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## Linear predictability vs. bull and bear market models in strategic asset allocation decisions: Evidence from UK data

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# Working Paper Series

## Linear Predictability Vs. Bull and Bear Market Models in Strategic Asset Allocation Decisions: Evidence from UK Data

Massimo Guidolin and Stuart Hyde

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## Manchester Business School

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## JEL Classification

G11; C53

## Abstract

Most papers in the portfolio choice literature have examined linear predictability frameworks based on the idea that simple but flexible Vector Autoregressive (VAR) models can be expanded to produce portfolio allocations that hedge against the bull and bear dynamics typical of financial markets through careful selection of predictor variables that capture business cycles and market sentiment. Yet, a distinct literature exists that shows that nonlinear econometric frameworks, such as Markov switching, are also natural tools to compute optimal portfolios arising from the existence of good and bad market states. This paper examines whether and how simple VARs can produce portfolio rules similar to those obtained under a simple Markov switching, by studying the effects of expanding both the order of the VAR and the number/selection of predictor variables included. In a typical stock-bond strategic asset allocation problem for U.K. data, we compute the out-of-sample certainty equivalent returns for a wide range of VARs and compare these measures of performance with those of nonlinear models. We conclude that most VARs cannot produce portfolio rules, hedging demands, or (net of transaction costs) out-of-sample performances that approximate those obtained from equally simple nonlinear frameworks.

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# Linear Predictability Vs. Bull and Bear Market Models in Strategic Asset Allocation Decisions: Evidence from UK Data.

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August 2, 2012

## Abstract

Most papers in the portfolio choice literature have examined linear predictability frameworks based on the idea that simple but flexible Vector Autoregressive (VAR) models can be expanded to produce portfolio allocations that hedge against the bull and bear dynamics typical of financial markets through careful selection of predictor variables that capture business cycles and market sentiment. Yet, a distinct literature exists that shows that nonlinear econometric frameworks, such as Markov switching, are also natural tools to compute optimal portfolios arising from the existence of good and bad market states. This paper examines whether and how simple VARs can produce portfolio rules similar to those obtained under a simple Markov switching, by studying the effects of expanding both the order of the VAR and the number/selection of predictor variables included. In a typical stock-bond strategic asset allocation problem for U.K. data, we compute the out-of-sample certainty equivalent returns for a wide range of VARs and compare these measures of performance with those of nonlinear models. We conclude that most VARs cannot produce portfolio rules, hedging demands, or (net of transaction costs) out-of-sample performances that approximate those obtained from equally simple nonlinear frameworks.

Key words: Predictability, Strategic Asset Allocation, Markov Switching, Vector Autoregressive Models, Out-of-Sample Performance.

JEL codes: G11, C53.

## 1. Introduction

Understanding the behaviour of asset returns and the portfolio allocation decisions associated with investments in those assets is of great importance to financial market participants. Intuitively, the optimal decision of an investor rests on the characteristics of the assets under consideration as well as the investment horizon. On the one hand, in the absence of return predictability, the seminal papers by Merton (1969) and Samuleson (1969) have shown that the optimal portfolio mix is independent of the selected investment horizon. That is, long-horizon investors should allocate their wealth and choose exactly the same asset allocation as a short-term investors do. On the other hand, the predictability of asset returns

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is now a well established, though widely debated, issue in empirical finance (see, e.g., Welch and Goyal, 2008). This evidence of predictability has given rise to a literature which has sought to examine how long-run portfolio allocation decisions are affected. The seminal work by Brennan et al. (1997) and Barberis (2000) demonstrates that an investor's investment horizon is no longer irrelevant once asset returns are predictable. In fact, even weak predictability can yield economically significant variations in asset allocation decisions. Moreover, much of the portfolio choice literature focusses on linear predictability patterns, irrespective of a growing strand of research suggesting that predictability patterns are nonlinear, for instance in the form of persistent and predictable bull and bear market states.<sup>1</sup>

Because of their simplicity, linear frameworks provide a natural benchmark in many empirical applications. Their adoption in asset allocation exercises is predicated on two related but simplistic ideas. First, though the portfolio choice literature typically focuses on models of limited scale (e.g., VAR(1) models with 2-3 predictors at most, see Barberis, 2000; Lynch, 2001), the results imply that larger scale models must exist that can produce improved realized portfolio performance. That is, models must exist that effectively exploit their flexibility in terms of number of lags and number and nature of predictors to capture any predictability patterns relevant to portfolio performance, rendering any nonlinear effects of secondary importance. Second, these small scale VAR(1) models would be indicative of the importance of linear predictability for dynamic portfolio selection because it is well known from standard graduate econometrics that any VAR( $p$ ) model can be re-written as an expanded VAR(1) provided the right state variables are used to augment the multivariate scope of the model. In contrast to the widespread reliance on these assumptions in the empirical portfolio choice literature, in this paper we seek to examine whether any such small and medium size VAR models as commonly employed in the empirical finance literature can exploit linear predictability to yield *realized, out-of-sample* portfolio outcomes equivalent to those of an investor that chooses to exploit nonlinear predictability patterns. A positive finding implies that it would be irrelevant whether or not portfolio managers would or could exploit any evidence of predictable bull and bear regimes in financial prices.

Our investigation is performed with reference to U.K. financial data. Similarly to U.S. data, there is now massive evidence that both U.K. equity index returns (often including futures on indices, see e.g., Abhyankar, 1999; Abhyankar, Copeland, and Wong, 1995; Ap Gwilym, Brooks, Clare, and Thomas, 1999; Cochran, 2004; Guidolin and Timmermann, 2003; McMillan, 2004b; Shively, 2003) and U.K. bond prices, returns, and implied interest rates (see e.g. Brüggemann and Riedel, 2011; Lekkos and Milas, 2004a, 2004b; McMillan, 2004a; McMillan and Speight, 2002) display regime switching behavior. Therefore asking whether standard, VAR-type linear predictability models may be sufficient to capture all relevant predictability dynamics with reference to U.K. financial data does not merely represent a robustness check as to whether the effects are any different from those documented in the U.S. market; on the contrary, this is a crucial question for the entire strand of the empirical finance literature that has investigated nonlinear predictability in the U.K. financial markets. Alternatively, our key research objective can be

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<sup>1</sup>A few papers investigate the implications for optimal portfolio choice of nonlinear dynamics in asset returns, such as Ang and Bekaert (2002, 2004), Detemple et al. (2003), Guidolin and Timmermann (2005, 2008).

viewed as striving to make a point concerning the economic value of the widespread use of regime switching models to model and forecast financial returns, with particular emphasis on U.K. data: we know from earlier papers (see e.g., Ang and Bekaert, 2004; Guidolin and Timmermann, 2005) that regimes strongly affect optimal portfolio choices, especially in the perspective of long horizon investors. Some papers have recently investigated whether regime models can actually improve the forecasting performance in applied econometric work (see e.g., Guidolin, Hyde, McMillan, and Ono, 2010, Sarantis, 2001, and references therein). However, for such evidence to be compelling, one also needs to show that traditional, simple linear predictability frameworks cannot produce in-sample, recursive dynamic asset allocation decisions that are similar to those deriving from frameworks that exploit bull and bear dynamics.

As a matter of fact, establishing the extent to which the implications of linear and nonlinear predictability differ with respect to asset allocation is important for investors, asset managers and financial advisors alike. While, ex-ante, one may expect the portfolio choices implied from an extensive set of small and medium scale VAR( $p$ ) models to be different from those implied by a relatively basic nonlinear benchmark model—in our paper a three-state Markov switching model—ex-post there is no guarantee that nonlinear models may produce a higher economic value in actual decisions than simpler linear models do. To test this hypothesis, we perform a strategic asset allocation (SAA) exercise familiar in the literature (see, e.g., Barberis 2000; Brennan et al., 1997): at each point in time, a risk-averse (constant relative risk aversion) investor allocates her wealth across three macro-asset classes, i.e., stocks, long-term default risk-free government bonds, and 1-month Treasury bills. Using monthly U.K. data for the long period 1957-2009, which includes the recent financial crisis, we examine long-horizon portfolio choices (up to a 10-year horizon) of an investor who recursively solves a portfolio problem in which utility derives from real consumption (i.e., cash flows obtained from dividend and coupon payments and from selling securities in the portfolio) and rebalancing is admitted at the same frequency as the data. That is, the investor decides upon an asset allocation at time  $t$  knowing that at times  $t + 1, t + 2, \dots$ , up to  $t + 119$  she will be allowed to change the structure of her portfolio weights to reflect the fact that, at least in principle, new information will become available at all these future points. Furthermore, our investor selects optimal portfolio weights taking into account the presence of both fixed and variable transaction costs. This means that—because a given vector of optimal weights at time  $t$  may implicitly imply a need to trade in all assets between time  $t$  and  $t + 1$ —our investor will also take into account the trading implications of her portfolio decisions and especially the impact of any transaction costs incurred on expected utility.

The pillars of our research design are easily described. First, a genuine focus on a long-horizon is key to discussing the economic value of different models or of return predictability. Second, allowing for continuous/frequent rebalancing of portfolio weights and investors who care for real consumption streams and real portfolio returns is consistent with the way predictability is exploited in practice, i.e., with full awareness of the fact that its existence not only affects today's choice but will keep affecting choice in all subsequent periods. Third, as previously stressed by Balduzzi and Lynch (1999) and Lynch and Balduzzi (2000), all SAA problems under predictability and active portfolio management ought to carefully consider

whether the forecastable variation in investment opportunity sets offers enough welfare gains to exceed the often large trading costs.

Our key empirical results can be summarized as follows. First, our relatively large set of small- and medium scale (up to 6 predictors are included in all possible combinations) VAR( $p$ ) models (with  $p = 1, 2, 4,$  and  $12$ ) fails to imply portfolio choices that approximate those from a rather simple (one may say, naive) non-linear benchmark, represented by a plain vanilla three-state Markov switching model. This is of course only an ex-ante perspective: “different” does not imply “worse” in the view of an applied portfolio manager and what could be wrong is not the family of VARs, but the proposed nonlinear benchmark. More importantly, the large family of VARs systematically fail to perform better than nonlinear models in recursive (pseudo) out-of-sample tests, in the sense that VARs generally produce lower realized real certainty equivalent returns (i.e. risk-adjusted performances that take into account the curvature of the utility function under which the portfolio choice program has been solved) than models that identify and forecast bull and bear dynamics. This means that VARs cannot provide a good approximation to the time series properties of U.K. financial returns neither ex-ante nor ex-post. Although these findings may seem obvious ex-ante to some of our Readers, what is not obvious is that in recursive out-of-sample tests such nonlinearities seem to be ex-post sufficiently real and strong to condemn most (sometimes, all) VARs to disappointing long-run portfolio ex-post performances.

While the vast majority of the portfolio choice literature has examined investment horizon effects and the economic value of predictability models using U.S. data, our paper contributes to a small but not negligible strand of literature that has taken a distinctive interest in U.K. financial markets. Besides the references cited above, one closely related paper in this literature is Li (2011) who has used one VAR framework to investigate the horizon effects in optimal portfolio weights deriving from predictability and model uncertainty using U.K. data.<sup>2</sup> Differently from Li’s paper, our goal is to examine a wide range of VAR models (capturing the main variants that have appeared in the literature) in relation to a single, relatively simple, nonlinear model. Here, the large family of VARs consists of all those one can form using combinations of 6 predictors and alternative lag orders,  $p = 1, 2, 4$  and  $12$ . The predictor variables are those typical in the U.K. literature for their well-established predictive power: the dividend yield, the term spread, the gilt-equity yield, the RPI inflation rate, the rate of industrial production growth and the change in oil prices. Further, the analysis of Li (2011) is limited to a single risky asset (equity) while we consider the potential for investors to allocate funds to both equity and bonds.<sup>3</sup> In addition, we are not aware of previous papers that have jointly solved consumption and portfolio choice problems under MS dynamics for U.K. data. Further, while Anderson, Fletcher and Marshall (2011), Basu, Oomen and Stemme (2010) and Fletcher (2011) all provide evidence on the economic value of predictability/conditioning information for dynamic asset allocation strategies in a mean-variance framework where U.K. stocks form the (or

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<sup>2</sup>Boudry and Gray (2003) provide further international evidence, using Australian data, to examine the impact of predictability from the dividend yield, the term spread and the relative bill yield on portfolio choices at different horizons.

<sup>3</sup>Gil-Bazo (2006) documents that extending the asset menu allows for the characteristics of each risky asset to influence the portfolio mix over the investment horizon.

part of the) asset menu—in- and out-of-sample, with and without transactions costs and with the ex-post imposition of bull and bear markets—their analysis is limited to a short one-period investment horizon and to equities. One last, related paper is Guidolin and Hyde (2010) who, using U.S. data, have also compared the realized, ex-post performance of a rich family of linear models with a simple, nonlinear one. However, their analysis ignores transaction costs and therefore surely over-estimates the true economic value of the asset allocation model investigated. This is problematic, because it is a priori ambiguous whether transaction costs may differentially affect linear vs. nonlinear models. Apart from the obvious difference in the data examined, our effort is easily distinguishable from theirs because our research design insists on the crucial importance of transaction costs on portfolio decisions in the presence of predictability, and because of our focus on truly long-horizon, 10-year portfolios.

The rest of the paper is structured as follows. Section 2 describes the research design of our paper. Section 3 describes the data, the three-state Markov switching benchmark, and some features of linear predictability as captured by VARs. Section 4 computes and presents optimal portfolio weights and hedging demands under the two classes of models. Section 5 computes realized, recursive out-of-sample portfolio performances for our key experiment. Section 6 performs robustness checks and presents results from additional tests. Section 7 concludes.

## 2. Methodology

### 2.1. Econometric Models

We perform recursive estimation, portfolio weight calculation and performance evaluation for three groups of models. First and foremost, we entertain a large class of VAR( $p$ ) models. These VARs consist of (sets of) linear relationships linking  $\mathbf{r}_{t+1}$ , a  $N \times 1$  vector of risky real asset returns at time  $t + 1$ , and  $\mathbf{y}_{t+1}$ , a  $M \times 1$  vector of predictor variables at time  $t + 1$ , to lags of both  $\mathbf{r}_{t+1}$  and  $\mathbf{y}_{t+1}$ . For instance, in the case of a simple VAR(1), we have

$$\begin{bmatrix} \mathbf{r}_{t+1} \\ \mathbf{y}_{t+1} \end{bmatrix} = \boldsymbol{\mu} + \mathbf{A} \begin{bmatrix} \mathbf{r}_t \\ \mathbf{y}_t \end{bmatrix} + \boldsymbol{\varepsilon}_{t+1} \quad \boldsymbol{\varepsilon}_{t+1} \sim IID N(\mathbf{0}, \boldsymbol{\Omega}), \quad (1)$$

where  $\boldsymbol{\mu}$  is a  $(N + M) \times 1$  vector of intercepts,  $\mathbf{A}$  is a  $(N + M) \times (N + M)$  coefficient matrix, and  $\boldsymbol{\varepsilon}_{t+1}$  is a  $(N + M) \times 1$  vector of IID, Gaussian residuals. The representation of a VAR(1) in equation (1) is without loss of generality as any  $p$ th order VAR can be re-written as a VAR(1) (see e.g., Hamilton, 1994, p. 259). In this paper we consider multiple values of  $p$ ,  $p = 1, 2, 4$ , and  $12$ .<sup>4</sup> Note that—assuming a specification that always includes the lagged values of real asset returns as in (1)—for a given value of  $p$  there are  $2^M$  different VARs one can obtain according to which of the  $M$  predictors are included in  $[\mathbf{r}'_{t+1} \mathbf{y}'_{t+1}]'$ . This means that one VAR( $p$ ) will effectively describe the dynamics of  $\mathbf{r}_{t+1}$  and the first of the predictors, a VAR( $p$ ) the dynamics of  $\mathbf{r}_{t+1}$  and the second predictor, a VAR( $p$ ) the dynamics of  $\mathbf{r}_{t+1}$

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<sup>4</sup>Because our data start only in 1953, the case of  $p = 12$  could be estimated only when one predictor at the time was included in isolation in the VAR(12) model, that therefore becomes a four-variate system.



and the first two predictors in  $\mathbf{y}_{t+1}$ , another VAR( $p$ ) the dynamics of  $\mathbf{r}_{t+1}$  and the first and the third among the  $M$  predictors, etc. Of course, (1) implies that knowledge of current and past values of asset returns, of the predictors, or of both implies some power to forecast in a linear fashion future, subsequent realizations of both real asset returns and the predictors themselves.<sup>5</sup>

The second class of models consists of nonlinear models in the  $k$ -state Markov switching class with constant transition probabilities (collected in a  $k \times k$  matrix  $\mathbf{P}$ ),

$$\mathbf{r}_{t+1} = \boldsymbol{\mu}_{S_{t+1}} + \boldsymbol{\varepsilon}_{t+1} \quad \boldsymbol{\varepsilon}_{t+1} \sim IID N(\mathbf{0}, \boldsymbol{\Omega}_{S_{t+1}}), \quad (2)$$

where the latent Markov state is  $S_{t+1} = 1, \dots, k$  and  $\boldsymbol{\mu}_{S_{t+1}}$  is a  $N \times 1$  vector of state dependent intercepts in regime  $S_{t+1}$ . We also allow for the  $N \times N$  covariance matrix of residuals  $\boldsymbol{\Omega}_{S_{t+1}}$  to be state-dependent, implying that the variance of the asset returns is also state-dependent, i.e.,  $Var[\mathbf{r}_{t+1}|S_{t+1}] = \boldsymbol{\Omega}_{S_{t+1}}$ . In particular, we speak of heteroskedastic MSIH models when both means, variances, and covariances, are a function of the Markov state  $S_{t+1}$ ; of homoskedastic MSI models with only mean asset returns are a function of the Markov state, but the covariance matrix is time homogeneous.

We have also considered a further benchmark class widely adopted in the empirical finance and forecasting literature, a simple Gaussian IID model,

$$\mathbf{r}_{t+1} = \boldsymbol{\mu} + \boldsymbol{\varepsilon}_{t+1} \quad \boldsymbol{\varepsilon}_{t+1} \sim IID N(\mathbf{0}, \boldsymbol{\Omega}), \quad (3)$$

which is obviously a single-state restriction of (2). Of course, (3) implies that real asset returns are not predictable, while (2) implies that real asset returns may follow nonlinear predictability patterns, driven by the fact that the latent state  $S_{t+1}$  displays a predictable Markov structure when the transition matrix  $\mathbf{P}$  differs from  $(1/k)\boldsymbol{\iota}_{N+M}\boldsymbol{\iota}'_{N+M}$ , where  $\boldsymbol{\iota}_{N+M} \equiv [1 \ 1 \ \dots \ 1]'$ , i.e., when the regimes themselves are persistent so that knowledge of  $S_{t+1}$  implies some ability to forecast  $S_{t+1}$ :  $\Pr(S_{t+1}|S_t) \neq 1/k$ .

## 2.2. The Portfolio Choice Problem

Consider the portfolio and consumption decisions of a finite horizon investor with time-separable, constant relative risk aversion (CRRA) preferences who maximizes the expected utility of her lifetime consumption,

$$\max_{\{C_\tau, \omega_{i,\tau}\}_{\tau=1}^{H-1}} \sum_{i=1}^N \sum_{t=1}^H \beta^t E_t \left[ \frac{C_t^{1-\gamma}}{1-\gamma} | \mathbf{Z}_t \right] \quad \beta \in (0, 1), \quad \gamma > 1, \quad (4)$$

where the discount factor  $\beta = 0.9975$  is the subjective rate of time preference (corresponding to an annualized real discount rate of less than 3%, approximately the mean of the real 1-month T-bill rate in our sample), the coefficient  $\gamma$  measures relative risk aversion,  $C_t$  is the investor's consumption at time  $t$

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<sup>5</sup>When the matrix  $\mathbf{A}$  (or more generally, the matrices  $\mathbf{A}_1, \mathbf{A}_2, \dots, \mathbf{A}_p$ ) is upper triangular, then we have that while current and past values of the predictors forecast future asset returns, current and past values of the asset returns cannot forecast the predictors, as in Barberis (2000). For the case of  $p = 1$  we have tested and rejected these restrictions, and as a result all the empirical results that follow concern the case of unrestricted VAR matrix  $\mathbf{A}$ .

and  $\mathbf{Z}_t$  is the relevant vector of state variables at time  $t$ .<sup>6</sup> The investor consumes a proportion of wealth,  $\kappa_t \equiv C_t/W_t$ , allocating the remainder to an investment portfolio consisting of the  $N$  real risky assets. The return on the portfolio,  $r_{p,t+1}$  is then given by  $\sum_{i=1}^N \omega_{i,t} r_{i,t+1}$  where the weights,  $\omega_{i,t}$ , allocated to each risky asset must sum to unity, i.e.,  $\sum_{i=1}^N \omega_{i,t} = 1$ . The intertemporal budget constraint faced by the investor is given by

$$W_{t+1} = (W_t - C_t)(1 + r_{p,t+1}) = W_t(1 - \kappa_t)R_{p,t+1}, \quad (5)$$

where  $R_{p,t+1}$  is the gross portfolio return,  $R_{p,t+1} \equiv 1 + r_{p,t+1}$ . It is easy to show (see Ingersoll, 1987) that the Bellman equation faced by the investor for a CRRA utility function that can be derived from (4) and the budget constraint (5) is

$$\frac{a(\mathbf{Z}_t, t) W_t^{1-\gamma}}{1-\gamma} = \max_{\kappa_t, \boldsymbol{\omega}_t} \left\{ \frac{\kappa_t^{1-\gamma} W_t^{1-\gamma}}{1-\gamma} + \frac{\beta(1-\kappa_t)^{1-\gamma} W_t^{1-\gamma}}{1-\gamma} E \left[ a(\mathbf{Z}_{t+1}, t+1) R_{p,t+1}^{1-\gamma} | \mathbf{Z}_t \right] \right\}, \quad (6)$$

where  $a(\mathbf{Z}_t, t)$  is a function that can be computed numerically. Given that this optimization problem is homogeneous of degree  $(1-\gamma)$  in wealth, the solution is invariant in wealth. Hence the Bellman equation can be simplified to:

$$\frac{a(\mathbf{Z}_t, t)}{1-\gamma} = \max_{\kappa_t, \boldsymbol{\omega}_t} \left\{ \frac{\kappa_t^{1-\gamma}}{1-\gamma} + \frac{\beta(1-\kappa_t)^{1-\gamma}}{1-\gamma} E \left[ a(\mathbf{Z}_{t+1}, t+1) R_{p,t+1}^{1-\gamma} | \mathbf{Z}_t \right] \right\}. \quad (7)$$

Equation (7) can be solved by backward iteration, starting with  $t = H - 1$  and setting  $a(\mathbf{Z}_H, H) = 1$  and then computing  $a(\mathbf{Z}_H, t)$  by solving the optimization problem in (7) using  $a(\mathbf{Z}_{t+1}, t+1)$  from the previous iteration. The backward, recursive structure of the solution reflects the fact that the investor incorporates in the optimal weights computed at time  $t$  the fact that they will be revised in the future at times  $t+1, t+2, \dots, t+H-1$ , as new information becomes available through the vector of state variables  $\mathbf{Z}_t$ . A variety of solution methods have been proposed in the literature. We employ Monte Carlo methods for integral (expected utility) approximation.<sup>7</sup>

### 2.3. Transaction Costs

We assume that the investor faces transaction costs that are proportional to wealth, so that her law of motion for wealth is

$$W_{t+1} = (W_t - C_t)(1 - f_t)(1 + r_{p,t+1}), \quad (8)$$

where  $f_t$  is the transaction cost per dollar of wealth. The law of motion for wealth in (8) implicitly assumes that consumption at time  $t$  and any transaction costs to be paid at time  $t$  are obtained by liquidating costlessly the risky and the riskless assets in the proportions  $\{\omega_{i,t}\}_{i=1}^N$ . This assumption is sensible for liquid assets, especially when they pay coupons or dividends that can be readily used to

<sup>6</sup>In the case of a VAR( $p$ ),  $\mathbf{Z}_t \equiv [\mathbf{r}'_t \mathbf{y}'_t \mathbf{r}'_{t-1} \mathbf{y}'_{t-1} \dots \mathbf{r}'_{t-p+1} \mathbf{y}'_{t-p+1}]'$  so that the state vectors consists of a combination of lagged values of asset returns and predictor variables. In a Markov switching framework  $\mathbf{Z}_t$  consists instead of the vector of state probabilities inferred at time  $t$ .

