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The Green Paradox and Learning-by-Doing in the Renewable Energy Sector

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Abstract

The green paradox conveys the idea that climate policies may have unintended side effects when taking into account the reaction of fossil fuel suppliers. In particular, carbon taxes that will be implemented in the future induce resource owners to extract more rapidly which increases present carbon dioxide emissions and accelerates global warming. Our results suggest that future carbon taxes may even decrease present emissions if resource owners face increasing marginal extraction costs and if there is a clean energy source that is a perfect substitute and exhibits learning-by-doing (LBD). If the marginal extraction cost curve is sufficiently at, resource owners respond to a future carbon tax with lowering total extraction and only slightly increase present extraction. Moreover, taxation leads to higher energy prices which induces the renewable energy firms to increase output not only in the future, but also in the present because of the anticipated benefits from LBD. This crowds out energy from the combustion of fossil fuels and may outweigh the initial increase in present extraction, leading to less emissions in the present.

JEL-Code: Q380, Q540, Q280, H230.

Keywords: climate change, exhaustible resources, learning-by-doing, green paradox.

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1. Introduction

Carbon taxes that effectively combat global warming do not seem to be politically feasible in the short run as the latest climate conferences in Copenhagen, Cancun and Warsaw have demonstrated. Therefore, policy makers are restricted to the taxation of carbon dioxide (CO_2) emissions in the future. However, the implementation of delayed carbon taxes leads to the thread of a partial expropriation of resource owners, inducing them to extract their stocks more rapidly. This reaction is referred to as green paradox because it causes higher CO_2 emissions in the present and accelerates climate change. Moreover, higher present emissions will cause temperatures to rise faster which leaves less time for adaptation to global warming. Partly, they may even make adaptation impossible as some climate change induced effects might take place too rapidly or turn out to be irreversible. Therefore, with higher levels of contemporary global warming, both climate change damage and adaptation costs are expected to increase which is why policy makers should take the green paradox into account when designing climate policies.

This paper shows that, contrary to the green paradox, delayed carbon taxes may even decrease current emissions if resource owners face increasing marginal extraction costs and if there is a clean energy source that is a perfect substitute and exhibits learning-by-doing (LBD). LBD originates e.g. from the routinization of the production process or from minor technological improvements. It can be thought of as endogenous technological change which essentially lowers the costs of future production depending on accumulated production or experience in the past.[1] If the marginal extraction cost curve is sufficiently flat, a delayed carbon tax induces resource owners to reduce the fossil fuel supply substantially and to shift only very few extraction into the present. Additionally, taxation yields higher future energy prices which induces the clean energy sector to expand production not only in the future, but also in the present due to the

[1] The static correspondent to LBD would be economies of scale. However, under economies of scale, the cost reduction in unit costs rather originates from the distribution of fixed costs on all units produced then from a more efficient way of production (as it is the case under LBD) as the output increases.

anticipated benefits from LBD. The latter reduces the current energy price and induces resource owners to postpone extraction. If this effect is sufficiently large, it will outweigh the initial increase in present extraction, leading to less emissions in the present.

Eichner and Pethig (2011) also found that delayed taxation of carbon (in form of a tighter emissions cap in the future) may even reduce current emissions. However, in their model the reason for this is the existence of a second country which does not implement any climate policy. Tightening the emissions cap in the future by the abating country has essentially two effects: First, in the abating country the consumer price for fossil fuels in the future increases, causing the intertemporally maximizing households to consume more fossil fuels in the present so that present emissions increase. Second, the world price for fossil fuels in the future decreases which is why households in the non-abating country have an incentive to substitute present consumption of fossil fuels by future consumption, leading to fewer emissions in the present. Under certain conditions concerning the elasticities of fossil fuel demand and intertemporal substitution, the second effect will outweigh the first one and current emissions will decrease, meaning that there is a reversal of the green paradox. A similar result is found by Ritter and Schopf (2014).

The term green paradox was first coined by Sinn (2008a) and relates the theory of exhaustible resources (Hotelling (1931), Dasgupta and Heal (1979) and Long and Sinn (1985)) with environmental policies. Starting with Sinclair (1992), the vast majority of this literature assumes constant or zero extraction costs for exhaustible resources (Ulph and Ulph (1994), Withagen (1994) and Sinn (2008b)) and the existence of a clean backstop technology that supplies an unlimited amount of energy, but at a higher price (Hoel and Kverndokk (1996), Tahvonen (1997), Chakravorty et al. (1997) and Strand (2007)). In this setting, energy will be supplied exclusively by the combustion of fossil fuel in the first phase until the resource stock is completely exhausted and the backstop technology sets in. Moreover, any policy that decreases demand for fossil fuel

in the future inevitably increases present emissions, leading to higher environmental harm since total emissions are unaffected.[2] However, when extraction costs are convex (van der Ploeg and Withagen (2012), Hoel and Jensen (2012)), the resource stock may not be completely exhausted anymore and there is a trade-off between higher present emissions and lower total emissions. Given this trade-off, Gerlagh (2011) distinguishes between a weak and a strong green paradox where the net present value of environmental damage decreases (weak) or increases (strong) in response to a climate policy.

With respect to the supply of renewable energy, assuming increasing rather than constant marginal costs may be more realistic. One reason for this is that the appropriateness of the locations for the installation of renewable energy facilities is decreasing in the number of facilities already installed. Under this assumption, both the dirty and the clean energy source are employed simultaneously (Grafton et al. (2012)). Furthermore, the renewable energy sector benefits significantly from LBD.[3] According to Arrow (1962), LBD establishes a negative relationship between future production costs and past accumulated production.[4]

One strand of the literature includes both the extraction decision of fossil fuel owners and LBD in the alternative energy sector in its models but does not focus on the green paradox (Tahvonen and Salo (2001), Chakravorty et al. (2012), Kalkuhl and Edenhofer (2012, 2013)). Closest to our paper is Chakravorty et al. (2011) who find that the presence of learning in the renewable energy sector reduces energy prices and may accelerate resource extraction. Our approach differs from their approach with respect to several dimensions: Firstly, concerning the taxation of carbon emissions, Chakravorty et al.

[2] However, Edenhofer and Kalkuhl (2011) show that also overall emissions may decrease if carbon taxes are set sufficiently high. In this case, fossil fuel owners will only it may not anymore be optimal for the resource owners to exhaust their stocks completely because the market price for fossil fuel may be lower than the carbon tax.

[3] Empirically, Duke and Kammen (1999) and McDonald and Schrattenholzer (2001) report lower production costs with increasing production for solar panels and wind energy. For example, McDonald and Schrattenholzer (2001) report learning rates for wind and solar energy to be between 5 and 35 %, meaning a cost reduction of 5 to 35 % when the cumulative production is doubled.

[4] With respect to renewable energy, this assumption was incorporated in the models of Fischer and Newell (2008) and Kverndokk and Rosendahl (2007).

(2011) focus rather on the impact on energy prices and tax incidence than on the green paradox. Secondly, their analysis does not consider the case of increasing marginal extraction costs which is why the resource stock is always exhausted in their framework. Thirdly, they do not analyze any policies that aim to promote renewable energy. Lastly and most importantly, they use a dynamic setting with more than two periods which can be solved only numerically via calibration. Even though the authors conduct an extensive sensitivity analysis, their conclusions remain subject to the choice of the parameter values. We take a different approach by focusing on a two period model which allows us to derive theoretical results without relying on numerical solutions.

Our paper contributes to the literature on the green paradox by analyzing climate policies in the presence of LBD in the renewable energy sector and optimal extraction of non-renewable resources. More precisely, we analyze subsidies for renewable energy as well as carbon taxes and derive conditions under which the green paradox arises.

As a reference, we assume extraction costs to be zero which always leads to full exhaustion of the resource stock, implying the change in environmental damage to depend only on the change in current emissions. First, we examine the effect of LBD on the extraction decision and show that in the absence of any climate policy the effect of more effective learning on current emissions is ambiguous. The reason for this is that on the one hand learning reduces the future production costs leading to higher renewable energy output in the future. This causes the future energy price to decline and induces resource owners to extract more rapidly. On the other hand, learning also tends to increase the current renewable energy output due to the anticipation of the benefits from LBD. This decreases the current energy price and incentivizes resource owners to postpone extraction. The overall effect of learning on current emissions is therefore ambiguous.

Second and with respect to climate policies, we find that (still assuming zero extraction costs) the implementation of present (future) carbon taxes decreases (increases) current emissions which is the standard result of the green paradox. Further, subsi-

dizing renewable energy may increase current emissions depending on the magnitude of the learning factor. A subsidy for present output of renewable energy rises energy production and lowers the present energy price, inducing resource owners to postpone extraction. However, this effect is counteracted by the indirect effect due to LBD according to which future production costs decrease as more experience is accumulated in the present. This leads to more renewable energy production and lower energy prices in the future and incentivizes resource owners to extract more rapidly. If the learning effect is sufficiently large, this indirect effect dominates the initial effect and present emissions increase. Therefore, also subsidizing renewable energy may cause the green paradox.

