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Bernanke/Blinder revisited - The New Keynesian model with credit channel


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bernanke/blinder revisited – the new keynesian model with credit channel

by Sven Offick and Hans-Werner Wohltmann

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Bernanke/Blinder revisited – The New Keynesian model with credit channel

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September 10, 2014

Abstract

This paper integrates a money and credit market into a static approximation of the baseline New Keynesian model based on a money-and-credit-in-the-utility approach, in which real balances and borrowing contribute to the household’s utility. In this framework, the central bank has no direct control over the interest rate on bonds. Instead, the central bank’s instrument variables are the monetary base and the refinancing rate, i.e. the rate at which the central bank provides loans to the banking sector. Our approach gives rise to a credit channel, in which current and expected future interest rates on the bond and loan market directly affect current goods demand. The credit channel amplifies the output effects of isolated monetary disturbances. Taking changes in private (inflation and interest rate) expectations into account, we find that – contrarily to Bernanke and Blinder (1988) – the credit channel may also dampen the output effects of monetary disturbances.

JEL classification: A20, E51, E52

Keywords: Money, Loan, Money-and-credit-in-the-utility, Credit channel, New Keynesian model, Monetary policy

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1 Introduction

The baseline model of New Keynesian Macroeconomics (NKM) – also known as Macroeconomics without LM curve\(^1\) – has several drawbacks that we aim to address in this paper: First, money does not play an explicit role.\(^2\) Second, the interest rate on bonds (henceforth: the bond rate) is directly controlled by the central bank and is the only monetary policy instrument. These frameworks are not able to replicate the recent global increase in central bank money (i.e. in the monetary base) that was associated with the asset-purchasing programs of several central banks.\(^3\) Third, the existing NKM framework does not provide a link between money growth and inflation. Consequently, this framework contradicts the monetaristic view “that in the long run inflation is a monetary phenomenon” (M. Friedman, 1968).

To address these issues, this paper integrates a money and loan market into an otherwise standard NKM framework. The demand side of the money and loan market follows from a money-and-credit-in-the-utility approach with non-separable preferences, where real balances and borrowing contribute to the household’s utility. This approach is an extension of the money-in-the-utility approach discussed in Galí (2008). To the best of our knowledge, we are the first to introduce a fully microfounded loan demand from the household’s optimization problem.\(^4\) The supply side of the money and loan market is deduced from simplified balance sheets of the banking system and heavily builds on Jarchow (2010).

In this framework, the central bank has no direct control over the bond rate, which is now determined by the money market. Instead, the central bank’s instrument variables are the monetary base and the refinancing rate, i.e. the rate at which the central bank provides loans to the banking sector.\(^5\) Thereby, the central bank is able to affect the economy only indirectly through its influence on the money and loan market. We find that changes in the refinancing rate and in the monetary base only lead to less than proportional changes in the bond rate and in broad money, respectively.

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1 See Romer (2010).
2 If introduced at all, money is typically introduced as residual, which does not feedback into the economy.
3 In particular, these programs were exercised by the Federal Reserve, Bank of England, Bank of Japan, and the European Central Bank.
4 Alternatively, credit is introduced by the firm’s demand for loans to (partly) finance its capital stock as in Bernanke, Gertler and Gilchrist (1999) or to prefinance wages as in De Fiore and Tristani (2013). In Cúrdia and Woodford (2010), a credit market arises from two types of households (borrowers and savers) that differ with respect to their time preference to consume.
5 The refinancing rate is either set discretionarily or follows a Taylor-type interest rule.
In the spirit of Bernanke and Blinder (1988), the money-and-credit-in-the-utility approach implicitly gives rise to a credit channel as an additional transmission mechanism of monetary disturbances. Through this channel, current and expected future interest rates on the bond and loan market directly affect current goods demand. Taking changes in private (inflation and interest rate) expectations into account, we find that – contrarily to Bernanke and Blinder (1988) – the credit channel may also dampen the output effects of monetary disturbances.

Following Bofinger, Mayer and Wollmershäuser (2006), we use a static approximation of our final model equations to study the comparative-static effects of monetary and non-monetary disturbances. In particular, we focus on the impacts of an increase in the monetary base and in the central bank’s inflation target on output, inflation and the money stock.

The remainder of the paper is organized as follows: Section 2 introduces and develops our model framework. Section 3 studies the impacts of monetary and non-monetary disturbances where the credit channel is turned off. Section 4 studies the amplification effect of the credit channel. The last section concludes.

2 The model

2.1 Money and loan supply

The money and loan supply functions can be deduced from simplified balance sheets of the banking system, including a central bank, a commercial bank, and a consolidated balance sheet, which are given in table 1. The notation is as follows: $\tilde{W} =$ net foreign assets, $O =$ claims on government, $F =$ loans to commercial banks, $\tilde{C} =$ currency in circulation, $Z =$ minimum reserves, $U =$ excess reserves, $B =$ government bond holding, $K =$ credits to the non-commercial banking sector, $D =$ deposits of private non-banking sector.

The central bank lends money to the government ($O$) and to the commercial banks.
Table 1: Simplified balance sheets

<table>
<thead>
<tr>
<th>Central Bank Assets</th>
<th>Central Bank Liabilities</th>
<th>Commercial Bank Assets</th>
<th>Commercial Bank Liabilities</th>
<th>Consolidated Balance Assets</th>
<th>Consolidated Balance Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tilde{W} )</td>
<td>( \tilde{C} )</td>
<td>( Z )</td>
<td>( D )</td>
<td>( \tilde{W} )</td>
<td>( \tilde{C} )</td>
</tr>
<tr>
<td>( O )</td>
<td>( Z )</td>
<td>( U )</td>
<td>( F )</td>
<td>( O )</td>
<td>( D )</td>
</tr>
<tr>
<td>( F )</td>
<td>( U )</td>
<td>( B )</td>
<td>( K )</td>
<td>( B )</td>
<td>( K )</td>
</tr>
</tbody>
</table>

The commercial banks hold minimum reserves (\( Z \)), disposable excess reserves (\( U \)), and government bonds (\( B \)) bearing an interest rate \( i_B \). They provide loans (\( K \)) to the private non-banking sector at the loan rate \( i_L \). Loans typically face a higher risk of default than bonds. Bank lending and bond holding are, therefore, imperfect substitutes implying \( i_B \neq i_L \). The deposits on the commercial bank’s liability side only contain non-interest-bearing sight deposits of the non-banking sector. That is, we use a narrow money aggregate \( (M = \tilde{C} + D) \).

For the minimum reserves (\( Z \)) and the currency holding of the non-banking sector (\( \tilde{C} \)), we assume \( Z = rD \) (\( 0 < r < 1 \)) and \( \tilde{C} = qD \) (\( 0 < q < 1 \)) to hold. \( r \) is the minimum reserve rate and \( q \) is the currency-holding rate of the non-banking sector. The adjusted (or exogenous) monetary base (\( B_{m}^{x} \)) follows from the central bank’s balance sheet:

\[
B_{m}^{x} = \tilde{C} + Z + U - F = \tilde{W} + O
\]  

(1)

In addition, the following balance sheet constraints hold:

\[
B + K = (1 - r)D + F - U \tag{2}
\]

\[
B_{m}^{x} + B + K = M \tag{3}
\]

Following Bernanke and Blinder (1988), we express the loan supply (\( K^{x} \)) as a proportional function of the commercial bank’s available funds \( (1 - r)D \):

\[
K^{x} = a^{(+)}(i_{L}, i_{B}, z, \varepsilon) (1 - r)D \tag{4}
\]

\( z \) is the refinancing rate set by the central bank for loans provided to the commercial banks. The loan supply coefficient \( a(\cdot) \) depends positively on the loan rate \( (i_{L}) \), negatively
on the bond rate \((i_B)\), capturing the opportunity costs of loan supply, and negatively on the refinancing rate \((z)\). We also include a credit shock \((\varepsilon_K)\), which may terminate relation (4) and allows us to study the effects of monetary disturbances in a model with unstable credit and money supply multipliers.

Analogously to the loan supply, the bond demand can be expressed as

\[
B^d = b(i_B, i_L, z)(1 - r)D
\]  

such that the commercial bank’s net liability position at the central bank is given by

\[F - U = |a(\cdot) + b(\cdot) - 1|(1 - r)D.\]

Using

\[D = \frac{1}{1 + q} M\]

and the consolidated balance sheet constraint (3), the money supply can be expressed as a function of the adjusted monetary base:

\[M^s = m^s(i_L, i_B, z, \varepsilon_K)B^x_m\]

where the money supply multiplier is given by

\[m^s(\cdot) = \frac{1 + q}{1 + q - |a(\cdot) + b(\cdot)|(1 - r)}\]

It is reasonable to assume that the loan supply is more sensitive to changes in the loan rate than the bond demand \((a_{i_L} > |b_{i_L}|)\). Likewise, we assume that the bond demand is more sensitive to changes in the bond rate than the loan supply \((b_{i_B} > |a_{i_B}|)\). Under these assumptions, the money supply positively depends on the loan rate and the bond rate \((m^s_{i_L} > 0, m^s_{i_B} > 0)\) and negatively on the refinancing rate \((m^s_z < 0)\) and on the credit shock \((m^s_{\varepsilon_K} < 0)\).

Using (4), (6), and (7), the loan supply can be written as a function of the adjusted money supply (A.1). Note that Bernanke and Blinder (1988) model the demand for excess reserves explicitly and deduce the bond demand from the loan supply and the demand for excess reserves.
monetary base as well:

\[ K^s = \frac{a(\cdot)(1 - r)m^s(\cdot)}{1 + q} B_m^e = k^s(i_L, i_B, z, \varepsilon_K) B_m^e \]  

(9)

where the loan supply multiplier is given by

\[ k^s(\cdot) = \frac{a(\cdot)(1 - r)}{1 + q - [a(\cdot) + b(\cdot)](1 - r)} \]  

(10)

For the partial derivatives with respect to the loan rate and the refinancing rate, it holds

\[ k_{is}^s > 0 \text{ and } k_{iz}^s < 0. \]

The effect of the bond rate on the loan supply is ambiguous, but it is reasonable to assume \( k_{is}^s < 0. \)\(^\text{10} \)

The shock term \( \varepsilon_K \) negatively affects the loan supply multiplier \((k_{is}^s < 0).\)

In log-linear form, the money and loan supply functions read

\[ m = b_m^e + a_1(i_L - \bar{i}_{L,0}) + a_2(i_B - \bar{i}_{B,0}) - a_3(z - \bar{z}_0) - a_4\varepsilon_K \]  

(11)

\[ k = b_m^e + r_1(i_L - \bar{i}_{L,0}) + r_2(i_B - \bar{i}_{B,0}) - r_3(z - \bar{z}_0) - r_4\varepsilon_K \]  

(12)

where \( m = \frac{dM}{M_0}, \) \( k = \frac{dK}{K_0}, \) and \( b_m^e = \frac{dB}{B_{m,0}} \) are given in percentage deviations from their respective initial steady state \((M_0 = \bar{M}_{0,0}, B_{m,0}, K_0 = \bar{K}_{0,0}).\) The interest rates \( i_L, \) \( i_B, \) and \( z \) are, on the other hand, expressed in absolute (not percentage) differences from their steady states \((\bar{i}_{L,0}, \bar{i}_{B,0}, \text{ and } \bar{z}_0).\) The corresponding coefficients \( a_1 = \frac{m^e_{i_L}}{m_0}, \) \( a_2 = \frac{m^e_{i_B}}{m_0}, \) \( a_3 = -\frac{m^e_z}{m_0}, \) \( r_1 = \frac{k^e_{i_L}}{k_0}, \) \( r_2 = -\frac{k^e_{i_B}}{k_0}, \) and \( r_3 = -\frac{k^e_z}{k_0} \) are then semi-elasticities.\(^\text{11} \)

Similarly, \( a_4 = \frac{m^e_{\varepsilon_K}}{m_0} \) and \( r_4 = -\frac{k^e_{\varepsilon_K}}{k_0}. \)

\[ \text{2.2 Money and loan demand and consumption} \]

