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Prices, Product Differentiation, and Heterogeneous Search Costs

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Prices, Product Differentiation, and Heterogeneous Search Costs

Abstract

We study price formation in the standard model of consumer search for differentiated products but allow for search cost heterogeneity. In doing so, we dispense with the usual assumption that all consumers search at least once in equilibrium. This allows us to analyze the manner in which prices affect the decision to search rather than to not search at all, which is an important but often neglected aspect of the price mechanism. Recognizing the role the equilibrium price plays in consumers' participation decisions turns out to be critical for understanding how search costs affect market power. This is because the two margins that determine prices - the *intensive search margin*, or search intensity, and *the extensive search* margin, or search participation - may be affected in opposing directions by a change in search costs. When search costs go up, fewer consumers decide to search, which modifies the search composition of demand such that demand can become more elastic. At the same time, the consumers who choose to search reduce their search intensity, which makes demand less elastic. Whether the effect on the extensive or the intensive search margin dominates depends on the range and shape of the search cost density. We identify conditions for higher search costs to result in higher, constant, or lower prices. Similar results are obtained when the marginal gains from search vary across consumers.

JEL-Code: D430, D830, L130.

Keywords: sequential search, search cost heterogeneity, differentiated products, existence and uniqueness of equilibrium.

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1 Introduction

Ever since Stigler (1961), search theory has become an important tool for understanding the functioning of real-world markets. Seminal contributions include the homogeneous product market models of Burdett and Judd (1983), Reinganum (1979), and Stahl (1989), in which the prominent phenomenon of price dispersion is given microfoundations based on search theory. In markets where products are horizontally differentiated, the pioneering works of Wolinsky (1986) and Anderson and Renault (1999) show that prices remain above costs even if firm entry is costless and that prices may initially fall with the degree of product differentiation. An important and well-known result in this literature that is common to homogeneous and differentiated product market models is that higher search costs lead to higher prices, thus benefiting firms at the expense of consumers.

Most of the previous literature has proceeded under either of two restrictive assumptions. The first restrictive assumption is that search costs are required to be "low enough," *de facto* implying that all consumers choose to search at least once in equilibrium rather than to not search at all (e.g. Stahl, 1989; Burdett and Judd, 1983; Wolinsky, 1986). As pointed out by Stiglitz (1989), the alternative assumption that search costs are large may cause the market to collapse. For example, in the setting of Diamond (1971), the only price that can be part of a market equilibrium is the monopoly price (the well-known "Diamond paradox"). If the search cost is relatively high, the surplus consumers derive at the monopoly price may be insufficient to cover the cost of the first search, in which case consumers rather do not search at all and the market fails to exist.

This problem need not arise when consumer search costs are heterogeneous. However, the second restrictive assumption is precisely that most models in the literature abstract from consumer search cost heterogeneity or only allow for unrealistic forms of it, typically with some consumers having a search cost equal to zero (usually referred to as the "shoppers"), while the rest faces a positive and identical search cost (the "non-shoppers").[1]

This paper studies price formation in Wolinsky's (1986) consumer search model for differentiated products while allowing for more general forms of consumer search cost heterogeneity. In Wolinsky's model, a continuum of firms compete in prices to sell their horizontally differentiated products to consumers who search sequentially with the purpose of finding a satisfactory good. An individual

[1] There are some exceptions to this in the literature on consumer search for homogeneous products. For example Bénabou (1993), Rob (1985), Rauh (2009), Stahl (1996), and Tappata (2009) study homogeneous product models with search cost heterogeneity. In these papers, however, search costs are restricted to be sufficiently low so that all consumers search at least once. In Janssen and Moraga-González (2004), Janssen et al. (2005), and Rauh (2004) consumer participation is endogenous but the search cost heterogeneity assumed is rather special.

consumer visits a first store and learns how well the product of the chosen store matches her preferences. If the match value at the visited store is sufficiently good, the consumer stops searching and buys the product; otherwise, she walks away and searches at another store, again facing a tradeoff between buying and visiting another store. This process continues until a store is found that offers a sufficiently good match value. In taking these decisions, consumers have correct conjectures about the equilibrium price.[2]

When allowing for more general forms of consumer search cost heterogeneity, we are naturally led to discard the assumption that search costs are "sufficiently low." This is because in search markets with arbitrary heterogeneity in consumer search costs, the price mechanism ought to affect not only the intensity with which consumers search (which we call the *intensive search margin*) but also the share of consumers who choose to search for a good deal in the first place (which we refer to as the *extensive search margin*). The literature, by assuming that all consumers search at least once, has typically focused on the effects of the intensive search margin on price determination and thereby neglected the role of the extensive search margin.[3]

Correspondingly, an individual consumer only chooses to search the market when the surplus she expects to obtain is strictly positive. An individual firm, facing demand only from the consumers who choose to search, sets its price to maximize the average profits. We show that the model has a unique equilibrium in pure strategies when the density of match values is increasing and log-concave and the search cost density is also log-concave.

Failing to recognize that the price might affect both the intensive and the extensive search margins turns out to be critical for a complete understanding of the functioning of consumer search markets. In fact, we show that the relationship between search costs and the equilibrium price critically depends on both the *range* of search costs as well as on the *properties* of the search cost density. Interestingly, it turns out that the equilibrium price can increase, remain constant, or decrease as search costs increase for all consumers.

When the range of search costs is sufficiently small, all consumers choose to search. In this case, an exogenous (small) increase in search costs has only a bearing on the intensive search margin.

[2]Arguably Wolinsky's framework has become the workhorse model of consumer search for differentiated products. Recent work that has sprouted from his article includes, for example, Bar-Isaac et al. (2012), which extends the model to the case of quality-differentiated firms and studies design-differentiation, Armstrong et al. (2009) and Haan and Moraga-González (2011), which study the emergence and the price effects of prominence, Moraga-González and Petrikaitė (2013), which examines the effect of search costs on mergers, and Zhou (2014), which studies multiproduct search.

[3]This point is also the central tenet in Anderson and Renault (2006), who, by allowing for arbitrary search costs, are able to reconcile the empirical observation that much of the advertising we observe in arguably search environments does impart only match information and not price information.

Consumers, facing higher search costs, all choose to search less and accept products that are less attractive. Firms, anticipating that average demand becomes less elastic, respond by raising their prices.

However, when the range of search costs is sufficiently large, a shock that makes search costs higher affects both the intensive and the extensive search margins. As a result of the effect of the shock on search intensity, as explained before, firms tend to respond by raising prices. However, as a result of its effect on consumer participation, firms may get an incentive to lower their prices instead. This occurs because fewer consumers choose to search the market for a satisfactory product when search costs increase and, as it turns out, those who make such a choice are the ones with relatively low search costs. This change in the composition of demand may be such that the average (over the consumers who choose to search) demand becomes more elastic. We derive necessary and sufficient conditions under which *higher search costs result in lower, constant, or higher prices.*

Intuitively, the conditions have to do with how the different percentiles of the search cost density are affected by a shock that raises search costs. When an increase in search costs relatively more strongly affects the low percentiles of the search cost density then the effect on the intensive search margin dominates and higher search costs result in higher prices. We show that a sufficient condition for this to occur is that the search cost density has the monotone increasing likelihood ratio property. By contrast, when an increase in search costs is more strongly felt at the upper percentiles of the search cost density, then the impact on the extensive search margin dominates and prices decrease as search costs increase. This occurs when the search cost density has a likelihood ratio that decreases monotonically. When the likelihood ratio is constant, the equilibrium price stays the same after search costs have gone up because the effects on the intensive and extensive search margins exactly offset each other.

The insight that higher search frictions can result in higher, equal, or lower prices is fairly general and does not depend on modeling assumptions such as the number of firms or the search protocol. In Section 2.2 we extend the analysis by considering the case of a finite number of firms, as in Anderson and Renault (1999). Elsewhere we examine the case in which consumers search non-sequentially.[4]

The paper also studies the role of other sources of heterogeneity. The most important insight we derive is that heterogeneity that has a bearing on the *marginal* benefits from search affects price setting in a way quite different from heterogeneity that affects the *total* gains from search. For

[4]The analysis of the non-sequential search case is available from the authors upon request. For an empirical application, see Moraga-González et al. (2014).

example, suppose that consumers have to pay a fixed search cost in order to start searching; after paying this cost, they search as usual. Assume further that consumers are heterogeneous in terms of the fixed search cost only. In this case, an increase in fixed search costs results in a drop in demand but this has no bearing on price setting because the average demand remains equally elastic. Similar insights arise when consumer heterogeneity is in terms of valuations. Therefore, when consumer valuations vary so that there is heterogeneity in the marginal gains from search, an increase in consumer valuations may result in higher, constant, or lower prices.

An important result of this paper is to derive conditions on search cost heterogeneity under which higher search costs may result in higher, equal, or lower prices. This result is relevant for the recent literature on obfuscation, which points out that firms have incentives to obfuscate their products by raising the costs consumers have to incur to inspect their offers (Ellison and Wolitzky, 2012; Wilson, 2010). Our result tells that under certain conditions firms may benefit from doing exactly the opposite, that is, by *lowering* search costs.

