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Do we need non-linear models to predict REIT returns?

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Brad Case, Massimo Guidolin and Yildiray Yildirim

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G11; C53

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Do We Need Non-Linear Models to Predict REIT Returns?

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We investigate whether the favorable performance of a fairly simple multistate multivariate Markov regime switching model relative to even very complex multivariate GARCH specifications, recently reported in the literature using measures of in-sample prediction accuracy, extends to pseudo out-of-sample forecasting accuracy. We recursively estimate the parameters of each model specification and use it to forecast returns and variances for the REIT, stock, and bond asset classes; REIT-stock, REIT-bond, and stock-bond covariances; and the entire return densities for the three assets. Out-of-sample forecast tests indicate that the four-regime MS model is, indeed, superior to all of the GARCH specifications in forecasting returns in each asset class, but is inferior in forecasting return variances. For the purpose of forecasting inter-asset return covariances, however, the MS model is surprisingly superior to all of the GARCH models.

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Introduction

A growing consensus that listed real estate investment trusts (REITs) provide investors exposure to the returns of the underlying real estate asset class⁴ has renewed interest in the investment return characteristics of REITs, and particularly in how they interact in a mixed-asset portfolio with non-REIT equities and other financial assets such as bonds.⁵

In this paper the authors investigate whether the fairly simple four-regime Markov switching (henceforth MS) structure identified by Case, Guidolin and Yildirim [2013, henceforth CGY] remains successful in out-of-sample (OOS) validation tests. In particular, we use their multivariate multistate MS framework to make recursive, pseudo OOS forecasts of the transitions among the four regimes identified and of the moments of the return distribution conditional on the predicted regime probabilities. In the limit, the exercise is extended to encompass the entire multivariate, joint density of asset returns. We compare the forecasting performance of the four-regime MS model by comparing its forecasts with similarly recursive pseudo OOS forecasts of moments and densities generated by several promising GARCH specifications for the returns of the REIT, stock, and bond asset classes.

The issue of whether and how it may be possible to accurately predict REIT returns as well as the association—under some conditions, their multivariate correlations—among REIT, stock and bond returns is obviously relevant both to academics and to practitioners alike. In particular, students of real estate finance have been facing the progressive development of increasingly sophisticated (multivariate) time series models used to better understand the dynamics of real estate valuations and especially to forecast such valuations, often in critical times (such as the recent 2008-2009 financial crisis preceded by and concurrent with a real estate bust of historical proportions). In this paper, we therefore tackle a critical question that many scholars and practitioners must have asked themselves as well as colleagues: do such complex time series frameworks offer concrete predictive benefits, and when? Although a number of related papers exist, we are not aware of this question having been researched before. Importantly, our key findings are often obtained using testing methodologies—particularly in the case of predictive density tests—that are also quite novel and complicated. Yet, our baseline back-testing design appears to fit common practice with

⁴ See Hoesli and Oikarinen [2012]; Stefek and Suryanarayanan [2012]; Boudry *et al.* [2012]; Oikarinen *et al.* [2011]; Yunus *et al.* [2012]; Lee and Chiang [2010]; Chiang [2009]; Li *et al.* [2009]; Morawski *et al.* [2008]; and Lee *et al.* [2008].

⁵ See Chaves *et al.* [2011]; Berkelaar and Kouwenberg [2010]; Fischer *et al.* [2010]; Fugazza *et al.* [2008, 2009, 2012]; Lee [2010]; Sa-Aadu *et al.* [2010]; Amenc *et al.* [2009]; Dolguero [2009]; MacKinnon and Al Zaman [2009].

trading and risk management desks, while our main empirical findings are easy to review and summarize.

In-sample evidence shows that even rather complex ARCH-type models (*e.g.*, a BEKK GARCH(1,1) with Student *t* innovations) fails to fit the evidence of deviations from the null hypothesis of zero skewness and of tails with a thickness consistent with the assumptions on the distribution of the model innovations (*e.g.*, identical to the normal distribution). We therefore fit to the vector of REIT, stock, and bond returns a four-state heteroskedastic MS model similar to the one already employed by CGY [2013] on a shorter sample. On the one hand, the regime switching model gives rather intriguing results. For instance, while two highly persistent regimes (the “REIT premium” and “normal” bull market states) describe the behavior of the U.S. financial market during almost 80% of the sample period, two other interesting states exist—an “investor’s dream” and a “nightmare” regime—that appear to frequently “communicate”, in the sense that with a non-trivial probability markets switch from one to the other state.

However, there is no clear-cut evidence that any of the most promising multivariate time series models estimated in this paper dominates the others in terms of in-sample evidence. For instance, different information criteria give varying indications as to the best fitting GARCH-type model. While the maximized log-likelihood suggests that the four-state MS model better fits the historical returns than any of the single-state GARCH models, information criteria suggest that this improvement in fit does not compensate for the increase in the number of parameters. Moreover, we find evidence that the MS model may not completely account for persistence in squared residuals. Yet, further diagnostic tests emphasize that only the MS model may successfully deal with the non-Gaussian distribution of standardized residuals.

Because it is not clear what the trade-off may be between the ability of models to successfully fit the volatility clustering typical of the data and their success at reproducing the skewed and fat-tailed (marginal) distributions of the data, from the perspective of a “user” of these models interested in their OOS forecasting performance, we proceed to assess the comparative performance of seven models recursively estimated in this paper at forecasting the means, variances, and covariances of asset returns. We find that the non-linear four-state MS model produces more accurate forecasts than other models as far as means (*i.e.*, point forecasts) and correlations are concerned. Such superiority is stronger for short, 1-month horizons; in the case of correlations, we report evidence that at a 12-month horizon multivariate GARCH models are better than the MS model. Diebold-Mariano tests of equal predictive accuracy confirm that in the case of the 1-month horizon, the differential performance in favor of the MS formulation is statistically

significant. On the contrary, as one may expect given their nature and historical development, multivariate GARCH models—among them, in particular a simple constant conditional correlation framework—perform better than the MS model does.

Finally, we also investigate the use of the competing models to predict the entire *density* of asset returns in OOS tests. This has become increasingly important to risk and money managers in the aftermath of the 2008-2009 financial crisis, when considerable time variation in higher order moments, such as skewness and kurtosis, was found to contribute to poor performance by many risk management systems. We find that none of the models appears to have correctly predicted the subsequent distribution of asset returns: in a real time exercise and for all models, we have indications that some characteristics of the return densities for each asset class have been predicted incorrectly. Nevertheless, the density forecast tests offer some support for the claim that the MS model or any of the GARCH models—especially the CCC specification—represent the best approach to forecast REIT and stock return densities; multivariate GARCH models, however, appear to be superior for predicting bond densities.

Literature Review

One strand of literature recognizes that taking into account the term structure of asset volatilities and correlations can lead to important differences in portfolio construction for short-horizon vs. long-horizon investors when asset returns are predictable. Rehring and Sebastian [2011] use vector autoregression (VAR) linear models to estimate the term structure of volatilities for direct real estate and listed REITs in the U.S. and U.K., as well as the term structure of direct-REIT correlations in each country. Rehring [2012] presents a similar analysis for direct real estate, stocks, bonds, and Treasury bills in the U.K.

The development of autoregressive conditional heteroskedasticity (ARCH) models has encouraged a blossoming of research into dynamic asset volatilities and, more recently, dynamic inter-asset correlations. In one notable example, Ward [2008] developed a Bayesian approach to estimating a GARCH model of dynamic REIT volatility and documented superior performance relative to the standard maximum likelihood GARCH model, a result that Ward attributed in part to the incorporation of parameter restrictions in the Bayesian estimation.

Chong *et al.* [2009] employed the dynamic conditional correlation GARCH model (Engle [2002]) to evaluate the dynamic volatilities of U.S. REITs, commodities, eight equity benchmarks, and six fixed income benchmarks, as well as the dynamic correlations of REITs with each of the other benchmarks. The authors confirmed that correlations tend to rise during periods of increased

return volatility, but also that correlations between REITs and U.S. government bonds decline during periods of increased interest rate volatility.

Wu *et al.* [2010] estimated a dynamic conditional correlation bivariate threshold GARCH model (Engle [2002]) to evaluate conditional REIT beta, finding that U.S. REITs exhibited defensive properties during 2005-2009 such that investors would benefit from increased holdings during market declines. On the other hand, Yang *et al.* [2012] employed the multivariate asymmetric generalized dynamic conditional correlation GARCH model developed by Cappiello *et al.* [2006] to estimate dynamic volatilities of and correlations among U.S. stocks, bonds, REITs, and commercial mortgage-backed securities (CMBS) during 1999-2008 and found evidence of an asymmetric REIT-stock correlation, suggesting reduced hedging effectiveness for REITs against the stock market decline.

Fei *et al.* [2010] also employed Cappiello *et al.*'s [2006] AG-DCC framework to model REIT volatility as well as REIT correlations with both stocks and house prices (as measured by the S&P/Case-Shiller Home Price Indices). Fei *et al.* found that the REIT-stock correlation was related positively to the contemporaneous credit spread, negatively to the contemporaneous term spread, and positively to the lagged unemployment rate. They also found that expected REIT returns tended to be higher during months when conditional REIT-stock correlations were the lowest, and lower when conditional correlations were higher.

Peng and Schulz [2012] employed several versions of the DCC model to evaluate volatilities and correlations among REITs, stocks, and bonds in eight countries. Peng and Schulz used OOS tests to show economic trading benefits, as well as superior accuracy in risk estimation arising from the dynamic models when compared to static volatility and correlation estimates.

Case, Guidolin and Yildirim [2013] showed that the dynamic time series properties of asset returns may be more successfully described using MS formulations than even quite complex formulations of autoregressive conditional heteroskedastic (ARCH) models, especially when returns are modeled jointly in a trivariate framework. In particular, CGY found that a fairly simple model with four regimes—a “typical” state with normal equity returns and relatively low volatilities for stocks and REITs along with relatively low bond returns; a “REIT-premium” state with much higher returns for REITs than for non-REIT stocks and relatively high returns for bonds; an “investor’s dream” state with spectacularly high returns for all three asset classes; and an “investor’s nightmare” state with spectacularly low returns—provided a better fit than conditional correlation

GARCH models. The analysis was, however, limited to the characterization of historically realized returns, i.e., it was performed only in-sample.

However, a considerable literature in applied econometrics and finance has often doubted of the actual value of multi-state, Markov switching models in forecasting applications. As discussed in Guidolin [2012], all dynamic time series models are as good as their forecasting performance is. In fact, there is no guarantee that a model that fits historical data well will also perform well in OOS tests, for three reasons. First, the extensive search for more complicated models using the same (or similar) data set(s) may suffer from a so-called “data-snooping bias,” as pointed out by Lo and MacKinlay [1989]. Certainly, most of the real estate finance literature has focused on rather similar data from U.S. financial markets. Second, large, richly parameterized models may contain an excessive number of parameters and inevitably exhibit excessive sampling variation in parameter estimates, which in turn may adversely affect their OOS performance. Third, a model that fits a historical data set well may not forecast the future well because of unforeseen structural changes in the process of asset returns (see the discussion in Dacco and Satchell [1999]). In the case of MS models this may derive from misspecification of the number of regimes. From both practical and theoretical standpoints, then, in-sample analysis alone is not adequate and it is necessary to examine the OOS predictive ability of nonlinear models, which is the goal of our paper.

Data

Our primary focus is the real estate equity asset class, for which we use monthly total returns for listed equity real estate investment trusts (REITs) in the U.S. as measured by the FTSE NAREIT All Equity REIT Index. We also investigate total returns in the stock and bond asset classes using monthly data for the CRSP NYSE/AMEX/NASDAQ Value-Weighted Market Index and the CRSP U.S. Treasury 10-Year Bond Index. The risk-free rate is represented by the CRSP 30-Day Treasury Bill Returns series. Our data span the historical period January 1972 – December 2010 for which REIT returns were available.

Table 1 displays summary statistics for all three asset classes for the full historical sample. Research by Case, Yang and Yildiray [2012] and others establishes that a “modern REIT era” began in late 1991, so Table 1 also shows summary statistics for the pre-modern era (January 1972 – December 1991) and the modern era (January 1992 – December 2010). Over each of the three periods, REITs provided the highest arithmetic average returns at 1.104 percent per month during the pre-modern era, 1.050 percent per month during the modern era, and 1.078 percent per month over the full historical period. Returns of non-REIT stocks were comparable during the pre-

modern era (1.060 percent per month) but considerably lower during the modern era (0.736 percent per month) and averaged 0.889 percent per month over the full historical period. Bond returns, too, were higher during the pre-modern era (0.769 percent per month) than during the modern era through 2010 (0.552 percent per month).

The standard deviation of monthly returns indicates that REITs were less volatile than stocks during the pre-modern era (4.144 percent vs 4.867 percent), and therefore provided stronger risk-adjusted returns with a Sharpe ratio of 0.116. During the modern era REITs were more volatile than stocks (5.763 percent vs 4.483 percent), but their Sharpe ratio (0.182 for REITs vs 0.164 for stocks) indicates that their higher mean return more than compensated for their added volatility. Over the full historical period REIT volatility was slightly higher than stock volatility (4.993 vs 4.660) but, again, the stronger Sharpe ratio (0.125 vs 0.094) indicates that higher mean REIT returns more than compensated for their slightly larger volatility. Bond returns showed exceptionally low volatility (2.061) and therefore the highest Sharpe ratio (0.268) during the modern era, a result that is not surprising given the disinflation of the early 1990s and the aggressively expansionary monetary policies of more recent years.

Table 1 also shows evidence of widespread deviation from normality in returns, as shown by the strong rejections implied by the Jarque-Bera test (the p-values are essentially zero across all assets and sub-samples). In the case of REITs, this is mostly due to their high excess kurtosis, while in the case of stocks and bonds the rejection of normality is equally imputable to their asymmetry (i.e., non-zero skewness) as to their thick tails. As is well known, modeling regimes in the distribution of asset returns represents a parsimonious and straightforward way to capture asymmetries and fat tails, which we shall pursue in what follows. Finally, while there is widespread evidence across asset classes and sub-samples that returns are characterized by volatility clustering, as shown by the high and statistically significant serial correlation (as tested through a Ljung-Box test with 12 lags) in squared stock returns, there are indications of non-zero serial correlation in levels of returns only for REIT and 1-month T-bills. Of course, volatility clustering is easily reproduced and predicted through the use of ARCH-type models, but rich multi-state switching models also offer chances to fit this feature of the data.

Methodology

The core of our analysis consists of generating a sequence of recursive pseudo OOS forecasts under alternative assumptions that we use to evaluate the performance of each model specification in terms of forecasting state-specific conditional asset return moments (means, variance, and

covariances) as well as densities. For each model specification, we first use the data through January 1990 to forecast the total mean return and variance for each asset, plus the covariance between each pair of assets, for February 1990 (one month ahead) through January 1995 (60 months ahead). We then expand our sample by one month, using data through February 1990 to forecast for March 1990 through February 1995. We proceed recursively in this fashion until we exhaust the data sample.

We evaluate each model using four measures of forecast accuracy—their root mean squared error (RMSE), forecast error variance, mean absolute error (MAE), and bias—relative to the historically realized sample estimates of moments of functions of returns (total returns, squared returns, and cross-products) over each forecast horizon. We next apply Diebold-Mariano tests to evaluate on a pairwise basis which model provided the superior forecast of each moment in terms of squared or absolute loss. Finally, we use a range of tests to evaluate the performance of each model in forecasting return densities. Details on each of these tests will be provided within each corresponding section to keep the paper compact.

