

Jahan-Parvar, Mohammad R.; Liu, Hening

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Ambiguity and equity premium in production economies

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The University of Manchester
Booth Street West
Manchester M15 6PB

+44(0)161 306 1320
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Author(s) and affiliation

Dr. Mohammad R. Jahan-Parvar

Department of Economics
East Carolina University
A426 Brewster Building
Greenville, NC 27858, USA
E-Mail: jahanparvarm@ecu.edu

Dr. Hening Liu

Manchester Business School
Booth Street West
Manchester M15 6PB
Fax: +44 161-275-4023
E-Mail: hening.liu@mbs.ac.uk

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Ambiguity, Equity premium, Markov switching, Production economy, Smooth ambiguity.

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C61; D81; G11; G12

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Ambiguity and Equity Premium in Production Economies

Mohammad R. Jahan-Parvar*
East Carolina University

Hening Liu^{†‡}
University of Manchester

September 2011

Abstract

We propose a novel one-sector stochastic growth model, where productivity growth follows a Markov-switching process with two regimes, and where households have generalized recursive smooth ambiguity preferences. The adopted class of preferences permits a three-way separation of risk aversion, ambiguity aversion, and the attitude toward intertemporal substitution. Ambiguity averse agents are ambiguous about the probability distribution of productivity growth. We show that in the absence of ambiguity aversion, the presence of a persistent high productivity regime combined with the elasticity of intertemporal substitution being greater than unity cannot generate a sizable risk premium. With a moderate coefficient of relative risk aversion, our model with ambiguity aversion can account for the low volatility of consumption growth observed in the data, and produce a high and volatile equity premium and a low and smooth risk-free rate. In addition, this model is able to generate, albeit weak, long horizon predictability in equity returns.

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*Department of Economics, East Carolina University, A426 Brewster Building, Greenville, NC 27858, USA. e-mail: jahanparvarm@ecu.edu.

[†]Accounting and Finance, Manchester Business School, University of Manchester, Booth Street West, Manchester M15 6PB, UK. e-mail: Hening.Liu@mbs.ac.uk.

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1 Introduction

The original work of Rouwenhorst (1995) explores the asset pricing implications of a standard one-sector neoclassical growth model. Although the neoclassical growth model seems to be a natural candidate for the analysis of the behavior of asset prices over the business cycle, Rouwenhorst (1995) finds that a parsimonious equilibrium asset pricing model with a single source of uncertainty driven by technology shocks has much difficulty in explaining a number of stylized facts about asset returns. Among these facts, we observe (1) the equity premium puzzle of Mehra and Prescott (1985), (2) the risk-free rate puzzle of Weil (1989), (3) the equity volatility puzzle of Shiller (1981), (4) cyclical variation of equity premium and equity volatility (see, for example, Fama and French (1988a), Fama and French (1988b), Fama and French (1989), Campbell and Shiller (1988), Poterba and Summers (1988) and Schwert (1989)), and (5) predictability of equity returns over long horizons (see, for example, Fama and French (1988a) and Fama and French (1988b)). The limitations of the standard neoclassical growth model in explaining these asset pricing phenomena point to other potential sources of uncertainty that can improve this model's performance.

In this paper, we develop a new dynamic stochastic general equilibrium (DSGE) model that takes into account model uncertainty (also known as ambiguity). We find that the model that allows regime switching in productivity growth, a three-way separation among risk aversion, ambiguity aversion and the elasticity of intertemporal substitution (EIS), and convex capital adjustment costs, can go a long way to explain a variety of asset pricing phenomena that are well-documented in the empirical literature. Although the role of ambiguity has been extensively examined in Lucas-tree type pure exchange economy (for examples see, among others, Leippold et al. (2008), Epstein and Schneider (2008), Ju and Miao (2011) and Collard et al. (2011)), this feature has not been explored in production economies.¹ Our paper is, therefore, the first to study rigorously asset pricing implications of model uncertainty in the otherwise standard neoclassical

¹ To our knowledge, Cagetti et al. (2002) is an exception.

growth model in the literature.

Our model has three main ingredients. First, we model productivity growth rates as following a Markov-switching process with two different regimes.² The underlying state, which governs productivity regimes, follows a discrete-time Markov chain and is assumed to be observable. We also postulate that there are two distinct regimes with a “high productivity growth” regime being persistent and a “low productivity growth” regime being relatively transitory. The regime switching process is employed to capture cyclical variation of the state of the economy, and reflects persistent economic booms and less persistent recessions.³ More important, in each period economic agents are ambiguous as to which probability distribution truly characterizes future productivity growth. This ambiguity will not vanish in the long run since the underlying state is regime-switching. Note that unlike Ju and Miao (2011) where ambiguity arises due to non-observability of the underlying state, ambiguity in our model is purely driven by multiplicity in the probability distribution of growth rates. As a result, the economic regimes together with capital dynamics and shocks to productivity growth summarize time-variation in economic uncertainty and are the key determinants of equilibrium allocations and asset prices.

Second, we assume that economic agents’ utility preferences are represented by the generalized recursive smooth ambiguity model recently proposed by Ju and Miao (2011) and axiomatized by Hayashi and Miao (2011). This class of preferences extends the recursive smooth ambiguity utility model of Klibanoff et al. (2009) by further disentangling risk aversion and intertemporal substitution. Recursive smooth ambiguity utility is a dynamic extension of the smooth ambiguity utility model first proposed by Klibanoff et al. (2005). The utility preferences adopted in this paper accommodate a three-way separation among risk aversion, ambiguity aversion and the attitude toward intertemporal substitution. To this end, this preferences can also be viewed as an extension of Epstein and Zin (1989) recursive utility to incorporate ambiguity aversion. Agents

² The impacts of regime shifts have been extensively explored to study asset prices in endowment economies (see, for example, Cecchetti et al. (1990), David (1997), Veronesi (1999), and Cecchetti et al. (2000)).

³ The assumption that the “good state” is more persistent than the “bad state” is standard in the literature. Examples include Cecchetti et al. (1990, 2000).

are risk averse in the usual sense; i.e., they dislike any mean-preserving spreads of wealth. Agents are ambiguity averse in the sense that they dislike any mean-preserving spread of conditional expected utility or continuation value in recursive formulation induced by the probability weights assigned to different models. In the formulation of the preferences, this is achieved by imposing concave transformations of the certainty equivalent. The concave transformation reflects agents' pessimistic view about conditional expected continuation value. Thus, in this model, the conditional distribution given a regime cannot be integrated over the transition probabilities to yield a predictive distribution. Klibanoff et al. (2005) point out that this method of modeling ambiguity has the advantage of relaxing the tight link between ambiguity and ambiguity aversion.⁴ As a result, we are able to do comparative statics analysis by holding the set of models fixed while varying ambiguity aversion.

In our model, due to ambiguity aversion, states with lower continuation value receive more weights. Hence agents are pessimistic and form distorted subjective beliefs. Cecchetti et al. (2000) find that in an endowment economy, distortion of subjective beliefs regarding the persistence of expansions and recessions is important to explain a variety of features of observed asset returns data. However, in the model of Cecchetti et al. (2000) beliefs are distorted exogenously. On the other hand, in our model the magnitude of the distortion is endogenously determined from the solution to the social planner's problem, where the smooth ambiguity utility inducing the distortion has a sound decision-theoretic underpinning. Our model is parsimonious in that the transition probabilities governing regime switching are assumed to be constant, and in addition there are no elements of learning. Thus, we abstract from time-varying beliefs, which, however, are the main focus of the literature on asset prices and learning (for example, see Veronesi (1999)). Ju and Miao (2011) examine an endowment economy model where agents have generalized recursive smooth ambiguity preferences and also engage in Bayesian learning. In their model, the posterior beliefs updated in Bayesian fashion are naturally embedded in the utility function as "second-

⁴ The separation between ambiguity and ambiguity aversion, however, cannot be achieved in the multiple priors utility model. See, for example, Chen and Epstein (2002) and Epstein and Schneider (2008).

order probabilities”, a term named by Gollier (2011). The key difference in our model is that we allow agents to observe economic regimes, and as a result it is transition probabilities that enter into the utility function as the second-order probabilities. In terms of the information structure, our model is much simpler than the model of Ju and Miao (2011).

Third, we assume convex capital adjustment costs following the specification of Campanale et al. (2010). In the light of early works on production-based asset pricing (e.g., Rouwenhorst (1995) and Tallarini (2000)), neoclassical growth models assuming no adjustment costs have difficulty in producing a sizable price of risk and therefore equity premium, even when the separation between risk aversion and intertemporal substitution is achieved. This is mainly because (1) the price of capital (Tobin’s q) always stays constant at 1, and (2) in the absence of any impediment to capital adjustment, agents can freely adjust capital to smooth consumption. The resulting volatility of the marginal rate of substitution is, therefore, low, even with high levels of risk aversion. In the presence of capital adjustment costs, the price of capital fluctuates, depending on the state of the economy. In addition, aggregate consumption risk also rises. The price of risk, on the other hand, depends crucially on the curvature of the utility function. In contrast to the habit formation model (Jermann (1998) and Boldrin et al. (2001)) and the disappointment aversion model (Campanale et al. (2010)), our model produce high local curvature due to ambiguity aversion.

We find that our production economy model with ambiguity aversion can not only account for the low volatility of consumption growth but also reproduce a number of salient feature typically observed in asset returns data, including (1) high mean equity premium, (2) volatile equity returns, (3) low mean risk-free rate, (4) low volatility of the risk-free rate, (5) dividend growth more volatile than consumption growth, and (6) albeit weak, long horizon predictability in equity returns. These results represent a remarkable improvement over the existing literature in production based asset pricing, given the simplicity of our model.

The success of our model crucially lies in the postulated ambiguous growth. Due to ambiguity

aversion, there is an extra multiplicative term in the pricing kernel (the intertemporal marginal rate of substitution) relative to that obtained from Epstein-Zin recursive utility. This term greatly increases the variation of the pricing kernel, and thus carries ambiguity premium (uncertainty premium). The quantitative analysis reveals that ambiguity aversion accounts for almost 70% of the historical mean equity premium (approximately 7.4% in our sample) observed in the US data. However, with no ambiguity aversion, regime-switching productivity growth in conjunction with an EIS greater than unity can generate a risk premium of only about 0.5% (unlevered).

Regarding the risk-free rate, due to endogenous pessimism, ambiguity-averse agents perceive future investment opportunities to be worse than ambiguity-neutral agents do. The current price of the risk-free asset paying one unit of consumption goods in the next period is high, leading to a low risk-free rate. Moreover, our model also circumvents the well-known “risk-free rate volatility puzzle”. That is, the existing production economy models (for example, Jermann (1998), Boldrin et al. (2001), Kaltenbrunner and Lochstoer (2010) and Campanale et al. (2010)) inevitably produce a counterfactually high volatility of the risk-free rate when attempting to generate a high equity premium. This is the case when those models require rather low EIS. In our model, this difficulty is resolved by assuming an EIS greater than unity. The tight tension between strong aversion toward intertemporal substitution and the friction impeding consumption smoothing is, therefore, relaxed.

Related literature

Our paper belongs to a growing but still limited body of literature known as “production-based asset pricing” models. We briefly discuss a select number of closely related papers in what follows. Naturally, this list is not exhaustive.

Jermann (1998) and Boldrin et al. (2001) rely on habit persistence to explain the equity premium puzzle. In the presence of capital adjustment costs, habit persistence induces strong risk aversion, which generates a sizable price of risk and therefore high equity premium. These two papers are the first successful attempts in matching the equity premium and macroeconomic

moments in the literature. However, both models generate a counterfactual high volatility in risk-free rates since under habit persistence, strong risk aversion also leads to strong aversion to intertemporal substitution, besides, this attitude is time-varying. Our model does not have much difficulty along these two dimensions for two reasons: (1) risk aversion and the attitude toward intertemporal substitution are disentangled, and we rely on ambiguity aversion to produce a high equity premium, and (2) the high productivity regime is very persistent, which implies small variations in the expectation of the pricing kernel.

Campanale et al. (2010) show that “disappointment aversion” (see Gul (1991)) in a DSGE model with “Chew-Dekel” class of preferences and convex capital adjustment costs is important to match the mean equity premium and price of risk with what is observed in the data. Because their model needs a very low EIS to generate a high equity premium and volatile equity returns, the model inevitably results in counterfactually high volatility in risk-free rates.

Kaltenbrunner and Lochstoer (2010) examine two types of DSGE models with Epstein-Zin preferences, one with permanent and the other with transitory productivity shocks. They find that long-run consumption risk can endogenously arise in the production economies considered.⁵ Although with a reasonable coefficient of relative risk aversion and a rather low EIS (this implies preference for late resolution of uncertainty), the model can generate a high equity premium, the volatility of risk-free rates is still too high to be reconciled with the data.⁶

Extending the long-run consumption risk model of Bansal and Yaron (2004) to a production economy framework, Croce (2010) examines the asset pricing and business cycle implications of long-run productivity risk. Croce (2010) considers cases where the EIS parameter is greater than 1, and thus the model can account for the low volatility of the risk-free rate observed in the data. Croce (2010) also finds that to generate a sizable equity premium (his model generates 3% equity premium), the required coefficient of relative risk aversion is 30. Croce argues that the model

⁵ In an endowment economy, long-run risks are defined as consumption and/or dividend growth having a small but persistent predictable component; see Bansal and Yaron (2004).

⁶ Both Campanale et al. (2010) and Kaltenbrunner and Lochstoer (2010) assume the time discount factor β is greater than one.

has already delivers an improvement over Tallarini (2000), where under the assumption of the EIS being equal to 1, extremely high values of relative risk aversion are required to justify the high Sharpe ratio observed in the data. However, the implied equity premium is still very low in Tallarini (2000)'s model.

Recently, Ai et al. (2010) introduce intangible capital into a production economy and find that the model can produce a high equity premium, a low and smooth risk-free rate and value premium as well. Kung and Schmid (2011) develop an endogenous growth model to link macroeconomic risk and output growth. With Epstein-Zin utility, their model implies a positive relationship between the volatility of macroeconomic variables and growth. Moreover, the model is able to replicate a wide range of stylized facts implied by asset returns data.

Güvenen (2009) considers a heterogeneous agents model with limited stock market participation and where agents differ in their attitudes towards intertemporal substitution. Güvenen shows that the model is consistent with salient features of asset returns including high equity premium, low variation in risk-free rates and countercyclical variation of equity premium and its volatility. But the model generates high volatility in consumption growth relative to what is observed in the data. Güvenen suggests that there is an obvious trade-off between asset pricing and business cycle performance of the model.

The rest of the paper is organized as follows. Section 2 briefly introduces the smooth ambiguity model and its generalizations to the dynamic setting. Section 3 presents the production economy model and the market equilibrium. Section 4 describes the calibration exercises and also discusses the quantitative results about financial and business cycle quantities. Section 5 concludes. A detailed explanation of our numerical algorithm is included in the Appendix.

