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The Commitment Role of Equity Financing

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The Commitment Role of Equity Financing

Abstract

Existing theories of a firm’s optimal capital structure seem to fail in explaining why many healthy and profitable firms rely heavily on equity financing, even though benefits associated with debt (like tax shields) appear to be high and the bankruptcy risk low. This holds in particular for firms that show a strong commitment towards their workforce and are popular among employees. We demonstrate that such financing behavior may be driven by implicit arrangements made between a firm and its managers/employees. Equity financing generally strengthens a firm’s credibility to honor implicit promises. Debt, on the other hand, has an adverse effect on the enforceability of these arrangements because too much debt increases the firm’s reneging temptation, as some of the negative consequences of breaking implicit promises can be shifted to creditors. Our analysis provides an explanation for why some firms only use little debt financing. Predictions made by our theory are in line with a number of empirical results, which seem to stay in contrast to existing theories on capital structure. Our findings also carry new implications for how policies (e.g., tax policy) affect firm behavior.

JEL-Code: C730, D240, D860, G320.

Keywords: relational contracts, capital structure, corporate finance, debt financing.

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1 Introduction

Many firms seem substantially underleveraged, given existing theories of a firm’s optimal financing structure. In these theories, debt is usually considered to increase the bankruptcy risk of a firm, which imposes substantial costs on creditors, suppliers or employees. Yet many profitable and healthy firms – with negligible bankruptcy risk – have very little (or even no) debt and seem to leave substantial tax shields on the table.¹

Leverage appears to be particularly low at firms with a strong commitment for treating their employees well and keeping promises made to their workforce. A prominent example is Lincoln Electric, a leading manufacturer of welding products (and subject of one of the most successful Harvard Business School case studies). Lincoln Electric has motivated its employees since 1934 with the promise to share a significant amount of profits with them. In addition, the firm had hardly any debt throughout most of its history, which, according to former CEO Donald F. Hastings, was a fundamental component of its whole implicit incentive system. This became apparent when Lincoln Electric substantially increased leverage in the mid nineties after an unsuccessful international expansion, making employees worried whether the firm would still keep its promises in the future (Hastings, 1999, p. 166). Another example is Southwest Airlines. This firm is well-known for being an employer with a strong commitment towards its workforce² and, compared to other domestic competitors, Southwest also has a much lower debt ratio.³

In this paper, we show how equity financing nurtures trust of a firm’s employees in their employer, fostering their beliefs that implicit promises will be honored. The commitment of a firm to honor these promises is modelled as credibility in relational contracts formed with a manager or with employees.⁴ Debt has a negative impact on the enforceability of relational contracts because too much debt increases the firm’s reneging temptation. The reason is that after reneging, the company is less profitable and hence faces a higher probability of becoming bankrupt – in which case creditors are not fully repaid if their claims exceed the liquidation value of the firm. Some of the negative consequences of breaking implicit promises can thus be shifted to creditors.

We develop a dynamic model where a risk-neutral firm (the principal) relies on capital and effort of a risk-neutral agent. The agent might be a manager of the firm, but could also be its total workforce. Capital can be financed by equity or debt, and the use of neither of them is exogenously restricted. As creditors and principal share the same time preferences, direct costs of debt and equity are identical. The agent’s effort is either high or low, and he has to be sufficiently incentivized to exert high effort. However, effort includes non-verifiable aspects like the agent’s motivation or commitment towards the firm, and relational contracts are required to provide incentives. Yet relational contracts only work if the agent trusts the firm to compensate him accordingly.

¹See Graham (2000), Graham and Leary (2011), or Strebulaev and Yang (2013), who argue that firms with no debt could generate tax benefits of up to 15% of the market value of equity by increasing leverage.
²For example, since 2010 it has always received an “Employees Choice Awards Best Place to Work”, which is awarded by Glassdoor.com.
³In 2012, Southwest’s debt-to-equity ratio was 0.48, compared to an industry ratio of 1.14; see Fool.com.
⁴Relational contracts are implicit arrangements that are sustained by repeated interaction and used when formal contracts are not feasible.; see Malcomson (2013) for an overview.
The agent’s trust is determined by the firm’s stake in the cooperative relationship, i.e., future profits with effort compared to future profits without effort. The difference between future profits with and without effort is driven by two aspects. First, generated per-period surplus is higher with effort. Second, there is always the chance that the firm goes bankrupt, and this event is more likely without effort. In addition to foregone profits when the firm no longer operates, the costs of a bankruptcy manifest in a liquidation value that is lower than the capital initially invested. However, loans can only be pledged against the firm’s liquidation value. This implies that leverage should not exceed a threshold which is determined by the amount creditors get back for sure in case of a bankruptcy. If debt was higher, the firm would not have to bear the full costs of reneging since the interest rate charged by creditors – who cannot observe the agent’s actual effort level – would be too low and not reflect the increased off-equilibrium bankruptcy risk. Consequently, low debt reduces incentives to deviate and raises the firm’s credibility in the relational contract.

There is evidence that leverage is generally low whenever issues like reputation and long-term commitment – which are necessary components of well-functioning relational contracts – are crucial determinants of a firm’s success. As mentioned before, Lincoln Electric is famous for having a highly motivated workforce. This is mainly attributed to its incentive system, which includes a large share of profits going to its employees. Since the incentive system not only includes the quantity and quality of work, but also dependability and cooperation (Hastings, 1999, p. 166), it provides a great example for the use of relational contracts. A failed international expansion in the late eighties forced Lincoln Electric to issue substantial debt for the first time in its history. According to former CEO Donald F. Hastings, this shocked employees who started to doubt that they could count on the firm keeping its promises in the future: “What we had not fully considered, however, was that the very fact of borrowing would be a major cultural shock to our employees. Most people at Lincoln saw the taking on of any debt as reckless, a view that had its roots in our incentive system […]. As soon as it became apparent that we were going to have to borrow to finance the foreign expansion, employees became concerned. They could see it affecting their incomes. And, as our level of debt increased, they grew deeply worried – and they let senior managers know it.” (Hastings, 1999, p. 166). More systematic evidence for an adverse impact of leverage on employee motivation is provided by Bae, Kang, and Wang (2011) and DeVaro, Kim, and Vikander (2014). The former observe a negative connection between a firm’s debt ratio and its popularity among employees, using several rankings that measure the latter aspect (including the Fortune Magazine’s list of “100 Best Companies to Work For”). The latter show that in case relational contracts are used to motivate CEOs, leverage has a negative impact on their performance and the bonus payments they receive.

In a number of extensions of our model, we show that the negative impact of debt on the enforceability of relational contracts is even stronger if some simplifying assumptions made in the main part of the paper are relaxed. In a first test we allow for many (non-verifiable) effort levels instead of just two. As long as the most profitable effort level can not be enforced, it is again optimal to have a relatively low debt ratio. An interesting aspect of this extension is that the connection

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5 After getting back on track, Lincoln Electric quickly reduced leverage again, from a 63% debt-to-equity-ratio in 1992 to less than 12% in 1998 (Hastings, 1999, p. 178).
“higher debt implies a higher bankruptcy risk” – which is at the root of most models deriving negative aspects of debt financing – is not needed for low leverage to be optimal. In fact, a higher debt level would not necessarily increase a firm’s bankruptcy risk, but only reduce enforceable effort.

Second, we relax the assumption of identical (direct) costs between debt and equity and instead assume that debt is cheaper than equity – which might be driven by the tax-deductability of interest. As long as the cost benefit of debt is not too large, the firm might forego some of the potential savings and instead keep leverage low – in order to better sustain its relational contracts. This can help to understand why many profitable firms (with seemingly negligible bankruptcy risk and large unused tax benefits associated with debt) have very low debt ratios (see Graham, 2000, or Graham and Leary, 2011).

Third, we assume that creditors also suffer from a bankruptcy if outstanding debt is below the firm’s liquidation value. As an example, we introduce a delay between the filing for bankruptcy and actual liquidation of the firm. In the benchmark model, the firm’s liquidation value can immediately be approached after a default, which in reality can take a substantial amount of time.\footnote{According to the World Development Indicators (WDI) database provided by the World Bank, there is a lot of cross-country variation in the average time needed to resolve insolvency. In some countries insolvency proceedings are completed after 0.4 years, while in others it takes up to 10 years to resolve insolvency.} If this assumption is relaxed, the enforceability of relational contracts is maximized if the firm has no debt at all; because now any debt allows the firm to share costs of a delay with its creditors.

\section*{2 Related Literature}

For many decades, the corporate finance literature has been concerned with the impact of a firm’s financing structure on its value. Various benefits and costs have been attributed to the use of either debt or equity, where the most prominent ones can be subsumed under tax incentives and agency problems (see Harris and Raviv, 1991, or Myers, 2003, for reviews of the literature). Agency problems exist between firms and outside investors and are caused by the unverifiability of activities pursued by entrepreneurs or managers. Myers (1977), Stiglitz and Weiss (1981, 1983), or Holmstrom and Tirole (1997) develop moral hazard models to show that using too much debt can increase the risk of a bankruptcy. Because a firm is protected by limited liability, its downside risk after a default is reduced. This induces firms to take insufficient measures to reduce their bankruptcy risk – for example by selecting too risky investments or by enforcing inadequate effort to make projects successful. Bankruptcy costs or “costs of financial distress” constitute one part of the so-called tradeoff theory (Modigliani and Miller, 1963), which states that a firm’s optimal debt ratio should balance the costs and benefits of debt. The latter are to a large extent associated with tax-deductible interest payments.\footnote{In addition to providing tax benefits, debt is supposed to be a response to agency problems between a firm and its management. For example, managers might have tendencies to overinvest, which are potentially induced by preferences for empire building (see Williamson, 1964, or Jensen and Meckling, 1976), short-termism of managers who focus on activities the market can easily observe (see Stein, 1989, or Bebchuk and Stole, 1993), or managers’ overconfidence into their own abilities (see Roll, 1986, or Heaton, 2002). If it is not possible to otherwise induce them to act in shareholders’ interest, debt can optimally reduce the free cash flow available to managers and hence...}
However, the empirical case for the classical tradeoff theory is not too strong. It rather seems that actual debt levels are much lower than predicted, and firms with potentially low costs of financial distress seem to be substantially underleveraged (see Graham, 2000, among others). As Myers (2003, pp. 21) put it, “... studies of the determinants of actual debt ratios consistently find that the most profitable companies in a given industry tend to borrow the least ... High profits mean low debt, and vice versa. But if managers can exploit valuable interest tax shields, as the tradeoff theory predicts, we should observe exactly the opposite relationship. High profitability means that the firm has more taxable income to shield, and that the firm can service more debt without risking financial distress.” Graham and Leary (2011) come to the conclusion that traditional explanations of a firm’s optimal capital structure are incomplete and that new approaches are needed. Those should also consider the effect of a firm’s financing structure on its relationship with non-financial stakeholders, for example employees. In an early contribution, Brander and Lewis (1986) emphasize the benefits of debt which can be used by firms as a negotiating tool in labor bargaining to demand larger concessions from its employees. More recently, Berk, Stanton, and Zechner (2010) show that risk-averse employees demand higher wages if leverage is increased. If their employer goes bankrupt, they become unemployed, and hence must be compensated for any increase in this risk induced by higher debt ratios. Titman (1984) analyzes the impact of a firm’s capital structure on implicit contracts with its customers/suppliers. If the company produces durable goods that require future maintenance or other services, the firm’s liquidation imposes costs on its customers. However, these costs are not taken into account by the firm. Equity can then serve as a commitment device to not liquidate the firm too early.

More related to our approach, Maksimovic and Titman (1991) develop a model where incentives to maintain high product quality are negatively affected by leverage (their idea might also be applied to the relationship between a firm and its employees). They provide conditions for a firm to produce a high-quality product that reduces its profits in the short run, but makes its customers believe that it is likely to maintain high quality in the future. Debt financing can reduce incentives to produce high product quality, since a quality reduction increases current cash flows at the expense of bondholders who then receive less in the future. In particular, if a firm is in financial distress, it may have an incentive to cut costs and reduce product quality in order to avoid immediate bankruptcy.

All the mentioned agency theories have in common that debt only has a negative (marginal) impact on firm value if it is associated with a substantial and immediate bankruptcy risk. And thus, these theories can not explain why leverage often is particularly low for healthy and profitable firms. Restrict their investment opportunities (see Jensen, 1986, Hart and Moore, 1995, or Zwiebel, 1996). In addition, debt shifts control rights to debtholders in bad states of the world. This threat of a potential loss of control is supposed to discipline managers and incentivize them to choose the right actions and repay investors (see Bolton and Scharfstein, 1990, Dewatripont and Tirole, 1994, Hart and Moore, 1998, or Inderst and Müller, 2003).

8 These also include the (so-far-not-mentioned) pecking order theory by Myers and Majluf (1984). There, asymmetric information between a firm and outside investors concerning the real value of the firm induces a ranking over sources to finance new projects. First, internal funds should be used, followed by debt and then equity. Although this theory might explain the reluctance of firms to issue debt, the empirical evidence is at best mixed (see Leary and Roberts, 2010).

9 This argument received empirical supported by Matsa (2010) or Benmelech, Bergman, and Enriquez (2012).
In our model, bankruptcy is a possibility, but merely an off-equilibrium phenomenon. Therefore, we add a new dimension to indirect costs of financial distress to our model and provide a new but complementary explanation for a firm’s reluctance to use debt financing.