In the more general model, we allow for increasing marginal extraction costs and get a result that is in contrast to the green paradox, namely that delayed taxation of carbon may even decrease current emissions. If marginal extraction costs are increasing, resource owners will not exhaust the full stock of resources entirely and any climate policy does not only affect the timing of the extraction, but also the volume of total extraction. In response to a future carbon tax, resource owners initially shift some extraction to the present and reduce overall extraction where both result in higher future energy prices. This induces the renewable energy sector to expand production in the future and - due to the anticipated gains from LBD - also in the present. The latter leads to falling energy prices in the present and incentivizes resource owners to postpone extraction. If the marginal extraction cost curve is sufficiently flat, the impact of taxation on future energy prices and therefore on the renewable energy sector is substantial. In this case, the incentive for resource owners to postpone extraction outweighs the initial increase of current emissions and current emissions decrease despite the fact that a carbon tax in the future was implemented. Thus, there is a reversal of the green paradox.

The paper is organized as follows: Section 2 presents the basic model with zero extraction costs while Section 3 analyzes the effect of LBD on present emissions in the absence of climate policies. In Section 4, we examine the impact of climate policies on

environmental damage. Section 5 extends the basic model by assuming convex extraction costs and analyzes the effect of LBD and climate policies on both present and total emissions. Finally, Section 6 concludes.

2. Basic Model

The model consists of two time periods where the first period may represent the next 5 to 10 years, the time necessary to accumulate experience, whereas the second period represents the remaining future. In the following, lower-case letters always refer to variables and functions in the first and capital letters to variables and functions in the second period. There are two sources of energy: A polluting energy source from fossil fuels (x) and a clean renewable energy source (y) which exhibits LBD.

2.1. Fossil Fuel Sector and Environmental Damage

Energy is produced by the combustion of fossil fuels. We normalize units such that one unit of fossil fuel is converted into one unit of energy, causing one unit of emissions. The market for fossil fuels is competitive. To begin with, we assume resource owners to have zero extraction costs which serves as a reference case for the analysis. This assumption will be relaxed in Section 5 where we assume extraction costs to be convex. With zero extraction costs, it is always optimal to exhaust the stock of resources $\bar{X}$ completely as long as the energy price is still positive.

Let $p > 0$ and $P > 0$ be the market prices of energy, the maximization problem of the resource owner reads

$$\max_{x,X} \pi_f = (p-t)x + \beta(P-T)X \quad \text{s.t.} \quad x + X \leq \bar{X} \tag{1}$$

where t and T are per unit carbon taxes and β denotes the discount factor.[5] The first-order condition (FOC) yields the arbitrage condition according to Hotelling's rule in a two-period framework

$$p - t = \beta(P - T). \tag{2}$$

The FOC states that in an interior solution the producer price in the present period equals the discounted future producer price, implying the resource owner to be indifferent between extracting today or in the future.

Let $\tilde{X} = x + X$ be the total amount of emissions, the environmental damage function can be represented as

$$ED = ED(x, \tilde{X}) \tag{3}$$

with $ED_x > 0$ and $ED_{\tilde{X}} > 0$.[6] Thus, the damage from global warming increases in both present and total emissions.[7] Since the resource stock is exhausted completely, we have $\tilde{X} = \bar{X}$ and the change of environmental damage only depends on the variation of current emissions.

2.2. Renewable Energy Sector

Renewable energy is produced in a competitive market where the representative firm faces increasing marginal costs in both periods, i.e. $c_y(y) > 0$, $c_{yy}(y) > 0$, $C_Y(y, Y) > 0$ and $C_{YY}(y, Y) > 0$. This reflects the fact that for each firm, the marginal productivity tends to decrease as the output of renewable energy facilities is expanded. Moreover, on sector level, the appropriateness of locations for the installation of renewable energy facilities tends to decrease once the most appropriate locations have already been used. Consider the example of onshore wind farms. While the first wind farm is constructed

[5] For convenience, we abstract from discounting within each period since it would not add any important insight.

[6] In the following, subscripts denote the first or second derivative with respect to the corresponding variable.

[7] Climate damage can be expected to rise in current emissions even in the absence of discounting as it may accelerate climate change. For a more elaborated discussion see Hoel (2011).

in the area where the wind blows strongest and most steadily, any further wind farm will have less favorable conditions.[8] Furthermore, some raw materials such as rare earth metals that are crucial in the production of renewable energy facilities may become increasingly scarce and expensive as the capacity of renewable energy expands, causing marginal costs to increase.

We incorporate LBD by assuming future costs to decline with experience accumulated in the first period, but at a decreasing rate, i.e. $C_y(y, Y) < 0$ and $C_{yy}(y, Y) > 0$ as well as $C_{yY}(y, Y) = C_{Yy}(y, Y) < 0$.[9] The latter states that also future marginal costs decrease with experience. Furthermore, we assume overall convexity of the cost function, implying $C_{yy}C_{YY} - C_{yY}^2 > 0$ in order to satisfy second order conditions. This condition basically states that the own convexity dominates the cross effects.[10] A typical functional form that incorporates increasing unit costs compared to the learning curve proposed by Wright (1936) can be represented by[11]

$$C(y, Y, b) = C(Y)y^{-b} \tag{4}$$

with $y > 1$, $C(Y)$ a convex function and $b > 0$ representing the learning factor that determines the magnitude of the cost reduction due to accumulated experience in the first period.[12] We assume that a higher learning factor decreases future marginal costs and strengthens the effect of accumulated experience on future costs, i.e. $C_{Yb} < 0$, $C_{yb} < 0$

[8] According to this argument, the convexity of the cost function in the second period should also depend on the amount of renewable energy produced in the first period. However, we abstract from those intertemporal relationships. One can think of all renewable energy facilities constructed in the first period being fully depreciated at the beginning of the second period and having to be replaced by new facilities.

[9] In reality, also the fossil fuel sector is likely to exhibit some LBD. However, since this sector is relatively mature, the learning rates in the renewable energy sector should be far higher. We incorporate this fact by normalizing the learning rate in the fossil fuel sector to zero and assuming learning to take place exclusively in the renewable energy sector.

[10] The same assumption is found in Reichenbach and Requate (2012) and Lehmann (2013).

[11] Wright (1936) proposed a learning curve with constant unit costs, i.e. $C(y, Y, b) = cYy^{-b}$.

[12] The interpretation of the learning factor b is the following: The unit costs fall by b whenever accumulated production is doubled.

and $C_{yYb} < 0$.[13] The relationship between future costs and first-period quantities is illustrated in Figure 1.

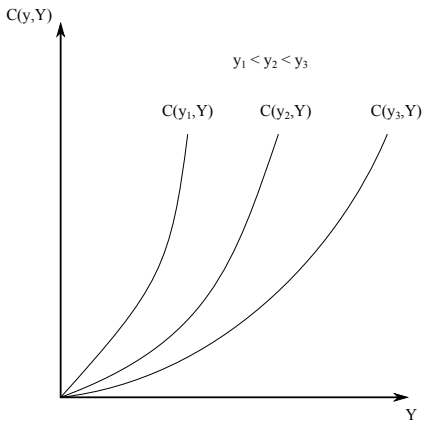

Figure 1: Cost Function of Renewable Energy with LBD

The representative firm takes the positive effect of LBD into account and maximizes its profit

$$\pi_r = (p+s)y - c(y) + \beta[PY - C(y,Y,b)] \tag{5}$$

where s is the per unit subsidy.[14] Differentiating with respect to y and Y yields the following FOCs for an interior solution[15]

$$\frac{\partial \pi_r}{\partial y} = 0 \qquad \Leftrightarrow \qquad p + s = c_y(y) + \beta C_y(y,Y,b) \tag{6}$$

$$\frac{\partial \pi_r}{\partial Y} = 0 \qquad \Leftrightarrow \qquad P = C_Y(y,Y,b). \tag{7}$$

[13]Note that for the functional form of equation (4), the term C_{Yb} is always negative while $C_{yb} = C(Y)y^{-b-1}[b\ln y - 1]$ is only negative for $b < 1/\ln y$ which means that b may not be too large. The latter also holds true for C_{yYb}.

[14]Note that we only consider a subsidy in the first period since we assume that the government would like to take advantage of LBD by triggering first period output.

[15]For an interior solution, we require $c_y(0) < p + s$ and $C_Y(0) < P$ which guarantees y and Y to be strictly positive quantities.

