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Working Paper
Risk Matters: A Comment

CESifo Working Paper, No. 4793

Provided in Cooperation with:
Ifo Institute – Leibniz Institute for Economic Research at the University of Munich

Suggested Citation: Born, Benjamin; Pfeifer, Johannes (2014) : Risk Matters: A Comment, CESifo Working Paper, No. 4793, Center for Economic Studies and Ifo Institute (CESifo), Munich

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Risk Matters: A Comment

Benjamin Born
Johannes Pfeifer

CESifo Working Paper No. 4793
Category 6: Fiscal Policy, Macroeconomics and Growth
May 2014

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Risk Matters: A Comment

Abstract

Jesús Fernández-Villaverde, Pablo A. Guerrón-Quintana, Juan F. Rubio-Ramírez and Martín Uribe (2011) find that risk shocks are an important factor in explaining emerging market business cycles. We show that their model needs to be recalibrated because it underpredicts the targeted business cycle moments by a factor of three once a time aggregation error is corrected. Recalibrating the corrected model for the benchmark case of Argentina, the peak response of output after an interest rate risk shock increases by 63 percent and the contribution of interest rate risk shocks to business cycle volatility more than doubles. Hence, risk matters more in the recalibrated model. However, the recalibrated model does worse in capturing the business cycle properties of net exports once an additional error in the computation of net exports is corrected.


Keywords: interest rate risk, stochastic volatility.

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Special thanks go to Antonio Ciccone and Juan Rubio-Ramírez. We are also grateful to Klaus Adam, Michael Evers, and Gernot Müller for very helpful suggestions and discussions. All remaining errors are of course our own.
Fernández-Villaverde et al. (2011) (FGRU subsequently) find that risk shocks – mean preserving spreads to shock distributions – are an important factor in explaining business cycles in emerging market economies. Their results and methods have already spurred further work on the role of risk shocks for macroeconomic fluctuation (e.g. Martin M. Andreasen (2012), Yusuf Soner Başkaya, Timur Hülăgü and Hande Küşük (2013), Susanto Basu and Brent Bundick (2012), Benjamin Born and Johannes Pfeifer (2013), Jesús Fernández-Villaverde, Pablo A. Guerrón-Quintana, Keith Kuester and Juan F. Rubio-Ramírez (2012), and Michael Plante and Nora Traum (2012)). To establish the importance of risk shocks in emerging market economies, FGRU use data for Argentina, Brazil, Ecuador, and Venezuela to calibrate a model of a small open economy subject to shocks to the level and volatility of interest rates. FGRU then report two sets of results for their calibrated model: The first set of results is used to gauge the success of the calibration by comparing six predicted moments – two of them untargeted – with the corresponding moments in the data (the untargeted moments being the volatility and cyclical properties of net exports). The second set shows the predicted response of main macroeconomic variables, like output and consumption, to risk shocks.

We argue that these results are affected by two coding issues. The first coding issue is in the time aggregation of flow variables from months to quarters. The correct time aggregation mechanically reduces the volatility of the main flow variables and the effect of risk shocks on these variables by a factor of three for any given calibration. However, as the volatilities of the main flow variables are targeted moments in the calibration, it is a priori unclear how the correct time aggregation changes the impact of risk shocks on aggregate variables once the model is recalibrated. When we recalibrate the corrected model for the benchmark case of Argentina, we actually find that risk shocks matter more for output: the peak effect of a risk shock on output turns out to be 63 percent higher than reported by FGRU.

A second coding issue is in the computation of net exports and affects the cyclicality and volatility of net exports. When we correct the computation, we find that net exports are predicted to be procyclical instead of approximately acyclical as reported in FGRU. This continues to be the case in the recalibrated and corrected model for the benchmark case of Argentina. Hence, the model fails to capture the empirically countercyclical behavior of
net exports documented in FGRU and the emerging market business cycle literature (see e.g. Mark Aguiar and Gita Gopinath, 2007; David K. Backus, Patrick J. Kehoe and Finn E. Kydland, 1992; Javier García-Cicco, Roberto Pancrazi and Martín Uribe, 2010; Pablo A. Neumeyer and Fabrizio Perri, 2005). Furthermore, the recalibrated corrected model predicts that net exports are substantially more volatile than in the data.