A few recent papers have put forward situations in which higher search costs do not necessarily lead to higher prices. The only paper in which the mechanism is similar to ours is Janssen et al. (2005). Janssen et al. study a homogenous product market with sequential search similar to Stahl's (1989) setting and show that prices will surely fall after search costs increase, provided the search cost is sufficiently high. Though the present paper deals with differentiated products, our analysis suggests that their outcome is the result of the special "shoppers and non-shoppers" assumption on consumer search cost heterogeneity. Chen and Zhang (2011), who enrich Stahl's (1989) setting by adding loyal consumers, show that a reduction in the search cost sometimes leads to higher equilibrium prices. In a different framework where search is price-directed, Armstrong and Zhou (2011) show that higher search costs lead to lower prices. In a model in which consumers search for various products from multiproduct firms, Zhou (2014) demonstrates that product externalities can lead firms to cut their prices when search costs go up. Finally, in a model with vertical relations, Janssen and Shelegia (2014) encounter situations where retail prices decrease as search costs rise.

The structure of the remainder of the paper is as follows. Section 2 presents the model of sequential search for differentiated products with heterogeneous consumers and studies the existence and uniqueness of a symmetric equilibrium. In Section 2.1 we study the effects of an increase in search costs on the equilibrium price. For robustness purposes, a duopoly version of the model is examined in Section 2.2. A discussion on the effects of fixed search cost heterogeneity is given in Section 3; there we also discuss the likely implications of valuation heterogeneity. We conclude in

Section 4. Most proofs have been placed in the Appendix to ease the reading of the paper.

2 Sequential search for differentiated products

We adopt the framework proposed in the seminal contribution of Wolinsky (1986) but we allow consumers to have heterogeneous search costs. Consider a market with infinitely many consumers and firms and, without loss of generality, normalize the number of consumers per firm to 1.[5] Firms produce horizontally differentiated products using the same constant returns to scale technology of production; let r be the marginal cost of the firms. Aiming at maximizing their expected profits, firms choose their prices simultaneously. We focus on symmetric Nash equilibria (SNE); let p^* denote an SNE price.

A consumer m has tastes for a product i described by an indirect utility function that is $u_{im} = \varepsilon_{im} - p_i$ if she buys product i at price p_i and zero if she does not buy the product. The parameter ε_{im} is a match value between consumer m and product i. We assume that the match value ε_{im} is the realization of a random variable distributed on the interval $[0, \overline{\varepsilon}]$ according to a differentiable cumulative distribution function (CDF) denoted by F. Match values ε_{im} are independently distributed across consumers and products. Moreover, they are private information of consumers so personalized pricing is not possible. Let f be the probability density function (PDF) of F. We assume that f is log-concave. For later use, we define the monopoly price as $p^m = \arg\max_p (p - r)(1 - F(p))$.

Consumers search sequentially in order to maximize expected utility. While searching, they have correct beliefs about the equilibrium price and can recall previously inspected products costlessly. Consumers differ in their (marginal) costs of search.[6] A buyer's search cost is drawn independently from a differentiable cumulative distribution function G with support $(\underline{c}, \overline{c})$ and positive density g everywhere. We refer to the difference between the upper and lower bound of the search cost distribution as the *range* of search costs. We require the lower bound of the search cost distribution $\underline{c}$ to be sufficiently low because otherwise no consumer would search and the market would collapse; without loss of generality, we normalize it to zero.

We now characterize the SNE price. In order to do so, we derive the payoff of a firm, say i, that deviates from the SNE price p^* by charging a price $p_i \neq p^*$. Next, we compute the first order condition (FOC), apply the symmetry condition $p_i = p^*$, and study the existence and uniqueness of

[5] In Section 2.2 we examine the case of a finite number of firms.

[6] In some settings consumers may need to incur a *fixed* search cost in order to start searching. For a discussion of the effects of fixed search cost heterogeneity, see Section 3. In that section we also analyze the role of valuation heterogeneity.

the SNE.

Consider the (expected) payoff to a firm i that deviates from the equilibrium by charging a price p_i. In order to compute firm i's demand, we first need to characterize consumer search behavior. Since consumers do not observe deviations before searching, we can rely on Kohn and Shavell (1974), who study the search problem of a consumer who faces a set of independently and identically distributed options with a known distribution. Kohn and Shavell show that the optimal search rule is static in nature and has the stationary reservation utility property. Accordingly, consider a consumer with search cost c and denote the solution to

$$h(x) \equiv \int_x^{\overline{\varepsilon}} (\varepsilon - x) f(\varepsilon) d\varepsilon = c \tag{1}$$

in x by $\hat{x}(c)$. The left-hand-side (LHS) of equation (1) is the expected benefit in symmetric equilibrium from searching one more time for a consumer whose best option so far is x. Its right-hand-side (RHS) is the consumer's cost of search. Hence $\hat{x}(c)$ represents the threshold match value above which a consumer with search cost c will optimally decide not to continue searching for another product. The function h is monotonically decreasing. Moreover, $h(0) = E[\varepsilon]$ and $h(\overline{\varepsilon}) = 0$. It is readily seen that for any $c \in [0, \min\{\overline{c}, E[\varepsilon]\}]$, there exists a unique $\hat{x}(c)$ that solves equation (1). Differentiating equation (1) successively, we obtain

$$\hat{x}'(c) = -\frac{1}{1 - F(\hat{x}(c))} < 0;$$
$$\hat{x}''(c) = \frac{f(\hat{x}(c)) \left[\hat{x}'(c)\right]^2}{1 - F(\hat{x}(c))} > 0,$$

which implies that $\hat{x}(c)$ is a decreasing and convex function of c on $[0, \min\{\overline{c}, E[\varepsilon]\}]$, with $\hat{x}(E[\varepsilon]) = 0$ and $\hat{x}(0) = \overline{\varepsilon}$.[7]

In order to compute firm i's demand, consider a consumer with search cost c who shows up at firm i to inspect its product after possibly having inspected other products. Let $\varepsilon_i - p_i$ denote the utility the consumer derives from the product of firm i. Obviously, if alternative i is not the best one so far, the consumer will discard it and search again. Therefore, only when the deal offered by firm i happens to be the best so far, the consumer will consider to stop searching and and buy the product right away. For this decision, the consumer compares the gains from an additional search with the costs of such a search. In this comparison, the consumer holds correct expectations about the equilibrium price so she expects the other firms to charge p^*. The expected gains from

[7] Consumers with search cost $c > E[\varepsilon]$, if any, will automatically drop from the market and can therefore be ignored right away. Therefore $\hat{x}(c)$ is well-defined for every consumer that matters for pricing.

searching one more firm, say firm j, are equal to $\int_{\varepsilon_i - p_i + p^*}^{\overline{\varepsilon}} [\varepsilon_j - (\varepsilon_i - p_i + p^*)] f(\varepsilon_j) d\varepsilon_j$. Comparing this to equation (1), it follows that, conditional on having arrived at firm i, the probability that buyer c stops searching at firm i is equal to $\Pr[\varepsilon_i - p_i > \hat{x}(c) - p^*] = 1 - F(\hat{x}(c) + p_i - p^*)$. With the remaining probability, the consumer finds the product of firm i not good enough and continues searching; with infinitely many firms, such a consumer will surely buy at another firm. Because a consumer with search cost c may visit firm i after having visited no, one, two, three, etc. other firms, the unconditional probability she stops searching and buys at firm i is

$$\frac{1 - F(\hat{x}(c) + p_i - p^*)}{1 - F(\hat{x}(c))}. \tag{2}$$

To obtain the payoff of firm i we need to integrate expression (2) over the consumers who decide to search the market for a satisfactory product; in other words, we need to integrate over those consumers who derive expected positive surplus from participation. To compute the surplus a consumer with search cost c obtains from participation, we note that she will stop and buy at the first firm she visits whenever the match value there is greater than $\hat{x}(c)$; otherwise she will drop the first option and continue searching. In the latter case she will encounter herself exactly in the same situation as before because, conditional on participating, she will continue searching until she finds a match value for which it is worth to stop searching. Denoting by $CS(c)$ her consumer surplus, we have that:[8]

$$CS(c) = \hat{x}(c) - p^*.$$

Setting this surplus equal to zero, we obtain the critical search cost value above which consumers will refrain from participating in the market. Using equation (1), solving $\hat{x}(c) - p^* = 0$ for c gives

$$\tilde{c}(p^*) = \int_{p^*}^{\overline{\varepsilon}} (\varepsilon - p^*) f(\varepsilon) d\varepsilon. \tag{3}$$

Depending on how large the range of search costs is, more or less consumers will choose to search the market for a satisfactory deal. Correspondingly, we define

$$c_0(p^*) \equiv \min\{\overline{c}, \tilde{c}(p^*)\}.$$

We refer to the decision to search as the *extensive search margin*. Because $\tilde{c}(p^*)$ is decreasing in p^*, if consumers expect a higher equilibrium price then fewer of them will choose to search the market for an acceptable product. The standard assumption in the search cost literature has been that $c_0(p^*) = \overline{c}$, which implies that all consumers search at least once. Because $c_0(p^*)$ might depend on the equilibrium price, this assumption is undeniably restrictive.

[8] For a formal derivation of this expression, see the Appendix.