Equation (6) implies that the firm chooses the first period quantity such that the marginal costs exceed the producer price since $\beta C_y(y, Y, b)$ is negative. The firm anticipates the future cost reduction when determining the present quantity and is therefore willing to accept a potential loss in the first period. Note that we implicitly assume that all learning is private and that there are no learning spillovers. However, introducing learning spillovers would not alter any of our qualitative results as long as LBD is at least partially private.[16] This issue will be discussed further at the end of Section 5.

2.3. Equilibrium

The demand for energy is falling in price. Since both energy sources are assumed to be perfect substitutes, the energy price is given by the inverse demand $p(x + y)$ with $p_x = p_y = p' < 0$ and $P(X + Y)$ with $P_X = P_Y = P' < 0$. Incorporating the inverse demand into the FOCs of the fossil fuel (equation (2)) and renewable energy sector (equations (6) and (7)) constitutes a system of three equations with three endogenous variables x, y and Y that depend on the resource stock $\bar{X}$, the learning factor b and the policy variables s, t and T.[17] We assume all agents to have perfect information and the government to be able to fully commit to its announced policies. Since there is no uncertainty, all outcomes in this deterministic setting are already certain at the beginning of the first period. Thus, in the following we apply comparative statics in order to analyze how the outcomes represented by the three endogenous variables x, y and Y alter in response to a change of the learning factor b or the climate policies s, t and T.

[16] Introducing learning spillovers would cause the gains from learning to be appropriated only partially by a single firm. However, each firm would still have an incentive to produce y in excess (such that marginal costs exceed the producer price) due to the anticipation of future cost reductions. Formally, equation (6) would change to $p + s = c_y(y) + \rho\beta C_y(y, Y, b)$ where ρ is the degree of private appropriability. See Fischer and Newell (2007) for a formal derivation of the appropriability rate.

[17] In fact, also the second period extraction X is an endogenous variable. However, since resource owners always exhaust their (exogenously given) stocks completely, second period extraction is only the residual between the total stock and present extraction and therefore immediately given when x is determined.

3. The Effect of Learning on Fossil Fuel Extraction

Before turning to the effect of climate policies on the extraction decision of fossil fuel owners, we should examine how the pure presence of LBD in the renewable energy sector affects the extraction decision. Since we are interested in the effect of LBD, represented by the learning factor b, we set the policy variables s, t and T equal to zero. An increase in the learning factor results in two initial effects which augment the amount of both present and future renewable energy output y and Y. However, both initial effects have different impact on first period extraction x.

The first initial effect of a higher learning factor b results in an expansion of future renewable energy output Y since it decreases future production costs. This affects the first period extraction x via two channels. First, the second period energy price decreases which induces resource owners to shift extraction to the present. Second, the renewable energy firms also increase y because the benefits from LBD are increasing with higher output of Y.[18] This reduces the first period energy price and causes resource owners to postpone extraction. Thus, there are two countervailing effects which is why the total effect of an initial increase in Y on x is ambiguous.

The second initial effect of a higher learning factor increases y because learning has become more effective and firms are therefore willing to invest more in future cost reduction via augmenting y. However, the effect of an increase of y on x works again via two channels. First, the first period energy price declines and resource owners will postpone extraction. The second channel originates from LBD where the renewable energy firms increase Y due to lower production costs. This reduces P and induces resource owners to extract more rapidly. Since both channels work in opposing directions, the effect of an initial increase in y on x is also ambiguous.

In total, the overall effect of an increasing learning factor on current emissions is unclear. Formally, we have a system of three equations originating from the three FOCs

[18] Formally, we have $\frac{dy}{dY} = \frac{\beta(C_{YY} - C_{yY})}{c_{yy} + \beta(C_{yy} - C_{yY})} > 0$.

(equations (2), (6) and (7)) with the three endogenous variables x, y and Y and the exogenous variable b. We totally differentiate this system and get[19]

$$\underbrace{\begin{pmatrix} p' + \beta P' & p' & -\beta P' \\ p' & p' - c_{yy}(y) - \beta C_{yy}(y,Y,b) & -\beta C_{yY}(y,Y,b) \\ -P' & -C_{Yy}(y,Y,b) & P' - C_{YY}(y,Y,b) \end{pmatrix}}_{M} \begin{pmatrix} dx \\ dy \\ dY \end{pmatrix} = \begin{pmatrix} 0 \\ \beta C_{yb}(y,Y,b)db \\ C_{Yb}(y,Y,b)db \end{pmatrix} \tag{8}$$

Using standard matrix algebra and Cramer's rule, the impact of an increase in b on x is given by[20]

$$\frac{dx}{db} = \frac{1}{det(M)} \left[\{\beta C_{yb}[\beta P' C_{yY} - p'(P' - C_{YY})]\} + \{C_{Yb}[\beta P'(p' - c_{yy} - \beta C_{yy}) - \beta p' C_{yY}]\} \right] \tag{9}$$

where $det(M)$ is the determinant of matrix M. We show in the appendix that the sign of $det(M)$ is negative given our assumptions concerning the cost function. The first term in curly brackets represents the effect of an increase in b on x which originates from a change in y whereas the second term displays the effect originating from the initial increase of Y. As can be seen, the sign of both effects is ambiguous which is why the effect of an increase in the learning factor on current emissions is ambiguous as well.

In the appendix, we show that also the overall effect of an increase of the learning factor on the renewable energy quantities y and Y is not clear even though the initial effects are positive. The reason for this is that each initial effect can be outweighed by the indirect consequences of the other effect. However, a higher learning factor always increases the total quantity of energy produced in each period. Therefore, the energy price unambiguously declines with higher b. Both results are in line with Chakravorty et al. (2011). Finally, Proposition 1 summarizes the findings

[19] Since $X = \bar{X} - x$, we have $\frac{\partial P}{\partial X} = -\frac{\partial P}{\partial x} = -P'$.

[20] In the following, we suppress the arguments of the functions for the sake of notational convenience.

Proposition 1

The overall effect of an increase in b on the energy quantities x, y and Y is ambiguous. However, total energy production in both periods rises with b, causing the energy price to decline.

4. The Effect of Climate Policies

We now turn to the comparative statics of the effects of the policy variables t, T and s on the endogenous variables x, y and Y. Analogously to the analysis above, we totally differentiate the FOCs (2), (6) and (7) holding the learning factor $b > 0$ constant and get the following system of equations

$$M \begin{pmatrix} dx \\ dy \\ dY \end{pmatrix} = \begin{pmatrix} dt - \beta dT \\ -ds \\ 0 \end{pmatrix} \tag{10}$$

where M is the matrix that was defined in the previous section.

The Effect of Taxation

First, we start with the implementation of a future tax T in order to reassess the standard result of the green paradox which predicts present emissions to increase. The effect of T on present fossil fuel extraction is given by

$$\frac{dx}{dT} = (-\beta)\frac{1}{det(M)}\Bigg[\underbrace{[p' - c_{yy} - \beta C_{yy}][P' - C_{YY}] - \beta C_{yY}^2}_{N}\Bigg]. \tag{11}$$

As the determinant of M is negative and the numerator N is unambiguously positive, future taxation always increases present fossil fuel extraction and the green paradox arises.[21] However, LBD attenuates the magnitude of the shift of extraction to the

[21] Even though $-\beta C_{yY}^2$ is negative, the numerator is unambiguously positive due to the assumption that own convexity dominates cross effects.

present. A reduction in second period extraction increases second period energy price which induces renewable energy firms to expand production in the future and - due to the anticipated gains from LBD - also in the present. The latter lowers the energy price in the present, incentivizing resource owners to postpone extraction. However, the reduction in present extraction can never outweigh the initial increase in present extraction as long as the stock of resources will be exhausted entirely. We will see in Section 5 that present extraction does not necessarily increase in response to delayed carbon taxation when resource owners will not exhaust the entire resource stock but only those resources which are economically viable.

Concerning the introduction of a present carbon tax t, it will definitely induce resource owners to postpone extraction - though the magnitude of the extraction shift is again attenuated by the presence of LBD. If both taxes were introduced simultaneously, the reaction of the resource owners would depend on the tax path. More precisely, current extraction will increase (decrease) as long as the future tax increases with a rate higher (lower) than the interest rate. This is the standard result of the green paradox.

The effect of taxation on the renewable energy quantities y and Y is ambiguous. For example, a delayed carbon tax induces resource owners to shift extraction to the present which lowers p. This leads to less current and - because of LBD - also to less future renewable energy output. At the same time, the tax increases the future energy price which causes y and Y to increase. The overall effect finally depends on the magnitude of the learning factor.