The money and loan demand and the demand for consumption goods follow from the maximization of the representative household’s lifetime utility subject to the period budget

\( \text{\textsuperscript{10}} \) Taking the partial derivative of the consolidated balance sheet equation (3) with respect to the bond rate \( i_B \) gives \( K_{is}^s = M_{i_B}^s - B_{i_B}^e, \) where the bond demand function can be written as

\[ B^d = \frac{b_l(\cdot)(1 - r)m(\cdot)}{1 + q} B_m^e = \frac{b_l(\cdot)(1 - r)}{1 + q - [a(\cdot) + b(\cdot)](1 - r)} B_m^e = \frac{b_l(\cdot)(1 - r)}{1 + q - [a(\cdot) + b(\cdot)](1 - r)} B_m^e \]

Hence, assuming \( b_l^i > m_{i_B}^s \) implies \( K_{is}^s < 0 \) and \( k_{is}^s < 0. \)

\( \text{\textsuperscript{11}} \) In the following, we may simply refer to elasticities instead of semi-elasticities. Note, however, that the steady state values of interest rates are rather small. An interest rate semi-elasticity is, therefore, much larger than the corresponding interest rate elasticity.
constraint

\[
C_t + \frac{T^n_t}{P_t} + \frac{M_t}{P_t} + \frac{B_t}{P_t} + (1 + i_{L,t-1}) \frac{K_{t-1}}{P_t} = \frac{W_t}{P_t} N_t + \frac{M_{t-1}}{P_t} + (1 + i_{B,t-1}) \frac{B_{t-1}}{P_t} + \frac{K_t}{P_t}
\] (13)

The budget constraint explicitly takes into account that the household may hold cash, invest into bonds, and take out a loan.\(^{12}\) The right hand side covers the household’s available funds, including income from labor \((W_t N_t)\) at the wage rate \(W_t\), cash holding from the previous period \((M_{t-1})\), interest and principal payments from bond holding \(((1 + i_{B,t-1})B_{t-1})\) at the beginning of period \(t\), and loans granted from commercial banks \((K_t)\). These funds are used for consumption \((P_t C_t)\) at price \(P_t\), lump-sum taxes \((T^n_t)\), cash holding \((M_t)\), bond investments \((B_t)\), and for interest and principal payments from borrowing in the last period \(((1 + i_{L,t-1})K_{t-1})\).\(^{13}\)

For the period utility function, we use a money-and-credit-in-the-utility approach with non-separable preferences:\(^{14}\)

\[
U \left( C_t, \frac{M_t}{P_t}, \frac{K_t}{P_t}, N_t \right) = X_t^{1-\sigma} - \frac{N_t^{1+\eta}}{1+\eta}
\] (14)

where \(X_t\) is a composite CES-index of consumption, real balances, and – additionally – borrowing:

\[
X_t = \left[ \alpha_1 C_t^{1-\nu} + \alpha_2 \left( \frac{M_t}{P_t} \right)^{1-\nu} + \alpha_3 \left( \frac{K_t}{P_t} \right)^{1-\nu} \right]^{\frac{1}{1-\nu}}
\] (15)

The parameter \(\nu\) denotes the inverse elasticity of substitution between the different components of the index \(X_t\). \(\alpha_1 > 0, \alpha_2 > 0, \) and \(\alpha_3 > 0\) capture the relative weight with which consumption, real balances, and borrowing contribute to the household’s period utility \((\alpha_1 + \alpha_2 + \alpha_3 = 1)\). The assumption that borrowing positively affects the household’s utility may sound counterintuitive at first, but – as money in the utility – captures the household’s liquidity gains from borrowing.\(^{15}\) The marginal utilities (partial derivatives of

\(^{12}\)Note that we follow Bernanke and Blinder (1988) and rule out credit rationing.

\(^{13}\)We assume that both bonds and loans are riskless and have a one-period term to maturity.

\(^{14}\)See Galí (2008) for an analogous approach, which, however, does not take borrowing into account.

\(^{15}\)\(\alpha_3 > 0\) also ensures that the loan demand depends negatively on the loan rate and positively on the bond rate, see equation (22).
\[
\frac{\partial U}{\partial C_t} = \alpha_1 X_t^{\nu-\sigma} C_t^{-\nu} > 0 \quad (\nu > 0, \sigma > 0, \alpha_1 > 0) \quad (16)
\]

\[
\frac{\partial U}{\partial (M_t/P_t)} = \alpha_2 X_t^{\nu-\sigma} \left(\frac{M_t}{P_t}\right)^{-\nu} > 0 \quad (\alpha_2 > 0) \quad (17)
\]

\[
\frac{\partial U}{\partial (K_t/P_t)} = \alpha_3 X_t^{\nu-\sigma} \left(\frac{K_t}{P_t}\right)^{-\nu} > 0 \quad (\alpha_3 > 0) \quad (18)
\]

\[
\frac{\partial U}{\partial N_t} = -N_t^\eta > 0 \quad (\eta > 0) \quad (19)
\]

Let \(\beta \in (0, 1)\) be the discount factor and \(E\) be the expectations operator. The maximization of the intertemporal utility function \(E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, M_t/P_t, K_t/P_t, N_t)\) with respect to \(B_t, M_t,\) and \(K_t\) subject to the period-by-period budget constraint (13) yields the following first-order conditions:

\[
\frac{1}{1 + i_{B,t}} = \beta E_t \left\{ \left(\frac{X_{t+1}}{X_t}\right)^{\nu-\sigma} \left(\frac{C_{t+1}}{C_t}\right)^{-\nu} \frac{P_t}{P_{t+1}} \right\} \quad (Euler\ equation) \quad (20)
\]

\[
\left(\frac{M_t}{P_t}\right)^{-\nu} = \frac{\alpha_1 C_t^{\nu-\sigma} i_{B,t}}{\alpha_2 C_t^{\nu-\sigma} 1 + i_{B,t}} \quad (real\ balances\ demand) \quad (21)
\]

\[
\left(\frac{K_t}{P_t}\right)^{-\nu} = \frac{\alpha_1 C_t^{\nu-\sigma} i_{L,t} - i_{B,t}}{\alpha_3 C_t^{\nu-\sigma} 1 + i_{B,t}} \quad (real\ loan\ demand) \quad (22)
\]

The demand for real balances is increasing in consumption and decreasing in the bond rate, whereas the demand for loans depends on both interest rates. It is decreasing in the loan rate and increasing in the bond rate and in consumption if we assume that the loan rate exceeds the bond rate in the initial steady state, i.e. \(\bar{i}_{L,0} > \bar{i}_{B,0}\). The positive dependency of the loan demand on the bond rate captures the fact that the household is willing to finance its bond demand by credit for \(\alpha_3 > 0\). In log-linear form, the money and loan demand functions read as

\[
m_t - p_t = c_t - \frac{\beta^2}{\nu(1 - \beta)} (i_{B,t} - \bar{i}_{B,0}) \quad (23)
\]

\[
k_t - p_t = c_t + \frac{1}{\nu \bar{i}_{L,0} - \bar{i}_{B,0}} \left\{ \lambda (i_{B,t} - \bar{i}_{B,0}) - (i_{L,t} - \bar{i}_{L,0}) \right\} \quad (24)
\]

where \(\beta = 1/(1 + \bar{i}_{B,0})\) and \(\lambda = \frac{1 + \bar{i}_{L,0}}{1 + \bar{i}_{B,0}} > 1\) for \(\bar{i}_{L,0} > \bar{i}_{B,0}\).

In line with Galí (2008), the Euler consumption equation, which reflects the principle
of intertemporal consumption smoothing, is given by

\[ \sigma c_t = \sigma E_t c_{t+1} - (i_{B,t} - E_t \pi_{t+1} - \bar{r}_{B,0}) + (\nu - \sigma) \{ E_t (c_{t+1} - x_{t+1}) - (c_t - x_t) \} \]  

(25)

where \( c_t - x_t \) = \( \chi_1 (i_{B,t} - \bar{r}_{B,0}) + \chi_2 (i_{L,t} - \bar{r}_{L,0}) \) and

\[ \chi_1 = \frac{1}{X_0^{1-\nu}} \frac{1}{\nu} \left[ \frac{\alpha_2}{1-\beta} \left( \frac{M_0}{P_0} \right)^{1-\nu} - \frac{\alpha_3}{i_{L,0}} \frac{1}{i_{B,0}} \lambda \left( \frac{K_0}{P_0} \right)^{1-\nu} \right] \leq 0 \]  

(26)

\[ \chi_2 = \frac{1}{X_0^{1-\nu}} \frac{1}{\nu} \left[ \frac{\alpha_3}{i_{L,0}} \frac{1}{i_{B,0}} \left( \frac{K_0}{P_0} \right)^{1-\nu} \right] > 0 \]  

(27)

Putting all together, the linearized Euler equation based on the non-separable utility function (14) can be expressed as

\[ c_t = E_t c_{t+1} - \frac{1}{\sigma} (i_{B,t} - E_t \pi_{t+1} - \bar{r}_{B,0}) + \frac{\nu - \sigma}{\sigma} \{ \chi_1 E_t \Delta i_{B,t+1} + \chi_2 E_t \Delta i_{L,t+1} \} \]  

(28)

where \( \Delta i_{B,t+1} = i_{B,t+1} - i_{B,t} \) and \( \Delta i_{L,t+1} = i_{L,t+1} - i_{L,t} \). If the elasticities of substitution \( \nu \) and \( \sigma \) are different from each other (\( \nu \neq \sigma \)), current consumption does not only depend on expected future consumption (\( E_t c_{t+1} \)) and on the real interest rate (\( i_{B,t} - E_t \pi_{t+1} \)), but also on expected future changes in the loan rate and the bond rate. \( \nu \) determines the interest rate semi-elasticity of money demand, see equation (23). Estimates of the money demand suggest that \( \nu \) is sufficiently large such that it is reasonable to assume that \( \nu \geq \sigma \).\(^{17}\) In the remaining of this paper, we assume that this condition holds.

In addition to the (ordinary) New Keynesian real interest rate effect, changes in the current bond and loan rate affect current consumption through three effects. The first two effects work in the same direction as the real interest rate effect and operate as follows: A decline in the current bond rate (loan rate) increases real balance demand (loan demand), increasing \( X_t \) (for any given \( C_t \)) and the marginal utility of current consumption, and, hence, leading to a rise in current consumption in order to smooth marginal utility over time. In the following, we denote this effect as real balance (credit) effect. The third effect

\(^{16}\)Note that \( c_t = \frac{dC_t}{C_0} \approx \log C_t - \log C_0 \) and \( x_t = \frac{dX_t}{X_0} \approx \log X_t - \log X_0 \). \( E_t \pi_{t+1} \) is the expected inflation rate of the next period, i.e. \( E_t \pi_{t+1} = E_t \left( \frac{P_{t+1} - P_t}{P_t} \right) \). We assume that inflation is zero in the initial steady state (\( \pi_0 = 0 \)). For a detailed derivation of the log-linearized IS equation, see Appendix B and C.

\(^{17}\)Estimates of the money demand reported in Walsh (2010) suggest a semi-elasticity between 1 and 10. Galí (2008) uses a value of 4. For \( \beta = 0.99 \), a semi-elasticity of \( \theta = \frac{\sigma^2}{\beta (1-\beta)} \leq 10 \) translates into \( \nu \geq 9.8 \).
works oppositely to the real interest rate effect. Recall that the bond rate directly affects the real loan demand. A decline in the bond rate reduces loan demand and, hence, current consumption. In the following, we denote this effect as (contractionary) credit-financed bond effect.

The same three effects can be identified for changes in the expected future bond and loan rate, only with opposite sign. An expected reduction in the future loan rate increases future demand for loans, increasing $E_t X_{t+1}$ (for any given $E_t C_{t+1}$) and future marginal utility of consumption $E_t \frac{\partial U}{\partial C_{t+1}}$, leading to a decline in current consumption. An expected reduction in the future bond rate increases, on the one hand, the demand for future real balances, and lowers, on the other hand, the demand for future loans.