The payoff to the deviant firm i is then:

$$\pi(p_i; p^*) = (p_i - r)D(p_i, p^*), \tag{4}$$

where demand $D(p_i, p^*)$ is[9]

$$D(p_i, p^*) = \int_0^{c_0(p^*)} \frac{1 - F(\hat{x}(c) + p_i - p^*)}{1 - F(\hat{x}(c))} g(c)dc. \tag{5}$$

The FOC is given by:

$$\int_0^{c_0(p^*)} \frac{1 - F(\hat{x}(c) + p_i - p^*)}{1 - F(\hat{x}(c))} g(c)dc - (p_i - r) \int_0^{c_0(p^*)} \frac{f(\hat{x}(c) + p_i - p^*)}{1 - F(\hat{x}(c))} g(c)dc = 0.$$

Applying symmetry, i.e., $p_i = p^*$, we can rewrite the FOC as:

$$p^* = r + \frac{G(c_0(p^*))}{\int_0^{c_0(p^*)} \frac{f(\widehat{x}(c))}{1 - F(\hat{x}(c))} g(c)dc}. \tag{6}$$

We now show that a candidate symmetric equilibrium price exists and is unique. First consider the case in which search costs are sufficiently low, i.e. $\overline{c} < \tilde{c}(p^*)$. Under this parameter constraint, $c_0(p^*) = \overline{c}$ and therefore expression (6) gives the candidate equilibrium price explicitly. In this case, obviously, there exists a unique candidate equilibrium price.[10]

When search costs are not restricted to be sufficiently small then $\overline{c} > \tilde{c}(p^*)$ and correspondingly $c_0(p^*) = \tilde{c}(p^*)$. In this case the equilibrium price is given implicitly by the solution to equation (6). We now show that equation (6) has a unique solution in this case as well. For this we define the function

$$L(p) \equiv G(\tilde{c}(p)) - (p - r) \int_0^{\tilde{c}(p)} \frac{f(\widehat{x}(c))}{1 - F(\widehat{x}(c))} g(c)\, dc$$

for $p \in [r, p^m]$, where p^m as before denotes the monopoly price. Note that

$$L(r) = G(\tilde{c}(r)) > 0.$$

Also observe that $L(p^m)$ can be written as

$$L(p^m) = \int_0^{\tilde{c}(p^m)} \frac{1 - F(\hat{x}(c)) - (p^m - r)f(\hat{x}(c))}{1 - F(\hat{x}(c))} g(c)\, dc. \tag{7}$$

[9] In writing this payoff we have assumed that $p_i < p^*$. For $p_i > p^*$ the payoff is slightly different because the expression $\hat{x}(c) + p_i - p^*$ that is part of $D(p_i, p^*)$ can exceed $\overline{\varepsilon}$. Denote by $\hat{c}(p_i)$ the solution of the equation $\hat{x}(c) + p_i - p^* = \overline{\varepsilon}$ for c. Since $\hat{x}(c)$ is strictly decreasing in c, we get $\hat{x}(c) + p_i - p^* < \overline{\varepsilon}$ for all $c > \hat{c}(p_i)$. Therefore the payoff would be

$$\pi(p_i; p^*) = (p_i - r) \int_{\hat{c}(p_i)}^{c_0(p^*)} \frac{1 - F(\hat{x}(c) + p_i - p^*)}{1 - F(\hat{x}(c))} g(c)\, dc.$$

The FOC at the symmetric equilibrium price is however exactly the same as the one we present below.

[10] To be more precise, the condition we need for this is $\overline{c} < \int_{p^*}^{\overline{\varepsilon}} (\varepsilon - p^*) f(\varepsilon) d\varepsilon$, where $p^* = r + \frac{1}{\int_0^{\overline{c}} \frac{f(\widehat{x}(c))}{1 - F(\widehat{x}(c))} g(c)dc}$.

The sign of this expression depends on the sign of the numerator of the fraction in the integrand. We now argue that $L(p^m) < 0$ because $1 - F(\hat{x}(c)) - (p^m - r)f(\hat{x}(c)) \leq 0$ for all $c \in [0, \tilde{c}(p^m)]$. In fact, note that by log-concavity of f, because $\hat{x}(c)$ decreases in c, it follows that $f(\hat{x}(c))/[1 - F(\hat{x}(c))]$ decreases in c, which implies that $1 - F(\hat{x}(c)) - (p^m - r)f(\hat{x}(c))$ increases in c. Because $\hat{x}(c(p)) = p$, if we set $c = \tilde{c}(p^m)$ in the expression $1 - F(\hat{x}(c)) - (p^m - r)f(\hat{x}(c))$, we get the monopoly pricing rule $1 - F(p^m) - (p^m - r)f(p^m) = 0$. We can now conclude that $L(p^m) < 0$ because the expression $1 - F(\hat{x}(c)) - (p^m - r)f(\hat{x}(c))$ is increasing in c and takes on value zero when we compute it at the upper bound of the integral.

Taken together, $L(r) > 0$ and $L(p^m) < 0$ imply that a candidate equilibrium price $p^* \in [r, p^m]$ exists. We finally note that

$$\begin{aligned}\frac{dL(p)}{dp} &= g(\tilde{c}(p))\frac{d\tilde{c}(p)}{dp} - (p-r)f(p)g(\tilde{c}(p))\frac{d\tilde{c}(p)}{dp} - \int_0^{\tilde{c}(p)} f(\hat{x}(c))g(c)\,dc \\ &= \frac{1 - F(p) - (p-r)f(p)}{1 - F(p)} g(\tilde{c}(p))\frac{d\tilde{c}(p)}{dp} - \int_0^{\tilde{c}(p)} f(\hat{x}(c))g(c)\,dc \qquad (8)\end{aligned}$$

is negative for any $p \in [r, p^m]$, which implies that there exists a unique candidate equilibrium price. This follows from the fact that $1 - F(p) - (p-r)f(p) \geq 0$ (because it is the first order derivative of the monopoly payoff $(p-r)(1 - F(p))$, which is log-concave) and $d\tilde{c}(p)/dp < 0$ (because, from equation (3), $\tilde{c}$ is decreasing in p). In particular, at the candidate equilibrium price p^* we must have $dL(p^*)/dp < 0$. Figure 1 illustrates these observations.

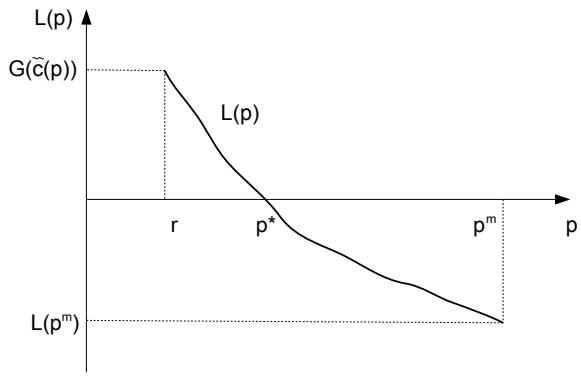


Figure 1: Existence and uniqueness of a candidate equilibrium price

Our next result provides conditions for existence of a symmetric equilibrium.

Proposition 1 *There may be two types of symmetric equilibria in the model of sequential search for differentiated products with heterogeneous search costs:*

(A) An SNE where all consumers conduct at least a first search, in which case $\overline{c} < \tilde{c}(p^*)$ and the equilibrium price is given by expression (6) after setting $c_0(p^*) = \overline{c}$, and where $\hat{x}(c)$ solves equation (1).

(B) An SNE where only some of the consumers search for a satisfactory product, in which case $\overline{c} > \tilde{c}(p^*)$ and the equilibrium price is given by expression (6) after setting $c_0(p^*) = \int_{p^*}^{\overline{\varepsilon}} (\varepsilon - p^*) f(\varepsilon) d\varepsilon$. In this type of equilibrium the fraction of consumers $G\left(\int_{p^*}^{\overline{\varepsilon}} (\varepsilon - p^*) f(\varepsilon) d\varepsilon\right)$ conducts at least a first search while the rest of the consumers do not search at all.

When the density of match values f is increasing and the search cost density g is log-concave, then the SNE exists and is unique.

The proof is in the Appendix. Since a direct verification of the second order conditions does not deliver clear-cut results, we proceed by showing that the demand function of an individual firm is a log-concave function of its own price. Once this is proven, we know that the firm profit function (4) is quasi-concave in its own price so that the unique candidate equilibrium price given by expression (6) is indeed an equilibrium. In order to prove that the demand function of a firm is log-concave in the firm's own price, we first show that the demand of a single consumer is log-concave both in price and in search cost and then make use of Theorem 6 in Prékopa (1973), thereby showing that integration over search costs preserves log-concavity.

2.1 The effect of higher search costs on the SNE price

In this subsection we study the impact of higher search costs on the equilibrium prices given by Proposition 1. In order to address this question, we parametrize the search cost distribution G by a positive parameter β and use the notation $G(c;\beta)$ and $g(c;\beta)$ to indicate that the distribution and density of search costs depend on β. Specifically, we assume that an increase in β implies an increase in search costs in a first-order stochastic dominance (FOSD) sense, i.e. $\partial G(c;\beta)/\partial\beta < 0$ for all c. Let $p^*(\beta)$ be the corresponding SNE price for a given β. We are interested in the behavior of $p^*(\beta)$ with respect to β.