As for the specific models under consideration, we investigate the forecasting performance of six multivariate specifications employed by CGY [2013]. We refer readers to that paper for details of model estimation; here we review only a few essential aspects of model structure. Our benchmark is a single-state Gaussian IID random walk for asset prices—that is, a representation of the no-predictability benchmark in finance:

$$\mathbf{r}_t = \boldsymbol{\mu} + \boldsymbol{\Omega}^{1/2} \boldsymbol{\varepsilon}_t \quad [1]$$

with $\boldsymbol{\varepsilon}_t \sim \text{NID}(\mathbf{0}, \mathbf{I}_3)$, where 3 is the number of assets under consideration and $\boldsymbol{\Omega}^{1/2}$ is a 3×3 Choleski factorization for the covariance matrix of returns, i.e., $(\boldsymbol{\Omega}^{1/2})(\boldsymbol{\Omega}^{1/2})' = \boldsymbol{\Omega} \equiv \text{Var}[\mathbf{r}_t \mathbf{r}_t']$. We add components to this baseline specification that earlier work has suggested may improve in-sample fit. First, we estimate a single-state homoskedastic Gaussian VAR(1):⁶

$$\mathbf{r}_t = \boldsymbol{\mu} + \boldsymbol{\Phi} \mathbf{r}_{t-1} + \boldsymbol{\Sigma} \boldsymbol{\varepsilon}_t \quad [2]$$

Next, we estimate two versions of the diagonal BEKK GARCH(1,1) model of Engle and Kroner [1995], specifying first a Gaussian and then a Student's t distribution for the innovations. Denoting the one-step-ahead conditional mean vector and covariance matrix of \mathbf{r}_t by $\boldsymbol{\mu}_{t|t-1} \equiv E_{t-1}[\mathbf{r}_t]$ and $\boldsymbol{\Omega}_{t|t-1} \equiv \text{Var}_{t-1}[\mathbf{r}_t]$, respectively, the model may be written as

⁶ Similarly to CGY [2013], increasing the number of lags in the VAR has been considered, but the null hypothesis of $p = 1$ was never formally rejected in favor of more lags.

$$\mathbf{r}_t = \boldsymbol{\mu}_{t|t-1} + \boldsymbol{\Omega}_{t|t-1}^{1/2} \boldsymbol{\varepsilon}_t \quad \boldsymbol{\varepsilon}_t \sim \text{IID } G(\mathbf{0}, \mathbf{I}_3), \quad [3]$$

where $\boldsymbol{\varepsilon}_t$ denotes a vector white noise process with unit variances and $G(\mathbf{0}, \mathbf{I}_3)$ refers to a generic standardized distribution, here either a multivariate normal or a multivariate t-Student. In the case of a simple diagonal BEKK GARCH(1,1) model, we have

$$\boldsymbol{\Omega}_{t|t-1} = \boldsymbol{\Psi} + \mathbf{A}(\hat{\boldsymbol{\varepsilon}}_{t-1} \hat{\boldsymbol{\varepsilon}}_{t-1}') \mathbf{A}' + \mathbf{B} \boldsymbol{\Omega}_{t-1|t-2} \mathbf{B}, \quad [4]$$

in which \mathbf{A} and \mathbf{B} are diagonal matrices. We also estimated the constant conditional correlation (CCC) GARCH model developed by Bollerslev [1990], as well as the dynamic conditional correlation (DCC) GARCH generalization introduced by Engle [2002]. Specifically, let $\mathbf{D}_{t|t-1}$ denote the $N \times N$ diagonal matrix with the conditional standard deviations, or the square root of the diagonal elements in $\boldsymbol{\Omega}_{t|t-1}$ along the diagonal. The conditional covariance matrix may then be uniquely expressed in terms of the decomposition

$$\boldsymbol{\Omega}_{t|t-1} = \mathbf{D}_{t|t-1} \boldsymbol{\Gamma}_{t|t-1} \mathbf{D}_{t|t-1}, \quad [5]$$

where $\mathbf{D}_{t|t-1}$ denotes the 3×3 matrix of conditional correlations. Of course, this decomposition does not result in any immediate simplifications from a modeling perspective, as the conditional correlation matrix must now be estimated. However, following Bollerslev [1990] and assuming that the temporal variation in the covariances is driven solely by the temporal variation in the corresponding conditional standard deviations, so that the conditional correlations are constant, $\boldsymbol{\Gamma}_{t|t-1} = \boldsymbol{\Gamma}$, dramatically reduces the number of parameters in [5].⁷

While the assumption of constant conditional correlations may often be a reasonable simplification over shorter time periods, it is arguably too simplistic in many situations of practical interest. To circumvent this, while retaining the key features of the decomposition $\boldsymbol{\Omega}_{t|t-1} = \mathbf{D}_{t|t-1} \boldsymbol{\Gamma}_{t|t-1} \mathbf{D}_{t|t-1}$, Engle [2002] developed the Dynamic Conditional Correlation (DCC) model in which the temporal variation in the conditional correlation is characterized by a simple scalar GARCH(1,1) model, with the covariance matrix for the standardized residuals targeted at their unconditional value:

$$\mathbf{Q}_{t|t-1} = (1 - \alpha - \beta) \bar{\mathbf{Q}} + \alpha (\hat{\mathbf{u}}_{t-1} \hat{\mathbf{u}}_{t-1}') + \beta \mathbf{Q}_{t-1|t-2}, \quad [6]$$

⁷ Moreover, the CCC assumption greatly simplifies the multivariate estimation problem, which may now proceed in two steps. In the first step N individual univariate GARCH models are estimated for each of the series in \mathbf{r}_t , resulting in an estimate for the diagonal matrix, $\hat{\mathbf{D}}_{t|t-1}$. In our paper, following much of the literature, we select a simple GARCH(1,1) for all the returns series. In a second step, defining the 3×1 vector of standardized residuals for each of the univariate series, $\hat{\mathbf{u}}_t \equiv \hat{\mathbf{D}}_{t|t-1}^{-1} (\mathbf{r}_t - \hat{\boldsymbol{\mu}}_{t|t-1})$, the elements in $\boldsymbol{\Gamma}$ may simply be estimated by the corresponding sample analogue, $\hat{\boldsymbol{\Gamma}} = T^{-1} \sum_{t=1}^T \hat{\mathbf{u}}_t \hat{\mathbf{u}}_t'$.

where $\bar{\mathbf{Q}}$ is the unconditional covariance matrix of the standardized errors $\hat{\mathbf{u}}_t \equiv \hat{\mathbf{D}}_{t|t-1}^{-1} \hat{\boldsymbol{\varepsilon}}_t$ where $\hat{\mathbf{D}}_{t|t-1}$ is the diagonal matrix of time t GARCH(1,1) standard deviations estimated in the first step. Although this recursion guarantees that the $\mathbf{Q}_{t|t-1}$ matrices are positive definite, the individual elements are not necessarily between -1 and +1. Thus, in order to arrive at an estimate for the conditional correlation matrix, the elements in $\mathbf{Q}_{t|t-1}$ must be standardized, resulting in the following estimate for the ij th correlation:

$$\hat{\rho}_{ij,t} \equiv \{\hat{\Gamma}_{t|t-1}\}_{ij} = \frac{\{\mathbf{Q}_t\}_{ij}}{\{\mathbf{Q}_t\}_{ii}^{1/2} \{\mathbf{Q}_t\}_{jj}^{1/2}} \quad [7]$$

Like the CCC model, the DCC model is also simple to implement in large dimensions, requiring only the estimation of three univariate models along with the estimation of the two exponential smoothing parameters, α and β .⁸

Finally we estimate the four-state Markov regime switching VAR(1) specification of CGY:⁹

$$\mathbf{r}_t = \boldsymbol{\mu}_{S_t} + \boldsymbol{\Phi} \mathbf{r}_{t-1} + \boldsymbol{\Omega}_{S_t}^{1/2} \boldsymbol{\varepsilon}_t \quad [8]$$

with $\boldsymbol{\varepsilon}_t \sim \text{NID}(\mathbf{0}, \mathbf{I}_3)$. $S_t = 1, 2, \dots, K$ is a latent state variable driving all the matrices of parameters appearing in [8]. $\boldsymbol{\mu}_{S_t}$ is a 3x1 vector that collects the 3 regime-dependent intercepts, while the 3x3 matrix $\boldsymbol{\Omega}_{S_t}^{1/2}$ represents the factor applicable to state S_t in a state-dependent Choleski factorization of the variance covariance matrix, $\boldsymbol{\Omega}_{S_t}$. When $K > 1$, alternative hidden states are possible and they will influence both the conditional mean and the volatility/correlation structures characterizing the multivariate process in [8]. These unobservable states are generated by a discrete-state, homogeneous, irreducible and ergodic first-order Markov chain

$$\Pr(S_t = j | \{S_j\}_{j=1}^{t-1}, \{r_j\}_{j=1}^{t-1}) = \Pr(S_t = j | S_{t-1} = i) = p_{ij}, \quad [9]$$

where p_{ij} is the generic $[i, j]$ element of the $K \times K$ transition matrix \mathbf{P} .¹⁰ All these multivariate models, including the MS VAR in [8]-[9], are estimated by maximum likelihood (MLE). In

⁸ As formally shown in Engle and Sheppard (2001), the parameters in the DCC model characterizing the dynamic dependencies in $\mathbf{Q}_{t|t-1}$, and in turn $\Gamma_{t|t-1}$, may be consistently estimated in a second step by maximizing the partial log-likelihood function.

⁹ On the basis of in-sample empirical results and a detailed model specification search, both described in CGY [2013], we constrain the VAR coefficients to be identical across regimes, allowing only the intercepts to be regime-specific.

¹⁰ The assumption of a first-order Markov process is not restrictive, since a higher order Markov chains can always be re-parameterized as a higher dimensional first-order Markov chain, i.e., substitutability exists between the order of the Markov chain driving S_t and the number of regimes K . See Guidolin [2012] for additional details. Ergodicity implies the existence of a stationary vector of (unconditional) state

particular, estimation and inference in the MS VAR case are based on the EM (Expectation-Maximization) algorithm, a filter that enables the iterative calculation of the one-step ahead forecast of the state vector $\xi_{t+1|t}$ given the information set and the construction of the log-likelihood function of the data. As explained in CGY (see also the textbook treatment in Hamilton [1994]), standard inferential procedures and tests—such as Wald’s and those based on the likelihood ratio principle—are available.¹¹

Empirical Model Estimates

A specific technical appendix not for publication summarizes the empirical estimates of the single-state models that we have entertained in the paper. Here we simply recall the most important features of the estimations that have been performed. The estimation of a simple Gaussian VAR(1) model over the full historical period January 1972 – December 2010. REIT reveals that returns are significantly and positively related to lagged values of stock and bond returns, while stock returns are positively related to lagged values of REIT returns; bond returns are related positively to their own lagged values and negatively to lagged REIT returns. The constant variances for REITs (23.425), stocks, (21.328), and bonds (5.128) are all close to the historically realized variances implied by Table 1, as are constant correlations. However, a multivariate Ljung-Box (L-B) test for serial correlation indicates that the model fails to account for persistence in squared residuals, which is to be expected given its homoskedastic nature.

The empirical estimates for the VAR(1) diagonal BEKK GARCH(1,1) model with Gaussian shocks show that, as with the previous model, REIT returns are related significantly and positively to lagged stock and bond returns, and bond returns are related positively to their own lagged values and negatively to lagged REIT returns; stock returns, however, show no statistically significant relationship with lagged values of any of the three asset classes. The implied unconditional mean returns, however, are smaller than the sample estimates for all assets. The conditional variance functions indicate that persistence is strongest for bonds (the implied half-life of a shock is 25

probabilities $\bar{\xi}$ satisfying $\bar{\xi} = \mathbf{P}'\bar{\xi}$. Irreducibility implies that $\bar{\xi} > \mathbf{0}$ meaning that all unobservable states are possible.

¹¹ The only exception concerns the number of non-zero rows of the transition matrix \mathbf{P} , *i.e.* the number of regimes K . In this case, even under the assumption of asymptotic normality of the estimator of the model’s parameter, standard tests are characterized by non-standard asymptotic distributions of the likelihood ratio test statistic due to the existence of nuisance parameters under the null hypothesis. We say that these parameters become a nuisance to the estimation. Under any number of regimes smaller than K there are a few structural parameters of the unrestricted model—the elements of the transition probability matrix associated with the rows that correspond to disappearing states—that can take any values without influencing the resulting likelihood function.

months) and weaker for REITs (17 months) and stocks (15 months), and that implied unconditional standard deviations are similar to sample estimates. This is evidence that log-likelihood maximization favors fitting in-sample variances over mean returns. The conditional covariance functions, too, indicate persistence in correlations; the implied unconditional REIT-stock correlation is similar to the sample estimate (0.589 vs 0.581), but the implied unconditional REIT-bond and stock-bond correlations are slightly higher than their sample estimates.

The maximized log-likelihood and the Akaike, Bayes-Schwartz, and Hannan-Quinn information criteria all suggest that the VAR(1) diagonal BEKK GARCH(1,1) model with Gaussian shocks better fits the historically realized data than the homoscedastic VAR(1) model does, even considering the increase in the required number of parameters. Moreover, a multivariate L-B test documented in the appendix suggests that the model has successfully addressed serial correlation in the residuals. The Jarque-Bera test, however, advises that the standardized residuals from the model are not normally distributed, contrary to what has been assumed. This is of course evidence of residual misspecifications that stem from un-modeled skewness and kurtosis dynamics in the data, consistent with the presence of asymmetries and thick tails in Table 1. Interestingly, such econometric misspecifications also cause a failure of the model to fit key financial aspects of the data under investigation: the implied (monthly) Sharpe ratios are all smaller than the empirical ones reported in Table 1.

Because a *t*-Student distribution generally helps in fitting fat-tailed return distributions (and this was one of the salient features of the non-normalities that have emerged above), we have also estimated the same VAR(1) diagonal BEKK GARCH(1,1) model with a Student's *t* distribution specified for the innovations. In this model REIT returns are related to lagged bond returns but not significantly to lagged stock returns, stock returns are related positively to lagged REIT returns, and bond returns are related only to their own lagged values. Implied unconditional mean returns are now close to their sample estimate for REITs (1.074 vs 1.078), slightly higher for stocks (0.941 vs 0.889), and slightly lower for bonds (0.532 vs 0.663). Interestingly, the model implies that unconditional volatility is a bit less for REITs than their sample estimate (4.430 vs 4.993) but higher for both stocks (4.798 vs 4.660) and bonds (2.745 vs 2.311); it also implies substantially greater volatility persistence for bonds (the half-life of a shock climbs to 46 months), though not for REITs or stocks. The estimated conditional covariance functions, too, are characterized by substantial persistence. The maximized log-likelihood (2902 vs 2849), as well as the AIC, BIC, and HQ criteria all indicate that the specification of a Student's *t* distribution for innovations provides a better fit of the historical data. However, it remains the case that the Sharpe ratios implied by the

model are often distant from the empirical ones. In a statistical perspective, a Cramer-Von Mises test of the assumption that the standardized residuals have a multivariate Student-t distribution with the estimated degrees of freedom parameter still yields an overwhelming rejection, with a p-value of zero. Unreported evidence shows that this occurs not only because the standardized residuals remain strongly asymmetrically distributed, but also because of non-zero excess kurtosis.