To sum up, the effects of the bond rate (current and future) on current consumption are ambiguous. If the interest rate spread $\bar{r}_{iL,0} - \bar{r}_{iB,0}$ is sufficiently large and/or the weight $\alpha_3$ relatively small compared to $\alpha_2$, we have (according to (26)) $\chi_1 > 0$ and the demand for credit-financed bonds is rather small. In this case, a reduction in the expected future bond rate negatively affects current consumption, whereas a reduction in the current bond rate positively affects current consumption.

### 2.3 Interest rate rule and the Phillips curve

In contrast to the standard New Keynesian model framework, we lay out an interest rate rule of Taylor-type which is not based on the bond rate $i_B$, but instead describes the reaction of the refinancing rate $z$ as central bank’s control variable:

$$z_t = z_0 + k_\pi (\pi_t - \pi^T) + k_x x_t + \varepsilon_{z,t} \tag{29}$$

where $k_\pi$, $k_x \geq 0$. $\pi^T$ is the target inflation rate of the central bank. $z_t$ positively reacts to the inflation gap ($\pi_t - \pi^T$) and to the output gap ($x_t = y_t - y^f$), where $y^f$ is the constant output level under fully flexible prices. $\varepsilon_{z,t}$ is an interest rate shock, which also describes the central bank’s discretionary policy in case $k_\pi = k_x = 0$.

The inflation rate is described by a simple (completely microfounded) Phillips curve of
the form

$$\pi_t = \tilde{\beta} E_t \pi_{t+1} + \delta x_t + \varepsilon_{x,t}$$  \hspace{1cm} (30)$$

where $0 < \tilde{\beta} \leq 1$, $\delta > 0$. $\varepsilon_{x,t}$ is a cost shock. Note that we do not include a cost channel, which would make inflation directly dependent on the interest rate.$^{19}$

### 2.4 The complete model framework

In the following, we use a static approximation of the Euler equation and the Phillips curve and study the comparative-static effects of monetary and non-monetary shocks on the money, loan and goods market.$^{20}$ On the goods market, we assume that the equilibrium condition $y = c$ holds. In a comparative-static analysis, we do not have any transitory dynamics. Therefore, changes in the price level are equivalent to changes in the inflation rate, i.e. $d\pi = dp$ with $\pi = p - p_0$. In differences, the complete New Keynesian model with money and credit in the utility encompasses the following set of equations:

$$dy = -b(di_B - d\pi^c) + c_1d(\Delta i_B^p) + c_2d(\Delta i_L^p) + d\varepsilon_y \hspace{1cm} (31)$$

$$d\pi = d\pi^c + \delta dy + d\varepsilon_\pi \hspace{1cm} (32)$$

$$dm - dp = dy - \theta d i_B \hspace{1cm} (33)$$

$$dm = db_m^x + a_1 di_L + a_2 di_B - a_3 dz - a_4 d\varepsilon_k \hspace{1cm} (34)$$

$$dk - dp = dy - q_1 di_L + q_2 di_B \hspace{1cm} (35)$$

$$dk = db_m^x + r_1 di_L - r_2 di_B - r_3 dz - r_4 d\varepsilon_k \hspace{1cm} (36)$$

$$dz = k_\pi(d\pi - d\pi^T) + k_z dy + d\varepsilon_z \hspace{1cm} (37)$$

$$dp = d\pi \hspace{1cm} (38)$$

Equations (31) and (32) describe the goods market (IS and Phillips curve), equations (33) and (34) describe the money market, and equations (35) and (36) describe the loan market.

$^{18}$See e.g. Walsh (2010) for a derivation of the Phillips curve.

$^{19}$For details on how to introduce a cost channel into the New Keynesian model, see Ravenna and Walsh (2006).

$^{20}$A static approximation of the Euler equation and the Phillips curve was introduced in Romer (2012) and Bofinger, Mayer and Wollmershäuser (2006). For simplification, we follow Bofinger, Mayer and Wollmershäuser (2006) and set $\tilde{\beta}$ in the Phillips curve equal to unity.
The expected changes in the bond and loan rate are given by $\Delta i^e_B = i^e_B - i_B$ and $\Delta i^e_L = i^e_L - i_L$, where the expected future bond rate $i^e_B$ and the expected future loan rate $i^e_L$ are assumed to be exogenous.\(^{21}\)

We assume that the expected future inflation rate $\pi^e$ is initially pinned down to the central bank’s target level $\pi^T$. In case $\pi^T$ changes, the expected inflation rate either changes in the same manner ($d\pi^e = d\pi^T$) or remains constant ($d\pi^e = 0$). In the former case, the public expects a permanent change in the central bank’s target level and expects the central bank to credibly achieve its announced target. In the latter case, the public expects a temporary change in the target level or finds the central bank’s announcement to be not credible.

In the following, we study based on the model equations (31) to (38) the comparative-static impacts of monetary disturbances ($db^x_m > 0$, $d\pi^T > 0$, $d\varepsilon_x < 0$) and of demand and supply disturbances ($d\varepsilon_y > 0$, $d\varepsilon_x > 0$). We further allow for changes in private expectations ($d\pi^e > 0$, $di^e_B \neq 0$, $di^e_L \neq 0$) and for credit supply shocks ($d\varepsilon_K > 0$). In the upcoming Section 3, we presume a traditional New Keynesian IS equation, i.e. $c_1 = c_2 = 0$. This is equivalent to the assumption that the elasticities of substitution $\sigma$ and $\nu$ are equal (see Section 2.2). In Section 4, we relax this assumption and study the impacts for $c_1 \neq c_2 \neq 0$, i.e. $\sigma \neq \nu$. In this case, the credit channel is active and serves as an additional transmission mechanism through which monetary disturbances affect the real economy. Contrarily, in case $\sigma = \nu$, only the traditional interest rate channel is active.

### 3 Monetary and non-monetary disturbances

#### 3.1 MM and KK curves

Figure 1 presents the model (31) to (38) in a $i_B/i_L$-plane in case $c_1 = c_2 = 0$. The MM curve describes the equilibrium on the money market and the KK curve describes the equilibrium on the loan market. To construct the MM and KK curve, we replace the output variable $dy$ by the IS equation (31) and the price variable $dp = d\pi$ by the Phillips curve (32). The MM curve has a negative slope. A rise in the loan rate positively affects the loan supply and, hence, the money supply. To restore equilibrium on the money market, the bond rate

\(^{21}\)A similar approach is used in a static approximation of the New Keynesian model of an open economy by Bofinger, Mayer and Wollmershäuser (2009) to model expected changes in the exchange rate.
needs to fall implying a rise (fall) in the money demand (supply). The KK curve has a positive slope if the following condition holds:

\[ r_2 + q_2 > b[1 + \delta + r_3(k_x + \delta_k)] \]  

(39)

\( r_2 \) and \( q_2 \) stand for the bond rate semi-elasticity of the loan supply and the loan demand, respectively. The parameter \( b = 1/\sigma \) is the intertemporal elasticity of substitution in consumption, and \( \delta \) is the slope of the Phillips curve. Empirically, inequality (39) is typically satisfied. Therefore, it is assumed that the KK curve in figure 1 has a positive slope. A rise in the bond rate lowers the loan supply and increases the demand for loans to finance the higher bond demand. To restore equilibrium, the loan rate needs to rise.\(^{22}\)

**Figure 1**: Response of the MM and KK curve without credit channel to an expansionary shock

![Graph showing the response of the MM and KK curves](image)

*Note: The left shift in the MM and KK curve is due to an expansionary shock (\( db_m^x > 0, \, d\pi^T > 0, \, dx < 0, \, dy > 0, \, dx < 0, \, dK < 0 \)). The parameter calibration is reported in the Appendix A. To mute the credit channel, we set \( c_1 = c_2 = 0 \).*

Figure 1 shows the reaction of the KK and MM curves in response to expansionary shock.\(^{22}\)*Note that the interest rate semi-elasticities \( r_2 \) and \( q_2 \) are typically larger than one (see also footnote 11), whereas the Phillips curve is rather flat (i.e. \( \delta << 1 \)) and \( \sigma \geq 1 \) (i.e. \( b \leq 1 \)). For more details on a reasonable parameter calibration, see Appendix A. Further note that a rise in the bond rate also induces a decline in output, which, in turn, has a negative effect on the loan demand. A negatively sloped KK curve would mean that this effect dominates such that a rise in the bond rate would induce an excess supply on the loan market.
shocks, either in form of an expansionary monetary innovation \((d\varepsilon_m > 0, d\pi^T > 0, d\varepsilon_z < 0)\) or in form of a disturbance of the IS and Phillips curve \((d\varepsilon_y > 0, d\varepsilon_\pi < 0)\) or in form of an expansionary credit supply shock \((d\varepsilon_K < 0)\). The MM and KK curves shift in the same direction. The (horizontal) shift in the MM curve is stronger than the shift in the KK curve for all disturbances if the following sufficient conditions hold:

\[ r_1 + q_1 > a_1 \quad (40) \]
\[ a_3 > r_3 \quad \text{or} \quad r_3 - a_3 \quad \text{sufficiently small if} \quad r_3 > a_3 \quad (41) \]

The first condition (40) says that the loan market is more elastic to changes in the loan rate than the money market.\(^{23}\) The second condition (41) says that the money supply is more elastic to changes in the refinancing rate than the loan supply.\(^{24}\) If not stated otherwise, we assume that both conditions hold for the remaining of the paper.\(^{25}\)

In case the KK has a positive slope, a stronger (horizontal) shift in the MM curve than in the KK curve makes sure that both interest rates \(i_B\) and \(i_L\) move in the same direction. The bond and loan rate decline whenever the MM and KK curves are shifted to the left as e.g. in case of expansionary monetary policy \((d\varepsilon_m > 0, d\pi^T > 0, d\varepsilon_z < 0)\). The decline in the bond rate leads, in turn, to a rise in output through the IS equation (31).

Analytically, the MM and KK curve are described by the following system of equations:

\[
\begin{pmatrix}
  a_1 & a_2 + \theta + b\phi_1 \\
  r_1 + q_1 & -(r_2 + q_2 - b\phi_2)
\end{pmatrix}
\begin{pmatrix}
  d\pi_m \\
  d\pi^T
\end{pmatrix}
= \begin{pmatrix}
  -1 \\
  -1
\end{pmatrix}
\begin{pmatrix}
  a_3 \\
  r_3
\end{pmatrix}
\begin{pmatrix}
  d\varepsilon_x \\
  d\varepsilon_z
\end{pmatrix}
+ \begin{pmatrix}
  -a_3k_\pi \\
  -r_3k_\pi
\end{pmatrix}
d\pi^T
\]

\[
+ \begin{pmatrix}
  1 + a_3k_\pi + b\phi_1 \\
  1 + r_3k_\pi + b\phi_2
\end{pmatrix}
d\varepsilon_x + \begin{pmatrix}
  \phi_1 \\
  \phi_2
\end{pmatrix}
\begin{pmatrix}
  1 + a_3k_\pi \\
  1 + r_3k_\pi
\end{pmatrix}
d\varepsilon_y + \begin{pmatrix}
  a_4 \\
  r_4
\end{pmatrix}
d\varepsilon_K
\]

\[ (42) \]

\(^{23}\)An analogous condition to (40) can be found in Brunner and Meltzer (1988) and in Jarchow (2010). Note that the money demand is independent from the loan rate.

\(^{24}\)Note that \(a_3 = \frac{|M^1|}{M_0} > r_3 = \frac{|K^1|}{K_0}\) implies \(k_\pi^1 < \frac{M_0}{M_0} < 1\). From the balance sheet constraint (3) follows \(k_\pi^2 < 1\) and empirically \(\frac{M_0}{M_0} < 1\). Hence, \(a_3 > r_3\) is not guaranteed.