We finally contribute to the literature on relational contracts. Theoretical foundations were laid by Bull (1987), MacLeod and Malcomson (1989), or Levin (2003), a good overview including more recent developments is provided by Malcomson (2013). Many economists now are convinced that the appropriate handling of relational contracts is crucial for the success of firms. Gibbons and Henderson (2013), for example, argue that different aspects of relational contracts are responsible for observed persistent performance differences among seemingly similar enterprises. Therefore, improving our understanding of the precise impact of relational contracts on firm success is crucial, in particular in the context of a firm’s organizational structure.

3 Model

We analyze a firm and the behavior of its risk-neutral owner (principal, she) and its risk-neutral agent (he). This agent might be a manager or the firm’s total workforce. For concreteness, we mostly refer to a single agent, but show below that the formal analysis and the main results are basically the same in a relationship between firm and total workforce. The firm needs two kinds of inputs for production, namely capital and the agent’s effort. The time horizon is infinite, time is discrete, and players share a common discount factor $\delta$, with $0 < \delta < 1$.

The firm may be liquidated in any period $t$. This event is denoted by $l_t = 1$ (how and why a liquidation might occur is described below), while $l_t = 0$ if the firm is not liquidated. The general state of the firm – whether it still exists or not – at the beginning of period $t$ is denoted by $d_t \in \{0, 1\}$. Thus, if $d_t = 1$, the firm has not been liquidated yet; if $d_t = 0$, it has been liquidated in one of the previous periods. A liquidation is irreversible, so that $d_t = d_{t-1}(1 - l_t)$.

3.1 Technology, Financing and Timing

In period $t = 0$, the principal has to make the capital investment $K > 0$ to get the firm running, i.e. to induce $d_0 = 1$ (the amount $K$ is exogenously given). She can either use equity (assuming she is not liquidity constrained and has sufficient own funds) or short-term debt to finance $K$. If she wants to use debt financing, she can enter a perfectly competitive credit market at the end of every period $t = 0, 1, 2, \ldots$, where all potential creditors are risk neutral and also have a discount factor $\delta$. The total debt level borrowed in period $t - 1$ and used in period $t$ is denoted by $D_t (\leq K)$, so that equity amounts to $K - D_t$. Interest is denoted by $r_t$ and paid at the end of period $t$. Hence, the firm repays $(1 + r_t)D_t$ before it can enter the credit market in period $t + 1$ to borrow $D_{t+1}$.

At the beginning of every period $t = 1, 2, \ldots$ – given the firm exists ($d_t = 1$) – the principal makes an employment offer to the agent. This offer involves a compensation package $W_t = (w_t, b_t) \in \mathbb{R}^2$.

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10In a related setting, Englmaier and Fahn (2013) allow $K$ to be endogenous and show that overinvestments enhance the enforceability of relational contracts.
where $w_t$ is a fixed component paid at the beginning of the period and $b_t$ a discretionary bonus paid later. The agent’s decision – about accepting the offer or not – is captured by $d_t^A$, where $d_t^A = 1$ describes an acceptance, and $d_t^A = 0$ a rejection. If $d_t^A = 0$ the firm is liquidated, with the consequences of a liquidation further described below. The agent consumes his outside utility $\tilde{u} \geq 0$ whenever he is not employed. Upon accepting a contract offer ($d_t^A = 1$), the agent chooses effort. Effort is binary, $e_t \in \{0, 1\}$, and associated with private effort costs $c(e_t)$ – with $c(0) = 0$ and $c(1) = c > 0$. The agent’s contribution to firm value is characterized by the function $f(e_t)$, with $f(1) > f(0) \geq 0$. We assume $f(1) - c > f(0)$, which implies that – given the firm is active in a given period – effort yields a higher per-period surplus than no effort. After observing $f(e_t)$, the principal has the choice to forward $b_t$ to the agent. The agent is supposed to be remunerated with the bonus whenever he has exerted the appropriate effort level.

In any period, the firm can experience a shock that leads to its liquidation, i.e. to $l_t = 1$. The likelihood of this shock depends on the period-$t$ effort level and equals $1 - \rho(e_t)$, with $\rho(1) > \rho(0) \geq 0$. High effort thus is not only beneficial because it gives a higher instantaneous surplus, but also because it increases the chances that the firm survives for another period. If the firm has not faced a negative shock, the principal is supposed to repay the loan $D_t$, plus interest $r_tD_t$, where $r_t$ is endogenously determined. If she complies and repays, she enters the credit market again to borrow $D_{t+1}$, and the game moves on to the next period. If the principal defaults and refuses to repay $(1 + r_t)D_t$, the creditors get access to the firm’s assets and liquidate the firm at the end of period $t$. The possibility of renegotiation would not affect our results as long as creditors were able to install a different principal to run the firm after a default. This new principal would face the same incentives concerning a voluntary default as the original one and hence take the same decisions. The liquidation value of assets is immediately realized and equals $\gamma K$, with $\gamma < 1$. The firm is protected by limited liability (in case of a bankruptcy), and creditors receive $\overline{D}_t = \min\{D_t(1 + r_t), \gamma K\}$ in case of a liquidation, while the principal keeps $\Pi_t = \max\{\gamma K - D_t(1 + r_t), 0\}$. This implies that $(1 - \gamma)K$ are the total bankruptcy costs the firm faces. These costs can relate to firm-specific assets that are less valuable outside the firm.

Finally, let us assume that the principal is the sole equity holder who receives residual profits, so we do not have to model dividend payments. If more shareholders – among whom profits are distributed – existed, the present analysis would remain unaffected as long as no fundamental conflict of interest among them were present, in particular concerning the choice whether to honor relational contracts. We also exclude the possibility to use equity shares to compensate the agent – because we want to focus on effort dimensions for which verifiable measures do not exist (as elaborated in the next section).

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11Note that a liquidation in case of a rejection would not be necessary if another manager could be hired in that case. This would leave the present analysis unaffected (since $d_t^A = 0$ will never happen in equilibrium) as long as all potential managers were aware of deviations from equilibrium behavior by the firm.

12This might be endogenized by assuming that $f(e_t)$ is a random variable, with $f(1)$ first-order stochastically dominating $f(0)$. Furthermore, if the realization of $f(e_t)$ was below a threshold, debt could not be repaid, triggering a liquidation.

13This assumption is relaxed below, where we show that a later realization of the liquidation value amplifies the committal role of equity financing (see Section 7.4).
The timing of actions for each period $t \geq 1$ can be summarized in the following figure (where P denotes the Principal and A the Agent):

3.2 Information Structure

We assume that effort $e_t$ as well as $f(e_t)$ can be observed by the principal and the agent but not by any third party, in particular not by creditors. Therefore, $e_t$ can not be part of explicit, court-enforceable incentive contracts, and a relational contract is required to motivate the agent. Note that $f(e_t)$ is not called output in our setting but rather “reflects everything the principal values (gross of wages), (and) [...] might be more appropriately called the agent’s total contribution” (Gibbons, 2010, p. 341). Therefore, the case $e = 0$ does not necessarily imply that the agent does not work at all. It rather captures all dimensions of the agent’s effort that are verifiable (and optimally implemented by the firm) – either because they reflect standard tasks or because sufficient verifiable measures exist. Consequently, $f(0) > 0$ is possible (and will generally be assumed throughout), implying that it can be profitable to run the firm with $e = 0$.

The bonus payment $b_t$ is neither verifiable nor can it be observed by creditors. The reason is that the bonus does not only have monetary components, but includes aspects like promotions, extra free time, work climate, or other non-monetary perks the agent values and that are costly to the firm. For simplicity, we also assume that the fixed wage $w_t$ can not be observed by creditors. However, it would be sufficient to assume that the part of the compensation package used to reward the agent for his non-verifiable effort is not observable. Otherwise, payments might serve as a signal to creditors whether someone deviated from equilibrium behavior, which is a case we want to rule out. Our results would still hold, though, if creditors were able to observe wage payments but the relationship between firm and agent was subject to moral hazard (i.e., if the firm was not able to observe effort and $f(e)$ stochastically depended on effort). The same would be true if long-term debt contracts were used. In that case, creditors could not immediately adjust the terms of the debt contract upon observing changes in the relationship between principal and agent.

As for creditors, short-term formal contracts are feasible in the following sense: The principal is not able to divert $D_t$ after borrowing it, so $D_t$ is always invested in the firm. In addition, repayment of $(1 + r_t)D_t$ is specified in the credit contract, with a default by the principal followed by the creditors getting access to the firm’s assets, as described above. We show in Appendix IV that the possibility to divert $D_t$ does not affect our results.
A liquidation of the firm is observed by all players. To simplify the analysis, we assume that all aspects of the relationship between firm and creditors (as further defined below) can be observed by the agent, as well as by the whole credit market. The former part is only imposed for notational convenience, the latter part allows us to assume without loss of generality that the principal only borrows from one creditor.

3.3 Histories, Strategies, and Equilibrium Concept

Before specifying histories, we introduce some additional notation. The amount of debt actually repaid by the principal is denoted by \( \hat{D}_t \), with \( \hat{D}_t \leq (1 + r_t)D_t \). The amount the firm attempts to borrow for period \( t + 1 \) is denoted by \( \hat{D}_{t+1} \). The amount actually granted by the creditor equals \( D_{t+1} \), with \( D_{t+1} \leq \hat{D}_{t+1} \).

The public events in period \( t \), i.e., those that are observable to all parties involved (principal, agent, creditors), are \( h_t = \{d_t, d_t^A, \hat{D}_t, l_t, \hat{D}_{t+1}, D_{t+1}\} \), with \( h^t = \{h_n\}_{n=0}^{t-1} \) being the public history at the beginning of period \( t \). Furthermore, \( H^t = \{h^t\} \) is the set of public histories until time \( t \), and \( H = \bigcup_t H^t \) the set of public histories.

In addition, principal and agent can observe \( \{w_t, e_t, b_t\} \). The shared history of principal and agent at the beginning of period \( t \) is \( h_{PA}^t = h^t \cup \{w_n, e_n, b_n\}_{n=1}^{t-1} \). Accordingly, \( H_{PA}^t = \{h_{PA}^t\} \) is the set of shared histories between principal and agent until time \( t \), and \( H_{PA} = \bigcup_t H_{PA}^t \) the set of shared histories.

The strategies of principal, agent and creditor are denoted by \( \sigma^P \), \( \sigma^A \) and \( \sigma^C \), respectively. The principal’s strategy \( \sigma^P \) determines the fixed wage, given \( h_{PA}^t \). Given \( h_{PA}^t \cup \{d_t, d_t^A, w_t, e_t\} \), it further specifies the bonus \( b_t \), the choice how much to pay back \( (\hat{D}_t) \), as well as \( \hat{D}_{t+1} \) (the decision about how much should be borrowed next period). The agent’s strategy \( \sigma^A \) determines \( d_t^A \) and \( e_t \), given \( h_{PA}^t \cup \{w_t\} \).

Since the creditor is not able to observe the features of the relationship between principal and agent, he has to form beliefs about the history \( h_{PA}^t \). It will be sufficient to consider the creditor’s belief concerning whether the game is in equilibrium or not, i.e. whether a deviation from equilibrium behavior has occurred. The reason is that trigger strategies are optimal in the relationship between principal and agent, as further described below. This belief is denoted \( \mu_t \in [0,1] \), and formed given the public history \( h^t \cup \{d_t, d_t^A, \hat{D}_t, l_t, \hat{D}_{t+1}\} \). Thus, the creditor’s strategy \( \sigma^C \) specifies \( D_{t+1} \), given \( h^t \cup \{d_t, d_t^A, \hat{D}_t, l_t, \hat{D}_{t+1}\} \) and \( \mu_t \). In addition, we assume that \( \mu_0 = 1 \).

Furthermore, mixed strategies do not expand the set of enforceable payoffs (see Mailath and Samuelson, 2006, p. 310, who show that this holds in games with a product structure) and are therefore not considered.

We apply the concept of sequential equilibrium, which consists of the players’ strategies and the creditor’s belief \( \mu_t \). Furthermore, a sequential equilibrium strategy maximizes the respective discounted payoff stream of a player, given other players’ strategies, and given \( \mu_t \). Finally, \( \mu_t \) has to be consistent with strategies, and is updated according to Bayes’ rule, whenever possible.\(^{14}\)

\(^{14}\)Consistency further requires that at information sets that are reached with zero probability, equilibrium strategies
In the following, our objective is to characterize a sequential equilibrium that maximizes the firm’s expected discounted profit stream in period \( t = 0 \).

### 3.4 Payoffs

At the beginning of a period \( t \geq 1 \) in which the firm is operating, so \( d_t = 1 \) and \( d_t^A = 1 \), the expected equilibrium payoff of the principal can be defined recursively and equals

\[
\Pi_t = f(e_t) - w_t - b_t + \rho(e_t) \mathbf{1}_{\{D_t = (1+r_t)D_t\}} \left[ -D_t (1 + r_t) + D_{t+1} + \delta \Pi_{t+1} \right] \\
+ \rho(e_t) \mathbf{1}_{\{\hat{D}_t < (1+r_t)D_t\}} \left[ -\hat{D}_t + \max \left\{ \gamma K - \left( D_t (1 + r_t) - \hat{D}_t \right), 0 \right\} \right] + (1 - \rho(e_t)) \Pi_t.
\]

There, recall that \( \Pi_t = \max \{ \gamma K - D_t (1 + r_t), 0 \} \) is the firm’s liquidation payoff, and that once the firm repays less than it owes to the creditor \( (\hat{D}_t < (1 + r_t)D_t) \), the latter gets access to the firm’s assets in order to capture as much as possible of the outstanding debt. Furthermore, since the investment \( K \) has to be made in period \( t = 0 \),

\[
\Pi_0 = -(K - D_1) + \delta \Pi_1.
\]

The agent’s equilibrium payoff in a period \( t \geq 1 \) in which the firm is operating and \( d_t^A = 1 \) is

\[
U_t = w_t + b_t - c(e_t) + \delta \left[ \rho(e_t) U_{t+1} + (1 - \rho(e_t)) \overline{U} \right],
\]

with \( \overline{U} = \frac{w}{1-\delta} \).