2 Smooth Ambiguity Preferences

In this section, we describe the framework that we use to model the agent's preferences and beliefs. This framework is embedded in a general equilibrium model with nontrivial production

in the next section. The static version of the utility preferences used in this paper is of the smooth ambiguity type, introduced by Klibanoff et al. (2005). Klibanoff et al. (2009) develop the recursive version of this class of preferences in a dynamic setting. In the spirit of Epstein and Zin (1989), Hayashi and Miao (2011) further generalize the model by disentangling risk aversion and intertemporal substitution. In this paper, we assume that the agent's preferences and beliefs are characterized by the generalized recursive smooth ambiguity model of Hayashi and Miao (2011). Interested readers could refer to these papers for more details.

2.1 The static and recursive formulation

We start with a formulation ordinally equivalent to the smooth ambiguity model of Klibanoff et al. (2005):

$$v^{-1}\mathbb{E}_{\zeta}v(u^{-1}(\mathbb{E}_{\pi_{\theta}}u \circ f)) \equiv v^{-1}\left(\int_{\Theta}v\left(u^{-1}\left(\int_S u(f)d\pi_{\theta}\right)\right)d\zeta(\theta)\right) \quad (1)$$

where f is an act that maps states to decisions (for example, a policy function), \mathbb{E} is an expectation operator, u is a von Neumann-Morgenstern utility function, v is an increasing function, ζ is a subjective probability measure over a set of parameters denoted by Θ in which each element induces a probability measure π_{θ} over the state space S . The decision maker prefers act f to act g if and only if

$$v^{-1}\mathbb{E}_{\zeta}v(u^{-1}(\mathbb{E}_{\pi_{\theta}}u \circ f)) \geq v^{-1}\mathbb{E}_{\zeta}v(u^{-1}(\mathbb{E}_{\pi_{\theta}}u \circ g)).$$

This formulation is ordinally equivalent to the model of Klibanoff et al. (2005) if we define $\phi \equiv v \circ u^{-1}$. In the above formulation, π_{θ} yields beliefs over outcomes given a certain parameter value, while ζ reflects the decision maker's uncertainty as to which probability distribution in the set of probability distributions induced by Θ truly governs the state space. As noted by Klibanoff et al. (2005), a key feature of the smooth ambiguity utility model is that it achieves a separation between ambiguity, identified as a characteristic of the decision maker's subjective beliefs, and ambiguity attitude, identified as a characteristic of the decision maker's tastes. In particular, ambiguity is captured by multiplicity of the subjective set of probability measures induced by

the set Θ . Attitudes toward pure risk and ambiguity are characterized, respectively, by the shape of u and v . The decision maker is risk averse if and only if u is concave, and he is ambiguity averse if and only if v is a concave transformation of u . According to Klibanoff et al. (2005), ambiguity aversion is defined to be an aversion to mean preserving spreads in the distribution over expected utility values, $\mathbb{E}_{\pi_\theta} u \circ f$, induced by ζ and the act f . This distribution represents the probabilities of different evaluations of the act f under different probability measures deemed as relevant. Gollier (2011) call this distribution the “second-order” distribution. This class of preferences implies that ambiguity averse decision makers prefer acts whose evaluation is more robust to the possible variation in probabilities than those who display risk aversion only. In addition, this class of preferences implies the *irreducibility of compound distributions*. That is, the model does not impose the compound reduction between ζ and the π_θ s in the support of ζ . In the special case of ϕ being linear, such reduction is feasible, and the decision maker displays ambiguity neutrality which is observationally equivalent to a subjective expected utility decision maker with a subjective prior ζ .

Klibanoff et al. (2009) embed the static model (1) in a dynamic setting and develop a recursive formulation of the smooth ambiguity model. In a discrete-time setting, the state space is denoted by S . The decision-maker’s information in period- t is summarized by history $s^t = \{s_0, s_1, s_2, \dots, s_t\}$ with the root node $s_0 \in S$ given and $s_t \in S$. The decision maker chooses among consumption plans $C \equiv (C_t)_{t \geq 0}$, each of which maps a history s^t to a payoff. That is, C_t is adapted to s^t and is a measurable function of s^t . The decision maker is uncertain as to which probability distribution governs the full state space S^∞ . This uncertainty is represented by a parameter space Θ , a set of candidate models or a state evolving over time according to a Markov chain. The decision maker is allowed to make inference on the set of parameters (if unobservable) based on history s^t . Suppose $\pi_\theta(s_{t+1}|s^t)$ denotes the probability distribution that the next observation will be s_{t+1} , given the parameter $\theta \in \Theta$ and the history s^t . We denote by ζ the decision maker’s prior on the parameter space Θ . Klibanoff et al. (2009) develop the following

recursive version of the smooth ambiguity model:

$$V_{s^t}(C) = u(C(s^t)) + \beta \phi^{-1} \left[\int_{\Theta} \phi \left(\int_{S_{t+1}} V_{(s^t, s_{t+1})}(C) d\pi_{\theta}(s_{t+1}|s^t) \right) d\zeta(\theta|s^t) \right] \quad (2)$$

where $V_{s^t}(C)$ is an indirect value function, $\beta \in (0, 1)$ is the subjective discount factor, $\zeta(\theta|s^t)$ denotes the Bayesian posterior updated given the history s^t , and u and ϕ are defined in the same way as in the static model. Collard et al. (2011) study the asset pricing implications of ambiguity using this model. The utility function (2) is always well defined for the specification $\phi(x) = -\exp(-x/\lambda)$, $\lambda > 0$. This specification has a straightforward connection with the robust control approach of Hansen (2007) and Hansen and Sargent (2010). However, in such a case the utility function is not homogeneous, which is a desirable property for numerical value function iteration.

2.2 The generalized recursive smooth ambiguity model

In this paper, we use the generalized recursive smooth ambiguity model, which further extends the recursive version of the smooth ambiguity model by allowing for the separation between risk aversion and intertemporal substitution. This class of preferences is recently proposed by Ju and Miao (2011) and axiomatized by Hayashi and Miao (2011). Inspired by Kreps and Porteus (1978) and Epstein and Zin (1989), Ju and Miao (2011) propose the following formulation:

$$V_t(C) = W(C_t, \mathcal{R}_t(V_{t+1}(C))), \quad \mathcal{R}_t(V_{t+1}) = v^{-1} \left(\mathbb{E}_{\zeta_t} [v \circ u^{-1} \mathbb{E}_{\pi_{\theta,t}} [u(V_{t+1})]] \right) \quad (3)$$

where $V_t(C)$ is the continuation value at date t , W is a time aggregator that associates period- t continuation value to the payoff generated from period- t consumption plan and some certainty equivalent of period- $t+1$ continuation value, \mathcal{R}_t is an uncertainty aggregator that maps period- $t+1$ continuation value to its period- t certainty equivalent, and u and v have the same interpretation as in the static setting. When $v \circ u^{-1}$ is linear, that is, the decision maker is ambiguity neutral, we obtain recursive utility of Epstein and Zin (1989). In that case, we can integrate the probability distribution $\pi_{\theta,t+1}$ over the Bayesian posterior ζ_t to obtain a predictive distribution, which is one

of the fundamental concepts in Bayesian analysis. When $v \circ u^{-1}$ is nonlinear, the decision maker displays aversion to uncertainty about which probability distribution governs the state space. Allowing for non-indifference to the timing of the resolution of uncertainty, Ju and Miao (2011) consider a time aggregator in the spirit of Kreps and Porteus (1978) and Epstein and Zin (1989) in the following form:

$$W(c, y) = [(1 - \beta)c^{1-\rho} + \beta y^{1-\rho}]^{\frac{1}{1-\rho}}, \quad \rho > 0, \neq 1 \quad (4)$$

with

$$\begin{aligned} u(x) &= \frac{x^{1-\gamma}}{1-\gamma}, \quad \gamma > 0, \neq 1 \\ v(x) &= \frac{x^{1-\eta}}{1-\eta}, \quad \eta > 0, \neq 1 \end{aligned}$$

where ρ is the inverse of the EIS parameter ψ , γ is the relative risk aversion parameter, and η is the ambiguity aversion parameter.

Applying the aggregator (4) to (3), we obtain

$$\begin{aligned} V_t(C) &= \left[(1 - \beta)C_t^{1-\rho} + \beta \{\mathcal{R}_t(V_{t+1}(C))\}^{1-\rho} \right]^{\frac{1}{1-\rho}} \\ \mathcal{R}_t(V_{t+1}(C)) &= \left(\mathbb{E}_{\zeta_t} \left[\left(\mathbb{E}_{\pi_{\theta,t}} \left[V_{t+1}^{1-\gamma}(C) \right]^{\frac{1-\eta}{1-\gamma}} \right) \right]^{\frac{1}{1-\eta}} \right) \end{aligned}$$

It is worth noting that the decision maker is ambiguity averse if and only if $\eta > \gamma$. If $\eta = \gamma$, the decision maker is ambiguity neutral and his preferences are represented by recursive utility of Epstein and Zin (1989) and Weil (1989).

In the limiting case $\rho = 1$, the utility model becomes

$$U_t = (1 - \beta) \ln C_t + \frac{\beta}{1 - \eta} \ln \left\{ \mathbb{E}_{\zeta_t} \exp \left(\frac{1 - \eta}{1 - \gamma} \ln \left(\mathbb{E}_{\pi_{\theta,t}} \left[\exp((1 - \gamma) U_{t+1}) \right] \right) \right) \right\}$$

where $U_t = \ln V_t$. Ju and Miao (2011) note that this specification is isomorphic to the risk sensitive preferences studied by Hansen (2007) and Hansen and Sargent (2010). Specifically, the two risk sensitivity adjustments for the distributions $\pi_{\theta}(s_{t+1}|s^t)$ and ζ_t , which are both in the form of

“log-exp”, capture the decision maker’s concern about the misspecification in $\pi_\theta (s_{t+1}|s^t)$ given a parameter θ (or alternatively, a hidden state) and in the Bayesian posteriors ζ_t , respectively.

Here, we modify notations to embed the utility function of Ju and Miao (2011) into the setting of this paper. We assume that uncertainty is represented by a state z evolving over time as a Markov chain with transition probabilities given. The Markov state can switch between a finite number of regimes. Each possible regime corresponds to a probability distribution over the state space. Unlike Ju and Miao (2011) who consider learning about a hidden state, we assume that the state is observable in order to keep our model parsimonious. Under the present information structure, the utility function can be written in the following

$$\begin{aligned} V_{z_t,t}(C) &= \left[(1-\beta)C_t^{1-\rho} + \beta \{ \mathcal{R}_{z_t,t}(V_{z_{t+1},t+1}(C)) \}^{1-\rho} \right]^{\frac{1}{1-\rho}} \\ \mathcal{R}_{z_t,t}(V_{z_{t+1},t+1}(C)) &= \left(\mathbb{E}_{z_t} \left[\left(\mathbb{E}_{\pi_{z,t}} \left[V_{z_{t+1},t+1}^{1-\gamma}(C) \right]^{\frac{1-\eta}{1-\gamma}} \right)^{\frac{1}{1-\eta}} \right] \right)^{\frac{1}{1-\eta}} \end{aligned}$$

where $V_{z_t,t}(C)$ is the period- t continuation value of consumption plans C given period- t state, and $\mathcal{R}_{z_t,t}(V_{z_{t+1},t+1}(C))$ is the certainty equivalent of future continuation value given period- t state. As we abstract from Bayesian learning, the conditional expectation operator $\mathbb{E}_{z_t}[\cdot]$ is taken with respect to the transition probabilities governing regime switching. In the inner conditional expectation operator $\mathbb{E}_{\pi_{z,t+1}}[\cdot]$, $\pi_{z,t+1}$ is the probability distribution of $\mathcal{S}_{t+1} \subset S^\infty$ given a regime and the history s^t . The EIS parameter, ψ , is given by $1/\rho$.

3 The Production Economy Model

We consider a standard real business cycle model with many infinitely-lived firms and a representative household agent. A single type of consumption good is produced by a constant-returns-to-scale production function that depends on productivity shocks.⁷ An exogenous stochastic process of productivity growth, which follows a Markov switching process, drives uncertainty in this economy. We assume there is one source of real friction, convex capital adjustment costs.

⁷ Boldrin et al. (2001) study a two-sector model where consumption good and investment good are produced separately.

Households

The representative household agent chooses consumption to maximize smooth ambiguity utility

$$V_{z_t,t}(C) = \left[(1 - \beta)C_t^{1-\rho} + \beta \{ \mathcal{R}_{z_t,t}(V_{z_{t+1},t+1}(C)) \}^{1-\rho} \right]^{\frac{1}{1-\rho}} \quad (5)$$

$$\mathcal{R}_{z_t,t}(V_{z_{t+1},t+1}(C)) = \left(\mathbb{E}_{z_t} \left[\left(\mathbb{E}_{\pi_z} \left[V_{z_{t+1},t+1}^{1-\gamma}(C) \right]^{\frac{1-\eta}{1-\gamma}} \right) \right]^{\frac{1}{1-\eta}} \right) \quad (6)$$

subject to the budget constraint

$$w_t N_t + \phi_{t-1} (P_t + D_t) = C_t + \phi_t P_t$$

where z is the state governing transitions between productivity growth regimes, π_z is the distribution of productivity growth given a regime, w_t is the wage rate, N_t is the supply of labor hours, ϕ_t denotes shares of the representative firm, P_t is the asset price, and D_t is dividends payout per share. Since leisure does not appear in the utility function, the supply of labor is assumed to be exogenous and equal to \bar{N} .

Firms

The consumption good is produced according to a constant returns to scale Cobb-Douglas production function:

$$Y_t = K_t^\alpha (A_t N_t)^{1-\alpha}, \quad (7)$$

where Y_t is the output, K_t is the capital stock, N_t is the amount of labor hours, and A_t is the aggregate productivity shock.⁸ Labor input is assumed to be exogenous and equal to \bar{N} .

Uncertainty in the economy is driven by the stochastic dynamics of productivity growth. The productivity growth rate $\Delta a_{t+1} \equiv \log \left(\frac{A_{t+1}}{A_t} \right)$ follows a Markov-switching process

$$\Delta a_t = \mu(z_t) + \sigma \epsilon_t, \quad \epsilon_t \sim N(0, 1) \quad (8)$$

where z_t evolves according to a Markov chain with two regimes. We denote the high productivity growth regime (or good regime) by $z_t = 1$, and low growth regime (or bad regime) by $z_t = 2$.

⁸ A_t can also be viewed as an exogenous, labor-enhancing technology level.