\(^{25}\)Note that in case of a discretionary policy (i.e. \(k_1 = k_2 = 0\)), the KK curve is steeper than the MM curve if in addition to (40) the sufficient condition \(a_2 + \theta > r_2 + q_2\) holds. Analogously to (40) it says that the money market is more elastic to changes in the bond rate than the loan market.
where

\[ \phi_1 = 1 + \delta + a_3(k_x + \delta k_x) \quad (43) \]
\[ \phi_2 = 1 + \delta + r_3(k_x + \delta k_x) \quad (44) \]

The first (second) equation in (42) represents the MM curve (KK curve). The corresponding system’s determinant reads

\[ \Gamma = -a_1(r_2 + q_2 - b\phi_2) - (r_1 + q_1)(a_2 + \theta + b\phi_1) \quad (45) \]

and is negative under the conditions (40) and (41).\(^{26}\) Let \( A \) denotes the (2 × 2)-system matrix of the system (42), then

\[ A^{-1} = \frac{1}{\Gamma} \begin{pmatrix} -(r_2 + q_2 - b\phi_2) & -(a_2 + \theta + b\phi_1) \\ -r_1 + q_2 & a_1 \end{pmatrix} \quad (46) \]

Pre-multiplying the system (42) by \( A^{-1} \), we are able to specify analytically the comparative-static effects of exogenous disturbances on the bond and loan rate.

### 3.2 Responses to monetary disturbances

This section studies the following monetary disturbances: (i) an increase in the monetary base with and without a contractionary credit shock, (ii) a reduction in the refinancing rate, and (iii) an increase in the inflation target with and without adjustment of inflation expectations. To start with, an isolated increase in the adjusted monetary base gives the following multipliers:

\[ \frac{di_L}{db_m} = \frac{1}{\Gamma} [r_2 + q_2 + a_2 + \theta + b(\phi_1 - \phi_2)] \quad (47) \]
\[ \frac{di_B}{db_m} = \frac{1}{\Gamma} (r_1 + q_1 - a_1) \quad (48) \]

Under the conditions (40) and (41), both multipliers are negative. For constant inflation expectations, the real interest rate \( i_B - \pi^e \) declines as well, such that – via the IS equation

\(^{26}\)In the discretionary case \((k_x = k_x = 0)\), condition (40) suffices to ensure \( \Gamma < 0 \).
and the Phillips curve – output and inflation increase. If the refinancing rate $z$ follows an interest rate rule of Taylor-type ($k_x > 0$, $k_\pi > 0$), it increases and dampens the effects on inflation, output, and on the bond and loan rate.

Without credit channel ($c_1 = c_2 = 0$) and constant inflation expectations, the effects of monetary disturbances on the real money stock are unambiguously determined by the LM curve. An expansionary increase in the monetary base increases the real money stock. This implies that the rise in the nominal money stock is stronger than the rise in inflation ($dm > d\pi$). The relative multiplier is given by

\[
\frac{dm}{db_m} \times \frac{dm/d\pi}{db_m/d\pi} > \frac{(1 + \delta)b + \theta}{\delta b} > 1
\]  

(49)

From the supply side, the decline in the bond and loan rate and the rise in the refinancing rate diminish the positive effect of the monetary base on the money stock. Hence, the increase of the monetary base has less than proportional effects on the money stock. In summary, it holds $db_m > dm > d\pi > 0$.

The response of the loan volume is, on the other hand, ambiguous. The reason is that the loan and bond rate always respond in the same direction, but have opposing effects on the loan demand and supply. This holds for all disturbances. We, therefore, do not discuss the loan response for the remaining of this section. Recall that we assume that in the initial steady state the loan rate is larger than the bond rate ($i_{L,0} > i_{B,0}$) such that the demand for loan-financed bond holding is rather small.\textsuperscript{27} In case the bond and loan rate decline and output rises, it is then reasonable to expect a rise in loan demand and in the loan volume. This is also confirmed by the simulation exercise in Section 4.3.\textsuperscript{28}

The expansionary effects of an increase in the monetary base are dampened if simultaneously a contractionary credit shock ($d\varepsilon_K > 0$) hits the economy. The contractionary credit shock reduces the credit and money supply multiplier and, therefore, the impact of the monetary base on the money stock and inflation.\textsuperscript{29} A sufficiently large credit shock

\textsuperscript{27}Note that the bond rate semi-elasticity $q_2$ is by construction larger than the corresponding loan rate semi-elasticity $q_1$ of the loan demand (see the microfoundation in Section 2) for $\tilde{r}_{L,0} > \tilde{r}_{B,0}$. Changes in the loan rate are, though, typically larger than changes in the bond rate.

\textsuperscript{28}Recall that we ignore credit rationing as in Bernanke and Blinder (1988) and Brunner and Meltzer (1988). Expansionary monetary shocks would typically lead to an excess demand on loan market. If we allow for credit rationing, the loan supply (short side of the market) would determine the loan volume, which would negatively affect output.

\textsuperscript{29}Note that similar to condition (41), it has to hold that $|r_4 - a_4|$ is sufficiently small if $r_4 > a_4$. This ensures that the shift in the MM curve is stronger than in the KK curve and that an isolated contractionary credit
neutralizes the expansionary monetary policy effects nearly completely as can be seen in the simulation exercise in Section 4.3 and which is currently experienced in the European Economic Monetary Union.

We obtain completely analogous results for output, inflation, the bond and loan rate, and the real and nominal money stock if we consider an exogenous increase in the target level \( \pi^T \) (without adjustment of inflation expectations) or an exogenous refinancing shock of the form \( d\varepsilon_z < 0 \). All three disturbances affect the model economy via the money and loan supply either directly or through a change in the refinancing rate. The relative multipliers are exactly the same:

\[
\frac{dm/db}{dm/d\pi^T} = \frac{dm/d\pi}{d\pi/d\pi^T} = \frac{dm/d\varepsilon_z}{d\pi/d\varepsilon_z} \quad (50)
\]

We next consider an anticipated increase in the inflation target, where now inflation expectations adjust. We assume that they adjust in exactly the same manner, i.e. \( d\pi^e = d\pi^T > 0 \). Under conditions (40) and (41), both, the (nominal) bond and loan rate then respond positively, while the real interest rate declines:

\[
\left. \frac{di_L}{d\pi^T} \right|_{d\pi^e > 0} = -\frac{1}{1}[(r_2 + q_2)(1 + b\phi_1) + (a_2 + \theta)(1 + b\phi_2) + b(k_x + \delta k_x)(a_3 - r_3)] > 0 \quad (51)
\]

\[
\left. \frac{di_B}{d\pi^T} \right|_{d\pi^e > 0} = -\frac{1}{1}[(r_1 + q_1)(1 + b\phi_1) - a_1(1 + b\phi_2)] > 0 \quad (52)
\]

\[
\left. \frac{di_B - d\pi^e}{d\pi^T} \right|_{d\pi^e > 0} = \frac{di_B}{d\pi^T} - 1 < 0 \quad \Leftrightarrow \quad (r_1 + q_1)(1 - a_2 - \theta) < a_1(1 + r_2 + q_2) \quad (53)
\]

The last inequality typically holds since \( a_2 \) and \( \theta \) are semi-elasticities with \( a_2 + \theta > 1 \). Then the real bond rate declines so that we have positive output effects implying that inflation overshoots its target level \( (d\pi > d\pi^e = d\pi^T) \). This stands in contrast to the baseline NKM framework, where we have no output effects in response to a fully anticipated change in the target level. The reason is that in our framework the central bank does not control the bond rate directly. Since the bond and loan rate always move in the same direction, disequilibria on the money market are partly reduced by changes in the loan rate. Changes in the refinancing rate, therefore, lead only to small changes in the bond rate. This result is confirmed by our simulation exercise in Section 4.3. Note that the response of the real shock (\( d\varepsilon_K > 0 \)) leads to an increase in the bond rate, and, hence, to a decline in output.

\[ ^{30}\text{This follows from the Phillips curve equation.} \]
money stock is now ambiguous since \( dy > 0 \) and \( di_B > 0 \).

To sum up, the effects of monetary disturbances are transmitted into the real economy via the money and loan market. An expansionary shock \((db_m > 0, d\pi^T > 0, d\varepsilon < 0)\) increases money and in general also loan supply, which puts downward pressure on the bond and loan rate and, hence, leads to an expansion in output. Changes in the monetary base, though, cause only less than proportional changes in broad money. In the presence of a credit shock, which reduces the credit and money supply multiplier, the effect on broad money is even further reduced. In comparison to the baseline New Keynesian model, the transmission of shocks through the money and loan market leads to smaller changes in the bond and loan rate. As a result, anticipated increases in the inflation target \((d\pi^e = d\pi^T > 0)\) have positive output effects.

### 3.3 Demand and cost shock

Non-monetary disturbances (as e.g. the demand shock \( d\varepsilon_y > 0 \) and the supply shock \( d\varepsilon_\pi > 0 \)) have the same effects on output and inflation as in the baseline NKM model. In case of the demand shock \( d\varepsilon_y > 0 \), we obtain a positive output effect of the form

\[
\frac{dy}{d\varepsilon_y} = -\frac{1}{\Gamma}[a_1(r_2 + q_2) + (r_1 + q_1)(a_2 + \theta)]
\]

which is accompanied by an increase in inflation. Contrarily to the above discussed monetary disturbances, the demand and cost shock do not hit the money and loan supply directly. The positive output and inflation response leads to an increase in the money and loan demand, and lowers the money and loan supply via an increase in the refinancing rate. Consequently, the bond and loan rate need to rise to restore equilibrium on the money and loan market. This, in turn, diminishes the output and inflation response. Thus, the feedback effects from the money and loan market stabilize the economy in the presence of a demand shock on the goods market.\(^{31}\)

A positive cost shock \((d\varepsilon_\pi > 0)\) under conditions (40) and (41) leads to stagflation \((dy < 0, d\pi > 0)\). Output and inflation, therefore, have opposing effects both on the money and loan demand. If \((1+\delta)dy+d\varepsilon_\pi > 0\), the cost shock leads to an increase in the money and

\(^{31}\)Note, however, that the bond rate may be more stabilizing if it directly follows an interest rate rule of Taylor-type, which is the case in the baseline New Keynesian model.
loan demand. The central bank faces the well-known trade-off between output and inflation stabilization. Typically, the refinancing rate rises, responding stronger to inflation than to output deviations. This, in turn, negatively affects money and loan supply. Consequently, the bond and loan rate need to rise to restore equilibrium on the money and loan market. Thus, the feedback effects from the money and loan market destabilize output, but stabilize inflation.

4 The credit channel

In this section, we allow expected changes in the bond and loan rate to affect goods demand directly through the IS equation, i.e. we allow for $c_1 \neq 0$. By expanding the IS equation with the expected change in the interest rates $\Delta i_L^e$ and $\Delta i_B^e$, current consumption does not only depend on the real interest rate $i_B - \pi^e$, but also on the loan and bond rate $i_L$ and $i_B$. In this case, the system matrix $A$ for system (42) has to be replaced by

$$
\tilde{A} = \begin{pmatrix}
   a_1 + \phi_1 c_2 & a_2 + \theta + \phi_1 (b + c_1) \\
   r_1 + q_1 + \phi_2 c_2 & -(r_2 + q_2 - \phi_2 (b + c_1))
\end{pmatrix}
$$

(55)

where the corresponding (negative) determinant reads as

$$
\tilde{\Gamma} = - (a_1 + \phi_1 c_2) (r_2 + q_2) - (a_2 + \theta) [(r_1 + q_1) + \phi_2 c_2] - (b + c_1) [(r_1 + q_1) \phi_1 - a_1 \phi_2] 
$$

(56)

$$
= \Gamma - c_1 [(r_1 + q_1) \phi_1 - a_1 \phi_2] - c_2 [(r_2 + q_2) \phi_1 + (a_2 + \theta) \phi_2]
$$

(57)

Furthermore, we need to include the future expected bond and loan rate as additional exogenous disturbances by adding

$$
\begin{pmatrix}
   \phi_1 \\
   \phi_2
\end{pmatrix}
\begin{pmatrix}
   c_1 \Delta i_B^e \\
   c_2 \Delta i_L^e
\end{pmatrix}
$$

(58)

to the right hand side of (42). The future expected interest rates $\Delta i_B^e$ and $\Delta i_L^e$ are then location parameters of the modified MM and KK curve.