As established above, the fact that all creditors can observe the public history implies that we can assume that the firm only borrows from one creditor. Hence, this creditor’s payoff in a period \( t \geq 1 \) where the firm is active equals

\[
V_t = \rho(e_t) \mathbf{1}_{\{D_t = (1+r_t)D_t\}} \left[ D_t (1 + r_t) - D_{t+1} + \delta V_{t+1} \right] \\
+ \rho(e_t) \mathbf{1}_{\{\hat{D}_t < (1+r_t)D_t\}} \left[ \hat{D}_t + \min \left\{ D_t (1 + r_t) - \hat{D}_t, \gamma K \right\} \right] + (1 - \rho(e_t)) \overline{D}_t,
\]

with \( \overline{D}_t = \min \{ D_t (1 + r_t), \gamma K \} \). In addition, \( V_0 = -D_1 + \delta V_1 \).

Without having a qualitative impact on our results, we will from now on impose

**Assumption 1**: \( \rho(1) = 1 \)

Assumption 1 implies that if the agent exerts high effort, the firm will not face an involuntary bankruptcy. In addition, we define \( \rho \equiv \rho(0) < 1 \). Assumption 1 simplifies the analysis without must approximate beliefs that are derived from strategies that assign positive probabilities to every action (Kreps and Wilson, 1982).
affecting our results, for which we only need the likelihood of a bankruptcy being higher given the agent exerts less effort.

\section*{4 Benchmark Cases}

Before deriving conditions for the enforceability of relational contracts, we analyze two benchmark cases. First, we assume that relational contracts are not feasible, in which case the firm might still be operated if $f(0)$ is sufficiently large. Second, we assume that \textit{formal} contracts to enforce $e = 1$ are feasible. In both cases, we show that given the firm never defaults voluntarily, the financing structure does not affect firm value. Here, we just present an overview of both cases and relegate a more elaborate analysis, as well as the proofs of the presented Lemmas, to Appendix II.

\subsection*{4.1 Benchmark I: $e = 0$}

First, we develop a profit-maximizing equilibrium with $e = 0$ in all periods, assuming that the firm has been started and the investment $K$ been made (recall that the principal also needs the agent to operate the firm if $e = 0$). For this to be interesting, $f(0)$ must be strictly positive and large enough so that running the firm is optimal compared to liquidating it when $e = 0$. In this case, the situation analyzed here describes the outcome when positive effort can not be enforced. Furthermore, it determines players’ off-equilibrium payoffs in a relational contract, and hence determines whether such a relational contract can be enforced at all.$^{15}$ However, there is one difference between payoffs in this section and off-equilibrium payoffs to enforce $e = 1$. Here, creditors are aware of $e = 0$ and set interest rates accordingly. In an equilibrium with $e = 1$, creditors might not realize that the game is out of equilibrium – they can only observe deviations if the firm’s debt policy changes accordingly.

A profit-maximizing equilibrium with $e = 0$ has the following features. The agent receives his outside payoff in every period, i.e., $w = \overline{w}$. The interest rate $r$ depends on the debt level (which is constant over time). If $D \leq \delta \gamma K$, the creditor receives $(1 + r)D$ for sure, and only has to be compensated for discounting. In this case, $r = \frac{1 - \delta}{\delta}$. If $D$ is larger, the creditor only gets $\gamma K < (1 + r)D$ in the event of a bankruptcy since the firm is protected by limited liability; then, the creditor must be compensated for the higher bankruptcy risk and demands an interest rate $r > \frac{1 - \delta}{\delta}$.

In addition, the principal might be credit constrained. This means that she would like to borrow more but is not granted additional credit - even if she is willing to pay a higher interest rate. Such a situation occurs if a \textit{voluntary default} is optimal for higher debt, i.e., the principal would refuse to repay $(1 + r)D$ even though she was able to do so. For a voluntary default not to be optimal, the condition $-rD + \delta \Pi(0) \geq \Pi$ must hold, where \( \Pi(0) = \frac{f(0) - \pi - \rho rD + (1 - \rho)\Pi}{1 - \delta \rho} \), i.e., the firm’s continuation value must be sufficiently large.

Driven by the firm’s limited liability, a voluntary default becomes more tempting once debt is above $\delta \gamma K$. This resembles the mechanism proposed by Myers (1977), Stiglitz and Weiss (1981, \footnote{If $f(0)$ is too small, players’ off-equilibrium payoffs in a relational contract are determined by an immediate liquidation of the firm.}}
1983), or Holmstrom and Tirole (1997). There, limited liability induces highly leveraged firms to take insufficient measures to reduce their bankruptcy risk, for example by selecting too risky investments or by enforcing inadequate effort to make projects successful. However, this aspect has been widely analyzed, and we aim at focusing on relatively healthy firms where owners have no incentives to take measures that induce a bankruptcy. Let us therefore assume that the negative (direct) effect of leverage on the principal’s incentives is absent, and, for the rest of this paper, impose

\[
\text{Assumption 2: } \delta (f(0) - \bar{u}) \geq K \frac{(1-\delta \rho) - \delta (1-\rho) \gamma \delta \rho}{\delta \rho}.
\]

Assumption 2 is sufficient for a voluntary default never to be optimal. It does not affect our qualitative results, though. In Appendix III, we show that the positive impact of equity on the enforceability of relational contracts, as derived below, is more pronounced once Assumption 2 is violated.

The next Lemma gives the main result of this section.

**Lemma 1:** Given \( e_t = 0 \) in every period the firm is active, it is optimal to make the investment \( K \) and start the firm in period \( t = 0 \). Furthermore, the firm’s value at the onset of the game, \( \Pi_0(0) \), is independent of its financing structure.

Here, we obtain a Modigliani-Miller-type irrelevance result\(^{16}\) as firm value is independent of its financing structure. The reason is that direct costs of debt and equity are identical, and that everyone – including creditor – is aware that \( e = 0 \). If debt is above \( \delta \gamma K \), it is becoming more expensive, which just reflects the real cost of capital. In addition, under Assumption 2, debt has no effect on the principal’s incentive to voluntarily default, which is why it does not affect the firm’s bankruptcy risk.

In the next section, we show that this result also holds if formal contracts can be used to motivate the agent to choose \( e = 1 \).

### 4.2 Benchmark II: Formal Contracts Feasible

Benchmark II analyzes the situation that effort \( e_t \) is verifiable and can be part of a formal short-term contract. In addition, we rule out the possibility that the firm is able to file for bankruptcy in order to avoid wage payments.

Consider the following agreement: Upon accepting the contract, the agent receives the fixed wage \( w = \bar{u} \). If \( e = 1 \), he also gets a bonus \( b = c \), whereas no bonus is paid in case of \( e = 0 \). Besides, the formal contract can be revealed to the creditor whose information disadvantage then is eliminated. Such a contract makes the agent accept the contract in every period, and further induces him to

\(^{16}\)See Modigliani and Miller (1958).
exert effort. Furthermore, it is a profit-maximizing contract, since the whole surplus goes to the principal.

We get

**Lemma 2:** Assume the agent’s effort is verifiable. Then, it is optimal to carry out the investment $K$ and start the firm in period $t = 0$. Furthermore, $e = 1$ is implemented in every period and firm value $\Pi_0(1)$ is independent of $D$.

The irrelevance of the firm’s financing structure on its value is again driven by a) identical direct costs of debt and equity financing and b) debt having no impact on the benefits of operating the firm (compared to a liquidation). This result adds one aspect to Modigliani and Miller’s irrelevance theorem: If the workforce of a firm has to be incentivized to exert extra effort, the financing structure has no impact on this relationship as long as enforcement is either not possible or if enforcement is not an issue because formal contracts can be written.

In the following we show that the financing structure affects the extent to which relational contracts can be enforced, though.

5 Relational Contracts

In this section, we derive conditions for a relational contract with $e_t = 1$ and $d_t^A = 1$ in every period and without a voluntary bankruptcy being optimal. The latter point implies $\hat{D}_t = D_t(1 + r_t)$, i.e., in equilibrium the firm repays its debt.

Note that a condition for starting the firm and making the investment $K$ being optimal is not needed in this section. The reason is that the relational contract to enforce positive effort will only be imposed by the principal if it yields higher profits than a contract with $e = 0$, as derived above. Since starting the firm is already optimal for $e = 0$ by assumption, it will be optimal here as well.

5.1 Constraints

The following constraints must be satisfied in an equilibrium with positive effort in every period. As before, an individual rationality constraint is needed to make sure that accepting the contract is optimal for the agent:

\[ U_t \geq \overline{U}, \quad \forall t \geq 1. \quad \text{(IR)} \]

Furthermore, it must be optimal for the agent to deliver effort $e_t = 1$, given that he trusts the principal to actually pay the bonus $b_t$ and given an equilibrium continuation payoff $U_{t+1}$. This is captured by an incentive compatibility (IC) constraint for every period $t$. There, if the agent deviates and chooses $e_t = 0$, he does not receive the bonus, and the equilibrium without any effort is played from then on where the agent only receives his outside option $\overline{u}$ in every period.\footnote{This is based on the presumption that once the agent deviated, the principal thinks he will not exert effort in the subsequent periods.}
Hence, we have

\[-c + b_t + \delta U_{t+1} \geq \delta U_t, \quad \forall t \geq 1.\] (IC)

For the principal, the following constraints must hold. First, it has to be optimal for her to make the promised payment \(b_t\), given the agent chose \(e_t = 1\). If she reneges and does not forward the bonus to the agent, all trust between players is lost, and no positive effort can subsequently be enforced (following Abreu, 1988, who shows that a player with an observable deviation from equilibrium behavior should optimally be punished by receiving their minmax-payoff). Yet we have to take the possibility into account that the principal might also (try to) change the firm’s financing structure after reneging on paying \(b_t\). We therefore denote the principal’s off-equilibrium repayment \(\hat{D}_{t}^0\), the amount she attempts to borrow in this case \(\tilde{D}_{t+1}^0\), and the amount she actually borrows off-equilibrium \(D_{t+1}^0\). Then, the principal’s so-called dynamic enforcement (DE) constraint equals

\[-b_t - D_t(1 + r_t) + (D_{t+1} + \delta \Pi_{t+1}(1)) \geq -\hat{D}_{t}^0 + \left(D_{t+1}^0 + \delta \hat{\Pi}_{t+1}(0)\right) \quad \forall t \geq 1.\] (DE)

Off-equilibrium profits \(\hat{\Pi}_{t+1}(0)\) might differ from \(\Pi(0)\), the latter denoting the profits in an equilibrium with \(e = 0\), as derived in Section 3.1. There, creditors were aware of the fact that \(e = 0\). Here, creditors can not observe \(e_t, f(e_t), w_t\) and \(b_t\), but can only use information from the firm’s debt policy – i.e., its choice of \(D_t\) and \(\hat{D}_{t+1}\) – to draw inferences on whether the game is in equilibrium or not. This potentially has an impact on profits, since the interest rate \(r\) for debt levels above \(\delta \gamma K\) is higher if creditors expect a higher bankruptcy risk. Concerning repayment \(\hat{D}_{t}^0\), though, only two cases have to be considered. Because any \(\hat{D}_{t}^0 \neq D_t(1 + r_t)\) triggers an immediate liquidation of the firm, the principal will either choose \(\hat{D}_{t}^0 = D_t(1 + r_t)\) or \(\hat{D}_{t}^0 = 0\). Hence, (DE) equals

\[-b_t - D_t(1 + r_t) + D_{t+1} + \delta \Pi_{t+1}(1) \geq \hat{\Pi}_{t} \quad \forall t \geq 1, \quad \text{or}\]

\[-b_t - D_t(1 + r_t) + D_{t+1} + \delta \Pi_{t+1}(1) \geq -D_t(1 + r_t) + D_{t+1}^0 + \delta \hat{\Pi}_{t+1}(0) \quad \forall t \geq 1,\] (2)

depending on what kind of deviation delivers higher profits.

Furthermore, repaying the loan – after rewarding the agent – has to be better than defaulting,

\(-D_t(1 + r_t) + (D_{t+1} + \delta \Pi_{t+1}(1)) \geq \Pi_t\), which is satisfied given (1). This implies that agency problems between firms and creditors (more precisely: agency problems between firms and outside investors) – which in the form of a potential diversion of initial funds or returns are at the core of many contributions on a firm’s optimal financing structure (for example see Ellingsen and Kristiansen, 2011, or Bergemann and Hege, 2005) – are still present in our setting. But they only remain relevant in an interaction with another agency problem – namely between firm and manager or employees.

Finally, we impose a creditor rationality (CR) constraint to guarantee that the creditor does not future as well. However, the analysis would be identical if the principal believed that the deviation was a singular event, which is driven by the agent not receiving a rent in any profit-maximizing equilibrium (derived below).
make a loss in equilibrium:

\[-D_t + \delta V_t \geq 0 \ \forall t \geq 0. \quad (CR)\]

This set of constraints must be satisfied in order to enforce an equilibrium with \(e_t = 1\) in every period \(t\), where the agent accepts the contract offer in every period, and where a voluntary bankruptcy is never optimal. In the next step, we show that this set of constraints can be simplified and condensed into one condition.

