthusian dynamics preceding the Industrial Revolution. However, given England's special position as the forerunner of the Industrial Revolution, it is unclear if a transitory post-Malthusian period is a general phenomenon. Using data from Denmark, Norway and Sweden, this research provides evidence for the existence of a post-Malthusian phase in the transition from stagnation to growth in Scandinavia.

**Keywords** Demography, Post-Malthusian Dynamics, Malthus, Pre-Industrial Scandinavia, Demographic Transition, Economic Growth

**JEL Classification Codes** C32, N3, O1

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# 1 Introduction

Recent long-run economic growth theory has focused on the transition of societies from millennia of economic stagnation to an era of sustained economic growth (Galor, 2011). While income per capita has stagnated for the vast part of human history, the world as a whole has experienced a ten-fold increase in income per capita in the last two centuries, following the Industrial Revolution and the Demographic Transition. However, the differential timing of this transition from stagnation to growth has led to large differences in income across the globe. In light of the fact that millions of people remain to live in severe poverty, gaining insights into the mechanisms that brought some countries out of stagnation and lead them into a state of sustained economic growth remains a central topic in the field of economic growth.<sup>1</sup>

Theories building upon a Malthusian interpretation of the pre-industrial era, such as unified growth theory (e.g. Galor and Weil, 2000; Galor, 2011), hypothesize that technological advances in production spurred population growth, offsetting the positive effect on income per capita through decreasing marginal returns to labour. According to this view, an accumulation of technological knowledge led to an increase in population size in the Malthusian era while income per capita remained stagnant.

Inspired by Boserup and Kaldor (1965) and Smith (1776), unified growth theories suggest that the increasing population size accelerated the technological growth rate, giving rise to a post-Malthusian era in which population growth did not counterbalance entirely the growth in income. Eventually, the population growth and associated acceleration of technological growth in the post-Malthusian regime gave rise to the modern growth regime, where the demand for human capital – as a consequence of the acceleration in technological development – elevated investment in the human capital of children, resulting in a negative association between income per capita and population growth.

This research provides evidence that post-Malthusian dynamics were operating in Scandinavia in the later part of its pre-industrial era. The present analysis further suggests that a negative association between income and fertility eventually replaced the post-Malthusian positive association between income and fertility. These findings are consistent with the hypothesis that the evolution of societies from stagnation to growth is characterized by a transition period of post-Malthusian dynamics.

While the vast majority of existing empirical investigations of Malthusian stagnation are not directly concerned with the post-Malthusian phase and the period of transition from stagnation to growth, Møller and Sharp (2014) formulate and test a simple post-Malthusian model with persistent technology shocks.<sup>2</sup> Using annual series of real wages, marriage rates, birth rates and death rates

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<sup>1</sup>The transition from stagnation to growth has been analyzed by (Galor and Weil, 1999, 2000; Galor and Moav, 2002; Hansen and Prescott, 2002; Lucas, 2002; Lagerlöf, 2003; Doepke, 2004; Galor, 2005; O’Rourke et al., 2014; Strulik and Weisdorf, 2008; Dalgaard and Strulik, 2010) and others. The Malthusian epoch has been analyzed separately by (Clark, 2007; Weisdorf, 2008; Vollrath, 2011; Tabata, 2013) and others.

<sup>2</sup>Time series-based analyses, most of which focus on England, include Bailey and Chambers (1993); Lee and Anderson (2002); Nicolini (2007); Crafts and Mills (2009); Rathke and Sarferaz (2010); Klemp (2012). Ashraf and Galor (2011) establish evidence in the cross-country context, for which the Malthusian theory predicts that technologically more advanced countries should be more populated but no richer.

for pre-industrial England, their empirical analysis find evidence consistent with post-Malthusian dynamics as early as the mid-sixteenth century.

However, in light of England's special position as the forerunner of the Industrial Revolution, the extent to which the existence of a post-Malthusian period can be considered a general phenomenon is still unknown. In particular, the occurrence of the Industrial Revolution in England may have instigated a fundamental change in the process of development in economies that took off later. Furthermore, in the period leading up to the Industrial Revolution, some characteristics of the English economy were markedly different from those of other European economies. In particular, England had a relatively high level of income per capita. For these reasons, the present research employs, for the first time, a set of high-quality economic and demographic historical data to investigate the existence of a post-Malthusian period in the Scandinavian countries.

The research adopts the method developed in Møller and Sharp (2014) to investigate the presence of a post-Malthusian period in the Scandinavian countries: Denmark, Norway and Sweden.<sup>3</sup> There are two main advantages to studying these countries, compared to existing analyses of Malthusian dynamics in other countries. Firstly, in light of the fact that official national statistics of births and deaths were kept from the 1730s, the quality of the demographic birth and death rates from Denmark, Norway and Sweden is known to have a high accuracy Gille (1949). In contrast to data sources for other countries, which are often based on subsets of the national population, such as individual towns or samples of parishes (e.g. Wrigley and Schofield, 1989), the Scandinavian data cover the entire Scandinavian populations and are therefore much more reliable. Secondly, these countries, often referred to as part of the Poor Periphery, were poorer, experienced lower economic growth rates, and industrialized later than England and many Central European economies. In particular, the Scandinavian countries industrialized and experienced demographic transitions during the late nineteenth century.<sup>4</sup> Therefore, the Scandinavian countries might be expected to exhibit dynamics that more closely resembles Malthusian dynamics than post-Malthusian dynamics.<sup>5</sup>

Table 1 gives a broad indication of how the Scandinavian countries compared to England in terms of economic development. The table reports that the Norwegian and Swedish levels of GDP per capita in 1700 were the same as for the United Kingdom in the 1500s for which the analysis in Møller and Sharp (2014), using English data, begins. Likewise, the income per capita of Denmark in 1820 was at the level of that in the United Kingdom in 1700. As in many European countries, factory production, modeled on that in the United Kingdom, as well as railroads arrived in Scandinavia in the mid-1800s. However, the onsets of economic take-offs occurred later in Scandinavia than in the United Kingdom. Denmark took off around the 1880s, largely through highly mechanized

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<sup>3</sup>The analysis in Møller and Sharp (2014) is the first formal time-series based econometric framework designed to describe the transition period and potential post-Malthusian dynamics. However, other empirical studies that focus on the strict Malthusian model have found results that are compatible with the existence of a non-Malthusian economy before the Industrial Revolution (Nicolini, 2007; Pfister and Fertig, 2010; Klemp, 2012, see e.g.).

<sup>4</sup>While unified growth theory is compatible with differential timings of demographic transitions in relation to transitions from economic stagnation to growth, a demographic transition tends to follow or coincide with an economic transition. Thus, the Scandinavian experience appears, on this account, consistent with common varieties of unified growth theory.

<sup>5</sup>It should also be noted that due to the more recent industrialization of the Scandinavian countries, compared to England, data on income per capita from the pre-industrial era could in general be expected to be of a higher quality.