Consider first the case of Proposition 1A. When the upper bound of the search cost distribution is sufficiently low, the SNE price is

$$p^*(\beta) = r + \frac{1}{\int_0^{\overline{c}(\beta)} \frac{f(\hat{x}(c))}{1-F(\hat{x}(c))} g(c,\beta) dc}. \tag{9}$$

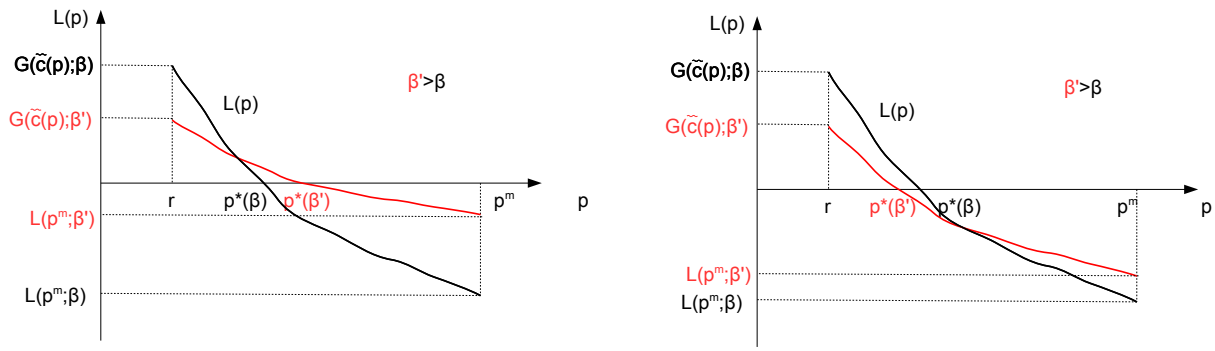
(Notice that we allow the upper bound of the search cost distribution to be increasing in β.) The effect of an increase in search costs on the equilibrium price follows from taking the derivative of equation

(9) with respect to β. Because f is log-concave, the hazard rate $f(\hat{x}(c))/(1-F(\hat{x}(c)))$ increases in $\hat{x}(c)$ and decreases in c. As a result, because the denominator of equation (9) is the expectation of this hazard rate, it falls as β goes up. This implies that the equilibrium price *unambiguously* increases as search costs rise.

We now move to the the case of Proposition 1B. In this case the equilibrium price $p^*(\beta)$ is given by the unique solution to equation

$$L(p;\beta) \equiv G(\tilde{c}(p);\beta) - (p-r)\int_0^{\tilde{c}(p)} \frac{f(\hat{x}(c))}{1-F(\hat{x}(c))} g(c,\beta)dc = 0. \tag{10}$$

Upon observing equation (10) we see that an increase in search costs affects two terms. The term $G(\tilde{c}(p);\beta)$ goes down because of FOSD, while the integral term goes up by the log-concavity of f. As a result, an increase in β has, potentially, an ambiguous effect on the SNE price. This is illustrated in Figure 2. In Figure 2(a) we depict a case for which higher search costs result in a higher SNE price. The black downward sloping curve shows the function $L(p;\beta)$. When β increases to β', the function $L(p;\beta)$ changes from the black to the red curve. In this case, the new equilibrium price increases in search costs. By contrast, in Figure 2(b) we observe the opposite case in which a higher β results in a lower SNE price.



(a) Higher $\beta \to$ higher price

(b) Higher $\beta \to$ lower price

Figure 2: The effect of higher search costs on the SNE price ($\bar{c}$ sufficiently large)

The economic intuition for why the effect of higher search costs on the SNE price is ambiguous when search costs are sufficiently high is as follows. An increase in search costs has two effects on demand. On the one hand, search becomes more costly and as a result of that consumers' reservation values fall. This decreases search activity and demand becomes less elastic. Because of this effect, firms have an incentive to raise their prices. On the other hand, when search costs increase a larger fraction of consumers choose not to search at all. This alters the composition of demand and possibly

makes average demand more elastic because the consumers who choose to leave the market are the ones with higher search costs. Because of this effect, firms may have an incentive to lower their prices. Interestingly, as the following example shows, these two effects exactly offset each other when match values and search costs are both uniformly distributed.

The uniform-uniform example. *Assume match values are uniformly distributed on* $[0, 1]$ *while search costs are uniformly distributed on* $[0, \beta]$*. In this case, it is straightforward to check that* $\hat{x}(c) = 1 - \sqrt{2c}$ *and that* $\tilde{c}(p^*) = (1 - p^*)^2/2$*. Then:*

(A) If $\beta \leq \frac{2}{9}(1-r)^2$*, a unique equilibrium exists in which all consumers search at least one time. The equilibrium price and profits are given by*

$$p^* = r + \sqrt{\frac{\beta}{2}} \text{ and } \pi^* = \sqrt{\frac{\beta}{2}},$$

while consumer surplus (CS) and social welfare (W) are equal to

$$CS = 1 - r - \frac{7}{3}\sqrt{\frac{\beta}{2}} \text{ and } W = 1 - r - \frac{4}{3}\sqrt{\frac{\beta}{2}}.$$

In this case, price and profits increase as search costs go up, while consumer surplus and social welfare decrease.

(B) If $\beta > \frac{2}{9}(1-r)^2$*, then a unique equilibrium exists in which a fraction of consumers* $1-\frac{2}{9\beta}(1-r)^2$ *does not search at all. The equilibrium price and profits are given by*

$$p^* = \frac{1}{3}(1 + 2r) \text{ and } \pi^* = \frac{2(1-r)^3}{27\beta},$$

while consumer surplus and social welfare are

$$CS = \frac{4(1-r)^3}{81\beta} \text{ and } W = \frac{10(1-r)^3}{81\beta}.$$

In this case, the equilibrium price is independent of the level of search costs, while profits, consumer surplus, and social welfare decrease in search costs.

An increase in the costs of search diminishes the fraction of consumers who choose to search and those who choose to search reduce their search intensity. With uniformly distributed search costs, the average demand remains equally elastic and, correspondingly the price does not change. We now proceed to a more general examination of the impact of an increase in search costs on the price.

Invoking the implicit function theorem, the effect of an increase in β on the equilibrium price in equation (10) is given by the sign of

$$\frac{dp^*(\beta)}{d\beta} = -\frac{\frac{\partial L(p^*;\beta)}{\partial \beta}}{\frac{\partial L(p^*;\beta)}{\partial p^*}}. \tag{11}$$

We have already argued above that the function $L(p;\beta)$ is monotone decreasing in p so the denominator of equation (11) is negative. In regard to the numerator of equation (11), we note using the notation $g_\beta(c;\beta) \equiv \partial g\left(c;\beta\right)/\partial\beta$ that

$$\begin{aligned}
\frac{\partial L(\cdot)}{\partial \beta} &= \int_0^{\tilde{c}(p^*)} g_\beta(c;\beta)dc - (p^* - r)\int_0^{\tilde{c}(p^*)} \frac{f(\widehat{x}(c))}{1-F(\widehat{x}(c))} g_\beta\left(c;\beta\right)dc; \\
&= \int_0^{\tilde{c}(p^*)} g_\beta(c;\beta)dc - \frac{\int_0^{\tilde{c}(p^*)} g(c;\beta)dc}{\int_0^{\tilde{c}(p^*)} \frac{f(\widehat{x}(c))}{1-F(\widehat{x}(c))} g\left(c;\beta\right)dc} \int_0^{\tilde{c}(p^*)} \frac{f(\widehat{x}(c))}{1-F(\widehat{x}(c))} g_\beta\left(c;\beta\right)dc; \\
&= \int_0^{\tilde{c}(p^*)} g(c;\beta)dc \left[\frac{\int_0^{\tilde{c}(p^*)} g_\beta(c;\beta)dc}{\int_0^{\tilde{c}(p^*)} g(c;\beta)dc} - \frac{\int_0^{\tilde{c}(p^*)} \frac{f(\widehat{x}(c))}{1-F(\widehat{x}(c))} g_\beta\left(c;\beta\right)dc}{\int_0^{\tilde{c}(p^*)} \frac{f(\widehat{x}(c))}{1-F(\widehat{x}(c))} g\left(c;\beta\right)dc} \right],
\end{aligned} \tag{12}$$

where the second equality follows from using the equilibrium condition (6). We therefore establish:

Proposition 2 *Let $G(c;\beta)$ be a parametrized search cost CDF with positive density on $[0,\overline{c}(\beta)]$ and with derivative $\partial G(\cdot)/\partial\beta < 0$. Then the comparative statics of the SNE price in Proposition 1 is as follows:*

(A) The equilibrium price given by Proposition 1A unambiguously increases in β.

(B) The equilibrium price given by Proposition 1B increases (decreases) in β if and only if

$$\frac{\int_0^{\tilde{c}(p^*)} g_\beta(c;\beta)dc}{\int_0^{\tilde{c}(p^*)} g(c;\beta)dc} - \frac{\int_0^{\tilde{c}(p^*)} \frac{f(\widehat{x}(c))}{1-F(\widehat{x}(c))} g_\beta\left(c;\beta\right)dc}{\int_0^{\tilde{c}(p^*)} \frac{f(\widehat{x}(c))}{1-F(\widehat{x}(c))} g\left(c;\beta\right)dc} > 0 (< 0). \tag{13}$$

Moreover, if g is from the family of power distributions on $[0,\beta]$ (which includes the uniform), then the equilibrium price is independent of β.

Proof. It remains to prove that when g is from the family of power distributions on $[0,\beta]$, then the equilibrium price does not depend on β. Let $g(c) = \alpha c^{\alpha-1}/\beta^\alpha$ on $[0,\beta]$, and let $\alpha > 0$. In this case, it is straightforward to check that the SNE price is given by the solution to

$$p^* = r + \frac{(\tilde{c}(p^*))^\alpha}{\int_0^{\tilde{c}(p^*)} \frac{f(\hat{x}(c))}{1-F(\hat{x}(c))} \alpha c^{\alpha-1} dc}, \tag{14}$$

which is clearly independent of β. ■

Condition (13) is a necessary and sufficient condition under which the SNE price will go down (or up) after search costs increase for all consumers. Intuitively, for prices to increase in search costs, we need that the effect on search intensity is stronger than the effect on consumer participation. This occurs when, in relative terms, the search cost shock is not felt very strongly at the higher percentiles of the search cost density. In those situations the average demand becomes less elastic after search costs increase. Our next result provides a sufficient condition under which this is indeed the case.