### 5.2 Auxiliary Analysis

We first show that the creditor does not get a rent in a competitive credit market. In addition, agents do not get a rent in any profit-maximizing equilibrium. The proofs to all Lemmas and the Proposition in this section can be found in Appendix I.

**Lemma 3:** Any profit-maximizing equilibrium can be replaced by one where the constraints (IRA), (IC) and (CR) bind in every period \(t\).

Lemma 3 follows from our objective to find profit-maximizing equilibria. In addition, there are no further frictions that might make it necessary to grant the agent a rent when providing incentives. Furthermore, because no (voluntary or involuntary) bankruptcy is observed in equilibrium, creditors expect the firm to always meet its obligations. Hence, the interest rate in an equilibrium with \(e_t = 1\) in every period equals

\[r_t = \frac{(1 - \delta)}{\delta}, \ \forall t \geq 1,\]

which holds irrespective of the debt level. Lemma 3 also implies that \(w_t = \bar{w}\) and \(b_t = c\) in every period \(t\), and compensation payments are constant over time.

In the next Lemma, we show that off-equilibrium, i.e., after reneging on the bonus, the principal will stick to the equilibrium borrowing policy.

**Lemma 4:** Assume the principal refused to pay the bonus \(b_t\) and subsequently did not default. Then, \(D^0_t = D_t\) in all periods \(t \geq \tau + 1\).

After reneging, the principal would like to use the informational advantage she has over the creditor concerning the firm’s survival probability, and replace as much equity as possible with debt. However, creditors would get suspicious if the principal tried to borrow more than expected and infer that the game is not in equilibrium anymore. This would induce them to adjust interest rates accordingly, and only accept conditions assuming \(e = 0\) from then on.

Given the stationary structure of the game, and given that the agent’s compensation is the same in every period, one expects debt to also be the same across periods. This is indeed the case, and formally proven in the following Lemma.
**Lemma 5:** In a profit-maximizing relational contract with \( e = 1 \) in every period, it is (weakly) optimal to have debt constant over time.

Together with Lemma 3, this allows us to subsequently omit all time subscripts because payments and debt are constant over time. Finally, we show that after reneging, the principal will not default but instead continue to operate the firm with \( e = 0 \).

**Lemma 6:** Assume the principal refused to pay the bonus \( b_t \). Then, she will not default and repay the loan in any future period.

Lemma 6 directly follows from Assumption 2, which states that a voluntary bankruptcy is not optimal in an equilibrium with \( e = 0 \). Here, the firm’s profits in case of a continuation of the firm after reneging are even (weakly) higher than above, since the information asymmetry between firm and creditors keeps interest rates too low off-equilibrium. Using Lemmas 3-6, the (DE) constraint is simplified, yielding a condition (DE’) that is necessary and sufficient for a profit-maximizing equilibrium with \( e = 1 \).

**Proposition 1:** The principal will implement \( e = 1 \) in every period if and only if the constraint

\[-c + \delta \left( \Pi(1) - \hat{\Pi}(0) \right) \geq 0 \]  

(De’)

is satisfied.

In words, the principal is only willing to honor her promises if discounted equilibrium profits exceed the gains of a deviation.

6 Debt Financing and the Enforceability of Relational Contracts

In this section, we explore the impact of the financing structure on firm value and particularly to what extent it affects the enforceability of relational contracts. To start with, we show that if the firm uses relational contracts to motivate the agent, its financing structure still has no direct effect on firm value.

**Lemma 7:** Assume condition (DE’) holds for any debt level \( D \) with \( 0 \leq D \leq K \). Then, \( \Pi_0(1) \) is independent of \( D \).

**Proof:** This follows from \( \Pi_0 = -(K - D) + \delta \Pi(1) = -(K - D) + \delta \frac{f(1) - \pi - c}{1 - \delta} \frac{1 - \delta D}{1 - \delta} = -K + \delta \frac{f(1) - \pi - c}{1 - \delta} \). Q.E.D.
The logic of this results resembles the one underlying Lemma 1. If the enforceability constraint is not an issue and direct costs of debt and equity financing are identical, the financing structure is irrelevant.

However, the financing structure of the firm can have an indirect impact on its profits, namely via its impact on the enforceability of relational contracts. The following Proposition states the main result of this paper: the (DE) constraint is easier to satisfy if debt is low.

**Proposition 2:** The enforceability of relational contracts is maximized for \( D \leq \delta \gamma K \). Generally, the (DE) constraint may or may not be satisfied; it is more likely to be satisfied for higher values of \( f(1) \) and \( \delta \), and for lower values of \( D, f(0), \rho, \gamma \) and \( \bar{u} \).

**Proof:** Here, we only prove the negative impact of \( D \) on the enforceability of relational contracts. The remaining parts can be found in Appendix I.

The (DE) constraint necessary to enforce \( e = 1 \) equals \( c \leq \delta \left( \Pi(1) - \bar{\Pi}(0) \right) \), where \( \bar{\Pi}(0) \) is the firm’s off-equilibrium payoff given creditors are not aware of the agent’s loss of trust. Furthermore, \( r = \frac{1 - \delta}{\delta} \) on and off the equilibrium path, and the principal does not liquidate the firm off-equilibrium.

Hence, the (DE) constraint is

\[
c \leq \delta \left( \frac{(f(1) - c - \bar{u})}{1 - \delta} - \frac{(f(0) - \bar{u})}{1 - \delta \rho} - (1 - \rho) \frac{D}{\delta} + \bar{\Pi} \right). \tag{4}
\]

First, assume \( D \leq \delta \gamma K \). Then, \( \bar{\Pi} = \gamma K - \frac{D}{\delta} \), and

\[
c \leq \delta \left( \frac{(f(1) - c - \bar{u})}{1 - \delta} - \frac{(f(0) - \bar{u})}{1 - \delta \rho} - \gamma K(1 - \rho) \frac{1 - \rho}{1 - \delta \rho} \right), \]

i.e., (DE) is independent of \( D \). For \( D > \delta \gamma K \), \( \bar{\Pi} = 0 \), and the (DE) constraint equals \( c \leq \delta \left( \frac{(f(1) - c - \bar{u})}{1 - \delta} - \frac{(f(0) - \bar{u})}{1 - \delta \rho} - \frac{D}{\delta} \frac{1 - \rho}{1 - \delta \rho} \right), \)

i.e., its right hand side is decreasing in \( D \). Q.E.D.

Intuitively, the principal can share parts of the bankruptcy costs with the creditor when \( D > \delta \gamma K \), which is driven by the following aspect. The creditor is not fully repaid in case of a bankruptcy because firm assets are not sufficient to fully cover all obligations. The optimal response of the creditor would be to adjust interest rates. Since the principal does not change her borrowing policy off-equilibrium, though, the creditor is not aware of a deviation by the principal. Hence, the principal pays too little interest to cover the real costs of capital after reneging. Therefore, the outside option becomes relatively more attractive – compared to equilibrium payoffs – for higher values of \( D \).

In addition, the values of \( f(1), \delta \) and all other parameter values determine the enforceability of relational contracts, and also interact with the firm’s leverage. For example, Proposition 2 implies that values of \( f(1) \) and \( \overline{f(1)} \) exist such that the debt level is irrelevant for the enforceability of relational contracts for \( f(1) < \overline{f(1)} \) and \( f(1) > \underline{f(1)} \). In the first case, relational contracts can never, in the latter case they can always be enforced. For \( \overline{f(1)} \leq f(1) \leq \underline{f(1)} \), there exist thresholds \( D(f(1)) \), with \( \delta \gamma K \leq D(f(1)) \leq K \), determined by the binding condition (4), such that relational contracts are feasible for \( D \leq D(f(1)) \), and not for \( D > D(f(1)) \).
Note that Proposition 2 does not depend on Assumption 2, which states that a voluntary bankruptcy is not optimal off-equilibrium. What is required for debt having a negative impact on relational contracts is that a bankruptcy is more likely off-equilibrium than in equilibrium. Whether the bankruptcy is voluntary or not is irrelevant. In Appendix III, we show that once Assumption 2 does not hold, the negative impact of debt on the enforceability of relational contracts actually becomes stronger. Then, a bankruptcy can instantaneously happen off-equilibrium (and not just with probability \((1 - \rho)\)), and bankruptcy costs are immediately realized with probability 1.

Finally, a lower liquidation payoff helps to enforce relational contracts and might have a positive (indirect) effect on firm value. A lower \(\gamma\) reduces the firm’s outside option and, thus, reduces the benefits from reneging. Therefore, the firm would gain from a reduction in \(\gamma\).\(^{18}\) Although the value \(\gamma\) is exogenously given in our model, one could think about activities that reduce the liquidation value. A dispersion of ownership, for example, might induce rent seeking activities in case of bankruptcy and consequently increase the associated efficiency loss.

\section{Robustness and Model Extensions}

In this section, we show that the negative impact of debt on the enforceability of relational contracts remains – and in some cases becomes more pronounced – once some assumptions are relaxed. First, we show that it does not make a difference if relational contracts are not formed with one agent, for example a manager, but with the firm’s workforce as a whole.

\subsection{Multilateral Relational Contracts}

Let us assume that the task of the principal is to manage the whole workforce of the firm. This workforce is of mass 1, and each of the individual agents chooses effort \(e_t \in \{0, 1\}\) in every period. While \(e = 1\) is associated with effort cost of \(c\), \(e = 0\) is not costly for the respective agent. The most crucial aspect of multilateral relational contracts (see Levin, 2002) is that the behavior of the firm towards one particular agent can be observed by all other employees. For this reason, reneging on one agent can (and optimally will) be punished by all of them: They subsequently lose trust and are not willing to exert effort anymore.

If we only consider symmetric equilibria (i.e., either all agents choose \(e_t = 1\), or none of them does), the formal analysis is identical to above, and a low debt level helps the firm to enforce relational contracts with its workforce. If we also allow for non-symmetric equilibria, the situation rather resembles the one where we allow for continuous effort, as is analyzed in the subsequent Section 7.2: If the (DE) constraint binds, relational contracts can only be enforced for some agents, and the number of those agents is maximized for \(D \leq \delta \gamma K\).

\(^{18}\)Note that even if an involuntary bankruptcy were possible in an equilibrium with \(e = 1\), i.e. if \(\rho(1) < 1\), this result would remain valid as long as \(\rho(1)\) was not too low.
7.2 Continuous Effort

Suppose several degrees of quality or productivity, i.e., a variety of effort levels, exist. As long as the most profitable relational contract can not be enforced for any debt level, it is generally optimal to have low debt ratios. Formally, we extend the model in the following way. Effort can assume higher values than 1 and is continuous for larger levels, i.e., $e_t \in \{0\} \cup [1, \infty)$. Note that we use this formulation to stay as close as possible to our benchmark model; nothing would change if effort were continuous from zero on. Effort costs are $c \cdot e_t$, and the agent’s contribution is $f(e_t) = \eta g(e_t)$, where $\eta > 0$ and $g(\cdot)$ is strictly increasing and concave. For simplicity, we assume that $\rho(e_t) = 1$ for all $e_t \geq 1$. All other assumptions are the same as before, in particular that $\rho(0) < 1$ and that $\eta g(0)$ is sufficiently large to satisfy Assumption 2.

Given it is larger than 1 (which we assume throughout), surplus-maximizing effort $e^*$ is characterized by $\eta g'(e^*) = c$. Concerning the enforceability of relational contracts, the formal analysis is the same as above, only that (IC) and (DE) constraints have to be satisfied for the respective effort level $e_t$. In addition, agents do not receive a rent, and relational contracts can be stationary (i.e., profit-maximizing effort is the same in every period). The reason is that if an effort level can be enforced in one period, it can also be enforced in all other periods. Moreover, the capital structure has no direct effect on profits $\Pi_0 = -K + \delta \frac{\eta g(e) - c \cdot e - \bar{\pi}}{1 - \delta}$. Then, an effort level $e \geq 1$ can be enforced if and only if it satisfies

$$c \cdot e \leq \delta \left( \frac{\eta g(e) - c \cdot e - \bar{\pi}}{1 - \delta} - \frac{\eta g(0) - \bar{\pi}}{1 - \delta \rho} - \frac{D}{\delta} + \frac{\rho \frac{1 - \rho}{\delta} D - (1 - \rho) \Pi}{1 - \delta \rho} \right), \quad (5)$$

where $r = \frac{1 - \delta}{\delta}$ and $\Pi = \max \{ \gamma K - \frac{D}{\delta}, 0 \}$. This yields

**Proposition 3:** The enforceability of relational contracts for any effort level $e \geq 1$ is maximized for $D \leq \delta \gamma K$. Generally, the (DE) constraint may or may not be satisfied for any effort level $e$ between 1 and $e^*$; it is more likely to be satisfied for higher values of $\eta$ and $\delta$, and for lower values of $D$, $g(0)$, $\rho$, $\gamma$ and $\bar{\pi}$. As long as implementable effort is below $e^*$, it is strictly optimal to have $D \leq \delta \gamma K$.

(The proof to Proposition 3 can be found in Appendix I.)