Table 1: GDP per capita for the United Kingdom and the Scandinavian Countries, 1500–1900

Year	1500	1600	1700	1820	1850	1870	1900
Denmark	738	875	1039	1274	1767	2003	3017
Norway	610	665	722	801	956	1360	1877
Sweden	651	700	750	819	1019	1359	2209
United Kingdom	714	974	1250	1706	2330	3190	4492

food production and export to the United Kingdom, and Norway and Sweden (which at that time constituted a union) took off soon afterwards through a more typical expansion of the manufacturing sector. However, the economic state of all three Scandinavian economies resembled that of the United Kingdom prior to their respective transitions into industrial production. Economically, they were predominantly agricultural, based largely on small owner occupied farms (i.e., yeoman agriculture); and demographically, they were typified by the “European Marriage Pattern” Hajnal (1965), whereby marriage only took place when income was sufficient to support the family in a new household, thus suggesting the positive relationship between income and fertility postulated by Malthus.<sup>6</sup>

Thus, in light of the fact that the Industrial Revolution started in England, the poorer Scandinavia may be expected to have exhibited more Malthusian characteristics in the period leading up to its industrialization, in comparison to England. Hence, it should be of great interest to test the post-Malthusian hypothesis.

The present results clearly suggest that the Scandinavian countries did undergo a post-Malthusian period prior to industrialization. Indeed, the results of the present research are strikingly similar to the findings for England presented in Møller and Sharp (2014): for all three Scandinavian countries the research finds evidence of a non-stagnating income per capita positively affecting population size and an absence of a feedback effect from population size to income per capita. Furthermore, the present analysis is markedly unambiguous in comparison to the existing evidence from England, which – in light of the superior quality of the Scandinavian data – further strengthen the reliability of the analysis and the validity of the findings. In particular, for all three countries, we find evidence of a preventive check (i.e., a positive effect of income per capita on the birth rate), and furthermore, for Denmark and Sweden the positive check (i.e., a negative effect of income per capita on the mortality rate) is supported.

The next section describes the theory, Section 3 derives the relevant statistical model, Section 4 describes the data, Section 5 presents the estimation results, and Section 6 concludes.

## 2 Empirical Framework

Unified growth theory hypothesizes that the transition of societies from an era of stagnation to a period of sustained economic growth is associated with a post-Malthusian phase in which scale effects of population growth prompt a rising rate of technological development that offsets the

<sup>6</sup>Møller and Sharp (2014) find evidence in favor of this in England.

adverse Malthusian effects of population size on income per capita (via diminishing returns), while there is a positive feedback effect from income per capita on population growth.<sup>7</sup> Thus, in contrast to a pure Malthusian phase where technological advances induces a growing population size while income remains stagnating, in the post-Malthusian phase, technological advances induce both a growing population and a higher income per capita.

Based on Møller and Sharp (2014), this section presents a simple post-Malthusian economic model, and shows that it has a direct correspondence with a set of parameter restrictions on a flexible class of time-series econometric models, the co-integrated vector autoregressive model.<sup>8</sup> Hence, the post-Malthusian hypothesis can be parsimoniously assessed using a joint test of a set of parameter restrictions on the flexible econometric model. Furthermore, the econometric model provides estimates of the central parameters of the post-Malthusian model.

## 2.1 The General Malthusian Model and the post-Malthusian Hypothesis

This section will outline a simple general Malthusian-based model that includes the standard Malthusian model as well as the post-Malthusian model as special cases. In the following section, the model is reformulated as a dynamic econometric model, and a test of the post-Malthusian hypothesis, i.e. of the absence of an effect of population size on income per capita, is derived.<sup>9</sup>

Consider a closed economy with an agricultural sector and endogenous population size. In every period, the economy produces a single homogeneous good using land and labour as inputs. The supply of land is exogenous and fixed over time whereas the evolution of labour supply is governed by households' fertility decisions and the death of individual's in the preceding periods. Income per capita is given by the real wage, which is determined on the aggregate labour market.

Under the assumptions of a standard a Cobb-Douglas constant-returns-to-scale production function, inelastic labour supply and labour market clearing, the natural logarithm of the real wage,  $w_t$ , is given by

$$w_t = c_0 - c_1 \ln N_t + \ln A_t \quad (1)$$

where (here and below) the coefficients are positive (unless otherwise stated),  $N_t$  is the total population size, and  $A_t$  is technology, or the level of labour demand, which determines the efficacy of the production process. In general,  $A_t$  may be thought of as comprising all determinants of labour demand at given real wages, such as capital, technology and knowledge.

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<sup>7</sup>For scale effects, see (Boserup and Kaldor, 1965; Kremer, 1993).

<sup>8</sup>Møller and Sharp (2014) demonstrate that, for the period from the mid-1500s to around 1800, English time-series data is indeed consistent with the post-Malthusian hypothesis. Thus, their analysis finds evidence of positive scale effects of population size offsetting the negative feedback from a larger population while growth in income per capita induce births. In essence, their results suggest that when real wages grow in the wake of technological improvements, population will grow (primarily through more births resulting from more marriages), but this will not only feedback negatively on real wages (cf. the diminishing marginal returns), but also positively by increasing the overall technological level, with the result that the net-feedback effect may be small, and so, non-stagnation (non-stationarity) emerges.

<sup>9</sup>For a full account of the model and the econometric details, the reader is referred to Møller and Sharp (2014).

Equation (1), therefore, describes how population affects living standards. Wages are suppressed by population growth due to diminishing returns (i.e.,  $c_1 > 0$ ), but increase with the technological sophistication.

The following two equations in turn describe the feedback effects from living standards to the demographic rates. In particular, equation (2) describes the so-called Malthusian “preventive check” whereby fertility is positively affected by real wages,

$$b_t = a_0 - a_1 w_t + \varepsilon_{bt}, \quad (2)$$

where  $b_t$  is the crude birth rate and  $\varepsilon_{bt}$  is a shock to the birth rate, as explained below. Furthermore, equation (3) describes the so-called Malthusian “positive check”, whereby mortality is negatively affected by real wages,

$$d_t = a_2 - a_3 w_t + \varepsilon_{dt}, \quad (3)$$

where  $d_t$  is the crude death rate and  $\varepsilon_{dt}$  is a shock to the death rate, as explained below.<sup>10</sup> Several explanations of the preventive check, ranging from rational economic behavior to purely biological circumstances, have been proposed in the literature on demographics and Malthusian dynamics (see e.g. the introduction in Lee (1997)). The positive check has a purely biological underpinning and is based on the adverse effects of lower income per capita on nutrition, infant mortality, and more (see e.g. Lee, 1997; Schultz, 1981).

Under an assumption of no international migration, equation (4) gives the log-approximation of the population growth rate, as determined by the difference between the birth rate and the mortality rate,

$$\ln N_t = \ln N_{t-1} + b_{t-1} - d_{t-1} \quad (4)$$

While relations (1)-(4) are standard in models of Malthusian dynamics, the present model includes a positive effect of population size on technological growth. Furthermore, the model allows for *persistent* shocks to basic technology, and thus to technology as a whole, meaning that changes to the stock of knowledge are permanent. Thus, the basic technological level is assumed to follow a random walk, capturing the accumulative nature of knowledge. Equations (5) and (6) model the positive effect of population size on technological growth and the persistent characteristic of the shocks to basic technology, respectively,

$$\ln A_t = c_3 \ln N_t + \ln X_t \quad (5)$$

$$\ln X_t = \ln X_{t-1} + \varepsilon_{Xt} \quad (6)$$

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<sup>10</sup>The terms “preventive check” and “positive check” can be understood in the context of a decrease in income. When income is reduced, and simple adaptive expectations of future income are lowered, individuals will tend to delay their marriage and the onset of having children. In this way population is intentionally kept down (“checked”), preventing an otherwise larger population from being positively checked by mortality in the future.

where  $0 \leq c_3 \leq c_1$  and  $X_t$  is exogenous technological knowledge. The parameter  $c_3$  accounts for the effect of population size on technology. Thus, as in accordance with unified growth theory, inspired by Boserup and Kaldor (1965) and Smith (1776),  $c_3$  may be large enough to offset the negative effect of population on income,  $c_1$ , that is due to decreasing returns to scale.