That is, $\mu(1) > \mu(2)$. The transition probability matrix, \mathbf{P} , is given by

$$\mathbf{P} = \begin{bmatrix} p_{11} & 1 - p_{11} \\ 1 - p_{22} & p_{22} \end{bmatrix}. \quad (9)$$

where p_{11} denotes the “good-to-good” transition probability, and p_{22} denotes the “bad-to-bad” transition probability. The assumed process is able to accommodate the presence of a persistent component of productivity growth when one of the two transition probabilities is relatively large and close to 1. Using the approach recently advanced by Garcia et al. (2008), one can show that the process (8) can be obtained as a result of Markov chain discretization of the long-run risk model of Croce (2010) with constant volatility. Although Croce (2010) also explores time variation in the conditional variance of productivity growth, we assume that the volatility term remains constant, and aim to generate desired results through ambiguity aversion.

The capital stock evolves according to

$$K_{t+1} = (1 - \delta_k) K_t + I_t - G(K_t, K_{t+1}) \quad (10)$$

$$G(K_t, K_{t+1}) = \left| \left(\frac{K_{t+1}}{K_t} - \omega \right) \right|^\iota K_t, \quad \iota > 1, \omega > 0 \quad (11)$$

where G is a convex capital adjustment cost function which introduces frictions. The functional form of the capital adjustment cost follows the formulation of Campanale et al. (2010). Firms are fully owned by the representative household. Each firm issues a single share of equity to the representative household. Each firm chooses labor inputs (which are assumed to be exogenous), the amount of investment expenditure and capital stock to maximize the present value of all current and future profits, that is,

$$V_0 = \sum_{t=0}^{\infty} \varrho_t (Y_t - w_t N_t - I_t), \quad \varrho_t \equiv \frac{1}{R_0^e R_1^e \dots R_t^e}$$

given the capital accumulation equation (10), where R_t^e is the return on equity that is to be determined in equilibrium (see below). The dividends to shareholders are given by

$$D_t = Y_t - w_t N_t - I_t$$

Market equilibrium

In equilibrium, output is equal to the sum of consumption and investment:

$$C_t + I_t = Y_t. \quad (12)$$

In financial markets, the equilibrium condition requires that the representative household holds all outstanding equity shares, and all other assets are in zero net supply. The first order conditions for the firms' optimization problem are standard and have the usual interpretation. The first order condition for labor inputs implies $Y_t - w_t N_t = \alpha Y_t$. As a result, dividends are given by $D_t = \alpha Y_t - I_t$.

Following the DSGE literature, for example Jermann (1998), Boldrin et al. (2001), and Campanale et al. (2010), we view the return on the capital stock to be the return on equity. We denote this return by R_t^e and treat the claim as an unlevered equity claim, given the production technology in the model. It can be shown that (see Campanale et al. (2010) or the Appendix to this paper)

$$\begin{aligned} R_{t+1}^e &= \frac{P_{t+1} + D_{t+1}}{P_t} \\ R_{t+1}^e &= \frac{D_{t+1} + [1 + G_{K_{t+2}}(K_{t+1}, K_{t+2})]K_{t+2}}{[1 + G_{K_{t+1}}(K_t, K_{t+1})]K_{t+1}}, \end{aligned} \quad (13)$$

where the subscript of G stands for the partial derivative, and $1 + G_{K_{t+i}}(K_{t+i-1}, K_{t+i})$ gives Tobin's q . Thus, unlike in the frictionless market, capital adjustment costs deliver time variation in Tobin's q .

Social planner's problem and the pricing kernel

The social planner's problem, in recursive form, is presented in the Appendix. The social planner chooses consumption and capital investment to maximize his welfare. Given that the utility function (5) satisfies homogeneity, the problem can be formulated in terms of stationary variables, which are defined in the Appendix. Ju and Miao (2011) show that the pricing kernel for the

generalized recursive smooth ambiguity utility is given by

$$M_{z_{t+1},t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\rho} \left(\frac{V_{z_{t+1},t+1}}{\mathcal{R}_{z_t,t}(V_{z_{t+1},t+1})} \right)^{\rho-\gamma} \left(\frac{\left(\mathbb{E}_{z_{t+1}} \left[V_{z_{t+1},t+1}^{1-\gamma} \right] \right)^{\frac{1}{1-\gamma}}}{\mathcal{R}_{z_t,t}(V_{z_{t+1},t+1})} \right)^{-(\eta-\gamma)} \quad (14)$$

where $z_{t+1} = 1, 2$, and $\mathbb{E}_{z_{t+1}}[\cdot]$ denotes the conditional expectation operator for the distribution of productivity growth conditioned the period- $(t+1)$ state z_{t+1} . When the agent is ambiguity neutral ($\eta = \gamma$), the last term in (14) vanishes, and the pricing kernel has the same functional form as in Croce (2010) and Kaltenbrunner and Lochstoer (2010). Furthermore, if the agent displays constant relative risk aversion ($\gamma = \rho$), then we obtain the familiar pricing kernel for the expected utility. As usual, the return on equity, R_{t+1}^e , satisfies the Euler equation

$$\mathbb{E}_{z_t} [M_{z_{t+1},t+1} R_{t+1}^e] = 1. \quad (15)$$

where \mathbb{E}_{z_t} is the period- t conditional expectation operator for the current state being z_t . The risk-free rate, $R_{f,t}$, is the reciprocal of the conditional expectation of the pricing kernel:

$$R_{f,t} = \frac{1}{\mathbb{E}_{z_t} [M_{z_{t+1},t+1}]} \quad (16)$$

The conditional equity premium, denoted by $\mathbb{E}_t (R_{t+1}^{ep})$, can be written as

$$\mathbb{E}_t (R_{t+1}^{ep}) = \mathbb{E}_t (R_{t+1}^e - R_{f,t}).$$

4 Calibration and Results

In this section, we first describe how we choose parameter values in the stochastic growth model. Then we calibrate preference parameters (ambiguity aversion and the EIS) to reproduce the first and second unconditional moments of equity returns and risk-free interest rates, and second unconditional moments of a select number of macroeconomic variables that are consistent with the US data.

Data on financial variables (equity returns, risk-free interest rates, dividends and price-

dividend ratios) are drawn from CRSP (Center for Research in Security Prices).⁹ The nominal risk-free rates correspond to the 3-month Treasury bill rates. The equity return is defined as the return on CRSP value-weighted index. Data on macroeconomic variables (consumption, investments, outputs and CPI) are from the data bank at the Federal Reserve Bank of St. Louis. Output is defined as the sum of investment, exports, consumption of perishables and non-perishables, and services. All nominal variables are deflated using the CPI data from FRED II data bank.

[Insert Table 1 about here]

We calibrate the model to a quarterly frequency, and we focus on time aggregated annual statistics. Table 1 summarizes the parameter values that are held invariant throughout the calibration exercise unless otherwise stated. As in the standard literature, we set the capital share (α) to 0.35. Exogenous labor supply, \bar{N} , is set to 0.20, which is similar to the value considered by Croce (2010).¹⁰ Quarterly depreciation rate of capital is set to $\delta = 0.015$, which implies a annualized depreciation rate of 6%. The two transition probabilities are set, respectively, to $p_{11} = 0.94$ and $p_{22} = 0.73$.¹¹ This implies that the average duration of a recession is about 4.5 quarters, and the average expansion is about 14 quarters, as is consistent with the results of Rouwenhorst (1995) and Hamilton (1989) about the post-war period. The value of σ is set to match the standard deviation of productivity growth rates. Data on productivity growth are obtained from an annual multifactor productivity index provided by the Bureau of Labor Statistics. The mean productivity growth rates within the two regimes, μ_1 and μ_2 , are set such that in the steady state, the implied mean productivity growth matches the data. This procedure yields $\mu_1 = 0.0089$ and $\mu_2 = -0.0158$. As a result, the calibrated parameter values of the regime-switching model suggest two distinct regimes where the good regime is persistent.

We set the exponent parameter in the capital adjustment costs function, ι , equal to 1.0985, to match consumption volatility for the sample period 1945–2009. The subjective discount factor is

⁹ The construction of the price-dividend ratio follows the methodology in Campanale et al. (2010) Appendix.

¹⁰ A broad set of calibration exercises show that the effect of \bar{N} on financial and business cycle quantities is insignificant.

¹¹ Recall that subscripts 1 and 2 stand for “good/high” and “bad/low” productivity growth regimes.

set at $\beta = 0.9935$. Throughout the paper, the coefficient of relative risk aversion, γ , is assumed to be 3. This value is even lower than that considered by Kaltenbrunner and Lochstoer (2010), who assume $\gamma = 5$, and much lower than the values assumed by Croce (2010) and Tallarini (2000), which are, respectively $\gamma = 30$ and $\gamma = 100$. Due to nonlinearities, our model does not admit an explicit analytical solution.¹² Thus, we solve the model numerically, using the value function iteration method, and then run Monte Carlo simulations to compute the required moments.¹³

4.1 Benchmark calibration

Table 2 reports the unconditional moments of the key macroeconomic and financial variables generated from three calibrated models, labeled “Ambiguous growth model (I—III)”. Results are generated from 10,000 simulations, each containing 480 quarters (equivalent to 120 years). All statistics are in annualized terms. The data for calibration spans from 1945 to 2009.

[Insert Table 2 about here]

We vary the ambiguity aversion parameter, η , and the EIS parameter, ψ , to produce a mean risk-free rate of 1.17% and a volatility of consumption growth of 2.08%, to exactly match those observed in the data.¹⁴ We then inspect how closely these calibration exercises can match other moments of asset returns. Our targeted level of consumption volatility is lower than that considered by Kaltenbrunner and Lochstoer (2010), although somewhat higher than that in Croce (2010). From Model I to Model III, we decrease the value of ψ while increasing the value of η , mainly to keep the mean risk-free rate constant at 1.17%. All else being equal, a high ambiguity aversion parameter or a high EIS parameter implies a low mean risk-free rate. Intuitively, on one hand, when the EIS parameter is high (e.g., $\psi = 2$), the agent is willing to substitute consumption intertemporally and thus is willing to save. The enhanced saving motive lowers the risk-free rate

¹² Garcia et al. (2008) develop an analytical framework for solving equilibrium asset prices in endowment economies with long-run risks and/or Markov switching. Unfortunately, their framework does not apply to production economies.

¹³ The code is written in Compaq Visual FORTRAN 6.6 and available upon request from the authors.

¹⁴ We discuss the methodology used for choosing the values for η and plausibility of these values in the Appendix. Notice that $\eta > \gamma$ implies that η values are greater than 10, based on the acceptable range for γ discussed in Mehra and Prescott (1985).

in equilibrium. On the other hand, a high degree of ambiguity aversion makes the agent assign more weights to states with lower continuation value. The endogenous belief distortion reflects the agent's pessimistic view about future investment opportunities. This effect increases the current price of the risk-free asset paying one unit of consumption good in the next period, and as a result risk-free rates are low. Here, the two key ingredients that allow us to simultaneously match consumption volatility and the mean risk-free rate are: (1) the separation between risk aversion and the attitude toward intertemporal substitution, and (2) ambiguity aversion. The former can effectively deliver a low volatility of consumption growth while keep the mean risk-free rate low. The latter greatly decreases risk-free rates, a channel that is absent in Epstein-Zin's recursive utility model adopted by Kaltenbrunner and Lochstoer (2010) and Campanale et al. (2010). Thus, the calibrated ambiguous growth models can successfully reproduce the low mean risk-free rate without assuming $\beta > 1$, a debatable assumption postulated by Kaltenbrunner and Lochstoer (2010) and Campanale et al. (2010).

The effects of the ambiguity aversion and EIS parameter on consumption volatility are, however, small for $\psi > 1$. This allows us to consider different values of preference parameters without changing consumption volatility much, as can be seen in Model I–III. The variability of consumption growth increases with the EIS parameter, as in other papers, and also with the ambiguity aversion parameter, though the latter effect is small. This is because the capital stock accumulates at a lower rate under ambiguity aversion (see Figure 5). The marginal capital adjustment costs are, therefore, low given convex capital adjustment costs. Thus, facing productivity shocks, the agent can rearrange consumption intertemporally at a lower cost. Besides consumption volatility, our model can also produce an autocorrelation of consumption growth of 0.26 that is close to the data. Nevertheless, the model predicts a much too high consumption volatility to output volatility ratio relative to what is observed in the data. As noted by previous papers (for example, Jermann (1998) and Kaltenbrunner and Lochstoer (2010)), this is a common issue for models with capital adjustment costs. The friction imputes much more variation into consumption than

output.

Thanks to the effect of adjustment costs, our model generates significant time variation in the price of capital. In the absence of adjustment costs, Tobin's q is always kept constant at 1, and thus all variation in equity returns is entirely due to the variation of dividends. It is shown in Table 2 that Model I–III all produce volatile equity returns; among them, Model I has a volatility of equity returns closest to that observed in the data while all the other models generate slightly lower levels of volatility of equity returns.

The remarkable results shown in Table 2 concern the mean equity premium and the volatility of the risk-free rate. Without exception, all calibrated ambiguous growth models generate high mean equity premiums over 7% as well as low levels of the volatility of risk-free rates. Among Model I–III, Model III produces the highest mean equity premium, 7.85%, exceeding that observed in the data (7.42%). More important, it is remarkable that in this paper the equity claim is defined as *unlevered* claim to aggregate dividends. In addition, all calibrated models generate a volatility of risk-free rates less than 0.8%, which is observed in the data. The results shown here represent a notable improvement over the existing literature (for example, see Jermann (1998), Boldrin et al. (2001), Kaltenbrunner and Lochstoer (2010), Campanale et al. (2010) and Croce (2010)) as those models encounter much difficulty in reconciling a high equity premium with a smooth risk-free rate. Jermann (1998) and Boldrin et al. (2001) rely on habit persistence to obtain high equity premium. However, the implied strong aversion toward intertemporal substitution inevitably leads to excessively volatile risk-free rates. Similarly, both Kaltenbrunner and Lochstoer (2010) and Campanale et al. (2010) consider rather low values for the EIS parameter. The benchmark calibration of Croce (2010) assumes that the EIS is greater than 1, and thus can reproduce the low volatility in risk-free rates, but the model lacks a channel to raise the mean equity premium beyond 3%. Turning to our ambiguous growth model, the key feature is that we rely on ambiguity aversion rather than the aversion toward intertemporal substitution to generate a sizable equity premium. As a result, a smooth risk-free rate can be naturally accommodated by assuming a

EIS greater than 1. Additionally, because the risk-free rate is the reciprocal of the expectation of the pricing kernel, our model correctly accounts for the dynamics of the pricing kernel.