Proposition 3 implies that once the firm is restricted in implementing the desired effort level – because condition (5) binds in equilibrium – it should have a debt level below $\delta \gamma K$ to maximize enforceable effort. The intuition resembles the case with binary effort. A debt level above $\delta \gamma K$ allows the firm to share some of the costs of reneging with its creditors and ceteris paribus increases its reneging temptation. More precisely, there exist three thresholds for $\eta$ once efficient effort is larger than one: $\eta_-$, $\eta_+$ and $\bar{\eta}$, with $\bar{\eta} > \eta_+ > \eta_-$. For $\eta < \eta_-$ and $\eta > \bar{\eta}$, firm value is independent of leverage. In the first case, this is because no positive effort can be enforced; in the second case, because first-best effort can be enforced for any debt level. For $\eta_- \leq \eta < \eta_-$, debt should not exceed $\delta \gamma K$, and effort is strictly below its first-best level, i.e., determined by the binding (DE)
constraint (5). Different from the case with binary effort, a debt level slightly above $\delta \gamma K$ would then not completely destroy relational contracts, but just reduce implementable effort. Finally, for $\overline{\eta} \leq \eta < \overline{\eta}$, there exist thresholds $D(\eta)$, with $\delta \gamma K \leq D(\eta) \leq K$, determined by the binding condition (5), such that first-best effort can be enforced for $D \leq D(\eta)$, and not for $D > D(\eta)$. So, the range of parameter values where debt actually matters is larger if effort is continuous, and the more difficult it becomes to enforce the most profitable effort level, the bigger is the importance of low debt to support the relational contract. The analysis in this chapter also sheds light on an aspect that allows for a sharper distinction between our model and existing theories. Here, the negative impact of debt on the maximum enforceable effort level, i.e., the quality of relational contracts, is not driven by debt increasing the likelihood of a bankruptcy. To see this, assume $\eta$ is above $\overline{\eta}$, i.e., relational contracts are feasible. Then, as long as $\eta \leq \overline{\eta}$, a debt level slightly above $\delta \gamma K$ (if $\eta \leq \overline{\eta}$) or above $D(\eta)$ (if $\overline{\eta} \leq \eta < \overline{\eta}$) does not increase the likelihood of a bankruptcy and consequently the (expected) cost of financial distress, but only reduces enforceable effort. Hence, the mechanism derived in this paper is different from the standard argument applied in the literature (which is also made in related stakeholder theories developed by Maksimovic and Titman, 1991, or Berk et. al, 2010), where additional debt always implies an increase in the expected cost of financial distress.

### 7.3 Debt Cheaper than Equity

Now, we relax the assumption that the costs of debt and equity are identical. Debt is often regarded as being cheaper than equity, in particular because interest payments are tax-deductable (see Graham, 1996, 2000, or Strebulaev and Yang, 2013 for estimates of the tax benefits of debt\(^{19}\)).

Thereby, we assume that debt is effectively cheaper than equity, without explicitly modelling all potential tax effects that have to be taken into account. The effective interest rate is $\hat{r} = \alpha r$, where $r$ is the interest creditors receive, and $0 < \alpha \leq 1$. In addition, we stick to the setting with continuous effort, as introduced in the previous section.

For convenience, the repayment obligation in case of a bankruptcy is still assumed to be $(1+r)D$ (for example because after a bankruptcy, the firm does not make any profits and thus pays no taxes anymore). If the firm only had to repay $(1+\hat{r})D$ after a bankruptcy, the following results would be identical, with the difference that the threshold, above which $\Pi = 0$, would be higher $(\delta/(\alpha + \delta(1-\alpha))\gamma K$ compared to $\delta \gamma K$).

Assuming that the direct costs of debt financing are lower than of equity financing, we obtain the following results.

**Proposition 4**: The enforceability of relational contracts decreases in $\alpha$ and is maximized for $D = \delta \gamma K$. Generally, the (DE) constraint may or may not be satisfied for any effort level $e$ between 1 and $e^*$; it is more likely to be satisfied for higher values of $\eta$ and $\delta$, and for lower values of $g(0)$, $\rho$, $\gamma$ and $\Pi$. The firm’s uniquely optimal debt level is determined by one of the following cases:

\(^{19}\)Strebulaev and Yang (2013), for example, claim that firms with no debt could generate tax benefits of 7-15% of the market value of equity by increasing leverage.
\[ e^*, \text{ the surplus-maximizing effort level, can be enforced for any value of } D; \text{ then, } D = K \]

- Equilibrium effort \( e \) is between 1 and \( e^* \); then, there exist values \( \alpha \) and \( \overline{\alpha} \), with \( 0 < \alpha < \overline{\alpha} < 1 \) such that \( D = \delta \gamma K \) for \( \alpha \geq \overline{\alpha} \) and \( D = K \) for \( \alpha \leq \alpha \). For intermediate levels of \( \alpha \), optimal debt assumes a value between \( \delta \gamma K \) and \( K \) and is determined by the binding (DE) constraint, with effort being characterized by
  \[
  (1 - \delta \rho) [c - \delta f'(e)] - \alpha (c - \delta \rho f'(e)) (1 - \delta) = 0.
  \]

- No effort level \( e \geq 1 \) can be enforced for any value of \( D \); then, \( D = K \)

(The proof to Proposition 4 can be found in Appendix I.)

A lower \( \alpha \) relaxes the firm’s dynamic enforcement constraint. However, this just reflects an income effect which would be less pronounced if we explicitly took profit taxes into account. Different from our previous analysis, the enforceability of relational contracts is not maximized for all debt levels below \( \delta \gamma K \), but uniquely for \( D = \delta \gamma K \). This is driven by the direct costs of debt being lower than the costs of equity. Hence, if debt were strictly below \( \delta \gamma K \), higher leverage would increase profits without negatively affecting relational contracts.

Moreover, if the cost advantage of leverage is large (indicated by a small \( \alpha \)), the firm solely focuses on exploiting this benefit.\(^{20}\) In this case, the firm would be able to enforce higher effort, however not pursue this opportunity because higher effort would require a reduction of leverage. Given \( \alpha \) is sufficiently large, though, the firm rather foregoes some of the cost advantage of debt in order to maintain the relational contracts with its workforce. This holds even though a lower \( \alpha \) relaxes the firm’s dynamic enforcement constraint.

In sum, the firm might decide to not make use of potential tax shields and react less to a variation in \( \alpha \) than predicted if neglecting the negative impact on the enforceability of relational contracts. As before, this is not driven by a higher bankruptcy risk being imposed by higher leverage, but because more debt would reduce enforceable effort.

### 7.4 Liquidation Value Realized in Later Periods

Finally, we assume that the firm can share some of its bankruptcy costs with creditors even if the outstanding debt is below the liquidation value \( \gamma K \). To analyze a particular case, we introduce a delay between the date when the firm defaults until its assets are liquidated. This is certainly more in line with reality, since the average time it takes to resolve a bankruptcy varies between 0.4 and 10 years, depending on a country’s institutions.\(^ {21} \) Reasons for a delay might be legal disputes that have to be settled or complicated bankruptcy laws. Furthermore, we make the simplifying assumption that the creditor is not compensated at all for waiting, which implies that he does not accumulate additional interest claims once bankruptcy has been declared.

\(^{20}\)Note that the extreme prediction that the firm chooses a debt ratio of 100% in this case is driven by Assumption 2 (saying that the firm is not credit constrained), which we impose to isolate the impact of debt on the enforceability of relational contracts.

\(^{21}\)See the World Development Indicators (WDI) database provided by the World Bank.
Formally, creditors and principal have to wait $T \geq 0$ periods after the firm has gone bankrupt before they can approach $\gamma K$. This implies that the firm’s outside option after reneging is

$$\Pi = \delta T_{\text{max}} \max \left\{ \gamma K - \frac{D}{\delta}, 0 \right\}.$$  

In addition, we stick to the assumption that effort is continuous, but return to the case that direct costs of debt and equity are identical. Furthermore, first-best effort is supposed to be larger than 1. Then, there is still no direct effect of debt on $\Pi_0$. The negative effect of debt on the enforceability of relational contracts is, however, larger once the assumption of immediate liquidation is relaxed. In particular, we have

**Proposition 5:** For $T \geq 1$, the enforceability of relational contracts is maximized for $D = 0$.

(The proof to Proposition 5 can be found in Appendix I.)

Having to wait for the liquidation of the firm presents additional costs of being out-of-equilibrium, and debt gives the firm the possibility to share these costs. Hence, debt reduces the firm’s commitment, and $D = 0$ might become strictly optimal.  

Proposition 5 implies that there are three thresholds for $\eta$: $\underline{\eta}$, $\bar{\eta}$ and $\overline{\eta}$, with $\bar{\eta} > \eta > \underline{\eta}$. For $\eta < \underline{\eta}$ and $\eta > \bar{\eta}$, firm value is independent of leverage. In the first case, relational contracts can never be enforced, in the second case first-best effort is always feasible. For $\underline{\eta} \leq \eta < \bar{\eta}$, the firm should set $D = 0$, and effort is below first-best and determined by the binding (DE) constraint. For $\bar{\eta} \leq \eta < \overline{\eta}$, there exist thresholds $D(\eta)$, with $0 \leq D(\eta) \leq K$, determined by the binding (DE) constraint, such that first-best effort is feasible for $D \leq D(\eta)$, and not for $D > D(\eta)$.

8 Empirical Relevance

The main result of this paper is the positive impact of equity financing on the enforceability of relational contracts. When assessing its empirical validity, though, we have to make precise what we expect to observe. Since effort and bonus payments are not verifiable, these aspects do not reflect easily measurable dimensions such as working hours or monetary compensation. A promising approach to get over such problems is taken by DeVaro, Kim, and Vikander (2014). They start with the observation that a firm’s profits depend on aspects like the general state of the economy which can not be affected by its CEO. For this reason, is is often argued that a CEO’s compensation should not vary with any exogenously given state of the world, at least if formal contracts on all relevant aspects of managerial performance are feasible. However, DeVaro, Kim, and Vikander (2014) observe that this state of the world is autocorrelated over time, and a high realization is a good predictor

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22 If the creditor’s obligations would increase by a factor $\frac{1}{\delta}$ (i.e., the creditor would be fully compensated for the delay), $T = 0$ would not necessarily be uniquely optimal. However, the maximum debt level would be lower than before, since a larger $T$ would be equivalent to a lower value of $\gamma$.
for (ceteris-paribus) higher future profits. Hence, if relational contracts are needed to motivate the CEO, the firm has more credibility if the state is high. Using Standard & Poor’s Execucomp database for the years 1993-2011, DeVaro, Kim, and Vikander (2014) show that the state of the world has a positive impact on bonus payments, providing evidence for the existence of relational contracts in CEO compensation. Furthermore, their findings support our theory, as leverage has a negative impact on bonus payments and on several measures of managerial performance.

Another approach relates to effort in our model rather capturing issues like employee motivation, whereas the bonus describes facets like a good working environment, flexibility of time-use, extra free time or other non-monetary perks a workforce values but that are (ceteris paribus) costly to a firm. Although these rather “soft” aspects are difficult to validate, there are some indirect measures that are good indicators of well-functioning relational contracts. For example, firms which provide good working conditions and have a motivated workforce should be more popular among its employees. On average, the share of equity financing should hence be positively related to a firm’s reputation for being a good employer. Bae, Kang, and Wang (2011) provide a systematic and quantitative analysis of this question and provide evidence in favor it. They compare firms ranking high on several indices measuring employee friendliness, like the Fortune Magazine’s list of “100 Best Companies to Work For”, by matching these firms to ones that are not on such lists, and show that the former use substantially more equity.

Our prediction is also supported by anecdotal evidence. Lincoln Electric’s workforce is extremely motivated not only because of a generous incentive system, but also because firm policy is supported by low debt.23 Southwest Airlines is known for treating its employees very well - who in turn are extremely satisfied with their jobs - and has a debt ratio that is substantially lower than that of its competitors (according to Fool.com, Southwest’s debt-to-equity ratio in 2012 was 0.48, while the ratios of Delta, US Airways and United amounted to 7.42, 6.02 and 7.61, respectively.) Furthermore, while there have been almost 200 airline bankruptcies since 197824, Southwest never had to face such difficulties.

Relatedly, our model predicts that in order to sustain relational contracts, firms do not fully reap potential tax benefits of debt. This relates to the puzzle why so many healthy and profitable companies with negligible bankruptcy risk appear to be substantially underlevered (Graham and Leary, 2011). We argue that the urge of firms to maintain well-functioning relational contracts might make them reluctant to exploit seemingly valuable tax shields.

Finally, our model can contribute to the question why so many firms operate at a zero (or almost zero) debt level (Strebulaev and Yang, 2013). These firms are more profitable, pay higher dividends, and seem to forego substantial tax shields. On the one hand, such an outcome is predicted by our model if bankruptcy costs can – even for low debt levels – always be shared with creditors (at least if the tax benefits of debt are not too large). On the other hand, understanding the precise impact of a certain debt level on a firm’s commitment might be hard to assess for employees because that would require full knowledge of the firm’s characteristics (like $f(1)$, $\gamma$ or $\rho$). Employees often will

23 See the quote of former CEO Donald Hastings in the introduction.
24 See airlines.org.
not possess such a degree of sophistication and only have a rough idea about what a given debt level implies for the trustworthiness of their employer. In terms of communication, signalling commitment with a zero-debt level arguably is much easier than trying to convince one’s employees that a certain amount of debt is not detrimental to the promises made to them. In this sense, Strebulaev and Yang’s (2013) observations might be driven by an attempt of firms to maximize commitment in their employment relationships.

9 Conclusion

This paper has developed a relational contracting model to analyze firms’ financing behavior from a new angle. An important feature of this model is that debt financing has no direct effect on firm value and bankruptcy risk. However, debt financing may indirectly affect firm value by reducing the enforceability of implicit arrangements between a firm and its workforce: More equity is associated with more credibility and more commitment when a firm makes promises. One straightforward implication is that enterprises with well-functioning relational contracts should be less leveraged, on average. Another implication is that even profitable enterprises may use very little or no debt in order to maintain their credibility towards managers/workers.