In the following Section 4.1, we assume that the expected future bond and loan rate do

---

32In addition to the conditions (40) and (41), we assume that $b + c_1 > 0$, which means that the net effect of the current bond rate on output is negative. These conditions suffice for a negative determinant.
not change \((d_{tL}^e = d_{tB}^e = 0)\), i.e. the households expect the bond and loan rate to return to their respective initial steady state. A decline in the bond or loan rate \((d_{tB}, d_{tL} < 0)\) is then equivalent to \(d\Delta \epsilon_{tL}^e > 0\) and \(d\Delta \epsilon_{tB}^e > 0\), which, in turn, induces positive output effects if \(c_1, c_2 > 0\). In the subsequent Section 4.2 we relax this assumption and show how changes in the expected bond and loan rate affect our results. For simplicity, we limit our analytical analysis to the case of discretionary policy, i.e. \(\delta_{\pi} = \delta_x = 0\). Note that the discretionary case implies \(\phi_1 = \phi_2 = 1 + \delta\). In Section 4.3, we show in a simulation exercise that our main derived results are robust with respect to the non-disccretionary case.

4.1 Responses to monetary disturbances

In this section, we consider an increase in the monetary base and in the target level (with and without adjustment of inflation expectations). To start with, we study the effects of an increase in the exogenous monetary base. For constant inflation and interest rate expectations \((d\pi^e = d\epsilon_{tL}^e = d\epsilon_{tB}^e = 0)\), we obtain

\[
\frac{d_{tL}}{d\pi_m} = \frac{1}{\Gamma}(r_2 + q_2 + a_2 + \theta) \\
\frac{d_{tB}}{d\pi_m} = \frac{1}{\Gamma}(r_1 + q_1 - a_1)
\]

As in the case without credit channel, both multipliers are negative under the weak assumption \((40)\), i.e. \(r_1 + q_1 > a_1\). Then \(d\Delta \epsilon_{tB}^e > 0\) and \(d\Delta \epsilon_{tL}^e > 0\), and the total output effect is given by

\[
\frac{dy}{d\pi_m} = -\frac{1}{\Gamma}\left\{(b + c_1)(r_1 + q_1 - a_1) + c_2(r_2 + q_2 + a_2 + \theta)] = -\frac{b(r_1 + q_1 - a_1) + \tilde{c}}{\Gamma - (1 + \delta)\tilde{c}} \right. \]

Note that \(b + c_1 > 0\) and \(r_1 + q_1 > a_1\) are sufficient conditions for a positive output multiplier. The coefficient

\[
\tilde{c} = c_1(r_1 + q_1 - a_1) + c_2(r_2 + q_2 + a_2 + \theta)
\]

captures the impact of the credit channel, where \(\tilde{c} = 0\) corresponds to the model without credit channel, i.e. to the case \(c_1 = c_2 = 0\). With increasing \(\tilde{c}\), \(\tilde{\Gamma} = \Gamma - (1 + \delta)\tilde{c}\) decreases.
and the output multiplier increases:

$$\frac{d}{dc} \left( \frac{dy}{d\tilde{c}_m} \right) = -\frac{a_1(r_2 + q_2) + (r_1 + q_1)(a_2 + \theta)}{(-\Gamma + (1 + \delta)\tilde{c})^2} > 0$$

(63)

This implies that for $\tilde{c} > 0$, the output effects of an isolated change in the monetary base are enhanced through the credit channel, whereas the output effects are dampened for $\tilde{c} < 0$. Recall from Section 2.2 that in case $d\tilde{r}_L = d\tilde{r}_B = 0$, the credit channel affects output via three transmission mechanisms. On the one hand, a decline in the bond and loan rate acts expansionary through the real balance and credit effect. On the other hand, a decline in the bond rate also has a contractionary effect on output through a lower credit-financed bond demand. The expansionary credit effect unambiguously implies $c_2 > 0$. If the expansionary real balance effect dominates the contractionary credit-financed bond effect, we also have $c_1 > 0$.\(^{33}\) Otherwise, $c_1 < 0$.

The strength of all three effects (on output) depends on the elasticities of the loan and money market $r_1$, $q_1$, $a_1$, $r_2$, $q_2$, $a_2$, and $\theta$. All elasticities are included in the definition of $\tilde{c}$ in (62). The larger $r_1 + q_1 - a_1$, the larger is the change in the bond rate to exogenous disturbances and, hence, the stronger are the real balances and credit-financed bond effect. Graphically, the larger $r_1 + q_1 - a_1$, the larger is the horizontal shift in the MM curve relatively to the shift in the KK curve. Analogously, the larger $r_2 + q_2 + a_2 + \theta$, the larger is the change in the loan rate to exogenous disturbances and, hence, the stronger is the credit effect. Graphically, the larger $r_2 + q_2 + a_2 + \theta$, the larger is the (vertical) shift in the MM and KK curve.

If $\tilde{c} < 0$, the contractionary credit-financed bond effect dominates the other two expansionary effects. This dampening case $\tilde{c} < 0$ can, however, generally be ruled out. As can be seen from definition (62), it would require $c_1 < 0$ (credit-financed bond effect dominates real balance effect), an implausible high loan rate semi-elasticity $r_1$ of the loan supply and a relatively low bond rate elasticity $r_2$ of the loan supply.\(^{34}\) In the remaining of the paper, we assume $\tilde{c} > 0$.

For $\tilde{c} > 0$, the qualitative response of the real and nominal money stock is the same as

\(^{33}\)In particular, $c_1 > 0$ if $\frac{\alpha_3}{\alpha_2} > \frac{\tilde{r}_B}{\tilde{r}_L - \tilde{r}_B}(1 + \tilde{r}_L)\nu$ (assuming $\nu > \sigma$). For a derivation, see Appendix D.

\(^{34}\)Note that $c_1 < 0$ requires a relatively large weight $\alpha_3$ on the loan demand compared to the weight $\alpha_2$ on real balances in the utility function. This, however, contradicts that the ratio $\tilde{M}/\tilde{K} > 1$, as can be seen in the calibration exercise in Appendix A.
without credit channel. However, both, the increase in inflation and in the nominal money stock are stronger than in the case $\tilde{c} = 0$ since the output response is stronger and the bond and loan rate response – in absolute terms – is less strong. The relative multiplier in case $c_1, c_2 \neq 0$ is given by

$$\frac{dm/db}{d\pi/db} = \frac{[(1 + \delta)b + \theta](r_1 + q_1 - a_1) + (1 + \delta)\tilde{c}}{\delta b(r_1 + q_1 - a_1) + \delta \tilde{c}}$$

(64)

which is decreasing in $\tilde{c}$, i.e. it holds

$$1 < \frac{dm/db}{d\pi/db}\bigg|_{\tilde{c} > 0} < \frac{dm/db}{d\pi/db}\bigg|_{\tilde{c} = 0}$$

(65)

That is, the overshooting of the money stock over the inflation rate is smaller when the credit channel is active ($\tilde{c} > 0$).

Next, we consider an increase in the target level. Under discretionary policy (i.e. $k_\pi = k_x = 0$), an increase in the inflation target without adjustment of inflation expectations does not have any effect, neither with nor without credit channel. This implies that an anticipated change in the inflation target of the form $d\pi^e = d\pi^T$ is equivalent to an isolated change in the inflation expectations. The effects on the loan and bond rate are then given by

$$\frac{di_L}{d\pi^e} = -\frac{1}{\Gamma}(r_2 + q_2 + a_2 + \theta)(1 + b(1 + \delta)) > 0$$

(66)

$$\frac{di_B}{d\pi^e} = -\frac{1}{\Gamma}(r_1 + q_1 - a_1)(1 + b(1 + \delta)) > 0$$

(67)

The (nominal) bond and loan rate respond positively to an increase in inflation expectations. Then $d\Delta i_B^e < 0$ and $d\Delta i_L^e < 0$, leading to contractionary output effects via the credit channel. By contrast and in line with Section 3, we have a decline in the real bond rate inducing a positive output effect. The total effect on output is given by

$$\frac{dy}{d\pi^e} = \frac{1}{\Gamma}(b\phi + \tilde{c})$$

(68)

where $\phi = (r_1 + q_1)[1 - (a_2 + \theta)] - a_1(1 + r_2 + q_2)$ is typically negative since $a_2 + \theta > 1$. If

35If $k_\pi = k_x = 0$, the model equations (31) to (38) are independent of $\pi^T$. 
\( \hat{c} \) is equal to zero or sufficiently small, output responds positively to an isolated increase in inflation expectations. With increasing \( \hat{c} \), the expansionary output effect decreases:

\[
\frac{d}{d\hat{c}} \left( \frac{dy}{d\pi} \right) = \Gamma + \frac{(1 + \delta)b\phi}{\Gamma^2} < 0 \tag{69}
\]

Since the loan and bond rate rise, the credit channel diminishes the output response of an increase in inflation expectations.

To sum up, the credit channel amplifies the output effects of isolated monetary disturbances without adjustments in (inflation and interest rate) expectations.\(^{36}\) Contrarily, the credit channel may dampen the output effects if inflation expectations adapt in the same direction, as e.g. in the case \( d\pi^T = d\pi^e > 0 \). The credit channel enhances the influence of the monetary base on broad money, but changes in broad money are still less than proportional to changes in the monetary base. These results hold as long as bond and loan rate expectations do not adjust.

### 4.2 Adjustment of interest rate expectations

Until now, we have assumed that the expectations on the future bond and loan rate do not change, i.e. households expect the bond and loan rate to return to their initial steady state level. In this subsection, we study how changes in the bond and loan rate expectations affect the credit channel as transmission mechanism of monetary disturbances.

Let us first consider an isolated change in the expected future bond and/or loan rate. The bond and loan rate respond as follows:

\[
\begin{bmatrix}
    di_L \\
    di_B
\end{bmatrix} = -\frac{1}{\Gamma} \begin{bmatrix}
    (1 + \delta)(r_2 + q_2 + a_2 + \theta) \\
    (1 + \delta)(r_1 + q_1 - a_1)
\end{bmatrix} (c_1 di_B^e + c_2 di_L^e) \tag{70}
\]

and the output effects are given by

\[
\frac{dy}{di_L^e} = -\frac{c_2}{\Gamma} \left[ a_1 (r_2 + q_2) + (a_2 + \theta)(r_1 + q_1) \right] > 0 \tag{71}
\]

\[
\frac{dy}{di_B^e} = \frac{c_1}{c_2} \frac{dy}{di_L^e} > 0 \quad \text{for} \quad c_1 > 0, \ c_2 > 0 \tag{72}
\]

\(^{36}\)This result is in line with Bernanke and Blinder (1988), who study the impact of the credit channel in a variant of the textbook IS/LM model and do not take changes in private expectations into account.
Recall from Section 2 that \(c_2\) is unambiguously positive, whereas the sign of \(c_1\) is ambiguous. A rise in the expected future loan rate lowers future loan demand, lowering future marginal utility of consumption, leading to an increase in current consumption. A rise in the expected future bond rate leads, on the one hand, to a lower demand for future real balances, and, on the other hand, to higher demand for future loans to finance the rise in future bond demand. If \(c_1 > 0\), the former effect dominates and a rise in the expected future bond rate is expansionary. In this case, a rise in the expected future bond (loan) rate leads to an actual but less than proportional rise in the bond (loan) rate and shifts the MM and KK curve to the right implying \(d\Delta\bar{i}_B^e > 0\) and \(d\Delta\bar{i}_L^e > 0\).37 That is, given \(c_1, c_2 > 0\), changes in the expected future bond or loan rate are self-fulfilling. If, on the other hand, \(c_1 < 0\), a rise in the expected future bond rate is contractionary and leads to a decline in the current bond rate.