Proposition 3 *Assume that $g(c;\beta)$ has the monotone increasing likelihood ratio (MILR) property. Then, no matter whether the range of search costs is small or large, the SNE price given by Proposition 1 unambiguously increases in β.*

We have shown in Proposition 2 that for the case when the density of search costs is from the family of power densities, the SNE price given in Proposition 1B is constant in parameter β. Note that the family of power densities has a *constant* likelihood ratio with respect to β. Moreover, Proposition 3 has demonstrated that when the search cost density has the MILR property, higher search costs result in higher prices. These observations lead us to conjecture that when the density has the monotone *decreasing* likelihood ratio property, the SNE price falls in search costs. Though we have been unable to provide a general proof of this result, we now provide two different settings—one analytical and one numerical—where this is indeed the case.

The family of MDLR search cost densities $\mathbf{g = (1+t)\left[(c/\beta)^t + 1\right]/(\beta(2+t))}$, $\mathbf{0 \le t \le 1}$

Proposition 4 *Assume that match values are distributed according to the uniform distribution and that the search cost density is*

$$g(c;\beta) = \frac{1+t}{\beta(2+t)}\left[1+\left(\frac{c}{\beta}\right)^t\right], \textit{with } 0 \le t \le 1,$$

which is log-concave and has monotone decreasing likelihood ratio (MDLR). Then, an increase in β (which implies a FOSD shift of the search cost CDF):

(A) Unambiguously increases the equilibrium price given by Proposition 1A.

(B) Unambiguously decreases the equilibrium price given by Proposition 1B.

For this family of search cost densities, an increase in the costs of search is felt relatively strongly at the upper percentiles of the distribution. This has a significant impact on the extensive search margin and, in spite of the fact that consumers search less, the average demand becomes more elastic. Firms then respond by raising prices.

For robustness purposes, we study numerically the price equilibrium for the case of the Kumaraswamy (1980) distribution, which has the MDLR property for some values of the parameters.

The Kumaraswamy (1980) density

Definition: *The Kumaraswamy distribution has CDF $G(\cdot)$ and PDF $g(\cdot)$ given by*

$$G(c) = 1 - \left[1 - \left(\frac{c}{\beta}\right)^a\right]^b, \; c \in [0, \beta], \; a, b > 0;$$
$$g(c) = \frac{ab}{\beta}\left(\frac{c}{\beta}\right)^{a-1}\left[1 - \left(\frac{c}{\beta}\right)^a\right]^{b-1}. \tag{15}$$

The Kumaraswamy (1980) distribution is often used as a substitute for the beta-distribution (see, e.g., Ding and Wolfstetter, 2011). This distribution turns out to be quite useful in our setting because its likelihood ratio is increasing (for $b > 1$), decreasing (for $0 < b < 1$), or constant (for $b = 1$) with respect to the shifter parameter β.[11] Note that the β parameter multiplies the search cost c and scales the support of the distribution. An increase in β therefore shifts the search cost distribution rightward, which signifies that search costs are higher for all consumers.

Table 1 reports some of the numerical results we obtain using the Kumaraswamy distribution. While computing the equilibrium we set $r = 0$ and assume match values are uniformly distributed on $[0, 1]$. Moreover, we set $a = 1$ and let b take on values 1/2, 1, and 3/2. For the latter two values, the search cost density is log-concave so, from Proposition 1, we know the equilibrium exists. For the case $b = 1/2$, the search cost density is not log-concave but we have checked numerically that the equilibrium also exists.

When $b = 3/2$ the density function satisfies the MILR property; in this case, as shown in Proposition 2, the equilibrium price increases in search costs. We have also computed profits and consumer surplus, which decrease in search costs. We also report consumer surplus conditional on searching, which in this case decreases as well. When $b = 1$ the price is constant and so is consumer surplus conditional on searching; however, profits, consumer surplus, and welfare fall in search costs. Finally, when $b = 1/2$ the density function satisfies the MDLR property. In this case, price decreases in search costs and consumer surplus conditional on searching therefore goes up; however, profits, consumer surplus, and welfare go down anyway.

The results reported in Table 1 do not change when we choose other values for parameter b. On the basis of the numerical results, we state the following:

[11] A proof can be obtained from the authors upon request. Observe that the uniform density case is obtained by setting $a = b = 1$ in the Kumaraswamy distribution above.

	$b = 3/2$			$b = 1$			$b = 1/2$		
	$\beta = 1$	$\beta = 2$	$\beta = 3$	$\beta = 1$	$\beta = 2$	$\beta = 3$	$\beta = 1$	$\beta = 2$	$\beta = 3$
p^*	0.3287	0.3311	0.3319	0.3333	0.3333	0.3333	0.3378	0.3354	0.3347
π	0.1045	0.0539	0.0363	0.0740	0.0370	0.0246	0.0393	0.0190	0.0125
CS	0.0729	0.0367	0.0245	0.0493	0.0246	0.0164	0.0250	0.0124	0.0082
$CS/\int_0^{\frac{(1-p)^2}{2}} gdc$	0.2294	0.2255	0.2244	0.2222	0.2222	0.2222	0.2153	0.2189	0.2200
Welfare	0.1775	0.0907	0.0609	0.1234	0.0617	0.0411	0.0643	0.0314	0.0208

Table 1: Sequential search for differentiated products (uniform-Kumaraswamy with $a = 1$)

Result: *Assume that match values are uniformly distributed on* $[0, 1]$ *and that search costs are distributed on the interval* $[0, \beta]$ *according to the Kumaraswamy distribution with parameter* $a = 1$*. Then:*

(A) The equilibrium price in Proposition 1A increases in β*.*

(B) The equilibrium price in Proposition 1B decreases in β *if* $0 < b < 1$*, is constant in* β *if* $b = 1$*, and increases in* β *if* $b > 1$*.*

The results obtained in this section are rather intuitive and they happen to be robust across model specifications. In fact, we next study a duopoly version of our model and demonstrate by numerically solving the model that the insights we have derived remain valid. Moreover, elsewhere we have studied a version of our model where consumers search non-sequentially and again similar results obtain (see also Footnote 4).

2.2 Duopoly

In this subsection we study the duopoly case. Except that there are only two firms in the market, the rest of the model details are exactly the same as before.[12]

We now present the derivations to compute the SNE price p^*. For this we derive the (expected) payoff to a firm i that deviates by charging a price $p_i < p^*$. In order to compute firm i's demand, consider a consumer with search cost c who visits firm i in her first search. This happens with probability 1/2. Let $\varepsilon_i - p_i$ denote the utility the consumer derives from the product of firm i. Notice that search behavior is exactly the same as in the previous section. The consumer expects the other firm to charge the equilibrium price p^*. The expected gains from searching one more time are equal to $\int_{\varepsilon_i - p_i + p^*}^{1} [\varepsilon_j - (\varepsilon_i - p_i + p^*)] f(\varepsilon_j) d\varepsilon_j$. It follows that the probability that the buyer visits firm i

[12] Extending this analysis to the case of N firms is straightforward.

first and stops searching at firm i is equal to

$$\frac{1}{2}\Pr[\varepsilon_i - p_i > \max\{\hat{x}(c) - p^*, 0\}] = \frac{1}{2}\left[1 - F(\hat{x}(c) + p_i - p^*)\right].$$

where $\hat{x}(c)$ continues to be the solution to equation (1). Consumer c may find the product of firm i not good enough at first and may therefore continue searching. However, upon visiting the rival firm j, it may happen that consumer c returns to firm i because such a firm offers her the best deal after all. This occurs with probability

$$\frac{1}{2}\Pr[\max\{\varepsilon_j - p^*, 0\} < \varepsilon_i - p_i < \hat{x}(c) - p^*] = \frac{1}{2}\int_{p_i}^{\hat{x}(c)+p_i-p^*} F(\varepsilon - p_i + p^*)f(\varepsilon)d\varepsilon. \tag{16}$$

With probability 1/2 consumer c first visits the other firm, firm j. In that case, she will walk away from product j when searching again is more promising than buying j right away. Upon visiting firm i, she will buy product i when she finds product i better than j. This occurs with probability $(1/2)\Pr[\max\{\varepsilon_j - p^*, 0\} < \min\{\hat{x}(c) - p^*, \varepsilon_i - p_i\}]$, which is equal to

$$\frac{1}{2}\left[F(\hat{x}(c))\left[1 - F(\hat{x}(c) + p_i - p^*)\right] + \int_{p_i}^{\hat{x}(c)+p_i-p^*} F(\varepsilon - p_i + p^*)f(\varepsilon)d\varepsilon\right].$$

To obtain the payoff of firm i we need to integrate over the consumers who decide to participate in the market. It can be shown that with two firms the surplus of a consumer with search cost c is given by the expression

$$CS(c) = \frac{1 - F(\hat{x}(c))^2}{1 - F(\hat{x}(c))}\left[\int_{\hat{x}(c)}^{\overline{\varepsilon}} (\varepsilon - p^*)f(\varepsilon)d\varepsilon - c\right] + 2\int_{p^*}^{\hat{x}(c)} (\varepsilon - p^*)F(\varepsilon)f(\varepsilon)d\varepsilon. \tag{17}$$

Setting this surplus equal to zero, we obtain the critical search cost value $\tilde{c}(p^*)$ above which consumers will refrain from participating in the market. Inspection of equation (17) reveals that the last consumer who chooses to search has a search cost c such that $\hat{x}(c) = p^*$. Using again the notation $c_0(p^*) \equiv \min\{\overline{c}, \tilde{c}(p^*)\}$, the expected payoff to firm i is:

$$\pi_i(p_i; p^*) = \frac{p_i - r}{2}\int_0^{c_0(p^*)}\left[(1 + F(\hat{x}(c)))\left(1 - F(\hat{x}(c) + p_i - p^*)\right) + 2\int_{p_i}^{\hat{x}(c)+p_i-p^*} F(\varepsilon - p_i + p^*)f(\varepsilon)d\varepsilon\right]g(c)dc. \tag{18}$$