Our analysis sheds new light on empirical evidence suggesting that many firms exploit tax benefits associated with debt financing only to a limited extent or not at all. This empirical evidence is at odds with many agency cost/tradeoff models, where a firm’s debt ratio has to be sufficiently large to actually trigger a bankruptcy and consequently cause costs of financial distress. Even if these costs are, on average, underestimated in empirical studies (see Molina, 2005), this does not explain why particularly healthy and profitable firms hardly use debt (Myers, 2003). Our model provides a rationale for this and shows that it can be optimal for a firm with low bankruptcy risk to exhibit low (or even zero) debt.

Finally, this article suggests that empirical predictions obtained from theories suggesting target capital structures (e.g., Hovakimian, Hovakimian, Tehranian, 2004) may be misleading, as our model predicts a lower (endogenously determined) debt threshold or target debt ratio. Around this threshold, standard incentive considerations as in a tradeoff context might be dominated by considerations about the enforceability of relational contracts. The latter point could explain why some firms are responding in a discontinuous way (or not at all) to changes in institutional (policy) incentives such as taxes.

Appendix I – Omitted Proofs

The proofs to Lemma 1 and 2 can be found in Appendix II.

Lemma 3: Any profit-maximizing equilibrium can be replaced by one where the constraints (IRA), (IC) and (CR) bind in every period t.
Proof: If (CR) constraints did not bind, interest payments $r_t$ could be slightly reduced, without violating any constraint but increasing $\Pi_0$. Concerning the (IC) constraint, assume that (IC) does not bind in any period $t'$. Now, reduce $b_t$ and increase $w_t$ by an $\varepsilon > 0$ sufficiently small such that (IC) for period $t'$ still holds. This has no impact on any (IRA) constraint, as well as on (IC) constraints in periods $t \neq t'$. Furthermore, profits and all constraints relating to the principal are not affected, with the exception of (DE) for period $t'$, which is relaxed.

Concerning (IRA), we can plug in the binding (IC) constraint, i.e., $\delta (U_{t+1} - \overline{U}) = c - b_t$, into the agent’s payoff $U_t = w_t + b_t - c + \delta U_{t+1}$, and get $U_t = w_t + \delta \overline{U}$. Now, assume (IRA) does not bind in any period $t'$. Then, $w_t$ can be slightly reduced, without violating any constraint, however increasing $\Pi_0$. Q.E.D.

Lemma 4: Assume the principal refused to pay the bonus $b_t$ and subsequently did not default. Then, $D_t^0 = D_t$ in all periods $t \geq \tau + 1$.

Proof: Note that a sequential equilibrium requires beliefs to be consistent in a sense that beliefs at information sets that are reached with zero probability equilibrium strategies must approximate beliefs that are derived from strategies that assign positive probabilities to every action. Thus, we have to show that $\tilde{D}_t^0 \neq \tilde{D}_t$ necessarily makes the creditor believe that the principal reneged and hence implies $\mu_t = 0$ for all $t \geq \tau$ and $e_t = 0$ for all $t > \tau$. However, these considerations are irrelevant for $\tilde{D}_t^0 \leq \delta \gamma K$. Then, the creditor’s claims will be fulfilled in any case. Hence, he will demand an interest rate of $r_\tau = \frac{(1-\delta)}{\delta}$, irrespective of $\mu_\tau$.

Now, suppose that $\tilde{D}_t^0 \neq \tilde{D}_t$ and $\tilde{D}_t^0 > \delta \gamma K$. If in this case $\mu_\tau > 0$ for $t \geq t$, the creditor would ask for an interest rate determined by $-D_\tau + \delta [\mu_\tau (1 + r_\tau) D_\tau + (1 - \mu_\tau) (\rho + r_\tau) D_\tau + (1 - \rho) \gamma K] = 0$. This would yield $r_\tau^0 = \frac{D_\tau (1-\delta (\mu_\tau + (1-\mu_\tau) \rho) - \delta (1-\mu_\tau) (1-\rho) \gamma K)}{\delta D_\tau (\mu_\tau + (1-\mu_\tau) \rho)}$, which is smaller than the “fair” off-equilibrium interest rate, $r_\tau = \frac{D_\tau (1-\delta (\mu_\tau + (1-\mu_\tau) \rho) - \delta (1-\mu_\tau) (1-\rho) \gamma K)}{\delta D_\tau}$. In addition, an interest rate $r_\tau^0$ would also yield a debt level $\tilde{D}_t^0 > \delta \gamma K$ optimal for the principal. Therefore, whenever $\tilde{D}_t^0 \neq \tilde{D}_t$, creditors will from then on only accept conditions based on the conjecture that the agent does not choose positive effort anymore. Q.E.D.

Lemma 5: In a profit-maximizing relational contract with $e = 1$ in every period, it is (weakly) optimal to have debt constant over time.

Proof: It is sufficient to show that profits have to be the same in every period, i.e., $\Pi_1(1) = \Pi_2(1) = \ldots \equiv \Pi(1)$. To the contrary, assume that there are two subsequent periods $t'$ and $t'+1$ with $\Pi_{t'}(1) \neq \Pi_{t'+1}(1)$ (if there are two periods with differing profits, there have to be two subsequent periods where this is the case). First, assume that $\Pi_{t'}(1) > \Pi_{t'+1}(1)$. Then, replace $D_t$ in all periods $t \geq t'+1$ by $D_{t-1}$. All these values are enforceable, given the original values constituted an equilibrium. Furthermore, $\Pi_{t'}(1)$ is increased, hence (DE) for period 1 relaxed (note that $D_{t+1}$ actually cancels out in (DE) because $\Pi_{t+1}(1) = f(1-\pi-c-\delta^{-1}D_{t+1}+D_{t+2}+\delta \Pi_{t+2}(1)), and \Pi_0(1)$
(weakly) increased as well.

If \( \Pi_t < \Pi_{t+1} \), \( e_t \) and \( D_t \) in all periods \( t \geq t' \) can be replaced by \( D_{t+1} \), thereby increasing \( \Pi' \) and hence \( \Pi_0 \). It follows immediately that \( D \) can be constant throughout. Q.E.D.

Lemma 6: Assume the principal refused to pay the bonus \( b_t \). Then, she will not default and repay the loan in any future period.

Proof: Because of stationarity, we just have to show that

\[
-rD + \delta \Pi(0) \geq \Pi,
\]

where

\[
\Pi(0) = \frac{f(0) - \pi - \rho r D + (1 - \rho) \Pi}{1 - \delta},
\]

\( \Pi = \max \{ 0, \gamma K - (1 + r)D \} \), and \( r = \frac{1 - \delta}{\delta} \). Hence, this condition becomes

\[
\delta (f(0) - \pi) \geq (1 - \delta) \Pi + \frac{1 - \delta}{\delta} D.
\]

For \( D \leq \delta \gamma K \), condition (6) equals \( \delta (f(0) - \pi) \geq (1 - \delta) \gamma K \), which holds. For \( D > \delta \gamma K \), condition (6) is \( \delta (f(0) - \pi) \geq \frac{1 - \delta}{\delta} D \). Due to Assumption 2 this is satisfied for debt levels \( D \leq K \). Q.E.D.

Proposition 1: The principal will implement \( e = 1 \) in every period if and only if the constraint

\[
-c + \delta \left( \Pi(1) - \hat{\Pi}(0) \right) \geq 0
\]

is satisfied.

Proof: Necessity of this constraint for the principal’s ability to implement \( e = 1 \) in every period follows from the previous analysis. The proof of sufficiency is also straightforward. Assume condition (7) holds. Then, setting \( b_t = c \) and \( w_t = \pi \), all \( t \), satisfies all constraints needed to implement \( e_t = 1 \) in all periods \( t \).

In addition, the optimality for the principal to actually implement \( e = 1 \) (given (7) holds) follows from the agent not getting any rent and from \( f(1) - c > f(0) \). Q.E.D.

Proposition 2: The enforceability of relational contracts is maximized for \( D \leq \delta \gamma K \). Generally, the (DE) constraint may or may not be satisfied; it is more likely to be satisfied for higher values of \( f(1) \) and \( \delta \), and for lower values of \( D \), \( f(0) \), \( \rho \), \( \gamma \) and \( \pi \).

Proof: The (DE) constraint, necessary to enforce \( e = 1 \), equals \( c \leq \delta \left( \Pi(1) - \hat{\Pi}(0) \right) \), where \( \hat{\Pi}(0) \) is the firm’s off-equilibrium payoff given creditors are not aware of the agent’s loss of trust. There, \( r = \frac{1 - \delta}{\delta} \) on and off the equilibrium path, and the principal does not liquidate the firm but continues to run it also off-equilibrium.

The (DE) constraint is

\[
c \leq \delta \left( \frac{(f(1) - c - \pi)}{1 - \delta} - \frac{(f(0) - \pi)}{1 - \delta \rho} - \frac{D}{\delta} + \frac{\rho(1 - \delta)}{1 - \delta \rho} \Pi \right).
\]
It follows that this constraint is relaxed for higher values of \( f(1) \) (and eventually holds for \( f(1) \) sufficiently large), and tightened for larger values of \( c, f(0), \rho \) and \( \overline{\pi} \). Furthermore, note that for \( D \leq \delta \gamma K \), the right hand side of (4) decreases in \( \gamma \), and is unaffected by \( \gamma \) for \( D > \delta \gamma K \).

Concerning the impact of \( \delta \) on condition (8), first note that its right hand side is positive given Assumption 2.

For \( D \leq \delta \gamma K \), we have
\[
\frac{(f(1) - c - \overline{\pi})}{1 - \delta} - \frac{(f(0) - \overline{\pi})}{1 - \delta} - \gamma K \left( \frac{1 - \rho}{1 - \delta} \right)
\]
where
\[
\frac{(f(1) - c - \overline{\pi})}{1 - \delta} - \frac{(f(0) - \overline{\pi})}{1 - \delta} + \frac{(1 - \rho)(1 - \delta)\gamma K}{1 - \delta} > 0.
\]
For \( D > \delta \gamma K \), we have
\[
\frac{(f(1) - c - \overline{\pi})}{1 - \delta} - \frac{(f(0) - \overline{\pi})}{1 - \delta} - \frac{D 1 - \rho}{\delta 1 - \delta} \geq \frac{(f(1) - c - \overline{\pi})}{1 - \delta} - \frac{(f(0) - \overline{\pi})}{1 - \delta} - \frac{K 1 - \rho}{\delta 1 - \delta} > 0.
\]
The first derivative of the right hand side of (8) with respect to \( \delta \) is:

- For \( D \leq \delta \gamma K \):
  \[
  \frac{f(1) - c - (f(0) - \overline{\pi})}{(1 - \delta)^2} + (1 - \rho) \left[ \frac{(f(0) - \overline{\pi}) + \gamma K (1 - \delta)(1 - \delta) + \delta (f(0) - \overline{\pi}) - (1 - \delta) \gamma K}{(1 - \delta)^2 (1 - \rho)} \right] > 0.
  \]
- For \( D > \delta \gamma K \):
  \[
  \frac{f(1) - c - (f(0) - \overline{\pi})}{(1 - \delta)^2} + (1 - \rho) \left[ \frac{(f(0) - \overline{\pi}) + \frac{D 1 - \rho}{\delta}}{(1 - \delta)^2 (1 - \delta) + \frac{\delta}{\delta 1 - \delta} [(1 - \delta) + \rho(1 - \delta)]} \right] > 0.
  \]

Q.E.D.

**Proposition 3:** The enforceability of relational contracts for any effort level \( e \geq 1 \) is maximized for \( D \leq \delta \gamma K \). Generally, the (DE) constraint may or may not be satisfied for any effort level \( e \) between 1 and \( e^* \); it is more likely to be satisfied for higher values of \( \gamma \) and \( \delta \), and for lower values of \( D, g(0), \rho, \gamma \) and \( \overline{\pi} \). As long as implementable effort is below \( e^* \), it is strictly optimal to have \( D \leq \delta \gamma K \).

**Proof:** For \( D \leq \delta \gamma K \), condition (5) is \( c \cdot e \leq \delta \left( \frac{g(e) - c - e - \overline{\pi}}{1 - \delta} - \frac{g(0) - \overline{\pi}}{1 - \delta} - \frac{(1 - \rho)\gamma K}{1 - \delta} \right) \), while it becomes \( c \cdot e \leq \delta \left( \frac{g(e) - c - e - \overline{\pi}}{1 - \delta} - \frac{g(0) - \overline{\pi}}{1 - \delta} - \frac{D 1 - \rho}{\delta 1 - \delta} \right) \) for \( D > \gamma K \). Hence, it is weakly optimal to have \( D \leq \delta \gamma K \) because otherwise, condition (5) would be relaxed for lower values of \( D \).

The constraint is relaxed for higher values of \( \gamma \) and \( \delta \), and for lower values of \( g(0), \rho, \gamma \) and \( \overline{\pi} \). Furthermore, taking an arbitrary effort level \( e \) with \( 1 \leq e \leq e^* \), the constraint is satisfied for \( e \) if \( \delta \to 1 \) and not if \( \delta \to 0 \).

To show that it is strictly optimal to have \( D \leq \delta \gamma K \) in case surplus-maximizing effort \( e^* \) can not be enforced, assume that \( D \geq \delta \gamma K \) (note that both representations of the (DE) constraint are identical for \( D = \delta \gamma K \)). Then, the problem is to maximize \( \Pi_0(e) = -K + \frac{\delta}{\delta 1 - \delta} \left( g(e) - c - e - \overline{\pi} \right) \), subject to \( c \cdot e \leq \delta \left( \frac{g(e) - c - e - \overline{\pi}}{1 - \delta} - \frac{g(0) - \overline{\pi}}{1 - \delta} - \frac{D 1 - \rho}{\delta 1 - \delta} \right) \) and \( D \geq \delta \gamma K \).