The Effect of Subsidizing Renewable Energy

We next analyze the effect of a subsidy for present renewable energy on present extraction.[22] Proceeding analogously as above, we find that a subsidy always increases

[22] In practice, many governments encourage renewable energy production making use of feed-in tariffs. According to REN (2013) p. 68, feed-in tariffs are the most widely adopted policy instrument to support renewable energy employed by 71 countries in 2013. However, the qualitative result of our analysis would not change since both subsidies and feed-in tariffs lead to an increase of the producer price relative to the no-policy scenario.

present output of renewable energy. With respect to current emissions, we have

$$\frac{dx}{ds} = (-1)\frac{1}{det(M)}[\beta P' C_{yY} - p'(P' - C_{YY})] \tag{12}$$

where the sign of the equation is ambiguous and depends, in particular, on the size of the term C_{yY} that represents LBD. For small absolute values of C_{yY}, a subsidy yields less x whereas higher values of C_{yY} will increase x. The magnitude of C_{yY} depends positively on the size of the learning factor b. To see this, consider the linear cost function $C(y, Y, b) = (a - by)Y$. In this case, we have $C_{yY} = -b$, implying the absolute value of C_{yY} to be increasing in b. For the functional form from equation (4), the absolute value of C_{yY} increases in b as long as b is not too large.[23]

Turning to the interpretation of the result, we distinguish between the direct effect and the indirect effect due to LBD which is shown in Figure 2. Given that a renewable energy subsidy increases present output, the direct effect causes p to decline and induces resource owners to postpone extraction (x decreases).

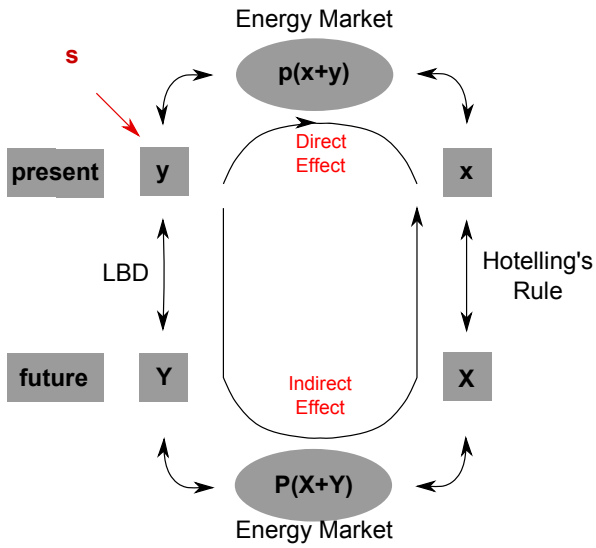

Figure 2: Effect of Subsidizing Renewable Energy on Current Extraction

[23] To see this, note that $C_{yYb} = C_Y(Y)y^{-b-1}(b \ln y - 1)$ which is negative for small b and becomes positive for $b > 1/\ln y$.

The indirect effect works in the opposite direction: An increase in y leads to lower future renewable energy costs which induces renewable energy firms to expand the production of Y. This causes the future energy price to fall and incentivizes resource owners to extract more rapidly (x increases). Whether or not the indirect effect outweighs the direct effect with respect to present extraction depends on the size of the term C_{yY} and therefore on the learning factor b. Figure 3 shows the change in the energy quantities in response to an introduction of a subsidy for different learning factors b.[24]

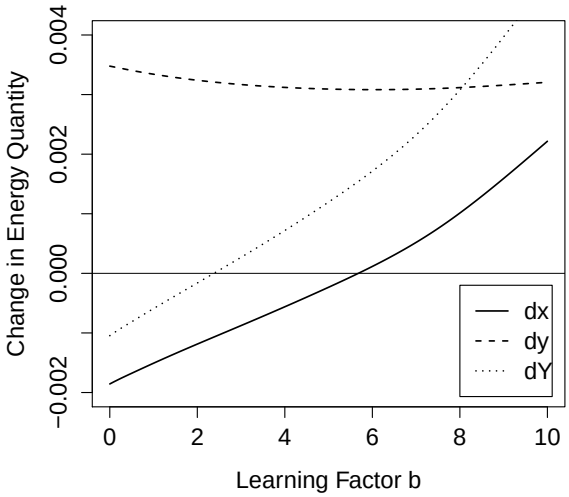

Figure 3: Effect of Subsidizing Renewable Energy on Energy Quantities

In the absence of learning ($b = 0$), we only observe the direct effect of the subsidy which increases y and reduces x and Y. The reduction of Y results from the fact that resource owners postpone extraction which reduces the future energy price and therefore also Y.

The indirect effect works diametrically to the direct effect. Thus, as learning sets in, dy declines in response to a decrease of Y while dY rises due to the cost reduction that was caused by higher present renewable output. Figure 3 indicates that, as b becomes

[24]See appendix for the used functional forms and parameter values.

larger, dY turns positive which also reinforces the output of y (due to the anticipation of cost benefits). More importantly, higher future renewable output reduces future energy prices, inducing fossil fuel owners to extract more rapidly. For b being large enough, the indirect effect dominates the direct effect with respect to x, leading to higher present extraction in response to a subsidy for renewable energy. Thus, despite the fact that the substitute of present energy from fossil fuel was subsidized, present extraction increases and the green paradox arises.

Proposition 2 summarizes the insights from this section.

Proposition 2

Let extraction costs be zero. Then

a) present emissions increase (decrease) if the carbon tax rises faster (slower) than the interest rate while LBD attenuates the magnitude of the change in emissions.

b) a present renewable energy subsidy increases present emissions if the learning factor is sufficiently high.

5. General Model with Increasing Marginal Extraction Costs

So far, we assumed resource owners to have zero extraction costs. If fossil fuel suppliers faced positive, but constant marginal extraction costs, the analysis from Section 3 and 4 would still be valid as long as the energy price exceeds the marginal extraction costs. Under this assumption, it would still be optimal to exhaust the available resource stock completely.[25] However, if resource owners face increasing marginal extraction costs, they are likely not to exhaust all of their physically available resources, but only those resources which are economically viable. In this case, any climate policy does not only

[25] However, if marginal extraction costs were sufficiently high, resource owners would only extract the amount until the producer price equals the marginal costs. In this case, resource owners will only extract a part of the physically available resources. This issue will be discussed more extensively at the end of this section.

affect the timing of extraction, but also the total volume of extraction. In fact, there is much evidence that the marginal extraction costs are increasing with the quantity that has already been extracted.[26] First, for each oil well or coal mine, extraction of the first units requires less energy than extraction of any further unit. Second and on a global level, once the lowest cost resources have already been exhausted, higher cost resources have to be extracted. For example, oil is increasingly exploited from deep water wells or energy intensive tar sands which exhibit far higher extraction costs than conventional oil wells.

For our analysis, let z be the accumulated extraction amount and $e(z)$ be the extraction cost function. For simplicity, we assume $e(z)$ to be zero up to a threshold x' with $x < x' < x + X$ and $e(z)$ to be positive and rising beyond that threshold. Thus, in the first period extraction costs are always zero whereas they are convex in the second period. This assumption does not affect any of our qualitative results but simplifies the analysis. Then, the maximization problem of the representative resource owner reads

$$\max_{x,X} \pi_{f'} = (p-t)x + \beta[(P-T)X - e(x+X)] \quad \text{s.t.} \quad x + X \leq \bar{X}. \tag{13}$$

If the resource constraint is not binding, the FOCs are given by

$$\frac{\partial \pi_{f'}}{\partial x} = 0 \qquad \Leftrightarrow \qquad p - t = \beta(P - T) \tag{14}$$

$$\frac{\partial \pi_{f'}}{\partial X} = 0 \qquad \Leftrightarrow \qquad P - T = e_{\tilde{X}}(\tilde{X}) \tag{15}$$

where $e_{\tilde{X}}(\tilde{X}) = e_x(\tilde{X}) = e_X(\tilde{X})$ are the marginal extraction costs evaluated at $\tilde{X}$ and $\tilde{X} = x + X$ is the total extraction amount. In the following, our analysis is based on the assumption that the resource stock is exhausted economically rather than physically,

[26] See Rogner (1997) for estimates of the marginal extraction cost curves for different hydrocarbon resources.

implying $\tilde{X} < \bar{X}$.[27] In fact, this assumption will be crucial for most of our further results and will be discussed more extensively at the end of this section. Equation (14) represents Hotelling's rule and corresponds perfectly with equation (2) due to the assumption concerning the extraction costs in the first period.[28] Equation (15) pins down the total extraction $\tilde{X}$ and states that in equilibrium the producer price defined as market price for energy minus per unit carbon tax must equal the marginal extraction costs. If the producer price was above the marginal extraction costs, resource owners could increase their profits by extracting more. If the producer price was below the marginal extraction costs, resource owners would benefit from reducing extraction. Since $\tilde{X}$ is endogenous, we have to consider both present and total emissions in order to assess the effect of learning and the climate policies on environmental damage.