<sup>7</sup>An appendix not for publication provides additional details on the numerical methods used in the solution of the portfolio problem. See also Guidolin and Timmermann (2007, 2008b) and Guidolin and Hyde (2010).

pay for transaction costs. In particular, we assume that there is both a fixed and a variable component to transaction costs. Therefore  $f_t$  is modelled as a function of the difference between the end- and the beginning-of-period wealth allocation to the assets,  $\{\omega_{i,t} - \omega_{i,t-1}\}_{i=1}^N$

$$f_t = \tau_f I_{\{\exists i \omega_{i,t} \neq \omega_{i,t-1}\}} + \tau_v \sum_{i=1}^N |\omega_{i,t} - \omega_{i,t-1}|, \quad (9)$$

where  $I_{\{\exists i \omega_{i,t} \neq \omega_{i,t-1}\}} = 1$  when the condition  $\omega_{i,t} \neq \omega_{i,t-1}$  is satisfied for at least one  $i = 1, \dots, N$  (i.e., there is trading in any asset  $i$  between  $t - 1$  and  $t$ ), and 0 otherwise. The first term is a fixed fraction of the total value of the portfolio that represents the fixed cost of rebalancing the portfolio, regardless of the size of the rebalancing. The second term is proportional to the change in the value of the asset holdings. Interestingly, under the new dynamic budget constraint (8), the inherited portfolio allocation from the previous period,  $\{\omega_{i,t-1}\}_{i=1}^N$ , becomes a state variable when either  $\tau_c$  or  $\tau_v$  (or both) is (are) greater than zero, since their values determine the transaction costs to be paid at time  $t$ . Similarly to Balduzzi and Lynch (1999) we initially set  $\tau_f = 0.1\%$ ,  $\tau_v = 0.5\%$ .

Under (8), the Bellman equation of the problem becomes:

$$\frac{a(\mathbf{Z}_t, \boldsymbol{\omega}_{t-1}, t)}{1 - \gamma} = \max_{\kappa_t, \boldsymbol{\omega}_t} \left\{ \frac{\kappa_t^{1-\gamma}}{1 - \gamma} + \frac{\beta (1 - \kappa_t)^{1-\gamma}}{1 - \gamma} E \left[ a(\mathbf{Z}_{t+1}, \boldsymbol{\omega}_t, t + 1) R_{p,t+1}^{1-\gamma} | \mathbf{Z}_t \right] \right\}. \quad (10)$$

where  $\boldsymbol{\omega}_{t-1}$  is a  $N \times 1$  vector that collects the starting portfolio weights. Also in this case, the Bellman equation may be solved by backward recursion, using Monte Carlo methods. The only difference with respect to the case of  $\tau_f = \tau_v = 0$  is that a Monte Carlo approximation of the expectation  $E[a(\mathbf{Z}_{t+1}, \boldsymbol{\omega}_t, t + 1) R_{p,t+1}^{1-\gamma} | \mathbf{Z}_t]$  requires now that we draw  $G$  random samples for both asset returns  $\{\mathbf{R}_{t+1,g}\}_{g=1}^G$  and the predictors  $\{\mathbf{Z}_{t+1,g}\}_{g=1}^G$  and recognize that the choice of  $\boldsymbol{\omega}_t$  also affects the term  $E[a(\mathbf{Z}_{t+1}, \boldsymbol{\omega}_t, t + 1) R_{p,t+1}^{1-\gamma} | \mathbf{Z}_t]$ . This turns the maximization in a fixed-point problem that can be easily solved on a  $(N - 1) \times (N - 1)$  grid for  $\boldsymbol{\omega}_t$ .

#### 2.4. Performance Measurement

Our (pseudo) out-of-sample (OOS) experiment has a recursive structure within which we consider both expanding and rolling window estimation schemes. Rolling window estimation schemes are important in our research design because of the possibility that vector autoregressive frameworks the parameters of which may be updated on a relatively frequent basis may represent a powerful and yet intuitive device through which portfolio managers may try and deal with the evidence of regimes and instability in predictive relationships.

In the expanding window case, we estimate all models (in the case of VARs, these are 188 different linear frameworks) using data for the period 1957:03-1973:01 and then proceed to compute portfolio weights at horizons  $H = 1, 60,$  and  $120$  months, in the latter case with continuous (i.e., monthly, at the same frequency as the data) rebalancing. Therefore the portfolio shares will be indexed as obtained in correspondence to 1973:01 and will refer to the holding period 1973:01-1973:02 in the case of  $H = 1,$

to 1973:02-1978:01 for  $H = 60$ , and to 1973:02-1983:01 for  $H = 120$ , even though rebalancing can be performed at the end of every month and this has been correctly taken into account by our investor. At this point, the estimation sample is extended by one additional month, to the period 1957:03-1973:02, producing again portfolio weights at horizons of 1, 60, and 120 months. This process of recursive estimation, forecasting, and portfolio solution is repeated until we reach the last possible sample, 1957:03-2009:12 (even though the recursive OOS predictive or portfolio performance can be only computed for portfolios indexed up to time 2009:12 minus  $H$  months).

In the rolling window case, we use a window of 10 years of data, since we need to estimate relatively complex and richly parameterized VAR(4). The exercise is identical to the one described for the expanding case, apart from the use of an initial, shorter 10-year sample, 1963:02-1973:01. On the next iteration, the estimation sample is simply rolled forward by one month, to the period 1963:03-1973:02, producing again portfolio weights at horizons of 1, 60, and 120 months. This process of recursive estimation, forecasting, and portfolio solution is repeated until we reach the last possible sample, 2000:01-2009:12. In the case of the rolling window OOS scheme, we are able to consistently estimate over time 170 linear models.<sup>8</sup>

To evaluate recursive *realized* OOS performance, we focus on two measures. First, we calculate the certainty equivalent return (CER), defined as the sure real rate of return that an investor is willing to accept rather than adopting a particular risky portfolio strategy. We compute/solve for CER as

$$\sum_{t=1}^{T-H} \beta^t E_t \left[ \frac{\hat{C}_t^{1-\gamma}(\hat{\omega}_t)}{1-\gamma} \right] = \sum_{t=1}^{T-H} \beta^t E_t \left[ \frac{\tilde{C}_t^{1-\gamma}}{1-\gamma} \right] \quad \text{and} \quad \tilde{C}_t = \frac{1 - \beta CER_H^{1-\gamma}}{1 - (\beta CER_H^{1-\gamma})^{(H-t+1)/\gamma}}, \quad (11)$$

where  $\tilde{C}_t$  is the monthly consumption flow an investor receives under a constant investment opportunity set simply composed of a riskless real asset that yields a monthly certainty equivalent of  $CER_H$ , and  $T$  is the length of the (pseudo) out-of-sample period used in our backtesting exercise. Given this definition, transaction costs are ignored for the purposes of computing CER even when the optimal weights  $\hat{\omega}_t$  reflect transaction costs. This means, that where transaction costs were indeed applied, the  $H$ -horizon  $CER_H$  needs to be interpreted as a net-of-transaction costs measure.

Second, we compute the out-of-sample Sharpe Ratio for each portfolio strategy, defined as

$$SR_H \equiv \frac{\prod_{t=1}^{T-H} (1 + r_{p,t}^H) - \prod_{t=1}^{T-H} (1 + r_t^f)}{\sqrt{\frac{1}{T-H} \sum_{t=1}^{T-H} (R_{p,t}^H - (T-H)^{-1} \sum_{t=1}^{T-H} R_{p,t}^H)^2}}, \quad (12)$$

where  $r_t^f$  is the real 1-month T-bill and  $r_{p,t+h}^H$  is real portfolio return on a  $H$ -horizon strategy.

Third, even though our paper explicitly entertains several sets of parameters related to the size of transaction costs, to get a feeling for the amount of trading required to implement each type of portfolio

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<sup>8</sup>We had to drop a few models characterized by  $p = 4$  (to be precise, 12) because with only ten years of monthly data, at at least one point in our recursive estimation exercise, parameter estimates could not be obtained because of near-singularity issues. As one would expect, this happened especially for cases when  $M$  was relatively large, i.e., when many predictors were simultaneously used with long lags. However, because the models causing such issues always corresponded to models that had yielded very poor realized portfolio performances in the expanding window case, we have decided to keep the 10-year rolling window parameterization unaltered, as this would be probably rather appealing to many investors, for whom a relatively short rolling window means a higher protection against breaks and parameter instability.

strategy, we compute portfolio turnover, defined as the average sum of the absolute value of the trades across the  $N$  available assets that is required net of the natural effect of real asset returns cumulating over time:

$$\text{Turnover} = \frac{1}{T-H} \sum_{t=1}^{T-H} \sum_{n=1}^N \left| \hat{\omega}_{n,t}^H - \frac{\prod_{t=1}^{T-H} (1 + r_{n,t}^H) \hat{\omega}_{n,t-1}^H}{\sum_{n=1}^N \prod_{t=1}^{T-H} (1 + r_{n,t}^H) \hat{\omega}_{n,t-1}^H} \right|.$$

This *adjusted* turnover index can be interpreted as the average percentage of wealth traded in each period on a discretionary basis, i.e., as a result of a decision to re-shuffle the portfolio structure and not simply as a result of the differential rate of growth of real asset values, given the initial weights,  $\{\hat{\omega}_{n,t-1}^H\}_{n=1}^N$ .<sup>9</sup>

### 3. Data and Preliminary Estimation Results

We use monthly data on asset returns and a standard set of predictive variables sampled over the period 1957:03 to 2009:12. The data are obtained from Datastream and Global Financial Database. The real asset return data are computed from the FTSE All Share equity index, the 10-year Government bond return and the short term Treasury bill return, all deflated by the retail price index (RPI) inflation rate. The six predictive variables are the dividend yield on equities (computed as a moving average of the past 12-month dividends on the FTSE All Share-weighted index divided by the lagged index), the RPI inflation rate, the term spread defined as the difference between long- (10 year) and short-term (3 month) government bond yields, the rate of industrial production growth, the gilt-equity yield (the ratio of the yield on irredeemable gilts (consols) to the equity dividend yield) and the change in spot oil prices (West Texas Intermediate). This choice of predictor variables is governed by the prior literature on return predictability in the U.K. financial markets, which provides evidence of the forecasting ability of the dividend yield (Black and Fraser, 1995; Pesaran and Timmermann, 2000; McMillan, 2003; Kellard et al. 2010), the term spread (Clare and Thomas, 1992; Black and Fraser, 1995) retail price inflation (Pesaran and Timmermann, 2000), industrial production (Pesaran and Timmermann, 2000; McMillan, 2003) the gilt-equity yield (Clare et al., 1994; Pesaran and Timmermann, 2000; Harris and Sanchez-Valle, 2000) and oil prices (Pesaran and Timmermann, 2000). With 6 predictors and  $p = 1, 2, 4$  and (to a limited extent) 12, this implies  $3 \times 2^6 = 192$  alternative VAR models. These 192 specifications are estimated using both an expanding window and a rolling window scheme. In addition, we examine 6 VAR(12) models (each predictor separately, only under the expanding scheme). This yields a total of 390 models.

Full-sample descriptive statistics for all series are reported in the top panel of Table 1. Mean real stock returns are close to 0.65% per month with mean real long-term bond returns around 0.25% implying annualized returns of 7.8% and 3.0% respectively. Estimates of volatility imply annualized values of around 19% for real stock returns and 5.6% for real bond returns, yielding unconditional monthly Sharpe ratios of 0.09 and 0.06, respectively. Real asset returns are characterized by relatively large (in absolute value) skewness and kurtosis and are clearly non-Gaussian, as signalled by the rejections of the (univariate) null of normality delivered by the Jarque-Bera test. Table 1 also reports summary statistics for the 6

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<sup>9</sup>For simplicity, we initialize  $\hat{\omega}_{n,1978:01}^H = \hat{\omega}_{n,1979:12}^H$  for all horizons and strategies.

predictors used in this paper, but fails to reveal any unexpected features. For instance, the retail inflation rate was on average 5.6% per year, the riskless term spread has been on average positive and close to 100 basis points (in annualized terms), the annualized dividend yield has been on average 4.4%, and industrial production has grown at the rather modest average yearly rate of 1.3%. In spite of its ups and downs, oil prices have on average grown at a 9.4% rate per year, while the gilt-equity ratio has been on average well in excess of 1. The second and third panels of Table 1 also show summary statistics disaggregated for the initial estimation sample (under the expanding scheme, 1957:03-1977:12) and for the recursive, OOS backtesting sample (1978:01-2009:12) and in fact reveal some evidence of pervasive instability in the key statistical properties of the data, which is consistent with our introductory remarks on whether and how simple linear models may effectively capture and forecast such shifts.

Visual inspection of the time series dynamics of the real value of stocks, bonds, and bills in the U.K. over our sample period, gives the customary view that on average the (real) returns on equities are much higher than for bonds, but that large boom and bust cycles are also frequent. For instance, the deep stock market declines of 1974, 2001-2002, and 2007-2008 are visible. While the growth in the real value of a pure investment in Treasury bills has been relatively smooth (at least since 1982), also the real value of gilts has been exposed to important fluctuations. However, both in the 2001-2002 and the 2007-2008 crises, the real value of have moved in a direction disconnected with stocks, presumably because of a “flight-to-safety” effect.

The rest of this Section is devoted to a number of related sets of estimation results that need brief comment as a way of introducing our main findings in Sections 4-6. In Section 3.1 we briefly discuss the properties and implications of our estimates of a simple three-state Markov switching heteroskedasticity (MSIH) model. The number of details and depth of description is kept to a minimum because the goal of our paper is not to analyze the portfolio choice implications of regime switching models (a task already undertaken by Ang and Bekaert, 2002, and Guidolin and Timmermann, 2005) able to capture bull and bear dynamics but instead whether standard VAR models can approximate the portfolio implications of MISH in the case of U.K. financial markets. In Section 3.2 we outline some evidence on the nature and strength of the linear predictability patterns—as picked up by simple VARs typical of the empirical finance literature—that characterize our data on U.K. stock and bond real returns. The objective here is not (and it could not be) to provide an exhaustive quantification of what linear predictability implies, but to at least provide some evidence for how this predictability may appear in a VAR vs. a MISH model.

### 3.1. *Regimes in U.K. Real Asset Returns*

As an initial step, we estimate and compare a number of alternative specifications for the Markov switching model in (2). Specifically we examine information criteria such as the Bayes-Schwartz information criterion (BIC), a standard measure that trades off in-sample fit for parsimony, and perform Davies (1977)-corrected likelihood ratio tests which account for nuisance parameter issues in the context of Markov switching models. All of these statistics confirm the number of regimes  $k$  equal to 3, consistent with the findings of

Guidolin and Timmermann (2005).<sup>10</sup>

The standard QMLE parameter estimates for the three-state model are reported in panel B of Table 2.<sup>11</sup> An understanding of the MSIH model can be achieved by analysis of the parameter estimates within each regime. The first regime is a bear state, and in a sense, this is an extreme or “rare events” regime in which expected real returns are highly negative and statistically different from zero. In this state, all assets are more volatile than they are unconditionally (in panel A of the Table, where monthly volatilities can be read off the main diagonal of the “Correlations/Volatilities” panel), with peaks of 24 percent for stocks and of 6% per gilts, in annualized terms. The bear state is short-lived with an average duration of around 3.3 months. In this regime, the three asset classes are only mildly correlated. In most cases, at least on average, when the U.K. financial markets exit this bear state, they switch to the intermediate, equity bull regime which is characterized by positive and statistically significant mean real returns on all assets. In this regime, all assets are less volatile than in the unconditional, single-state case. This regime is highly persistent with an average duration of 23.5 months and characterizes approximately 80% of any long sample. This means that for the majority of the time, U.K. financial markets are characterized by positive real returns on all assets and moderate volatility, which fits historical experience.

The third regime is another bull state, but with three distinct features: the dominant asset class in terms of mean real returns is long-term government bonds, while stocks have an estimated mean coefficient which fails to be significant at conventional levels. All assets are more volatile in this state than in the single-state, unconditional benchmark; real returns on long-term bonds are highly correlated with both stocks (0.39) and 1-month T-bills (0.32). We label this regime as a “bond bull state” with high volatility. Clearly, the data lead to specifying this third regime because they need the flexibility to specify heterogeneous dynamics for bond and stock returns during bull regimes. Further checks confirm that the poor performance of simpler, two-state models fitted to our data largely derives from this need to allow for differential dynamics in stock and bond returns. Yet this third regime is rare, accounting for less than 8% of the sample, with an average duration of 3 months. Interestingly, as shown by the transition probabilities, there is a significant link between the bear state and the gilt bull/high volatility state. This link is further illustrated by plotting the smoothed probabilities for each of the three regimes. Figure 1 highlights that the first regime represents an extreme state largely restricted to bear markets and to well known recessionary periods in the 1970s, early 1980s, the invasion of Iraq and the first Gulf War in 1990 and the recent financial crisis. The second (bull) state is exceptionally persistent and has in fact characterized long periods of the recent U.K. financial history, such as most of the 1960s, the mid 1980s and 1988-1990, 1991-2001 and then 2004-2007. Finally, the third state occurs rarely, often following

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<sup>10</sup>Details on the model selection process are available from the Author(s) upon request. We emphasize that however the goal of our paper is not to isolate the best fitting Markov switching model but instead to show that—even if the Markov switching benchmark were to be misspecified—simple linear VAR models cannot produce qualitatively similar forecasts and, as a result, portfolio decisions.

<sup>11</sup>Panel A reports the estimates for a single-state model, which here is the Gaussian IID benchmark in (3) and shows results that are consistent with the summary statistics in Table 1. Moreover, Table 1 shows that gilt real returns are positively but moderately correlated with both real bill and stock returns. On the contrary, real stock returns and real bill returns are essentially uncorrelated.

or preceding the bear regime. These are periods of declining short-term rates and of increasing gilt prices that lead—consistently with our characterization of the regime—to high and statistically significant real bond returns. Interestingly, the third regime captures the October 1987 crash but immediately returns to the bull regime and similarly the third regime does not occur following the bear regime in 1991.

### 3.2. *Linear Predictability*

While in the MSIH framework, it is the predictability of future regimes captured by the constant transition probability matrix, in the case of VAR( $p$ ) models such predictability is characterized in two ways: by the choice of which predictors are included; by the number of lags,  $p$ . As we have explained above, one of the key objectives of this paper is to entertain (within a given set) all possible VAR models that can be built for alternative choices among six predictors, and different selections of the parameter  $p$ . It is of course impossible to document parameter estimates for all such possible 262 linear models. In this section we therefore select results on a qualitative basis and report recursive OLS estimates of the VAR parameters for two cases that (with some hindsight) play an important role in our research design: a full (i.e., including all predictors) VAR(1) model estimated on an expanding window basis; a leaner VAR(1) model that includes only the log-change in oil prices as a predictor, estimated on a 10-year rolling basis.<sup>12</sup>

Figure 2 shows recursive, expanding window estimates along with the associated 90% confidence intervals for the coefficients associated to the six predictive variables in a full VAR(1) model. In each of the six panels, we plot the time variation of the coefficient (the solid, boldfaced curve) and of its 90% confidence interval for predicting real stock returns (on the left axis) and for predicting real gilt and bill returns (on the right axis). Visibly, most 90% confidence intervals do include most of the time (often, for all the periods investigated, between January 1972 and December 2009) zero, which is equivalent to state that the null hypothesis of a zero coefficient could not be rejected using a test of hypothesis with 10% size. In fact, in 16 cases out of 18 possible cases (3 real return series times six alternative predictors), a null hypothesis of no predictive power for subsequent returns cannot be rejected for all the months in our sample. The only exceptions concern real bill returns that—especially after the early 1990s—can be predicted (with p-values generally between 0.05 and 0.1) by past log-changes in oil prices with a negative coefficient (i.e., higher growth in oil prices lead a reduction in short-term real rates), and real stock returns that—between 1974 and 1987—can be predicted (again with p-values between 0.05 and 0.1) by past dividend yields with a positive coefficient, as one generally finds in the empirical finance literature. Real bond returns were never really predictable in our sample, at least through the lenses of this relatively large-scale VAR(1) model. Figure 2 is therefore useful to emphasize that linear forecastability of real asset returns generally tends to be rather weak in our U.K. sample, although some interesting exceptions can be found. Moreover, such results on the weak predictive power for standard macroeconomic instruments

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<sup>12</sup>The full VAR(1) model is selected because it is also a way to provide an overall summary of the strength of the predictability patterns in the data. The smaller VAR(1) model based on oil prices, is selected as a representative of typical results of rolling window estimates because of its strong performance in terms of realized portfolio CERs and therefore reflects some degree of hindsight on our part.



for the subsequent behavior of financial markets tends to fit rather well earlier evidence reported by Pesaran and Timmermann (2000) and Harris and Sanchez-Valle (2000). However, it must be noticed that although generally non-significant, for a number of assets the estimated coefficients are relatively large so it is conceivable that in the aggregate, recursive estimation of the model may indeed generate interesting and potentially useful time-varying forecasts for the moments of real asset returns (in particular, their means, see below for related comments). In fact, the deeper question that Figure 2 raises and that is at the core of our paper is whether or not recursive estimation of a simple linear vector autoregressive model may represent a sufficiently effective strategy to support optimal SAA decisions in the presence of instability of the underlying economic relationships.