A simultaneous change in the expected future bond and loan rate of equal size on output is given by

\[
y = \frac{dy}{d\bar{i}_L^e} \left(1 + \frac{c_1}{c_2}\right) \bar{i}_L^e
\]

(73)

That is, an equal-sized increase in the expected bond and loan rate \((d\bar{i}_L^e = d\bar{i}_B^e > 0)\) has expansionary output effects if38

\[
c_1 + c_2 > 0 \quad \iff \quad 1 > \frac{\alpha_3}{\alpha_2} \left(\bar{i}_{L,0} - \bar{i}_{B,0}\right)^{\nu - 1} \quad \text{(assuming } \nu > \sigma) \tag{74}
\]

Note that \(\bar{i}_{L,0}, \bar{i}_{B,0} \ll 1\) and \(\nu > 1\). Hence, only for \(\alpha_2\) sufficiently close to zero (no utility from holding real balances), this condition may not hold.

The question arises in what direction do the loan and bond rate expectations adjust in case of an expansionary monetary policy as discussed in the previous subsection? If an isolated expansionary monetary policy \((db_m^e > 0, d\pi^T > 0, d\varepsilon_z < 0)\) without adjustment of inflation expectations \((d\pi^e = 0)\) is accompanied by an equal-sized downward adjustment of the loan and bond rate expectations \((d\bar{i}_L^e = d\bar{i}_B^e < 0)\), the amplification effect of the credit

37 From (70) we obtain (due to \(\Gamma < 0\) and \(r_1 + q_1 > a_2\)) \(\frac{dy}{d\bar{i}_L^e} > 0, \frac{dy}{d\bar{i}_B^e} > 0\) (since \(c_2 > 0\)), and \(\frac{dy}{d\bar{i}_m^e} > 0, \frac{dy}{d\bar{i}_n^e} > 0\) if \(c_1 > 0\).

38 See Appendix D, for a derivation of condition (74).
channel is reduced. In the special case of static expectations, i.e. $d\pi^e_L = d\pi^e_B < 0$ and $d\pi^e_B = d\pi^e_B < 0$, the credit channel is completely mute.

If, on the other hand, one thinks of $i^e_L$ and $i^e_B$ as long-run interest rate expectations rather than short-run expectations, it may be more reasonable to assume that expected interest rates rise rather than fall due to higher expected future inflation. We, therefore, study now the impacts of a rise in the inflation target that is accompanied with a rise in expected future inflation and an equal-sized rise in the expected future bond and loan rate ($d\pi^T = d\pi^e = d\pi^e_L = d\pi^e_B$). In general, the output effects are given by

$$dy = \frac{dy}{d\pi^T} d\pi^T + \frac{dy}{d\pi^e} d\pi^e + \frac{dy}{d\pi^e_B} d\pi^e_B + \frac{dy}{d\pi^e_L} d\pi^e_L$$

(75)

Recall that $dy/d\pi^T = 0$ in the discretionary case. Inserting the multipliers (68), (71), and (72), and assuming $d\pi^T = d\pi^e = d\pi^e_L = d\pi^e_B > 0$ then gives (cf. (68))

$$dy = \frac{1}{\bar{\Gamma}} [b\phi + \tilde{c} - (c_1 + c_2)\varphi] d\pi^T$$

(76)

where $\phi = (r_1 + q_1)[1-(a_2+\theta)]-a_1(1+r_2+q_2) < 0$ and $\varphi = a_1(r_2+q_2)+(a_2+\theta)(r_1+q_1) > 0$.

We have shown that an increase in inflation expectations ($d\pi^e > 0$) and an increase in interest rate expectations ($d\pi^e_L = d\pi^e_B > 0$) are expansionary in isolation (cf. equation (68) and (73)). Hence, a simultaneous increase is expansionary as well. Contrarily, the effect of the credit channel on output is ambiguous as equation (76) highlights. On the one hand, the credit channel diminishes the expansionary output effect through the upward-adjustment of the inflation expectations (represented by $\tilde{c}/\bar{\Gamma} < 0$). On the other hand, the upward-adjustment of the bond and loan rate expectations amplifies the output effects (represented by $-(c_1 + c_2)\varphi/\bar{\Gamma} > 0$). Given the parameter calibration in Appendix A, we find that the net amplification effect is positive. That is, without credit channel ($c_1 = c_2 = 0$) the total (positive) output effect is smaller than with credit channel ($c_1, c_2 \neq 0$).

To sum up, an isolated change in the expected future bond and/or loan rate typically leads to a less than proportional change in the current bond and/or loan rate (self-fulfilling

\footnote{This follows from (73) and (74) in case $d\pi^e_L = d\pi^e_B < 0$.}

\footnote{To save space, we consider adjustment in inflation and interest rate expectations only in case of changes in the central bank’s inflation target level. These expectation adjustments may, of course, also occur for changes in the refinancing rate or in the monetary base.}
expectations). Then an (equal-sized) increase in the bond and loan rate expectations leads to \( d\Delta i^e_L > 0 \) and \( d\Delta i^e_B > 0 \) and has expansionary effects on output. The expansionary effects of an increase in inflation expectations, on the other hand, are diminished by the credit channel (see previous section). The effects of a simultaneous adjustment of inflation and interest rate expectations may, therefore, be amplified or dampened by the credit channel.

4.3 Simulated responses under non-discretionary policy

In the previous two sections, we have, for simplicity, assumed that the central bank follows a discretionary policy. In this section, we show in a simulation exercise how the model responds if the refinancing rate follows an interest rate rule of Taylor-type \((k_\pi, k_x > 0)\).

Table 2 shows the responses of the model with credit channel for various shocks, including

<table>
<thead>
<tr>
<th>Shock Type</th>
<th>Effect on Output</th>
</tr>
</thead>
</table>
| \( db_m^e > 0, \) | \( d\varepsilon_z < 0, d\pi^T > 0, d\pi^e = d\pi^T > 0, d\pi^e_T = d\pi^e_L = d\pi^e_B > 0, d\pi^e_y > 0, d\varepsilon_T > 0, \) and \( d\varepsilon_K > 0. \) The parameter calibration is given in Appendix A. For reference purposes, we also include the responses of the model without credit channel \((c_1 = c_2 = 0).\)

Our simulation exercise confirms our main analytical results: (i) The credit channel amplifies the (output) effects of isolated monetary disturbances. (ii) If the monetary shock is accompanied by a sufficiently large adjustment in inflation expectations, the credit channel dampens the output effects, as in the case \( d\pi^e = d\pi^T > 0. \) (iii) If the monetary shock is accompanied by adjustments in inflation and interest rate expectations (in the same direction), the credit channel may either amplify or dampen the output effects. In case \( d\pi^e = d\pi^T = d\pi^e_L = d\pi^e_B > 0, \) table 2 reports an amplification of the output response.

Contrarily to our results of the previous subsection, table 2 shows that an isolated increase in the inflation expectations now leads to contractionary output effects in the model with credit channel if the refinancing rate follows an interest rate rule of Taylor-type. Due to the rise in inflation, the refinancing rate increases, which puts further upward pressure on the loan an bond rate and enhances the contractionary effect of the credit channel on output.\(^42\) The effects of an anticipated increase in the target level of the form

\(^41\) Note that setting \( \nu = \sigma \) turns off the credit channel as well but would also affect various other elasticities, e.g. the bond rate semi-elasticity of money demand.

\(^42\) The net effect on output turns contractionary only if the effect of the refinancing rate on the money and loan supply is sufficiently strong. During the simulation, we set the refinancing rate semi-elasticity of the
\( \pi_T = \pi^e > 0 \) remain, on the other hand, expansionary. Table 2 also confirms our results from Section 3 that the central bank has a limited influence on the bond rate and on the money stock. Changes in the monetary base and in the refinancing rate lead to less than proportional changes in the money stock and in the bond rate, respectively. This holds in the model with and without credit channel. The influence of monetary policy is even further reduced if the economy is hit by a contractionary credit shock. In case \( db_m = d\varepsilon_K > 0 \), the credit shock neutralizes the expansionary effects of an increase in the monetary base nearly completely.\(^{43}\)

In our simulation exercise, we also include non-monetary disturbances. In the presence of a demand shock, we find that the credit channel dampens the output response. In case of a cost shock, we find that the credit channel amplifies the output response, but dampens the inflation response.

5 Summary

This paper introduces a money and loan market into a static approximation of the New Keynesian framework. The demand side of the money and loan market follows from a money-and-credit-in-the-utility approach, where real balances and borrowing contribute to the household’s utility. The supply side of the money and loan market is deduced from simplified balance sheets of the banking system. In this framework, the central bank has no direct control over the bond rate. Instead, the central bank’s instrument variables are the monetary base, the refinancing rate, and the central bank’s inflation target. An increase in the monetary base or in the target level or a reduction in the refinancing rate increases money and loan supply, which puts downward pressure on the (current) bond and loan rate and, hence, has positive output effects. The introduced money-and-credit-in-the-utility approach implicitly gives rise to a credit channel as an additional transmission mechanism of monetary shocks in which the (current and future) bond and loan rate directly affect current goods demand.

Our main results are as follows: First, the central bank’s influence on the bond rate

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\(^{43}\)During the simulation, we set \( r_4 = 1 \) and \( a_4 = 0.9 \). Consequently, an equal-sized increase in the monetary base and in the credit shock has no immediate impact on the loan demand and only a small effect on the money supply.
and on a broader money aggregate is reduced. Changes in the refinancing rate and in the monetary base lead to less than proportional changes in the bond rate and in broad money, respectively. In the presence of a credit shock which reduces the credit and money supply multiplier, the central bank’s influence is even further reduced. Second, in line with Bernanke and Blinder (1988), the credit channel amplifies the (output) effects of isolated monetary disturbances. Third, if the monetary shock is accompanied by a sufficiently large adjustment in inflation expectations, the credit channel dampens the output effects. Fourth, if, on the other hand, the monetary shock is accompanied by adjustments in interest rate expectations (in the same direction), the amplification effect of the credit channel further increases.
Table 2: Responses in the non-discretionary case with and without credit channel

<table>
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<tr>
<th></th>
<th>$dy$</th>
<th>$d\pi$</th>
<th>$di_L$</th>
<th>$di_B$</th>
<th>$d(i_B - \pi^c)$</th>
<th>$dz$</th>
<th>$dm$</th>
<th>$d(m - p)$</th>
<th>$dk$</th>
<th>$d(k - p)$</th>
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<td>-0.03</td>
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<td>0.41</td>
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<td>(-0.04)</td>
<td>(-0.04)</td>
<td>(0.02)</td>
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<td>(0.19)</td>
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<td>(0.46)</td>
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<td>(0.16)</td>
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<td>(0.04)</td>
<td>(0.03)</td>
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</tbody>
</table>

Note: The parameter calibration is reported in Appendix A. Numbers in brackets give the responses of the model without credit channel ($c_1 = c_2 = 0$). Due to rounding, some entries are slightly inconsistent.
Appendix

A Calibration

Table A.1 shows the calibrated parameters that we choose in our simulation exercise in Section 4.3 and for the construction of the MM and KK curve in Section 3. We broadly follow the textbook calibrations given in Galí (2008) and Walsh (2010). \( \beta \) is set to 0.99 implying a quarterly steady state bond rate of \( \bar{i}_{\beta,0} = 0.01 \). The interest rate spread \( \bar{i}_{L,0} - \bar{i}_{B,0} = 0.01 \) then implies that the loan rate is \( \bar{i}_{L,0} = 0.02 \). We assume \( \sigma = \eta = 2 \) (implying \( b = 0.5 \)) and a Calvo parameter \( \omega \) set to 0.75 yielding a Phillips curve slope of \( \delta = 0.34 \). \( \nu \) is set to 25 such that the bond rate semi-elasticity of money demand is given by \( \theta = \beta^2 / [\nu(1 - \beta)] = 3.9 \), which is close to the value proposed by Galí (2008). Estimates of \( \theta \) reported in Walsh (2010) range from 1 to 10. For the semi-elasticities of the loan demand, we obtain \( q_1 = 1/[\nu(\bar{i}_{L,0} - \bar{i}_{B,0})] = 4 \) and \( q_2 = q_1(1 + \bar{i}_{L,0})/(1 + \bar{i}_{B,0}) = 4.04 \). Close to the values reported in Walsh (2010), \( \alpha_1 \) is set to 0.95. According to

\[
\left( \frac{\bar{M}_0}{\bar{K}_0} \right)^{\nu} \frac{\bar{i}_{L,0} - \bar{i}_{B,0} \alpha_2}{\bar{i}_{B,0} \alpha_3}
\]

\( \alpha_2 > \alpha_3 \) ensures that \( \bar{M}_0 > \bar{K}_0 \). We choose \( \alpha_2 = 0.04 \) and \( \alpha_3 = 0.01 \) implying \( \bar{M}_0/\bar{K}_0 = 1.06 \). Empirical data for the U.S. suggests a ratio of 1.1 to 1.5, which would imply an even smaller weight \( \alpha_3 \).\(^{44}\) These weights imply \( c_1 = 0.003 \) and \( c_2 = 0.46 \). For the semi-elasticities of the money and loan supply, we choose values that are equal or similar to the corresponding money and loan demand elasticities and satisfy the conditions \( a_3 > r_3 \), \( r_1 + q_1 > a_1 \), and \( r_4 > a_4 \).