In order to reduce the system of four equations (equations (6), (7), (14) and (15)) and four endogenous variables (x, y, Y and $\tilde{X}$) to three, we totally differentiate equation (15) taking into account the inverse demand function $P = P(X + Y)$ as well as the fact that $dX = d\tilde{X} - dx$ and get

$$d\tilde{X} = \frac{P'}{P' - e_{\tilde{X}\tilde{X}}(\tilde{X})}(dx - dY) + \frac{1}{P' - e_{\tilde{X}\tilde{X}}(\tilde{X})}dT. \tag{16}$$

This equation is plugged in when we totally differentiate the remaining three FOCs yielding a system of three equations with three endogenous variables (x, y and Y).[29]

5.1. The Effect of Learning

As before, we begin our analysis with examining the effect of an increase of the learning factor b on the environmental damage. Setting the policy variables s, t and T equal to

[27] If the resource constraint was binding and the resource stock was exhausted physically, we would have $e(x + X) = e(\bar{X})$ which is constant. Then, the FOCs (14) and (15) reduce to $p - t = \beta(P - T)$ and we would be back in the setting of Sections 3 and 4.

[28] If extraction costs in the first period were positive as well, Hotelling's rule would read $p - t - e_x(x) = \beta(P - T - e_x(x))$. However, this would not alter any of our qualitative results.

[29] In the appendix, we derive equation (16) in more detail.

zero, totally differentiating equations (6), (7) and (14) and substituting equation (16) where necessary yields

$$\underbrace{\begin{pmatrix} p' + \alpha\beta P' & p' & -\alpha\beta P' \\ p' & p' - c_{yy}(y) - \beta C_{yy}(y,Y,b) & -\beta C_{yY}(y,Y,b) \\ -\alpha P' & -C_{yY}(y,Y,b) & \alpha P' - C_{YY}(y,Y,b) \end{pmatrix}}_{M'} \begin{pmatrix} dx \\ dy \\ dY \end{pmatrix} = \begin{pmatrix} 0 \\ \beta C_{yb}(y,Y,b)db \\ C_{Yb}(y,Y,b)db \end{pmatrix} \tag{17}$$

with $det(M') < 0$ and $\alpha = 1 - \frac{P'}{P' - e_{\tilde{X}\tilde{X}}(\tilde{X})} = \frac{e_{\tilde{X}\tilde{X}}(\tilde{X})}{e_{\tilde{X}\tilde{X}}(\tilde{X}) - P'} \in [0,1)$. Given that second period energy demand is price sensitive ($P' < 0$), the factor α essentially measures the steepness of the marginal extraction cost curve, which is equivalent to the supply curve of fossil fuels, evaluated at $\tilde{X}$.

The effect of an increase of b on the energy quantities x, y and Y is again ambiguous due to the two initial effects of b on y and Y. For example, the effect of an increase in b on x reads

$$\begin{aligned} \frac{dx}{db} =& \frac{1}{det(M')}\beta C_{yb}[\alpha\beta P' C_{yY} - p'(\alpha P' - C_{YY})] + \\ & \frac{1}{det(M')} C_{Yb}[\alpha\beta P'(p' - c_{yy} - \beta C_{yy}) - \beta p' C_{yY}] \end{aligned} \tag{18}$$

where the upper line represents the initial effect of a higher learning factor on y and the lower line the initial effect on Y. Relative to the case without extraction costs, equation (18) differs from equation (9) only with respect to the factor α. If α was 1, there would be no difference in the effect of a higher learning factor on x. However, if α was close to zero, a higher learning factor would unambiguously reduce current emissions.[30]

From the last section, we have seen that a higher learning factor initially increases both renewable energy quantities y and Y. As a first effect, a higher y decreases p and

[30] We show in the appendix that also the renewable energy quantities y and Y unambiguously increase for α being sufficiently small.

leads resource owners to postpone extraction regardless of the size of α. However, this effect is counteracted by the second effect according to which a higher b increases Y and lowers P. A fall in P affects both the Hotelling rule according to equation (14) and the total extraction amount according to equation (15) where both induce resource owners to reduce second period supply X. The amount X can be reduced via both increasing present extraction and reducing total extraction. Formally, we have $x + X = \tilde{X}$ which implies that $dX = d\tilde{X} - dx$. The magnitude of $d\tilde{X}$ and dx finally depends on α.

Consider first the case where $\alpha \to 1$, implying that either $e_{\tilde{X}\tilde{X}}(\tilde{X})$ goes to infinity or $P' \to 0$. In the following we will not focus on the latter case since this would imply demand for energy to be almost completely inelastic.[31] For $e_{\tilde{X}\tilde{X}}(\tilde{X})$ going to infinity, we are essentially in the setting of the model from Section 3 because the fossil fuel supply curve evaluated at $\tilde{X}$ is completely inelastic (a vertical line) and so the resource owners will almost always extract the same amount $\tilde{X}$ regardless of the price P. Consequently, resource owners hardly ever reduce total extraction ($d\tilde{X} \to 0$) and will increase present extraction until Hotelling's rule is satisfied ($dX \to -dx$). As α becomes smaller, resource owners would more and more reduce X via lowering $\tilde{X}$. This partially offsets the initial fall in P, so that the incentive to increase present extraction becomes smaller. For $\alpha \to 0$, we have $dX \to d\tilde{X}$ which implies $dx \to 0$. In this case, the effect of an initially increased Y on x becomes negligible while an initially increase in y still lowers x which is why a higher learning factor unambiguously reduces current emissions. In the appendix, we show that there is at most one cut-off value for α for which the term $\frac{dx}{db}$ potentially switches its sign. For any values of α below the cut-off value, current emissions unambiguously decrease in response to a higher learning factor. For values of α above the threshold, the effect of more effective learning on current emissions may be positive or negative.

We next turn to the question of how the total extraction and therefore total emissions

[31] If demand for energy was completely inelastic, an increase in Y would not have any impact on x, y or $\tilde{X}$ because P is not affected.

are affected by more effective learning. We derive expressions for $\frac{dx}{db}$ and $\frac{dY}{db}$ from equation (17) and plug in both terms into equation (16) which yields

$$\frac{d\tilde{X}}{db} = (1-\alpha)\frac{1}{det(M')}\Big[\beta C_{yb}[p'C_{YY} - p'C_{yY}] + C_{Yb}[-p'\beta C_{yY} + p'(c_{yy} + \beta C_{yy})]\Big] < 0. \quad (19)$$

For any $\alpha \in [0,1)$, the total amount of extraction is decreasing in the learning factor while the decrease is larger for smaller values of α.Finally, Proposition 3 summarizes the findings.

Proposition 3

Let $\alpha = \frac{e_{\tilde{X}\tilde{X}}(\tilde{X})}{e_{\tilde{X}\tilde{X}}(\tilde{X}) - P'} \in [0,1)$ be a measurement of the steepness of the marginal extraction cost curve evaluated at $\tilde{X}$. If extraction costs are convex and the resource stock is exhausted economically, an increase of the learning factor impacts present and total emissions and therefore environmental damage depending on α according to the following table:

Table 1: Effect of a higher Learning Factor

	α large	*α small*
x	-/+	-
$\tilde{X}$	-	-
$ED(x, \tilde{X})$	-/+	-

As learning becomes more effective, total emissions always decline regardless of the size of α. If α is sufficiently large, the impact of a higher learning factor on present emissions is still ambiguous. If present emissions decrease as well, there will be unambiguously less environmental damage. If present emissions increase, the effect on environmental

damage is unclear since there is a trade off between higher present and fewer total emissions. In this case, the change of environmental damage depends on the specific functional form of $ED(x, \tilde{X})$ as well as on the magnitude of the changes in x and $\tilde{X}$. For α sufficiently small, both present and total emissions unambiguously decline and the effect on total damage is negative.

5.2. The Effect of Climate Policies

We now turn to the analysis of the policy instruments. Therefore, we proceed analogously as before and get

$$M' \begin{pmatrix} dx \\ dy \\ dY \end{pmatrix} = \begin{pmatrix} dt - \alpha\beta dT \\ -ds \\ -(1-\alpha)dT \end{pmatrix}. \tag{20}$$

Note that a future carbon tax also affects directly the decision of the second period output of renewable energy in form of the last entry of the vector on the right-hand side compared to equation (8) from Section 4.[32]

Concerning the effect of the policies s, t and T on total emissions $\tilde{X}$, we show in the appendix that all three policies reduce total extraction as long as $\alpha < 1$. Since all climate policies reduce the producer price for fossil fuels in the future, resource owners will lower total extraction.