It may be objected that it is likely that the weak forecasting power of standard predictors that emerges in Figure 2 may derive from the unwise choice of including all predictors simultaneously. If sets of predictors tend to contain similar information on future investment opportunities, standard econometric estimates will suffer from multicollinearity issues even though the model may yield valuable forecasts. To guard against this possibility and to also showcase one qualitative example of the behavior of parameter estimates in a rolling window implementation, Figure 3 shows instead recursive, rolling window estimates and 90% confidence intervals for the coefficients in a VAR(1) model with  $M = 1$ , the changes in log-oil spot price. In this case there is only one predictor, and as such we can also afford to report recursive estimates of intercepts and own- and cross-asset vector autoregressive coefficients, for a total of five panels. Within each panel, coefficient estimates and their confidence bands are presented in a way similar to Figure 2. Clearly, the inclusion of lagged real asset returns in our VAR models is far from irrelevant: real bond returns are predicted by past stock returns (especially during the 1970s and 1980s); real bond returns are also strongly serially correlated, with highly significant coefficients in the range 0.2-0.5; real bill returns are also serially correlated, as one would expect from the empirical term structure literature. Interestingly, there is now evidence that past log-changes in oil prices forecast subsequent real returns on both gilts and bills with negative coefficients, and this evidence has grown stronger over time.

In unreported tabulations (available upon request from the Authors) we have also examined the relative forecasting performance of different linear predictive models. Although an extensive analysis based on standard statistical criteria may actually be misleading because this paper is focussed on a portfolio choice-driven loss function, we have evidence of a rather poor average root mean squared forecast error (RMSFE) across different linear models, especially as far as real bond returns are concerned. At short forecast horizons, we obtain some evidence generally favorable to relatively large-scale (with  $M$  of at least 3 or 4) VARs with long lags, once more with differences across models that are particularly large especially for real equity return forecasts. However, at longer horizons and especially for cumulative real asset returns (as an investor with a horizon of  $H > 1$  month cares for the cumulant of real returns and not for point forecasts), the balance shifts towards smaller models with  $p = 1$  or 2. Interestingly, rolling window schemes produce more accurate forecasts than the expanding window. Therefore, there is at least preliminary evidence that for intermediate and long horizons, when cumulated asset returns matter, it is

relatively lean, not overly parameterized models that generate the most accurate predictions. It remains to be seen whether such fair forecasting performance may translate in realized OOS portfolio performances that may compete with models that are explicitly designed to capture regimes.

## 4. Optimal Strategic Asset Allocation and Hedging Demands

### 4.1. Recursive Portfolio Weights

Table 3 reports summary statistics for the optimal SAA weights for the case  $\gamma = 5$ . The table presents three statistics computed on our (pseudo) OOS sample, 1972:01-2009:12: the mean recursive portfolio weights, their sample standard deviation, and their 90% empirical range, i.e., the values of the weights that leave 5% of the recursive weights in each of the two tails. Reporting the range avoids undue reliance on sample standard deviations as measures of dispersion when the weights have distributions which are highly non-normal. These statistics are computed and presented for the heteroskedastic MSIH model, its homoskedastic counterpart (see below for additional comments), the Gaussian IID benchmark, and a variety of VAR models that are selected in consideration of their pseudo-out-of sample portfolio performance at a 120-month horizon. It is immediately clear from Table 3 that different VAR models—depending on the predictors they include, on the number of lags, and on whether they are estimated using either a rolling window or an expanding scheme—fail to imply homogeneous summary statistics for portfolio weights. If one focusses on a comparison of different VARs, it appears that everything is possible. For instance, while a VAR(1) estimated on rolling data and that includes oil and the dividend yield as predictors implies that on average a long-horizon investor with  $\gamma = 5$  should allocate 269% of her portfolio to gilts, 200% to stocks, borrowing 369% of her wealth at the bill rate, a rich expanding window VAR(1) that includes all predictors but oil implies that the same investor should on average invest 121% in bonds, 62% in stocks, borrowing 53% of her wealth at the bill rate, which is a much less extreme set of SAA positions. This is of course an exciting discovery for our main goal: because VARs seem to be flexible enough to generate many alternative patterns for dynamic asset allocation, the question is then whether any of these VARs may approximate (or out-perform) the OOS performance of the Markov switching framework(s). However, Table 3 also illustrates the existence of structural differences across MSIH, the no predictability benchmark, and at least *the best performing VARs*, according to all types of summary statistics. The bull and bear framework generally implies—and this applies also to short-horizon,  $H = 1$  positions—less extreme positions than VAR models. Contrary to common findings under linear predictability, VAR models do not imply excessively high weights in stocks. Rather, in the case of recursive long-horizon mean weights, the differences mostly concern long-term bonds: while the Gaussian IID benchmark implies that on average gilts should be shorted (-70%) and MSIH suggests an even more extreme short position (-185%), the best performing VARs estimated in this paper imply a very high and hardly plausible average weight for bonds, between 152 and 419 percent, depending on the VAR specification examined.<sup>13</sup> On the contrary,

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<sup>13</sup>The weights mentioned in the main text are the 1-month optimal weights, since this allows a three-way comparison involving the Gaussian IID results. However, most VARs imply a long-run demand for stocks that largely exceeds the

while all VAR models yield a negative, large average demand for T-bills (i.e., a VAR investor ought to leverage her portfolio to be able to invest more than 100% in stocks and especially bonds), MSIH delivers portfolios that are only modestly short in T-bills.

Table 3 also reports sample measures of dispersion of recursive portfolio weights. Given its structure, MSIH delivers bond and bill weights which are much less volatile than the VAR weights but this does not apply to the weights computed for stocks. These findings also apply to the 90% empirical range of optimal weights. As one should expect, the recursive Gaussian IID weights are always the least volatile for all assets. These results are crucial because they show that the (a priori sensible and) widespread belief that regime switching asset allocation frameworks may imply “excessively” volatile portfolio weights may be misleading when applied to long-run SAA under realistic continuous, dynamic rebalancing.

#### 4.2. Hedging Demands

Table 3 also computes average hedging demands—the difference between long- and short-horizon optimal portfolio weights that ought to hedge portfolio performance against future, stochastic variation in investment opportunities—over our back-testing period. As already characterized by Guidolin and Hyde (2010) for US data, MSIH tends to imply rather small hedging demands which may be explained by the fact that although the Markov chain underlying the three-state process described in Section 3.1 presents some interesting persistence, over a 10-year horizon such persistence dissipates entirely so that the SAA will be solely based on the ergodic probability distribution implied by the MSIH model.<sup>14</sup> Although some heterogeneity exists, most VAR models (especially those that perform best in OOS portfolio experiments) yield positive and rather large (i.e., easily in excess of 50%) hedging demands for stocks, meaning that, at least in relative terms, stocks are much less risky in the long- than in the short run. This is consistent with the findings of Barberis (2000) when parameter uncertainty is ignored. On the contrary, under VAR predictability the hedging demand for gilts tends to be predominantly negative. Finally, as far as bills are concerned, the sign of the corresponding average hedging demands tends to be different—for instance, in some cases positive on average and in other cases negative—across alternative VAR models.<sup>15</sup> Because linear predictability frameworks yield rather different implications as far hedging demands are concerned, it is then possible to also interpret the key questions of this paper—whether or not simple VARs may be flexible enough to characterize bull and bear states—as an attempt to see whether taking into account the typical, non-negligible hedging demands reported in the literature may actually improve the ex-post, realized OOS performance in SAA problems.

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1-month weight and a long-run demands for 1-month T-bills that are negative and large.

<sup>14</sup>To complete our reasoning, in spite of the different state probability vector that governs the predicted MSIH dynamics at  $H = 1$  and 120 months, the resulting optimal weights will be similar because an investor that is allowed continuous rebalancing will over-weight the portfolio implications of the regime(s) perceived as most likely in short-run, strongly discounting the information that this very regime(s) contains as for the long-run dynamics of asset returns. See Guidolin and Timmermann (2005) for additional details on how continuous rebalancing changes the nature of SAA problems under Markov switching.

<sup>15</sup>Hedging demands are zero by construction in the Gaussian IID case, when stochastic opportunities are assumed to be constant over time.

Figures 4-6 go beyond the mean hedging demands in Table 3 by plotting recursive optimal weights at  $H = 1$  and 120 months and the resulting hedging demands. In Figure 4, the typical regime switching dynamic may be recognized in both short- and long-horizon portfolio weights. As a result, hedging demands are relatively small and are at most characterized by occasional spikes (positive for bills and negative for stocks) that tend to be associated with transitions from the third, gilt bull/high volatility regime to the third, bear state. In the case of gilts, the resulting hedging demand is effectively almost zero for long periods of time, especially those characterized by the second, bull state. In Figure 4 it is also clear that although the changes in portfolio compositions implied by MSIH are occasionally large, as the regimes are persistent, there are also long periods (e.g., 1991-1998) that are characterized by homogeneous states, thus limiting the amount of portfolio re-shuffling that ends up being required. These decisions of switching structure are of course also affected by the transaction costs  $\tau_f$  and  $\tau_v$ . In particular, the regime switching demand for stocks oscillates between approximately 0% in the bear state to 300-400% in the bull state, with the third regime (and the rare periods of uncertainty on the prevalent state, like early 2002) characterized by a lower demand for equities.

Figure 5 plots instead SAA weights and implied hedging demands for the two VAR models already investigated above, i.e., a full VAR(1) estimated on an expanding window and 10-year rolling VAR(1) in which oil log-price changes is the only predictor specified. Clearly, especially if the right-scales are compared to those in Figure 4, both types of VAR models imply a strong time variation in the optimal weights. Because linear predictability is largely driven by a few, highly persistent variables, in spite of their high variability, optimal weights also tend to display strong persistence. As a result, hedging demands are also large in absolute value, although persistent switches in their signs appear. Figure 6 completes the picture by comparing the dynamics of MSIH vs. VAR portfolio weights—in the latter case, as represented by a model in which oil log-price changes are the only predictor—under two assumptions concerning transaction costs: zero costs ( $\tau_f = \tau_v = 0$ , left axis scale) vs.  $\tau_f = 0.1\%$ ,  $\tau_v = 0.5\%$  (right axis scale). In the case of regime switching weights (the three plots in the left-most column), the impact of transaction costs is clearly limited, especially as far as long-term weights are concerned. The intuition is that moderate costs can hardly dissuade investors from optimally re-shuffling their portfolios provided the horizon over which such costs may be spread is sufficiently long. However, in the case of 1-month weights, a few cases can be detected where the change in portfolio composition is either delayed or more timid; when this occurs, we notice some important departures of long-term weights from short-term ones, which would imply sizeable hedging demands not previously visible in Figure 4. Under linear predictability, differences between the cases of zero vs. non-zero transaction costs are instead clearly visible in the three plots in the right-most column of Figure 6. In this case, especially once the differences between the right (when transaction costs are taken into account) and the left scales are taken into account the presence of frictions produces a considerable impact on the structure and dynamics of portfolio weights. In any event, the realized OOS portfolio performances reported in Section 5—the core of our paper—will be calculated considering different hypotheses for the level of transaction costs, including the case of no frictions.

## 5. Realized Recursive Portfolio Performance: Main Findings

Up to this point, we have shown that VAR models produce dynamic (short- and long-run) SAA weights and hedging demands that depart from those that are typically implied by a (simple, three-state) model that accounts for nonlinear patterns. Although this suggests that simple linear frameworks may miss important predictability patterns in the data for applications, one should always exercise caution before concluding that such differences between the SAA patterns deriving from linear vs. nonlinear models mean that the former would cause a welfare loss to investors. As it is well-known from forecasting theory, a better in-sample fit does not imply that a model will out-perform another in OOS experiments. Hence a portfolio manager always needs to examine evidence on the recursive, OOS performance of both linear and nonlinear SAA models before selecting one or actually concluding that neglecting nonlinearities in the form of regimes may damage investors' portfolios. This is exactly what we set out to do in this section: use the recursive experiment outlined in Section 2.4 to assess whether VAR models can yield realized OOS performances that are equivalent (or even superior) to MSIH's.

Before proceeding to the analysis, two considerations are relevant, that we formulate in the form of questions, followed by a few related comments. First, how strong an OOS realized under-performance by VAR models should we expect before concluding that in simply modeling linear predictability patterns one has to withstand a loss relative to regime switching frameworks that are useful in detecting and forecasting bull and bear states? Second, at what horizon should such evidence (if any) be most troublesome to real-life investors and portfolio managers? As for the first question, given the multitude of VAR models examined in our research design, using that standard frequentist inferential logic that admits a type-I error in formal tests of hypotheses, we may expect that at least by sheer chance, at least  $\alpha$  percent among all the VAR models ought to be able to out-perform the MSIH benchmark, even when the underlying data generating process does in fact contain bull and bear patterns of switching moments. For instance, if we were to pick  $\alpha = 5\%$  as commonly done in applied empirical work, then one would expect that out of a total of 390 VAR models, approximately 20 may outperform MSIH by chance; a tighter  $\alpha = 1\%$  should anyway deliver that at most 4 models may still outperform MSIH by sheer luck. Obviously, this makes a possible finding that *none* of the 390 VAR models out-performs MSIH even more impressive. As for the horizon, our explicit goal consists of the realized, OOS evaluation of the potential welfare costs of mistakenly assuming that multivariate linear frameworks may be rich enough to surrogate the features of bull and bear cycles. Therefore in what follows we explicitly focus on realized OOS performance at  $H = 60$  and 120 months expecting—under the null that bull and bear states are really characterizing the time series dynamics of U.K. asset returns—that at most 4-20 VAR models among the rich set of 390 entertained in our research design may eventually outperform MSIH in terms of standard criteria of welfare used in the applied portfolio management literature.<sup>16</sup>

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<sup>16</sup>The reason for entertaining both  $H = 60$  and 120 months is that, as already discussed in Section 2.3, the actual recursive evaluation of realized portfolio performance can only be implemented for portfolio formed between 1978:01 through 2009:12 minus  $H$  months. Because stopping our evaluation in 1999:12 may be considered not as representative as the case in which one stops the evaluation at least on 2004:12, in what follows considerable space is also devoted to  $H = 60$  months strategies.

Table 4 reports the key results of the paper.<sup>17</sup> The table refers to the baseline case of  $\tau_f = 0.1\%$ ,  $\tau_v = 0.5\%$ . For the case of  $\gamma = 5$ , we report the best 10 performing models (plus a few additional benchmarks, if required) when all models are ranked according to their realized, OOS real CER. The top panel concerns the 120-month horizon, the middle panel the intermediate 60-month horizon, and the lower panel the 1-month horizon. In the view of a long-horizon investor, MSIH ranks second out of 393 alternative models (390 plus the no predictability IID benchmark and two regime switching models) with an annualized CER of 2.4%; the attached 95% confidence interval is relatively tight, [2.2%, 2.6%], which means that it is likely that a  $\gamma = 5$  investor would be ready to pay at least an annualized real, constant return of over 2% to perform SAA using the MISH model.<sup>18</sup> In fact, the best performing model is the Gaussian IID model—where an investor may ignore predictability altogether—with a real CER of 6.8% and a relatively narrow confidence interval. All VAR models consistently rank below both the bull and bear frameworks and the Gaussian IID model, with the best VAR yielding a negative CER of -2.7%. Such a negative compensatory welfare measure means that an investor would be ready *to pay to be spared* the experience of having to invest for the long run using a multivariate linear framework. This means that our set of VARs never includes any model that is capable of producing CERs which exceed the CER of MSIH. In terms of the top ranking VARs at the long-horizon there is evidence of clear structure. They are very parsimonious ( $p = 1$ ) and typically include either oil price inflation or the dividend yield as predictors. Also they are most frequently estimated using the rolling window scheme. For instance, a rather simple rolling VAR that includes only lagged real asset returns and oil price inflation as predictors of future real returns turns out to be best among all VARs and produces a CER of -2.7% with a bootstrapped 95% confidence interval of [-2.7%, -2.2%]. Therefore, even the two confidence intervals for MSIH and the best VAR fail to overlap. On the other hand, Table 4 also shows the median performance statistics for all expanding window VAR models and rolling window VAR models entertained in our paper. For long horizon investors, they both imply highly disappointing negative CERs of -2.8 percent, that is, the investor would need to be paid to follow the median strategy.

Table 4 also shows that the MSIH delivers a rather moderate long-horizon annualized real mean return (6.7%) in comparison to the best mean return from a VAR (12.3%) and yet a relatively high volatility (44.1% per annum vs. 36.7% for the best VAR). Consequently the MSIH yields a relatively disappointing Sharpe ratio (0.11 in monthly terms vs. 0.29 for the best VAR) and this is considerably lower than the Sharpe ratio that a Gaussian IID investor would have been able to secure (0.48) by ignoring predictability. While it should not be surprising that a power utility investor attaches a higher CER to the MSIH despite the Gaussian IID delivering a higher Sharpe ratio, it is also important to understand what may account for this difference. This derives from the role of higher-order moments ignored by the mean-variance investor. Over the long horizon, the MSIH has a large and positive excess kurtosis (2.2) that is in excess of the tail

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<sup>17</sup>In Table 4, the reported 95% confidence bands have been bootstrapped by applying a block bootstrap to each series of recursive, realized performance statistics.

<sup>18</sup>Although this is a small real welfare gain measure, it still exceeds the annualized 1.8% real bill rate for our full-sample period that we may want to use as an estimator of the long-run, real riskless rate for the UK economy that an investor may earn by leaving 100% of her wealth in cash.

thickness exhibited by the best VAR models or the no predictability model (between -1.2 for the Gaussian IID model and 1.4). However, MSIH also implies a high positive skewness (1.7) that the VARs and no predictability model cannot generate; in fact, both the VAR and the no predictability models generate approximately symmetric long-horizon portfolio returns. Because positive skewness (right asymmetries) in realized portfolio returns benefits a power utility investor, the implication is that MSIH is rewarded by a relatively high CER not because of its pure mean-variance reward ratio, but because MSIH is a way for a long-run investor to enjoy the potential benefits of large and positive realized performances occurring with a higher probability than negative realized performances.

The middle panel of Table 4 shows instead results for the intermediate  $H = 60$  month horizon. These results are even stronger than those commented above and are largely consistent with what Guidolin and Hyde (2010) have reported for US data:<sup>19</sup> MSIH in this case is the best performing strategy with a rather strong annualized real CER of 9.9% (the 95% bootstrapped confidence interval is [9.1%, 10.8%]) that tops the 7.3% yielded by the no predictability, Gaussian IID model (with confidence interval [6.8%, 7.8%]). In fact, because the two confidence intervals fail to overlap one may even interpret this evidence as inconsistent with the null hypothesis that—when statistical uncertainties are taken into account—the two strategies fail to yield different realized performances. The best performing VARs appear to be very distant from both MSIH and the IID model: the best performing rolling VAR(1) in which past oil log-price changes forecast asset returns imply a real CERs of 0.3%, with a confidence interval that includes zero CERs. Also for  $H = 60$ , the median performance of both expanding (-0.6%) and rolling (-0.5%) window VARs are largely disappointing. Many other findings already commented in the case of  $H = 120$  extend to the intermediate horizon: for instance, MSIH has in this case a relatively high monthly Sharpe ratio (0.44) that however fails to exceed the ratio for the no predictability benchmark (0.75), but MSIH implies a high, positive excess skewness that explains how its real CER may dominate although the Sharpe ratio does not. The lower panel of Table 4 reports model performances for the best 10 models when the investor’s horizon is short. Although this is admittedly less interesting for our paper, MSIH still comes in first in a ranking that compares it to VAR models, with a real CER of 2.1% per annum. Once more, the first best realized recursive performance is obtained when all predictability patterns (linear and nonlinear) are simply ignored: the Gaussian IID real annualized CER is 4.4%. At short horizons, the CER performance from all the VAR models turns again negative and gravely disappointing, in the sense that the top 10 VAR models paradoxically include models with real CERs below -10% per year and with confidence intervals so wide to span real CERs as small as -20%; moreover, the median real annualized CER of all VAR models are in the order of -67%, which is of course the poorest of the performances. Even to short-horizon investors, it seems that ignoring bull and bear dynamics in favor of VAR models may be highly dangerous, with large performance losses possible.<sup>20</sup> Although the results in the lower panel of

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<sup>19</sup>Some Readers may in fact find these results more convincing as these derive from recursive realized performances also computed over the OOS period 2000:01-2004:12. Eventually, it may be a matter of taste whether a 5-year horizon may be considered sufficiently long to be qualified as a realistic long horizon.

<sup>20</sup>The good real CER performance of MSIH is now generated by properties of portfolio returns which are different from those commented for the  $H = 60$  and 120 month cases. Now MSIH gives the best annualized mean performance among all

Table 4 strengthens our earlier conclusion that it is hard for VARs to compete with models that take into account regimes, we leave for future research the task of exploring why and how ignoring predictability may actually lead to superior 1-month recursive performance.

Table 4 also reports adjusted turnover coefficients which clearly show that one potential source of out-performance of both MSIH and the IID benchmark over linear predictability models is given by the high turnover of the latter. For all the horizons in the table, the ranking is clear: as one would expect, given the implied slow updating of sample moments, the no predictability framework implies negligible turnover and hence, transaction costs (e.g., the index is 0.13 when  $H = 120$  months);<sup>21</sup> MSIH yields intermediate, but still rather plausible values of turnover (e.g., 0.41 for  $H = 120$ , even though this more than 3 times the index value obtained for the Gaussian IID case); VAR models require instead absurd values of turnover (e.g., the median expanding window VAR gives an adjusted turnover index of 5.7; the median rolling window VAR of 4.6) and are as such heavily penalized by transaction costs (even when these are accounted for ex-ante, i.e., as part of an investor’s optimization).

## 6. Additional Tests and Robustness Analysis

We perform robustness checks of the key result of Section 5 that no VAR model may produce a realized OOS performance comparable to that of bull and bear models. In Section 6.1, we examine the ranking of alternative models based on performance criteria different from realized, real CER. In Section 6.2, we comment on backtesting results obtained for a simpler three-state MSI homoskedastic framework. In Section 6.3, we return to the performance criteria examined in Section 5, but assuming different preferences for our investor. In Section 6.4, we deepen our understanding for the role played by transaction costs and examine configurations with lower (in fact, no) transaction costs as well as with higher (fixed) costs.

### 6.1. Other Performance Criteria

Table 5 has a structure similar to Table 4 and it is based on identical assumptions concerning the parameters  $\gamma$ ,  $\tau_f$ , and  $\tau_v$ . However, in each of its three panels, strategies are sorted on the basis of their recursive OOS *bias-corrected Sharpe ratio*, computed as:

$$SR_H^{bc} \equiv \frac{\prod_{t=1}^{T-H} (1 + r_{p,t}^H) - \prod_{t=1}^{T-H} (1 + r_t^f)}{\left(1 + \frac{3}{4(T-H)} + \frac{100}{128(T-H)^2}\right) \sqrt{\frac{1}{T-H} \sum_{t=1}^{T-H} (R_{p,t}^H - (T-H)^{-1} \sum_{t=1}^{T-H} R_{p,t}^H)^2}}$$

The motivation for the correction in the Sharpe ratio displayed in the formula above lies in Jobson and Korkie’s (1981) proof that the standard Sharpe ratio computed as the ratio between the sample mean excess return and sample variance is an (upward-) biased estimator of the true but unknown reward-to-risk ratio in finite samples, even when it is assumed that portfolio returns are normally distributed. Moreover, 

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models (18%), although its volatility is comparable to those typical of VARs (e.g., 36% vs. 52% for the best VAR). This delivers MSIH Sharpe ratios that are higher than the typical VAR yearly Sharpe ratios (0.45 vs. odd, negative ratios).