To shorten the expressions we will from now on write $\hat{x}$ instead of $\hat{x}(c)$ but the reader should keep

in mind the dependency of $\hat{x}$ on c. Taking the FOC gives

$$0 = \int_0^{c_0(p^*)} \left[(1+F(\hat{x}))\left[1-F(\hat{x}+p_i-p^*)\right] + 2\int_{p_i}^{\hat{x}+p_i-p^*} F(\varepsilon-p_i+p^*)f(\varepsilon)d\varepsilon \right] g(c)dc$$
$$- (p_i - r)\int_0^{c_0(p^*)} (1+F(\hat{x}))\, f(\hat{x}+p_i-p^*)g(c)dc$$
$$- 2(p_i-r)\int_0^{c_0(p^*)} \left[\int_{p_i}^{\hat{x}+p_i-p^*} f(\varepsilon-p_i+p^*)f(\varepsilon)d\varepsilon + F(\hat{x})f(\hat{x}+p_i-p^*) - F(p^*)f(p_i) \right] g(c)dc. \tag{19}$$

Applying symmetry $p_i = p^*$ gives

$$0 = \int_0^{c_0(p^*)} \left[(1+F(\hat{x}))\,(1-F(\hat{x})) + 2\int_{p^*}^{\hat{x}} F(\varepsilon)f(\varepsilon)d\varepsilon \right] g(c)dc$$
$$- 2(p^*-r)\int_0^{c_0(p^*)} \left[(1+F(\hat{x}))\, f(\hat{x}) + \int_{p^*}^{\hat{x}} f(\varepsilon)^2 d\varepsilon - F(\hat{x})f(\hat{x}) + F(p^*)f(p^*) \right] g(c)dc. \tag{20}$$

In order to check how the equilibrium price changes when search costs go up, we proceed by solving the FOC in equation (20) numerically. We again use the uniform distribution for the match values and the Kumaraswamy distribution for the search costs. The focus is on the case in which the upper bound of the search cost distribution β is sufficiently high, conform Proposition 1B.

For the case of the uniform distribution we have that

$$CS(c) = \hat{x}(c) - p^* - \frac{\hat{x}(c)^3 - p^{*3}}{3},$$

whereas the critical search cost value above which consumers refrain from searching the market is

$$\tilde{c}(p^*) = \frac{(1-p^*)^2}{2}$$

In Table 2, we set $r = 0$ and let search costs be distributed according to the Kumaraswamy distribution with parameter $a = 1$ and various levels of the parameters b and β. The table shows once again that prices decrease when search costs increase when $b = 1/2$, in which case the search cost density has the MDLR property. For the $b = 1$ case (uniform distribution), once more prices are independent of the search cost upper bound. Finally, when $b = 3/2$ and the search cost density has the MILR property, we get the standard result that prices increase with higher search costs.

3 Discussion

In this section we discuss the implications of other forms of consumer heterogeneity, such as fixed search cost heterogeneity and valuation heterogeneity.

	$b = 3/2$			$b = 1$			$b = 1/2$		
	$\beta = 1$	$\beta = 2$	$\beta = 3$	$\beta = 1$	$\beta = 2$	$\beta = 3$	$\beta = 1$	$\beta = 2$	$\beta = 3$
p^*	0.4713	0.4715	0.4716	0.4717	0.4717	0.4717	0.4722	0.4720	0.4719
π	0.0370	0.0188	0.0126	0.0255	0.0127	0.0085	0.0132	0.0065	0.0043
CS	0.0225	0.0113	0.0076	0.0153	0.0076	0.0051	0.0078	0.0038	0.0025
$CS/\int_0^{\tilde{c}(p^*)} gdc$	0.1116	0.1107	0.1105	0.1100	0.1100	0.1100	0.1084	0.1092	0.1095
Welfare	0.0966	0.0491	0.0329	0.0665	0.0332	0.0221	0.0343	0.0168	0.0112

Table 2: Duopoly model (uniform-Kumaraswamy with $a = 1$)

Fixed search cost heterogeneity

Sometimes consumers also face a *fixed* search cost, for instance when they first have to start up a computer, log onto a website, or drive to a mall before their actual search begins. Suppose that consumers have to pay a fixed search cost up front, denoted k, in order to start searching; after paying this cost, consumers proceed as in the model of Section 2, and face a marginal search cost equal to c. Suppose further that consumers are heterogeneous only in their fixed search cost. Let $\mathcal{K}(k)$ be the distribution of the fixed search cost, with support $[0, \overline{k}]$.

It is straightforward to check that the search rule given by the solution to equation (1) remains the same. Correspondingly, the surplus of a consumer with fixed search cost k is equal to

$$CS(k) = \hat{x}(c) - p^* - k.$$

For a consumer to enter the market this surplus must be strictly positive. Setting $CS(k) = 0$ and solving for k gives a critical fixed search cost, denoted $\tilde{k}(p^*)$, above which consumers will abstain from searching for an acceptable product. Defining $k_0(p^*) \equiv \min\{\overline{k}, \tilde{k}(p^*)\}$, the payoff analogous to the payoff of the deviant firm i in equation (4) is given by

$$\begin{aligned}\pi(p_i; p^*) &= (p_i - r) \int_0^{k_0(p^*)} \frac{1 - F(\hat{x}(c) + p_i - p^*)}{1 - F(\hat{x}(c))} d\mathcal{K}(k) \\ &= \left[1 - \mathcal{K}(\tilde{k}_0(p^*))\right] (p_i - r) \frac{1 - F(\hat{x}(c) + p_i - p^*)}{1 - F(\hat{x}(c))}. \end{aligned} \tag{21}$$

Inspection of equation (21) immediately reveals that a change in the distribution of the fixed search cost has no bearing on equilibrium pricing because, even though it affects demand, it does it in a way that does not modify the elasticity of demand.

The main distinction between heterogeneity in the fixed search cost and heterogeneity in the marginal search cost is that the former does not affect the marginal (net) benefits from search while

the latter does. Because the marginal benefits from search for the consumers who choose to search are not affected by an increase in fixed search costs, the elasticity of demand remains the same. This illustrates the point that the mechanism we have identified in this paper does not just arise because consumer participation is endogenous. In fact, for the elasticity of average demand to increase or decrease as search costs increase, consumers' marginal benefits from search need to change as well.

Valuation heterogeneity

The same point can be made when analyzing how changes in the distribution of consumer valuations affect pricing. Suppose now that consumers vary in their valuations for the products. For this purpose, define the utility of a consumer m as

$$u_{im} = v_m + \mu_m \varepsilon_{im} - p_i,$$

where v_m denotes the intrinsic utility consumer m derives from the product (irrespective of the match value) and μ_m stands for the marginal utility of an increase in the match between the consumer and the product.

Suppose, to start with, that consumers only differ in their marginal valuations for the match values. This means we can simplify the utility specification to $u_{im} = \mu_m \varepsilon_{im} - p_i$. Denote the distribution of marginal valuations by $\mathcal{M}(\mu)$, with support $[\underline{\mu}, \overline{\mu}]$.

In this case the search rule has to be slightly modified because the gains from searching one more time equal $\mu h(x)$, where $h(x)$ is given by equation (1). Accordingly, denote the solution to

$$\mu \int_x^{\overline{\varepsilon}} (\varepsilon - x) f(\varepsilon) d\varepsilon = c \tag{22}$$

in x by $\hat{x}(\mu)$. Upon observing equation (22), it is obvious that variation in marginal valuations has the same effects as variation in marginal search costs. We can therefore conclude that a decrease in the marginal valuations of consumers may result in lower, constant, or higher prices.

Suppose now that consumers only differ in their intrinsic valuations so they all have the same search cost and the same match marginal utility. Let the utility be $u_{im} = v_m + \varepsilon_{im} - p_i$. Denote by $\mathcal{V}(v)$ the distribution of intrinsic valuations, with support $[\underline{v}, \overline{v}]$. Again the search rule given by the solution to equation (1) remains unmodified. Correspondingly the surplus of a consumer with valuation v is

$$CS(v) = v - \hat{x}(c) - p^*.$$

This does not differ much from the case above where consumers have different fixed search costs. Defining $v_0(p^*) \equiv \max\{\underline{v}, \tilde{v}(p^*)\}$, the payoff corresponding to the payoff of the deviant in equation

(4) is in this case:

$$\pi(p_i; p^*) = [1 - \mathcal{V}(v_0(p^*))]\,(p_i - r)\frac{1 - F(\hat{x}(c) + p_i - p^*)}{1 - F(\hat{x}(c)}. \tag{23}$$

We conclude that a change in the distribution of intrinsic valuations has no bearing on equilibrium pricing. Changes in the distribution of intrinsic valuations vary the size of the market but not the elasticity of demand.

4 Conclusions

This paper has studied price determination in a model of search for differentiated products. The novelty of our study has been to allow for arbitrary search cost heterogeneity. We have also revisited the question how an increase in search costs affects the level of prices.