We next examine the effect of the policy variables on the energy quantities x, y and Y. Concerning an introduction of t and s, our results from Section 4 do not alter. In fact, a present carbon tax always reduces present fossil fuel extraction while the effect on output of renewable energy in both periods is ambiguous and depends on the magnitude of the learning effect.[33] With the introduction of a subsidy for renewable energy, the quantity

[32] The reason for this is that a future carbon tax influences not only Hotelling's rule, meaning the decision between extracting today or in the future, but also the total amount of emissions which causes X to be endogenous and not only to be the residual between the resource stock and present extraction. Since X is endogenous and depends on T, also P and therefore Y are directly affected by T.

[33] However, if α is close to zero, both renewable energy quantities unambiguously increase. The reason is that a present carbon tax only affects p while P is hardly affected because resource owners reduce

y increases unambiguously while the effect on x and Y depends on the magnitude of the learning effect. However, for α close to zero, we definitely observe a decline in x and a rise in Y.[34] The results are summarized in Proposition 4 below.

Turning to the introduction of a future tax, the effect of T on y and Y is still ambiguous as in the reference case with zero extraction costs. However, the effect of T on present extraction can now be written as

$$\begin{aligned}\frac{dx}{dT} =& \frac{1}{det(M')}(-\alpha\beta)[(p' - c_{yy} - \beta C_{yy})(\alpha P' - C_{YY}) - \beta C_{yY}^2] + \\ & \frac{1}{det(M')}(\alpha - 1)[-\beta p' C_{yY} + \alpha\beta P'(p' - c_{yy} - \beta C_{yy})].\end{aligned} \tag{21}$$

For $\alpha \to 1$, the lower term virtually vanishes and we observe x to increase with T which is the standard result of the green paradox. On the other hand, as α approaches zero, the effect of an increase in T on x becomes negative. In the appendix, we show that $\frac{dx}{dT}$ monotonically increases with α which guarantees the existence of a cut-off value for α where $\frac{dx}{dT} > 0$ if α exceeds this value. For α being below this cut-off value, we have $\frac{dx}{dT} < 0$ which is contradictory to the green paradox.

The reason for this result is the following: An introduction of T has essentially two effects. First, second period producer price $P - T$ initially decreases which induces resource owners to reduce X via increasing present extraction and reducing total extraction. Second, as X decreases, second period energy price P increases which leads to higher output of Y and induces the renewable energy firm to produce more y due to the anticipation of the gains from LBD. This decreases p and crowds out current energy from fossil fuels as it induces resource owners to reduce first period extraction. For α

present extraction via lowering $\tilde{X}$ rather than postponing extraction. Thus, the effect of an increase in t initially leads to a rise in y, but not to a fall in Y as it was the case with zero extraction costs. Moreover, higher y causes higher output of Y due to the effect of LBD which is why both energy quantities increase.

[34] The reason is that as y increases in response to a subsidy, resource owners will lower x via reducing $\tilde{X}$, letting P and therefore Y virtually unaffected. Nevertheless, Y increases due to the LBD effect. This causes P to decrease, but again resource owners will rather reduce $\tilde{X}$ than increase x so that the overall effect of a renewable energy subsidy is a decline in x and a rise in Y.

sufficiently small, this reduction outweighs the initial increase in first period extraction as can be seen in Figure 4.

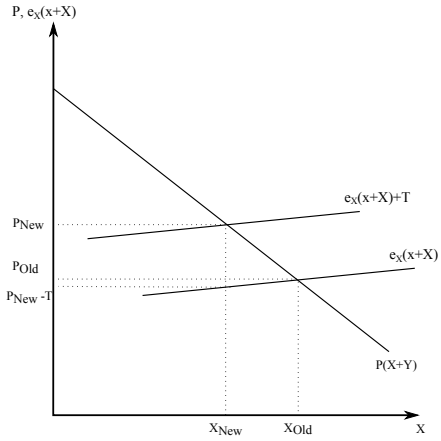

Figure 4: Delayed Carbon Taxation with Flat Marginal Extraction Cost Curve

In the absence of taxation, we find the equilibrium where, according to equation (15), the marginal extraction cost curve intersects the second period energy demand curve (X_{Old} and P_{Old}). Taxation leads to an upward shift of the marginal extraction cost curve. The new equilibrium is characterized by the second period fossil fuel quantity X_{New}. More importantly, observe that the difference between new and old energy price $P_{New} - P_{Old}$ is relatively large which means that renewable energy firms will expand their production substantially. Moreover, the difference between old and new producer price $P_{Old} - (P_{New} - T)$ is very small so that resource owners have only very few incentives to increase present extraction according to Hotelling's rule. Thus, as α is sufficiently small, it is likely that the crowding out of first period extraction by the expansion of renewable energy production outweighs the initial increase in present extraction. Consequently, present extraction decreases despite the fact that a future tax is introduced. A result that is in stark contrast to the standard result of the green paradox. Finally, Proposition

4 summarizes the insights of this section.

Proposition 4

Let $\alpha = \frac{e_{\tilde{X}\tilde{X}}(\tilde{X})}{e_{\tilde{X}\tilde{X}}(\tilde{X})-P'} \in [0,1)$ *be a measurement of the steepness of the marginal extraction cost curve evaluated at* $\tilde{X}$*. If extraction costs are convex and the resource stock is exhausted economically, then the effect of climate policies on present and total emissions depends on* α *according to the following table:*

Table 2: Effect of Climate Policies

Variable	*Effect of t*		*Effect of T*		*Effect of s*	
	α *large*	α *small*	α *large*	α *small*	α *large*	α *small*
x	-	-	+	-	-/+	-
$\tilde{X}$	-	-	-	-	-	-
$ED(x, \tilde{X})$	-	-	-/+	-	-/+	-

The introduction of a present carbon tax unambiguously decreases environmental damage regardless of the size of α. If α is large, the effect of a subsidy on present emissions depends on the size of the learning factor as in Section 4 while future carbon taxes tend to increase present emissions. Then, according to Gerlagh (2011), we can either observe a weak green paradox where the effect on environmental damage is negative or a strong green paradox, where the increased damage from higher present emissions outweighs the decreased damage from lower total emissions. If α is small, a subsidy and also a future carbon tax reduce present as well as total emissions and therefore the environmental damage. The main result of this paper is that present emissions may decrease in response to delayed carbon taxation which is a reversal of the green paradox.

For this result to hold, three conditions have to be satisfied: First, learning should be private to the extent that renewable energy firms, at least partially, anticipate future cost

reduction due to LBD when choosing present quantity which leads to crowding out of present energy from fossil fuel. Second, the marginal extraction cost curve evaluated at $\tilde{X}$ should be very flat so that resource owners only slightly increase present extraction. Third, the resource stock needs to be exhausted economically rather than physically which causes the total extraction amount to decline in response to any climate policy including delayed carbon taxation.

With respect to learning, our analysis assumed that all gains from learning are private and are therefore perfectly internalized by the representative firm. In reality, LBD is likely to be influenced by both own accumulated experience (internal learning) and total experience of the whole sector (external learning). Then, firms will produce an inefficiently low amount of output because they do not internalize the positive externality in form of learning spillovers on their competitors.[35] The empirical literature on learning demonstrates the existence of learning spillovers, but emphasizes that internal learning is the predominant source of learning.[36] Thus, the benefits from LBD will be at least partially internalized by the renewable energy firms and the first condition that drives our result is satisfied.

Concerning the marginal extraction cost curve, there are surprisingly only few studies that deal with the estimation of long run supply curves for fossil fuels. Since oil is relatively unimportant in the production of electricity, we restrict our attention to natural gas and coal. We evaluate the slope of the marginal extraction cost curve at the market price of the resource that is likely to prevail in the future. Given equation (15), this is equivalent to evaluating the supply curve at $\tilde{X}$. For natural gas, Bauer et al. (2013) estimate the market price to be around 6 USD per GJ energy by 2050.[37] They distinguish the estimated supply curves between low, medium and high resource availability. While in the low resource availability scenario the supply curve becomes steep beyond 5 USD,

[35] In fact, this would be an economic justification for subsidizing renewable energy.
[36] See Irwin and Klenow (1994), Foster and Rosenzweig (1995) and Gruber (1998).
[37] In the following, USD always refers to real United States Dollar per GJ energy in the year 2005.

the supply curves are still flat around 6 USD in the other two scenarios.[38] Further, the supply curve estimated by Rogner (1997) only becomes steep beyond a price of around 10 USD. For coal, the market price is expected to be around 3 USD by 2050. At this price, the three estimated supply curves of Bauer et al. (2013) as well as the supply curve of Rogner (1997) are still flat. Thus, even though all those estimates are subject to enormous uncertainty, there is at least some evidence that the marginal extraction cost curve is flat at the prevailing future market price which indicates that also the second condition is likely to be satisfied.