<sup>21</sup>The turnover index for the Gaussian IID model is different for different values of  $H$  in Table 4 because the recursive OOS assessment in each of the panels is based on different back-testing samples.



Table 5 offers evidence on three additional realized performance measures: first, what is commonly called the *Sortino ratio*, defined as a Sharpe ratio in which the mean excess portfolio return at the numerator is divided by the portfolio downside semi-standard deviation (see Fishburn, 1977),<sup>22</sup> second, we also show the *success rate*, which is the fraction of the OOS months used in our backtesting design in which a given strategy outperforms the IID benchmark, which is when an investor gives up on trying to exploit any kinds of predictability patterns.

The finding already commented in Section 4 re-appears, as it should be, in Table 5: at a  $H = 120$  month horizon, no Markov switching model appears among the top 10 performing models. In fact, MSIH turns out to be among the worst models we have experimented with in this paper, with a corrected (annualized) Sharpe ratio of 0.11 that needs to be contrasted to a 0.37 from the Gaussian IID benchmark and 0.31 from the best performing VAR model, in this case an expanding VAR(1) in which there is only one predictor (the dividend yield). In fact, when they are compared with the 95% bootstrapped confidence interval for the MSIH corrected Sharpe ratio of [0.12, 0.32], the confidence intervals for the Gaussian IID model ([0.35, 0.39]) and the best VAR models ([0.29, 0.33]) point to the possibility that hypotheses of identical Sharpe ratios between MSIH and the other models may be easily rejected. The Sharpe ratios of the median expanding (rolling) window VAR models is 0.19 (0.22) and as such they appear to be in line with the ratio obtained for MSIH. Of course, this is additional bad news for MSIH as this implies that even selecting at random any model among the VARs, one may be obtain an OOS realized Sharpe ratio that is very similar to the one from MSIH. However—on this the bias correction has a very little effect—this hardly surprising as we have understood already how an investor may extract utility-relevant welfare from holding MSIH portfolios: from the benefits of roughly identical volatilities, slightly lower realized means, but considerably higher skewness. Of course, faced with such a menu of trade-offs, a mean-variance investor who is not concerned with skewness or any other higher order moments, will instead select VAR portfolios.<sup>23</sup> Yet, it is difficult to under-emphasize the fact that even the VAR SAA weights underlying Table 5 have been computed assuming the investor is characterized by CRRA preferences with  $\gamma = 5$  and that his “felicity function” depends on real consumption flows and not real terminal wealth: for a long-run investor, it is not completely clear why to such an investor the (corrected) realized Sharpe ratios computed on portfolio returns ought to matter when a power utility objective implies that the entire realized density of portfolio returns are important.

When shorter investment horizons are considered, Table 5 gives a less stark contrast with Table 4. In particular, for  $H = 60$ , even though the no predictability benchmark keeps yielding the highest corrected Sharpe ratio (0.74), MSIH comes in second with an adjusted ratio of 0.44. VAR models yield good

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<sup>22</sup>When portfolio returns are normally distributed, total variance and semi-variance (which conditions on returns being below their mean) are identical. Deviations from normality imply instead that total and downside semi-variance differ. As such the Sortino ratio will contain information that is supplementary to that in the Sharpe ratio only if portfolio returns are sufficiently non-normal.

<sup>23</sup>Moreover, because we understand (see e.g., Campbell and Viceira, 2002) that for short horizons and when asset returns have a normal distribution, power utility is locally mean-variance, it is not surprising to find that under the Gaussian IID benchmark the resulting portfolio may turn out to be maximizing the resulting Sharpe ratio. However, note that our power utility objective depends on consumption and not wealth, which makes this intuition only partially appropriate.

corrected Sharpe ratios that however are systematically inferior to those that a Markov switching model guarantees. For  $H = 1$  month, we find that real CER and corrected Sharpe ratios rankings are very similar, with MSIH outperforming both a no predictability IID model and all VARs.<sup>24</sup>

However, also for long investment horizons, Table 5 returns to give strong indications in favor of bull and bear frameworks when strategies are ranked on the basis of the Sortino ratio. For instance, at  $H = 120$  months, MSIH gives one of the best 5 Sortino ratios (1.23 vs. 0.87 for the Gaussian IID benchmark) which is considerable higher than the ratios typical of median of expanding (0.46) and rolling window (0.55) strategies. Not even the VAR strategies with the highest Sortino ratios achieve at best ratios of 0.7-0.8). Of course, given the earlier evidence that MSIH yields realized portfolio returns that are strongly skewed to the right, this is hardly surprising, as a positively asymmetric distribution for realized portfolio returns may easily imply a lower variability below the mean than above it. These rankings are confirmed—and get even stronger—in the case of  $H = 60$  and 1 month. This is evidence that as soon as realized OOS performances are assessed on the basis not of symmetric mean-variance related criteria, but of asymmetric criteria, including those implied by power utility preferences, MSIH becomes hard or even impossible to outperform by VAR models that ignore the existence of regimes. The last three columns of Table 5 show that for most VAR models, the source of their under-performance does not come from their inability to yield higher returns than the Gaussian IID benchmark, but instead from the fact that they occasionally under-perform it causing large realized losses.

## 6.2. *Constant Variances and Correlations*

One sensible objection to the evidence presented in Tables 4 and 5 is that so far our research design has compared the realized OOS performances from a large set of homoskedastic VAR models with a heteroskedastic regime switching model in which the Markov state variable drives not only the time variation in mean real asset returns but also their covariance matrix. Although we have found the data strongly requesting such a three-state heteroskedastic specification, it may be objected that the incapability of VAR models to perform as well as the MSIH does may derive from their homoskedastic nature. Therefore, even though the bulk of the SAA literature has reported results that suggest that time-varying stochastic volatility would only exercise second-order effects on optimal portfolio decisions (see e.g., Chacko and Viceira, 2005), we have also proceeded to estimate a three-state MSI model in which only mean real asset returns are Markov switching.<sup>25</sup> Figure 7 shows in a way comparable to Figure 4 recursive optimal SAA weights from the homoskedastic regime switching model. For comparison, we also

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<sup>24</sup>As a matter of fact, while in Table 4 MSIH failed to outperform the Gaussian IID model in real CER terms, in Table 5 MSIH generates a corrected, annualized Sharpe ratio of 0.28 that exceeds the 0.09 of the Gaussian IID model.

<sup>25</sup>Such a model is rejected on our U.K. data both by the information criteria and by standard likelihood-ratio tests of the restriction of a constant, regime-independent covariance matrix. Interestingly, it can be shown that even though the covariance matrix is constant over time, a MSI model is anyway heteroskedastic because of the dependence of variances and covariances from the regime-dependent means. Empirically, such a MSI model is characterized by regimes that have an economic interpretation that is very similar to that reported in Section 3. However, the three regimes are considerably less persistent. Detailed parameter estimates are available from the Authors upon request.

plot the same Gaussian IID weights that have been used already in Sections 3-5. Transaction costs are imposed according to the baseline specification already commented above. As far as the MSI weights are concerned, the patterns are qualitatively similar to those shown already in Figure 4, although the oscillations in portfolio structure appear to be slightly more abrupt and larger than in Figure 4, especially as far as stocks are concerned. Another important difference between MSI and MSIH is that the former model implies a larger higher mean weight to stocks and lower weights to gilts and also bills. In essence, the fact that MSI cannot capture regimes in variances and covariances, seems to make stocks look less risky than under MSIH. Finally, as Table 3 shows, MSI entails larger (in absolute value) hedging demands than MSIH does; such hedging demands are on average positive for bills and gilts, and negative for stocks that hence under MSI stocks look increasingly risky as the horizon grows.

Tables 4 and 5 have reported already realized OOS performances for MSI in addition to MSIH. MSI always ranks below MSIH, but—consistently with earlier findings in the literature—the distance to MSIH is generally modest. In particular, at  $H = 120$  months, MSI ranks third in general and still above all VAR models that have examined in this paper. Its annualized real CER is 2.04% vs. 2.40% for MSIH; however, the 95% bootstrapped confidence intervals for the two regime switching frameworks do not overlap (they are [1.94%, 2.15%] and [2.24%, 2.57%]) which may be taken as an indication that the two performance measures are distinguishable in a statistical sense. In fact, the Sharpe ratio of MSI is extremely disappointing (0.05) even more so than the ratio for MSIH and the explanation for such a divergence between MSI and MSIH has the same explanation offered in Section 6: in the long run, MSI yields a disappointing mean, a relatively high variance and excess kurtosis (this are due to the massive long positions in stocks that tend to dominate over time, as shown in Figure 6), but also a positive and high excess skewness that benefits the average, perceived welfare of an investor. Clearly, MSI implies a higher adjusted turnover than MSIH does, although the level achieved (0.70) remains between one fourth and one fifth the turnover typical of VARs. Similar results extend to the  $H = 60$  and 1 month panels in Table 4, where MSI always follows MSIH in terms of realized OOS performance measures, but at a rather short distance and always performing better than all the VAR models we have experimented with here.<sup>26</sup> Table 5 reinforces this idea that MSI delivers slightly worse performance than MSIH, although its basic ranking properties vs. VAR models are the same as those commented in Section 6.1. For instance, at  $H = 120$  MSI has a dismal corrected Sharpe ratio of 0.08 but an excellent Sortino ratio of 0.70. All in all, we conclude that there is no evidence that our focus on a heteroskedastic MSIH affects the results in a qualitative sense: none (or at most, very few) of the linear predictability-based strategies represents a serious competitor to bull and bear frameworks, which is a powerful indication that regime shifts characterized the dynamics of U.K. returns and that VARs cannot surrogate such a dynamics neither by expanding the set of predictors involved ( $M$ ) nor by using an increasing number of lags ( $p$ ).

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<sup>26</sup>At  $H = 60$  months, MSI also outperforms the Gaussian IID benchmark: its real CER is 8% vs. 7.3% for the no predictability benchmark. However, its Sharpe ratio (0.32) is almost half the the Gaussian IID ratio (0.74).

### 6.3. Transaction Costs

So far, we have assumed  $\tau_f = 0.1\%$ ,  $\tau_v = 0.5\%$  and allowed an investor to take such costs into account ex-ante, i.e., before deciding her optimal portfolio shares. It is natural to ask whether our key finding that VAR models cannot surrogate the realized performance of nonlinear models that take bull and bear states into account may be affected by our peculiar choices of the parameters  $\tau_f$  and  $\tau_v$ . Table 6 starts by presenting results for the case in which no transaction costs are imputed, i.e.,  $\tau_f = \tau_v = 0$  similarly to Guidolin and Hyde (2010). Our conclusions hold intact and the only minor difference is that for  $H = 1$  we see the homoskedastic MSI model outperformed by a 4 different VARs. However, for intermediate and long horizons, we find the usual result that no linear framework can produced realized OOS real CERs comparable to those offered by the nonlinear models (and the Gaussian IID benchmarks). As one would expect, in the absence of transaction costs all realized performance measures are sensibly higher than when frictions are taken into account:<sup>27</sup> for instance, at a 120-month horizon, MSIH yields a real annualized CER of 5.4%, MSI of 5.1% and both measures are on the one hand inferior to the performance of the Gaussian IID model (8.9%) but also exceed by far the real CER of the best VAR (0.8%), which is again a rolling window model that only includes oil log-price changes as the non-return predictor.<sup>28</sup>

Table 7 focusses instead on the case of  $\tau_f = 0.5\%$ ,  $\tau_v = 1\%$ , i.e., when variable transaction costs are doubled and fixed transaction costs are raised by five times. This is obviously the most relevant case for a retail investor that as such pays very high (even unrealistically so, although it is difficult to sort through the myriad of costs that retail traders pay on their operations) costs. For simplicity, Table 7 shows results only for the two extreme cases  $H = 120$  and 1 month, although results for  $H = 60$  were qualitatively similar to those reported for the long-horizon case. The effects of raising transaction costs are counter-intuitive on the surface: the biggest impact is indeed recorded on the adjusted turnover rates, which decline by approximately one-quarter in the case of both regime switching and VAR models. This means that higher frictions taken into account on an ex-ante basis do make investors extremely cautious before trading which, in the presence of high transaction costs especially of fixed type, end up *improving* the realized ex-post real CER performance of the Markov switching models: at  $H = 120$  months, MSIH scores now a real CER of 5% (the 95% bootstrapped confidence interval is [3.7%, 4.1%]) which is slightly better than the no predictability benchmark (4.7% with confidence interval [3.4%, 3.7%]) and once more than all the VAR models we have entertained in this paper. The Sharpe ratios of MSIH and MSI do remain inferior to those provided by the Gaussian IID model, but the difference is now smaller while the Markov switching ratios are now very similar to the best mean-variance trade-offs of the best VARs.

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<sup>27</sup>Note however that there is no mathematical necessity in this statement because transaction costs are imputed not only ex-post on the realized trading profits, but also ex-ante. Therefore, because frictions may dissuade an investor from trading thus saving the related costs, it is in principle possible that performances may even be enhanced by considering transaction costs, if these lower the adjusted portfolio turnover. However, this never occurs in our paper.

<sup>28</sup>We have also investigated alternative performance criteria similar to those tabulated in Table 5, finding qualitatively similar results. It remains the case that nonlinear models yield disappointing Sharpe ratios but excellent Sortino ratios, which is consistent with the strong asymmetry properties of realized returns.

#### 6.4. Risk Aversion

Although the choice of  $\gamma = 5$  appears typical of most empirical finance literature, it is important to check the robustness of our empirical results to changes in the coefficient of relative risk aversion imputed to the investor. Table 8 returns to the baseline transaction cost specification ( $\tau_f = 0.1\%$ ,  $\tau_v = 0.5\%$ ) of Table 4, but concerns the case of  $\gamma = 2$ , which is a relatively low degree of risk aversion destined to make an investor that exploits predictability rather aggressive in exploiting any signal she may believe to forecast subsequent asset returns. Moreover,  $\gamma = 2$  ought to justify a strong tilt of all optimized portfolios towards the riskier assets, in our case, chiefly stocks. Table 8 shows that all these effects materialize in our experiments and that these are especially obvious—as one should expect—in the  $H = 1$  case. However, in a long-run perspective it remains the case that only one VAR model (an expanding window model with  $p = 4$  and that includes the gilt/equity ratio, RPI inflation, and oil price inflation as predictors) outperforms MSIH with a real annualized CER of 0.5%; all other models, including MSIH and the no predictability benchmark imply negative realized CERs but do outperform the residual 99.7% of the VAR models. In fact, low risk aversion makes investors so aggressive and adjusted turnover statistics so large to generally yield rather disappointing performances, both in a CER and in a Sharpe ratio perspective. It seems that aggressive investors should be really cautious before having their trades entirely driven by any type of econometric framework, although MSIH remains better performing than all VARs but one.

Table 9 completes this robustness check presenting OOS realized performance statistics for the case of  $\gamma = 10$ , that corresponds instead to a rather high degree of risk aversion. In this case, at least for a long-horizon investor, the results are qualitatively identical to those that have already appeared in Table 4: MSIH yields the second best real CER (2%) just after the Gaussian IID benchmark (3.9%). However, these findings come from much less active investors that become obviously very cautious both in terms of following market signals and in terms of overall composition of their portfolios. For instance, all the adjusted turnover statistics drop compared to Table 4. In any event, it is reassuring that the key empirical finding of this paper—VAR models cannot easily surrogate bull and bear dynamics in out-of-sample tests—are robust to changes in the selected risk aversion coefficients.

## 7. Conclusion

We have investigated whether it is possible for VAR models (linear predictability) to yield strategic asset allocation decisions equivalent to those obtained from a simple regime switching model. Secondly we have sought to establish whether the realized portfolio performance from this large family of linear predictability models is any better than that provided by explicitly modeling nonlinearity. Using U.K. asset return data on stocks, bonds and T-bills over the long period 1957-2009 we show that VAR models with linear predictability are unable to generate outcomes similar to those from a simple three-state Markov switching model. We have therefore concluded that regimes (more generally, nonlinearities) are not only statistical features of U.K. real stock and bond returns, but also that these generate optimal SAA

decisions and economic values that cannot be simply replicated using naïve linear predictability models.

Although they were rather sharp and essentially robust to a number of robustness checks (concerning risk aversion, the homoskedastic or heteroskedastic nature of the Markov switching framework, the performance criteria used to rank models, and the exact configuration and level of transaction costs), our key results are of course dictated by a number of choices that we have made. Though we have no specific prior on the fact that any of these choices may affect our conclusions, a number of different parameterizations would be interesting and represent interesting directions for future research. For instance, our nonlinear benchmark could have been replaced by a more powerful and complex nonlinear model, such as Markov switching models with ARCH effects and/or time-varying transition probabilities (if required by the data). The set of VAR models on which we have based our analysis could have expanded to encompass many more predictor variables than we have used in this paper. Finally, while in this paper we have considered an investor that is actually contemplating resorting to a VAR modeling strategy to support her long-horizon SAA decisions, it is very unlikely that this investor may actually decide to specify and estimate one particular VAR model and to stick to it over time: An investor is likely to use statistical criteria to judge the likely performance of competing VAR models at each point in time, with the possibility of occasionally switching among different VARs.

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## Appendix: Solution of asset allocation problems by Monte Carlo methods

### *Markov Switching Model*

Given the optimization problem is solved backwards at each time  $t$  (since the portfolio can be rebalanced every month), such that  $a(\boldsymbol{\pi}_{t+1}^i, t+1)$  is known for all values of  $i = 1, 2, \dots, Q$  on a discretization grid. Here  $a(\cdot)$  is not a function of the state variables  $\mathbf{Z}_{t+1}$  but the regime probabilities  $\boldsymbol{\pi}_{t+1}$ . Computing a Monte Carlo approximation of the expectation

$$E_t \left[ \{\boldsymbol{\omega}_t \mathbf{R}_{t+1,g}\}^{1-\gamma} a(\boldsymbol{\pi}_{t+1}^i, t+1) \right]$$

requires drawing  $G$  random samples of asset returns  $\{\mathbf{R}_{t+1,g}(\boldsymbol{\pi}_{t+1}^i)\}_{g=1}^G$  from the  $t+1$  one-step joint density conditional on the period- $t$  parameter estimates  $\hat{\boldsymbol{\theta}}_t = \left( \left\{ \hat{\boldsymbol{\mu}}_S, \hat{\boldsymbol{\Omega}}_S \right\}_{S=1}^k, \hat{\mathbf{P}} \right)$  assuming that, at each point  $\boldsymbol{\pi}_t^i$  is updated to  $\boldsymbol{\pi}_{t+1}(\boldsymbol{\pi}_t^i)$ . The algorithm consists of the following steps:

1. For each possible value of the current regime  $S_t$  simulate  $G$  returns  $\{\mathbf{R}_{t+1,g}(S_{t+1})\}_{g=1}^G$  in calendar time from the regime switching model:

$$\mathbf{R}_{t+1,g}(S_{t+1}) = \boldsymbol{\mu}_{S_{t+1}} + \boldsymbol{\varepsilon}_{t+1,g} \quad \boldsymbol{\varepsilon}_{t+1,g} \sim N(\mathbf{0}, \boldsymbol{\Omega}_{S_{t+1}})$$

The simulation enables regime switching as governed by the transition probability matrix  $\hat{\mathbf{P}}_t$ . For example, starting in state 1, the probability of switching to state 2 between  $t$  and  $t+1$  is  $\hat{p}_{12} \equiv \mathbf{e}'_1 \hat{\mathbf{P}}_t \mathbf{e}'_2$ , while the probability of remaining in state 1 is  $\hat{p}_{11} \equiv \mathbf{e}'_1 \hat{\mathbf{P}}_t \mathbf{e}'_1$ . Hence, at each point in time,  $\hat{\mathbf{P}}_t$  governs possible state transitions.

2. Combine the simulated returns  $\{\mathbf{R}_{t+1,g}\}_{g=1}^G$  into a random sample size  $G$ , using the probability weights contained in the vector  $\boldsymbol{\pi}_t^i$ :

$$\mathbf{R}_{t+1,g}(\boldsymbol{\pi}_t^i) = \sum_{j=1}^k (\boldsymbol{\pi}_t^i \mathbf{e}_j) \mathbf{R}_{t+1,g}(S_t = j)$$

3. Update the future regime probabilities perceived by the investor using the standard Hamilton-Kim filtering formula

$$\boldsymbol{\pi}_{t+1}(\boldsymbol{\pi}_t^i) = \frac{(\boldsymbol{\pi}_t^i)' \hat{\mathbf{P}} \odot \boldsymbol{\eta}(\mathbf{R}_{t+1,g}(\boldsymbol{\pi}_t^i); \hat{\boldsymbol{\theta}}_t)}{\left( (\boldsymbol{\pi}_t^i)' \hat{\mathbf{P}} \odot \boldsymbol{\eta}(\mathbf{R}_{t+1,g}(\boldsymbol{\pi}_t^i); \hat{\boldsymbol{\theta}}_t) \right) \boldsymbol{\nu}_k}$$

This gives an  $G \times k$  matrix  $\{\boldsymbol{\pi}_{t+1}(\boldsymbol{\pi}_t^i)\}_{g=1}^G$ , whose rows correspond to simulated vectors of perceived regime probabilities at time  $t + 1$ .

4. For all  $g = 1, 2, \dots, G$  calculate the value  $\tilde{\boldsymbol{\pi}}_{t+1,g}^i$  on the discretization grid ( $i = 1, 2, \dots, Q$ ) closest to  $\boldsymbol{\pi}_{t+1,g}(\boldsymbol{\pi}_t^i)$  using the distance measure  $\sum_{j=1}^{k-1} |\boldsymbol{\pi}_{t+1}^i \mathbf{e}_j - \boldsymbol{\pi}_{t+1,g} \mathbf{e}_j|$ , i.e.

$$\tilde{\boldsymbol{\pi}}_{t+1,g}^i(\boldsymbol{\pi}_t^i) \equiv \arg \min \sum_{j=1}^{k-1} |\mathbf{x} \mathbf{e}_j - \boldsymbol{\pi}_{t+1,g} \mathbf{e}_j|.$$

Knowledge of the vector  $\{\tilde{\boldsymbol{\pi}}_{t+1,g}^i(\boldsymbol{\pi}_t^i)\}_{g=1}^G$  allows us to build  $\left\{a(\boldsymbol{\pi}_{t+1}^{(i,g)}, t+1)\right\}_{g=1}^G$ , where  $\boldsymbol{\pi}_{t+1}^{(i,g)} \equiv \tilde{\boldsymbol{\pi}}_{t+1,g}^i(\boldsymbol{\pi}_t^i)$  is a function of the assumed, initial vector of regime probabilities  $\boldsymbol{\pi}_t^i$ .