In the case of the VAR(1) CCC-GARCH(1,1) and VAR(1) DCC-GARCH(1,1) estimation results, the conditional mean functions are the same as the ones obtained in the homoskedastic Gaussian VAR(1) model.¹² Because the attempt to achieve a better fit of the data by resorting to a non-normal distribution for the model's innovations has failed to give appreciable results, the VAR(1) CCC and DCC models are again estimated assuming that the innovations have a multivariate normal distribution, in line with the bulk of the literature (see *e.g.*, Case *et al.* [2012]). In both tables, the models fail to provide an unequivocally superior fit to historical data according to the maximized log-likelihood, AIC, BIC, and HQ criteria. For instance, the log-likelihood function fails to increase when one moves from the BEKK model to the CCC. More interestingly, a richer DCC implies a lower optimized log-likelihood than a BEKK does. However, when it comes to ranking models based on information criteria, we obtain a different perspective: the H-Q gets worse (i.e., it increases) both when going from a BEKK to the CCC, and when comparing the BEKK with the DCC. This is possible because the DCC requires more parameters than does the BEKK. This represents evidence in favor of the sample fit provided by a *t*-Student BEKK GARCH(1,1) model, given a common conditional mean function.

In the case of REITs, the CCC model suggests slightly greater volatility persistence and an implied unconditional volatility (4.755) higher than the *t*-Student diagonal BEKK model (4.430) but lower than the sample estimate (4.993). For stocks, the implied unconditional volatility from the CCC model (4.863) is greater than either the *t*-Student BEKK model (4.798) or the sample estimate (4.660). The CCC-implied unconditional bond volatility (2.285) is lower than what is implied by the *t*-Student BEKK model (2.745) and slightly lower than the sample estimate (2.311). The DCC model produces the highest implied unconditional volatilities for both REITs (5.189) and stocks (5.148) and the implied volatility for bonds (2.625) is higher than the sample estimate (2.311) though not as high as that implied by the *t*-Student model.

¹² This is because a quasi maximum likelihood estimation approach allows us to estimate the model in two steps: first, we estimate the conditional mean (autoregressive parameters) by OLS; second, conditioning on the first-step estimates, we estimate the models for conditional variances and covariances.

Table 2 summarizes the empirical estimates for the four-state VAR(1) homoskedastic Markov regime switching model. Relative to the model estimated by CGY [2013] on a slightly shorter historical time period (January 1972 – December 2008), the results show some differences that can generally be attributed to the addition of two years during which historically realized returns were extraordinarily strong for REITs and bonds but not for stocks. Regimes 1 and 2 correspond roughly to the “REIT premium” and “typical” bull market states identified by CGY, but the “REIT premium” is actually estimated to be relatively common, with ergodic probabilities of 0.410 for Regime 1 and 0.379 for Regime 2, respectively; moreover, implied unconditional means are higher for bonds than for stocks in both states (0.783 vs 0.217 in Regime 1, and 0.729 vs 0.636 in Regime 2), and more attractive for bonds than for REITs (0.660) in Regime 2. Implied unconditional volatility is sharply lower for REITs than for stocks in Regime 2 (2.600 vs 3.581) and only slightly higher in Regime 1 (4.815 vs 4.798), so that implied unconditional Sharpe ratios are higher for REITs in both states (0.092 vs <0 in Regime 1 and 0.060 vs 0.037 in Regime 2). Regime-conditional correlations, however, are similar to those estimated by CGY and all larger in Regime 2 than in Regime 1 (0.654 vs 0.454 for REITs with stocks, 0.410 vs -0.132 for REITs with bonds, and 0.493 vs -0.215 for stocks with bonds).

As in CGY, Regime 3 can be characterized as an “investor’s dream” state with spectacularly high implied (unconditional) regime-specific mean returns (3.764 percent per month for REITs, 2.904 for stocks, and 3.028 for bonds) and moderately low implied unconditional volatilities (3.478, 3.720, and 3.212 respectively), resulting in spectacularly strong implied unconditional Sharpe ratios (0.936 for REITs, 0.644 for stocks, and 0.784 for bonds). Implied unconditional correlations, too, are low at 0.291 for REITs with stocks, 0.002 for REITs with bonds, and 0.258 for stocks with bonds, which implies strong hedging opportunities specific to this regime. Finally, Regime 4 can be characterized as an “investor’s nightmare” state with spectacularly low mean returns (especially for REITs), high volatilities (especially for REITs, 15.713 percent per month), and a high REIT-stock correlation (0.810). Indeed, the implied unconditional mean returns (-6.894 percent per month for REITs, -4.367 for stocks, and -2.198 for bonds) are even more severe than estimated by CGY, presumably because their historical period through the end of 2008 did not include the early-2009 months of the financial crisis.

Figure 1 depicts the smoothed state probabilities for all four regimes. As the figure indicates, what used to be considered the “normal” bull market prevailed during three-quarters of the 265 months from August 1975 through August 1997, interrupted by only four months of “nightmare” (October 1979, March 1980, and October-November 1987), a four-month stretch identified only marginally

as the “REIT premium” state (June-September 1979), and repeated short bursts of the “dream” state. Starting in September 1997, however, the “REIT premium” state has been the norm, prevailing in 137 of 160 (more than 85 percent) of months. Both the “normal” state and the “investor’s dream” have been comparatively rare since the long “REIT premium” stretch began: the “normal” state has prevailed only during February-March 2003 and December 2003-February 2004—and in those episodes only uncertainly—while the “dream” has characterized only May-November 2003, March 2004, and to some extent March 2009 (the initial recovery from the recent crisis).¹³ Investors were hit by acutely bear states during April 2004 and during 6 of the 7 months of the crisis in September 2008-April 2009.

The estimated transition matrix shown in Table 2 indicates that the “REIT premium” state (Regime 1) is the most persistent, with episodes lasting 43.5 months on average, and is readily entered from the “investor’s nightmare” state (Regime 4, with an estimated 0.180 transition probability)—as, indeed, appears to have occurred in May 2009. The “normal” state (Regime 2) is less persistent, with episodes lasting 9 months on average, but is often accessed starting from the “investor’s dream” state (Regime 3, with a 0.213 transition probability). Episodes of both the “investor’s dream” and the “nightmare” states tend to be short (lasting, on average, just 3.8 months for Regime 3 and 2.3 months for Regime 4); thankfully, however, the dream is relatively likely to follow the nightmare with a transition probability of 0.259, while the probability of entering the extreme bear state is significantly different from zero only from the dream at just 0.049 (in which case perhaps it may be called a “post-Bacchanalian hangover”).

The maximized log-likelihood (2903) suggests that the four-state MS model better fits the data than any of the single-state models, but the AIC, BIC, and HQ criteria suggest that this improvement in fit does not compensate for the large increase in the number of parameters to be estimated (57 vs 24-26 parameters in the case of multivariate GARCH models). Moreover, the multivariate Ljung-Box test for serial correlation suggests that the MS model has not completely accounted for persistence in the squared residuals (although the corresponding L-B test commands a p-value between 5 and 10 percent only). Interestingly, however, the Jarque-Bera test statistic indicates that the MS model has dealt successfully with the non-Gaussian distribution of standardized residuals, while each of the single-state models continued to display signs of non-normality. It is

¹³ It is tempting to think of the near-disappearance of episodes of strong stock market returns since August 1997 as part of the “new normal” popularized by PIMCO starting in 2009, but the “new normal” scenario fails to account either for the fact that weakness in stock returns seems to have begun as early as September 1997 (averaging just 5.74 percent annualized returns through December 2010) or for the strength of REIT returns over the same period (11.12 percent in annualized terms).

not clear what the trade-off may be between the ability of models to successfully fit the volatility clustering in the data and their success at re-producing their skewed and fat-tailed (marginal) distributions. Moreover, it remains unclear which of these features may be more promising in the OOS forecasting perspective that we pursue.

Accuracy of Recursive Moment Forecasts

While the diagnostic tests discussed in the previous section hint to the fact that the four-state VAR(1) Markov switching model may outperform single-state models in terms of in-sample fit over the our historical sample, the primary interest of this paper is to evaluate OOS forecast accuracy. To this end, we conducted recursive pseudo out-of-sample tests: specifically, we estimated the full set of parameters for each model using the realized historical data for only the period January 1972 through January 1990, and used those parameter estimates to forecast the moments of the conditional joint return distribution—return means, variances, and inter-asset covariances—for the months from February 1990 through January 1995 (that is, one-month forecasts through 60-month forecasts).¹⁴ We then re-estimated the parameters of each model using the updated realized historical data through February 1990, and used the updated parameter estimates to forecast the joint return moments for March 1990 through February 1995, repeating this process and eventually truncating the forecast period until we had finally used historical data for January 1972 through November 2010 to forecast only December 2010. This is a rather typical and natural back-testing, real time recursive exercise intended to tease out of the data whether any model offers predictive advantages.

Mean Forecasts

The first panel (A) of Table 3 summarizes the performance of each model in recursively forecasting REIT returns, with predictive accuracy measured on the basis of the root mean squared forecast error (RMSE), forecast error variance, MAE, and prediction bias (see Table 3 for a definition of these measures). Strikingly, the four-state MS model provides very substantially superior one-month forecasts than any of the single-state models in terms of RMSE (4.961 vs a minimum of 5.535 for the homoskedastic VAR(1) and DCC-GARCH models), error variance (24.602 vs 30.633 for the same models), and MAE (3.437 vs 3.789 for the CCC-GARCH model). The MS model was the only one that produced forecast returns with an upward bias, which was greater in absolute value (0.088) than that of the IID benchmark (0.062) as well as the homoskedastic and DCC-GARCH

¹⁴ In what follows, we display and discuss only the one-month and 12-month forecasts. Results for longer-horizon forecasts were qualitative similar to those obtained for 12-month forecasts, but are of dubious usefulness to portfolio and risk managers even though they are fully legitimate in a statistical dimension.

VAR(1) models (0.000), but less than those of the remaining GARCH models. The MS model also produced the most accurate 12-month forecast according to the RMSE criterion, but underperformed the homoskedastic VAR(1) model, the IID benchmark, and the Gaussian diagonal BEKK GARCH model according to the error variance and MAE criteria; moreover, not surprisingly, the differences in 12-month forecast accuracy measures were not nearly as large as for one-month forecasts.

An unreported figure (see the appendix) depicts the one-month REIT return forecasts produced by the *t*-Student BEKK specification, which was only marginally more successful than (and visually indistinguishable from) the Gaussian BEKK GARCH model, the CCC-GARCH model, and even the IID random walk benchmark; the graph shows that the models fail to capture essentially any of the sharp monthly variation in historically realized returns. The same set of plots depicts the REIT returns forecast by the homoskedastic and DCC-GARCH VAR(1) specifications (utilizing the same conditional mean functions), which much more successfully reflect monthly swings. Figure 2, however, shows that the four-state MS specification is dramatically more successful in forecasting monthly REIT returns.¹⁵ This is especially visible in correspondence to the dramatic swings of the 2008-2009 crisis, but is also true during less turbulent months, *e.g.*, March 1998 (DCC-GARCH forecasts a return of 0.35%, MS forecast is 1.26%, the realized return was 1.79%), December 2001 (DCC-GARCH -0.50%, MS 1.36%, realized 2.44%), or August 2003 (DCC-GARCH -1.57%, MS -0.28%, realized 0.54%).

Panel B of Table 3 shows that the outperformance of the MS model in terms of forecast accuracy is even more pronounced for non-REIT stock returns than for REITs: the MS model produces not merely a smaller RMSE (4.105 vs a minimum of 4.502 for the CCC-GARCH model), error variance (16.853 vs 20.222, again for the CCC-GARCH specification), and MAE (3.191 vs 3.429 for the *t*-Student BEKK model), but also less bias (practically 0, *i.e.*, no bias vs -0.168 for the homoskedastic VAR(1) model and the DCC-GARCH). As panel C shows, the MS also outperforms the other models in forecasting one-month bond returns according to all four measures. Moreover, panel B indicates that MS outperforms all of the single-state models in terms of 12-month mean stock return forecast accuracy according to all four measures; panel C concerning bonds, however, shows that the homoskedastic VAR(1) model provides better 12-month forecasts of bond returns according to the RMSE and error variance criteria.

¹⁵ Similar plots concerning 12-month forecasts have been omitted to save space.

In short, the four-state VAR(1) MS model produced dramatically better one-month forecasts of REIT, stock, and bond returns than any of the single-state models by every criterion except (for REIT returns only) bias; the model also performed comparatively well at 12-month return forecasts, especially in the stock market. This progressive improvement in the ability to forecast stock and bonds returns when going from the *t*-Student BEKK VAR(1) model, to the Gaussian DCC VAR(1), to the multi-state non-linear models is visible.

Variance Forecasts

The first panel of Table 4 summarizes the performance of each model in recursively forecasting the (conditional) variance of REIT returns, using the same four measures (RMSE, error variance, MAE, and bias) but applied to the square of realized monthly returns. Interestingly, the four-state MS model produces larger forecast errors than any of the GARCH models according to the RMSE, error variance, and bias criteria, outperforming only the IID benchmark and the homoskedastic VAR(1) model (and, indeed, underperforming even those simple models in terms of bias). The CCC-GARCH specification appears to have provided the most accurate one-month variance forecasts, with the smallest RMSE and error variance and the second smallest bias. Only the MAE criterion suggested comparatively successful forecasts from the MS model, which is an indication that while the MS model may generally track the time-variation in squared asset returns, it has more difficulties with forecasting spikes in volatility, for which multivariate GARCH models are surely better designed.

An unreported set of plots (see Appendix) depicts one-month REIT volatility forecasts produced by the homoskedastic Gaussian VAR(1) model, which is visually indistinguishable from the IID random walk benchmark. In the plot, for simplicity the (latent) volatility is approximated by the square root of squared realized asset returns, i.e., by the absolute value of returns. As the graph shows, the two approaches fail to capture essentially any of the sharp monthly variation in squared returns; the increase in REIT volatility during the 2008-2009 crisis is depicted as only a small rise in forecast volatility (only due to the fact that these homoskedastic models have been recursively re-estimated), and the subsequent sudden decline is not captured at all. The same plots (right column) shows the much more successful one-month REIT volatility forecast by the four-state MS specification. As Figure 3 makes clear, however, the CCC-GARCH model (only marginally better than, and visually indistinguishable from, the *t*-Student and Gaussian diagonal BEKK specifications and the DCC-GARCH specification) much more satisfactorily captures the very dramatic increase and subsequent decline of REIT volatility during and after the recent financial crisis. It is somewhat interesting that a constant conditional correlation model outperforms the DCC at

forecasting variances; as we shall see, this is clearly not the case when one forecasts conditional covariances.

The second and third panels of Table 4 show that the outperformance by the GARCH modeling approaches—especially, though not dramatically, the CCC formulation—also holds for forecasting stock and bond volatilities. In short, while the four-state MS model with heteroskedastic components more successfully forecasts volatility than the simple homoskedastic VAR(1), single-state GARCH specifications clearly outperform the four-state approach except under a MAE criterion. Although this may be considered relatively unsurprising given the rudimentary structure of the conditional heteroskedasticity function featured by the four-state MS, such differences ought to be taken into account by portfolio and risk managers.

Covariance Forecasts

Table 5 uses the same four criteria to evaluate the performance of each model in forecasting REIT-stock, REIT-bond, and stock-bond covariances. It is perhaps common wisdom that GARCH-type models perform well at forecasting conditional return variances, even relative to an MS model with four regimes. The authors were quite surprised, therefore, to find that the GARCH models appear to fall well short of the MS model in predicting conditional covariances. For one-month forecasts of REIT-stock covariances, the best-performing GARCH model appears to have been the *t*-Student diagonal BEKK specification, which sharply improved over the IID random walk in terms of RMSE (0.469 vs 0.500), error variance (0.220 vs 0.249), and MAE (0.184 vs 0.196). The four-state MS, however, showed significant improvement according to all three criteria with an RMSE of just 0.457, error variance of 0.206, and MAE of 0.166.