With respect to the condition of economical exhaustion, it seems to be likely that resource owners will stop extracting fossil fuels in the future because extraction costs are too high rather than because they have already exhausted all of their available resources. Translated to the two period setting of our model, economical exhaustion requires that the marginal extraction costs of the last unit in the second period equals the producer price.[39] Whether or not this holds true is an empirical question and depends, in particular, on the time horizon of the second period. As the time horizon of the second period becomes larger, the likelihood of economical exhaustion increases. First, as more time has passed, more fossil fuels have already been extracted, causing marginal extraction costs to be higher. Second, the costs of fossil fuel substitutes decrease in time which translates into lower market prices for fossil fuels in the future. In conclusion, the assumption of economical exhaustion is likely to hold true, if the length of the second period is sufficiently large.[40]

[38] For the market price, see Figure 10 and for supply curves, see Figure 1 in the supplementary material of Bauer et al. (2013).

[39] If the producer price exceeded the marginal extraction costs, resource owners would exhaust their entire stock and we are back in the setting of Section 4.

[40] If we had set up a model with infinite time horizon, the condition of economical exhaustion would have been met with certainty because there is no fixed termination point of the game. Nevertheless, given the existence of a substitute technology and convex extraction costs, there will be a point in time when extraction of fossil fuels will cease. In the last period, resource owners could potentially shift the marginal extraction unit into the next period and receive the prevailing market price. However, this would not be profitable because this would violate the arbitrage condition according to which the last extraction unit would yield zero profits in both the last and next to last period.

Since all three conditions are likely to hold true, the green paradox may not arise in response to the implementation of future carbon taxes.

6. Conclusion

We analyze the extraction behavior of fossil fuel owners in the presence of a clean substitute technology that exhibits LBD and ask under which conditions the green paradox arises. We find that the effect of more effective learning in terms of a higher learning factor on present emissions is ambiguous. Concerning the standard instruments to combat climate change, we find that subsidizing renewable energy may provoke the green paradox while present carbon taxes always reduce present emissions. Contrary to the standard result of the green paradox, future carbon taxes may reduce present emissions under certain conditions.

The effect of a higher learning factor on current emissions is ambiguous since there are two initial effects that have ambivalent impact on current emissions. On the one hand, a higher learning factor reduces future production costs leading to an increase in future renewable energy output. This reduces the future energy price and induces resource owners to shift extraction into the present. On the other hand, a higher learning factor also triggers present renewable energy production because learning has become more effective. This causes the present energy price to decline and incentivizes fossil fuel owners to postpone extraction. Thus, the overall effect of learning on current extraction is ambiguous. However, if the marginal extraction cost curve evaluated at the prevailing energy price is sufficiently flat and the resource stock is exhausted economically rather than physically, a higher learning factor will definitely reduce present and total emissions, leading to less environmental harm.

Subsidizing present renewable energy causes the green paradox if the learning factor is sufficiently high. While the direct effect of a subsidy unambiguously reduces present extraction, this effect is potentially outweighed by the indirect effect due to LBD. The

indirect effect causes future renewable energy supply to increase in response to higher present renewable output, inducing fossil fuel owners to extract more rapidly. If the marginal extraction cost curve is sufficiently flat around the prevailing energy price, subsidizing renewable energy always reduces current and total emissions regardless of the size of the learning factor.

A present carbon tax always reduces current emissions irrespective of the learning factor and the steepness of the marginal extraction cost curve. Future carbon taxes always increase current emissions when extraction costs are zero. Current emissions also rise in response to delayed carbon taxation if the marginal extraction cost curve is steep around the prevailing energy price. However, if the marginal extraction cost curve is flat, future carbon taxes are likely to reduce current emissions which is in contrast to the standard result of the green paradox. The reason is that in response to a future carbon tax, resource owners will lower total extraction substantially and increase present extraction only modestly. At the same time, the future energy price increases which induces renewable energy firms to expand production in the future and, because of the anticipation of higher future output, also today. The latter crowds out present energy from the combustion of fossil fuels and outweighs the initial increase in present emissions. Three conditions have to be met in order to obtain this result: The gains from learning should be, at least partially, private, the marginal extraction cost curve should be flat around the prevailing energy price in the future and the resource stock should be exhausted economically rather than physically. We found some evidence that all requirements are likely to be satisfied.

The policy implication of our paper is that a carbon tax that will be introduced only in the future does not necessarily increase present emissions and therefore environmental damage. This is an important insight since environmental policies are, at least in the short run, restricted to the effect that effective carbon taxes seem to be politically unfeasible for the next few years. Thus, if policy makers are restricted to employ carbon taxes

only in the future, our analysis suggests that this may not necessarily be accompanied by unintended side effects like the green paradox.

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A. Appendix

A.1. Proof of Propositions

The Determinants M and M'

First, we show that both determinants are negative. Applying Cramer's rule yields

$$det(M) = (p' + \beta P')(p' - c_{yy} - \beta C_{yy})(P' - C_{YY}) + 2\beta p' P' C_{yY}$$
$$- \beta P'^2 (p' - c_{yy} - \beta C_{yy}) - (p' + \beta P')\beta C_{yY}^2 - p'p'(P' - C_{YY})$$

which can be transformed to

$$det(M) = (p' + \beta P')[c_{yy} C_{YY} + \beta C_{yy} C_{YY} - \beta C_{yY}^2] + p' P' (2\beta C_{yY} - c_{yy} - \beta C_{yy} - \beta C_{YY}) < 0. \tag{A.1}$$

Since the second term is always negative and the first term is negative due to the assumption that the own convexity of the cost function dominates its cross effects, the sign of $det(M)$ is always negative.

For the determinant of M' from Section 5, we proceed the same steps as above and get

$$det(M') = (p' + \alpha\beta P')(c_{yy} C_{YY} + \beta C_{yy} C_{YY} - \beta C_{yY}^2) +$$
$$\alpha p' P' (2\beta C_{yY} - c_{yy} - \beta C_{yy} - \beta C_{YY}) < 0 \tag{A.2}$$

which is equivalent to $det(M)$ for $\alpha = 1$. Since $\alpha \in [0, 1)$, the determinant $det(M')$ is always negative.

Proof of Proposition 1

The effect of an increase in b on y and Y is given by

$$\frac{dy}{db} = \frac{1}{det(M)}\left[\beta C_{yb}[(p' + \beta P')(-C_{YY}) + p'P'] + C_{Yb}[-\beta p'P' + (p' + \beta P')\beta C_{yY}]\right]$$
$$\frac{dY}{db} = \frac{1}{det(M)}\left[\beta C_{yb}[-p'P' + (p' + \beta P')\beta C_{yY}] + C_{Yb}[(p' + \beta P')(p' - c_{yy} - \beta C_{yy}) - p'p']\right]$$

where both terms have ambiguous sign. The effect on first period price reads

$$\begin{aligned}\frac{dp}{db} &= p'\left[\frac{dx}{db} + \frac{dy}{db}\right] \\ &= p'\frac{1}{det(M)}\left[\beta C_{yb}[\beta P'(C_{yY} - C_{YY})] + C_{Yb}[\beta P'(\beta C_{yY} - c_{yy} - \beta C_{yy})]\right] \qquad \text{(A.3)}\end{aligned}$$

which is unambiguously negative.

Proof of Proposition 2

We first report the effects of T and s on the endogenous variables x, y and Y that have not been reported in the main part:

$$\frac{dy}{dT} = (-\beta)\frac{1}{det(M)}[\beta P' C_{yY} - p'(P' - C_{YY})] \lesseqgtr 0 \qquad \text{(A.4)}$$
$$\frac{dY}{dT} = (-\beta)\frac{1}{det(M)}[P'(p' - c_{yy} - \beta C_{yy}) - p' C_{yY}] \lesseqgtr 0 \qquad \text{(A.5)}$$
$$\frac{dy}{ds} = -\frac{1}{det(M)}[p'(P' - C_{YY}) - \beta P' C_{YY}] > 0 \qquad \text{(A.6)}$$
$$\frac{dY}{ds} = -\frac{1}{det(M)}[(p' + \beta P')C_{yY} - p'P'] \lesseqgtr 0 \qquad \text{(A.7)}$$

With the exception of $\frac{dy}{ds}$ all effects of climate policies on y and Y are ambiguous. To show part a) of Proposition 2, remember that from equation (10), we had

$$M \begin{pmatrix} dx \\ dy \\ dY \end{pmatrix} = \begin{pmatrix} dt - \beta dT \\ -ds \\ 0 \end{pmatrix}.$$

Therefore, we can write

$$dx = \underbrace{\frac{1}{det(M)} \Big[[p' - c_{yy} - \beta C_{yy}][P' - C_{YY}] - \beta C_{yY}^2 \Big]}_{<0} [dt - \beta dT]$$

and we can conclude that present emissions decrease as long as $dt > \beta dT = \frac{1}{1+r} dT$ and vice versa. Part b) of Proposition 2 was already shown in the main part.