[Insert Figure 1 about here]

Since both the mean and volatility of equity premium can be closely matched, we naturally fit the historical Sharpe ratio, which is equal to 0.42 for 1945–2009. This level is higher than that considered by Kaltenbrunner and Lochstoer (2010), which accounts for the period of Great Depression. In fact, Model I–III predict slightly higher Sharpe ratios than in the data. In addition, because we can match the mean and volatility of the risk-free rate, the model implied mean and volatility of equity returns are also close to the data. The last row in Table 2 presents the price of risk defined by $\sigma(M)/\mathbb{E}(M)$. Even though the coefficient of relative risk aversion and the volatility of consumption growth are the same across Model I–III, the price of risk varies from 0.70 to 1.20. These values are comparable to those obtained by Ju and Miao (2011) in their endowment economy model and greatly exceed those in Kaltenbrunner and Lochstoer (2010). Since $\mathbb{E}(M)$ is generally close to 1, the high price of risk is entirely attributed to large variation of the marginal rate of substitution. As dividends are endogenously determined in equilibrium, we are also interested in the volatility of dividend growth. Table 2 reveals that the annualized unconditional standard deviation of dividend growth is equal to about 5% across all the calibrated models, a level close to that in the data.¹⁵ Figure 1 also plots the simulated consumption growth and dividend growth in quarterly frequency. It is obvious that our model predicts more volatile dividend growth than consumption growth.

4.2 Inspecting the mechanism

To better understand the effects of ambiguity aversion, we vary the degree of ambiguity aversion with other preference parameters held constant. As noted above, one major advantage of using smooth ambiguity utility rather than multiple prior utility is that the concepts of ambiguity

¹⁵ As an example, the no leverage case in Jermann (1998) generates about 50% of dividend volatility observed in the data, and other cases generate substantially higher values. Models I–III in Table 2, on the other hand, match between 67% to 72% of dividend volatility observed in the data.

and ambiguity aversion are disentangled. This allows us to do comparative statics analysis. In particular, we assume $\eta = \gamma = 3$, which implies ambiguity neutrality. As in Table 2, we consider three alternative values for the EIS parameter, $\psi = 2, 1.5$ and 1.2 , which are the same as in Table 2. To further inspect the effect of separating risk aversion and intertemporal substitution, we also compute the case of expected utility, that is, $\psi = 1/\gamma = 1/3$. By comparing results under ambiguity neutrality and ambiguity aversion, we are able to inspect the impacts of ambiguity aversion on the moments of asset returns and the price of risk.

To facilitate our understanding of the effects of ambiguity aversion, we also simulate time series of the key macroeconomic and financial moments and/or variables generated from the models with and without ambiguity aversion. Specifically, we simulate macroeconomic quantities including productivity regimes, productivity growth rates, capital, investment, and consumption; and financial quantities including the price of risk, dividend growth, the price-dividend ratio, conditional equity premium, and the risk-free rate. We perform the simulation in response to recurring productivity growth shocks, which are artificially generated from the regime-switching process (8). In the simulation, we assume that the economy starts from the high productivity regime. Given simulated productivity shocks, capital, investment and consumption decisions adjust accordingly as the outcome of equilibrium allocations. Financial quantities are then computed as functions of the state variables.

[Insert Figure 2 about here]

The price of risk

In general, the key for both production economy and endowment economy models to generate a high equity premium is to impute a significant amount of variation into the marginal rate of substitution. Rouwenhorst (1995) finds that in a frictionless economy with the standard CRRA preferences, it is difficult to explain a high equity premium even though risk aversion is sufficiently high. This is because the agent can easily alter production plans to remove consumption risk.

The role of capital adjustment costs, as in other papers (Jermann (1998), Boldrin et al. (2001), Campanale et al. (2010), and Croce (2010)), is to make it more costly to smooth consumption, and thus to generate more consumption risk.

A fundamental difference between our production economy model and endowment economy models studied by Ju and Miao (2011) and Collard et al. (2011) is that in general equilibrium with nontrivial production, ambiguity aversion itself cannot explain the substantial equity premium observed in the data even when risk aversion and intertemporal substitution are disentangled. It must act in conjunction with capital adjustment costs to generate sufficiently high equity premium. This intuition is confirmed in Figure 2, which shows simulated values of productivity regimes and the model implied conditional moments. We observe that even without capital adjustment costs, the price of risk still rises with ambiguity aversion. However, ambiguity aversion has no effect on the equity premium, which always stays close to 0. In a frictionless economy, Tobin's q is constant and equal to 1. Little variation in the price of capital leads to that the equity claim carries little risk and thus almost no risk premium.

[Insert Figure 3 about here]

In the presence of adjustment costs, ambiguity aversion substantially increases both the price of risk and the conditional equity premium. Panel B of Figure 3 plots simulated conditional price of risk values, $\sigma_t(M) / \mathbb{E}_t(M)$. Relative to the frictionless economy case shown in Figure 2, the effect of ambiguity aversion on the price of risk becomes much more pronounced. Panel A of Figure 3 shows productivity regimes (the “circle” marker) with uppermost circles indicating the good regime and bottommost circles indicate the bad regime.

These two figures reveal a second important regularity. It is clear from the top and middle panels in Figures 2 and 3 that while the price of risk seems to be countercyclical under ambiguity neutrality, it is strongly procyclical under ambiguity aversion. Under ambiguity neutrality, the agent's beliefs about regime switching strictly conform to the transition probabilities and are thus not distorted. When the economy is currently experiencing the good regime, it is less likely

that the state will switch to the bad regime in the near future. Thus, the degree of economic uncertainty is low, resulting in a small risk premium. When the economy is in recessions, as the bad regime is more transitory there is a good chance that it will jump to the good regime in the future. Since the level of uncertainty is high, the equity claim must pay off a higher return to generate a commensurate risk premium, and the equity premium is, therefore, relatively high.

However, this pattern of cyclical variation is reversed under ambiguity aversion. The explanation lies in the asymmetric effects of ambiguity aversion in the presence of different regimes. Ambiguity aversion leads to endogenous distortion of the agent's beliefs about regime switching. Given such distortion, the rise in the price of risk incurred by ambiguity aversion becomes more pronounced in the presence of the good regime compared with the bad regime. The ambiguity averse agent endogenously puts more weights on states with lower continuation value, which are more likely to be associated with the bad regime. When the agent does so, there is more scope for this to take effect if the economy is in the good regime, due to its high persistence. In this case, ambiguity aversion has a more significant effect, that is, it greatly magnifies the impact of the agent's fear of unfavorable states. On the other hand, if the economy is currently in the low productivity regime, ambiguity aversion will have a much less significant impact on the price of risk. Since the bad regime is relatively transitory, the agent is confident that it is very likely that the future state will switch to the high productivity regime and future investment opportunities will improve. This implies much less scope for his fear of those unfavorable states. Thus, the effect of ambiguity aversion becomes much smaller.

Equity premium and equity volatility

[Insert Table 3 about here]

[Insert Table 4 about here]

By comparing Table 2 and Table 3, we observe that it is ambiguity aversion that indeed drives the high equity premium in the model. Under ambiguity neutrality, the Epstein-Zin recursive utility model with the EIS greater than 1 can at best generate a mean equity premium of about

2%. The mean equity premium under expected utility is even lower at 1.6%. We can decompose the equity premium obtained under ambiguity into three components:

$$\mathbb{E}(R^{ep,AA}) = \mathbb{E}(R^{ep,EU}) + \underbrace{[\mathbb{E}(R^{ep,EZ}) - \mathbb{E}(R^{ep,EU})]}_{\text{Epstein-Zin}} + \underbrace{[\mathbb{E}(R^{ep,AA}) - \mathbb{E}(R^{ep,EZ})]}_{\text{ambiguity aversion}}$$

where “AA”, “EZ”, and “EU”, respectively, stand for “ambiguity aversion”, “Epstein-Zin” and “expected utility”. On the right hand side, the first equity premium component arises in the expected utility model, and the second term is attributed to the separation of risk aversion and intertemporal substitution, and the third term is due to ambiguity aversion. Table 4 presents results on the equity premium decomposition. The results show that the separation between risk aversion and intertemporal substitution in fact contributes quite modestly to the equity premium. Instead, it is ambiguity aversion that accounts for the high equity premium generated by the model. This finding is in line with the endowment economy model of Ju and Miao (2011).

We can relate equity premium to the price of risk through the following equation derived from Euler equation (15):

$$\mathbb{E}_t(R_{t+1}^e) - R_{f,t} = -\frac{\sigma_t(M_{t+1})}{\mathbb{E}_t(M_{t+1})} \sigma_t(R_{t+1}^e) \rho_t(M_{t+1}, R_{t+1}^e).$$

We find that the latter two multiplicative terms, $\sigma_t(R_{t+1}^e)$ and $\rho_t(M_{t+1}, R_{t+1}^e)$, both decrease (in magnitude) with ambiguity aversion. However, this effect is dominated by the substantial increase in the price of risk. Thus, the net effect results in an increase in equity premium under ambiguity aversion. The less significant correlation between the pricing kernel and the equity return is due to the last multiplicative term in equation (14), which arises as a distortion driven by ambiguity aversion. This term reflects the impact of the endogenous pessimistic distortion in pricing of assets. When the degree of ambiguity aversion is high, the agent’s attitude makes his pricing of assets depend more on his pessimistic view while less on the business conditions that determine the return on the equity claim.

In addition, by comparing Table 2 and Table 3, we also notice that the volatility of equity

returns is slightly lower with ambiguity aversion. This finding stands in contrast to Ju and Miao (2011) where ambiguity aversion increases the volatility of equity returns. The volatility of equity returns is mainly determined by changes in the price-dividend ratio and the volatility of dividend growth. In an endowment economy such as the one studied by Ju and Miao (2011), the dividend process is exogenously specified. Thus, the variation of the price-dividend ratio is, to a great extent, due to the variation in the pricing kernel. This mechanism is often employed to explain the “equity volatility puzzle” of Shiller (1981). In a production economy, dividends are endogenously determined in the equilibrium, and the volatility of dividend growth depends on preference parameters. In addition, the volatility of the price of capital largely depends on the marginal capital adjustment costs. In a frictionless economy, the variation in the share price becomes minimal.

[Insert Figure 4 about here]

[Insert Figure 5 about here]

To see whether the volatility of price of capital and that of the dividend growth are dampened under ambiguity aversion, we plot in Figure 4 the simulated series of price-dividend ratios (Panel A) and dividend growth (Panel B). Note that the model implied price-dividend ratio is highly persistent. More important, it reveals that both sources of volatility are mitigated under ambiguity aversion. First, the price of capital becomes less volatile because the marginal capital adjustment costs are lower under ambiguity aversion. The ambiguity averse agent is not willing to invest in the capital stock because (1) he is ambiguous about future investment opportunities, and (2) given adjustment costs, consumption smoothing is costly. Thus, ambiguity aversion decreases the speed of capital accumulation through less investments.¹⁶ This intuition is confirmed in Figure 5. As a result, under ambiguity aversion, the marginal capital adjustment costs and the volatility of equity returns are both lower than in the ambiguity neutral case. Second, dividends

¹⁶ Cagetti et al. (2002) find that in a frictionless economy the preference for robustness speeds up the accumulation of the capital stock due to precautionary savings motive. Their result holds when it is costless for the agent to consume the capital stock. Once the capital adjustment costs are accounted for, their effect will dominate that of the precautionary savings motive.

in a production economy are given by $D_t = \alpha Y_t - I_t$. With total outputs assumed fixed, less volatile investments lead to lower volatility in dividend growth. This explains why in Figure 4 dividend growth is less responsive to productivity shocks for the ambiguity averse agent.

Turning to the equity premium, Panel C of Figure 3 plots simulated conditional equity premia, $\mathbb{E}_t(R_{t+1}^e - R_{f,t})$. Ambiguity aversion increases conditional equity premium mainly through increasing the price of risk. In particular, the effect is more pronounced when the good regime is present. Thus, the effect of ambiguity aversion on conditional equity premium is asymmetric for the good regime and the bad regime, as in the case of the price of risk (Panel B of Figure 3). Our model, therefore, predicts procyclical variation of equity premium, which seems counterfactual given the empirical evidence documented in, for example, Fama and French (1988a), Fama and French (1989) and Campbell and Shiller (1988). We strongly suspect that introducing a hidden state and Bayesian learning might help in producing countercyclical equity premia.

4.3 The volatility of risk-free rates

A major difficulty encountered in previous studies (Jermann (1998), Boldrin et al. (2001), Kaltenbrunner and Lochstoer (2010) and Campanale et al. (2010)) is that production economy models striving to replicate the high equity premium observed in the data will also generate an excessively volatile risk-free rate. Since the risk-free rate is the reciprocal of the conditional expectation of the pricing kernel, if the model implied risk-free rate is too volatile to be reconciled with the data, the model must have mismatched the dynamic behavior of the pricing kernel. In our model, ambiguity aversion can not only increase the equity premium but also reduce the volatility of the risk-free rate. This pattern can be observed by comparing Table 2 and Table 3. For $\psi = 1.2, 1.5,$ and 2 , $\sigma(R_f)$ decreases with the degree of ambiguity aversion η . It is worth noting that when the EIS is low as in the expected utility case ($\psi = 1/3$), the model generates an excessively volatile risk-free rate, with its unconditional standard deviation being equal to 3%.

[Insert Figure 6 about here]

Our model generates procyclical risk-free rates. To gain a better understanding, we plot in Figure 6 simulated risk-free rates. First, we notice that ambiguity aversion reduces the risk-free rate in all cases because the agent is pessimistic about future investment opportunities and willing to save. Second, the effect of ambiguity aversion is stronger in the presence of the good regime than the bad regime, because the good regime is more persistent and admits more scope of the impact of ambiguity aversion. This explains the observed smaller variation in risk-free rates when the agent is ambiguity averse.

4.4 Predictability of equity returns

To explore the ability of our ambiguous growth model to generate predictable equity returns, Table 5 presents results from regressing log equity returns and log excess returns onto log price-dividend ratios for the historical data and the model. We simulate 10,000 sets of time series data on both the dependent and independent variables and obtain the reported results by taking the average of estimated coefficients and the R^2 s, in order to avoid small sample bias. The estimation results for the historical data suggest that both equity returns and excess returns are predictable. First, the slope coefficients are all negative, which implies that high prices relative to dividends lead to low expected returns. Second, the estimated slope coefficient and R^2 s increase with the horizon, and the R^2 s rise to sizable values for 5-year ahead returns. A large body of literature has documented similar findings.¹⁷ Ju and Miao (2011) show that in an endowment economy, their baseline model with Bayesian learning and ambiguity aversion is able to generate predictability in returns, and further reproduce the pattern of increasing regression slopes with the horizon. But the model produces a negative relationship between the R^2 s and the horizon.