\(^{44}\)We use quarterly and seasonally adjusted data from the U.S. economy obtained from the Board of Governance of the Federal Reserve System. Data ranges from 1981Q1 to 2014Q2. For the money stock, we use the time series 'M2' (Series ID: M2). For the loan volume, we use either the time series 'Bank Credit of All Commercial Banks' (Series ID: TOTBKCR) or 'Loans and Leases in Bank Credit, All Commercial Banks' (Series ID: TOTLL).
Table A.1: Parameter calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Definition</th>
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<tbody>
<tr>
<td><strong>Calibrated parameters</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.99</td>
<td>Discount factor</td>
</tr>
<tr>
<td>$\omega$</td>
<td>0.75</td>
<td>Calvo parameter</td>
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<tr>
<td>$\sigma$</td>
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<td>Inverse of the elasticity of intertemporal substitution</td>
</tr>
<tr>
<td>$\eta$</td>
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<td>Inverse of the Frisch elasticity of labor supply</td>
</tr>
<tr>
<td>$\nu$</td>
<td>25</td>
<td>Inverse of the elasticity of intratemporal substitution</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>0.95</td>
<td>Utility weight on consumption</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>0.04</td>
<td>Utility weight on real balances</td>
</tr>
<tr>
<td>$\alpha_3$</td>
<td>0.01</td>
<td>Utility weight on borrowing</td>
</tr>
<tr>
<td>$i_{L,0} - i_{B,0}$</td>
<td>0.01</td>
<td>Steady state interest rate spread</td>
</tr>
<tr>
<td>$k_{\pi}$</td>
<td>1.5</td>
<td>Inflation coefficient in the Taylor rule</td>
</tr>
<tr>
<td>$k_x$</td>
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<td>Output coefficient in the Taylor rule</td>
</tr>
<tr>
<td>$a_1$</td>
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<td>Loan rate semi-elasticity of money supply</td>
</tr>
<tr>
<td>$a_2$</td>
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<td>Bond rate semi-elasticity of money supply</td>
</tr>
<tr>
<td>$a_3$</td>
<td>4.0</td>
<td>Refinancing rate semi-elasticity of money supply</td>
</tr>
<tr>
<td>$a_4$</td>
<td>0.9</td>
<td>Credit-shock coefficient in money supply</td>
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<tr>
<td>$r_1$</td>
<td>4.0</td>
<td>Loan rate semi-elasticity of loan supply</td>
</tr>
<tr>
<td>$r_2$</td>
<td>4.04</td>
<td>Bond rate semi-elasticity of loan supply</td>
</tr>
<tr>
<td>$r_3$</td>
<td>3.9</td>
<td>Refinancing rate semi-elasticity of loan supply</td>
</tr>
<tr>
<td>$r_4$</td>
<td>1.0</td>
<td>Credit-shock coefficient in loan supply</td>
</tr>
<tr>
<td><strong>Implicitly given parameters</strong></td>
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<tr>
<td>$\delta$</td>
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<td>Slope of the Phillips curve</td>
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<tr>
<td>$i_{B,0}$</td>
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<td>Steady state bond rate</td>
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<tr>
<td>$i_{L,0}$</td>
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<td>Steady state loan rate</td>
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<td>Bond rate semi-elasticity of loan demand</td>
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<tr>
<td>$c_1$</td>
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<td>Expected bond rate semi-elasticity of goods demand</td>
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<tr>
<td>$c_2$</td>
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<td>Expected loan rate semi-elasticity of goods demand</td>
</tr>
<tr>
<td>$\tilde{c}$</td>
<td>7.34</td>
<td>Credit-channel coefficient</td>
</tr>
</tbody>
</table>
B First-order conditions (FOCs)

From the Lagrangian

\[ L(C_{t+k}, B_{t+k}, M_{t+k}, K_{t+k}, N_{t+k}, \Psi_{t+k}) = E_t \sum_{k=0}^{\infty} \beta^k \left\{ \frac{X_{t+k}^{1-\sigma}}{1-\sigma} - \frac{N_{t+k}^{1+\eta}}{1+\eta} - \Psi_{t+k} \left( \frac{T_{t+k}^n}{P_{t+k}} + \frac{M_{t+k-1}}{P_{t+k}} - \frac{M_{t+k}}{P_{t+k}} \right) - W_{t+k} N_{t+k} + \frac{B_{t+k}}{P_{t+k}} \right\} \]

with Lagrange multiplier \( \Psi_{t+k} \), preference index

\[ X_{t+k} = \left[ \alpha_1 C_{t+k}^{1-\nu} + \alpha_2 \left( \frac{M_{t+k}}{P_{t+k}} \right)^{1-\nu} + \alpha_3 \left( \frac{K_{t+k}}{P_{t+k}} \right)^{1-\nu} \right]^{\frac{1}{\sigma}} \]

and period utility function

\[ U(C_{t+k}, M_{t+k} P_{t+k}, K_{t+k} P_{t+k}, N_{t+k} P_{t+k}) = \frac{X_{t+k}^{1-\sigma}}{1-\sigma} - \frac{N_{t+k}^{1+\eta}}{1+\eta} \]

we obtain the FOCs with respect to \( C_t, B_t, M_t, K_t, \) and \( N_t \):

\[ \frac{\partial L}{\partial C_t} = \frac{\partial U}{\partial C_t} - \Psi_t = \alpha_1 X_t^{1-\sigma} C_t^{1-\nu} - \Psi_t = 0 \]

Then

\[ \Psi_t = \alpha_1 X_t^{1-\sigma} C_t^{1-\nu} = \frac{\partial U}{\partial C_t} \]

\[ \frac{\partial L}{\partial B_t} = -\Psi_t \frac{1}{P_t} + \beta E_t \left( \Psi_{t+1} (1 + i_{B,t}) \frac{1}{P_{t+1}} \right) = -\alpha_1 X_t^{1-\sigma} C_t^{1-\nu} \frac{1}{P_t} + \beta (1 + i_{B,t}) E_t \left( \alpha_1 X_{t+1}^{1-\sigma} C_{t+1}^{1-\nu} \frac{1}{P_{t+1}} \right) = 0 \]

\[ \iff \frac{1}{1 + i_{B,t}} X_t^{1-\sigma} C_t^{1-\nu} \frac{1}{P_t} = \beta E_t \left( \frac{X_{t+1}^{1-\sigma} C_{t+1}^{1-\nu} \frac{1}{P_{t+1}}}{P_t} \right) \]

\[ \iff \frac{1}{1 + i_{B,t}} = \beta E_t \left\{ \frac{X_{t+1}}{X_t} \right\}^{1-\sigma} \left( \frac{C_{t+1}}{C_t} \right)^{1-\nu} \frac{P_t}{P_{t+1}} \]
\[
\frac{\partial L}{\partial M_t} = \frac{\partial U}{\partial M_t} + \frac{1}{P_t} + \beta E_t \frac{1}{P_{t+1}}
\]
\[
\frac{\partial L}{\partial M_t} = \alpha_2 X_t^{\nu-\sigma} \left( \frac{M_t}{P_t} \right)^{\nu} \frac{1}{P_t} - \alpha_1 X_t^{\nu-\sigma} C_t^{-\nu} \frac{1}{P_t} + \beta E_t \left( \alpha_1 X_t^{\nu-\sigma} C_t^{-\nu} \frac{1}{P_t} \right)
\]
\[
\frac{\partial L}{\partial M_t} = \alpha_2 X_t^{\nu-\sigma} \left( \frac{M_t}{P_t} \right)^{\nu} \frac{1}{P_t} - \alpha_1 X_t^{\nu-\sigma} C_t^{-\nu} \frac{1}{P_t} + \alpha_1 \frac{1}{1 + i_{B,t}} X_t^{\nu-\sigma} C_t^{-\nu} \frac{1}{P_t} = 0
\]
\[
\frac{\partial L}{\partial N_t} = \frac{\partial U}{\partial N_t} - \frac{\partial U}{\partial C_t} \frac{1}{1 + i_{B,t}} = 0
\]

\[
\partial L = \partial U \frac{1}{P_t} - \Psi_t \frac{1}{P_t} + \beta E_t \frac{1}{P_{t+1}}
\]
\[
\partial L = \alpha_2 X_t^{\nu-\sigma} \left( \frac{M_t}{P_t} \right)^{\nu} \frac{1}{P_t} - \alpha_1 X_t^{\nu-\sigma} C_t^{-\nu} \frac{1}{P_t} + \beta E_t \left( \alpha_1 X_t^{\nu-\sigma} C_t^{-\nu} \frac{1}{P_t} \right)
\]
\[
\partial L = \alpha_2 X_t^{\nu-\sigma} \left( \frac{M_t}{P_t} \right)^{\nu} \frac{1}{P_t} - \alpha_1 X_t^{\nu-\sigma} C_t^{-\nu} \frac{1}{P_t} + \alpha_1 \frac{1}{1 + i_{B,t}} X_t^{\nu-\sigma} C_t^{-\nu} \frac{1}{P_t} = 0
\]

\[
\frac{\partial L}{\partial K_t} = \frac{\partial U}{\partial K_t} - \frac{\partial U}{\partial C_t} \frac{1}{1 + i_{B,t}}
\]
\[
\frac{\partial L}{\partial K_t} = \alpha_3 X_t^{\nu-\sigma} \left( \frac{K_t}{P_t} \right)^{\nu} \frac{1}{P_t} + \alpha_1 X_t^{\nu-\sigma} C_t^{-\nu} \frac{1}{P_t}
\]
\[
\frac{\partial L}{\partial K_t} = \alpha_3 X_t^{\nu-\sigma} \left( \frac{K_t}{P_t} \right)^{\nu} \frac{1}{P_t} + \alpha_1 X_t^{\nu-\sigma} C_t^{-\nu} \frac{1}{P_t}
\]
\[
\frac{\partial L}{\partial K_t} = \alpha_3 \left( \frac{K_t}{P_t} \right)^{\nu} = \alpha_1 C_t^{-\nu} \left( \frac{1 + i_{L,t}}{1 + i_{B,t}} - 1 \right)
\]
\[
\frac{\partial L}{\partial K_t} = \frac{\alpha_1 i_{L,t} - i_{B,t}}{\alpha_3 C_t^{-\nu}}
\]

\[
\frac{\partial L}{\partial N_t} = -N_t^\eta + \frac{W_t}{P_t} = -N_t^\eta + \alpha_1 X_t^{\nu-\sigma} C_t^{-\nu} \frac{W_t}{P_t} = 0
\]
\[
\frac{\partial L}{\partial N_t} = -\frac{W_t}{P_t} = \frac{\partial U/\partial N_t}{\partial U/\partial C_t}
\]

**C Log-linearization**

Use small letters \( (m_t = \log M_t, p_t = \log P_t, c_t = \log C_t) \) and neglect constants.
C.1 Money demand equation

The FOC (B.14) implies

\[-\nu(m_t - p_t) = -\nu c_t + \log \frac{i_{B,t}}{1 + i_{B,t}}\]  (C.1)

From the Taylor approximation

\[\log \frac{i_{B,t}}{1 + i_{B,t}} \approx \log \frac{i_{B,0}}{1 + i_{B,0}} + \frac{1 + i_{B,0}}{i_{B,0}} \frac{1}{(1 + i_{B,t})^2} (i_{B,t} - i_{B,0})\]  (C.2)

we obtain

\[m_t - p_t = c_t - \frac{1}{\nu \iota_{B,0}} \frac{1}{1 + i_{B,0}} (i_{B,t} - \bar{i}_{B,0}) = c_t - \frac{\beta^2}{\nu (1 - \beta)} (i_{B,t} - \bar{i}_{B,0})\]  (C.3)

where \(\beta = \frac{1}{1 + i_{B,0}}\) and \(\bar{i}_{B,0} = \frac{1 - \beta}{\beta}\).