The same result holds for REIT-bond covariances, although in this case the best-performing GARCH model appears to have been the CCC-GARCH specification. Although the differences are not as dramatic, the MS model produced smaller values of RMSE (0.196 vs 0.200), error variance (0.038 vs 0.040), and MAE (0.081 vs 0.083). As the last panel shows, the same is true of forecast stock-bond covariances with slightly lower RMSE (0.119 vs 0.124), error variance (0.014 vs 0.015), and MAE (0.067 vs 0.069).

An unreported series of plots concerning the one-month REIT-stock correlation forecasts from the homoskedastic VAR(1) model (which is visually indistinguishable from the IID benchmark) and the very similar CCC-GARCH model,¹⁶ give evidence of the secular decline in REIT-stock correlations

¹⁶ In the plots, we focus on correlations because these are more meaningful than forecasting covariances. However, the cost to be paid is that realized correlations in real time are either -1 or +1 and therefore simply

during the 1990s that has been observed in realized data,¹⁷ but the decline is modest, from about 0.685 during the beginning of the forecast period to about 0.555 for the forecasts just prior to the liquidity crisis; moreover, neither shows the increase in REIT-stock correlations since October 2000 that has been observed in other papers,¹⁸ and neither shows any appreciable increase as a result of the liquidity crisis. Figure 4 shows instead the REIT-stock correlations forecast by the *t*-Student diagonal BEKK model (visually indistinguishable from the DCC-GARCH model) and the Gaussian diagonal BEKK model. Both show a much stronger decline during the 1990s from about 0.7 to as low as 0.01, a strong increase following October 2000 to as high as 0.7, and a jump to above 0.8 during the 2008-2009 crisis followed by a sharp decline to below 0.5 as the crisis subsided. The trends in these sets of correlation forecasts would appear quite familiar to investment managers—a truth that is, however, distressing, because they mirror correlations estimated using primitive methods such as a trailing 24-month correlation coefficient, also shown (for comparison) in Figure 4.

An unreported figure concerning the one-month REIT-stock correlation forecasts produced by the four-state Markov switching model reveals that, like the other models, the MS model forecasts the secular downward trend through the 1990s; in the MS framework, however, this trend can be explained most readily in terms of the declining state probability of the “normal” state (Regime 2), with a conditional REIT-stock correlation of 0.654, and the increasing predicted probability of the “REIT premium” state (Regime 1), characterized by a conditional REIT-stock correlation of just 0.454. Indeed, the MS forecast correlation reached its lowest values in November 1998 – February 1999, shortly following the end of the period during which Regime 2 became dominant and early in the period during which Regime 1 surged. From April 2004 through the beginning of the 2008 financial crisis the forecast correlation remained abnormally low, averaging just 0.433. During the financial crisis, the predicted correlation jumped to 0.675, but from March 2009 it declined rapidly to average just 0.477 from June 2009 through the end of our sample—in correspondence to a steadily declining probability of the “nightmare” state. Perhaps surprisingly, there is substantially greater monthly variation in MS conditional correlation forecasts for the REIT-bond and stock-bond pairs. The model implies a sharp decline in both forecast correlations subsequent to the financial crisis, consistent with the increasing estimated probability of the “REIT premium” regime.

correspond to a sign indicator function that we omit from the plots. This is due to the fact that dividing realized cross products of returns by their absolute values gives either -1 or +1 depending on whether the returns had the same sign or not.

¹⁷ See Case *et al.* [2012], Westerheide [2006], Conover *et al.* [2002], Clayton and MacKinnon [2001], and Chandrashekar [1999].

¹⁸ See Case *et al.* [2006], Huang and Zhong [2012], and Westerheide [2006].

Diebold-Mariano Tests

Diebold and Mariano's (1995, henceforth D-M) statistic compares the accuracy of two sets of forecasts using a selected loss function, $L(r_{t+h}, \hat{r}_{t+h|t}^i) = L(\hat{\varepsilon}_{t+h|t}^i)$ $i = 1, \dots, 7$ (to index models), where r_{t+h} is the return series to be predicted h periods into the future, $\hat{r}_{t+h|t}^i$ is a series of h -step forecasts, and $\hat{\varepsilon}_{t+h|t}^i$ is the implied series of forecast errors. The DM test statistic is

$$DM_P \equiv \frac{1}{\sqrt{P}} \frac{1}{\hat{\sigma}_P} \sum_{t=T-P+1}^{T-h} [L(\hat{\varepsilon}_{t+h|t}^{m_1}) - L(\hat{\varepsilon}_{t+h|t}^{m_2})],$$

where $\hat{\sigma}_P$ is a (corrected) estimator of the sample standard deviation of the differences in loss functions that has a complex expression that can be found, *e.g.*, in Corradi and Swanson [2006], and P is the length of the OOS period over which our real time assessment is performed. Under some technical assumptions, as both P and the overall sample size $T \rightarrow \infty$, under the null hypothesis, $DM_P \rightarrow^D N(0,1)$ and under the alternative hypothesis the test will have asymptotically unit power.¹⁹ Two commonly used loss functions are the squared, $L(\hat{\varepsilon}_{t+h|t}^i) = (\hat{\varepsilon}_{t+h|t}^i)^2$, and the absolute error loss, $L(\hat{\varepsilon}_{t+h|t}^i) = |\hat{\varepsilon}_{t+h|t}^i|$. Clearly, while the former loss reflects the same functional form as the MSE criterion, the latter is affine to the MAE. Tables 6-8 show the D-M statistic computed from the series of forecasts for REIT, stock, and bond (mean) returns (Table 6), variances (Table 7), and covariances (Table 8) produced by each pair of models for the one-month horizon.²⁰ In each panel, the figures above the diagonal are the D-M test statistics for the squared error loss function, while those below the diagonal are the statistics for the absolute error loss function; in both cases a positive (negative) statistic implies that the model in the column is superior (inferior) to the model in the row.

Table 6 presents the D-M test results for point forecasts. The results are striking: across all three assets, in every test pairing the four-state regime switching model against any of the other models, the MS model provides superior one-month-ahead forecasts regardless of whether the squared or absolute error loss function is used. In every case, the p-value implies rejection of the null hypothesis (of equal accuracy) at a high level of statistical significance, while not a single statistic comparing two non-MS models is significantly different from zero.

Table 7 presents the D-M statistics comparing one-month variance forecasts. The variance forecasts produced by the MS model appear to be superior only to those implied by the Gaussian IID random walk and the homoskedastic Gaussian VAR(1) model, and then only according to the

¹⁹ Although the result has been reported for one-step ahead errors, all results carry over to the case of $h > 1$.

²⁰ We have also performed D-M tests for 12-month forecasts but found results that are both qualitatively similar to those in Tables 6-8 and that in general reveal few or no statistically significant differences.

absolute error loss function and only for stocks and bonds. Of the four GARCH specifications, none appears to be consistently superior but according to the absolute error loss function the CCC specification outperforms all three other GARCH models in predicting stock variances, and outperforms the *t*-Student and DCC specifications in predicting bond variances; furthermore, the sign of the D-M statistic suggests superior CCC-GARCH variance forecasts in most of the pairwise comparisons that do not reach common levels of statistical significance.

Turning to Table 8 showing the results for return covariances, the results are again surprising: the D-M statistics indicate that the covariance forecasts produced by the four-state MS model are superior to those produced by every other model tested, and the test statistics are statistically significant for every comparison using the absolute error loss function for REIT-stock and REIT-bond covariance forecasts, as well as several of the comparisons of stock-bond covariance forecasts.

To summarize the D-M test results, the four-state MS model appears to produce more accurate one-month forecasts of both returns and covariances than any of the other models tested. This result is consistent with the conclusions suggested by the comparisons of RMSE results, forecast error variance, and MAE shown in Tables 3 and 5. GARCH-type specifications, however, appear to produce superior one-month variance forecasts, and the CCC specification appears to be the strongest performer among the class of GARCH models—a result that is, similarly, consistent with the forecast error statistics shown in Table 4.

Tests of Density Forecast Accuracy

The severe declines in most asset values during the 2008-2009 financial crisis has attracted the attention of both academics and practitioners to higher-moment statistics summarizing the shapes of return densities, especially skewness (positive or negative asymmetry in the return distribution) and kurtosis (tail probabilities greater or less than those implied by a normal distribution with identical variance). While accurate forecasts of means, variances, and covariances will be useful for many purposes in portfolio management, for many risk management purposes it will be important to evaluate each model's success in forecasting the entirety of the return density.

Following Diebold *et al.* [1998] and Berkowitz [2001], we evaluate the density forecasts produced by our seven multivariate models using seven statistics to test whether the probability integral transform $F_t(r_t | \mathfrak{I}_{t-1}, \boldsymbol{\theta}) \equiv \int_{-\infty}^{r_t} f_t(r_t | \mathfrak{I}_{t-1}, \boldsymbol{\theta}) dr_t$ is distributed i.i.d. $U[0,1]$, which will be the case if the sequence of density forecasts corresponds to the true data generating process. Here \mathfrak{I}_{t-1} is the

conditioning information set as of time $t-1$, for instance one lag of past information in the case of VAR(1) and first-order Markov processes. First, we use Kolmogorov and Cramer-von Mises tests of the null hypothesis that the scores $\hat{U}_t \equiv F_t(r_t | \mathfrak{T}_{t-1}, \hat{\theta})$ are distributed $U[0,1]$. Second, we apply Ljung-Box tests of zero serial correlation up to lag 12 on both the transformed scores $z_t \equiv \Phi^{-1} \left[\int_{-\infty}^{r_t} f_t(u) du \right]$ and their squares to test for serial correlation; we should fail to reject the null hypothesis of no serial correlation if the scores (and therefore their transforms) are independently distributed. Third, we jointly test whether the transformed scores have zero mean and unit variance, as they should if $U_t \sim U[0,1]$, which Diebold, Gunther and Tay [1998] and Berkowitz [2001] have shown to be equivalent to correct specification of the model. Finally, we use Jarque-Bera and Cramer-von Mises tests for whether the transformed scores are normally distributed.

Table 9 presents the results of all seven tests on one-month density predictions for REITs, stocks, and bonds, respectively, produced by our seven modeling frameworks. With respect to the Kolmogorov and Cramer-von Mises tests of uniformity of the distribution of the scores, we fail to reject the null hypothesis (that the models are correctly specified) in every case except the four-state MS forecasts of bond return densities. The Ljung-Box tests likewise fail to reject the null hypothesis that the transformed scores are independently distributed; applied to the squared transformed scores for forecasted stock return densities, however, the Ljung-Box test fails to reject the null except in the cases of the CCC-GARCH specification and the MS model.

The joint test of zero mean and unit variance of the transformed scores rejects the null hypothesis (of correct model specification) for the IID random walk benchmark and the homoskedastic VAR(1) model, and also for all four GARCH models in forecasting stock return densities, as well as for the MS model in forecasting bond return densities. The test fails to reject the null for the MS model in forecasting REIT and stock return densities, and for all four GARCH models in forecasting REIT and bond return densities.

The Jarque-Bera and Cramer-von Mises tests of normality of the transformed scores reject the null hypothesis (of correct model specification) in most cases: none of the specifications, it seems, successfully models the true data generating process. On the other hand, all four GARCH models (along with the IID model) are fairly successful at forecasting bond densities according to the Cramer-von Mises test, and the DCC-GARCH model also fails to reject the null hypothesis according to a Jarque-Bera test.

To summarize, none of the models appears to have successfully replicated the data generating process: in every case there is some evidence that some (unspecified) characteristic of the return density has been forecast incorrectly. Nevertheless, the density prediction tests offer some support for (1) either the MS model or any of the GARCH models—especially the CCC specification—as the best approach to forecast REIT return densities; (2) any of the GARCH models—especially the DCC specification—as the best approach to forecasting bond return densities; and (3) the MS model as the best approach to forecast stock return densities.

Conclusion

The objective of this paper is to investigate whether the favorable performance of a fairly simple multistate Markov regime switching model relative to even very complex multivariate GARCH specifications, observed by Case, Guidolin and Yildirim [2013] using measures of in-sample prediction accuracy, extends to real time, pseudo OOS forecasting accuracy. To evaluate this, we recursively estimated the parameters of each model specification and used it to forecast means and variances for the REIT, stock, and bond asset classes; REIT-stock, REIT-bond, and stock-bond covariances; and the entire return densities for the three assets.

Our OOS tests indicated that the four-regime MS model was, indeed, superior to all of the GARCH specifications in forecasting returns on each asset class, showing a smaller root mean squared forecast error, smaller error variance, smaller mean absolute error, and (for stock and bond returns) smaller bias over one-month forecast horizons. Diebold-Mariano tests confirmed that the MS model outperformed each alternative specification in pairwise comparisons using either the squared or absolute error loss functions.

Perhaps not surprisingly, our forecast tests indicated that, for the purpose of predicting return variances, the MS model is inferior (except under a mean absolute error criterion) to carefully constructed GARCH models, especially the CCC-GARCH specification. Diebold-Mariano tests generally confirmed the inferiority of the MS approach and the superiority of the GARCH approach, especially the CCC specification, for the purpose of forecasting volatility.

The most surprising result of our analysis, given the underperformance of the multistate MS model in forecasting variances, is its superior performance in forecasting inter-asset covariances. Our OOS tests indicated that the MS model produced the smallest RMSE error, smallest error variance, and smallest MAE, and Diebold-Mariano tests confirmed the superiority of the MS model in pairwise comparisons, with statistically significant differences in all comparisons of REIT-stock and REIT-bond covariance forecasts using an absolute error loss criterion.

While predictive density tests indicated some failure to forecast correctly the entire return distribution, they provided some support for the MS model as the superior approach to forecasting equity densities and a not inferior approach to forecasting REIT densities, while suggesting the superiority of a GARCH approach (especially the DCC-GARCH specification) for forecasting bond return densities.

To summarize, if the primary objective is to form superior forecasts of asset return variances in a multivariate framework, the analysis presented in this paper suggests that a GARCH model—especially the CCC-GARCH specification—systematically outperforms a four-state heteroskedastic Markov switching model. In contrast, if the primary objective is to forecast asset returns, the MS model appears to be superior to any of the GARCH specifications tested in this paper. Moreover, and most surprisingly, the MS framework appears to be comparable to any of the GARCH specifications for the purpose of forecasting covariances.

Of course, it would be interesting to try to combine the two main classes of multivariate non-linear models examined in this paper—i.e., multistate MS models with GARCH—to yield regime switching models. Although recent work by Pelletier [2006] shows how that can be accomplished in the case of simple CCC GARCH models, the technical and numerical difficulties of the general BEKK and DCC cases recommend leaving this exciting extension to future work.

A practical extension of our analysis would be to evaluate the economic benefits (after trading costs) of a framework in which multistate MS modeling, combined with GARCH, is used to guide monthly tactical reallocations among asset classes within a strategic allocation fixed by implied unconditional expected returns, variances, and covariances. Preliminary evidence in Bianchi and Guidolin [2013] and Sa-Aadu *et al.* [2010] show that portfolios that are regularly rebalanced to follow the signals from a simple MS model yield realized risk-adjusted performances that cannot be replicated by any homoskedastic VAR framework, even within a large family that includes a range of alternative predictors. However, their results represent largely examples and only place a lower bound to the measure of actual economic value that non-linear modeling in tactical re-allocations among REITS, stocks, and bonds may yield.