Traditional consumer search models have typically assumed that all consumers search at least once in equilibrium. By doing so, the existing literature has neglected an important role of the price mechanism, namely, that the price ought to affect the number of consumers who choose to search for a product in the first place. Assuming that all consumers search the market cannot easily be reconciled with the idea that search costs, to the extent that they are related to consumer demographics such as income, age, marital status etc., are likely to differ across individuals. In this paper we have shown that recognizing this role of the price mechanism turns out to be critical for our understanding of the effect of higher search costs on prices and profits. The reason is that an increase in search costs typically affects two margins: the extensive search margin, which reflects consumers' decisions on whether to start searching, and the intensive search margin, which corresponds to consumers' search intensity. In addition to having studied the existence and uniqueness of the symmetric equilibrium, the main results of the paper have been on characterizing conditions on search cost densities under which higher search costs result in higher, equal, or lower prices.

We have identified a critical property of search cost densities that plays a decisive role, namely, whether the likelihood ratio is increasing or decreasing in the parameter that shifts the search cost distribution. When the likelihood ratio is decreasing, an increase in search frictions affects consumers with high search costs relatively more strongly than it affects consumers with low search costs. In this case, the effect on the extensive search margin is stronger than the effect on the intensive search margin. Correspondingly, the average (over the consumers who continue choosing to search for an acceptable product) demand becomes more elastic and thereby prices decrease. When the search cost density has the monotone increasing likelihood ratio property, higher search costs impact the

intensive search margin more strongly than the extensive search margin and this results in higher prices. These insights are quite robust and hold under different modeling assumptions. The paper has also analyzed the roles played by other forms of consumer heterogeneity, and we have obtained similar results when marginal valuations differ across consumers.

APPENDIX

Derivation of consumer surplus. As mentioned in the main text, a consumer with search cost c will stop and buy after the first search when $\varepsilon > \hat{x}(c)$; otherwise she will drop the first option and continue searching, in which case she will encounter herself exactly in the same situation as before because, conditional on participating, the consumer will continue searching until she finds a match value for which it is worth to stop searching. Denoting by $CS(c)$ her consumer surplus, recursively, we must have:

$$CS(c) = -c + (1 - F(\hat{x}(c)))\frac{\int_{\hat{x}(c)}^{\overline{\varepsilon}}(\varepsilon - p^*)f(\varepsilon)d\varepsilon}{1 - F(\hat{x}(c))} + F(\hat{x}(c))CS(c).$$

Solving for $CS(c)$ gives

$$CS(c) = \frac{\int_{\hat{x}(c)}^{\overline{\varepsilon}}(\varepsilon - p^*)f(\varepsilon)d\varepsilon - c}{1 - F(\hat{x}(c))}.$$

Using the value of c from equation (1) we obtain

$$CS(c) = \frac{\int_{\hat{x}(c)}^{\overline{\varepsilon}}(\varepsilon - p^*)f(\varepsilon)d\varepsilon - \int_{\hat{x}(c)}^{\overline{\varepsilon}}(\varepsilon - \hat{x}(c))f(\varepsilon)d\varepsilon}{1 - F(\hat{x}(c))} = \frac{\int_{\hat{x}(c)}^{\overline{\varepsilon}}(\hat{x}(c) - p^*)f(\varepsilon)d\varepsilon}{1 - F(\hat{x}(c))} = \hat{x}(c) - p^*,$$

which is the expression we give in the text. ■

Proof of Proposition 1. It remains to prove that the equilibrium exists when f is increasing and g is log-concave. For this, we prove that the demand function of a firm i in equation (5) is a log-concave function of its own price p_i. Given this the firm profit function (4) is quasi-concave in its own price so that the unique candidate equilibrium price given by equation (6) is indeed an equilibrium.

We start by showing that the integrand in equation (5) is log-concave in c and in p_i under the conditions that f is increasing and g is log-concave. For this we first note that the product of log-concave functions is log-concave. The term $(1 - F(\hat{x}(c))^{-1}$ in the integrand of equation (5) is readily seen to be log-concave in c provided that f is increasing. Under log-concavity of g, the term $(1 - F(\hat{x}(c) + p_i))g(c)$ is log-concave in c and in p_i provided that the expression $q(p_i, c) \equiv 1 - F(\hat{x}(c) + p_i - p^*)$ is log-concave in c and in p_i. Let us now show that this is indeed the case.

We first note that, because $\hat{x}(0) = \overline{\varepsilon}$, for deviations such that $p_i - p^* > 0$ (see footnote 9) it will be the case that $q(p_i, c) = 0$ for values of c close to 0. However, from Prékopa (1971) we know that if a function is log-concave on a convex set and equal to zero elsewhere then the function is log-concave in the entire space.

The set on which $q(p_i, c) > 0$ is given by

$$S = \{(p_i, c) : p_i \in [r, p^m],\ p_i < \overline{\varepsilon} + p^* - \hat{x}(c)\}.$$

Because $\hat{x}(c)$ is decreasing and convex in c, the expression $\overline{\varepsilon} + p^* - \hat{x}(c)$ is increasing and concave. This implies that S is a convex set.

It then remains to prove that $q(p_i, c)$ is log-concave in c and p_i in S. For this we need to prove that the function $m(p_i, c) \equiv \ln[1 - F(\hat{x}(c) + p_i - p^*)]$ is concave in c and p_i in S, where ln denotes the natural logarithm. Taking derivatives we have:

$$\frac{\partial m}{\partial c} = -\frac{f(\hat{x}(c) + p_i - p^*)}{1 - F(\hat{x}(c) + p_i - p^*)}\hat{x}'(c);$$
$$\frac{\partial m}{\partial p_i} = \frac{f(\hat{x}(c) + p_i - p^*)}{1 - F(\hat{x}(c) + p_i - p^*)}.$$

To construct the Hessian matrix, we now compute the necessary second order derivatives:

$$\begin{aligned}\frac{\partial^2 m}{\partial c^2} &= -\frac{1}{[1 - F(\hat{x}(c) + p_i - p^*)]^2}\left\{\left[f'(\hat{x}(c) + p_i - p^*)\left[\hat{x}'(c)\right]^2 + f(\hat{x}(c) + p_i - p^*)\hat{x}''(c)\right][1 - F(\hat{x}(c) + p_i - p^*)]\right.\\ &\quad \left. + \left[f(\hat{x}(c) + p_i - p^*)\hat{x}'(c)\right]^2\right\};\\ &= -\frac{1}{[1 - F(\hat{x}(c) + p_i - p^*)]^2}\left\{\left[\hat{x}'(c)\right]^2\left[f'(\hat{x}(c) + p_i - p^*)[1 - F(\hat{x}(c) + p_i - p^*)] + [f(\hat{x}(c) + p_i - p^*)]^2\right]\right.\\ &\quad \left. + f(\hat{x}(c) + p_i - p^*)\hat{x}''(c)[1 - F(\hat{x}(c) + p_i - p^*)]\right\}.\end{aligned}$$

The sign of this expression depends on the sign of the part in curly brackets. We note that, because f is log-concave, the first summand of the expression in curly brackets is positive. Moreover, because $\hat{x}''(c) > 0$, the second summand is also positive. Therefore we conclude that $\partial^2 m/\partial c^2 < 0$.

We now observe that

$$\frac{\partial^2 m}{\partial p_i^2} = -\frac{1}{[1 - F(\hat{x}(c) + p_i - p^*)]^2}\left\{f'(\hat{x}(c) + p_i - p^*)[1 - F(\hat{x}(c) + p_i - p^*)] + [f(\hat{x}(c) + p_i - p^*)]^2\right\},$$

which is negative again by the log-concavity of f.

Finally we derive

$$\frac{\partial^2 m}{\partial p_i \partial c} = -\frac{\hat{x}'(c)}{[1 - F(\hat{x}(c) + p_i - p^*)]^2}\left\{f'(\hat{x}(c) + p_i - p^*)[1 - F(\hat{x}(c) + p_i - p^*)] + [f(\hat{x}(c) + p_i - p^*)]^2\right\}.$$

Defining

$$\psi(c, p_i) \equiv \frac{f'(\hat{x}(c) + p_i - p^*)[1 - F(\hat{x}(c) + p_i - p^*)] + [f(\hat{x}(c) + p_i - p^*)]^2}{[1 - F(\hat{x}(c) + p_i - p^*)]^2},$$

the Hessian matrix is

$$H = \begin{pmatrix} -\left[\hat{x}'(c)\right]^2\psi(c, p_i) - \frac{f(\hat{x}(c)+p_i-p^*)\hat{x}''(c)}{1-F(\hat{x}(c)+p_i-p^*)} & -\hat{x}'(c)\psi(c, p_i) \\ -\hat{x}'(c)\psi(c, p_i) & -\psi(c, p_i). \end{pmatrix}$$

It is straightforward to check that the determinant of H is equal to $\frac{f(\hat{x}(c)+p_i-p^*)\hat{x}''(c)}{1-F(\hat{x}(c)+p_i-p^*)}\psi(c, p_i)$, which is strictly positive. As a result the function $m(c, p_i)$ is strictly concave in c and p_i in S. Because S is a convex set, the integrand of equation (5) is log-concave in c and in p_i.