[Insert Table 5 about here]

Table 5 reports the model-generated results for different horizons, i.e., 1 year, 2, 3 and 5 years and for the ambiguous growth model I–III. We aggregate simulated quarterly returns into an-

¹⁷For example, see Campbell and Shiller (1988), Campbell and Shiller (1988b) Cecchetti et al. (2000), Welch and Goyal (2008) and Campbell and Yogo (2006), among many others.

nualized returns and run regressions onto the end-of-year price-dividend ratio. Panel A contains the results when log equity returns are used as the dependent variable. As in the data, the estimated regression slopes are all negative: high price-dividend ratios predict low expected equity returns. Further, the ambiguous growth model is not only able to generate increasing slopes (in magnitude) with the horizon but also to replicate the pattern of increasing R^2 s. However, the overall predictability is weak with the R^2 s being about 0.08. Panel B presents the results when we use log excess returns as the dependent variable. It is clear that predictability is moderately stronger than in the previous case: the R^2 s are higher if variation in risk-free rates is accounted for. But for long horizons, i.e., 3 and 5 years, the model implied R^2 s are still far below those in the data.

4.5 Implied consumption dynamics

[Insert Figure 7 about here]

The production economy models differ from endowment models in the sense that the joint dynamic behavior of aggregate consumption and dividends are endogenously determined in these models. Here, we examine the implications of our ambiguous growth model for the dynamics of aggregate consumption. Figure 7 shows simulated series of consumption growth (Panel A), expected consumption growth (Panel B), and the conditional variance of consumption growth (Panel C). Expected consumption growth is procyclical while the conditional variance of consumption growth is strongly countercyclical. Moreover, expected consumption growth clearly features regime switching. Ju and Miao (2011) assume that expected consumption growth has two distinct regimes in their endowment economy. Early works assuming regime shifts in expected consumption growth include Cecchetti et al. (1990) and Cecchetti et al. (2000). Our production economy model, therefore, can lend theoretical support to this line of models in a general equilibrium framework.

Bansal and Yaron (2004)'s long-run risk model exogenously specifies the consumption process in such a way that expected consumption growth and conditional variance of consumption

growth are both highly persistent. In doing so, they are able to explain a variety of asset pricing puzzles with the help of the long-lasting effects of innovations to expected consumption growth as well as to the conditional variance of consumption growth. Table 6 presents the autocorrelation coefficients up to 5 lags for model generated expected consumption growth and the conditional variance of consumption growth. It shows that both simulated variables exhibit moderate persistence where the 1st autocorrelations for expected consumption growth and the conditional variance of consumption growth are, respectively, about 0.67 and 0.70. Although our model produces time-varying expected consumption growth and stochastic conditional variance, the model is not able to provide sufficient theoretical justification for long run risks in consumption.

The implied moderate persistence can, however, lend support to the consumption growth process estimated solely on the basis of the time series properties of consumption growth data. According to Beeler and Campbell (2009) and Constantinides and Ghosh (2010), the historical consumption growth data suggest that the strength of persistence for expected consumption growth is much less than in Bansal and Yaron’s calibration. In a recent contribution, Collard et al. (2011) take into account this feature and show that model uncertainty regarding whether expected consumption growth is highly or moderately persistent is important for producing a high equity premium and volatile equity returns. They propose a “two- ρ ” model that includes the long-run risk (high-persistence) model of Bansal and Yaron (2004) and a low-persistence model.¹⁸ In Table 6, we present the autocorrelations implied by their low-persistence model for comparison. It is clear that our model can endogenously reproduce the dynamic behavior of expected consumption growth that is close to the implied dynamics of the exogenous specification employed by Collard et al. (2011). Thus, our model can provide theoretical justification for the low-persistence model in Collard et al’s calibration.

¹⁸ Collard et al. (2011) assume the persistence parameter is 0.30 in their calibration at annual frequency. This value is close to the GMM estimate provided by Constantinides and Ghosh (2010). Collard et al. argue that this value makes it difficult to distinguish the high-persistence and low-persistence models when the agent cannot directly observe the state variables.

5 Conclusion

In this paper, we develop a stochastic growth model with ambiguity and ambiguity aversion to explain a number of stylized facts about asset returns data. Productivity growth is specified to be a regime-switching process. Our model accommodates ambiguity and ambiguity aversion using the framework of smooth ambiguity preferences. Specifically, we use the generalized recursive smooth ambiguity utility model proposed by Ju and Miao (2011). Ambiguity aversion results in endogenously distorted beliefs that represent the agent's pessimistic view about future investment opportunities. The calibration exercise shows that the production economy model can match the mean equity premium and risk-free rate, the volatility of equity returns and the volatility of risk-free rates observed in the data. In addition, our model can generate long-horizon predictability of equity returns, though the predictability is still weak compared to the data.

The improvement of the model relative to the existing literature critically depends on the utility preferences adopted, which permit a three-way separation of risk aversion, ambiguity aversion, and the attitude toward intertemporal substitution. Without the separation between risk aversion and intertemporal substitution, the model generates too high and excessively volatile risk-free rates. Without ambiguity aversion, the model cannot produce a sufficiently high equity premium and a low enough risk-free rate. We find that the mean equity premium is only about 2 percent for Epstein-Zin recursive utility where the EIS is greater than unity.

This work may be extended in several directions. In this paper, unlike Ju and Miao (2011), we assume that the agent observes the state of the economy. It could be fruitful to study the impact of the presence of a hidden state and the role of Bayesian learning in a production economy in future research. Although the present model is successful in matching the usual set of asset-returns moments, the model predicts a too high ratio between consumption growth volatility and output growth volatility compared to the data. This is mainly due to the specification of capital adjustment costs. Other sources of market frictions such as multiple sectors or labor market frictions could be explored to see whether they can help in this regard. Recently, Kung

and Schmid (2011) study the role of innovation through research and development (R&D) in both long term growth expectations and in asset prices. It is worthwhile to investigate whether introducing ambiguity and ambiguity aversion into an endogenous growth model would enrich the fundamental relationship between macroeconomic risk and economic growth.

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6 Appendix

6.1 The Social Planner's Problem

Define the following stationary variables:

$$\{c_t, i_t, y_t, k_t, v_t\} = \left\{ \frac{C_t}{A_{t-1}}, \frac{I_t}{A_{t-1}}, \frac{Y_t}{A_{t-1}}, \frac{K_t}{A_{t-1}}, \frac{V_t}{A_{t-1}} \right\}$$

The social planner's problem can be written as

$$v(k_t, \Delta a_t, \zeta_t) = \max_{c_t, k_{t+1}} \left\{ (1 - \beta) c_t^{1-\rho} + \beta e^{(1-\rho)\Delta a_t} \left(\mathbb{E}_{z_t} \left[\left(\mathbb{E}_{\pi_z, t+1} \left[v_{t+1}^{1-\gamma}(k_{t+1}, \Delta a_{t+1}, \zeta_{t+1}) \right] \right)^{\frac{1-\eta}{1-\gamma}} \right] \right)^{\frac{1-\rho}{1-\eta}} \right\}^{\frac{1}{1-\rho}} \quad (17)$$

subject to the following constraints:

$$c_t + i_t = y_t \equiv e^{(1-\alpha)\Delta a_t} k_t^\alpha \bar{n}^{1-\alpha} \quad (18)$$

$$e^{\Delta a_t} k_{t+1} = (1 - \delta_k) k_t + i_t - \left| \frac{e^{\Delta a_t} k_{t+1}}{k_t} - \omega \right|^\iota k_t \quad (19)$$

$$\Delta a_t = \mu(s_t) + \sigma \epsilon_t, \quad \epsilon_t \sim N(0, 1) \quad (20)$$

$$c_t \geq 0, k_{t+1} \geq 0 \quad (21)$$

Ju and Miao (2011) show that the pricing kernel is given by

$$M_{z_{t+1}, t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\rho} \left(\frac{V_{z_{t+1}, t+1}}{\mathcal{R}_{z_t, t}(V_{z_{t+1}, t+1})} \right)^{\rho-\gamma} \left(\frac{\left(\mathbb{E}_{z_{t+1}} \left[V_{z_{t+1}, t+1}^{1-\gamma} \right] \right)^{\frac{1}{1-\gamma}}}{\mathcal{R}_{z_t, t}(V_{z_{t+1}, t+1})} \right)^{-(\eta-\gamma)},$$

which is equivalent to

$$M_{z_{t+1}, t+1} = \beta e^{-\rho \Delta a_t} \left(\frac{c_{t+1}}{c_t} \right)^{-\rho} \left(\frac{v_{z_{t+1}, t+1}}{\mathcal{R}_{z_t, t}(v_{z_{t+1}, t+1})} \right)^{\rho-\gamma} \left(\frac{\left(\mathbb{E}_{z_{t+1}} \left[v_{z_{t+1}, t+1}^{1-\gamma} \right] \right)^{\frac{1}{1-\gamma}}}{\mathcal{R}_{z_t, t}(v_{z_{t+1}, t+1})} \right)^{-(\eta-\gamma)}$$

where $z_{t+1} = 1, 2$.

6.2 Numerical Algorithm

Due to nonlinearities, the model does not admit an analytical solution. We use value function iteration (VFI) to solve the model and run Monte Carlo simulations to compute macroeconomic

and financial moments. Here, we present the VFI algorithm employed in this paper.

1. Compute the steady-state in the deterministic economy, assuming that the productivity growth rate is constant and equal to Δa_{ss} where Δa_{ss} is given by

$$\begin{aligned} p_{ss} &= \frac{1 - p_{22}}{2 - p_{22} - p_{11}} \\ \Delta a_{ss} &= p_{ss}\mu_1 + (1 - p_{ss})\mu_2 \end{aligned}$$

2. Discretization of the state space: we use (1) $N_{\Delta a}$ equidistant points for Δa on the interval $[\underline{\Delta a}, \overline{\Delta a}] = [\Delta a_{ss} - \underline{\lambda}_a\sigma, \Delta a_{ss} + \overline{\lambda}_a\sigma]$ where the constants $\underline{\lambda}_a$ and $\overline{\lambda}_a$ are set such that the interval of the grid is wide enough to contain the set of quadrature nodes used in the algorithm; (2) N_k equidistant points for k on the interval $[\underline{k}, \overline{k}] = [0.1k_{ss}, 1.9k_{ss}]$ where k_{ss} is the value of capital at the deterministic steady state.
3. Our goal here is to compute $v(\Delta a, k, 1)$ and $v(\Delta a, k, 2)$ on the grid $[\underline{\Delta a}, \overline{\Delta a}] \times [\underline{k}, \overline{k}]$. At this point, we guess value functions $v(\Delta a, k, 1)$ and $v(\Delta a, k, 2)$ that are arrays of $N_a \times N_k$.
4. Computing $\mathbb{E}_{s_t, t} [v^{1-\gamma}(k_{t+1}, \Delta a_{t+1}, s_{t+1})]$. Notice that

$$\begin{aligned} \mathbb{E}_z [v^{1-\gamma}(\Delta a', k', s')] &= p_{11}\mathbb{E}_{z'} [v^{1-\gamma}(\Delta a', k', z') \mid z' = 1] \\ &\quad + (1 - p_{11})\mathbb{E}_{z'} [v^{1-\gamma}(\Delta a', k', z') \mid z' = 2] \end{aligned}$$

when $z = 1$

$$\begin{aligned} \mathbb{E}_z [v^{1-\gamma}(\Delta a', k', z')] &= (1 - p_{22})\mathbb{E}_{z'} [v^{1-\gamma}(\Delta a', k', z') \mid z' = 1] \\ &\quad + p_{22}\mathbb{E}_{z'} [v^{1-\gamma}(\Delta a', k', z') \mid z' = 2] \end{aligned}$$

when $z = 2$

To compute the inner conditional expectations, notice that these variables are in fact invariant with respect to Δa (because Δa is integrated out). For example, we can approximate

the expectation using the Gauss-Hermite quadrature method

$$\mathbb{E} [v^{1-\gamma} (\Delta a', k', z') | z' = 1] = \int_{\underline{\Delta a}}^{\overline{\Delta a}} v^{1-\gamma} (\Delta a', k', s') \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(\Delta a - \mu(1))^2}{2\sigma^2}} d\Delta a'$$

where k' is some point on the capital grid. However, since the quadrature nodes generally yield values of $\Delta a'$ outside the grid of Δa , we need to interpolate $v^{1-\gamma} (\Delta a', k', z')$ on those values. Note that the initial guess and each iteration only give the value of $v (\Delta a, k, z)$ on the grid of the state space. To achieve this task, we first construct Chebyshev polynomials in Δa on the grid, and then for every k' on the grid we regress $\log (v (:, k', 1))$ onto the Chebyshev polynomials in Δa and obtain the Chebyshev regression coefficients. These coefficients allow us to interpolate $\log (v (\Delta a', k', 1))$ for a possible value of $\Delta a'$. The inner expectation $\mathbb{E}_{s'} [v^{1-\gamma} (\Delta a', k', s') | z' = 2]$ can be approximated in the same way.

5. Interpolate expectations with respect to capital. To solve the optimization problem, we choose the policy function k' to maximize the value function. The algorithm therefore requires any admissible value of capital. As a result, we need to evaluate the right-hand side of the recursion for every admissible value of the policy function k' . In particular, we need to approximate the following two expectations

$$\mathbb{E}_{z'} [v^{1-\gamma} (\Delta a', k', z') | z' = 1]$$

$$\mathbb{E}_{z'} [v^{1-\gamma} (\Delta a', k', z') | z' = 2]$$

for any admissible k' . To do this, we first create Chebyshev polynomials in k' on the grid, and then regress the two expectations onto the Chebyshev polynomials in k' . The regression coefficients obtained allow us to interpolate the expectations for any admissible value of k' .

6. Maximization and iteration. Given the states $(\Delta a, k, z)$, we search k' on the grid that maximizes the value function. To refine the optimal policy function, we employ a numerical optimization procedure to search for the optimal k' . The objective function is updated once an iteration is completed. The stopping rule is that the new value function and the old

value function has a standard sup-norm $\frac{|v'-v|}{|v|} < 1.e - 8$.

In order to improve convergence of our algorithm, we employ a multigrid scheme. That is, we first solve the model numerically on a small number of grids for k (i.e., 300 grid points). Then the algorithm is extended to a larger number of capital grids (e.g., 600 grid points) by first interpolating the solution in the previous round and then using the interpolated values as starting values for the next round of the algorithm. This procedure continues until the grid of k is fine enough and the algorithm converges. The maximal number of capital grids that we use is 2000. Once we solve the social planner's problem, we obtain equilibrium allocations as functions of the state variables. Then we use Monte Carlo simulations to compute financial and business cycle quantities.

6.3 The Size of Ambiguity Aversion Parameter

In Table 2, we report calibration results based on three values for ambiguity aversion parameter, η , which takes values 21, 25, and 31. We calibrate this parameter following the guidelines discussed in Halevy (2007), Ju and Miao (2011), and Chen et al. (2011). That is, we elicit the ambiguity through introspection and thought experiments related to Ellsberg (1961) paradox.