The exogenous shocks,  $\varepsilon_{bt}$ ,  $\varepsilon_{dt}$  and  $\varepsilon_{Xt}$ , reflect unsystematically fluctuating influences on births, deaths and knowledge respectively. In the econometric analysis these variables are treated as independent and identically distributed Gaussian random variables.

The case in which there are no scale effects ( $c_3 = 0$ ) corresponds to a standard model of Malthusian Stagnation. In contrast, the post-Malthusian hypothesis posits that the positive scale effect, i.e.  $c_3$ , counteracts the diminishing returns to labour, determined by  $c_1$ . Thus, the post-Malthusian hypothesis is that the general Malthusian model is operating and that furthermore  $c_3 = c_1$ . In light of the empirical application, this equality should be viewed as an approximation (see Møller and Sharp, 2014).

### 3 Time-Series Implications of the Model

The above Malthusian model has implications for the time series of the observed demographic rates and the real wage. Thus, solving equation (1)-(6) with respect to the vector  $x_t = (w_t, b_t, d_t)'$  results in a set of restrictions on a Vector Auto-Regressive (VAR) model with  $k$  lags (see Lütkepohl, 2005) written on the form,

$$\Delta x_t = \Pi x_{t-1} + \Gamma_1 \Delta x_{t-1} + \dots + \Gamma_{k-1} \Delta x_{t-(k-1)} + \mu + \varepsilon_t \quad (7)$$

The general Malthusian model implies a set of restrictions on the long-run matrix,  $\Pi$ . In particular, the model implies the following  $\Pi$ -matrix,

$$\Pi = \begin{pmatrix} 0 & -\bar{c}_1 & \bar{c}_1 \\ a_1 & -(1 + a_1 \bar{c}_1) & a_1 \bar{c}_1 \\ -a_3 & a_3 \bar{c}_1 & -(1 + a_3 \bar{c}_1) \end{pmatrix}, \quad (8)$$

where  $\bar{c}_1 \equiv c_1 - c_3$ . These restrictions on the  $\Pi$ -matrix therefore correspond to the general Malthusian model based on the birth and death rates and the real wage. The post-Malthusian hypothesis is that the positive effect of population growth on labour demand, given by the scale effect parameter,  $c_3$ , offsets the negative effects of population growth on income per capita, given by the decreasing-returns-to-scale-effect parameter,  $c_1$ . Thus, imposing the additional restriction  $\bar{c}_1 = 0$  yields a test of the post-Malthusian hypothesis. When this restriction is imposed in (8), the matrix  $\Pi$  has a reduced rank of 2, implying that  $x_t$  is non-stationarity and co-integrated (Johansen, 1996). This means that there is no steady state of income per capita, and hence, for the demographic rates, i.e. a state towards which these variables would converge in the absence of any disturbances (shocks). However, the restrictions also imply that, although these variables drift in this non-stationary manner, they do drift together, that is, they co-integrate. This therefore describes a post-Malthusian regime in which income continues to spur population growth (as in the pure



Malthusian phase) but is no longer stagnant (stationary). The reduced rank restriction can be parameterized as,  $\Pi = \alpha\beta'$ , where  $\alpha$  and  $\beta$  are both  $p \times r$  matrices with rank,  $r$ , (i.e., the “co-integration rank”), here equal to 2. In particular, the post-Malthusian hypothesis therefore implies that,

$$\alpha = \begin{pmatrix} 0 & 0 \\ -1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \beta' = \begin{pmatrix} -a_1 & 1 & 0 \\ a_3 & 0 & 1 \end{pmatrix}, \quad (9)$$

The elements of the  $\alpha$  matrix are known in the literature of co-integrated VAR-models as “adjustment coefficients”, and they reflect how closely income and the demographic rates drift together.<sup>11</sup> In the empirical implementation, the adjustment coefficients remain free parameters, reflecting the fact that adjustment to equilibrium will realistically require more than one period (see Møller and Sharp, 2014). Thus, in the empirical implementation, the  $\alpha$  matrix is given by

$$\alpha = \begin{pmatrix} 0 & 0 \\ -\rho_1 & 0 \\ 0 & -\rho_2 \end{pmatrix}. \quad (10)$$

## 4 Data

The Malthusian model is estimated using yearly observations of real wages, crude birth rates (i.e., the number of births per thousand inhabitants) and crude death rates (i.e., the number of deaths per thousand inhabitants) for Denmark, Norway and Sweden. The crude demographic rates for Denmark are taken from (Mitchell, 1998, Table A6), for Norway from the official Norwegian statistics bureau, Statistics Norway,<sup>12</sup> and for Sweden from Lund University Macroeconomic and Demographic Database.<sup>13</sup> The time series of the Danish real wages consist of wages of urban workers given by (Hansen, 1984, Table 20) deflated by the consumer price index given by (Mitchell, 1998, Table H2). For Norway wages in the industrial sector, given by Grytten (2009), are used, and for Sweden the series of day rates for male agricultural workers, given by (Jörberg, 1972, pp. 710–714), are used and deflated by the historical consumer price index maintained by the Swedish central bank, The Riksbank.<sup>14</sup>

The time series for the regression samples are plotted in Figure 1–3. The figure indicates the existence of linear deterministic trends in some of the series. Furthermore, the co-integrating relations between wages and the vital rates, may be affected by linear time trends. Thus, the empirical specification will include linear time trends, if statistically significant, in the variables and in the co-integrating relations to account for these secular effects.

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<sup>11</sup>For a detailed interpretation of these coefficients see Møller and Sharp (2014).

<sup>12</sup>The data can be found at [www.ssb.no/histstat/tabeller/3-13.html](http://www.ssb.no/histstat/tabeller/3-13.html).

<sup>13</sup>The data can be found at [www.ekh.lu.se/en/research/shna1560-2010/lu-madd/population](http://www.ekh.lu.se/en/research/shna1560-2010/lu-madd/population).

<sup>14</sup>The consumer price index is CPI 1. The data can be found at [www.riksbank.se/templates/Page.aspx?id=26813](http://www.riksbank.se/templates/Page.aspx?id=26813).