5. Solve the program

$$\max_{\boldsymbol{\omega}_t(\boldsymbol{\pi}_t^i)} G^{-1} \sum_{g=1}^G \left\{ [\boldsymbol{\omega}_t \mathbf{R}_{t+1,g}]^{1-\gamma} a(\boldsymbol{\pi}_{t+1}^{(i,g)}, t+1) \right\}$$

For large values of  $G$  this provides an arbitrarily precise Monte Carlo approximation to  $E[\{\boldsymbol{\omega}_t \mathbf{R}_{t+1,g}\}^{1-\gamma} a(\boldsymbol{\pi}_{t+1}^{(i,g)}, t+1)]$ . The value function evaluated at the optimal portfolio weights  $\hat{\boldsymbol{\omega}}_t(\boldsymbol{\pi}_t^i)$  gives  $a(\boldsymbol{\pi}_t^i, t)$  for the  $i$ th point on the initial grid. We also check whether  $\boldsymbol{\omega}_t \mathbf{R}_{t+1,g}$  is negative and reject all corresponding sample paths.

The algorithm is applied to all possible values  $\boldsymbol{\pi}_t^i$  on the discretization grid until all values of  $a(\boldsymbol{\pi}_t^i)$  are obtained for  $i = 1, 2, \dots, Q$ . It is then iterated backwards. We take  $a(\boldsymbol{\pi}_t^i, t+1)$  as given and use the actual vector of smoothed probabilities  $\boldsymbol{\pi}_t$ . The resultant vector  $\hat{\boldsymbol{\omega}}_t$  gives the optimal portfolio allocation at time  $t$ , while  $a(\boldsymbol{\pi}_t, t)$  is the optimal value function. In our application,  $Q$  is selected as  $5^2 = 25$  which fits the standard formula  $5^{k-1}$  as in Guidolin and Timmermann (2008) and the number of Monte Carlo simulations is 30,000.

#### VAR model

Again the optimization problem is solved by backward iteration for each point  $t$  so that  $a(\mathbf{Z}_{t+1}, t+1)$ . A Monte Carlo approximation of the expectation

$$E_t \left[ \{\boldsymbol{\omega}_t \mathbf{R}_{t+1,g}\}^{1-\gamma} a(\mathbf{Z}_{t+1}^i, t+1) \right]$$

now requires drawing  $G$  random samples of the state variables  $\{\mathbf{Z}_{t+1,g}^i\}_{g=1}^G$  from the  $t + 1$  one-step joint density conditional on the period- $t$  parameter estimates  $\hat{\boldsymbol{\theta}}_t = (\hat{\boldsymbol{\mu}}, \hat{\mathbf{A}}, \hat{\boldsymbol{\Omega}})$ . The algorithm is similar but much simpler than for the Markov Switching model. The  $G$  returns  $\{\mathbf{R}_{t+1}(\mathbf{Z}_t^i)\}_{g=1}^G$  need to be simulated from the VAR model. In this case  $Q = 20$  delivers quite accurate results (because of the linearity of the prediction framework) and we set again  $G = 30,000$ .

**Table 1**  
**Summary Statistics for Portfolio Returns and Predictors**

|   | Mean     | Median   | Std. Dev. | Uncond.<br>Sharpe Ratio | Minimum | Maximum | Skewness  | Kurtosis | J-B test   |
|---|----------|----------|-----------|-------------------------|---------|---------|-----------|----------|------------|
| <b>Full Sample Period (1957:03 - 2009:12)</b>                     |          |          |           |                         |         |         |           |          |            |
| Real Stock Returns  | 0.650*** | 1.116*** | 5.456     | 0.091                   | -27.00  | 51.40   | 0.766     | 16.148*  | 4,599***   |
| Real Gilt Returns   | 0.253*** | 0.251*** | 1.610     | 0.063                   | -5.986  | 7.796   | 0.399     | 6.004**  | 253.50***  |
| 1-month T-bill Real Returns                                       | 0.152*** | 0.216*** | 0.582     | 0                       | -3.492  | 1.967   | -1.541**  | 9.447*** | 1,341***   |
| Retail Inflation rate   | 0.464*** | 0.043*** | 0.640     | —                       | -1.622  | 4.309   | 1.720***  | 10.208** | 1,674***   |
| Gilt/Equity Ratio   | 1.922*** | 2.073*** | 0.559     | —                       | 0.586   | 3.315   | -0.296*** | 2.152*** | 28.074***  |
| Riskless Term Spread (annualized)                                 | 0.996*** | 1.034*** | 1.761     | —                       | -4.175  | 6.729   | 0.208     | 3.687    | 16.931***  |
| Oil Price Inflation   | 0.783**  | 0.000    | 8.407     | —                       | -32.70  | 134.6   | 6.589*    | 105.84*  | 282,196*** |
| Industrial production growth                                      | 0.106*   | 0.102*** | 1.358     | —                       | -7.752  | 9.748   | 0.037     | 12.848*  | 2,546***   |
| Dividend Yield (annual 12-month MA)                               | 4.440*** | 4.350*** | 1.277     | —                       | 2.060   | 12.04   | 0.982**   | 6.360*   | 397.46***  |
| <b>Initial Training/Estimation Sample (1957:03 - 1977:12)</b>     |          |          |           |                         |         |         |           |          |            |
| Real Stock Returns  | 0.529    | 0.887*   | 6.300     | 0.088                   | -21.16  | 51.40   | 1.807     | 19.237*  | 3,021***   |
| Real Gilt Returns   | -0.004   | 0.043    | 1.469     | 0.013                   | -5.957  | 6.671   | 0.040     | 6.670*   | 147.09***  |
| 1-month T-bill Real Returns                                       | -0.023   | 0.014    | 0.626     | 0                       | -3.492  | 1.967   | -1.490*   | 8.919*   | 479.32***  |
| Retail Inflation rate   | 0.570*** | 0.495*** | 0.714     | —                       | -1.622  | 4.281   | 1.421*    | 7.837*   | 343.58***  |
| Gilt/Equity Ratio   | 1.772*** | 1.571*** | 0.649     | —                       | 0.779   | 3.110   | 0.233**   | 1.617*** | 23.233***  |
| Riskless Term Spread (annualized)                                 | 2.022*** | 1.640*** | 1.550     | —                       | -0.760  | 6.729   | 0.861***  | 3.387    | 33.976***  |
| Oil Price Inflation   | 0.807    | 0.000    | 8.517     | —                       | -2.280  | 134.6   | 14.949    | 234.18*  | 593,210*** |
| Industrial production growth                                      | 0.211**  | 0.000    | 1.578     | —                       | -7.752  | 9.748   | 0.122     | 12.288   | 942.44***  |
| Dividend Yield (annual 12-month MA)                               | 5.036*** | 5.050*** | 1.282     | —                       | 2.910   | 12.04   | 1.566*    | 8.729    | 465.34***  |
| <b>Recursive, Pseudo Out-of-Sample Period (1978:01 - 2009:12)</b> |          |          |           |                         |         |         |           |          |            |
| Real Stock Returns  | 0.737*** | 1.305*** | 4.772     | 0.097                   | -27.00  | 13.32   | -0.951*   | 6.191    | 211.59***  |
| Real Gilt Returns   | 0.436*** | 0.412*** | 1.682     | 0.095                   | -5.986  | 7.796   | 0.505     | 5.542*   | 114.75**   |
| 1-month T-bill Real Returns                                       | 0.277*** | 0.291*** | 0.515     | 0                       | -3.199  | 1.576   | -1.558*   | 10.49*   | 1,010***   |
| Retail Inflation rate   | 0.388*** | 0.360*** | 0.571     | —                       | -1.435  | 4.309   | 1.955*    | 13.29*   | 1,856***   |
| Gilt/Equity Ratio   | 2.029*** | 2.112*** | 0.456     | —                       | -0.677  | 3.590   | -0.677*** | 3.590    | 33.409***  |
| Riskless Term Spread (annualized)                                 | 0.265*** | 0.211*** | 1.523     | —                       | -4.175  | 3.783   | -0.250    | 2.911    | 3.944      |
| Oil Price Inflation   | 0.766*** | 0.732*** | 0.834     | —                       | -32.70  | 45.80   | 0.256     | 6.420    | 183.41***  |
| Industrial production growth                                      | 0.030    | 0.103    | 1.173     | —                       | -6.980  | 7.503   | -0.268    | 10.78    | 932.03***  |
| Dividend Yield (annual 12-month MA)                               | 4.016*** | 3.925*** | 1.091     | —                       | 2.060   | 6.860   | 0.391**   | 2.539    | 12.646***  |

Table 2

Full-Sample Estimates of Three-State Heteroskedastic Markov Switching Multivariate Model for Real Stock, Bond, and 1-month T-Bill Returns

| Panel A - SINGLE STATE MODEL                         |                             |                                  |                                 |
|--|-----------------------------|----------------------------------|---------------------------------|
|  | Real 1-month T-bill Returns | Real Gilt Returns                | Real Stock Returns              |
| <b>1. Mean returns</b>                               | 0.1524**                    | 0.2530**                         | 0.6501**                        |
| <b>2. Correlations/Volatilities</b>                  |                             |                                  |                                 |
| Real 1-month T-bill Returns                          | 0.5822**                    |                                  |                                 |
| Real Gilt Returns                                    | 0.3450**                    | 1.6104**                         |                                 |
| Real Stock Returns                                   | -0.0179                     | 0.2662**                         | 5.4556**                        |
| Panel B - THREE-STATE MODEL                          |                             |                                  |                                 |
|  | Real 1-month T-bill Returns | Real Gilt Returns                | Real Stock Returns              |
| <b>1. Mean returns</b>                               |                             |                                  |                                 |
| Bear State   | -0.4330**                   | -1.4559**                        | -2.9216*                        |
| Equity Bull/Low Volatility State                     | 0.2353**                    | 0.2649**                         | 1.0382**                        |
| Gilt Bull/High Volatility State                      | 0.0661                      | 3.1809**                         | 1.7514                          |
| <b>2. Correlations/Volatilities</b>                  |                             |                                  |                                 |
| Bear State   |                             |                                  |                                 |
| Real 1-month T-bill Returns                          | 0.9750**                    |                                  |                                 |
| Real Gilt Returns                                    | 0.1780*                     | 1.8223**                         |                                 |
| Real Stock Returns                                   | -0.2773*                    | -0.0251                          | 7.0367**                        |
| Equity Bull/Low Volatility State                     |                             |                                  |                                 |
| Real 1-month T-bill Returns                          | 0.4407**                    |                                  |                                 |
| Real Gilt Returns                                    | 0.3875**                    | 1.1578**                         |                                 |
| Real Stock Returns                                   | 0.0857                      | 0.2264*                          | 4.0477**                        |
| Gilt Bull/High Volatility State                      |                             |                                  |                                 |
| Real 1-month T-bill Returns                          | 0.7084**                    |                                  |                                 |
| Real Gilt Returns                                    | 0.3237**                    | 2.1592**                         |                                 |
| Real Stock Returns                                   | -0.4291**                   | 0.3916**                         | 12.318**                        |
| <b>3. Transition probabilities</b>                   | Bear State                  | Equity Bull/Low Volatility State | Gilt Bull/High Volatility State |
| Bear State   | 0.6953**                    | 0.2242**                         | 0.0805                          |
| Equity Bull/Low Volatility State                     | 0.0221                      | 0.9575**                         | 0.0204                          |
| Gilt Bull/High Volatility State                      | 0.1422*                     | 0.0867                           | 0.6711**                        |
| Panel C - MARKOV CHAIN PROPERTIES, THREE-STATE MODEL |                             |                                  |                                 |
|  | Bear State                  | Equity Bull/Low Volatility State | Gilt Bull/High Volatility State |
| <b>Ergodic Probabilities</b>                         | 0.1210                      | 0.7998                           | 0.0792                          |
| <b>Average Duration (in months)</b>                  | 3.3                         | 23.5                             | 3.0                             |

\*\* = significant at 1% size or lower; \* = significant at 5% size.

Table 3

Summary Statistics for Realized, Recursive Optimal Portfolio Weights Computed under Power Utility Preferences ( $\gamma = 5$ )

| CER Rank  | Model                   | Lags | Predictors Included     |             |           |    |                     |    | 1-month T-bills |        |         | Long-term Bonds (Gilts) |        |         | Stocks |       |         |
|---|-------------------------|------|-------------------------|-------------|-----------|----|---------------------|----|-----------------|--------|---------|-------------------------|--------|---------|--------|-------|---------|
|   |                         |      | Gilt/Equity Yield Ratio | Term Spread | Inflation | IP | Oil Price Inflation | DY | T=1             | T=120  | Hedging | T=1                     | T=120  | Hedging | T=1    | T=120 | Hedging |
| <b>Sample Mean of Portfolio Weights</b>               |                         |      |                         |             |           |    |                     |    |                 |        |         |                         |        |         |        |       |         |
| 1   | MS Heteroskedastic      | 0    | —                       | —           | —         | —  | —                   | —  | -0.111          | -0.089 | 0.022   | -1.849                  | -1.846 | 0.002   | 2.960  | 2.935 | -0.024  |
| 2   | MS Homoskedastic        | 0    | —                       | —           | —         | —  | —                   | —  | -1.905          | -1.674 | 0.231   | -2.276                  | -2.174 | 0.102   | 5.187  | 4.853 | -0.333  |
| 3   | IID (No predictability) | 0    | —                       | —           | —         | —  | —                   | —  | 1.169           | —      | —       | -0.697                  | —      | —       | 0.528  | —     | —       |
| 4   | VAR Rolling             | 1    | N                       | N           | N         | N  | Y                   | Y  | -2.671          | -3.692 | -1.022  | 3.271                   | 2.688  | -0.583  | 0.400  | 2.004 | 1.605   |
| 5   | VAR Rolling             | 1    | N                       | N           | Y         | Y  | N                   | N  | -2.994          | -2.628 | 0.366   | 3.433                   | 3.063  | -0.370  | 0.561  | 0.565 | 0.004   |
| 6   | VAR Rolling             | 1    | N                       | N           | N         | N  | Y                   | N  | -3.555          | -2.683 | 0.871   | 4.009                   | 3.098  | -0.911  | 0.546  | 0.585 | 0.040   |
| 7   | VAR Expanding           | 1    | Y                       | Y           | Y         | Y  | N                   | Y  | -0.530          | -0.837 | -0.307  | 1.517                   | 1.213  | -0.304  | 0.013  | 0.624 | 0.611   |
| 8   | VAR Rolling             | 1    | N                       | N           | N         | Y  | N                   | Y  | -2.697          | -3.752 | -1.055  | 3.266                   | 2.708  | -0.558  | 0.432  | 2.044 | 1.613   |
| 9   | VAR Rolling             | 1    | N                       | Y           | N         | N  | Y                   | N  | -3.706          | -3.120 | 0.585   | 4.186                   | 3.515  | -0.671  | 0.519  | 0.605 | 0.086   |
| 101   | VAR Expanding           | 4    | N                       | N           | N         | Y  | N                   | N  | -1.723          | -0.904 | 0.819   | 2.183                   | 1.416  | -0.767  | 0.540  | 0.487 | -0.052  |
| <b>Sample Standard Deviation of Portfolio Weights</b> |                         |      |                         |             |           |    |                     |    |                 |        |         |                         |        |         |        |       |         |
| 1   | MS Heteroskedastic      | 0    | —                       | —           | —         | —  | —                   | —  | 0.222           | 0.225  | 0.048   | 1.978                   | 1.975  | 0.050   | 1.820  | 1.818 | 0.072   |
| 2   | MS Homoskedastic        | 0    | —                       | —           | —         | —  | —                   | —  | 1.091           | 1.084  | 0.250   | 1.292                   | 1.273  | 0.259   | 2.229  | 2.173 | 0.352   |
| 3   | IID (No predictability) | 0    | —                       | —           | —         | —  | —                   | —  | 1.161           | —      | —       | -0.662                  | —      | —       | 0.574  | —     | —       |
| 4   | VAR Rolling             | 1    | N                       | N           | N         | N  | Y                   | Y  | 6.714           | 6.636  | 1.441   | 6.518                   | 5.883  | 0.888   | 1.086  | 2.085 | 1.434   |
| 5   | VAR Rolling             | 1    | N                       | N           | Y         | Y  | N                   | N  | 6.351           | 5.972  | 1.395   | 6.338                   | 5.942  | 1.351   | 0.842  | 0.842 | 0.163   |
| 6   | VAR Rolling             | 1    | N                       | N           | N         | N  | Y                   | N  | 6.479           | 5.760  | 1.034   | 6.403                   | 5.695  | 0.996   | 0.796  | 0.814 | 0.141   |
| 7   | VAR Expanding           | 1    | Y                       | Y           | Y         | Y  | N                   | Y  | 6.661           | 5.940  | 0.790   | 6.448                   | 5.888  | 0.763   | 0.563  | 0.753 | 0.393   |
| 8   | VAR Rolling             | 1    | N                       | N           | N         | Y  | N                   | Y  | 6.643           | 6.747  | 1.470   | 6.538                   | 6.036  | 0.809   | 0.984  | 2.021 | 1.443   |
| 9   | VAR Rolling             | 1    | N                       | Y           | N         | N  | Y                   | N  | 6.762           | 6.160  | 1.284   | 6.665                   | 6.064  | 1.100   | 0.892  | 0.943 | 0.348   |
| 101   | VAR Expanding           | 4    | N                       | N           | N         | Y  | N                   | N  | 8.779           | 8.209  | 1.504   | 8.561                   | 8.122  | 1.326   | 0.935  | 0.823 | 0.211   |
| <b>Empirical 90% Range</b>                            |                         |      |                         |             |           |    |                     |    |                 |        |         |                         |        |         |        |       |         |
| 1   | MS Heteroskedastic      | 0    | —                       | —           | —         | —  | —                   | —  | 0.777           | 0.784  | 0.091   | 5.179                   | 5.155  | 0.079   | 4.957  | 5.051 | 0.133   |
| 2   | MS Homoskedastic        | 0    | —                       | —           | —         | —  | —                   | —  | 2.978           | 3.025  | 0.641   | 3.634                   | 3.568  | 0.773   | 6.548  | 6.510 | 0.945   |
| 3   | IID (No predictability) | 0    | —                       | —           | —         | —  | —                   | —  | 0.633           | —      | —       | 0.725                   | —      | —       | 0.758  | —     | —       |
| 4   | VAR Rolling             | 1    | N                       | N           | N         | N  | Y                   | Y  | 21.761          | 20.784 | 3.649   | 21.260                  | 19.744 | 2.755   | 3.104  | 5.665 | 3.654   |
| 5   | VAR Rolling             | 1    | N                       | N           | Y         | Y  | N                   | N  | 21.776          | 19.671 | 5.068   | 22.152                  | 19.930 | 4.891   | 2.187  | 2.258 | 0.547   |
| 6   | VAR Rolling             | 1    | N                       | N           | N         | N  | Y                   | N  | 20.715          | 19.263 | 3.198   | 21.384                  | 19.426 | 3.034   | 2.465  | 2.561 | 0.383   |
| 7   | VAR Expanding           | 1    | Y                       | Y           | Y         | Y  | N                   | Y  | 19.900          | 17.543 | 2.384   | 18.986                  | 17.646 | 2.313   | 1.739  | 2.398 | 1.304   |
| 8   | VAR Rolling             | 1    | N                       | N           | N         | Y  | N                   | Y  | 20.633          | 20.625 | 3.518   | 20.757                  | 19.205 | 2.621   | 2.839  | 5.747 | 3.549   |
| 9   | VAR Rolling             | 1    | N                       | Y           | N         | N  | Y                   | N  | 20.986          | 19.244 | 4.002   | 21.674                  | 19.378 | 3.771   | 2.893  | 3.010 | 0.821   |
| 101   | VAR Expanding           | 4    | N                       | N           | N         | Y  | N                   | N  | 26.894          | 25.215 | 4.266   | 25.855                  | 24.486 | 3.728   | 2.865  | 2.527 | 0.570   |

Table 4

Best Models Ranked According to Average Recursive Certainty Equivalent Return Obtained from Optimal Strategic Asset Allocation Choices Under Power Utility Preferences ( $\gamma = 5$ ): Baseline Transaction Cost Level