The Lagrangian in this case is
\[
L = -K + \delta \frac{f(e) - c - e - \overline{\pi}}{1 - \delta} + \lambda \left( \frac{g(e) - c - e - \overline{\pi}}{1 - \delta} - \frac{g(0) - \overline{\pi}}{1 - \delta} - D \frac{1 - \rho}{(1 - \delta)} - c \cdot e \right) + \mu [D - \delta \gamma K],
\]
and first-order conditions are
\[
\frac{\partial L}{\partial e} = \delta \frac{g(e) - c - \overline{\pi}}{1 - \delta} (1 + \lambda) - \lambda c = 0
\]
\[ \frac{\partial L}{\partial D} = -\lambda \frac{1-\rho}{(1-\delta \rho)} + \mu = 0 \]

Hence, if effort is below its first-best (which, for example, is the case if \( \delta \) is too low), implying \( \lambda > 0 \), \( \mu \) has to be strictly positive as well. Q.E.D.

**Proposition 4:** The enforceability of relational contracts decreases in \( \alpha \) and is maximized for \( D = \delta \gamma K \). Generally, the (DE) constraint may or may not be satisfied for any effort level \( e \) between 1 and \( e^\ast \); it is more likely to be satisfied for higher values of \( \eta \) and \( \delta \), and for lower values of \( g(0) \), \( \rho \), \( \gamma \) and \( \bar{\alpha} \). The firm’s uniquely optimal debt level is determined by one of the following cases:

- \( e^\ast \), the surplus-maximizing effort level, can be enforced for any value of \( D \); then, \( D = K \)
- Equilibrium effort \( e \) is between 1 and \( e^\ast \); then, there exist values \( \bar{\alpha} \) and \( \bar{\pi} \), with \( 0 < \bar{\alpha} < \bar{\pi} < 1 \) such that \( D = \delta \gamma K \) for \( \alpha \geq \bar{\pi} \) and \( D = K \) for \( \alpha \leq \bar{\pi} \). For intermediate levels of \( \alpha \), optimal debt assumes a value between \( \delta \gamma K \) and \( K \) and is determined by the binding (DE) constraint, with effort being characterized by \( (1 - \delta \rho)(c - \delta f'(e)) - \alpha (c - \delta p f'(e)) \) \( (1 - \delta) = 0 \).
- No effort level \( e \geq 1 \) can be enforced for any value of \( D \); then, \( D = K \)

**Proof:** Note that the interest rate \( r \) has the same value as before: In an equilibrium with \( e \geq 1 \), we have \( r = \frac{1-\delta}{\delta} \). In an equilibrium with \( e = 0 \), \( r = \frac{1-\delta}{\delta} \) for \( D \leq \delta \gamma K \), and \( r = \frac{D(1-\delta \rho) - \delta (1-\delta) \gamma K}{\delta \rho D} \) for \( D > \delta \gamma K \).

Given effort \( e \geq 1 \), profits at the beginning of the game are

\[ \Pi_0(e) = -(K-D) + \delta \Pi(e) = -(K-D) + \delta \frac{\eta g(e) - c \cdot e \cdot \bar{\pi} - \hat{r} D}{1-\delta} = -K + D(1-\alpha) + \delta \frac{\eta g(e) - c \cdot e - \bar{\pi}}{1-\delta}. \]

Given \( e = 0 \) in equilibrium, profits are \( \Pi_0(0) = -(K-D) + \delta \Pi = -(K-D) + \delta \frac{\eta g(0) - \bar{\pi} - e^\ast D + (1-\rho) \Pi}{1-\delta \rho} \), where \( \Pi = \max \{0, \gamma K - (1 + r) D\} \).

For \( D \leq \delta \gamma K \), we have \( \Pi_0(0) = -K + D \frac{\rho(1-\delta)(1-\alpha)}{1-\delta \rho} + \delta \frac{\eta g(0) - \bar{\pi} + (1-\rho) \gamma K}{1-\delta \rho} \).

For \( D > \delta \gamma K \), we have \( \Pi_0(0) = -K + D(1-\alpha) + \delta \frac{\eta g(0) - \bar{\pi} + (1-\rho) \gamma K}{1-\delta \rho} \).

It follows that once enforcing relational contracts either is not possible at all, or once first-best effort \( e^\ast \) is feasible for any value of \( D \), it is optimal to have \( D = K \).

Concerning the enforceability of relational contracts for \( e \geq 1 \), the (DE) constraint is

\[ -c \cdot e + \delta \frac{\eta g(e) - c \cdot e - \bar{\pi}}{1-\delta} - \delta \frac{\eta g(0) - \bar{\pi}}{1-\delta \rho} - \alpha D \frac{1-\rho}{(1-\delta \rho)} - \delta \frac{(1-\rho) \Pi}{1-\delta \rho} \geq 0, \]  

with \( \Pi = \max \{0, \gamma K - \frac{D}{\rho}\} \).

The constraint is relaxed for higher values of \( \eta \) and \( \delta \), and for lower values of \( g(0) \), \( \rho \), \( \gamma \) and \( \bar{\pi} \). Furthermore, taking an arbitrary effort level \( e \) with \( 1 \leq e \leq e^\ast \), the constraint is satisfied for \( e \) if \( \delta \to 1 \) and not if \( \delta \to 0 \).

For \( D \leq \delta \gamma K \), (9) equals \( -c \cdot e + \delta \frac{\eta g(e) - c \cdot e - \bar{\pi}}{1-\delta} - \delta \frac{\eta g(0) - \bar{\pi}}{1-\delta \rho} + \frac{(1-\rho) D}{1-\delta \rho} (1-\alpha) - \delta \frac{(1-\rho) \gamma K}{1-\delta \rho} \geq 0. \)
For $D > \delta\gamma K$, (9) equals $-c\cdot e + \delta \frac{n_g(e) - c - e - \pi}{1 - \delta} - \delta \frac{n_g(0) - \pi}{1 - \delta} - \alpha D \frac{1 - \rho}{1 - \delta}$, which is plugged into condition (11): $D \geq \delta\gamma K$, for $D > \delta\gamma K$ it decreases in $D$. This implies that the left hand side of (9) (for a given effort level $e \geq 1$) is maximized for $D = \delta\gamma K$.

Since $\Pi_0(e)$ is increasing in $D$ for a given effort level, it is hence optimal to have $D \geq \delta\gamma K$.

Assuming an effort level $e \geq 1$ can be enforced, the problem is to maximize $\Pi_0(e) = -K + D(1 - \alpha) + \delta \frac{n_g(e) - c - e - \pi}{1 - \delta}$, subject to $-c\cdot e + \delta \frac{n_g(e) - c - e - \pi}{1 - \delta} - \delta \frac{n_g(0) - \pi}{1 - \delta} - \alpha D \frac{1 - \rho}{1 - \delta} > 0$, $D \geq \delta\gamma K$ and $D \leq K$.

The Lagrangian of this problem is

$$L = -K + D(1 - \alpha) + \delta \frac{n_g(e) - c - e - \pi}{1 - \delta} + \lambda \left[ \delta \frac{n_g(e) - c - e - \pi}{1 - \delta} - \delta \frac{n_g(0) - \pi}{1 - \delta} - \alpha D \frac{1 - \rho}{1 - \delta} - c\cdot e \right] + \mu_1 [D - \delta\gamma K] + \mu_2 [K - D]$$

with first-order conditions

$$\frac{\partial L}{\partial e} = \left. \frac{\delta n_g'(e) - c}{1 - \delta} \right| (1 + \lambda) - \lambda c = 0,$$

and

$$\frac{\partial L}{\partial D} = (1 - \alpha) - \lambda \alpha \frac{1 - \rho}{1 - \delta} + \mu_1 - \mu_2 = 0.$$ (10)

First-best effort implies $\lambda = 0$, as well as $\mu_2 > 0$. Therefore, first-best effort will only be enforced if this is possible with maximum leverage.

Now, assume that $\lambda > 0$, i.e., effort is inefficiently low. Then, condition (10) gives $\lambda = \frac{\delta n_g'(e) - c}{c - \delta n_g'(e)}$, which is plugged into condition (11):

$$\frac{(1 - \rho)[c - \delta n_g'(e)] - \alpha(c - \delta n_g'(e))(1 - \delta)}{(1 - \rho)[c - \delta n_g'(e)]} + \mu_1 - \mu_2 = 0.$$ (11)

Denote the effort level that satisfies this condition for $\mu_1 = \mu_2 = 0$ by $\overline{e}$. $\overline{e}$ is unique and decreasing in $\alpha$. For $\alpha \to 0$, this condition cannot hold for $\mu_2 = 0$. For $\alpha \to 1$, $\overline{e} = e^*$, which is not feasible given that $\lambda > 0$. Hence, the values $\overline{e}$ and $\overline{\mu}$ as stated in the Proposition exist, and effort as well as debt have the proclaimed properties, given $e^*$ cannot be enforced but positive effort is optimal. Q.E.D.

**Proposition 5:** For $T \geq 1$, the enforceability of relational contracts is maximized for $D = 0$.

**Proof:** The (DE) constraint now equals

$$c\cdot e \leq \delta \left( \frac{n_g(e) - c - e - \pi}{1 - \delta} - \frac{n_g(0) - \pi}{1 - \delta} - \frac{D}{\delta} + \frac{\rho \frac{1 - \delta}{\delta} D - (1 - \rho) \delta T \max \{\gamma K - \frac{D}{\delta} \pi, 0\}}{1 - \delta} \right),$$

which gives two cases:

- $\frac{D}{\delta} \leq \gamma K$, and the (DE) constraint is
  $$c\cdot e \leq \delta \left( \frac{n_g(e) - c - e - \pi}{1 - \delta} - \frac{n_g(0) - \pi}{1 - \delta} - \frac{D}{\delta} \frac{1 - \rho}{1 - \delta} \frac{1 - \delta T}{1 - \delta} \frac{\delta}{1 - \delta} \gamma K \right),$$
  where its right hand side is decreasing in $D$ for $T \geq 1$.

- $\frac{D}{\delta} > \gamma K$, and the (DE) constraint is
  $$c\cdot e \leq \delta \left( \frac{n_g(e) - c - e - \pi}{1 - \delta} - \frac{n_g(0) - \pi}{1 - \delta} - \frac{D}{\delta} \frac{1 - \rho}{1 - \delta} \right),$$
  where its right hand side is decreasing in $D$. Q.E.D.
Appendix II – Benchmark Cases

Benchmark I: $e_t = 0$ in all Periods

As a benchmark case, we develop a profit-maximizing equilibrium with $e_t = 0$ in all periods, assuming that the firm has been started and the investment $K$ been made. For this to be interesting, $f(0)$ has to be strictly positive and large enough so that running the firm is optimal (compared to liquidating it).

Recall that the principal also needs the agent to operate the firm if $e = 0$. The reason is that $e = 0$ does not imply that the agent is completely unproductive, only that he is not willing to exert extra, non-verifiable, effort. Hence, the agent must receive an offer that induces him to accept the contract in every period. This is captured by the agent’s individual rationality constraint,

$$U_t \geq \overline{U}, \text{ all } t \geq 1. \quad (IR)$$

$U_t = w_t + \delta \left[ \rho U_{t+1} + (1 - \rho) \overline{U} \right]$ is the payoff of the agent in an equilibrium with $e_t = 0$ (where obviously no bonus payment is required).

Since we are interested in profit-maximizing equilibria, (IR) will bind in every period (if it did not bind in any period $t$, $w_t$ could be reduced), and $w_t = \overline{u}$. In addition, we focus on an equilibrium where debt is constant in every period (this assumption can be made without loss of generality). Therefore, we can omit time subscripts, and from now on denote profits in a profit-maximizing equilibrium with $e = 0$ by $\Pi(0)$. Then,

$$\Pi(0) = \frac{f(0) - \overline{u} - \rho r D + (1 - \rho) \Pi}{1 - \delta \rho}. \quad (IR)$$

To determine the interest rate $r$, note that the creditor must not make a loss in expectation. Hence, the creditor rationality constraint

$$-D + \delta V \geq 0 \quad (CR)$$

has to hold, where $V = \frac{\rho r D + (1 - \rho) \overline{D}}{1 - \delta \rho}$ and $\overline{D} = \min \{D(1 + r), \gamma K\}.$

The creditor rationality constraint (CR) has to bind in any profit-maximizing equilibrium (assuming a competitive credit market), and the interest rate equals

$$r = \frac{D (1 - \delta \rho) - \delta (1 - \rho) \overline{D}}{\delta \rho D}.$$ 

The actual value of $r$ depends on the amount $D$. If $D \leq \delta \gamma K$, then $\overline{D} = (1 + r)D$ and $r = \frac{1 - \delta}{\delta}$, since the interest rate only covers discounting. The reason is that the debt level is sufficiently low for the creditor to be fully repaid (including interest), no matter whether the firm is liquidated or not. If $D > \delta \gamma K$ (and given a voluntary bankruptcy is not optimal), we have $r = \frac{D(1 - \delta \rho) - \delta (1 - \rho) \gamma K}{\delta \rho D}.$

Note that if a voluntary bankruptcy were optimal, we would have $V = \overline{D}$, and no debt level higher than $\delta \gamma K$ would satisfy (CR).
In this case, debt is so high that the creditor would lose parts of his claims in case of a bankruptcy, and the interest rate not only captures time preferences, but also the risk of not getting fully repaid. Note that \( r_t \) also determines the opportunity cost of equity financing.