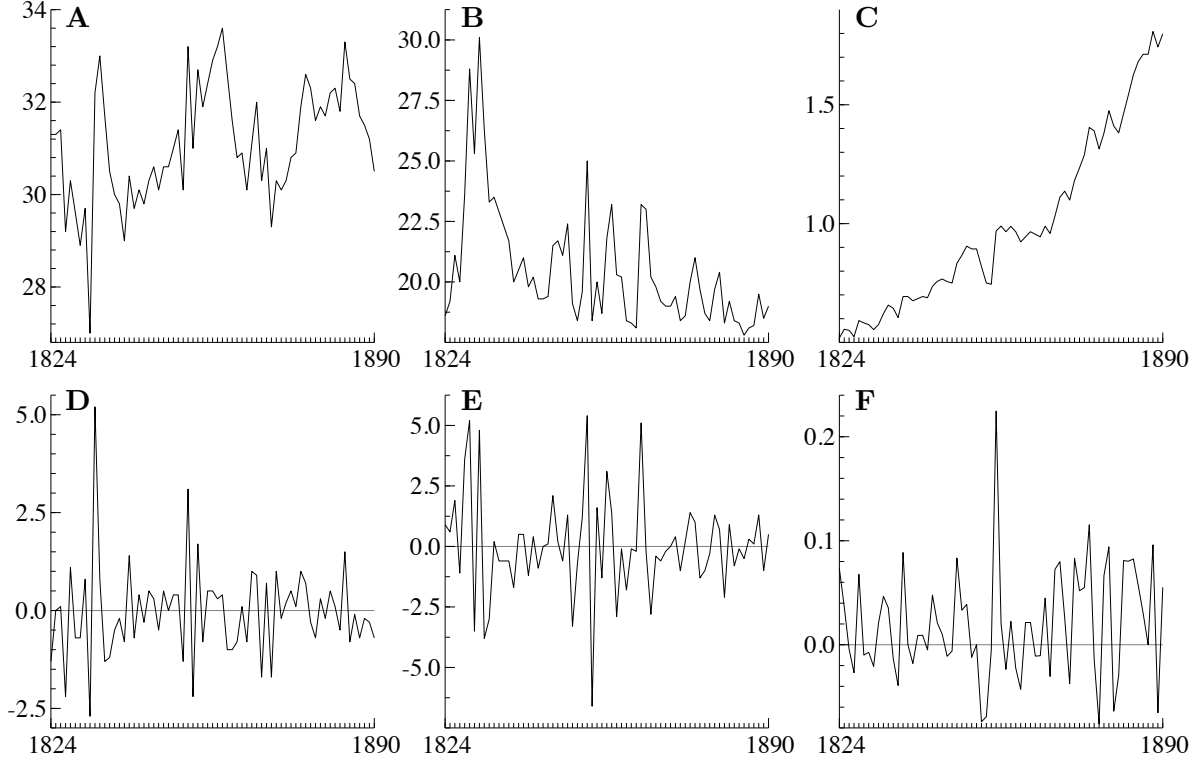


Figure 1: Plots of the Danish data in levels and first differences. A) Level of the crude birth rate. B) Level of the crude death rate. C) Level of the wage rate. D) First difference of the crude birth rate. E) First difference of the crude death rate. F) First difference of the wage rate.

## 5 Analysis

The analysis is performed in two steps. The first step consists of the setup of statistically well-specified VAR models for economically meaningful time periods that will function as the underlying general statistical models against which the post-Malthusian hypothesis will be tested. The second step consists of testing the post-Malthusian hypothesis and furthermore the determination of the point in time in which the post-Malthusian dynamics are rejected.

### 5.1 Unrestricted VAR Models

Two factors determine the appropriate sample periods to analyze the post-Malthusian hypothesis. First, in accordance with the fact that the Scandinavian countries has experienced demographic transitions, unified growth theory predicts that the post-Malthusian dynamics are followed by a reversal in the association between income and fertility. Therefore, to prevent an erroneous rejection of the post-Malthusian model based on the inappropriate inclusion of a part of the modern period, the end year of the analyzed samples are initially determined to be one decade before before the fertility transition (as given in Reher, 2004). However, the analysis can also uncover the point in time in which post-Malthusian dynamics are no longer describing the data. To this end, the regression analysis is performed recursively for various sample lengths, holding the initial year fixed,

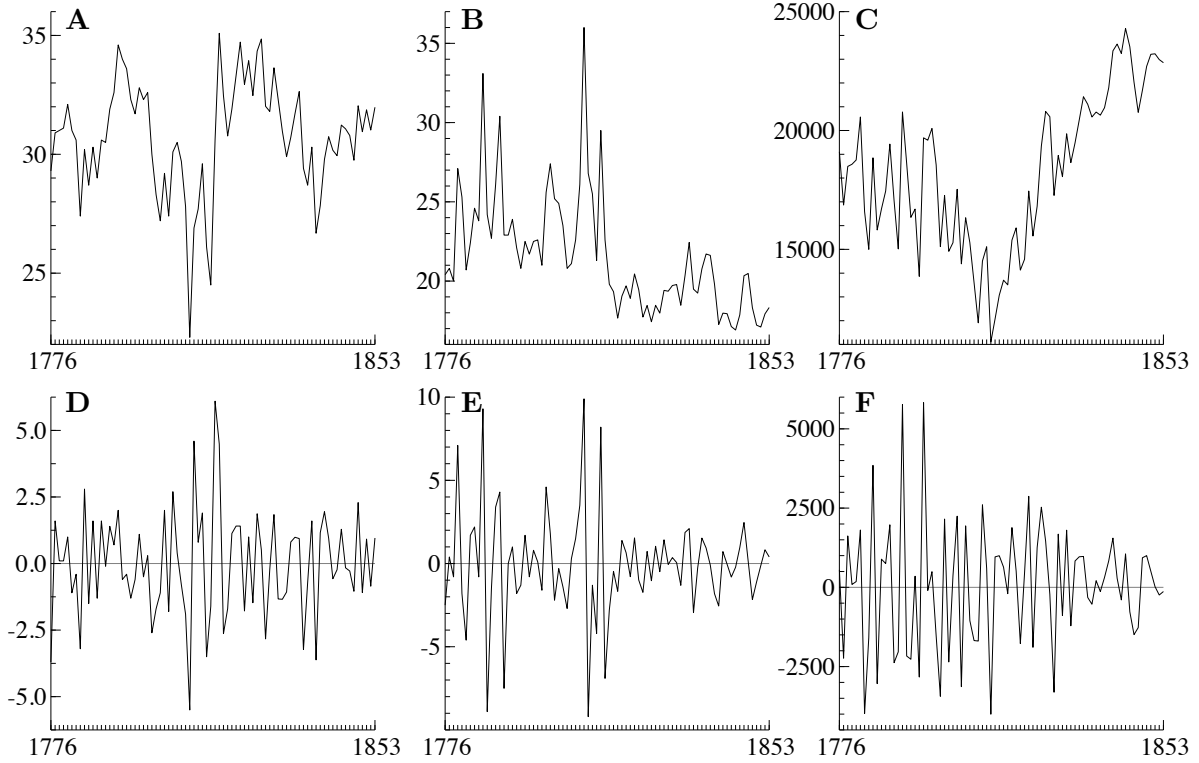


Figure 2: Plots of the Norwegian data in levels and first differences. A) Level of the crude birth rate. B) Level of the crude death rate. C) Level of the wage rate. D) First difference of the crude birth rate. E) First difference of the crude death rate. F) First difference of the wage rate.

thus indicating the constancy of the estimated parameters as well as the significance of the test of the post-Malthusian hypothesis.