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Table 1

Summary Statistics for US REITs, Stock, and Long-Term Government Bond Returns

The table reports basic summary statistics for U.S. stock, REIT, and (long-term, government) bond returns. For benchmarking purposes, also information on 1-month T-bill yields is provided. In the table, statistics in parenthesis are the p-values associated to the null hypothesis of a zero value for the parameter or statistic under investigation. When possible, the p-values are computed for two-tailed tests of hypothesis. In the case of kurtosis, the null hypothesis is of a kurtosis that equals the Gaussian benchmark of 3. Jarque-Bera is a test of distributional normality based on deviations of skewness and kurtosis coefficients from the null of normality. LB(12) is the Ljung-Box test for zero serial correlation up to order 12 for levels and squared returns, respectively. The null hypothesis of a zero median return is tested using a Wilcoxon signed-rank test. In the case of volatility, the null hypothesis is that the volatility of an asset class is the same as 1-month T-bills, and the hypothesis is tested using a variance ratio test.

	Mean	Volatility	Sharpe ratio	Median	Skewness	Excess Kurtosis	Jarque-Bera	LB(12) - Levels	LB(12) - Squares
Full Sample (1972-2010)									
REIT Returns	1.078 (0.000)	4.993 (0.000)	0.125 —	1.308 (0.000)	-0.756 (0.166)	8.43 (0.048)	1430.3 (0.000)	71.997 (0.000)	244.2 (0.000)
Stock Returns	0.889 (0.000)	4.660 (0.000)	0.094 —	1.280 (0.000)	-0.572 (0.063)	2.115 (0.096)	112.72 (0.000)	1.644 (0.999)	24.777 (0.016)
Govt. Bond Returns	0.663 (0.000)	2.311 (0.000)	0.091 —	0.615 (0.000)	0.310 (0.126)	1.311 (0.069)	40.992 (0.000)	14.461 (0.272)	90.471 (0.000)
1-Month T-Bill Returns	0.452 (0.001)	0.264 —	— —	0.429 (0.000)	0.749 (0.005)	1.172 (0.080)	70.546 (0.000)	4031.2 (0.000)	3219.8 (0.000)
Pre-Modern Era (1972-1991)									
REIT Returns	1.104 (0.011)	4.144 (0.000)	0.116 —	1.136 (0.000)	-0.349 (0.220)	2.253 (0.067)	55.619 (0.000)	20.993 (0.050)	18.611 (0.098)
Stock Returns	1.060 (0.001)	4.867 (0.000)	0.089 —	1.049 (0.000)	-0.386 (0.056)	2.465 (0.024)	66.718 (0.000)	9.046 (0.699)	7.273 (0.839)
Govt. Bond Returns	0.769 (0.000)	2.525 (0.000)	0.057 —	0.604 (0.000)	0.458 (0.013)	1.076 (0.168)	19.979 (0.001)	15.561 (0.212)	57.518 (0.000)
1-Month T-Bill Returns	0.625 (0.000)	0.228 —	— —	0.594 (0.000)	1.177 (0.000)	1.573 (0.045)	80.125 (0.000)	1336.3 (0.000)	1175.3 (0.000)
Modern Era (1992-2010)									
REIT Returns	1.050 (0.006)	5.763 (0.000)	0.182 —	1.618 (0.000)	-0.869 (0.219)	8.787 (0.068)	762.27 (0.000)	75.082 (0.000)	137.33 (0.000)
Stock Returns	0.736 (0.014)	4.483 (0.000)	0.164 —	1.339 (0.001)	-0.840 (0.038)	1.532 (0.192)	49.128 (0.000)	9.744 (0.638)	55.739 (0.000)
Govt. Bond Returns	0.552 (0.000)	2.061 (0.000)	0.268 —	0.633 (0.000)	-0.070 (0.428)	1.134 (0.192)	12.414 (0.002)	15.794 (0.201)	14.881 (0.248)
1-Month T-Bill Returns	0.270 (0.000)	0.153 —	— —	0.300 (0.000)	-0.345 (0.005)	-1.184 (0.000)	17.840 (0.000)	1906.6 (0.000)	1679.6 (0.000)

Table 2
Estimates of VAR(1) Four-State Markov Switching Heteroskedastic Model

The rightmost column reports unconditional means, volatilities, correlations, and Sharpe ratios implied by the model conditional in each of the four possible regimes. The Sharpe ratios are computed with reference to the average 1-month T-bill yield. Unconditional moments are computed under the (counter-factual) assumption that the system never leaves a given state and they should be interpreted with caution. The elements in the last column of the estimate Markov transition matrix are not associated to standard errors because of the presence of an adding-up constraint in estimation (by which the elements in each of row of the transition probability must sum to a total probability of 1). The joint Jarque-Bera test is computed using a Doornik-Hansen type square root of correlation matrix transformation to compute standardized residuals which are independent of the ordering of the asset return variables in the estimated vector system. In the table, boldfaced coefficients are significant at 5% or lower size.

Four-State VAR(1) Markov Switching Model						
	Unconditional Means, Vols & Correlations (monthly)					
Regime 1:						
Conditional mean functions	$r_{REIT,t} = 0.704 + 0.077r_{REIT,t-1} + 0.008r_{Stock,t-1} + \mathbf{0.256}r_{Bond,t-1} + (h_{2,REIT,t})^{1/2}u_{REIT,t}$ <div>(0.403) (0.052) (0.048) (0.074)</div>					0.785
	$r_{Stock,t} = 0.289 + \mathbf{0.133}r_{REIT,t-1} - 0.089r_{Stock,t-1} + 0.005r_{Bond,t-1} + (h_{2,Stock,t})^{1/2}u_{Stock,t}$ <div>(0.382) (0.057) (0.057) (0.087)</div>					0.217
	$r_{Bond,t} = \mathbf{0.509} - 0.010r_{REIT,t-1} - \mathbf{0.076}r_{Stock,t-1} + \mathbf{0.092}r_{Bond,t-1} + (h_{2,Bond,t})^{1/2}u_{Bond,t}$ <div>(0.126) (0.023) (0.025) (0.045)</div>					0.783
Conditional variance functions	$h_{REIT,t} = \mathbf{23.182} \quad h_{Stock,t} = \mathbf{23.020} \quad h_{bond,t} = \mathbf{2.834}$			4.815	4.798	1.683
Conditional correlation functions	$\rho[REIT, Stock] = \mathbf{0.454} \quad \rho[REIT, Gov. bond] = \mathbf{-0.132} \quad \rho[Stock, Gov. bond] = \mathbf{-0.215}$			0.454	-0.132	-0.215
	Implied Sharpe ratios:			0.092	-0.027	0.261
Regime 2:						
Conditional mean functions	$r_{REIT,t} = \mathbf{0.532} + 0.077r_{REIT,t-1} + 0.008r_{Stock,t-1} + \mathbf{0.256}r_{Bond,t-1} + (h_{3,REIT,t})^{1/2}u_{REIT,t}$ <div>(0.229) (0.052) (0.048) (0.074)</div>					0.660
	$r_{Stock,t} = \mathbf{0.743} + \mathbf{0.133}r_{REIT,t-1} - 0.089r_{Stock,t-1} + 0.005r_{Bond,t-1} + (h_{3,Stock,t})^{1/2}u_{Stock,t}$ <div>(0.310) (0.057) (0.057) (0.087)</div>					0.636
	$r_{Bond,t} = \mathbf{0.490} - 0.010r_{REIT,t-1} - \mathbf{0.076}r_{Stock,t-1} + \mathbf{0.092}r_{Bond,t-1} + (h_{3,Bond,t})^{1/2}u_{Bond,t}$ <div>(0.174) (0.023) (0.025) (0.045)</div>					0.729
Conditional variance functions	$h_{REIT,t} = \mathbf{6.759} \quad h_{Stock,t} = \mathbf{12.821} \quad h_{bond,t} = \mathbf{3.622}$			2.600	3.581	1.903
Conditional correlation functions	$\rho[REIT, Stock] = \mathbf{0.654} \quad \rho[REIT, Gov. bond] = \mathbf{0.410} \quad \rho[Stock, Gov. bond] = \mathbf{0.493}$			0.654	0.410	0.493
	Implied Sharpe ratios:			0.060	0.037	0.118

Table 2 (continued)
Estimates of VAR(1) Four-State Markov Switching Heteroskedastic Model

Four-State VAR(1) Markov Switching Model										
Regime 3:										
Conditional mean functions	$r_{REIT,t} = \mathbf{3.121} + 0.077r_{REIT,t-1} + 0.008r_{Stock,t-1} + \mathbf{0.256}r_{Bond,t-1} + (h_{4,REIT,t})^{1/2}u_{REIT,t}$ (0.678) (0.052) (0.048) (0.074)						3.764			
	$r_{Stock,t} = \mathbf{3.363} + \mathbf{0.133}r_{REIT,t-1} - 0.089r_{Stock,t-1} + 0.005r_{Bond,t-1} + (h_{4,Stock,t})^{1/2}u_{Stock,t}$ (0.808) (0.057) (0.057) (0.087)						2.904			
	$r_{Bond,t} = \mathbf{1.773} - 0.010r_{REIT,t-1} - \mathbf{0.076}r_{Stock,t-1} + \mathbf{0.092}r_{Bond,t-1} + (h_{4,Bond,t})^{1/2}u_{Bond,t}$ (0.540) (0.023) (0.025) (0.045)						3.028			
Conditional variance functions	$h_{REIT,t} = \mathbf{12.098}$			$h_{Stock,t} = \mathbf{13.838}$		$h_{bond,t} = \mathbf{10.320}$		3.478	3.720	3.212
Conditional correlation functions	$\rho[REIT, Stock] = \mathbf{0.291}$			$\rho[REIT, Gov. bond] = 0.002$		$\rho[Stock, Gov. bond] = \mathbf{0.258}$		0.291	0.002	0.258
Implied Sharpe ratios:								0.936	0.644	0.784
Regime 4:										
Conditional mean functions	$r_{REIT,t} = -5.808 + 0.077r_{REIT,t-1} + 0.008r_{Stock,t-1} + \mathbf{0.256}r_{Bond,t-1} + (h_{1,REIT,t})^{1/2}u_{REIT,t}$ (4.577) (0.052) (0.048) (0.074)						-6.894			
	$r_{Stock,t} = -4.869 + \mathbf{0.133}r_{REIT,t-1} - 0.089r_{Stock,t-1} + 0.005r_{Bond,t-1} + (h_{1,Stock,t})^{1/2}u_{Stock,t}$ (3.158) (0.057) (0.057) (0.087)						-4.367			
	$r_{Bond,t} = -0.214 - 0.010r_{REIT,t-1} - \mathbf{0.076}r_{Stock,t-1} + \mathbf{0.092}r_{Bond,t-1} + (h_{1,Bond,t})^{1/2}u_{Bond,t}$ (1.221) (0.023) (0.025) (0.045)						-2.198			
Conditional variance functions	$h_{REIT,t} = \mathbf{246.892}$			$h_{Stock,t} = \mathbf{89.963}$		$h_{bond,t} = \mathbf{18.584}$		15.713	9.485	4.311
Conditional correlation functions	$\rho[REIT, Stock] = \mathbf{0.810}$			$\rho[REIT, Gov. bond] = 0.034$		$\rho[Stock, Gov. bond] = -0.115$		0.810	0.034	-0.115
Implied Sharpe ratios:								-0.458	-0.491	-0.578
Estimated Transition Matrix										
	Regime 1	Regime 2		Regime 3		Regime 4		Ergodic Probs.		
Regime 1	0.977 (0.080)	0.011 (0.068)		0.000		0.012 (0.010)		0.410		
Regime 2	0.008 (0.008)	0.889 (0.010)		0.099		0.004 (0.010)		0.379		
Regime 3	0.002 (0.009)	0.213 (0.036)		0.737		0.049 (0.011)		0.176		
Regime 4	0.180 (0.033)	0.003 (0.017)		0.259		0.558 (0.122)		0.035		
Log-Likelihood	2903.077			Number of parameters			57			
Akaike information criterion	-12.3732			Saturation ratio			24.58			
Bayes-Schwartz information criterion	-11.9536			Joint Jarque-Bera on stdzed res.			15.005			
Hannan-Quinn information criterion	-12.2682			Mult. Ljung-Box(12) autocorrel.			130.23*			

Table 3

Summary Measures of Predictive Accuracy for the Mean (Point Forecasts)

For 7 multivariate models in this paper, we report the MSE, RMSE, the error variance, the prediction bias, and the MAE of the recursive, pseudo OOS point forecasts obtained from real time parameter estimation and updating. MSE is the mean squared forecast error, and MAE is the mean absolute forecast error,

$$\text{MSE}_{m,i} = \frac{1}{T-h} \sum_{t=1}^{T-h} (e_{t,h}^m)^2, \quad \text{MAE}_{m,i} = \frac{1}{T-h} \sum_{t=1}^{T-h} |e_{t,h}^m|$$

where m indicates the model under examination, h is the forecast horizon, and $e_{t,h}$ represents the forecast error, $e_{t,h}$ obtained as a difference between realized monthly returns and predicted ones. RMSE is the square root of MSE. The best performance measures across the set of 7 models are boldfaced.

Panel A: REIT Returns (Point Forecasts, in % terms)

Model	Mean Squared Error		Root Mean Squared		Error Variance		Bias		Mean Absolute Error	
	<i>H = 1 month</i>	<i>H = 12 month</i>	<i>H = 1 month</i>	<i>H = 12 month</i>	<i>H = 1 month</i>	<i>H = 12 month</i>	<i>H = 1 month</i>	<i>H = 12 month</i>	<i>H = 1 month</i>	<i>H = 12 month</i>
Gaussian IID Benchmark	31.860	32.415	5.644	5.693	31.856	32.414	-0.062	0.030	3.804	3.817
Gaussian VAR(1)	30.633	32.363	5.535	5.689	30.633	32.361	-0.000	0.038	3.796	3.814
Gaussian VAR(1) Diag. BEKK GARCH (1,1)	31.774	32.245	5.639	5.697	31.758	32.452	-0.126	0.029	3.796	3.816
t-Student VAR(1) Diag. BEKK GARCH (1,1)	31.765	32.571	5.636	5.708	31.749	32.575	-0.124	0.010	3.791	3.820
Gaussian VAR(1) CCC GARCH (1,1)	31.776	32.636	5.637	5.713	31.762	32.634	-0.121	0.037	3.789	3.828
Gaussian VAR(1) DCC GARCH (1,1)	30.633	32.568	5.535	5.707	30.633	32.567	-0.000	0.031	3.791	3.835
Four-State Markov Switching VAR(1)	24.610	32.464	4.961	5.670	24.602	32.463	0.088	0.042	3.437	3.824

Table 3 (continued)
Summary Measures of Predictive Accuracy for the Mean (Point Forecasts)

Panel B: Stock Returns (Point Forecasts, in % terms)

Model	Mean Squared Error		Root Mean Squared		Error Variance		Bias		Mean Absolute Error	
	<i>H = 1 month</i>	<i>H = 12 month</i>	<i>H = 1 month</i>	<i>H = 12 month</i>	<i>H = 1 month</i>	<i>H = 12 month</i>	<i>H = 1 month</i>	<i>H = 12 month</i>	<i>H = 1 month</i>	<i>H = 12 month</i>
Gaussian IID Benchmark	20.286	20.100	4.504	4.483	20.241	20.063	-0.212	-0.192	3.442	3.415
Gaussian VAR(1)	20.461	20.066	4.523	4.480	20.433	20.037	-0.168	-0.172	3.484	3.413
Gaussian VAR(1) Diag. BEKK GARCH (1,1)	20.274	20.216	4.503	4.496	20.223	20.174	-0.225	-0.206	3.429	3.424
t-Student VAR(1) Diag. BEKK GARCH (1,1)	20.275	20.185	4.503	4.493	20.226	20.151	-0.222	-0.185	3.434	3.417
Gaussian VAR(1) CCC GARCH (1,1)	20.271	20.117	4.502	4.485	20.222	20.077	-0.222	-0.200	3.430	3.415
Gaussian VAR(1) DCC GARCH (1,1)	20.461	20.117	4.523	4.485	20.433	20.086	-0.168	-0.176	3.484	3.420
Four-State Markov Switching VAR(1)	16.853	20.046	4.105	4.477	16.853	20.027	-0.000	-0.140	3.191	3.412