Given \( e = 0 \), a voluntary default is not optimal if

\[
-rD + \delta \Pi(0) \geq \bar{\Pi}. \tag{12}
\]

More precisely, for \( D \leq \delta \gamma K \), condition (12) is \( \frac{\delta}{1 - \delta} (f(0) - \bar{\pi}) \geq \gamma K \), whereas for \( D > \delta \gamma K \), it becomes \( \delta (f(0) - \bar{\pi}) \geq \frac{D(1 - \delta \rho) - \delta (1 - \rho) \gamma K}{\delta \rho} \). Condition (12) also determines whether and to what extent the firm is credit constrained. Being credit constrained, the firm would like to borrow more but is not granted additional credit - even if it is willing to pay a higher interest rate. This is driven by debt making a voluntary default more tempting and that – given a voluntary default is optimal – the creditor is not willing to lend more than \( \delta \gamma K \). This is summarized in

**Lemma A1:** Assume the firm has been started and \( e = 0 \) in every period. Then, the following conditions determine the firm’s borrowing capacity:

- If \( \delta (f(0) - \bar{\pi}) \geq K \frac{(1 - \delta \rho) - \delta (1 - \rho) \gamma}{\delta \rho} \), the firm is not credit constrained and can borrow any amount \( D \leq K \).
- If \( (1 - \delta) \gamma K \leq \delta (f(0) - \bar{\pi}) < K \frac{(1 - \delta \rho) - \delta (1 - \rho) \gamma}{\delta \rho} \), the firm is credit constrained and can borrow any amount \( D \leq \delta \gamma K \).
- If \( \delta (f(0) - \bar{\pi}) < (1 - \delta) \gamma K \), the firm is credit constrained and can borrow any amount \( D \leq \delta \gamma K \).

**Proof:** The last bullet point describes the situation where a voluntary bankruptcy is optimal in any case. Then, \( V = D \), and no debt higher than \( \delta \gamma K \) can be enforced (because no finite interest rate could satisfy the (CR) constraint for larger \( D \)).

Furthermore, since the right hand side of \( \delta (f(0) - \bar{\pi}) \geq \frac{D(1 - \delta \rho) - \delta (1 - \rho) \gamma K}{\delta \rho} \) is increasing in \( D \), if it is satisfied for \( D = K \), it holds for all \( D \leq K \). Hence the firm is not credit constrained for \( \delta (f(0) - \bar{\pi}) \geq K \frac{(1 - \delta \rho) - \delta (1 - \rho) \gamma}{\delta \rho} \). For intermediate values of \( \delta (f(0) - \bar{\pi}) \), the maximum amount of debt is characterized by condition (12), which holds with equality (taking into account that \( K \frac{(1 - \delta \rho) - \delta (1 - \rho) \gamma}{\delta \rho} \geq (1 - \delta) \gamma K \)). Q.E.D.

Lemma A1 also determines Assumption 2: If \( \delta (f(0) - \bar{\pi}) \geq K \frac{(1 - \delta \rho) - \delta (1 - \rho) \gamma}{\delta \rho} \), a voluntary default is not optimal, and the firm is not credit constrained.

This allows us to prove
Lemma 1: Given $e_t = 0$ in every period the firm is active, it is optimal to make the investment $K$ and start the firm in period $t = 0$. Furthermore, the firm’s value at the onset of the game, $\Pi_0(0)$, is independent of its financing structure.

Proof: We start with the second aspect and show that $\Pi_0$ is independent of $D$. Note that $\Pi_0 = -(K - D) + \delta f^{(0)} - \pi - \rho D + (1 - \rho) \frac{\Pi}{1 - \delta}$.

If $\frac{D}{\delta} \leq \gamma K$, we have $\hat{\Pi}_0 = -(K - D) + \delta f^{(0)} - \pi - \rho D + (1 - \rho) \frac{(\gamma K - D)}{1 - \delta}$.

If $\frac{D}{\delta} > \gamma K$, we have $\hat{\Pi}_0 = -(K - D) + \delta f^{(0)} - \pi - \rho D + (1 - \rho) \frac{\gamma K}{1 - \delta}$. Hence, $\Pi_0$ is independent of $D$.

Making the investment $K$ and starting the firm is profitable if and only if $\Pi_0(0) = -(K - D) + \delta f^{(0)} - \pi - \rho D + (1 - \delta \rho) \gamma K \geq 0$. This is equivalent to $\delta (f^{(0)} - \pi) \geq K (1 - \delta \rho) - \delta (1 - \rho) \gamma K$, which is implied by Assumption 2. Q.E.D.

Benchmark II: Formal Contracts Feasible

Consider the following agreement: Upon accepting the contract, the agent receives the fixed wage $w$. If $e = 1$, he also gets a bonus $b \geq 0$, whereas no bonus is paid in case of $e = 0$. Besides, the formal contract can be revealed to the creditor whose information disadvantage then is eliminated.

As before, any contract must induce the agent to accept the contract, instead of selecting $d^A = 0$ and consume his outside option. The individual rationality constraint of the agent, given by

$$U \geq U,$$

must be satisfied. There, $U = \frac{w - c + b}{1 - \delta}$ is the agent’s discounted equilibrium payoff stream at the beginning of a period where the firm is still active.

In addition, the bonus must incentivize the agent to choose $e = 1$ in every period. This is captured by an incentive compatibility (IC) constraint:

$$-c + b + \delta U \geq \delta U.$$

It is straightforward to show that $w = \pi$ and $b = c$ is the (not uniquely) cheapest way to induce high effort, and hence characterizes a profit-maximizing contract. This implies that the principal gets the whole surplus, and equilibrium discounted profit streams at the beginning of a period are $\Pi(1) = \frac{f^{(1)} - \pi}{1 - \delta}$.

This allows us to prove

Lemma 2: Assume the agent’s effort is verifiable. Then, it is optimal to carry out the investment $K$ and start the firm in period $t = 0$. Furthermore, $e = 1$ is implemented in every period and firm value $\Pi_0(1)$ is independent of $D$. 

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**Proof:** Given \( e = 1 \) is imposed in every period and bankruptcy is not observed in equilibrium, \( r = \frac{1-\delta}{\delta} \), irrespective of the debt level. Then, \( \Pi_0(1) = -(K-D) + \delta \Pi(1) = -K + \frac{f(1)-c-u}{1-\delta} \), proving that \( \Pi_0 \) is independent of \( D \). The optimality of \( e = 1 \) follows from \( f(1) - c > f(0) \) and Assumption 1 and 2. Given profits for \( e = 1 \) are higher than for \( e = 0 \), and given it is optimal to start the firm if \( e = 0 \), it is obviously also optimal to start the firm if \( e = 1 \). Q.E.D.

**Appendix III – Relational Contracts when Assumption 2 does not hold**

Here, we show that relaxing Assumption 2 – which states that a voluntary default is not optimal and hence the firm is not credit constrained – has no qualitative impact on our results, but rather strengthens them. Thereby, we analyze the impact of the financing structure on the (DE) constraint and consequently on the enforceability of relational contracts.

For convience, we still assume that in an equilibrium with \( e = 1 \), a voluntary bankruptcy is not optimal, so \( f(1) - c - u \) is sufficiently large.

As long as a default is not optimal off-equilibrium, the principal’s outside option is characterized by \( -rD + \delta \hat{\Pi}(0) \), where the second term describes off-equilibrium profits (i.e., given \( e = 0 \)), however with the firm sticking to its equilibrium debt policy. If the principal defaults after refusing to pay \( b \), her outside option is \( \Pi = \max \{ 0, \gamma K - D(1+r) \} \). Given Assumption 2, \( -rD + \delta \hat{\Pi}(0) \geq \Pi \) holds in any case. Once it is not imposed anymore, we have to distinguish between several cases and derive (DE) constraints for each of them.

Since \( r = \frac{1-\delta}{\delta} \) and \( \hat{\Pi}(0) = \frac{f(0)-\rho u + \frac{1-\delta}{\delta} D(1-\delta)}{\delta} \), the condition \( -rD + \delta \hat{\Pi}(0) \geq \Pi \) is identical to

\[
\frac{\delta}{1-\delta} (f(0) - \bar{u}) \geq \Pi + \frac{D}{\delta}.
\]  

(13)

For \( D \leq \delta \gamma K \), (10) becomes \( \delta (f(0) - \bar{u}) \geq (1-\delta) \gamma K \), and \( \delta (f(0) - \bar{u}) \geq \frac{1-\delta}{\delta} D \) for \( D > \delta \gamma K \).

This gives the following cases:

- \( \delta (f(0) - \bar{u}) \geq \frac{1-\delta}{\delta} K \). Then, a voluntary bankruptcy off-equilibrium is not optimal under any circumstances, and the situation is same as in the main part of this paper: The (DE) constraint equals

  \[
  -c + \delta \left( \frac{(f(1)-c-u)}{1-\delta} - \frac{(f(0)-u)}{1-\delta} \right) \geq 0, \text{ for } D \leq \delta \gamma K; \\
  -c + \delta \left( \frac{(f(1)-c-u)}{1-\delta} - \frac{(f(0)-u)}{1-\delta} \right) \geq D \frac{1-\rho}{\delta}, \text{ for } D > \delta \gamma K.
  \]

- \( (1-\delta) \gamma K \leq \delta (f(0) - \bar{u}) \geq \frac{1-\delta}{\delta} K \). Then, a voluntary bankruptcy off-equilibrium might be optimal. More precisely, defining \( \delta (f(0) - \bar{u}) = \frac{1-\delta}{\delta} D \), a voluntary bankruptcy is optimal for \( D > \overline{D} \), and not otherwise. In this case, the (DE) constraint equals
\[ -c + \delta \left( \frac{f(1)-c-p}{1-\delta} - \frac{f(0)-p+(1-\rho)\gamma K}{1-\delta \rho} \right) \geq 0, \text{ for } D \leq \delta \gamma K; \]
\[ -c + \delta \left( \frac{f(1)-c-p}{1-\delta} - \frac{f(0)-p}{1-\delta \rho} - \frac{D}{\delta} \frac{1-\delta \rho}{1-\delta} \right) \geq 0, \text{ for } D > \delta \gamma K \text{ and } D \leq D; \]
\[ -c + \delta \frac{f(1)-c-p}{1-\delta} - \frac{D}{\delta} \geq 0, \text{ for } D > \delta \gamma K \text{ and } D > D. \]

- \( \delta (f(0) - \overline{u}) < (1 - \delta)\gamma K. \) Then, a voluntary bankruptcy off-equilibrium is always optimal. In this case, the (DE) constraint equals

\[ -c + \delta \frac{f(1)-c-p}{1-\delta} - \gamma K \geq 0, \text{ for } D \leq \delta \gamma K; \]
\[ -c + \delta \frac{f(1)-c-p}{1-\delta} - \frac{D}{\delta} \geq 0, \text{ for } D > \delta \gamma K. \]

Thus, as long as \( D \leq \delta \gamma K, \) the (DE) constraint is unaffected by what happens off-equilibrium. Once \( D > \delta \gamma K, \) though, the negative impact of debt on the enforceability of relational contracts becomes substantially stronger when a voluntary bankruptcy might be optimal off-equilibrium (because \( \frac{1-\rho}{1-\delta \rho} < 1 \)). The reason is that the latter is equivalent to setting \( \rho = 0, \) which maximizes the negative effect of \( D \) on the left hand side of the (DE) constraint.

**Appendix IV – Principal is able to Divert \( D \)**

Here, we show that even if the principal is able to divert the funds borrowed from the creditor, our results do not change, at least after the firm has been started. Assume the following: After borrowing \( D, \) the principal can consume \( D \) instead of investing it into the firm, which however triggers a default. For this not to be optimal, the condition \( D + \delta \Pi(1) \geq D + \max \{ \gamma K - D, 0 \} \) must hold (the last term does not include interest payments because the default would happen in the same period as the diversion of funds). Then, we have two potential cases:

- \( D \leq \gamma K. \) And the condition becomes \( \delta \frac{f(1)-c-p}{1-\delta} - \frac{1-\delta}{1-\delta} D \geq \gamma K - D, \) or \( \delta \frac{f(1)-c-p}{1-\delta} \geq \gamma K, \) which is necessarily satisfied if starting the firm (and making the investment \( K \)) is optimal

- \( D > \gamma K. \) And the condition becomes \( \delta \frac{f(1)-c-p}{1-\delta} - \frac{1-\delta}{1-\delta} D \geq 0, \) which is tightest for \( D = K, \) so \( \delta \frac{f(1)-c-p}{1-\delta} \geq \gamma K, \) which is also satisfied if starting the firm has been optimal.

- Also with \( f(0), \) it is never optimal to divert \( D \) (due to Assumption 2).

If the principal is able to already steal \( D \) at the beginning of the game, without making the investment \( K, \) the situation is slightly different. Assuming that creditors have no possibility to regain anything in this case, the relevant condition becomes \(- (K - D) + \delta \Pi(1) \geq D, \) and \(- K + \delta \frac{f(1)-c-p}{1-\delta} - \frac{1-\delta}{1-\delta} D \geq 0. \)
This is tightest for \( D = K, \) and then becomes \(- 2K + \delta \frac{f(1)-c-p}{1-\delta} \geq 0. \) Only in this extreme case we need an extra condition. However, once the firm (or project) has been started and necessary investments made, the agency problem between firm and creditor is always resolved automatically.
References