| CER Rank | Model                   | Lags | Predictors Included     |             |           |    |                     | Horizon                          | Annualized Mean |              |                      | Annualized Volatility |            |                      | Sharpe Ratio         |              |                      | Annualized CER       |              |                      | Skewness | Kurtosis | Average turnover (adjusted) |                      |
|----------|-------------------------|------|-------------------------|-------------|-----------|----|---------------------|----------------------------------|-----------------|--------------|----------------------|-----------------------|------------|----------------------|----------------------|--------------|----------------------|----------------------|--------------|----------------------|----------|----------|-----------------------------|----------------------|
|          |                         |      | Gilt/Equity Yield Ratio | Term Spread | Inflation | IP | Oil Price Inflation |                                  | DY              | Mean returns | 95% Conf. Int. -- LB | 95% Conf. Int. -- UB  | Volatility | 95% Conf. Int. -- LB | 95% Conf. Int. -- UB | Sharpe ratio | 95% Conf. Int. -- LB | 95% Conf. Int. -- UB | CER (% Ann.) | 95% Conf. Int. -- LB |          |          |                             | 95% Conf. Int. -- UB |
| 1        | IID (No predictability) | 0    | —                       | —           | —         | —  | —                   | 120                              | 9.249           | 8.440        | 9.969                | 15.477                | 14.687     | 16.180               | 0.480                | 0.450        | 0.503                | 6.774                | 6.028        | 7.531                | 0.146    | 1.751    | 0.130                       |                      |
| 2        | MS Heteroskedastic      | 0    | —                       | —           | —         | —  | —                   | 120                              | 6.701           | 4.553        | 8.879                | 44.089                | 38.528     | 49.303               | 0.111                | 0.071        | 0.143                | 2.397                | 2.240        | 2.567                | 1.671    | 5.183    | 0.410                       |                      |
| 3        | MSI Homoskedastic       | 0    | —                       | —           | —         | —  | —                   | 120                              | 3.992           | 2.074        | 5.870                | 40.861                | 34.036     | 46.586               | 0.053                | 0.007        | 0.087                | 2.041                | 1.936        | 2.150                | 2.088    | 7.250    | 0.697                       |                      |
| 4        | VAR Rolling             | 1    | N                       | N           | N         | N  | Y                   | N                                | 120             | 12.331       | 10.801               | 13.787                | 36.349     | 33.792               | 38.563               | 0.289        | 0.266                | 0.310                | -2.669       | -2.744               | -2.183   | -0.012   | 4.120                       | 3.314                |
| 5        | VAR Rolling             | 1    | N                       | N           | N         | N  | N                   | Y                                | 120             | 12.782       | 11.399               | 14.294                | 34.927     | 32.417               | 37.031               | 0.314        | 0.295                | 0.337                | -2.676       | -2.736               | -2.327   | -0.067   | 4.243                       | 3.036                |
| 6        | VAR Expanding           | 1    | Y                       | Y           | N         | N  | Y                   | Y                                | 120             | 12.567       | 10.740               | 14.306                | 40.778     | 37.645               | 44.080               | 0.263        | 0.237                | 0.283                | -2.681       | -2.739               | -1.992   | 0.123    | 4.448                       | 3.788                |
| 7        | VAR Rolling             | 1    | N                       | N           | N         | Y  | Y                   | N                                | 120             | 11.774       | 10.190               | 13.278                | 37.012     | 34.129               | 39.324               | 0.269        | 0.245                | 0.291                | -2.698       | -2.754               | -2.310   | 0.013    | 4.054                       | 3.639                |
| 8        | VAR Rolling             | 1    | N                       | N           | N         | N  | N                   | Y                                | 120             | 14.866       | 12.926               | 16.870                | 44.421     | 42.363               | 45.955               | 0.294        | 0.262                | 0.327                | -2.699       | -2.754               | -2.312   | -0.320   | 3.401                       | 3.243                |
| 9        | VAR Rolling             | 1    | N                       | N           | N         | Y  | N                   | N                                | 120             | 12.269       | 10.755               | 13.659                | 35.895     | 33.400               | 38.316               | 0.291        | 0.267                | 0.309                | -2.700       | -2.753               | -2.421   | -0.002   | 4.139                       | 3.393                |
| 10       | VAR Rolling             | 1    | N                       | N           | N         | Y  | Y                   | N                                | 120             | 12.611       | 11.032               | 14.201                | 37.435     | 34.850               | 39.551               | 0.288        | 0.264                | 0.313                | -2.701       | -2.756               | -2.463   | -0.022   | 4.026                       | 3.682                |
|          |                         |      |                         |             |           |    |                     | Median Expanding VAR performance | 120             | 9.715        | 8.106                | 11.293                | 40.661     | 37.839               | 43.093               | 0.194        | 0.166                | 0.220                | -2.776       | -2.802               | -2.729   | 0.229    | 4.111                       | 5.670                |
|          |                         |      |                         |             |           |    |                     | Median Rolling VAR performance   | 120             | 11.435       | 9.770                | 13.220                | 42.829     | 40.757               | 44.696               | 0.224        | 0.195                | 0.255                | -2.773       | -2.798               | -2.723   | -0.036   | 3.698                       | 4.623                |
| 1        | MS Heteroskedastic      | 0    | —                       | —           | —         | —  | —                   | 60                               | 21.207          | 19.010       | 23.462               | 43.844                | 39.035     | 48.470               | 0.442                | 0.440        | 0.446                | 9.871                | 9.076        | 10.765               | 1.112    | 4.789    | 0.555                       |                      |
| 2        | MSI Homoskedastic       | 0    | —                       | —           | —         | —  | —                   | 60                               | 24.832          | 21.290       | 28.549               | 72.088                | 62.837     | 80.402               | 0.319                | 0.310        | 0.332                | 7.969                | 7.272        | 8.718                | 1.491    | 5.998    | 0.895                       |                      |
| 3        | IID (No predictability) | 0    | —                       | —           | —         | —  | —                   | 60                               | 8.035           | 7.611        | 8.460                | 8.293                 | 7.823      | 8.734                | 0.749                | 0.740        | 0.760                | 7.284                | 6.841        | 7.761                | 0.076    | 1.981    | 0.022                       |                      |
| 4        | VAR Rolling             | 1    | N                       | N           | N         | N  | Y                   | N                                | 60              | 11.509       | 9.470                | 13.495                | 40.601     | 36.453               | 44.403               | 0.239        | 0.210                | 0.263                | 0.228        | -0.059               | 1.590    | 0.340    | 3.766                       | 3.586                |
| 5        | VAR Rolling             | 1    | N                       | N           | N         | N  | N                   | Y                                | 60              | 10.910       | 9.203                | 12.796                | 35.558     | 32.240               | 38.442               | 0.256        | 0.229                | 0.285                | 0.076        | -0.174               | 2.096    | 0.269    | 3.663                       | 3.324                |
| 6        | VAR Rolling             | 1    | N                       | N           | N         | N  | N                   | Y                                | 60              | 16.500       | 13.258               | 19.718                | 54.388     | 50.148               | 57.699               | 0.270        | 0.228                | 0.310                | -0.178       | -0.364               | 2.408    | 0.144    | 1.918                       | 3.555                |
| 7        | VAR Rolling             | 1    | N                       | N           | Y         | Y  | N                   | N                                | 60              | 10.464       | 8.305                | 12.589                | 38.638     | 34.744               | 42.035               | 0.224        | 0.187                | 0.256                | -0.203       | -0.381               | 1.615    | 0.286    | 3.618                       | 4.045                |
| 8        | VAR Rolling             | 1    | N                       | N           | N         | Y  | N                   | N                                | 60              | 10.390       | 8.418                | 12.249                | 36.216     | 32.908               | 39.600               | 0.237        | 0.200                | 0.263                | -0.253       | -0.407               | 1.152    | 0.278    | 3.719                       | 3.769                |
| 9        | VAR Rolling             | 1    | N                       | Y           | N         | N  | Y                   | N                                | 60              | 11.664       | 9.597                | 14.421                | 41.850     | 38.219               | 45.689               | 0.235        | 0.203                | 0.276                | -0.277       | -0.425               | 0.418    | 0.334    | 3.639                       | 3.647                |
| 10       | VAR Rolling             | 1    | N                       | N           | N         | Y  | Y                   | N                                | 60              | 11.046       | 8.980                | 13.497                | 41.174     | 37.441               | 44.880               | 0.224        | 0.191                | 0.260                | -0.280       | -0.417               | 0.959    | 0.333    | 3.693                       | 3.996                |
|          |                         |      |                         |             |           |    |                     | Median Expanding VAR performance | 60              | 9.133        | 6.533                | 11.757                | 46.023     | 41.469               | 50.040               | 0.159        | 0.114                | 0.199                | -0.578       | -0.618               | -0.512   | 0.365    | 4.990                       | 6.383                |
|          |                         |      |                         |             |           |    |                     | Median Rolling VAR performance   | 60              | 11.001       | 8.727                | 13.197                | 49.425     | 44.609               | 52.971               | 0.186        | 0.155                | 0.215                | -0.535       | -0.585               | -0.439   | 0.254    | 4.170                       | 5.418                |
| 1        | IID (No predictability) | 0    | —                       | —           | —         | —  | —                   | 1                                | 2.619           | -0.759       | 6.205                | 7.832                 | 6.986      | 8.672                | 0.102                | -0.370       | 0.505                | 4.359                | 4.070        | 4.637                | -0.249   | 5.555    | 0.179                       |                      |
| 2        | MS Heteroskedastic      | 0    | —                       | —           | —         | —  | —                   | 1                                | 18.117          | 12.459       | 23.633               | 36.234                | 32.065     | 29.040               | 0.450                | 0.332        | 0.751                | 2.136                | -0.650       | 4.827                | -0.339   | 4.428    | 0.573                       |                      |
| 3        | MSI Homoskedastic       | 0    | —                       | —           | —         | —  | —                   | 1                                | 21.457          | 12.672       | 30.289               | 42.914                | 50.837     | 46.563               | 0.457                | 0.213        | 0.611                | -0.787               | -4.244       | 3.078                | 0.006    | 3.545    | 1.077                       |                      |
| 4        | VAR Rolling             | 2    | N                       | N           | N         | Y  | N                   | N                                | 1               | -42.777      | -54.411              | -31.635               | 51.500     | 72.125               | 60.089               | -0.866       | -0.780               | -0.557               | -0.995       | -7.659               | 6.878    | 0.677    | 13.267                      | 4.298                |
| 5        | VAR Rolling             | 2    | N                       | Y           | N         | Y  | N                   | N                                | 1               | -44.934      | -56.169              | -35.635               | 54.091     | 81.958               | 66.735               | -0.864       | -0.708               | -0.561               | -5.206       | -13.531              | 3.885    | 1.695    | 22.236                      | 4.542                |
| 6        | VAR Expanding           | 2    | Y                       | Y           | Y         | N  | N                   | Y                                | 1               | -47.151      | -58.219              | -37.451               | 48.142     | 65.101               | 56.910               | -1.017       | -0.922               | -0.690               | -5.618       | -13.791              | 9.033    | 0.307    | 9.015                       | 5.002                |
| 7        | VAR Rolling             | 2    | N                       | Y           | Y         | N  | N                   | N                                | 1               | -45.324      | -55.760              | -33.418               | 50.870     | 63.072               | 56.974               | -0.927       | -0.913               | -0.619               | -6.308       | -13.037              | 5.596    | -0.572   | 5.864                       | 3.943                |
| 8        | VAR Rolling             | 2    | N                       | N           | N         | Y  | N                   | Y                                | 1               | -48.604      | -58.763              | -35.003               | 54.193     | 76.990               | 65.579               | -0.931       | -0.787               | -0.562               | -8.448       | -14.898              | 3.680    | 0.879    | 16.288                      | 4.686                |
| 9        | VAR Expanding           | 2    | Y                       | Y           | Y         | N  | Y                   | Y                                | 1               | -49.139      | -59.328              | -39.247               | 50.345     | 64.437               | 57.255               | -1.012       | -0.949               | -0.717               | -10.409      | -21.011              | 8.108    | 0.130    | 7.960                       | 4.889                |
| 10       | VAR Expanding           | 2    | Y                       | Y           | Y         | N  | N                   | N                                | 1               | -47.822      | -59.070              | -37.549               | 49.626     | 64.646               | 56.791               | -1.000       | -0.942               | -0.693               | -11.051      | -23.251              | 8.472    | 0.208    | 8.793                       | 5.087                |
|          |                         |      |                         |             |           |    |                     | Median Expanding VAR performance | 1               | -45.003      | -55.522              | -35.429               | 57.308     | 49.544               | 65.134               | -0.817       | -1.157               | -0.572               | -66.588      | -67.647              | 7.943    | -0.068   | 9.533                       | 4.680                |
|          |                         |      |                         |             |           |    |                     | Median Rolling VAR performance   | 1               | -42.993      | -54.850              | -32.519               | 62.163     | 52.023               | 72.415               | -0.721       | -1.089               | -0.474               | -66.588      | -67.647              | 6.237    | 0.554    | 13.796                      | 4.244                |

Table 5

Best Models Ranked According to Average Long-Horizon Recursive Sharpe Ratio Obtained from Optimal Strategic Asset Allocation Choices Under Power Utility Preferences ( $\gamma = 5$ ): Baseline Transaction Cost Level

| CER Rank | Model                   | Lags | Predictors Included     |             |           |    |                     |    | Horizon | Corrected Sharpe Ratio |                      |                      | Sharpe Ratio |                      | Sortino Ratio        |        | Success Rate         |                      |       |                      |                      |
|----------|-------------------------|------|-------------------------|-------------|-----------|----|---------------------|----|---------|------------------------|----------------------|----------------------|--------------|----------------------|----------------------|--------|----------------------|----------------------|-------|----------------------|----------------------|
|          |                         |      | Gilt/Equity Yield Ratio | Term Spread | Inflation | IP | Oil Price Inflation | DY |         | Ratio                  | 95% Conf. Int. -- LB | 95% Conf. Int. -- UB | Ratio        | 95% Conf. Int. -- LB | 95% Conf. Int. -- UB | Ratio  | 95% Conf. Int. -- LB | 95% Conf. Int. -- UB | Rate  | 95% Conf. Int. -- LB | 95% Conf. Int. -- UB |
| 1        | IID (No predictability) | 0    | —                       | —           | —         | —  | —                   | —  | 120     | 0.468                  | 0.445                | 0.486                | 0.480        | 0.450                | 0.503                | 0.874  | 0.862                | 0.947                | —     | —                    | —                    |
| 2        | VAR Expanding           | 1    | N                       | N           | N         | N  | N                   | Y  | 120     | 0.314                  | 0.288                | 0.332                | 0.315        | 0.288                | 0.333                | 0.656  | 0.632                | 0.696                | 0.385 | 0.325                | 0.444                |
| 3        | VAR Expanding           | 1    | N                       | N           | N         | N  | Y                   | Y  | 120     | 0.310                  | 0.287                | 0.337                | 0.311        | 0.288                | 0.338                | 0.679  | 0.640                | 0.698                | 0.265 | 0.209                | 0.328                |
| 4        | VAR Rolling             | 1    | N                       | Y           | N         | N  | N                   | N  | 120     | 0.312                  | 0.286                | 0.336                | 0.313        | 0.287                | 0.337                | 0.617  | 0.575                | 0.660                | 0.227 | 0.178                | 0.274                |
| 5        | VAR Rolling             | 1    | N                       | N           | N         | N  | N                   | N  | 120     | 0.313                  | 0.295                | 0.336                | 0.314        | 0.295                | 0.337                | 0.583  | 0.566                | 0.612                | 0.531 | 0.470                | 0.593                |
| 6        | VAR Rolling             | 1    | N                       | N           | Y         | N  | N                   | N  | 120     | 0.310                  | 0.291                | 0.333                | 0.311        | 0.292                | 0.334                | 0.594  | 0.557                | 0.643                | 0.592 | 0.537                | 0.648                |
| 7        | VAR Expanding           | 1    | N                       | N           | N         | Y  | N                   | Y  | 120     | 0.298                  | 0.271                | 0.326                | 0.299        | 0.272                | 0.327                | 0.638  | 0.589                | 0.674                | 0.531 | 0.468                | 0.583                |
| 8        | VAR Expanding           | 1    | Y                       | N           | N         | N  | N                   | Y  | 120     | 0.297                  | 0.273                | 0.322                | 0.298        | 0.274                | 0.323                | 0.621  | 0.593                | 0.659                | 0.508 | 0.442                | 0.569                |
| 245      | MS Heteroskedastic      | 0    | —                       | —           | —         | —  | —                   | —  | 120     | 0.105                  | 0.095                | 0.119                | 0.111        | 0.071                | 0.143                | 1.228  | 0.751                | 1.660                | 0.577 | 0.523                | 0.635                |
| 268      | MSI Homoskedastic       | 0    | —                       | —           | —         | —  | —                   | —  | 120     | 0.080                  | -0.041               | 0.178                | 0.053        | 0.007                | 0.087                | 0.696  | -0.436               | 1.279                | 0.527 | 0.482                | 0.674                |
|          |                         |      |                         |             |           |    |                     |    | 120     | 0.194                  | 0.166                | 0.219                | 0.194        | 0.166                | 0.220                | 0.464  | 0.424                | 0.508                | 0.463 | 0.406                | 0.524                |
|          |                         |      |                         |             |           |    |                     |    | 120     | 0.224                  | 0.194                | 0.254                | 0.224        | 0.195                | 0.255                | 0.554  | 0.516                | 0.608                | 0.500 | 0.441                | 0.558                |
| 1        | IID (No predictability) | 0    | —                       | —           | —         | —  | —                   | —  | 60      | 0.741                  | 0.739                | 0.745                | 0.749        | 0.740                | 0.760                | 0.785  | 0.760                | 0.815                | —     | —                    | —                    |
| 2        | MS Heteroskedastic      | 0    | —                       | —           | —         | —  | —                   | —  | 60      | 0.441                  | 0.439                | 0.445                | 0.442        | 0.440                | 0.446                | 1.613  | 1.590                | 1.614                | 0.753 | 0.703                | 0.803                |
| 3        | MSI Homoskedastic       | 0    | —                       | —           | —         | —  | —                   | —  | 60      | 0.318                  | 0.309                | 0.332                | 0.319        | 0.310                | 0.332                | 1.831  | 1.733                | 1.950                | 0.613 | 0.562                | 0.662                |
| 4        | VAR Rolling             | 1    | Y                       | N           | N         | N  | N                   | N  | 60      | 0.277                  | 0.246                | 0.313                | 0.277        | 0.247                | 0.314                | 0.714  | 0.607                | 0.808                | 0.575 | 0.522                | 0.631                |
| 5        | VAR Rolling             | 1    | N                       | N           | N         | N  | N                   | Y  | 60      | 0.269                  | 0.227                | 0.309                | 0.270        | 0.228                | 0.310                | 0.854  | 0.735                | 0.962                | 0.503 | 0.448                | 0.554                |
| 6        | VAR Expanding           | 1    | N                       | N           | N         | N  | Y                   | Y  | 60      | 0.269                  | 0.239                | 0.298                | 0.270        | 0.240                | 0.299                | 0.794  | 0.709                | 0.900                | 0.516 | 0.463                | 0.566                |
| 7        | VAR Rolling             | 1    | N                       | N           | N         | N  | Y                   | Y  | 60      | 0.265                  | 0.231                | 0.298                | 0.265        | 0.232                | 0.299                | 0.846  | 0.734                | 0.952                | 0.503 | 0.446                | 0.559                |
| 8        | VAR Expanding           | 1    | N                       | N           | N         | N  | N                   | Y  | 60      | 0.269                  | 0.230                | 0.298                | 0.269        | 0.231                | 0.299                | 0.771  | 0.659                | 0.872                | 0.522 | 0.463                | 0.572                |
| 9        | VAR Rolling             | 1    | Y                       | N           | Y         | N  | N                   | N  | 60      | 0.266                  | 0.231                | 0.292                | 0.267        | 0.232                | 0.293                | 0.697  | 0.595                | 0.790                | 0.556 | 0.512                | 0.605                |
| 10       | VAR Rolling             | 1    | Y                       | N           | N         | N  | Y                   | N  | 60      | 0.265                  | 0.236                | 0.297                | 0.266        | 0.237                | 0.298                | 0.705  | 0.618                | 0.809                | 0.528 | 0.482                | 0.575                |
|          |                         |      |                         |             |           |    |                     |    | 60      | 0.158                  | 0.113                | 0.198                | 0.159        | 0.114                | 0.199                | 0.450  | 0.323                | 0.575                | 0.397 | 0.343                | 0.452                |
|          |                         |      |                         |             |           |    |                     |    | 60      | 0.185                  | 0.154                | 0.214                | 0.186        | 0.155                | 0.215                | 0.535  | 0.458                | 0.611                | 0.438 | 0.387                | 0.489                |
| 1        | MS Heteroskedastic      | 0    | —                       | —           | —         | —  | —                   | —  | 1       | 0.277                  | 0.136                | 0.259                | 0.339        | 0.294                | 0.680                | 0.380  | 0.179                | 0.382                | 0.668 | 0.619                | 0.714                |
| 2        | MSI Homoskedastic       | 0    | —                       | —           | —         | —  | —                   | —  | 1       | 0.208                  | 0.069                | 0.200                | 0.257        | 0.253                | 0.560                | 0.307  | 0.100                | 0.310                | 0.620 | 0.572                | 0.670                |
| 3        | IID (No predictability) | 0    | —                       | —           | —         | —  | —                   | —  | 1       | 0.094                  | -0.241               | 0.430                | 0.102        | -0.370               | 0.505                | -0.078 | -0.087               | 0.005                | —     | —                    | —                    |
| 4        | VAR Rolling             | 1    | N                       | Y           | Y         | N  | N                   | N  | 1       | -0.662                 | -0.656               | -0.673               | -0.404       | -0.795               | -0.188               | -0.862 | -0.842               | -1.014               | 0.148 | 0.107                | 0.194                |
| 5        | VAR Rolling             | 1    | N                       | Y           | N         | N  | N                   | N  | 1       | -0.786                 | -0.699               | -1.120               | -0.359       | -0.732               | -0.149               | -0.984 | -0.953               | -1.374               | 0.166 | 0.127                | 0.203                |
| 6        | VAR Rolling             | 1    | Y                       | Y           | N         | N  | N                   | N  | 1       | -0.788                 | -0.741               | -1.025               | -0.361       | -0.602               | -0.155               | -1.070 | -1.064               | -1.450               | 0.185 | 0.142                | 0.224                |
| 7        | VAR Rolling             | 1    | N                       | Y           | Y         | Y  | N                   | N  | 1       | -0.623                 | -0.652               | -0.503               | -0.467       | -0.872               | -0.255               | -0.808 | -0.778               | -0.821               | 0.132 | 0.094                | 0.163                |
| 8        | VAR Rolling             | 1    | Y                       | Y           | N         | Y  | N                   | N  | 1       | -0.721                 | -0.726               | -0.777               | -0.408       | -0.714               | -0.192               | -0.972 | -1.013               | -1.141               | 0.187 | 0.144                | 0.226                |
| 9        | VAR Rolling             | 1    | N                       | Y           | N         | Y  | N                   | N  | 1       | -0.721                 | -0.766               | -0.807               | -0.414       | -0.741               | -0.214               | -0.921 | -1.030               | -1.078               | 0.174 | 0.132                | 0.213                |
| 10       | VAR Rolling             | 1    | N                       | Y           | N         | N  | Y                   | N  | 1       | -0.688                 | -0.726               | -0.772               | -0.441       | -0.806               | -0.205               | -0.960 | -1.075               | -1.178               | 0.145 | 0.105                | 0.182                |
|          |                         |      |                         |             |           |    |                     |    | 1       | -0.670                 | -0.733               | -0.653               | -0.817       | -1.157               | -0.572               | -0.870 | -0.957               | -0.864               | 0.119 | 0.086                | 0.152                |
|          |                         |      |                         |             |           |    |                     |    | 1       | -0.565                 | -0.672               | -0.469               | -0.721       | -1.089               | -0.474               | -0.798 | -0.882               | -0.740               | 0.142 | 0.106                | 0.178                |