Proof of Proposition 3

First, we derive equation (16). Totally differentiating equation (15) yields

$$P' dX + P' dY - dT = e_{\tilde{X}\tilde{X}}(\tilde{X}) d\tilde{X}. \tag{A.8}$$

Substituting $dX = d\tilde{X} - dx$ and solving for $d\tilde{X}$ leads to equation (16).

From the text, we had

$$\begin{aligned} \frac{dx}{db} =& \frac{1}{det(M')} \beta C_{yb} [\alpha \beta P' C_{yY} - p'(\alpha P' - C_{YY})] + \\ & \frac{1}{det(M')} C_{Yb} [\alpha \beta P' (p' - c_{yy} - \beta C_{yy}) - \beta p' C_{yY}]. \end{aligned}$$

Note first that $\frac{dx}{db}\big|_{\alpha=0} < 0$ and that the sign of $\frac{dx}{db}\big|_{\alpha=1}$ is ambiguous. Further, $\frac{dx}{db}$ depends linearly on α. Thus, if $\frac{dx}{db}\big|_{\alpha=1}$ was positive, there would be at most one cut-off value of α for which the sign of $\frac{dx}{db}$ changes. Moreover, if $\frac{dx}{db}\big|_{\alpha=1}$ was negative, no such cut-off

value would exist.

The effect of learning on the energy quantities y and Y is given by

$$\frac{dy}{db} = \frac{1}{det(M')}\Big[\beta C_{yb}[(p' + \alpha\beta P')(-C_{YY}) + \alpha p' P'] + C_{Yb}[-\alpha\beta p' P' + (p' + \alpha\beta P')\beta C_{yY}]\Big]$$
$$\frac{dY}{db} = \frac{1}{det(M')}\Big[\beta C_{yb}[-\alpha p' P' + (p' + \alpha\beta P')\beta C_{yY}] + C_{Yb}[(p' + \alpha\beta P')(p' - c_{yy} - \beta C_{yy}) - p' p']\Big]$$

where the sign of both differentials is ambiguous. However, for $\alpha \approx 0$, both differentials are positive. A higher learning factor decreases the energy price as

$$\begin{aligned}\frac{dp}{db} &= p'\left[\frac{dx}{db} + \frac{dy}{db}\right] \\ &= p'\frac{1}{det(M')}\Big[\beta C_{yb}[\alpha\beta P'(C_{yY} - C_{YY})] + C_{Yb}[\alpha\beta P'(\beta C_{yY} - c_{yy} - \beta C_{yy})]\Big] < 0.\end{aligned} \tag{A.9}$$

Further, a higher learning factor also reduces total emissions:

$$\frac{d\tilde{X}}{db} = \frac{P'}{P' - e_{\tilde{X}\tilde{X}}(\tilde{X})}\left(\frac{dx}{db} - \frac{dY}{db}\right) = (1-\alpha)\left(\frac{dx}{db} - \frac{dY}{db}\right) \tag{A.10}$$
$$\frac{d\tilde{X}}{db} = (1-\alpha)\frac{1}{det(M')}\Big[\beta C_{yb}[p' C_{YY} - p' C_{yY}] + C_{Yb}[p'(c_{yy} + \beta C_{yy} - \beta C_{yY})]\Big] < 0.$$

Proof of Proposition 4

We report the effect of all climate policies on the single energy quantities x, y and Y and on total emissions $\tilde{X}$. For the effect of s and t on $\tilde{X}$ we make use of equation (A.10).

Effect of Present Carbon Taxation

$$\frac{dx}{dt} = \frac{1}{det(M')}\Big[(p' - c_{yy} - \beta C_{yy})(\alpha P' - C_{YY}) - \beta C_{yY}^2\Big] < 0$$
$$\frac{dy}{dt} = \frac{1}{det(M')}\Big[\alpha\beta P' C_{yY} - p'(\alpha P' - C_{YY})\Big] \lesseqgtr 0$$
$$\frac{dY}{dt} = \frac{1}{det(M')}\Big[-p' C_{yY} + \alpha P'(p' - c_{yy} - \beta C_{yy})\Big] \lesseqgtr 0$$
$$\frac{d\tilde{X}}{dt} = (1-\alpha)\frac{1}{det(M')}\Big[(p' - c_{yy} - \beta C_{yy})(-C_{YY}) - \beta C_{yY}^2 + p' C_{yY}\Big] < 0$$

Effect of Renewable Subsidy

$$\frac{dx}{ds} = \frac{1}{det(M')}(-1)\Big[\alpha\beta P' C_{yY} - p'(\alpha P' - C_{YY})\Big] \lesseqgtr 0$$
$$\frac{dy}{ds} = \frac{1}{det(M')}(-1)\Big[(p' + \alpha\beta P')(-C_{YY}) + \alpha p' P'\Big] > 0$$
$$\frac{dY}{ds} = \frac{1}{det(M')}(-1)\Big[(p' + \alpha\beta P') C_{yY} - \alpha p' P'\Big] \lesseqgtr 0$$
$$\frac{d\tilde{X}}{ds} = (1-\alpha)\frac{1}{det(M')}(-1)\Big[p'[C_{YY} - C_{yY}]\Big] < 0$$

$$
\begin{aligned}
\frac{dx}{dT} =& \frac{1}{det(M')}(-\alpha\beta)[(p' - c_{yy} - \beta C_{yy})(\alpha P' - C_{YY}) - \beta C_{yY}^2] + \\
& \frac{1}{det(M')}(\alpha - 1)[-\beta p' C_{yY} + \alpha\beta P'(p' - c_{yy} - \beta C_{yy})] \lesseqgtr 0 \\
\frac{dy}{dT} =& \frac{1}{det(M')}(-\alpha\beta)[\alpha\beta P' C_{yY} - p'(\alpha P' - C_{YY})] + \\
& \frac{1}{det(M')}(\alpha - 1)[-\alpha\beta p' P' + (p' + \alpha\beta P')\beta C_{yY}] \lesseqgtr 0 \\
\frac{dY}{dT} =& \frac{1}{det(M')}(-\alpha\beta)[-p' C_{yY} + \alpha P'(p' - c_{yy} - \beta C_{yy})] + \\
& \frac{1}{det(M')}(\alpha - 1)[(p' + \alpha\beta P')(p' - c_{yy} - \beta C_{yy}) - p'p'] \lesseqgtr 0
\end{aligned}
$$

The effect of a future carbon tax on total emissions is given by

$$
\begin{aligned}
\frac{d\tilde{X}}{dT} =& (1 - \alpha)\left(\frac{dx}{dT} - \frac{dY}{dT}\right) + \frac{1}{P' - e_{\tilde{X}\tilde{X}}(\tilde{X})} \\
=& \frac{1}{det(M')(P' - e_{\tilde{X}\tilde{X}}(\tilde{X}))}\left[p'[C_{YY}(c_{yy} + \beta C_{yy}) - \beta C_{yY}^2]\right] + \\
& \frac{1}{det(M')(P' - e_{\tilde{X}\tilde{X}}(\tilde{X}))}\left[p'P'(\beta C_{yY} - c_{yy} - \beta C_{yy})\right] < 0.
\end{aligned}
$$

Simplifying $\frac{dx}{dT}$ yields

$$
\frac{dx}{dT} = \frac{1}{det(M')}\beta\left[\alpha\left[(C_{YY} - P')(p' - c_{yy} - \beta C_{yy}) + \beta C_{yY}^2 - p' C_{yY}\right] + p' C_{yY}\right] \quad \text{(A.11)}
$$

As can be seen $\frac{dx}{dT}\big|_{\alpha=0} < 0$ while $\frac{dx}{dT}\big|_{\alpha=1} > 0$. Moreover, $\frac{\partial}{\partial\alpha}\frac{dx}{dT} = \frac{1}{det(M')}\beta\big[(C_{YY} - P')(p' - c_{yy} - \beta C_{yy}) + \beta C_{yY}^2 - p' C_{yY}\big] > 0$ which guarantees the existence of exactly one cut-off value for $\alpha \in (0, 1)$ where $\frac{dx}{dT} < (>) 0$ if α is below (above) this value.

A.2. Functional Form and Parameter Values for Figure 3

For Figure 3 in the main part, we used the following functional forms and parameters:

Table 3: Functional Forms

Function	Functional Form
$c(y)$	$c(y) = dy + c/2y^2$
$C(y, Y, b)$	$C(y, Y, b) = (dY + c/2Y^2)\left(\frac{A+y}{A}\right)^{(-b)}$
$p(x + y)$	$p(x + y) = a(x + y)^{(-\eta)}$
$P(\bar{X} - x + y)$	$P(\bar{X} - x + y) = a(\bar{X} - x + y)^{(-\eta)}$

Table 4: Parameter Values

Parameter	c	d	A	a	$\bar{X}$	η	β
Value	0.1	0.5	40	10	20	1	0.5