Panel C: Bond Returns (Point Forecasts, in % terms)

Model	Mean Squared Error		Root Mean Squared		Error Variance		Bias		Mean Absolute Error	
	<i>H = 1 month</i>	<i>H = 12 month</i>	<i>H = 1 month</i>	<i>H = 12 month</i>	<i>H = 1 month</i>	<i>H = 12 month</i>	<i>H = 1 month</i>	<i>H = 12 month</i>	<i>H = 1 month</i>	<i>H = 12 month</i>
Gaussian IID Benchmark	4.142	4.208	2.035	2.051	4.129	4.191	-0.116	0.129	1.544	1.549
Gaussian VAR(1)	3.916	4.176	1.979	2.044	3.900	4.160	-0.124	-0.129	1.517	1.544
Gaussian VAR(1) Diag. BEKK GARCH (1,1)	4.140	4.188	2.035	2.047	4.118	4.170	-0.150	-0.132	1.541	1.544
t-Student VAR(1) Diag. BEKK GARCH (1,1)	4.136	4.202	2.034	2.049	4.114	4.185	-0.151	-0.133	1.541	1.547
Gaussian VAR(1) CCC GARCH (1,1)	4.150	4.192	2.037	2.047	4.128	4.175	-0.149	-0.129	1.543	1.547
Gaussian VAR(1) DCC GARCH (1,1)	3.916	4.220	1.979	2.054	3.900	4.202	-0.124	-0.133	1.517	1.546
Four-State Markov Switching VAR(1)	3.312	4.188	1.820	2.047	3.331	4.177	-0.050	-0.107	1.391	1.542

Table 4

Summary Measures of Predictive Accuracy for the Variance (Point Forecasts of Squared Realized Returns)

For 7 multivariate models in this paper, we report the MSE, RMSE, the error variance, the prediction bias, and the MAE of the recursive, pseudo OOS point forecasts obtained from real time parameter estimation and updating. MSE is the mean squared forecast error, and MAE is the mean absolute forecast error,

$$\text{MSE}_{m,i} = \frac{1}{T-h} \sum_{t=1}^{T-h} (e_{t,h}^m)^2, \quad \text{MAE}_{m,i} = \frac{1}{T-h} \sum_{t=1}^{T-h} |e_{t,h}^m|,$$

where m indicates the model under examination, h is the forecast horizon, and $e_{t,h}$ represents the forecast error, $e_{t,h}$ obtained as a difference between realized *squared* monthly returns and predicted variances. RMSE is the square root of MSE. The best performance measures across the set of 7 models are boldfaced.

Panel A: REIT Squared Returns (in % terms)

Model	Mean Squared Error		Root Mean Squared		Error Variance		Bias		Mean Absolute Error	
	<i>H = 1 month</i>	<i>H = 12 month</i>	<i>H = 1 month</i>	<i>H = 12 month</i>	<i>H = 1 month</i>	<i>H = 12 month</i>	<i>H = 1 month</i>	<i>H = 12 month</i>	<i>H = 1 month</i>	<i>H = 12 month</i>
Gaussian IID Benchmark	1.048	1.111	1.024	1.052	1.022	1.076	0.160	0.172	0.309	0.317
Gaussian VAR(1)	1.052	1.104	1.026	1.051	1.022	1.075	0.173	0.171	0.306	0.316
Gaussian VAR(1) Diag. BEKK GARCH (1,1)	0.878	1.107	0.937	1.052	0.875	1.076	0.054	0.176	0.323	0.318
t-Student VAR(1) Diag. BEKK GARCH (1,1)	0.873	1.099	0.934	1.048	0.868	1.069	0.069	0.173	0.315	0.314
Gaussian VAR(1) CCC GARCH (1,1)	0.871	1.095	0.933	1.047	0.868	1.069	0.058	0.162	0.320	0.316
Gaussian VAR(1) DCC GARCH (1,1)	0.897	1.092	0.947	1.045	0.892	1.062	0.067	0.172	0.323	0.315
Four-State Markov Switching VAR(1)	1.038	1.104	1.019	1.050	1.006	1.074	0.178	0.172	0.302	0.317

Table 4 (continued)

Summary Measures of Predictive Accuracy for the Variance (Point Forecasts of Squared Realized Returns)

Panel B: Stock Squared Returns (in % terms)

Model	Mean Squared Error		Root Mean Squared		Error Variance		Bias		Mean Absolute Error	
	<i>H</i> = 1 month	<i>H</i> = 12 month	<i>H</i> = 1 month	<i>H</i> = 12 month	<i>H</i> = 1 month	<i>H</i> = 12 month	<i>H</i> = 1 month	<i>H</i> = 12 month	<i>H</i> = 1 month	<i>H</i> = 12 month
Gaussian IID Benchmark	0.121	0.124	0.348	0.353	0.121	0.124	-0.004	-0.005	0.209	0.210
Gaussian VAR(1)	0.121	0.124	0.348	0.352	0.121	0.124	0.002	-0.005	0.206	0.210
Gaussian VAR(1) Diag. BEKK GARCH (1,1)	0.111	0.125	0.333	0.354	0.111	0.125	0.001	-0.003	0.185	0.212
t-Student VAR(1) Diag. BEKK GARCH (1,1)	0.111	0.122	0.333	0.350	0.111	0.122	0.001	-0.002	0.185	0.206
Gaussian VAR(1) CCC GARCH (1,1)	0.110	0.123	0.332	0.351	0.110	0.123	-0.000	-0.002	0.182	0.213
Gaussian VAR(1) DCC GARCH (1,1)	0.111	0.123	0.334	0.351	0.111	0.123	-0.005	-0.002	0.186	0.206
Four-State Markov Switching VAR(1)	0.118	0.124	0.343	0.352	0.116	0.124	0.049	0.003	0.186	0.205

Panel C: Bond Squared Returns (in % terms)

Model	Mean Squared Error		Root Mean Squared		Error Variance		Bias		Mean Absolute Error	
	<i>H</i> = 1 month	<i>H</i> = 12 month	<i>H</i> = 1 month	<i>H</i> = 12 month	<i>H</i> = 1 month	<i>H</i> = 12 month	<i>H</i> = 1 month	<i>H</i> = 12 month	<i>H</i> = 1 month	<i>H</i> = 12 month
Gaussian IID Benchmark	0.006	0.006	0.078	0.079	0.006	0.006	-0.012	-0.012	0.052	0.052
Gaussian VAR(1)	0.006	0.006	0.077	0.079	0.006	0.006	-0.010	-0.012	0.051	0.053
Gaussian VAR(1) Diag. BEKK GARCH (1,1)	0.006	0.006	0.076	0.079	0.006	0.006	0.000	-0.011	0.046	0.053
t-Student VAR(1) Diag. BEKK GARCH (1,1)	0.006	0.006	0.077	0.079	0.006	0.006	-0.002	-0.011	0.047	0.052
Gaussian VAR(1) CCC GARCH (1,1)	0.006	0.006	0.076	0.079	0.006	0.006	0.002	-0.012	0.045	0.052
Gaussian VAR(1) DCC GARCH (1,1)	0.006	0.006	0.076	0.079	0.006	0.006	-0.001	-0.011	0.046	0.052
Four-State Markov Switching VAR(1)	0.006	0.006	0.077	0.080	0.006	0.006	0.007	-0.006	0.044	0.050

Table 5

Summary Measures of Predictive Accuracy for Covariances (Forecasts of Cross-Realized Returns)

For 7 multivariate models in this paper, we report the MSE, RMSE, the error variance, the prediction bias, and the MAE of the recursive, pseudo OOS point forecasts obtained from real time parameter estimation and updating. MSE is the mean squared forecast error, and MAE is the mean absolute forecast error,

$$\text{MSE}_{m,i} = \frac{1}{T-h} \sum_{t=1}^{T-h} (e_{t,h}^m)^2, \quad \text{MAE}_{m,i} = \frac{1}{T-h} \sum_{t=1}^{T-h} |e_{t,h}^m|,$$

where m indicates the model under examination, h is the forecast horizon, and $e_{t,h}$ represents the forecast error, $e_{t,h}$ obtained as a difference between realized *cross products* of monthly returns for pairs of assets and predicted covariances. RMSE is the square root of MSE. The best performance measures across the set of 7 models are boldfaced.

Panel A: REIT-Stock Covariance (in % terms)

Model	Mean Squared Error		Root Mean Squared		Error Variance		Bias		Mean Absolute Error	
	$H = 1 \text{ month}$	$H = 12 \text{ month}$	$H = 1 \text{ month}$	$H = 12 \text{ month}$	$H = 1 \text{ month}$	$H = 12 \text{ month}$	$H = 1 \text{ month}$	$H = 12 \text{ month}$	$H = 1 \text{ month}$	$H = 12 \text{ month}$
Gaussian IID Benchmark	0.250	0.261	0.500	0.511	0.249	0.261	0.027	0.028	0.196	0.199
Gaussian VAR(1)	0.250	0.261	0.500	0.511	0.249	0.260	0.025	0.027	0.194	0.199
Gaussian VAR(1) Diag. BEKK GARCH (1,1)	0.221	0.262	0.470	0.511	0.221	0.260	0.012	0.034	0.188	0.197
t-Student VAR(1) Diag. BEKK GARCH (1,1)	0.220	0.258	0.469	0.508	0.220	0.257	0.022	0.032	0.184	0.195
Gaussian VAR(1) CCC GARCH (1,1)	0.224	0.258	0.474	0.508	0.224	0.257	0.013	0.033	0.186	0.194
Gaussian VAR(1) DCC GARCH (1,1)	0.223	0.258	0.472	0.508	0.223	0.257	0.019	0.034	0.187	0.195
Four-State Markov Switching VAR(1)	0.209	0.270	0.457	0.519	0.206	0.263	0.056	-0.080	0.166	0.259

Table 5 (continued)

Summary Measures of Predictive Accuracy for Covariances (Forecasts of Cross-Realized Returns)

Panel B: REIT-Bond Covariance (in % terms)

Model	Mean Squared Error		Root Mean Squared		Error Variance		Bias		Mean Absolute Error	
	<i>H</i> = 1 month	<i>H</i> = 12 month	<i>H</i> = 1 month	<i>H</i> = 12 month	<i>H</i> = 1 month	<i>H</i> = 12 month	<i>H</i> = 1 month	<i>H</i> = 12 month	<i>H</i> = 1 month	<i>H</i> = 12 month
Gaussian IID Benchmark	0.040	0.041	0.200	0.203	0.040	0.041	-0.010	-0.011	0.084	0.085
Gaussian VAR(1)	0.040	0.041	0.200	0.203	0.040	0.041	-0.008	-0.011	0.084	0.085
Gaussian VAR(1) Diag. BEKK GARCH (1,1)	0.042	0.041	0.204	0.203	0.042	0.041	-0.002	-0.010	0.084	0.085
t-Student VAR(1) Diag. BEKK GARCH (1,1)	0.042	0.041	0.206	0.203	0.042	0.041	-0.003	-0.010	0.085	0.085
Gaussian VAR(1) CCC GARCH (1,1)	0.040	0.041	0.200	0.203	0.040	0.041	-0.007	-0.010	0.083	0.085
Gaussian VAR(1) DCC GARCH (1,1)	0.042	0.041	0.205	0.203	0.042	0.041	-0.002	-0.010	0.085	0.085
Four-State Markov Switching VAR(1)	0.038	0.042	0.196	0.204	0.038	0.041	-0.002	-0.027	0.081	0.091

Panel C: Stock-Bond Covariance (in % terms)

Model	Mean Squared Error		Root Mean Squared		Error Variance		Bias		Mean Absolute Error	
	<i>H</i> = 1 month	<i>H</i> = 12 month	<i>H</i> = 1 month	<i>H</i> = 12 month	<i>H</i> = 1 month	<i>H</i> = 12 month	<i>H</i> = 1 month	<i>H</i> = 12 month	<i>H</i> = 1 month	<i>H</i> = 12 month
Gaussian IID Benchmark	0.016	0.016	0.126	0.127	0.015	0.015	-0.025	-0.029	0.074	0.074
Gaussian VAR(1)	0.016	0.016	0.126	0.127	0.015	0.015	-0.025	-0.029	0.074	0.074
Gaussian VAR(1) Diag. BEKK GARCH (1,1)	0.015	0.016	0.124	0.127	0.015	0.015	-0.006	-0.028	0.069	0.074
t-Student VAR(1) Diag. BEKK GARCH (1,1)	0.015	0.016	0.124	0.126	0.015	0.015	-0.007	-0.026	0.070	0.073
Gaussian VAR(1) CCC GARCH (1,1)	0.016	0.016	0.126	0.126	0.016	0.015	-0.013	-0.026	0.071	0.073
Gaussian VAR(1) DCC GARCH (1,1)	0.015	0.016	0.124	0.126	0.015	0.015	-0.007	-0.026	0.070	0.073
Four-State Markov Switching VAR(1)	0.014	0.018	0.119	0.134	0.014	0.015	-0.010	-0.052	0.067	0.076

Table 6

Diebold Mariano Tests of Equal Predictive Accuracy for 1-Month Forecasts of Mean Asset Returns

For each pair of the 7 multivariate models entertained in this paper, the table reports the D-M statistic and the corresponding p-values computed from a HAC-adjusted estimator of the variance matrix of the loss function differences. Above the main diagonal the loss function is the squared loss; below the main diagonal the loss function is absolute loss. Above the main diagonal, a negative (positive) D-M statistic implies that the model in the row is inferior (positive) to the one in the column. Below the main diagonal, a negative (positive) D-M statistic implies that the model in the column is superior (inferior) to the one in the row. In the table, significant D-M statistics are boldfaced.