Table 6

Best Models Ranked According to Average Long-Horizon Recursive Certainty Equivalent Return Obtained from Optimal Strategic Asset Allocation Choices Under Power Utility Preferences ( $\gamma = 5$ ): No Transaction Costs

| CER Rank | Model                   | Lags | Predictors Included    |             |           |    |                     |    |         | Annualized Mean |                      |                      | Annualized Volatility |                      |                      | Sharpe Ratio |                      |                      | Annualized CER |                      |                      | Average turnover (adjusted) |          |          |
|----------|-------------------------|------|------------------------|-------------|-----------|----|---------------------|----|---------|-----------------|----------------------|----------------------|-----------------------|----------------------|----------------------|--------------|----------------------|----------------------|----------------|----------------------|----------------------|-----------------------------|----------|----------|
|          |                         |      | Git/Equity Yield Ratio | Term Spread | Inflation | IP | Oil Price Inflation | DY | Horizon | Mean returns    | 95% Conf. Int. -- LB | 95% Conf. Int. -- UB | Volatility            | 95% Conf. Int. -- LB | 95% Conf. Int. -- UB | Sharpe ratio | 95% Conf. Int. -- LB | 95% Conf. Int. -- UB | CER (% Ann.)   | 95% Conf. Int. -- LB | 95% Conf. Int. -- UB |                             | Skewness | Kurtosis |
| 1        | IID (No predictability) | 0    | —                      | —           | —         | —  | —                   | —  | 120     | 10.342          | 9.445                | 11.136               | 15.531                | 14.585               | 16.368               | 0.548        | 0.523                | 0.569                | 8.926          | 8.171                | 9.692                | 0.143                       | 1.631    | 0.195    |
| 2        | MS Heteroskedastic      | 0    | —                      | —           | —         | —  | —                   | —  | 120     | 7.721           | 5.304                | 10.183               | 49.359                | 42.856               | 55.425               | 0.119        | 0.081                | 0.151                | 5.406          | 5.249                | 5.577                | 1.998                       | 4.907    | 0.430    |
| 3        | MSI Homoskedastic       | 0    | —                      | —           | —         | —  | —                   | —  | 120     | 4.726           | 2.563                | 6.849                | 45.953                | 37.964               | 52.727               | 0.063        | 0.019                | 0.095                | 5.046          | 4.940                | 5.157                | 2.077                       | 5.096    | 0.942    |
| 4        | VAR Rolling             | 1    | N                      | N           | N         | N  | Y                   | N  | 120     | 16.492          | 14.807               | 18.075               | 36.831                | 34.636               | 39.058               | 0.398        | 0.375                | 0.416                | 0.835          | 0.767                | 1.447                | -0.069                      | 3.044    | 4.171    |
| 5        | VAR Rolling             | 1    | N                      | N           | N         | N  | N                   | N  | 120     | 16.902          | 15.055               | 18.374               | 34.968                | 32.198               | 37.501               | 0.431        | 0.411                | 0.441                | 0.833          | 0.766                | 1.380                | 0.002                       | 3.148    | 3.754    |
| 6        | VAR Expanding           | 1    | Y                      | Y           | N         | N  | Y                   | Y  | 120     | 16.826          | 14.767               | 19.101               | 41.979                | 38.988               | 45.219               | 0.357        | 0.332                | 0.382                | 0.826          | 0.762                | 1.237                | 0.317                       | 3.592    | 4.882    |
| 7        | VAR Rolling             | 1    | N                      | N           | N         | Y  | Y                   | N  | 120     | 15.844          | 14.018               | 17.676               | 37.459                | 34.903               | 39.448               | 0.374        | 0.349                | 0.402                | 0.804          | 0.747                | 1.399                | 0.161                       | 2.962    | 4.658    |
| 8        | VAR Rolling             | 1    | N                      | N           | N         | N  | N                   | Y  | 120     | 19.105          | 16.932               | 21.451               | 45.327                | 43.144               | 46.880               | 0.381        | 0.350                | 0.419                | 0.804          | 0.756                | 1.364                | -0.085                      | 2.415    | 4.065    |
| 9        | VAR Rolling             | 1    | N                      | N           | N         | Y  | N                   | N  | 120     | 16.371          | 14.654               | 18.199               | 36.093                | 33.711               | 38.500               | 0.403        | 0.381                | 0.425                | 0.803          | 0.747                | 1.266                | 0.079                       | 3.030    | 4.290    |
| 10       | VAR Rolling             | 1    | N                      | N           | Y         | Y  | N                   | N  | 120     | 16.768          | 14.947               | 18.601               | 37.615                | 35.071               | 39.931               | 0.397        | 0.374                | 0.420                | 0.802          | 0.739                | 1.237                | 0.076                       | 2.979    | 4.724    |
|          |                         |      |                        |             |           |    |                     |    | 120     | 15.392          | 13.156               | 17.565               | 44.423                | 42.095               | 46.500               | 0.299        | 0.283                | 0.370                | 0.726          | 0.698                | 0.783                | 0.083                       | 2.575    | 6.260    |
|          |                         |      |                        |             |           |    |                     |    | 120     | 15.530          | 13.268               | 17.616               | 44.392                | 42.082               | 46.476               | 0.311        | 0.284                | 0.354                | 0.728          | 0.699                | 0.788                | 0.081                       | 2.583    | 6.134    |
| 1        | MS Heteroskedastic      | 0    | —                      | —           | —         | —  | —                   | —  | 60      | 23.778          | 21.321               | 26.297               | 51.260                | 45.616               | 56.720               | 0.428        | 0.427                | 0.431                | 12.458         | 11.654               | 13.369               | 0.470                       | 4.782    | 0.594    |
| 2        | MSI Homoskedastic       | 0    | —                      | —           | —         | —  | —                   | —  | 60      | 27.873          | 23.919               | 32.015               | 84.033                | 73.302               | 93.602               | 0.310        | 0.301                | 0.323                | 10.528         | 9.821                | 11.291               | 0.686                       | 4.793    | 0.958    |
| 3        | IID (No predictability) | 0    | —                      | —           | —         | —  | —                   | —  | 60      | 8.999           | 8.530                | 9.470                | 9.593                 | 9.047                | 10.105               | 0.748        | 0.741                | 0.757                | 9.935          | 9.489                | 10.414               | -0.823                      | 1.856    | 0.036    |
| 4        | VAR Rolling             | 1    | N                      | N           | N         | N  | Y                   | N  | 60      | 16.144          | 13.519               | 18.863               | 54.512                | 49.169               | 59.176               | 0.263        | 0.238                | 0.287                | 6.476          | 6.187                | 8.306                | 0.124                       | 3.970    | 5.379    |
| 5        | VAR Rolling             | 1    | N                      | N           | N         | N  | N                   | N  | 60      | 15.368          | 13.007               | 17.756               | 48.979                | 44.033               | 53.374               | 0.277        | 0.254                | 0.298                | 6.401          | 6.082                | 8.240                | 0.109                       | 3.948    | 4.986    |
| 6        | VAR Rolling             | 1    | N                      | N           | N         | N  | N                   | Y  | 60      | 21.594          | 18.191               | 25.162               | 71.954                | 67.210               | 76.045               | 0.275        | 0.244                | 0.307                | 5.839          | 5.639                | 9.083                | -0.088                      | 2.512    | 5.332    |
| 7        | VAR Rolling             | 1    | N                      | N           | Y         | Y  | N                   | N  | 60      | 15.039          | 12.433               | 17.685               | 52.748                | 47.588               | 57.397               | 0.251        | 0.223                | 0.276                | 5.825          | 5.633                | 8.105                | 0.025                       | 3.881    | 6.068    |
| 8        | VAR Rolling             | 1    | N                      | N           | N         | Y  | N                   | N  | 60      | 15.007          | 12.573               | 17.478               | 50.164                | 45.046               | 54.765               | 0.263        | 0.239                | 0.286                | 5.803          | 5.621                | 7.668                | -0.012                      | 3.866    | 5.654    |
| 9        | VAR Rolling             | 1    | Y                      | N           | N         | N  | N                   | N  | 60      | 19.704          | 16.736               | 22.724               | 61.800                | 57.007               | 66.163               | 0.289        | 0.262                | 0.316                | 5.787          | 5.611                | 7.041                | -0.154                      | 2.837    | 5.137    |
| 10       | VAR Rolling             | 1    | N                      | Y           | N         | N  | Y                   | N  | 60      | 16.282          | 13.571               | 19.071               | 55.990                | 50.665               | 60.835               | 0.258        | 0.232                | 0.284                | 5.782          | 5.611                | 6.525                | 0.173                       | 4.129    | 5.471    |
|          |                         |      |                        |             |           |    |                     |    | 60      | 14.45016        | 11.248278            | 17.751575            | 63.475                | 57.620               | 68.567               | 0.206        | 0.174                | 0.244                | 5.429          | 5.386                | 5.501                | 0.148                       | 3.403    | 9.574    |
|          |                         |      |                        |             |           |    |                     |    | 60      | 15.84432        | 13.074664            | 18.744175            | 67.416                | 61.935               | 72.349               | 0.223        | 0.177                | 0.237                | 5.476          | 5.420                | 5.582                | -0.075                      | 2.806    | 8.127    |
| 1        | IID (No predictability) | 0    | —                      | —           | —         | —  | —                   | —  | 1       | 5.624           | 2.312                | 8.832                | 9.335                 | 8.330                | 10.349               | 0.407        | 0.059                | 0.677                | 5.401          | 1.972                | 8.717                | -0.901                      | 6.869    | 0.214    |
| 2        | MS Heteroskedastic      | 0    | —                      | —           | —         | —  | —                   | —  | 1       | 72.746          | 55.876               | 89.285               | 47.946                | 43.331               | 52.455               | 1.479        | 1.247                | 1.667                | 2.160          | -31.980              | 34.563               | -0.501                      | 4.307    | 0.688    |
| 3        | VAR Rolling             | 2    | Y                      | Y           | N         | N  | N                   | N  | 1       | 51.466          | 25.985               | 79.301               | 76.345                | 55.865               | 98.969               | 0.650        | 0.432                | 0.783                | 1.259          | -1.699               | 4.196                | 3.677                       | 24.272   | 5.329    |
| 4        | VAR Rolling             | 2    | Y                      | N           | N         | N  | N                   | N  | 1       | 46.732          | 23.687               | 71.227               | 67.932                | 53.451               | 82.935               | 0.661        | 0.409                | 0.837                | 0.673          | -2.855               | 4.209                | 2.474                       | 23.635   | 5.054    |
| 5        | VAR Rolling             | 2    | N                      | N           | N         | Y  | N                   | Y  | 1       | 34.093          | 11.452               | 58.700               | 68.220                | 52.048               | 85.022               | 0.473        | 0.185                | 0.669                | -1.414         | -5.657               | 2.952                | 3.757                       | 29.862   | 6.029    |
| 6        | MSI Homoskedastic       | 0    | —                      | —           | —         | —  | —                   | —  | 1       | 89.430          | 62.938               | 116.090              | 76.725                | 70.222               | 82.810               | 1.142        | 0.870                | 1.380                | -2.360         | -6.039               | 1.761                | -0.207                      | 3.466    | 1.723    |
| 7        | VAR Rolling             | 2    | N                      | Y           | N         | Y  | N                   | N  | 1       | 38.970          | 15.643               | 64.946               | 71.005                | 50.112               | 93.065               | 0.523        | 0.276                | 0.678                | -2.429         | -10.054              | 4.382                | 2.164                       | 24.371   | 5.813    |
| 8        | VAR Rolling             | 2    | Y                      | Y           | N         | Y  | N                   | N  | 1       | 51.200          | 24.716               | 79.740               | 78.513                | 59.109               | 99.360               | 0.629        | 0.387                | 0.784                | -3.701         | -11.193              | 3.177                | 2.413                       | 17.348   | 6.909    |
| 9        | VAR Rolling             | 2    | N                      | N           | N         | Y  | N                   | N  | 1       | 37.987          | 16.444               | 61.601               | 65.620                | 47.113               | 83.876               | 0.551        | 0.310                | 0.713                | -3.763         | -13.061              | 4.828                | 1.464                       | 10.326   | 5.447    |
| 10       | VAR Rolling             | 2    | Y                      | N           | N         | Y  | N                   | N  | 1       | 46.755          | 22.445               | 72.133               | 71.452                | 57.373               | 86.130               | 0.629        | 0.359                | 0.816                | -3.956         | -9.966               | 2.473                | 1.754                       | 17.708   | 6.069    |
|          |                         |      |                        |             |           |    |                     |    | 1       | 49.574          | 26.679               | 73.741               | 67.028                | 49.898               | 83.076               | 0.773        | 0.505                | 0.930                | -89.588        | -90.647              | 6.537                | 2.931                       | 26.026   | 6.020    |
|          |                         |      |                        |             |           |    |                     |    | 1       | 37.558          | 14.734               | 62.054               | 68.188                | 50.172               | 85.050               | 0.519        | 0.254                | 0.775                | -88.259        | -89.637              | 4.080                | 2.005                       | 22.630   | 5.365    |



Table 7

Best Models Ranked According to Average Long-Horizon Recursive Certainty Equivalent Return Obtained from Optimal Strategic Asset Allocation Choices Under Power Utility Preferences ( $\gamma = 5$ ): Higher Fixed Transaction Costs

| CER Rank | Model                   | Lags | Predictors Included     |             |           |    |                     |    |              | Horizon | Annualized Mean      |                      |            | Annualized Volatility |                      |              | Sharpe Ratio         |                      |              | Annualized CER       |                      |        | Skewness | Kurtosis | Average turnover (adjusted) |
|----------|-------------------------|------|-------------------------|-------------|-----------|----|---------------------|----|--------------|---------|----------------------|----------------------|------------|-----------------------|----------------------|--------------|----------------------|----------------------|--------------|----------------------|----------------------|--------|----------|----------|-----------------------------|
|          |                         |      | Gilt/Equity Yield Ratio | Term Spread | Inflation | IP | Oil Price Inflation | DY | Mean returns |         | 95% Conf. Int. -- LB | 95% Conf. Int. -- UB | Volatility | 95% Conf. Int. -- LB  | 95% Conf. Int. -- UB | Sharpe ratio | 95% Conf. Int. -- LB | 95% Conf. Int. -- UB | CER (% Ann.) | 95% Conf. Int. -- LB | 95% Conf. Int. -- UB |        |          |          |                             |
| 1        | MS Heteroskedastic      | 0    | —                       | —           | —         | —  | —                   | —  | 120          | 9.754   | 3.853                | 7.513                | 37.575     | 25.171                | 32.211               | 0.211        | 0.081                | 0.177                | 5.021        | 3.740                | 4.067                | 1.544  | 3.540    | 0.324    |                             |
| 2        | MSI Homoskedastic       | 0    | —                       | —           | —         | —  | —                   | —  | 120          | 5.853   | 1.755                | 4.967                | 31.099     | 22.237                | 30.436               | 0.130        | -0.003               | 0.103                | 4.720        | 3.436                | 3.650                | 1.929  | 5.299    | 0.482    |                             |
| 3        | IID (No predictability) | 0    | —                       | —           | —         | —  | —                   | —  | 120          | 8.718   | 8.266                | 8.819                | 11.844     | 9.022                 | 10.846               | 0.582        | 0.714                | 0.645                | 0.402        | 18.859               | 20.052               | -1.025 | 5.343    | 0.014    |                             |
| 4        | VAR Rolling             | 1    | 0                       | 0           | 0         | 0  | 1                   | 0  | 120          | 12.859  | 8.101                | 10.340               | 38.262     | 33.792                | 38.563               | 0.288        | 0.186                | 0.221                | -1.599       | -1.744               | -1.183               | -0.176 | 3.793    | 2.404    |                             |
| 5        | VAR Expanding           | 1    | 1                       | 1           | 1         | 0  | 0                   | 1  | 120          | 12.317  | 8.044                | 10.618               | 41.447     | 36.965                | 42.934               | 0.253        | 0.168                | 0.205                | -1.613       | -1.763               | -1.599               | -0.005 | 4.078    | 2.794    |                             |
| 6        | VAR Expanding           | 1    | 0                       | 1           | 1         | 1  | 1                   | 1  | 120          | 12.041  | 7.234                | 9.639                | 42.174     | 37.463                | 44.107               | 0.242        | 0.144                | 0.177                | -1.618       | -1.779               | -1.630               | 0.079  | 3.988    | 2.981    |                             |
| 7        | VAR Rolling             | 1    | 0                       | 0           | 0         | 1  | 0                   | 0  | 120          | 12.692  | 8.066                | 10.244               | 34.779     | 33.400                | 38.316               | 0.313        | 0.187                | 0.220                | -1.662       | -1.753               | -1.421               | -0.038 | 3.940    | 2.417    |                             |
| 8        | VAR Expanding           | 1    | 0                       | 1           | 1         | 0  | 0                   | 1  | 120          | 11.719  | 7.292                | 9.670                | 41.139     | 36.021                | 42.920               | 0.241        | 0.152                | 0.183                | -1.663       | -1.775               | -1.610               | 0.138  | 4.193    | 2.683    |                             |
| 9        | VAR Expanding           | 1    | 0                       | 1           | 0         | 1  | 0                   | 1  | 120          | 12.198  | 8.548                | 11.192               | 41.888     | 38.576                | 43.845               | 0.248        | 0.174                | 0.214                | -1.663       | -1.774               | -1.600               | 0.053  | 3.877    | 2.921    |                             |
| 10       | VAR Rolling             | 1    | 0                       | 0           | 0         | 1  | 0                   | 0  | 120          | 12.397  | 6.989                | 9.428                | 42.564     | 37.835                | 42.271               | 0.248        | 0.137                | 0.180                | -1.664       | -1.780               | -1.663               | -0.211 | 3.576    | 3.449    |                             |
|          |                         |      |                         |             |           |    |                     |    | 120          | 11.045  | 6.079                | 8.470                | 42.452     | 37.839                | 43.093               | 0.217        | 0.112                | 0.154                | -1.747       | -1.802               | -1.729               | 0.012  | 3.687    | 4.103    |                             |
|          |                         |      |                         |             |           |    |                     |    | 120          | 12.091  | 7.328                | 9.915                | 43.107     | 40.757                | 44.696               | 0.238        | 0.135                | 0.181                | -1.753       | -1.798               | -1.723               | -0.173 | 3.576    | 3.360    |                             |
| 1        | MS Heteroskedastic      | 0    | —                       | —           | —         | —  | —                   | —  | 1            | 18.226  | 9.344                | 17.725               | 29.626     | 25.623                | 30.919               | 0.554        | 0.293                | 0.514                | 3.217        | -3.650               | 1.827                | 0.297  | 4.660    | 0.449    |                             |
| 2        | IID (No predictability) | 0    | —                       | —           | —         | —  | —                   | —  | 1            | 2.816   | -1.751               | 3.104                | 9.099      | 8.087                 | 10.068               | 0.109        | -0.442               | 0.127                | 3.131        | 0.628                | 1.170                | -0.097 | 4.190    | 0.030    |                             |
| 3        | MSI Homoskedastic       | 0    | —                       | —           | —         | —  | —                   | —  | 1            | 22.823  | 9.504                | 22.717               | 44.159     | 41.085                | 48.401               | 0.476        | 0.187                | 0.432                | 1.784        | -6.244               | 9.105                | 0.134  | 3.426    | 0.728    |                             |
| 4        | VAR Expanding           | 1    | 1                       | 1           | 0         | 1  | 0                   | 1  | 1            | 33.609  | -43.514              | -33.216              | 73.891     | 12.981                | 17.677               | 0.430        | -3.493               | -1.982               | 0.485        | -28.970              | 28.442               | 4.874  | 14.892   | 3.746    |                             |
| 5        | VAR Expanding           | 1    | 0                       | 0           | 1         | 0  | 1                   | 0  | 1            | -2.223  | -41.609              | -30.917              | 41.068     | 13.894                | 19.023               | -0.099       | -3.126               | -1.721               | 0.111        | -29.142              | 28.011               | 8.243  | 48.797   | 3.004    |                             |
| 6        | VAR Expanding           | 1    | 1                       | 1           | 0         | 0  | 1                   | 1  | 1            | 12.756  | -42.648              | -32.237              | 61.123     | 13.102                | 18.910               | 0.179        | -3.394               | -1.801               | -0.050       | -29.326              | 27.741               | 6.473  | 25.837   | 3.039    |                             |
| 7        | VAR Expanding           | 1    | 0                       | 0           | 0         | 0  | 0                   | 0  | 1            | -8.784  | -40.621              | -29.549              | 21.379     | 12.303                | 17.933               | -0.496       | -3.450               | -1.749               | -1.395       | -30.146              | 26.798               | 3.212  | 12.443   | 3.316    |                             |
| 8        | VAR Expanding           | 1    | 0                       | 0           | 0         | 1  | 0                   | 0  | 1            | 12.312  | -53.717              | -43.409              | 63.099     | 15.628                | 20.106               | 0.166        | -3.554               | -2.250               | -5.143       | -32.645              | 20.546               | 5.919  | 22.864   | 4.472    |                             |
| 9        | VAR Rolling             | 1    | 0                       | 0           | 0         | 0  | 0                   | 0  | 1            | 9.827   | -61.436              | -47.153              | 54.036     | 18.240                | 22.798               | 0.148        | -3.468               | -2.148               | -7.436       | -21.509              | 4.943                | 4.855  | 19.084   | 5.256    |                             |
| 10       | VAR Rolling             | 1    | 0                       | 0           | 1         | 1  | 0                   | 0  | 1            | -4.829  | -43.286              | -30.403              | 36.429     | 14.626                | 20.527               | -0.183       | -3.084               | -1.570               | -7.516       | -34.227              | 19.962               | 5.929  | 26.009   | 2.797    |                             |
|          |                         |      |                         |             |           |    |                     |    | 1            | -6.876  | -57.015              | -43.620              | 43.087     | 16.515                | 21.711               | -0.202       | -3.563               | -2.093               | -71.588      | -71.941              | -46.204              | 5.619  | 24.817   | 4.711    |                             |
|          |                         |      |                         |             |           |    |                     |    | 1            | -4.831  | -56.567              | -41.679              | 47.777     | 17.341                | 24.138               | -0.139       | -3.367               | -1.802               | -48.657      | -54.878              | -33.530              | 5.376  | 22.823   | 4.308    |                             |



Figure 1

Smoothed Regime Probabilities from Three-State Heteroskedastic Markov Switching Model

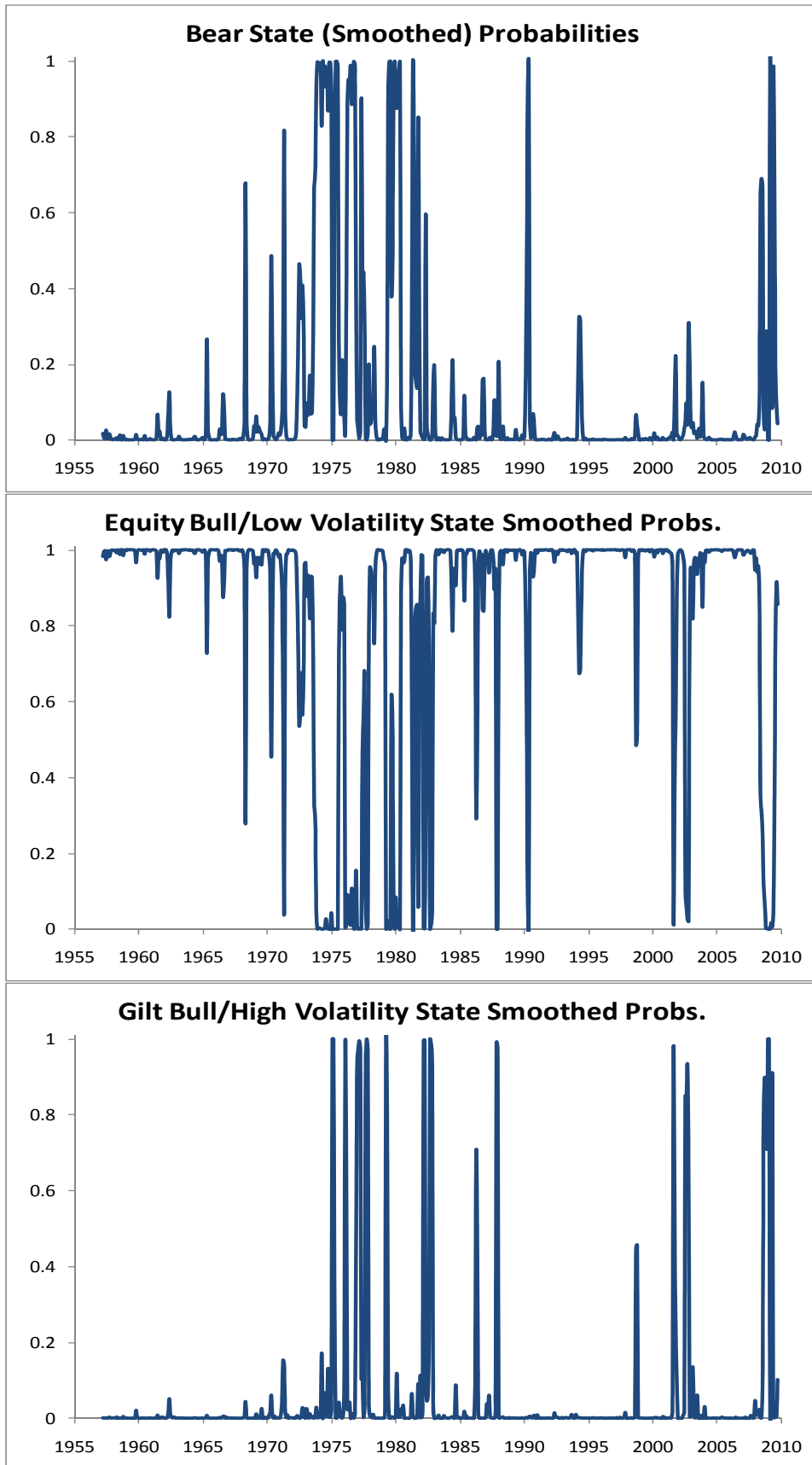


Figure 2

Recursive Expanding Window Estimates and 90% Confidence Intervals for Coefficients Associated to Predictive Variables in a Full-VAR(1) Model