Second, in order to maintain valid statistical inference when testing the post-Malthusian hypothesis, it is crucial to first formulate a well-specified unrestricted VAR model, i.e. a statistical model, that approximately satisfy the statistical assumptions, that the errors are indeed unsystematic, in the sense of not exhibiting autocorrelation, non-normality or autoregressive conditional heteroscedasticity (ARCH). However, one may add that in practice, non-normality due to excess kurtosis of the errors is less problematic than non-normality due to excess skewness (Juselius, 2006). Moreover, moderate ARCH effects have also been shown to have little impact on the determination of the co-integrating rank,  $r$  (Rahbek et al., 2002). Thus, the present analysis is primarily concerned with the assumption of no autocorrelation in the errors since this is clearly the most important one. Below, residual-based mis-specification tests are used to assess these important aspects of the statistical model specification.

In practice, obtaining a well-specified model requires taking account of events that may obscure the estimation of the post-Malthusian dynamics, for example by excluding extraordinary time periods. In the present cases, although the full sample periods start in year 1749 for Norway and Sweden, and year 1818 for Denmark, initial sub-periods are omitted due to two historical circumstances that prevent the specification of statistically satisfactory unrestricted VAR models.

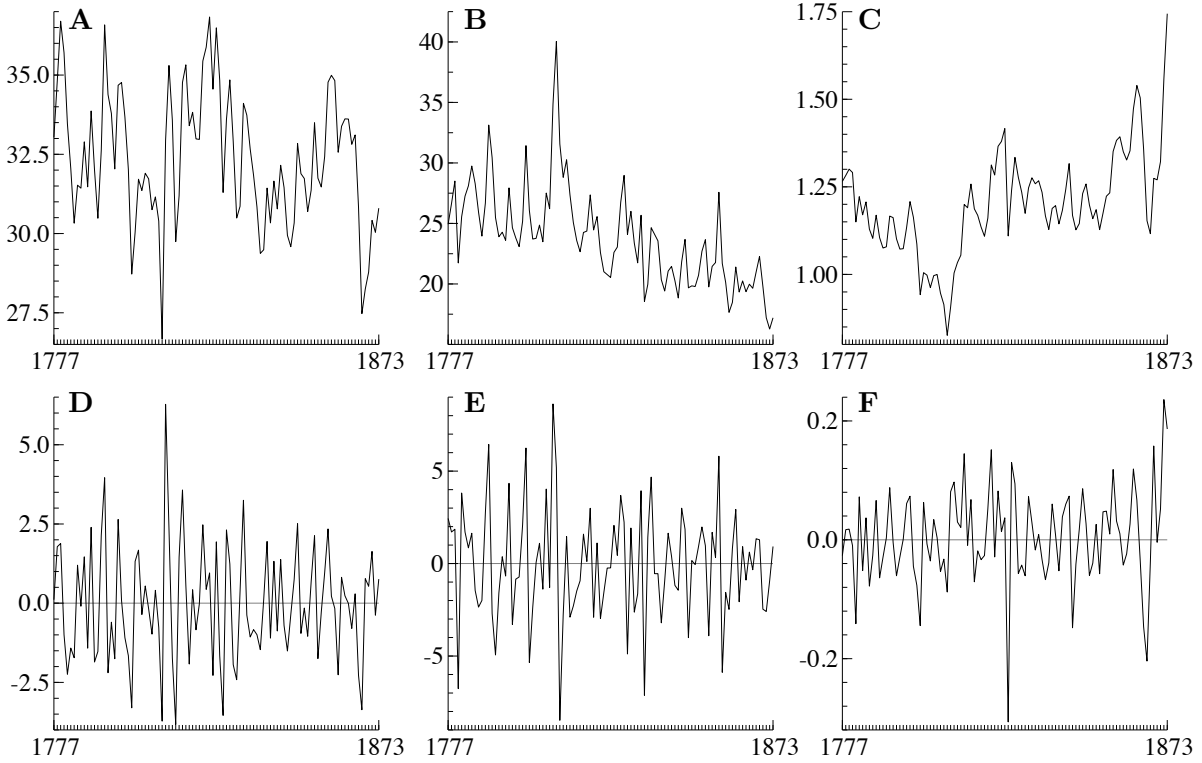


Figure 3: Plots of the Swedish data in levels and first differences. A) Level of the crude birth rate. B) Level of the crude death rate. C) Level of the wage rate. D) First difference of the crude birth rate. E) First difference of the crude death rate. F) First difference of the wage rate.

In particular, in the case of Denmark, the initial three-year period 1818–1820 is omitted due to the prominent wage inflation following the bankruptcy of the Danish state in 1813. Furthermore, in the case of Norway and Sweden, the initial period 1749–1775 is omitted due to the severe harvest failures and the associated disease and famine in 1772 and 1773 (Drake, 1969, p. 68).<sup>15</sup>

Following standard procedure in applied time-series econometrics (see e.g. Juselius, 2006), the analysis account for extraordinary events by including dummy variables for the unusually large residuals. This approach will work satisfactory if these residuals result from "large" exogenous events that are unrelated to the theory.<sup>16</sup>

By confronting non-sample information, i.e. historical references it is often relatively clear to identify what exogenous event took place. Interestingly, the periods under investigation prompt the use of very few dummy variables in order to obtain a statistically satisfactory specification. Furthermore, in all cases the cause corresponding can be unambiguously attributed to exogenous events. Specifically, in the case of Denmark, the model includes a dummy variable accounting for the effects of the Second Schleswig War in 1864, in which Denmark lost a large proportion

<sup>15</sup>We estimated separate models for the period 1749 to 1772 in Norway and Sweden and obtained results consistent with those from the later periods. However, in light of this short sample period and the associated low statistical power, these results are omitted.

<sup>16</sup>In general, the inclusion of dummy variables, modeled as deterministic components, improves the validity of the statistical inference when historical event that it is supposed to proxy can be regarded as exogenous.

Table 2: Results of trace tests of co-integration for rank  $r = 2$ 

	Denmark		Norway		Sweden	
	No	Yes	No	Yes	No	Yes
Deterministic components						
<i>P</i> -value	0.67	0.01	0.32	0.01	0.32	0.15

of its land and population. In the case of Norway, a dummy variable proxies the effects of the blockade of Norway by England in 1813 – during the Napoleonic wars – and the consequent famine. Furthermore, the model also includes a level shift dummy accounting for the shift in fertility in the years 1830–1854, most likely due to a marked change in the age composition over this time period as described by (Drake, 1969, p. 45). In the case of Sweden, the model includes dummy variables accounting for the effects of the Russo-Swedish War of 1808–9, and the smallpox outbreak of 1825.

The unrestricted VAR models for the three Scandinavian countries are well-specified according to the results of a range of residual-based tests for no autocorrelation, normality and homoscedasticity, presented in Table A.1–A.3.<sup>17</sup> Importantly, autocorrelation in the errors is convincingly rejected in the presence of deterministic components. In particular, the test for no autocorrelation is not rejected at the 5% significance level in 14 out of the 15 performed tests. Furthermore, the tests for homoscedasticity are not rejected in the vast majority of cases for Norway and Sweden, while it is rejected in the Danish case. The univariate tests of normality are non-rejected in most cases and in the few cases of indications of non-normality, it can be attributed to excess kurtosis. Overall, in light of the long time periods, the results of these misspecification tests indicate that the unrestricted VAR models are remarkably well-specified.