	Gaussian IID Benchmark	Gaussian VAR(1)	Gaussian VAR(1) Diag. BEKK GARCH (1,1)	t-Student VAR(1) Diag. BEKK GARCH (1,1)	Gaussian VAR(1) CCC GARCH (1,1)	Gaussian VAR(1) DCC GARCH (1,1)	Four-State Markov Switching VAR(1)
REIT Returns							
Gaussian IID Benchmark		1.216 (0.225)	0.672 (0.502)	0.670 (0.503)	0.675 (0.500)	1.216 (0.225)	2.426 (0.016)
Gaussian VAR(1)	-0.152 (0.880)		-1.072 (0.285)	-1.066 (0.288)	-1.068 (0.286)	NA	2.752 (0.006)
Gaussian VAR(1) Diag. BEKK GARCH (1,1)	-1.493 (0.137)	-0.101 (0.919)		0.227 (0.820)	-0.076 (0.939)	1.072 (0.285)	2.342 (0.020)
t-Student VAR(1) Diag. BEKK GARCH (1,1)	-1.683 (0.094)	-0.131 (0.896)	-0.467 (0.641)		-0.254 (0.800)	1.066 (0.288)	2.338 (0.020)
Gaussian VAR(1) CCC GARCH (1,1)	-1.510 (0.132)	-0.098 (0.922)	0.048 (0.962)	0.501 (0.617)		1.068 (0.286)	2.338 (0.020)
Gaussian VAR(1) DCC GARCH (1,1)	-0.152 (0.880)	NA	0.101 (0.919)	0.131 (0.896)	0.098 (0.922)		2.752 (0.006)
Four-State Markov Switching VAR(1)	-5.272 (0.000)	-5.927 (0.000)	-4.956 (0.000)	-4.952 (0.000)	-4.959 (0.000)	-5.927 (0.000)	
Stock Returns							
Gaussian IID Benchmark		-0.372 (0.710)	0.134 (0.894)	0.112 (0.911)	0.144 (0.886)	-0.372 (0.710)	3.672 (0.000)
Gaussian VAR(1)	0.838 (0.403)		0.380 (0.704)	0.378 (0.706)	0.388 (0.699)	NA	4.465 (0.000)
Gaussian VAR(1) Diag. BEKK GARCH (1,1)	-1.570 (0.118)	-1.098 (0.274)		-0.067 (0.947)	0.061 (0.951)	-0.380 (0.704)	3.469 (0.001)
t-Student VAR(1) Diag. BEKK GARCH (1,1)	-1.042 (0.299)	-1.020 (0.309)	1.278 (0.202)		0.112 (0.911)	-0.378 (0.706)	3.467 (0.001)
Gaussian VAR(1) CCC GARCH (1,1)	-1.478 (0.141)	-1.049 (0.275)	0.183 (0.855)	-1.040 (0.299)		-0.388 (0.699)	3.466 (0.001)
Gaussian VAR(1) DCC GARCH (1,1)	0.838 (0.403)	NA	1.098 (0.274)	1.020 (0.309)	1.094 (0.275)		4.465 (0.000)
Four-State Markov Switching VAR(1)	-3.580 (0.000)	-4.886 (0.000)	-3.268 (0.001)	-3.342 (0.001)	-3.291 (0.001)	-4.886 (0.000)	
Bond Returns							
Gaussian IID Benchmark		1.205 (0.230)	0.080 (0.936)	0.288 (0.774)	-0.372 (0.711)	1.205 (0.230)	3.635 (0.000)
Gaussian VAR(1)	-0.768 (0.443)		-1.204 (0.230)	-1.160 (0.247)	-1.229 (0.220)	NA	6.426 (0.000)
Gaussian VAR(1) Diag. BEKK GARCH (1,1)	-0.510 (0.610)	0.683 (0.495)		-0.080 (0.936)	1.204 (0.230)	0.425 (0.671)	3.655 (0.000)
t-Student VAR(1) Diag. BEKK GARCH (1,1)	-0.603 (0.547)	0.666 (0.506)	-0.150 (0.881)		-1.590 (0.113)	1.160 (0.247)	3.579 (0.000)
Gaussian VAR(1) CCC GARCH (1,1)	-0.063 (0.950)	0.736 (0.463)	1.006 (0.316)	1.172 (0.243)		1.229 (0.220)	3.634 (0.000)
Gaussian VAR(1) DCC GARCH (1,1)	-0.768 (0.443)	NA	-0.683 (0.495)	-0.666 (0.506)	-0.736 (0.463)		6.426 (0.000)
Four-State Markov Switching VAR(1)	-3.868 (0.000)	-5.687 (0.000)	-3.728 (0.000)	-3.684 (0.000)	-3.727 (0.000)	-5.687 (0.000)	

Table 7

Diebold Mariano Tests of Equal Predictive Accuracy for 1-Month Forecasts of Asset Return Variances

For each pair of the 7 multivariate models entertained in this paper, the table reports the D-M statistic and the corresponding p-values computed from a HAC-adjusted estimator of the variance matrix of the loss function differences. Above the main diagonal the loss function is the squared loss; below the main diagonal the loss function is absolute loss. Above the main diagonal, a negative (positive) D-M statistic implies that the model in the row is inferior (positive) to the one in the column. Below the main diagonal, a negative (positive) D-M statistic implies that the model in the column is superior (inferior) to the one in the row. In the table, significant D-M statistics are boldfaced.

	Gaussian IID Benchmark	Gaussian VAR(1)	Gaussian VAR(1) Diag. BEKK GARCH (1,1)	t-Student VAR(1) Diag. BEKK GARCH (1,1)	Gaussian VAR(1) CCC GARCH (1,1)	Gaussian VAR(1) DCC GARCH (1,1)	Four-State Markov Switching VAR(1)
REIT Returns							
Gaussian IID Benchmark		-1.738 (0.084)	1.241 (0.216)	1.267 (0.207)	1.239 (0.216)	1.215 (0.225)	0.552 (0.581)
Gaussian VAR(1)	-3.829 (0.000)		1.250 (0.213)	1.275 (0.204)	1.248 (0.213)	1.226 (0.221)	0.713 (0.477)
Gaussian VAR(1) Diag. BEKK GARCH (1,1)	0.698 (0.486)	0.878 (0.381)		1.880 (0.061)	1.146 (0.253)	-1.338 (0.182)	-1.262 (0.208)
t-Student VAR(1) Diag. BEKK GARCH (1,1)	0.307 (0.760)	0.491 (0.624)	-4.703 (0.000)		0.361 (0.718)	-1.521 (0.129)	-1.290 (0.198)
Gaussian VAR(1) CCC GARCH (1,1)	0.521 (0.603)	0.695 (0.488)	-2.640 (0.009)	3.062 (0.002)		-1.289 (0.199)	-1.261 (0.209)
Gaussian VAR(1) DCC GARCH (1,1)	0.709 (0.479)	0.904 (0.367)	-0.185 (0.853)	1.577 (0.116)	0.547 (0.585)		-1.229 (0.220)
Four-State Markov Switching VAR(1)	-1.365 (0.174)	-0.660 (0.510)	-1.151 (0.251)	-0.730 (0.466)	-0.940 (0.348)	-1.184 (0.238)	
Stock Returns							
Gaussian IID Benchmark		0.378 (0.706)	2.230 (0.027)	2.227 (0.027)	2.060 (0.040)	1.998 (0.047)	0.788 (0.432)
Gaussian VAR(1)	-5.306 (0.000)		2.155 (0.032)	2.153 (0.032)	1.998 (0.047)	1.933 (0.054)	0.767 (0.444)
Gaussian VAR(1) Diag. BEKK GARCH (1,1)	-3.734 (0.000)	-3.380 (0.001)		-0.516 (0.607)	1.012 (0.313)	-0.824 (0.411)	-1.697 (0.091)
t-Student VAR(1) Diag. BEKK GARCH (1,1)	-3.732 (0.000)	-3.384 (0.001)	-0.462 (0.645)		1.032 (0.303)	-0.727 (0.468)	-1.678 (0.095)
Gaussian VAR(1) CCC GARCH (1,1)	-3.588 (0.000)	-3.273 (0.001)	-2.326 (0.021)	-2.141 (0.033)		-1.697 (0.091)	-1.739 (0.083)
Gaussian VAR(1) DCC GARCH (1,1)	-3.160 (0.002)	-2.828 (0.005)	1.381 (0.169)	1.750 (0.081)	4.043 (0.000)		-1.481 (0.140)
Four-State Markov Switching VAR(1)	-4.244 (0.000)	-3.997 (0.000)	0.164 (0.870)	0.192 (0.848)	0.586 (0.559)	-0.105 (0.916)	
Bond Returns							
Gaussian IID Benchmark		2.215 (0.028)	1.519 (0.130)	1.093 (0.275)	1.236 (0.218)	1.272 (0.205)	0.126 (0.900)
Gaussian VAR(1)	-9.813 (0.000)		1.301 (0.194)	0.803 (0.423)	0.994 (0.321)	0.997 (0.320)	-0.043 (0.966)
Gaussian VAR(1) Diag. BEKK GARCH (1,1)	-5.536 (0.000)	-4.820 (0.000)		-0.599 (0.550)	-0.003 (0.997)	-0.020 (0.984)	-0.622 (0.534)
t-Student VAR(1) Diag. BEKK GARCH (1,1)	-3.668 (0.000)	-2.954 (0.003)	2.446 (0.015)		1.100 (0.273)	1.276 (0.203)	-0.519 (0.604)
Gaussian VAR(1) CCC GARCH (1,1)	-4.683 (0.000)	-4.096 (0.000)	-1.781 (0.076)	-5.544 (0.000)		-0.048 (0.962)	-0.737 (0.462)
Gaussian VAR(1) DCC GARCH (1,1)	-4.001 (0.000)	-3.339 (0.001)	0.648 (0.518)	-3.216 (0.002)	5.919 (0.000)		-0.748 (0.455)
Four-State Markov Switching VAR(1)	-3.895 (0.000)	-3.441 (0.001)	-1.091 (0.277)	-1.628 (0.105)	-0.600 (0.549)	-1.300 (0.195)	

Table 8

Diebold Mariano Tests of Equal Predictive Accuracy for 1-Month Forecasts of Asset Return Covariances

For each pair of the 7 multivariate models entertained in this paper, the table reports the D-M statistic and the corresponding p-values computed from a HAC-adjusted estimator of the variance matrix of the loss function differences. Above the main diagonal the loss function is the squared loss; below the main diagonal the loss function is absolute loss. Above the main diagonal, a negative (positive) D-M statistic implies that the model in the row is inferior (positive) to the one in the column. Below the main diagonal, a negative (positive) D-M statistic implies that the model in the column is superior (inferior) to the one in the row. In the table, significant D-M statistics are boldfaced.

	Gaussian IID Benchmark	Gaussian VAR(1)	Gaussian VAR(1) Diag. BEKK GARCH (1,1)	t-Student VAR(1) Diag. BEKK GARCH (1,1)	Gaussian VAR(1) CCC GARCH (1,1)	Gaussian VAR(1) DCC GARCH (1,1)	Four-State Markov Switching VAR(1)
REIT-Stocks Covariance							
Gaussian IID Benchmark		-0.449 (0.654)	1.264 (0.208)	1.287 (0.199)	1.348 (0.179)	1.193 (0.234)	1.687 (0.093)
Gaussian VAR(1)	-5.082 (0.000)		1.250 (0.213)	1.272 (0.204)	1.329 (0.185)	1.181 (0.239)	1.667 (0.097)
Gaussian VAR(1) Diag. BEKK GARCH (1,1)	-1.302 (0.194)	-0.876 (0.382)		0.823 (0.411)	-0.826 (0.409)	-1.899 (0.059)	0.763 (0.446)
t-Student VAR(1) Diag. BEKK GARCH (1,1)	-1.863 (0.064)	-1.442 (0.151)	-5.848 (0.000)		-0.928 (0.354)	-2.250 (0.025)	0.729 (0.467)
Gaussian VAR(1) CCC GARCH (1,1)	-1.780 (0.076)	-1.268 (0.206)	-0.509 (0.611)	0.700 (0.485)		0.324 (0.746)	1.056 (0.292)
Gaussian VAR(1) DCC GARCH (1,1)	-1.426 (0.155)	-1.032 (0.303)	-0.999 (0.319)	1.600 (0.111)	0.046 (0.964)		0.885 (0.377)
Four-State Markov Switching VAR(1)	-6.143 (0.000)	-5.577 (0.000)	-4.982 (0.000)	-4.345 (0.000)	-5.381 (0.000)	-4.306 (0.000)	
REIT-Bonds Covariance							
Gaussian IID Benchmark		1.149 (0.252)	-1.339 (0.182)	-1.345 (0.180)	-0.569 (0.570)	-1.339 (0.182)	1.506 (0.133)
Gaussian VAR(1)	-3.980 (0.000)		-1.352 (0.178)	-1.353 (0.177)	-0.660 (0.510)	-1.348 (0.179)	1.499 (0.135)
Gaussian VAR(1) Diag. BEKK GARCH (1,1)	0.175 (0.861)	0.717 (0.474)		-1.327 (0.186)	1.151 (0.251)	-1.299 (0.195)	1.467 (0.144)
t-Student VAR(1) Diag. BEKK GARCH (1,1)	0.946 (0.345)	1.428 (0.155)	3.101 (0.002)		1.200 (0.231)	1.366 (0.173)	1.445 (0.150)
Gaussian VAR(1) CCC GARCH (1,1)	-1.788 (0.075)	-1.243 (0.215)	-1.304 (0.194)	-1.788 (0.075)		-1.189 (0.236)	1.760 (0.080)
Gaussian VAR(1) DCC GARCH (1,1)	0.893 (0.373)	1.370 (0.172)	2.985 (0.003)	-0.946 (0.345)	1.742 (0.083)		1.444 (0.150)
Four-State Markov Switching VAR(1)	-4.287 (0.000)	-3.855 (0.000)	-3.290 (0.001)	-3.402 (0.001)	-2.556 (0.011)	-3.371 (0.001)	
Stock-Bond Covariance							
Gaussian IID Benchmark		2.291 (0.023)	1.147 (0.252)	0.955 (0.340)	0.123 (0.902)	0.838 (0.403)	3.053 (0.003)
Gaussian VAR(1)	-2.291 (0.023)		1.117 (0.265)	0.924 (0.356)	0.004 (0.997)	0.808 (0.420)	3.037 (0.003)
Gaussian VAR(1) Diag. BEKK GARCH (1,1)	-1.147 (0.252)	-1.117 (0.265)		-0.791 (0.430)	-1.221 (0.223)	-0.759 (0.449)	1.547 (0.123)
t-Student VAR(1) Diag. BEKK GARCH (1,1)	-0.955 (0.340)	-0.924 (0.356)	0.791 (0.430)		-0.987 (0.325)	-0.645 (0.519)	1.515 (0.131)
Gaussian VAR(1) CCC GARCH (1,1)	-0.123 (0.902)	-0.004 (0.997)	1.221 (0.223)	0.987 (0.325)		0.861 (0.390)	2.944 (0.004)
Gaussian VAR(1) DCC GARCH (1,1)	-0.838 (0.403)	-0.808 (0.420)	0.759 (0.449)	0.645 (0.519)	-0.861 (0.390)		1.470 (0.143)
Four-State Markov Switching VAR(1)	-3.053 (0.003)	-3.037 (0.003)	-1.547 (0.123)	-1.515 (0.131)	-2.944 (0.004)	-1.470 (0.143)	

Table 9
Predictive Density Tests for 1-Month Forecasts of Asset Returns

In the table, we evaluate the density forecasts produced by our models using seven statistics to test whether the probability integral transform scores from each model and for each asset return series are distributed i.i.d. $U[0,1]$, which should be the case if the sequence of density forecasts is generated by the true data generating process. P-values are in parenthesis. Boldfaced statistics indicate that the null of correct specification cannot be rejected