C.2 Credit demand equation

The FOC (B.19) implies

\[-\nu(k_t - p_t) = -\nu c_t + \log \frac{i_{L,t} - i_{B,t}}{1 + i_{B,t}}\]  (C.4)

where

\[\log \frac{i_{L,t} - i_{B,t}}{1 + i_{B,t}} \approx \log \frac{i_{L,0} - i_{B,0}}{1 + i_{B,0}} + \frac{1 + i_{B,0}}{i_{L,0} - i_{B,0}} \frac{1}{1 + i_{B,t}} (i_{L,t} - \bar{i}_{L,0})\]

\[-\frac{1 + i_{B,0}}{\bar{i}_{L,0} - \bar{i}_{B,0}} \frac{1 + \bar{i}_{L,0}}{1 + i_{B,t}}^2 (i_{B,t} - \bar{i}_{B,0})\]  (C.5)

Then

\[k_t - p_t = c_t + \frac{1}{\nu \iota_{L,0} - \iota_{B,0}} \frac{1}{1 + i_{B,0}} \left( \frac{1 + \bar{i}_{L,0}}{1 + i_{B,t}} (i_{B,t} - \bar{i}_{B,0}) - (i_{L,t} - \bar{i}_{L,0}) \right)\]  (C.6)
C.3 Euler equation

Rewrite the FOC (B.9) as

\[ \beta \frac{1 + i_{B,t}}{E_t \left( \frac{P_{t+1} - P_t}{P_t} + 1 \right)} E_t \left\{ \left( \frac{X_{t+1}}{X_t} \right)^{\nu - \sigma} \left( \frac{C_{t+1}}{C_t} \right)^{-\nu + \sigma} \left( \frac{C_t}{C_t} \right)^{-\sigma} \right\} = 1 \]  (C.7)

Then

\[ i_{B,t} - E_t \pi_{t+1} - \tilde{\tau}_{B,0} + (\nu - \sigma)(E_t x_{t+1} - x_t) \]

\[ - (\nu - \sigma)(E_t c_{t+1} - c_t) - \sigma(E_t c_{t+1} - c_t) = 0 \]  (C.8)

\[ \Leftrightarrow \sigma c_t = \sigma E_t c_{t+1} - (i_{B,t} - E_t \pi_{t+1} - \tilde{\tau}_{B,0}) + (\nu - \sigma)\{E_t(c_{t+1} - x_{t+1}) - (c_t - x_t)\} \]  (C.9)

where \( \pi_{t+1} = \frac{P_{t+1} - P_t}{P_t} \). Since

\[ x_t = \frac{dX_t}{X_0} = \frac{X_0^{-1}}{X_0} \left\{ \alpha_1 \tilde{C}_0^{1 - \nu} c_t + \alpha_2 \left( \frac{M_0}{P_0} \right)^{1 - \nu} (m_t - p_t) + \alpha_3 \left( \frac{K_0}{P_0} \right)^{1 - \nu} (k_t - p_t) \right\} \]  (C.10)

\[ = \frac{1}{X_0^{1 - \nu}} \left\{ \alpha_1 \tilde{C}_0^{1 - \nu} c_t + \alpha_2 \left( \frac{M_0}{P_0} \right)^{1 - \nu} \left[ c_t - \frac{\beta^2}{\nu(1 - \beta)}(i_{B,t} - \tilde{\tau}_{B,0}) \right] + \alpha_3 \left( \frac{K_0}{P_0} \right)^{1 - \nu} \left[ c_t + \frac{1}{\nu i_{L,0} - \tilde{\tau}_{L,0}} \{\lambda(i_{B,t} - \tilde{\tau}_{B,0}) - (i_{L,t} - \tilde{\tau}_{L,0})\} \right] \right\} \]  (C.11)

where \( \lambda = \frac{1 + i_{L,0}}{1 + \tilde{\tau}_{B,0}} \), we obtain

\[ x_t = \frac{1}{X_0^{1 - \nu}} \left\{ \alpha_1 \tilde{C}_0^{1 - \nu} + \alpha_2 \left( \frac{M_0}{P_0} \right)^{1 - \nu} + \alpha_3 \left( \frac{K_0}{P_0} \right)^{1 - \nu} \right\} c_t \]

\[ - \frac{1}{X_0^{1 - \nu}} \frac{1}{\nu i_{L,0} - \tilde{\tau}_{L,0}} \left\{ \alpha_2 \frac{\beta^2}{1 - \beta} \left( \frac{M_0}{P_0} \right)^{1 - \nu} - \alpha_3 \frac{1}{i_{L,0} - \tilde{\tau}_{B,0}} \lambda \left( \frac{K_0}{P_0} \right)^{1 - \nu} \right\} (i_{B,t} - \tilde{\tau}_{B,0}) \]

\[ - \frac{1}{X_0^{1 - \nu}} \frac{1}{\nu i_{L,0} - \tilde{\tau}_{B,0}} \left\{ \alpha_3 \frac{1}{i_{L,0} - \tilde{\tau}_{L,0}} \left( \frac{K_0}{P_0} \right)^{1 - \nu} (i_{L,t} - \tilde{\tau}_{L,0}) \right\} \]  (C.12)

\[ = c_t - \chi_1 (i_{B,t} - \tilde{\tau}_{B,0}) - \chi_2 (i_{L,t} - \tilde{\tau}_{L,0}) \]  (C.13)

implying

\[ c_t - x_t = \chi_1 (i_{B,t} - \tilde{\tau}_{B,0}) + \chi_2 (i_{L,t} - \tilde{\tau}_{L,0}) \]  (C.14)
Then
\[
c_t = E_t c_{t+1} - \frac{1}{\sigma} (i_{B,t} - E_t \pi_{t+1} - \bar{i}_{B,0}) + \frac{\nu - \sigma}{\sigma} E_t \Delta (c_{t+1} - x_{t+1}) \tag{C.15}
\]
\[
e_t c_{t+1} - \frac{1}{\sigma} (i_{B,t} - E_t \pi_{t+1} - \bar{i}_{B,0}) + \frac{\nu - \sigma}{\sigma} \{ \chi_1 E_t \Delta i_{B,t+1} + \chi_2 E_t \Delta i_{L,t+1} \} \tag{C.16}
\]

D Sign of \(c_1\) and \(c_1 + c_2\)

The definition of \(c_1\) implies in case \(\nu > \sigma\):
\[
c_1 > 0 \iff \chi_1 > 0 \iff \alpha_2 \frac{1}{i_{B,0}} \left( \frac{M_0}{P_0} \right)^{1-\nu} - \alpha_3 \frac{1 + i_{L,0}}{i_{L,0} - i_{B,0}} \left( \frac{K_0}{P_0} \right)^{1-\nu} > 0 \tag{D.1}
\]
\[
\iff \frac{\alpha_2}{\alpha_3} \left( \frac{M_0/P_0}{K_0/P_0} \right)^{1-\nu} \bar{i}_{B,0} > \frac{1 + i_{L,0}}{i_{L,0} - i_{B,0}} \tag{D.2}
\]

where
\[
\left( \frac{M_0}{P_0} \right)^{1-\nu} = \left( \frac{\alpha_1}{\alpha_2} \right)^{\frac{1-\nu}{\nu}} \left( \frac{i_{B,0}}{1 + i_{B,0}} \right)^{-\frac{1-\nu}{\nu}} \tag{D.3}
\]
\[
\left( \frac{K_0}{P_0} \right)^{1-\nu} = \left( \frac{\alpha_1}{\alpha_3} \right)^{\frac{1-\nu}{\nu}} \left( \frac{i_{L,0} - i_{B,0}}{1 + i_{B,0}} \right)^{-\frac{1-\nu}{\nu}} \tag{D.4}
\]
\[
\left( \frac{M_0/P_0}{K_0/P_0} \right)^{1-\nu} = \left( \frac{\alpha_3}{\alpha_2} \right)^{\frac{1-\nu}{\nu}} \left( \frac{i_{B,0}}{i_{L,0} - i_{B,0}} \right)^{-\frac{1-\nu}{\nu}} \tag{D.5}
\]

Then
\[
c_1 > 0 \iff \frac{\alpha_2}{\alpha_3} \left( \frac{\alpha_3}{\alpha_2} \right)^{\frac{1-\nu}{\nu}} \left( \frac{i_{B,0}}{i_{L,0} - i_{B,0}} \right)^{-\frac{1-\nu}{\nu}} \bar{i}_{B,0} \frac{(1 + i_{L,0})}{i_{L,0} - i_{B,0}} > \bar{i}_{B,0} \tag{D.6}
\]
\[
\iff \left( \frac{\alpha_2}{\alpha_3} \right)^{\frac{1-\nu}{\nu}} \left( \frac{i_{B,0}}{i_{L,0} - i_{B,0}} \right)^{-\frac{1-\nu}{\nu}} > 1 + i_{L,0} \tag{D.7}
\]
\[
\iff \frac{\alpha_2}{\alpha_3} > \frac{i_{B,0}}{i_{L,0} - i_{B,0}} \left( 1 + i_{L,0} \right)^{\nu} \tag{D.8}
\]
Analogously, it holds for $\nu > \sigma$:

$$c_1 + c_2 > 0 \iff \chi_1 + \chi_2 > 0 \iff$$

$$\frac{\alpha_2}{(1 + i_{B,0})i_{B,0}} \left( \frac{M_0}{P_0} \right)^{1-\nu} - \frac{\alpha_3}{i_{L,0} - i_{B,0}} \frac{1 + i_{L,0}}{1 + i_{B,0}} \left( \frac{K_0}{P_0} \right)^{1-\nu} + \frac{\alpha_3}{i_{L,0} - i_{B,0}} \left( \frac{K_0}{P_0} \right)^{1-\nu} > 0$$

(D.9)

$$\iff \frac{\alpha_2}{(1 + i_{B,0})i_{B,0}} \left( \frac{M_0}{P_0} \right)^{1-\nu} > \frac{\alpha_3}{i_{L,0} - i_{B,0}} \left( \frac{1 + i_{L,0}}{1 + i_{B,0}} - 1 \right) \left( \frac{K_0}{P_0} \right)^{1-\nu}$$

(D.10)

$$\iff \frac{\alpha_2}{\alpha_3} \left( \frac{M_0/P_0}{K_0/P_0} \right)^{1-\nu} > i_{B,0}$$

(D.11)

$$\iff 1 > \frac{\alpha_3 i_{B,0} (i_{L,0} - i_{B,0})^{\nu-1}}{\alpha_2}$$

(D.12)

(D.13)
References