The classic example of Ellsberg paradox is the static two urns case. Suppose that there are two urns filled with black and white marbles. Subjects are told that one urn has 50 white and 50 black marbles. The second urn may contain either 100 white or 100 black marbles. The exact composition of the second urn is unknown to the subjects. Subjects win a prize worth d dollars if they pull a black marble from an urn, otherwise they do not win or lose anything. Halevy (2007) reports that the majority of subjects prefer a bet on urn 1 over a bet on urn 2. We observe that if subjects are asked to bet on a white marble instead of black, they still prefer urn 1, see Halevy (2007). The standard Von Neumann-Morgenstern expected utility framework fails to explain this behavior, regardless of the level of risk aversion or beliefs held by the subjects. However, the static form of Klibanoff et al. (2005) smooth ambiguity aversion utility presented in equation (1) is capable of explaining this paradox. What is needed is that subjects show ambiguity aversion,

in the sense that v in equation (1) is more concave than u . Thus, the difference between certainty equivalents of betting on the first and second urns measure the ambiguity premium, which is a useful tool for calibrating η .

We formally define ambiguity premium as

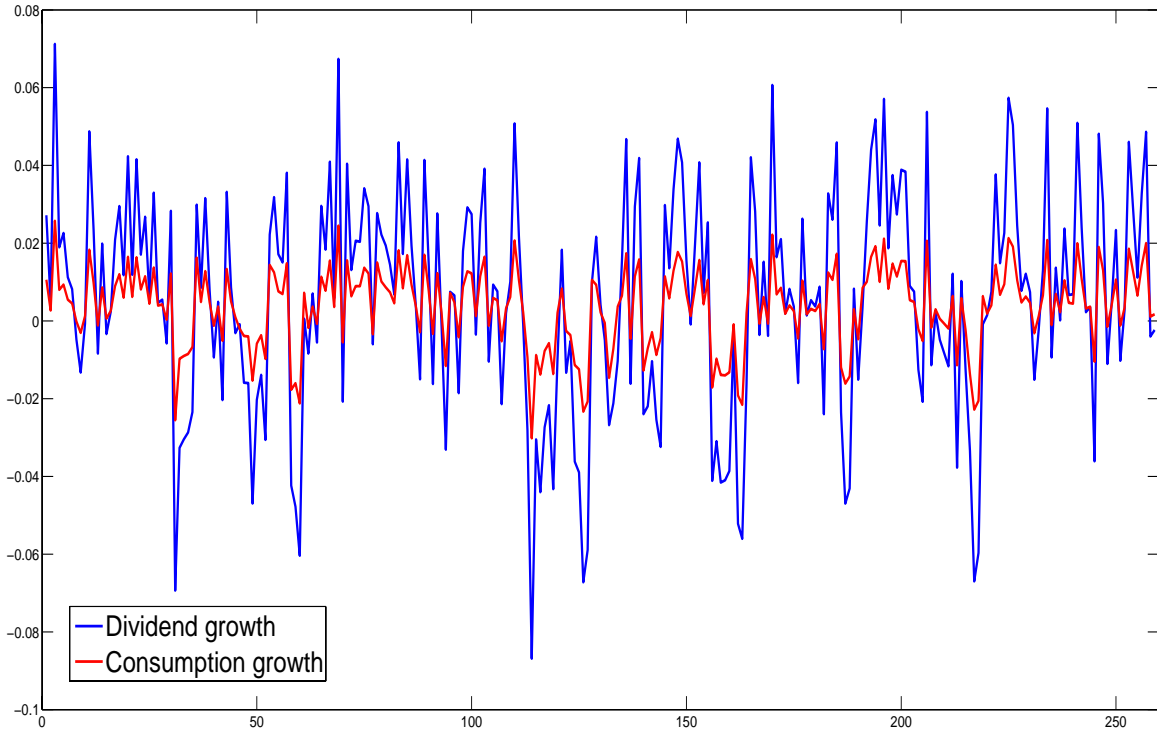
$$u^{-1} \left(\int_{\Theta} \int_S u(f) d\pi_{\theta} d\zeta(\theta) \right) - v^{-1} \left(\int_{\Theta} v \left(u^{-1} \left(\int_S u(f) d\pi_{\theta} \right) \right) d\zeta(\theta) \right). \quad (22)$$

Denote a subject's wealth level as w . Let u and v be defined as in equation (1). Following Chen et al. (2011), suppose that the subjective prior for the bet is $\zeta(\theta) = (0.5, 0.5)$. A bet on the second urn generates two probability measures over the color of marbles: $(0, 1)$ and $(1, 0)$. Thus, for $\eta > \gamma$, ambiguity premium is

$$(0.5(d+w)^{1-\gamma} + 0.5w^{1-\gamma})^{\frac{1}{1-\gamma}} - (0.5(d+w)^{1-\eta} + 0.5w^{1-\eta})^{\frac{1}{1-\eta}}. \quad (23)$$

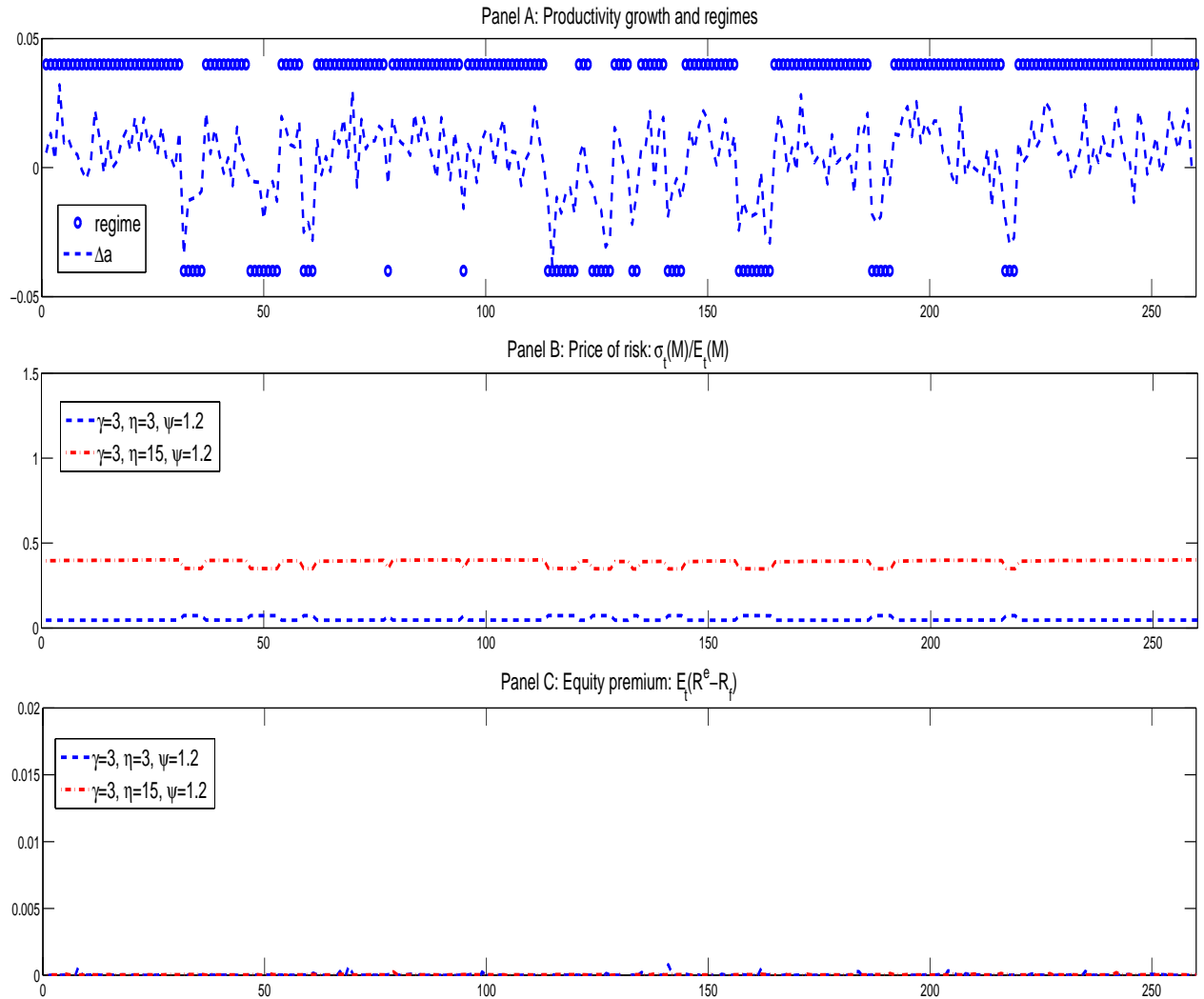
We can express ambiguity premium as a percentage of the expected value of the bet $d/2$. The size of this premium depends on the size of the bet or the prize-wealth ratio, d/w . Chen et al. (2011) compute values for ambiguity premia, given values for γ ranging between 0.5 and 15, η ranging between 40 and 110, and d/w ratios of 1% and 0.5%. Both Camerer (1999) and Halevy (2007) report ambiguity premia around 10-20% of the expected value of a bet in Ellsberg paradox experiments. Similar to Ju and Miao (2011) and Chen et al. (2011), our calibration crucially depends on the size of the bet. Ju and Miao (2011) use values for η ranging between 2 and 15, and Chen et al. (2011) calibrate η in 2 to 100 range. Thus, our choice of η between 21 and 31 is reasonable.

Figure 1: Simulated dividend growth and consumption growth
Simulated dividend growth and consumption growth ($\gamma=3, \eta=31, \psi=1.2$)



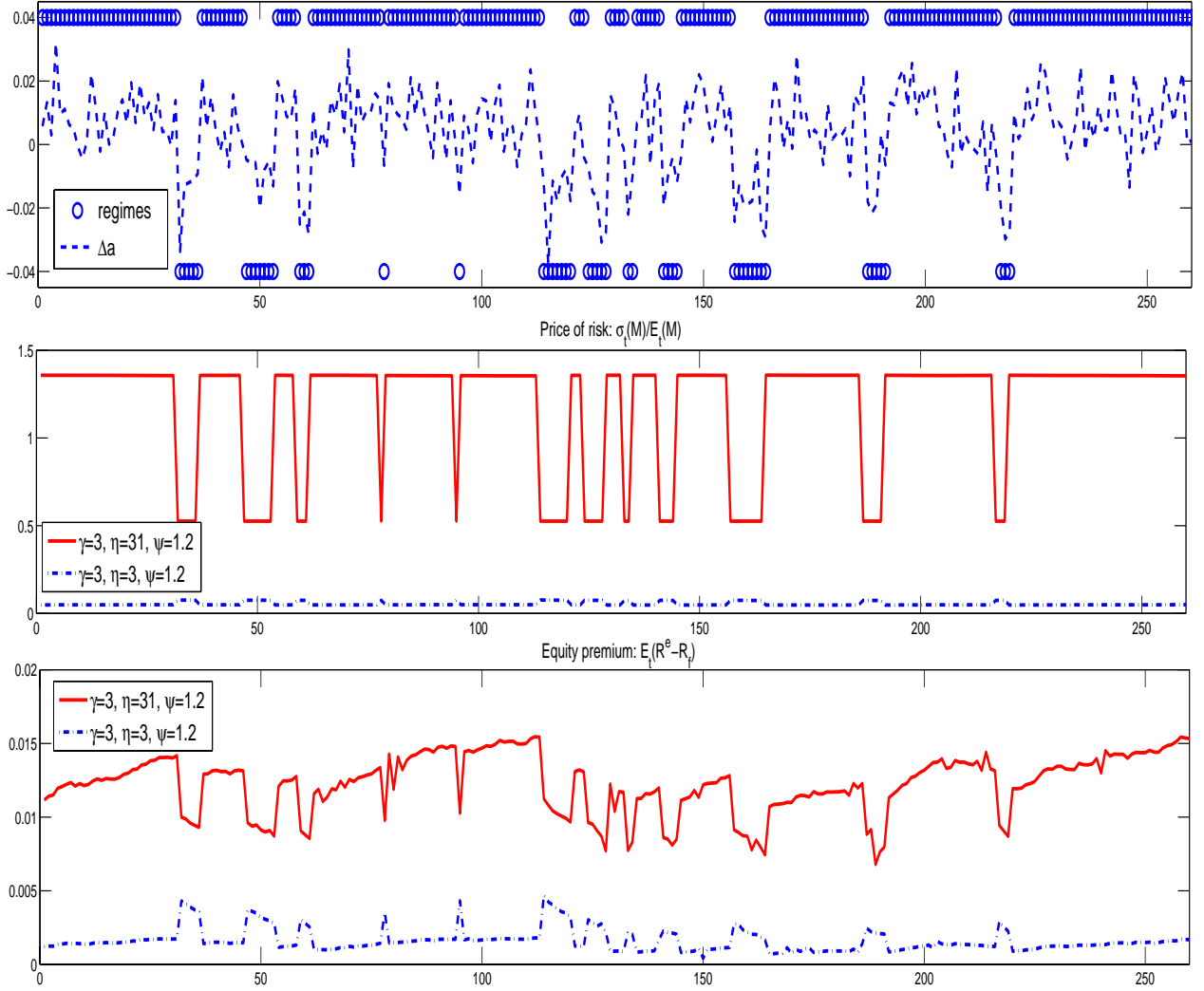
This figure plots simulated dividend growth rates and consumption growth rates for the ambiguous growth model III ($\gamma = 3, \eta = 31, \psi = 1.2$). Parameter values of the model are given in Table 1. The number of periods in simulation is 260 quarters. We first simulate artificial productivity regimes and productivity growth rates as states from the regime-switching model (8). The simulation starts from the economy being initially in the good regime. Dividends and consumption are then computed as functions of the state variables, as given by $D_t = \alpha Y_t - I_t$ and $C_t = Y_t - I_t$.