As established in Section 2.1, a necessary implication of the post-Malthusian hypothesis is that  $\Pi$ -matrix of the VAR model has a rank of  $r = 2$ , implying that the systems of variables are non-stationary. For all three countries the analysis finds evidence in favor of non-stationarity in the data and a rank of the  $\Pi$ -matrix of  $r = 2$ , consistent with this. Table 2 presents the  $p$ -value for the trace test of non-stationarity ( $r < 3$ ) against stationarity ( $r = 3$ ) both with and – as a robustness check – without deterministic components.<sup>18</sup>

Overall, the unrestricted VAR models for each of the three countries are well-specified and the data appear non-stationary. Therefore, the analysis can proceed to test the restrictions associated with the Post-Malthusian model on the VAR models.

Table 3: Estimation results

	Denmark	Norway	Sweden
Preventive check ( $a_1$ )	3.900* (1.133)	9.097* (2.184)	7.656* (2.982)
Positive check ( $a_3$ )	5.651* (2.514)	1.831 (1.530)	9.567* (3.553)
First adjustment coefficient ( $\rho_1$ )	.527* (.125)	.665* (.092)	.528* (.097)
Second adjustment coefficient ( $\rho_2$ )	.429* (.126)	0.769* (.067)	.722* (.103)
Number of years	68	79	98
$P$ -value of post-Malthusian test	0.86	0.30	0.06

\* Significant at the 5% significance level.

## 5.2 Results

### 5.2.1 Denmark

The analysis for Denmark is performed using three lags on the effective sample 1824–1890. The analysis includes linear trends that were found to be insignificant in the co-integrating relations, whereby they are restricted not to be in the co-integrating relations. The results of the regression under the restrictions given by in equations (9) and (10) are shown in Table 3, column 1.

The joint test of the restrictions corresponding to the post-Malthusian model in Denmark do not reject the theory ( $p=0.86$ ), meaning that the restricted statistical model describes the data as well as the highly flexible unrestricted VAR model. Thus, the analysis provides strong evidence in favor of post-Malthusian dynamics in Denmark operating in the time period 1824–1890. The estimates of the coefficients associated with the preventive and the positive check are both highly significant with the expected sign.

Figure 4 depicts the results of a series of regressions with the same basic structure on samples stopping in different years. The parameters seem remarkably stable over time, the model is accepted with a high  $p$ -value for all samples, and the restrictions are accepted for samples up until the First World War. The estimate of the preventive check parameter becomes insignificant for periods stopping after the turn of the 19th century. Under the inclusion of data after this point in time, the estimate flips sign, consistent with the notion of a negative association between income and fertility due to the existence and increasing influence of a child quantity-quality trade-off in the process of industrialization Galor (2011).

<sup>17</sup>All estimation is based on PcGive, OxMetrics 6.10 and CATS in RATS (see respectively Doornik and Hendry, 2001; Doornik, 2009; Dennis et al., 2006)).

<sup>18</sup>The test distribution has been simulated using CATS in RATS to take account of deterministic components (structural breaks and shifts in the time series). It should be noted that the inevitable measurement errors in the data bias the test towards rejection. While test was accepted more unambiguously for shorter samples, the results of tests based on longer samples are presented here to give a better indication of the stability of the post-Malthusian regime over longer time periods, until it eventually breaks down.

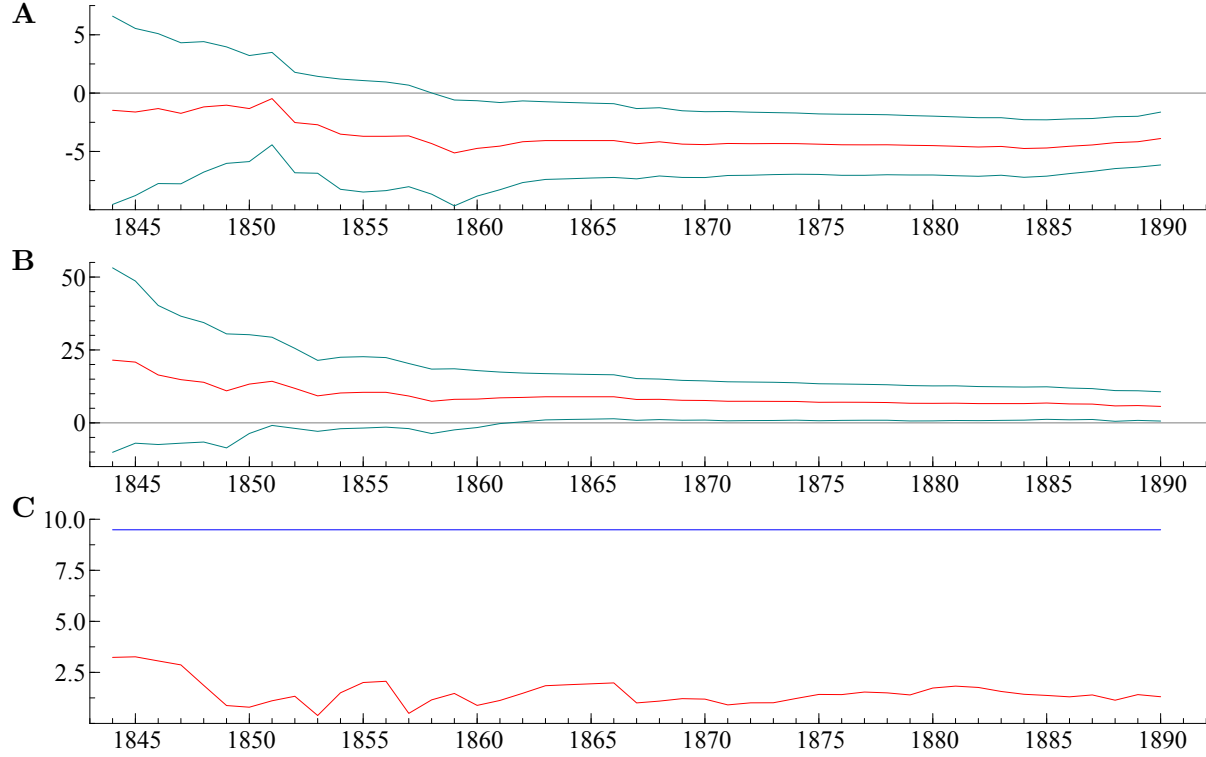


Figure 4: Results of recursive estimations for the Danish data. The figure depicts the results of a series of regression analyses for various sample lengths, holding the initial year fixed. A) The point estimate and confidence interval of the positive check. B) The point estimate and limits of the 95% confidence interval of the preventive check. C) The test statistic of the test of the post-Malthusian hypothesis, where values above the blue line indicates a rejection of the hypothesis at the 5% significance level.

### 5.2.2 Norway

The analysis for Norway is performed using three lags on the effective sample 1776–1853. The results of the regression under the restrictions given by in equations (9) and (10) are shown in Table 3, column 2. The analysis includes linear trends in the variable and in the co-integrating relations. The joint test of the restrictions corresponding to the post-Malthusian model in Norway does not reject the theory ( $p = 0.30$ ). Thus, the analysis provides evidence in favor of post-Malthusian dynamics in Norway over the time period 1776–1853. As in the case of Denmark, the regression provides clear evidence of the existence of the preventive check. Although statistically insignificant, the coefficient capturing the positive check has the expected sign.