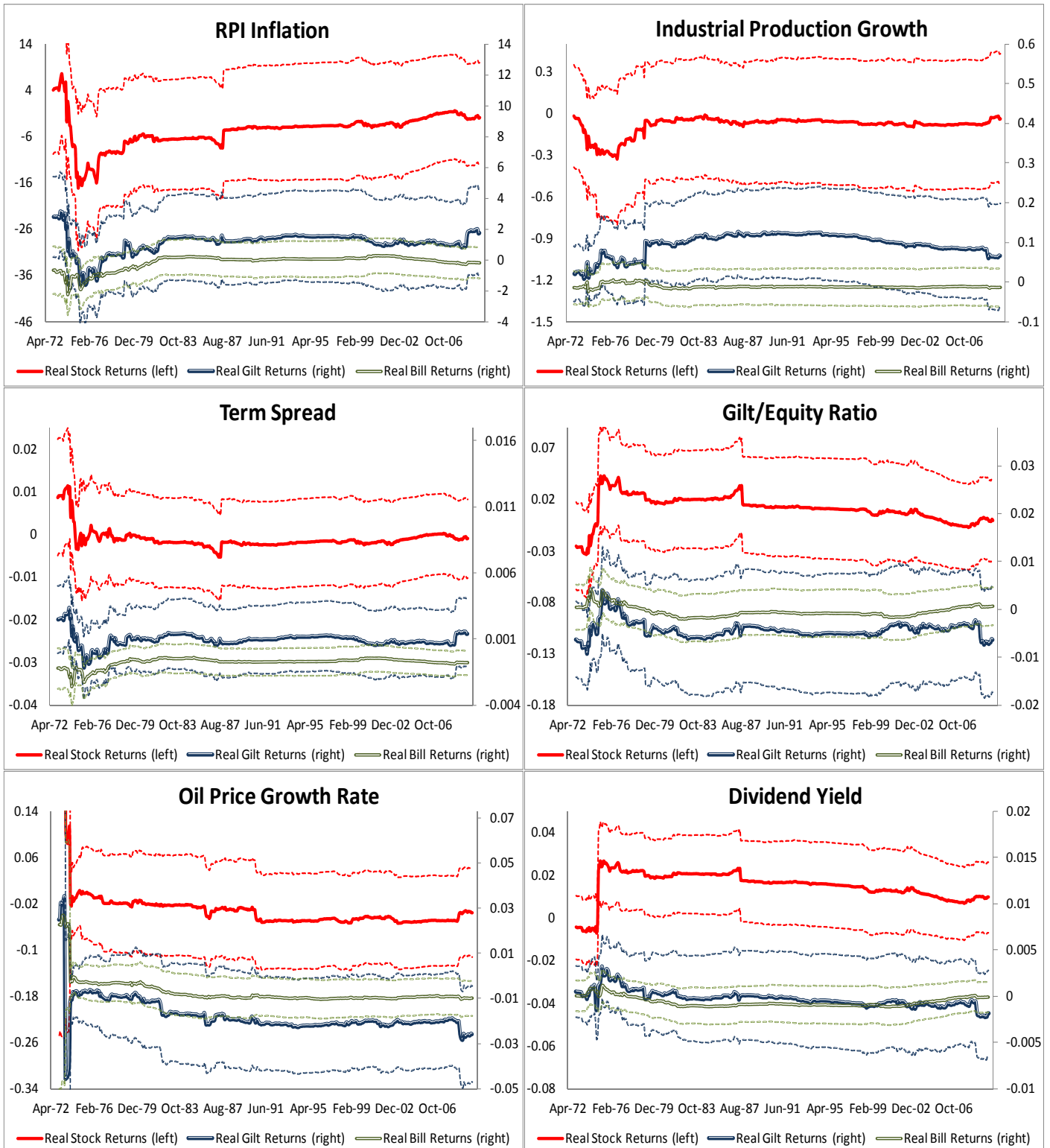


Figure 3

Recursive Rolling Window Estimates and 90% Confidence Intervals for Coefficients in a VAR(1) Model with Oil Log-Price Growth Predictor

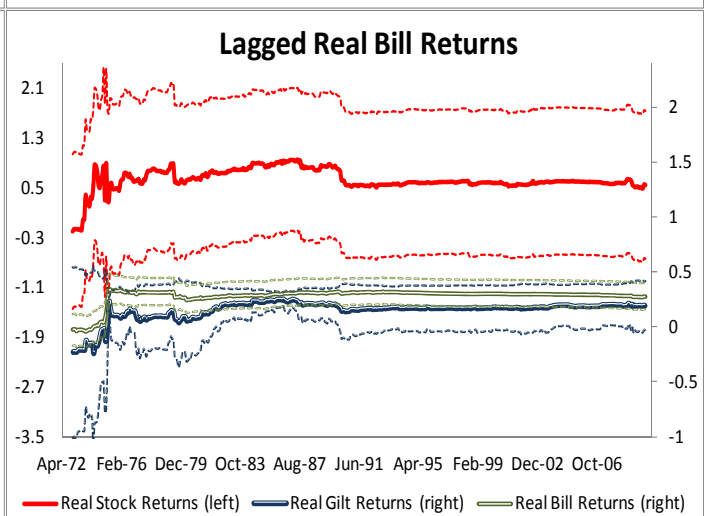
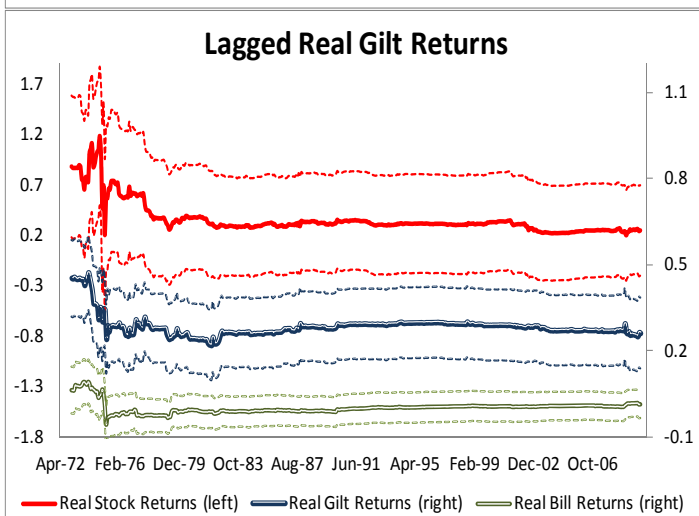
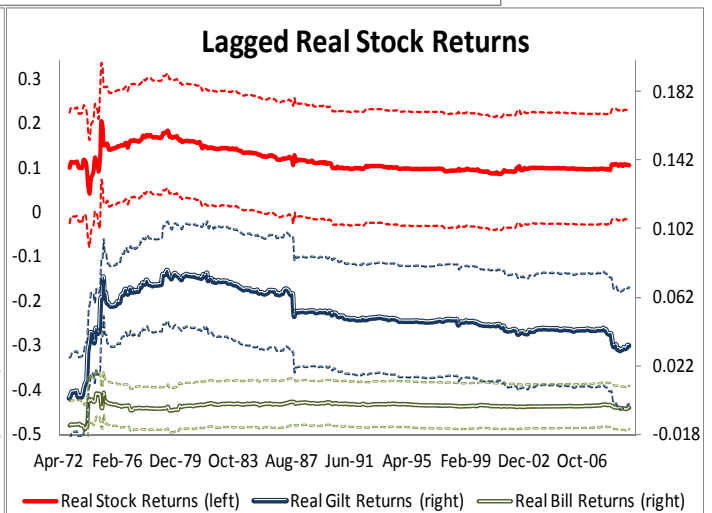
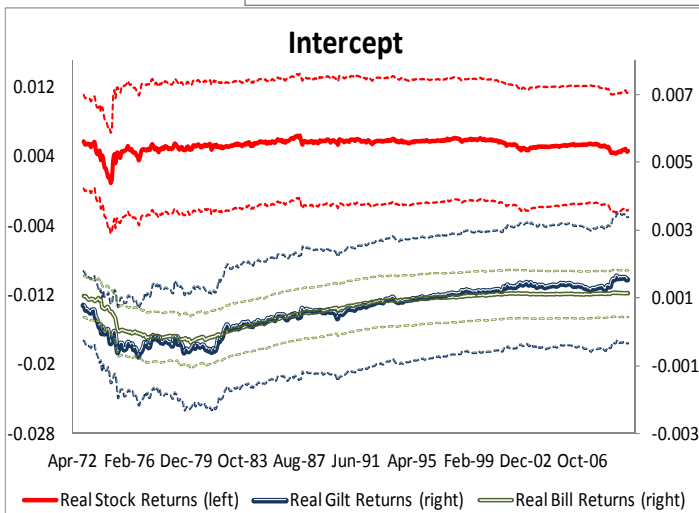
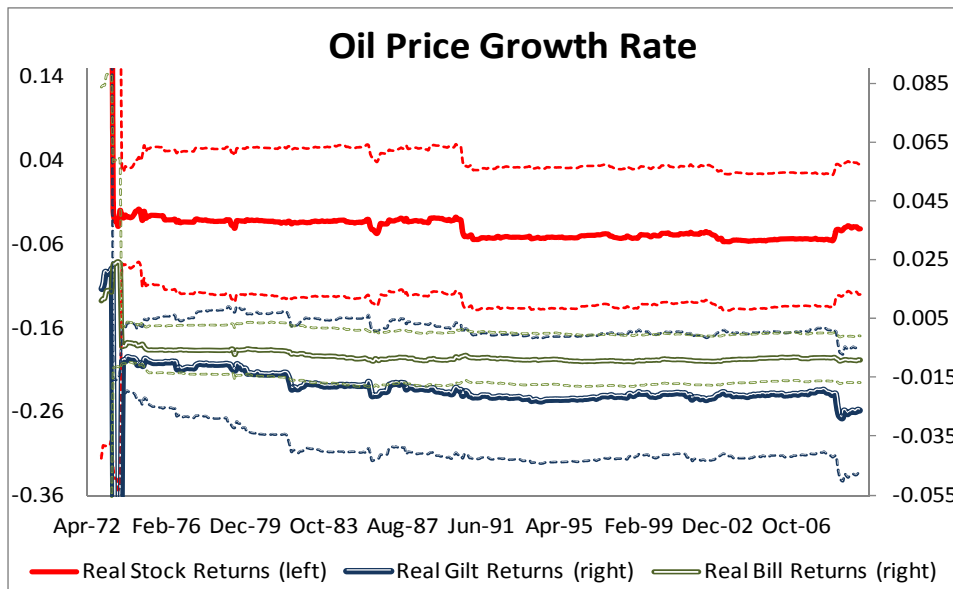


Figure 4

Recursive Optimal Weights under Three-State Heteroskedastic Markov Switching ( $\gamma = 5$ )

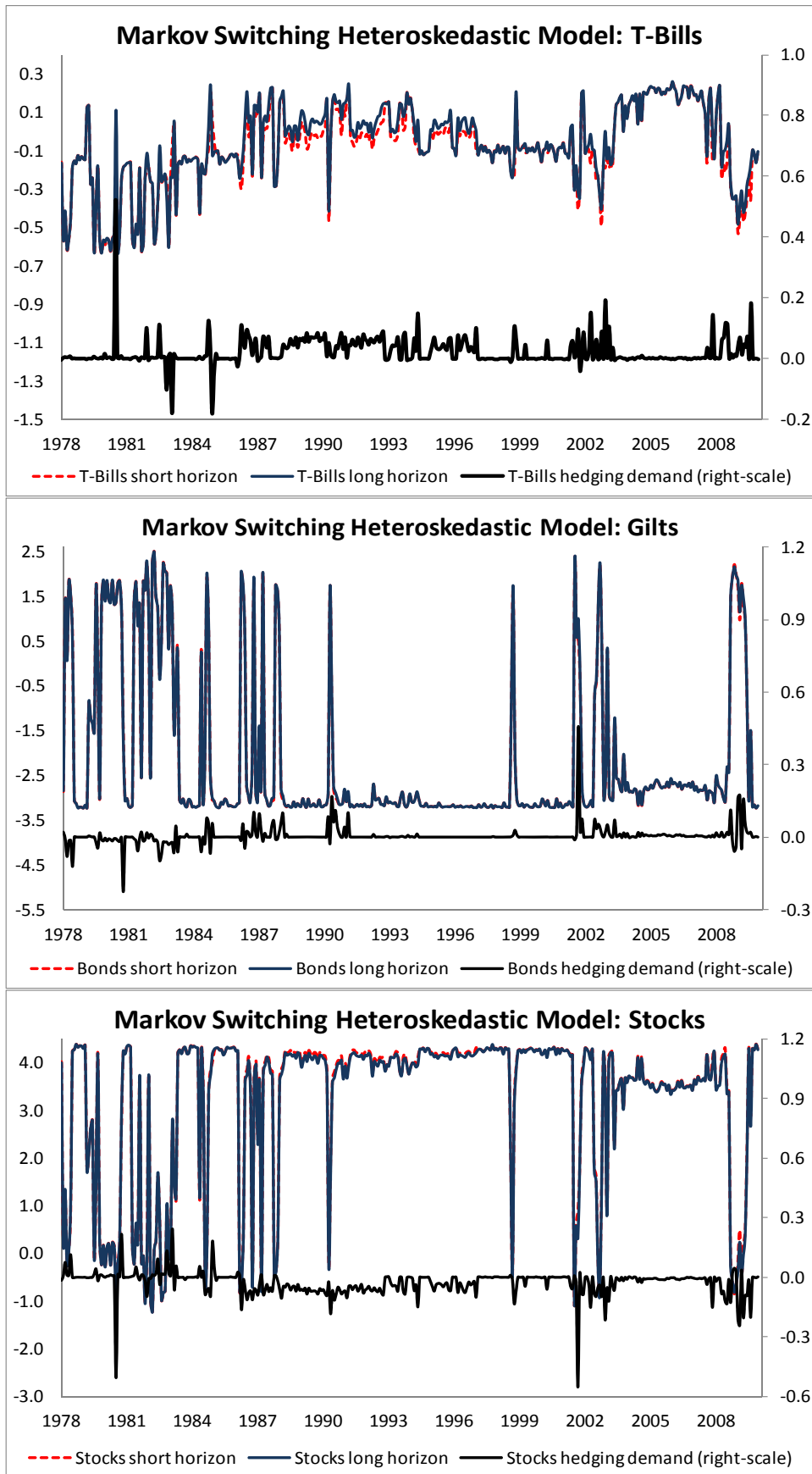


Figure 5

Recursive Optimal Weights under Two Representative Vector Autoregressive Models ( $\gamma = 5$ )

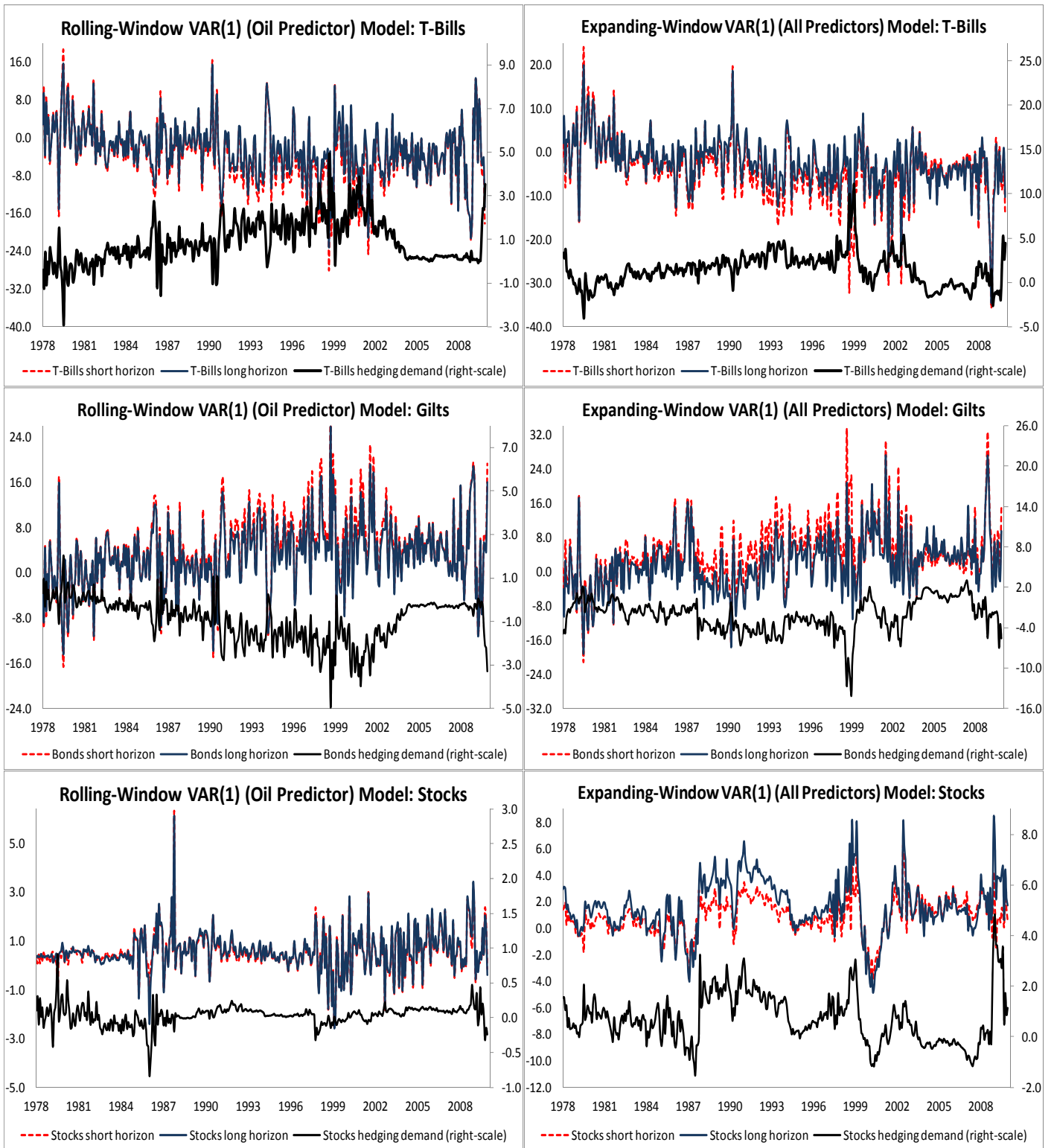


Figure 6

Recursive Optimal Portfolio Weights with and without Transaction Costs: Comparing the Best Performing VAR vs. Markov Switching Models ( $\gamma = 5$ )

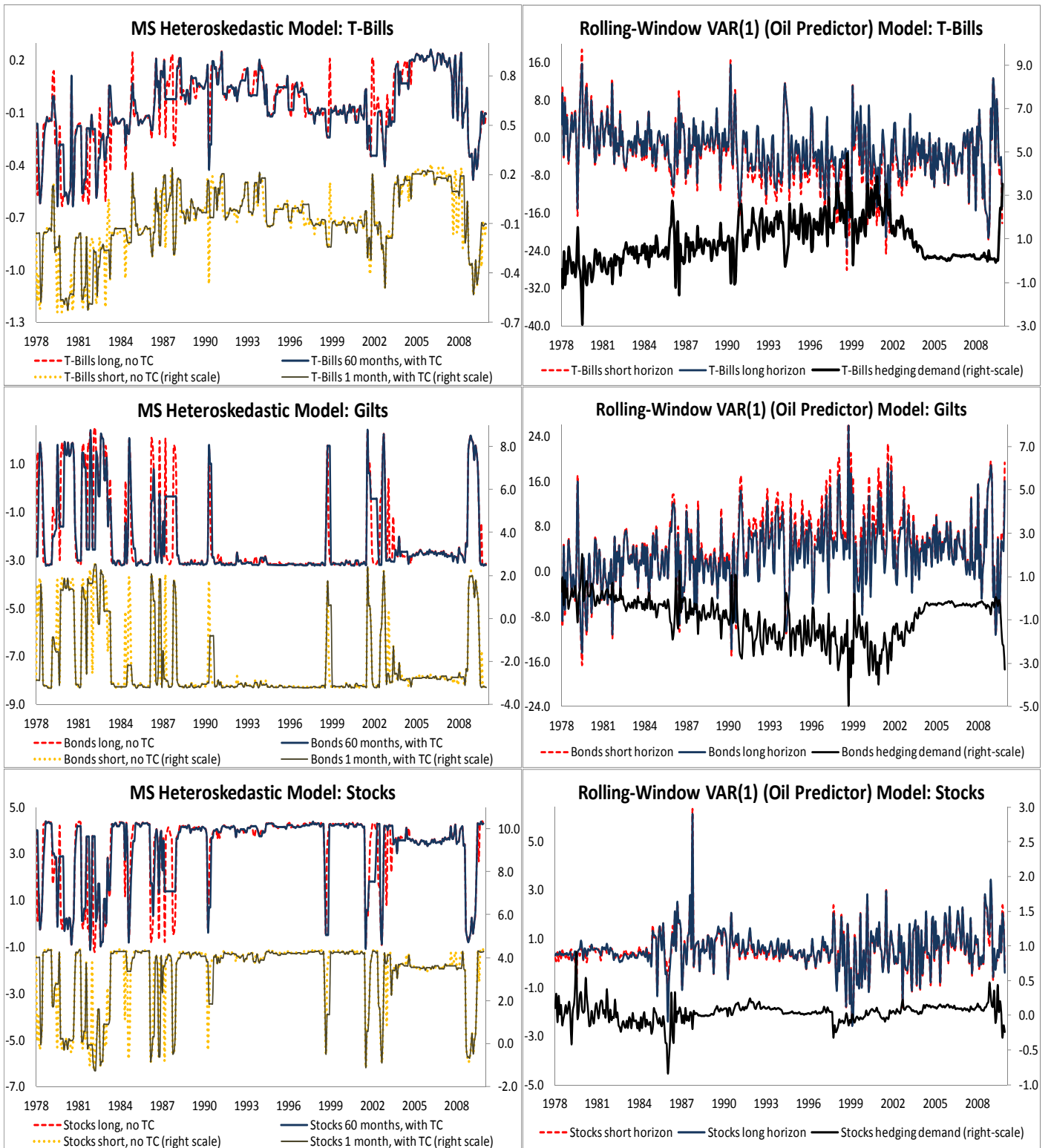




Figure 7

Recursive Optimal Portfolio Weights under Three-State Homoskedastic Markov Switching vs. a No Predictability Gaussian IID Benchmark ( $\gamma = 5$ )