Figure 2: Simulated regimes, the price of risk and conditional equity premium: no capital adjustment costs



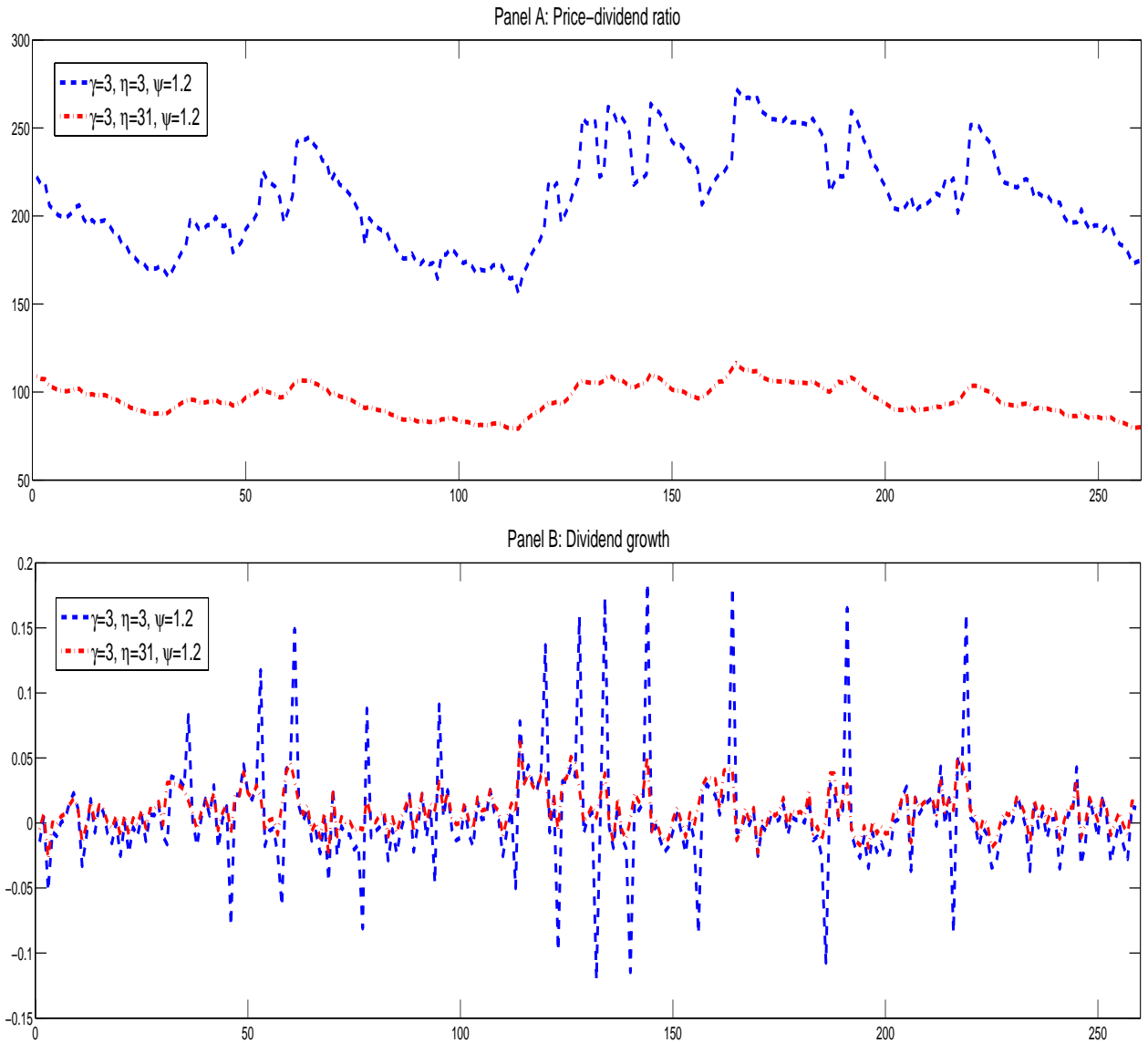
This figure plots simulated productivity growth and regimes (Panel A), the price of risk (Panel B), and the conditional equity premium (Panel C) under the assumption of no capital adjustment costs. Parameter values of the model are given in Table 1. We first simulate artificial productivity regimes and productivity growth rates as state variables from the regime-switching model (8). The simulation starts from the economy being initially in the good regime. In Panel A, productivity regimes are marked with “circle” where the uppermost circles indicate the high productivity regime, and the bottommost circles indicate the low productivity regime. The price of risk and conditional equity premium are computed as functions of the state variables where the pricing kernel is computed according to (14), and the equity returns is computed according to (13). The moments are calculated using Gaussian quadrature method. The results are depicted for two sets of preference parameters: ambiguity aversion (Model III: $\gamma = 3, \eta = 15, \psi = 1.2$), and ambiguity neutrality ($\gamma = 3, \eta = 3, \psi = 1.2$). The number of periods in simulation is 260 quarters.

Figure 3: Simulated regimes, the price of risk and conditional equity premium: with capital adjustment costs



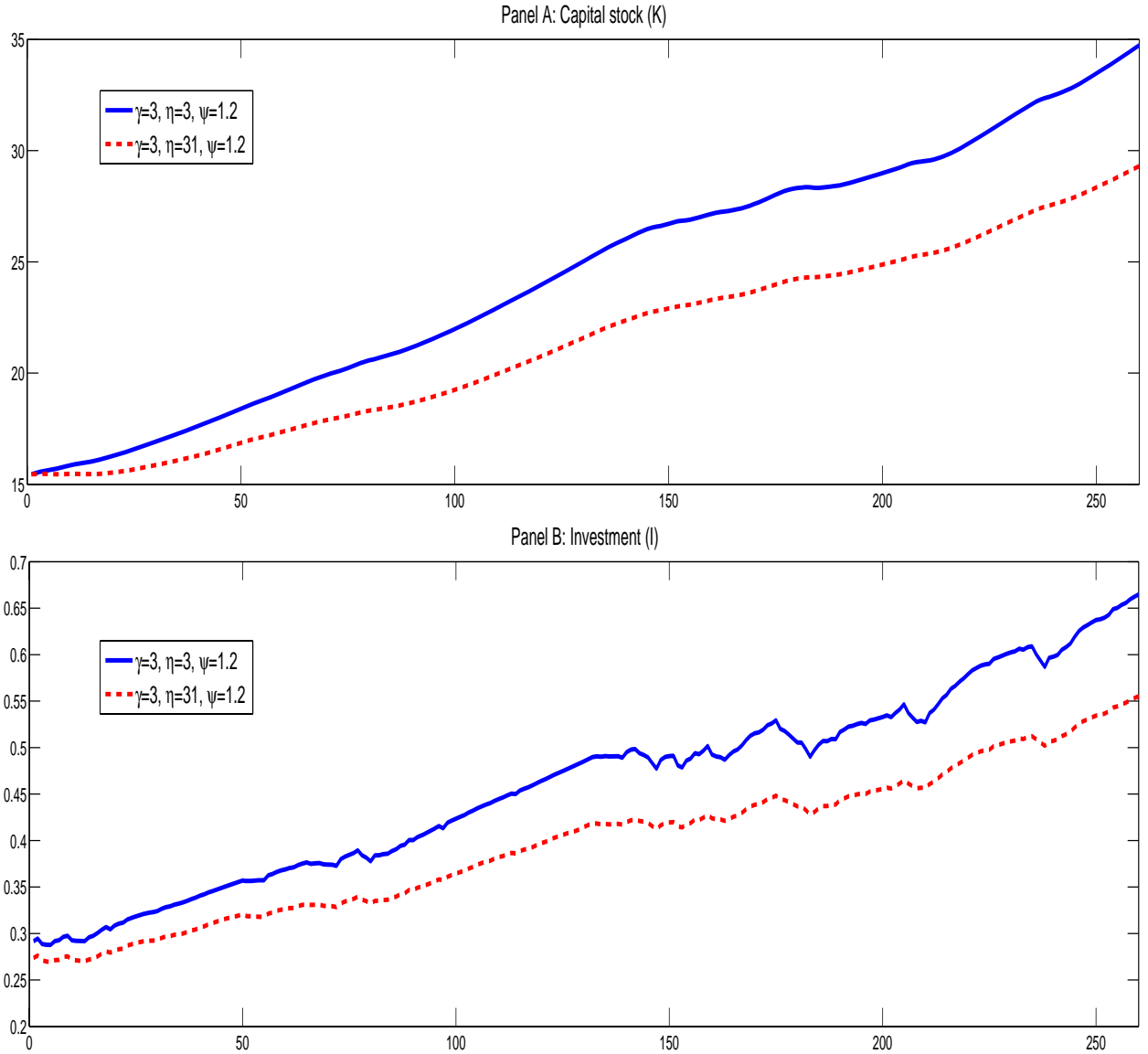
This figure plots simulated productivity growth and regimes (Panel A), the price of risk (Panel B) and the conditional equity premium (Panel C) in the presence of capital adjustment costs for the parameter values given in Table 1. We first simulate artificial productivity regimes and productivity growth rates as states from the regime-switching model (8). The simulation starts from the economy being initially in the good regime. In Panel A, productivity regimes are marked with “circle” where the uppermost circles indicate the high productivity regime, and the bottommost circles indicate the low productivity regime. The price of risk and the conditional equity premium are computed as functions of the state variables where the pricing kernel is computed according to (14), and the equity returns is computed according to (13). The moments are calculated using Gaussian quadrature method. The results are depicted for two sets of preference parameters: ambiguity aversion (Model III: $\gamma = 3, \eta = 31, \psi = 1.2$), and ambiguity neutrality ($\gamma = 3, \eta = 3, \psi = 1.2$). The number of periods in simulation is 260 quarters.

Figure 4: Simulated price-dividend ratios and dividend growth



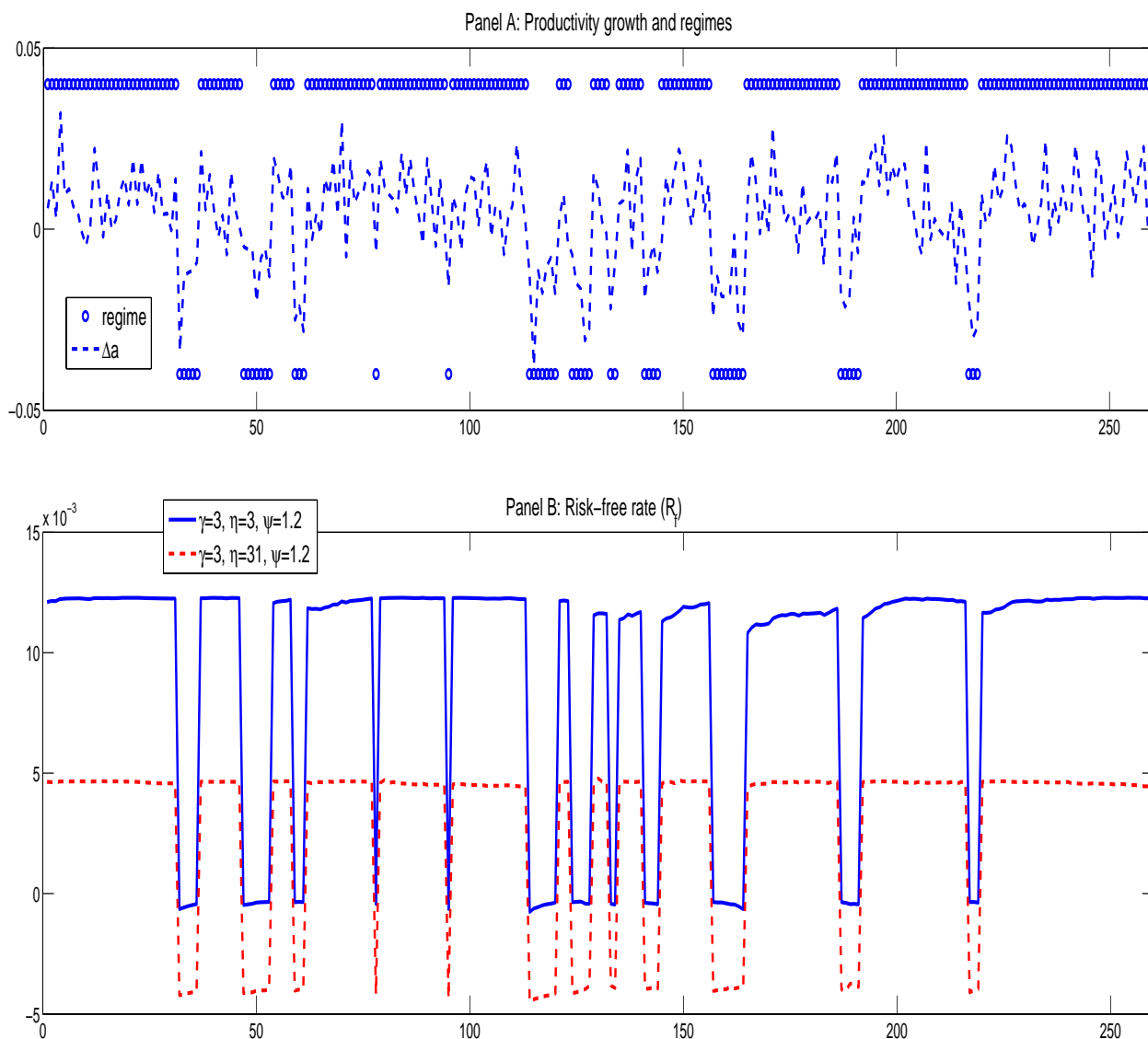
This figure plots simulated price-dividend ratios (Panel A) and dividend growth (Panel B) for the parameter values given in Table 1. We first simulate artificial productivity regimes and productivity growth rates as states from the regime-switching model (8). The simulation starts from the economy being initially in the good regime. The price-dividend ratio and dividend growth are computed and depicted for two sets of preference parameters: ambiguity aversion (Model III: $\gamma = 3, \eta = 31, \psi = 1.2$), and ambiguity neutrality ($\gamma = 3, \eta = 3, \psi = 1.2$). The number of periods in simulation is 260 quarters.