Figure 5 depicts the results of a series of regressions with the same basic structure on samples stopping in different years. In the present case, the parameters of interest seem very stable, and the model can be accepted throughout the period. In fact, the p-value is above 5% for periods stopping as late as 1920. However, for samples including the few years leading up to that point in time, the post-Malthusian model gives a poor fit to the data, and the estimate of the preventive check parameter becomes insignificant for periods stopping after 1910. Under the inclusion of data after

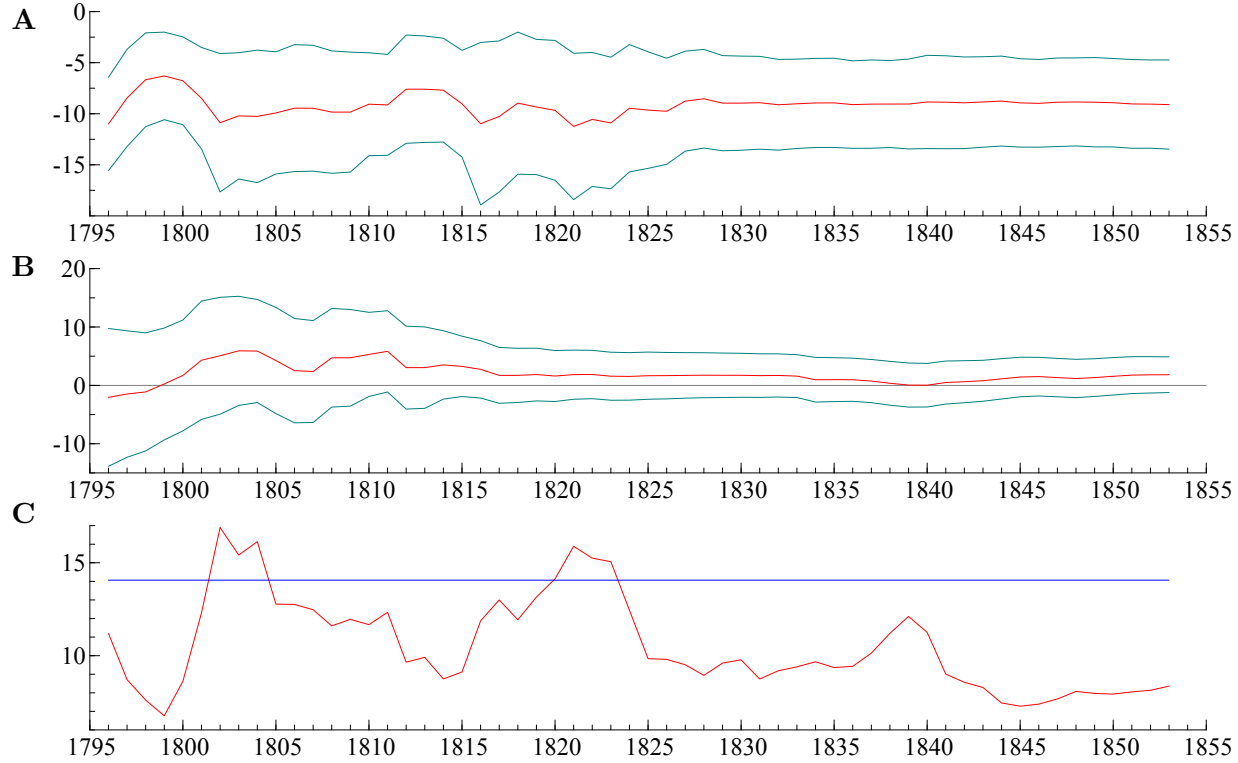


Figure 5: Results of recursive estimations for the Norwegian data. The figure depicts the results of a series of regression analyses for various sample lengths, holding the initial year fixed. A) The point estimate and limits of the 95% confidence interval of the positive check. B) The point estimate and confidence interval of the preventive check. C) The test statistic of the test of the post-Malthusian hypothesis, where values above the blue line indicates a rejection of the hypothesis at the 5% significance level.

this point in time, the estimate flips sign, consistent with the notion of an increasingly influential child quantity-quality trade-off.

### 5.2.3 Sweden

The analysis for Sweden is performed using three lags on the effective sample 1777–1873. The analysis includes linear trends in the variable and in the co-integrating relations. The results of the regression under the restrictions given by in equations (9) and (10) are shown in Table 3, col. 2.

The joint test of the restrictions corresponding to the post-Malthusian model in Sweden do not reject the theory ( $p = 0.06$ ). In fact, as figure 4 shows, the  $p$ -value is much higher for earlier samples (for example, it is 0.29 for the period until 1870), but extending the data until 1873 gives the largest sample for which it is above 5 percent. Thus, the analysis provides evidence in favor of post-Malthusian dynamics in Norway over the time period 1776–1853. As in the case of Denmark, the estimates of the coefficients associated with the preventive and the positive check are both highly significant with the expected sign.



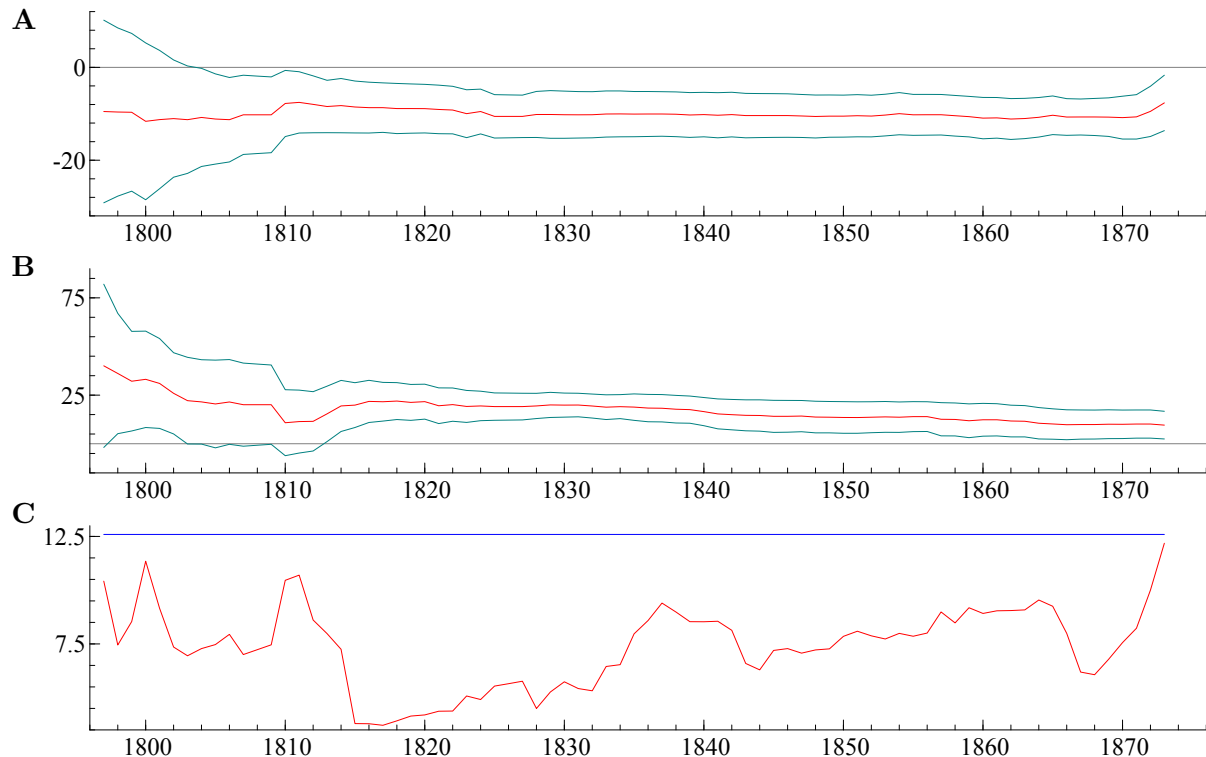


Figure 6: Results of recursive estimations for the Swedish data. The figure depicts the results of a series of regression analyses for various sample lengths, holding the initial year fixed. A) The point estimate and confidence interval of the positive check. B) The point estimate and limits of the 95% confidence interval of the preventive check. C) The test statistic of the test of the post-Malthusian hypothesis, where values above the blue line indicates a rejection of the hypothesis at the 5% significance level.