	Gaussian IID Benchmark	Gaussian VAR(1)	Gaussian VAR(1) Diag. BEKK GARCH (1,1)	t-Student VAR(1) Diag. BEKK GARCH (1,1)	Gaussian VAR(1) CCC GARCH (1,1)	Gaussian VAR(1) DCC GARCH (1,1)	Four-State Markov Switching VAR(1)
Kolmogorov Test of $U[0,1]$ Hypothesis							
REITs	0.532 (0.939)	0.630 (0.822)	1.091 (0.185)	1.051 (0.220)	0.788 (0.563)	0.989 (0.282)	1.309 (0.065)
Stocks	1.224 (0.100)	0.979 (0.293)	0.979 (0.293)	0.969 (0.305)	1.243 (0.091)	0.895 (0.400)	1.065 (0.207)
Bonds	1.213 (0.105)	1.327 (0.059)	0.940 (0.340)	0.950 (0.327)	0.874 (0.429)	0.794 (0.554)	1.390 (0.042)
Cramer-Von Mises Test of $U[0,1]$ Hypothesis							
REITs	0.044 (0.907)	0.071 (0.737)	0.331 (0.111)	0.260 (0.175)	0.123 (0.479)	0.250 (0.187)	0.386 (0.079)
Stocks	0.289 (0.145)	0.309 (0.127)	0.212 (0.245)	0.168 (0.338)	0.376 (0.083)	0.200 (0.266)	0.208 (0.252)
Bonds	0.408 (0.069)	0.411 (0.067)	0.168 (0.338)	0.161 (0.357)	0.142 (0.413)	0.121 (0.487)	0.569 (0.027)
Joint Test of Hypothesis of Zero Mean and Unit Variance for Transformed Scores							
REITs	13.128 (0.001)	17.202 (0.000)	3.313 (0.191)	1.343 (0.511)	0.147 (0.929)	1.974 (0.373)	2.406 (0.300)
Stocks	25.991 (0.000)	28.897 (0.000)	12.765 (0.002)	9.466 (0.009)	17.208 (0.000)	8.376 (0.015)	0.451 (0.798)
Bonds	6.843 (0.033)	8.062 (0.018)	1.083 (0.582)	0.964 (0.618)	1.203 (0.548)	0.716 (0.699)	9.065 (0.011)
Jarque-Bera Test of Normality of the Transformed Scores							
REITs	51.592 (0.000)	37.270 (0.000)	41.008 (0.000)	46.788 (0.000)	24.288 (0.000)	41.758 (0.000)	45.364 (0.000)
Stocks	48.992 (0.000)	36.234 (0.000)	49.020 (0.000)	57.034 (0.000)	44.435 (0.000)	40.573 (0.000)	7.061 (0.029)
Bonds	17.716 (0.000)	11.410 (0.003)	12.478 (0.002)	12.481 (0.002)	8.125 (0.017)	5.232 (0.073)	12.885 (0.002)
Cramer-Von Mises Test of Normality of the Transformed Scores							
REITs	0.151 (0.024)	0.140 (0.032)	0.146 (0.027)	0.154 (0.021)	0.108 (0.089)	0.128 (0.047)	0.209 (0.041)
Stocks	0.293 (0.000)	0.213 (0.004)	0.266 (0.001)	0.266 (0.001)	0.314 (0.000)	0.233 (0.002)	0.138 (0.035)
Bonds	0.125 (0.051)	0.160 (0.018)	0.110 (0.068)	0.107 (0.090)	0.104 (0.100)	0.125 (0.052)	0.191 (0.007)
Ljung-Box (12) Test on Transformed Scores							
REITs	19.896 (0.069)	18.836 (0.093)	14.016 (0.300)	13.257 (0.351)	12.588 (0.400)	11.204 (0.512)	20.040 (0.066)
Stocks	9.355 (0.672)	11.548 (0.483)	10.449 (0.577)	11.073 (0.523)	6.563 (0.885)	13.882 (0.308)	7.946 (0.789)
Bonds	14.226 (0.287)	13.652 (0.323)	12.314 (0.421)	11.367 (0.498)	11.292 (0.504)	8.655 (0.732)	5.773 (0.927)
Ljung-Box (12) Test on Squared Transformed Scores							
REITs	88.690 (0.000)	82.802 (0.000)	4.807 (0.964)	5.518 (0.938)	6.854 (0.867)	7.579 (0.817)	5.673 (0.059)
Stocks	42.996 (0.000)	44.600 (0.000)	39.619 (0.000)	42.393 (0.000)	8.827 (0.718)	37.739 (0.000)	20.983 (0.051)
Bonds	10.952 (0.533)	14.651 (0.261)	6.087 (0.912)	5.405 (0.943)	4.895 (0.961)	5.243 (0.949)	16.619 (0.164)

Figure 1

Smoothed State Probabilities from a VAR(1) Four-State Markov Switching Heteroskedastic Model

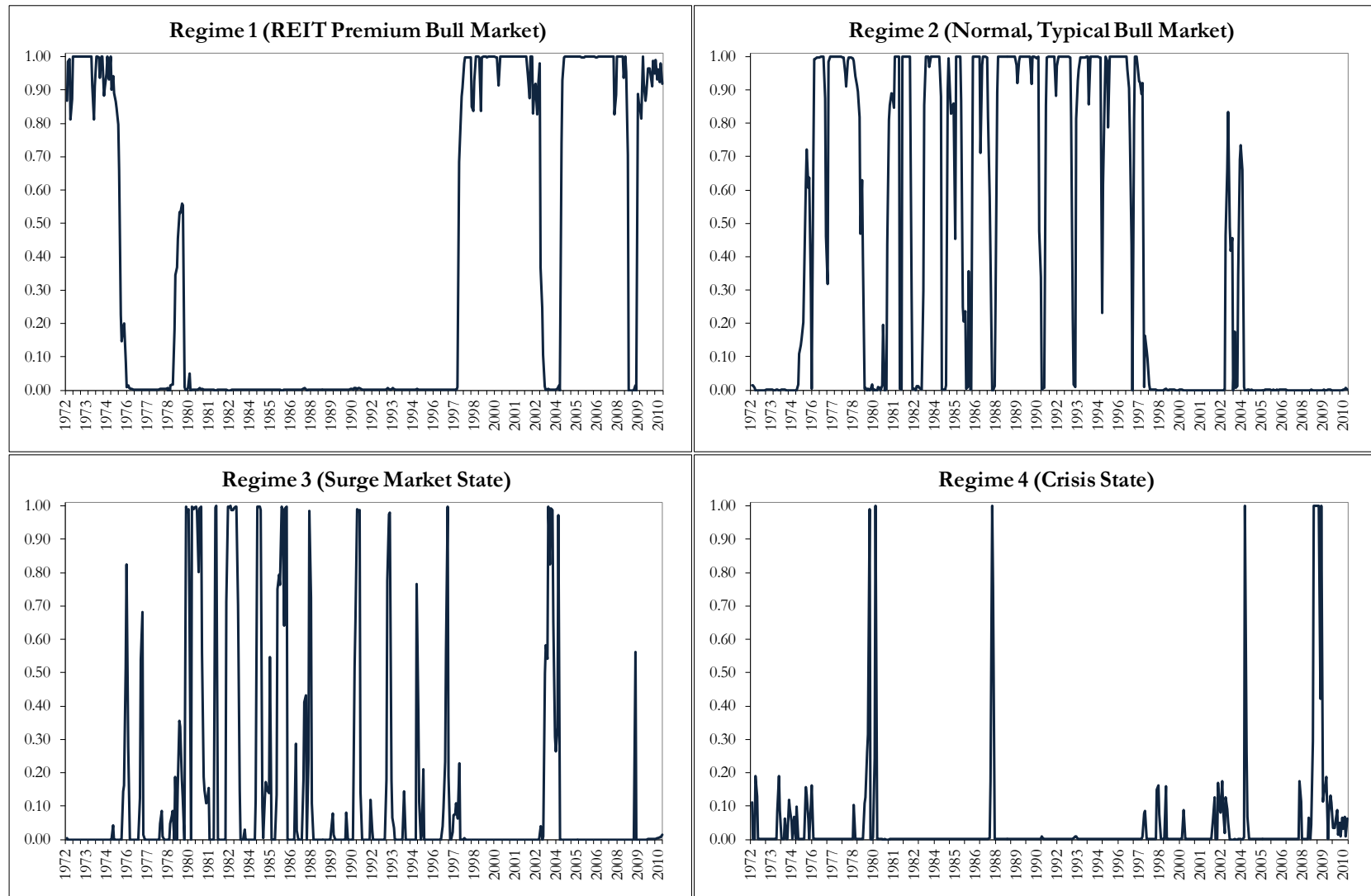


Figure 2

Recursive, Pseudo Out-of-Sample 1-Month Ahead Point Forecasts from Four-State VAR(1) Markov Switching Heteroskedastic Model

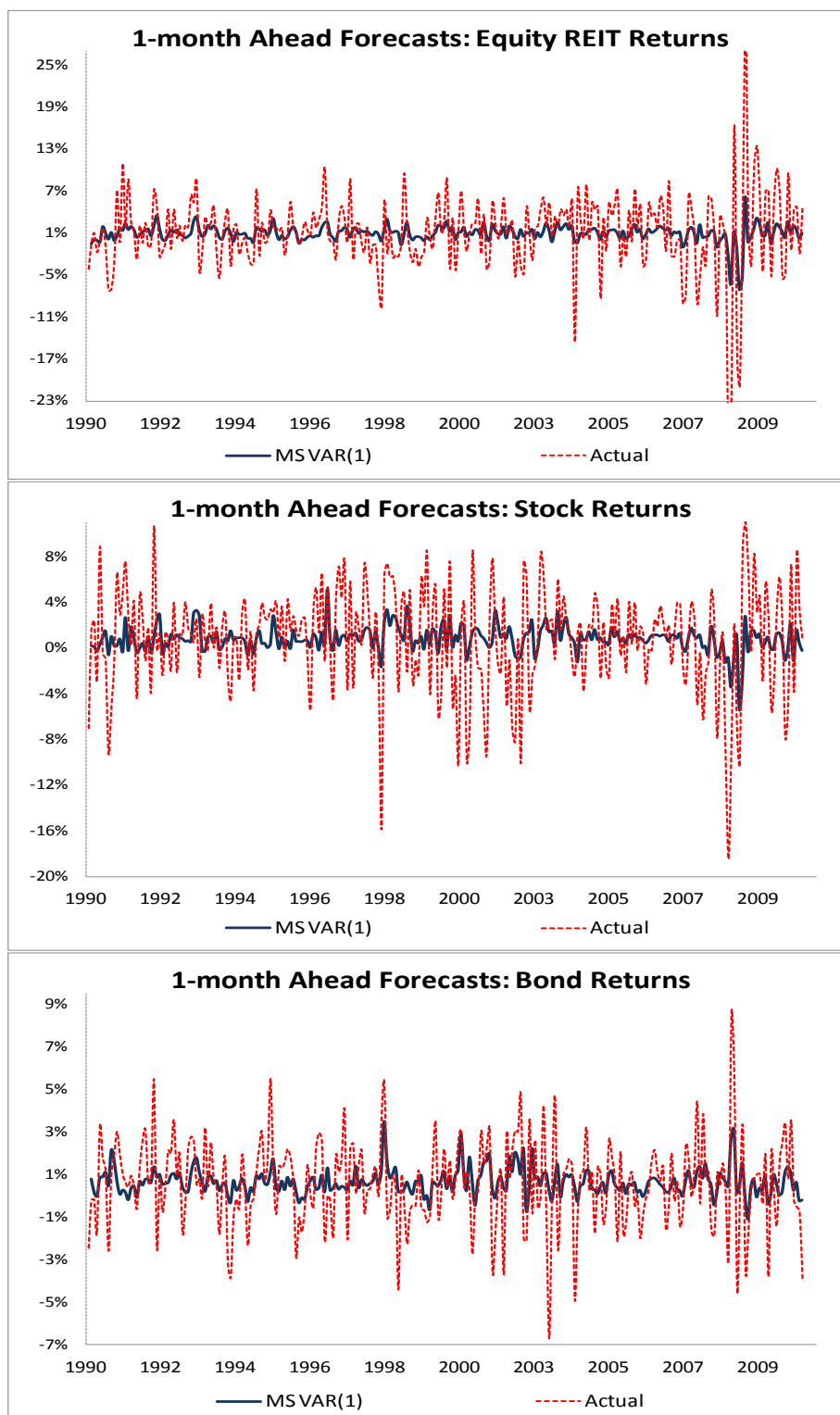


Figure 3
Recursive, Pseudo Out-of-Sample 1-Month Ahead Volatility Forecasts from
Gaussian CCC GARCH(1,1) Model

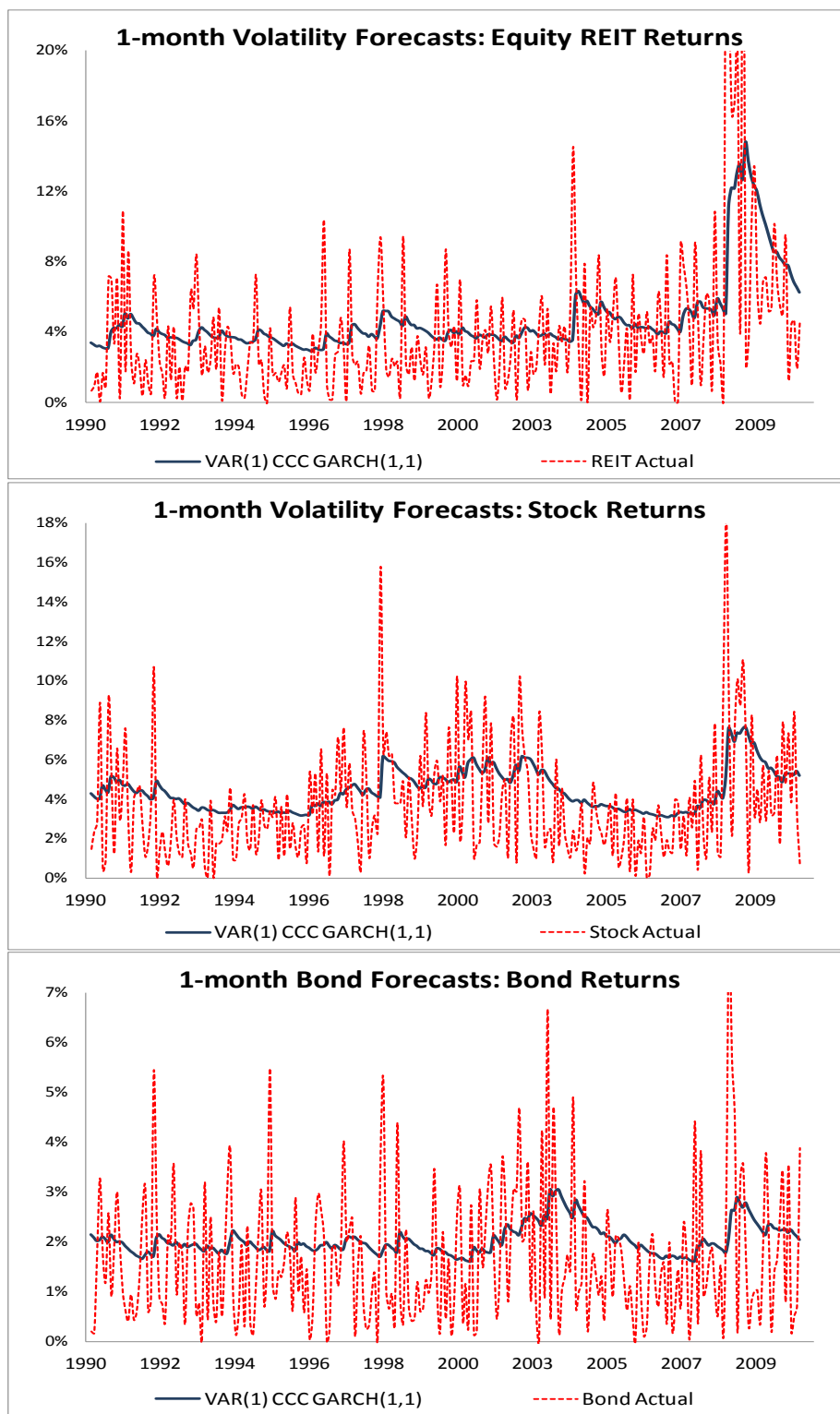


Figure 4

Recursive, Pseudo Out-of-Sample 1-Month Ahead Correlation Forecasts from
Gaussian vs t-Student VAR(1) BEKK GARCH(1,1) Models