Figure 5: Simulated capital stock and investment



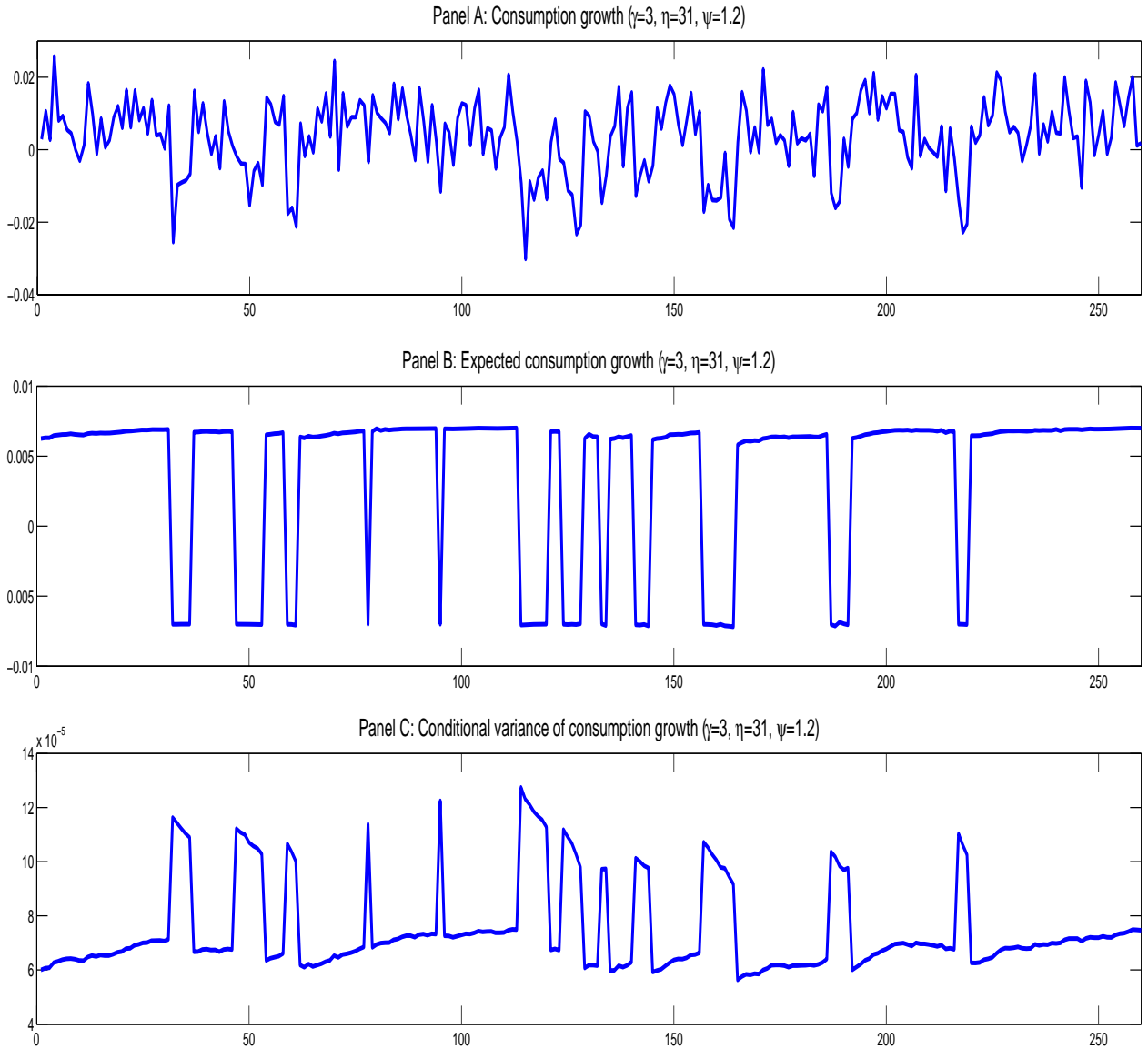
This figure plots the capital stock K (Panel A) and investment I (Panel B) in response to artificial productivity regimes and productivity growth rates simulated from the regime-switching model (8). Parameter values of the model are given in Table 1. The simulation starts from the economy being initially in the good regime. Results are computed and depicted for two sets of preference parameters: ambiguity aversion (Model III: $\gamma = 3, \eta = 31, \psi = 1.2$), and ambiguity neutrality ($\gamma = 3, \eta = 3, \psi = 1.2$). The number of periods in simulation is 260 quarters.

Figure 6: Simulated regimes and risk-free rates



This figure plots simulated productivity growth and regimes (Panel A) and the risk-free rate (Panel B) for the parameter values given in Table 1. We first simulate artificial productivity regimes and productivity growth rates as states from the regime-switching model (8). The simulation starts from the economy being initially in the good regime. In Panel A, productivity regimes are marked with “circle” where the top circles indicate the high productivity regime, and the bottom ones indicate the low productivity regime. The risk-free rate is computed according to (16) where the conditional expectation is evaluated using Gaussian quadrature method. The results are depicted for two sets of preference parameters: ambiguity aversion (Model III: $\gamma = 3, \eta = 31, \psi = 1.2$), and ambiguity neutrality ($\gamma = 3, \eta = 3, \psi = 1.2$). The number of periods in simulation is 260 quarters.

Figure 7: Simulated consumption dynamics



This figure plots simulated consumption growth (Panel A), conditional expected consumption growth (Panel B) and the conditional variance of consumption growth (Panel C) for the parameter values given in Table 1. We first simulate artificial productivity regimes and productivity growth rates as states from the regime-switching model (8). The simulation starts from the economy being initially in the good regime. Consumption is calculated as $C_t = Y_t - I_t$. Conditional moments of consumption growth are calculated using Gaussian quadrature method. The results are depicted for two sets of preference parameters: ambiguity aversion (Model III: $\gamma = 3, \eta = 31, \psi = 1.2$), and ambiguity neutrality ($\gamma = 3, \eta = 3, \psi = 1.2$). The number of periods in simulation is 260 quarters.

Table 1: **Calibration values of model parameters.**

Parameter	Description	Value
γ	Coefficient of risk aversion	3
α	Capital share	0.35
β	Time discount parameter	0.9935
\bar{N}	Exogenous labor input	0.20
ι	Exponent of adjustment costs function	1.0985
δ	Depreciation rate of capital	0.015
μ_1	Mean growth rate (regime 1)	0.0089
μ_2	Mean growth rate (regime 2)	-0.0158
σ	Standard deviation	0.0085
p_{11}	Transition probability (regime 1 to regime 1)	0.94
p_{22}	Transition probability (regime 2 to regime 2)	0.73

Table 2: **Ambiguous growth models: calibration results**

Statistic	U.S. data 1945–2009	Ambiguous growth		
		Model I	Model II	Model III
		$\eta = 21$	$\eta = 25$	$\eta = 31$
		$\psi = 2.0$	$\psi = 1.5$	$\psi = 1.2$
		$\gamma = 3$	$\gamma = 3$	$\gamma = 3$
Panel A: Macroeconomic moments				
$\sigma_{\Delta c}$	2.08	2.08	2.08	2.08
$\sigma_{\Delta c}/\sigma_{\Delta y}$	0.70	1.24	1.24	1.24
$Corr(\Delta c_{t+1}, \Delta c_t)$	0.20	0.26	0.26	0.26
Panel B: Financial moments				
$E[R_f]$	1.17	1.17	1.17	1.17
$\sigma(R_f)$	0.82	0.52	0.60	0.70
$E(R^{ep})$	7.42	7.52	7.19	7.85
$\sigma(R^{ep})$	17.55	16.68	16.29	16.05
Sharpe ratio	0.42	0.45	0.44	0.49
$E[R^e]$	8.23	8.69	8.36	9.00
$\sigma(R^e)$	17.23	16.69	16.30	16.09
$\sigma_{\Delta D}$	7.34	5.28	5.00	4.92
$\sigma(M)/E(M)$	n.a.	0.70	0.93	1.23

This table reports key annualized moments for three calibrated stochastic growth models (Model I–III) with ambiguity aversion where the representative agent has the generalized recursive smooth ambiguity utility and capital adjustment costs are present. The coefficient of relative risk aversion, γ , is 3 across all the three models. The parameter $\psi = 1/\rho$ denotes the elasticity of intertemporal substitution. The statistics reported for Model I–III are under the parameterization given in Table 1. The model is calibrated to match the standard deviation of consumption growth and the unconditional mean of the risk free rate. The equity returns are for an unlevered claim on aggregate dividends. The statistics for the model are calculated based on 10,000 simulations. The statistics calculated based on US historical data (1945–2009) are also presented in the table. The macroeconomic moments reported include: (1) the volatility of consumption growth $\sigma_{\Delta c}$ (in log), (2) the relative volatility of consumption to output $\sigma_{\Delta c}/\sigma_{\Delta y}$ (both in log terms), and (3) the 1st autocorrelation in consumption. The financial moments reported include: (1) the mean risk-free rate $E[R_f]$, (2) the volatility of the risk-free rate $\sigma(R_f)$, (3) the mean equity premium $E(R^{ep})$, defined by $\mathbb{E}(R_{t+1}^e - R_{f,t})$, (4) the volatility of equity premium $\sigma(R^{ep})$, defined by $\sigma(R_{t+1}^e - R_{f,t})$, (5) Sharpe ratio, defined by $E(R^{ep})/\sigma(R^{ep})$, (6) the mean equity return $E[R^e]$, (7) the volatility of equity returns $\sigma(R^e)$, (8) the volatility of aggregate dividends growth $\sigma_{\Delta D}$, and (9) the price of risk $\sigma(M)/E(M)$. The statistics $\sigma_{\Delta c}$, $E[R_f]$, $\sigma(R_f)$, $E(R^{ep})$, $\sigma(R^{ep})$, $E[R^e]$, $\sigma(R^e)$ and $\sigma_{\Delta D}$ are in percentage.

Table 3: **Epstein-Zin utility: calibration results**

Statistic	U.S. data 1945–2009	Epstein-Zin			
		$\gamma = 3$ $\psi = 2.0$	$\gamma = 3$ $\psi = 1.5$	$\gamma = 3$ $\psi = 1.2$	$\gamma = 3$ $\psi = 1/3$
$E[R_f]$	1.17	3.28	3.60	3.90	7.76
$\sigma(R_f)$	0.82	0.63	0.80	0.97	3.01
$E(R^{ep})$	7.42	2.06	2.20	2.10	1.62
$\sigma(R^{ep})$	17.55	18.55	17.78	17.61	18.23
Sharpe ratio	0.42	0.11	0.12	0.12	0.09
$\sigma(M)/E(M)$	n.a.	0.06	0.05	0.05	0.03

This table reports key annualized moments for four simulations of the stochastic growth models where the representative agent has Epstein-Zin recursive utility and capital adjustment costs are present. The coefficient of relative risk aversion, γ , is 3 across all the three models. The parameter $\psi = 1/\rho$ denotes the elasticity of intertemporal substitution. The statistics reported for the model are under the parameterization given in Table 1. The equity returns are for an unlevered claim on aggregate dividends. Each set of unconditional moments is calculated based on 10,000 simulations. The first three models ($\psi = 2, 1.5$ and 1.2) correspond to the case where the EIS is greater than 1. The fourth model corresponds to the expected utility case where the EIS is the reciprocal of the risk aversion parameter. The financial moments reported include: (1) the mean risk-free rate $E[R_f]$, (2) the volatility of the risk-free rate $\sigma(R_f)$, (3) the mean equity premium $E(R^{ep})$, (4) the volatility of equity premium $\sigma(R^{ep})$, (5) Sharpe ratio, and (6) the price of risk $\sigma(M)/E(M)$. The statistics $E[R_f]$, $\sigma(R_f)$, $E(R^{ep})$ and $\sigma(R^{ep})$ are in percentage.

Table 4: **Equity premium decomposition**

Statistic	Ambiguous growth		
	Model I	Model II	Model III
	$\eta = 21$	$\eta = 25$	$\eta = 31$
	$\psi = 2.0$	$\psi = 1.5$	$\psi = 1.2$
	$\gamma = 3$	$\gamma = 3$	$\gamma = 3$
$E(R^{ep,EZ}) - E(R^{ep,EU})$	0.44	0.58	0.48
$E(R^{ep,AA}) - E(R^{ep,EZ})$	5.46	4.99	5.75

This table reports the equity premium decomposition results for the three calibrated models, Model I–III, in Table 2. The formula for the decomposition is

$$\mathbb{E}(R^{ep,AA}) = \mathbb{E}(R^{ep,EU}) + \underbrace{\left[\mathbb{E}(R^{ep,EZ}) - \mathbb{E}(R^{ep,EU}) \right]}_{\text{Epstein-Zin}} + \underbrace{\left[\mathbb{E}(R^{ep,AA}) - \mathbb{E}(R^{ep,EZ}) \right]}_{\text{ambiguity aversion}}$$

where $\mathbb{E}(R^{ep,EU})$ is the mean equity premium for expected utility, whose value is given in the rightmost column of Table 3; $\mathbb{E}(R^{ep,EZ}) - \mathbb{E}(R^{ep,EU})$ is attributed to Epstein-Zin utility with ψ being greater than 1, whose values are obtained from Table 3 by taking the difference between the mean equity premium for $\psi > 1$ and that for $\psi = 1/3$; $\mathbb{E}(R^{ep,AA}) - \mathbb{E}(R^{ep,EZ})$ is the contribution to the equity premium solely due to ambiguity aversion, whose values are obtained by subtracting the mean equity premium for Epstein-Zin utility from that for the generalized recursive smooth ambiguity utility.

Table 5: Long-horizon predictability: calibration results

Horizon(s)	Data (1945-2009)		Model I $\eta = 21, \psi = 2$		Model II $\eta = 25, \psi = 1.5$		Model III $\eta = 31, \psi = 1.2$	
	Slope	R^2	Slope	R^2	Slope	R^2	Slope	R^2
Panel A Dependent variable: $r_{t,t+s}^e$								
1 year	-0.0082	0.0114	-0.0534	0.0340	-0.0636	0.0401	-0.0709	0.0463
2 years	-0.0276	0.0898	-0.0672	0.0469	-0.0792	0.0533	-0.0887	0.0592
3 years	-0.0370	0.1093	-0.0789	0.0582	-0.0922	0.0643	-0.1024	0.0698
5 years	-0.0664	0.1502	-0.0966	0.0764	-0.1122	0.0823	-0.1221	0.0867
Panel B Dependent variable: $r_{t,t+s}^e - r_{t,t+s}^f$								
1 year	-0.0168	0.0536	-0.0609	0.0368	-0.0706	0.0444	-0.0758	0.0521
2 years	-0.0659	0.0749	-0.0817	0.0532	-0.0948	0.0629	-0.1031	0.0724
3 years	-0.0818	0.1399	-0.0997	0.0667	-0.1159	0.0773	-0.1263	0.0870
5 years	-0.1328	0.3550	-0.1281	0.0865	-0.1494	0.0972	-0.1619	0.1058

This table reports predictive regression results for the three calibrated models, Model I-III, presented in Table 2. We report the regression slopes and the R^2 's estimated from regressing annualized equity returns and excess returns (both in log terms) onto the end-of-year log price-dividend ratio. The results are obtained by taking the average of OLS estimates for 10,000 simulated sample paths. The horizon of returns includes 1 year, 2, 3 and 5 years. The statistics estimated from US historical data (1945-2009) are also presented in the table.

Table 6: **Consumption dynamics**

Ambiguous growth				
Statistic		Model I	Model II	Model III
	Collard, et al	$\eta = 25$	$\eta = 30$	$\eta = 36$
	Calibration	$\psi = 2.0$	$\psi = 1.5$	$\psi = 1.2$
Panel A: Expected consumption growth (quarterly)				
AC(1)	0.74	0.67	0.67	0.67
AC(2)	0.55	0.45	0.44	0.44
AC(3)	0.41	0.30	0.30	0.30
AC(4)	0.30	0.20	0.20	0.20
AC(5)	0.22	0.13	0.13	0.13
Panel B: Conditional variance of consumption growth (quarterly)				
AC(1)		0.74	0.71	0.68
AC(2)		0.57	0.52	0.47
AC(3)		0.46	0.39	0.33
AC(4)		0.38	0.30	0.24
AC(5)		0.33	0.24	0.18

This table reports results for consumption dynamics implied by the three calibrated models, Model I–III, in Table 2. Panel A presents the autocorrelation coefficients up to the 5th lag for simulated expected consumption growth in quarterly frequency. For the purpose of comparison, we also compute the autocorrelation coefficients implied by the calibration of the “low-persistence” model in Collard et al. (2011). Panel B contains the autocorrelation coefficients up to the 5th lag for the simulated conditional variance of consumption growth. The statistics generated from Model I–III are based on averaging over 10,000 simulations. The moments are calculated using Gaussian quadrature method.