Figure 6 depicts the results of a series of regressions with the same basic structure on samples stopping in different years. In the present case, the parameters of interest seem very stable, and the model can be accepted throughout the period. Extending the sample beyond 1890, the estimate flips sign, consistent with the notion of an increasingly influential child quantity-quality trade-off.

Table A.4 in the appendix presents the estimates of the preventive and positive checks as elasticities and compare them to existing estimates from other studies. The present estimates are of the same order of magnitude, but typically large, compared to those found in other studies.

Overall, the results for all three Scandinavian countries support the post-Malthusian model in which technological progress raises income and spurs population growth while offsetting diminishing returns to labour.

## 6 Conclusion

Theories of economic growth hypothesize that the transition from stagnation to sustained growth is associated with a post-Malthusian phase in which income increases and spurs population growth

while scale effects offset diminishing returns to labour. Evidence suggests that England was characterized by post-Malthusian dynamics as early as two centuries preceding the Industrial Revolution. However, given England's special position as the forerunner of the Industrial Revolution, it is unclear if the existence of a post-Malthusian period is a general phenomenon. Using data from Denmark, Norway and Sweden, this research provides evidence for the existence of a post-Malthusian phase in the transition from stagnation to growth in Scandinavia.

The present analysis concludes that the Scandinavian countries were characterized by post-Malthusian dynamics in the period leading up to their industrialization. Thus, population growth was still positively affected by income per capita but it was not stagnant, possibly due to positive scale effects from population on the technological level that counterbalanced the Malthusian negative feedback effects arising from diminishing returns to labour. Furthermore, the breakdown of the post-Malthusian dynamics was associated with a reversal of the association between income per capita and fertility, consistent with the notion of an increasingly influential child quantity-quality trade-off.

These results are strikingly similar to the recent evidence of post-Malthusian dynamics in England, to the extent that the elasticities of the preventive and positive checks are of similar magnitudes, that estimates are remarkably stable over a long period, and that the breakdown of the post-Malthusian period is marked by a reversal of the association between income per capita and fertility (Møller and Sharp, 2014). Hence, given this high degree of similarity and the fact that the Scandinavian data are of superior quality, increasing confidence can be put in the generality of the results obtained in Møller and Sharp (2014) as a relevant description of the economic and demographic dynamics during the transition from stagnation to growth. Furthermore, the analysis establishes, that in spite of Denmark, Norway and Sweden being less developed than England in the pre-industrial era, these economies were indeed more post-Malthusian than Malthusian before their economic take-offs.

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## A Mis-specification Tests of the Unrestricted VAR Models

The mis-specification tests presented below are based on the unrestricted VAR model with dummy variables and are estimated using CATS in RATS, version 2.

Table A.1: Lag-specific tests for autocorrelation and heteroscedasticity (ARCH)

	<i>P</i> -value of test of absence of	
	Auto- correlation	Heterosce- dasticity
DENMARK		
Lag 1	0.99	0.00
Lag 2	0.35	0.00
Lag 3	0.37	0.00
Lag 4	0.25	0.00
NORWAY		
Lag 1	0.82	0.57
Lag 2	0.31	0.07
Lag 3	0.59	0.11
Lag 4	0.04	0.00
SWEDEN		
Lag 1	0.05	0.98
Lag 2	0.46	1.00
Lag 3	0.33	0.96
Lag 4	0.24	0.86

Table A.2: Univariate statistics

								<i>P</i> -value of test of absence of	
	Mean	S.D.	Skewness	Kurtosis	Max.	Min.	<i>R</i> <sup>2</sup>	ARCH	Normality
DENMARK									
$\Delta w$	0.00	0.04	0.12	2.61	0.11	-0.08	0.48	0.69	0.87
$\Delta b$	0.00	0.86	0.53	4.00	2.57	-1.93	0.46	0.01	0.08
$\Delta d$	-0.00	1.52	1.22	5.37	5.31	-2.75	0.46	0.05	0.00
NORWAY									
$\Delta w$	0.00	0.09	-0.16	4.53	0.28	-0.33	0.34	0.91	0.01
$\Delta b$	-0.00	1.40	-0.61	3.61	3.50	-4.45	0.49	0.42	0.08
$\Delta d$	0.00	1.51	0.61	3.45	4.70	-2.76	0.78	0.12	0.08
SWEDEN									
$\Delta w$	0.00	0.05	-0.10	3.37	0.15	-0.14	0.39	0.90	0.39
$\Delta b$	0.00	1.25	-0.20	3.29	2.76	-3.90	0.52	0.58	0.43
$\Delta d$	-0.00	2.06	0.59	4.53	6.51	-5.77	0.52	0.56	0.01

Table A.3: Overall tests for normality

	$P$ -value of overall test of normality
Denmark	0.00
Norway	0.00
Sweden	0.01

## B Comparison of Elasticities

Table A.4: Estimated Absolute Elasticities from Various Sources

	Preventive check	Positive check
PRESENT STUDY		
Denmark 1821–1890	0.13	0.28
Norway 1775–1853	0.30	0.08
Sweden 1775–1873	0.25	0.47
OTHER STUDIES		
England <sup>a</sup> 1560–1760	0.21–0.32	0.10–0.22
England <sup>b</sup> 1701–1759	0.31	Not analyzed
England <sup>c</sup> 1548–1834	0.14	0.10
England <sup>d</sup> 1540–1870	0.12	0.08
Europe <sup>e</sup> 1540–1870	0.14	0.16

Sources: a) Møller and Sharp (2014), b) Klemp (2012), c) Lee (1981), d) Lee and Anderson (2002), e) Galloway (1988).

The results for all three Scandinavian countries support the post-Malthusian model in which income spur is non-stagnating and spurs fertility and deters mortality. Table A.4 presents the estimates for the preventive and positive checks as long-run elasticities, calculated by dividing the estimated coefficients by their respective averages. Furthermore, the table includes the estimated absolute elasticities of the preventive check (i.e., the positive effect of wages on births) and the positive check (i.e., the negative effect of wages on deaths).

The table establishes that the estimates produced by the present analysis are of the same order of magnitude, but typically large, compared to those found in other studies. The preventive check is somewhat weaker in Denmark compared to the other nations. Furthermore, the present analysis finds evidence that the preventive check became weaker as Denmark moved into the nineteenth century.