We now invoke Theorem 6 in Prékopa (1973) showing that the integral taken over a convex subset of the real line of a log-concave function is also log-concave, which implies that the demand function (5) is log-concave in p_i. Therefore, an equilibrium exists and is unique. ■

Proof of Proposition 3. It remains to prove that the SNE price given by Proposition 1B will also increase under the MILR property. In order to prove this statement, we make use of the following version of Theorem 9 in Menezes and Monteiro (2009).[13]

Theorem (Menezes and Monteiro, 2009). Let $f_1,\ f_2,\ f_3,\ f_4$ be non-negative functions on $[a,b]$ such that $f_1(x)\,f_2(y) \leq f_3(x \vee y)\,f_4(x \wedge y)$ for all $x,\ y \in [a,b]$, where $x \vee y \equiv \max\{x,y\}$, $x \wedge y \equiv \min\{x,y\}$. Then

$$\int_a^b f_1(x)\,dx \int_a^b f_2(x)\,dx \leq \int_a^b f_3(x)\,dx \int_a^b f_4(x)\,dx.$$

Let $\gamma > \beta$. We prove that

$$\frac{\int_0^{c_0(p^*)} g(c;\beta)dc}{\int_0^{c_0(p^*)} \frac{f(\hat{x}(c))}{1-F(\hat{x}(c))} g(c;\beta)dc} \leq \frac{\int_0^{c_0(p^*)} g(c;\gamma)dc}{\int_0^{c_0(p^*)} \frac{f(\hat{x}(c))}{1-F(\hat{x}(c))} g(c;\gamma)dc}$$

by using the theorem. Let

$$f_1(c) = g(c,\beta), \quad f_2(c) = \frac{f(\hat{x}(c))}{1-F(\hat{x}(c))} g(c,\gamma),$$
$$f_3(c) = g(c,\gamma), \quad f_4(c) = \frac{f(\hat{x}(c))}{1-F(\hat{x}(c))} g(c,\beta).$$

We show that for all $c,\ d \in [0, \widetilde{c}(p)]$

$$f_1(c)\,f_2(d) \leq f_3(c \vee d)\,f_4(c \wedge d), \tag{A24}$$

i.e.,

$$g(c,\beta)\frac{f(\hat{x}(d))}{1-F(\hat{x}(d))} g(d,\gamma) \leq g(c \vee d,\gamma)\frac{f(\hat{x}(c \wedge d))}{1-F(\hat{x}(c \wedge d))} g(c \wedge d,\beta).$$

Take first $c < d$; we have $c \vee d = d,\ c \wedge d = c$. So we need to prove that

$$g(c,\beta)\frac{f(\hat{x}(d))}{1-F(\hat{x}(d))} g(d,\gamma) \leq g(d,\gamma)\frac{f(\hat{x}(c))}{1-F(\hat{x}(c))} g(c,\beta).$$

This is equivalent to

$$g(d,\gamma)g(c,\beta)\left(\frac{f(\hat{x}(d))}{1-F(\hat{x}(d))} - \frac{f(\hat{x}(c))}{1-F(\hat{x}(c))}\right) \leq 0,$$

which is true because f is log-concave ($f(\hat{x}(c))/[1-F(\hat{x}(c))]$ is decreasing in c and $c < d$).

[13] We are indebted to Paulo Monteiro for alerting us about this theorem and showing us how to use it for this proof. For the proof of the theorem, we refer the reader to the original source Menezes and Monteiro (2009). (We have nevertheless developed a more detailed proof, which is available from us upon request.)

Now take $c \geq d$; we have $c \vee d = c,\ c \wedge d = d$. So we need to prove that

$$g(c,\beta)\frac{f(\hat{x}(d))}{1-F(\hat{x}(d))}g(d,\gamma) \leq g(c,\gamma)\frac{f(\hat{x}(d))}{1-F(\hat{x}(d))}g(d,\beta). \tag{A25}$$

If $g(c,\beta) = 0$ then the inequality clearly holds for any d. If $g\,(d,\beta) = 0$ then $d = 0$. Then the fact $g\,(d,\beta) = 0$ means that the derivative of $G\,(c,\beta)$ is 0 at 0. Since from FOSD (implied by the MILR property) $G\,(c,\gamma) \leq G\,(c,\beta)$, the derivative of $G\,(c,\gamma)$ must also be 0 at 0. Therefore, $g\,(d,\gamma) = 0$, so the inequality holds.

Next, by dividing equation (A25) by $g(c,\beta)$ and $g(d,\beta)$ and reorganizing, we get

$$\frac{f(\hat{x}(d))}{1-F(\hat{x}(d))}\left(\frac{g(d,\gamma)}{g(d,\beta)} - \frac{g(c,\gamma)}{g(c,\beta)}\right) \leq 0.$$

This holds for all $c \geq d$ if and only if $g\,(c,\beta)$ has the MILR property. ■

Proof of Proposition 4. Consider the case in which match values are distributed uniformly on $[0,1]$ and search costs are distributed on $[0,\beta]$ according to the following distribution function:

$$G(c;\beta) = \frac{c}{\beta(2+t)}\left[1+t+\left(\frac{c}{\beta}\right)^t\right],\ \text{with } 0 \leq t \leq 1,$$

Notice that an increase in β shifts the search cost distribution to the right, so higher β implies a FOSD shift of the search cost distribution. The corresponding density is given in the proposition.

We now make two observations about this density function. First, the family of densities g is log-concave, which together with the uniform density for the match values ensures that a SNE exists and is unique (Proposition 1). To see this, we note that

$$\frac{\partial^2 \ln[g(c;\beta)]}{\partial c^2} = -\frac{t\left[1-t+\left(\frac{c}{\beta}\right)^t\right]\left(\frac{c}{\beta}\right)^t}{c^2\left[1+\left(\frac{c}{\beta}\right)^t\right]^2} < 0.$$

In addition, we notice that g has the MDLR property. To see this, we note that

$$\frac{g'_\beta}{g} = \frac{-\frac{1+t+(1+t)^2\left(\frac{c}{\beta}\right)^t}{\beta^2(2+t)}}{\frac{1+t}{\beta(2+t)}\left[1+\left(\frac{c}{\beta}\right)^t\right]} = -\frac{1+(1+t)\left(\frac{c}{\beta}\right)^t}{\beta\left[1+\left(\frac{c}{\beta}\right)^t\right]}$$

Taking the derivative with respect to c gives

$$\frac{\partial(g'_\beta/g)}{\partial c} = -\frac{t^2\left(\frac{c}{\beta}\right)^{t-1}}{\beta^2\left[1+\left(\frac{c}{\beta}\right)^t\right]^2} < 0.$$

We now compute the equilibrium prices in the cases of Proposition 1. We have that:

(A) When $\beta \leq \frac{8(1-r)^2(1+t)^4}{(6t^2+13t+6)^2}$ the equilibrium price is

$$p^*(\beta) = r + \frac{\sqrt{\beta}(2+t)(1+2t)}{2\sqrt{2}(1+t)^2},$$

which clearly increases in β.

(B) When $\beta > \frac{8(1-r)^2(1+t)^4}{(6t^2+13t+6)^2}$, the equilibrium price is given by the solution to equation (10). Using the formulas above for the distribution and the density this equation is:

$$L(p;\beta) \equiv \frac{(1-p)^2}{2\beta(2+t)}\left[1+t+\left(\frac{(1-p)^2}{2\beta}\right)^t\right] - (p-r)\int_0^{(1-p)^2/2} \frac{1+t}{\sqrt{2c}\beta(2+t)}\left[1+\left(\frac{c}{\beta}\right)^t\right] dc = 0.$$

After integrating and simplifying we obtain that the price is given by the solution to

$$\widetilde{L}(p;\beta) \equiv (1-p)\left[1+t+\left(\frac{(1-p)^2}{2\beta}\right)^t\right] - 2(t+1)(p-r)\left[1+\frac{(1-p)^{2t}}{2^t\beta^t(1+2t)}\right] = 0.$$

From the analysis above, we know that $\widetilde{L}(p;\beta) = 0$ has a unique solution; let $p(\beta)$ be such a solution. Applying the implicit function theorem we have that

$$\frac{dp}{d\beta} = -\frac{\partial\widetilde{L}/\partial\beta}{\partial\widetilde{L}/\partial p}$$

where

$$\frac{\partial\widetilde{L}}{\partial\beta} = \frac{2^{1-t}t(1+t)(p-r)(1-p)^{2t}}{\beta^{t+1}(1+2t)} - \frac{t(1-p)^{2t+1}}{2^t\beta^{t+1}} \tag{A26}$$

$$\frac{\partial\widetilde{L}}{\partial p} = \frac{2^{2-t}t(t+1)(p-r)(1-p)^{2t-1}}{\beta^t(1+2t)} - 2(t+1)\left(\frac{(1-p)^{2t}}{2^t\beta^t(1+2t)}+1\right) - \frac{2^{1-t}t(1-p)^{2t}}{\beta^t} - \left(\frac{(1-p)^2}{2\beta}\right)^t - t - 1 \tag{A27}$$

From the equilibrium condition equation $\widetilde{L}(p;\beta) = 0$ we obtain that

$$p - r = \frac{(1-p)\left(1+t+\left(\frac{(1-p)^2}{2\beta}\right)^t\right)}{2(t+1)\left(1+\frac{(1-p)^{2t}}{2^t\beta^t(1+2t)}\right)}.$$

Substituting $p-r$ in equations (A26) and (A27) by this expression and simplifying we obtain

$$\frac{\partial\widetilde{L}}{\partial\beta} = -\frac{t^2(1-p)^{2t+1}}{\beta\left((1-p)^{2t}+2^t\beta^t(1+2t)\right)} < 0$$

$$\frac{\partial\widetilde{L}}{\partial p} = -\frac{2t^2(1-p)^{2t}}{(1-p)^{2t}+2^t\beta^t(1+2t)} - \frac{(3+4t)(1-p)^{2t}}{2^t\beta^t(1+2t)} - 3(t+1) < 0.$$

From this, we conclude that $dp/d\beta < 0$. As a result, the equilibrium price unambiguously decreases in β. ■

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