The rest of the paper proceeds as follows. Section I deals with the time aggregation and Section II with the computation of net exports. Section III concludes.1

I. Time Aggregation

A. Correct Time Aggregation Keeping the Model Calibration at the Values in FGRU

FGRU set up their model in monthly terms, but report results at quarterly frequency as most data are available at quarterly frequency only. They aggregate monthly output, consumption, investment, and hours worked to quarterly frequency by summing up monthly percentage deviations.2 For flow variables expressed in percentage deviation terms, the correct way to aggregate is to average the monthly values. For example, if monthly GDP is 100 in steady state, a one percent GDP deviation from steady state for one month corresponds to a deviation of one third of a percent (1/300) for quarterly GDP. The correct time aggregation mechanically reduces the predicted volatility of the main flow variables and the predicted effect of risk shocks on these variables by a factor of three for any given set of model parameters. The effect on the predicted volatility of output, consumption, and investment is illustrated in Table 1.3,4 The table shows the predicted moments reported in FGRU, the

1Minor points and technical descriptions of the algorithms used are relegated to the appendix. We use FGRU’s first stage estimates for the exogenous processes and the same notation, focusing on their benchmark case of Argentina with uncorrelated shocks (termed M1 in FGRU).

2Specifically, for the moment computations, the percentage deviations are from the deterministic steady state and for the impulse response functions (IRFs) from the ergodic mean in the absence of shocks (EMAS) (see Appendix A.A2 for details). We use the term EMAS for FGRU’s concept of “starting from the ergodic mean and in the absence of shocks” (p. 10 in their technical appendix). The EMAS is the fixed point of the third order approximated policy functions in the absence of shocks. Sometimes, it is referred to as the “stochastic steady state” (e.g. Michel Juillard and Ondra Kamenik, 2005), because it is the point of the state space where, in absence of shocks in that period, agents would choose to remain although they are taking future volatility into account.

3For Argentina, the results of FGRU could be exactly replicated due to FGRU’s computer code providing the pseudo-random number generator seed used. For the other countries, small differences are introduced by a different seed for the pseudo-random number generator. This explains why the relative volatilities do not stay exactly constant from the first to the second and the fourth to the fifth column.

4The effect of correct time aggregation keeping all model parameters at the values calibrated by FGRU on the impact of risk shocks on output, consumption, investment, and hours is illustrated in Appendix C.
Table 1—Effect of Correct Time Aggregation (TA) on Second Moments Keeping the Model Parameters at the Values Calibrated in FGRU

<table>
<thead>
<tr>
<th></th>
<th>Argentina</th>
<th>Ecuador</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>FGRU</td>
<td>Correct TA</td>
<td>Data</td>
<td>FGRU</td>
</tr>
<tr>
<td>$\sigma_Y$</td>
<td>5.30</td>
<td>1.77</td>
<td>4.77</td>
<td>2.23</td>
</tr>
<tr>
<td>$\sigma_C/\sigma_Y$</td>
<td>1.54</td>
<td>1.53</td>
<td>1.31</td>
<td>2.13</td>
</tr>
<tr>
<td>$\sigma_I/\sigma_Y$</td>
<td>3.90</td>
<td>3.90</td>
<td>3.81</td>
<td>9.05</td>
</tr>
<tr>
<td></td>
<td>Venezuela</td>
<td></td>
<td></td>
<td>Brazil</td>
</tr>
<tr>
<td></td>
<td>FGRU</td>
<td>Correct TA</td>
<td>Data</td>
<td>FGRU</td>
</tr>
<tr>
<td>$\sigma_Y$</td>
<td>4.56</td>
<td>1.47</td>
<td>4.72</td>
<td>4.52</td>
</tr>
<tr>
<td>$\sigma_C/\sigma_Y$</td>
<td>0.51</td>
<td>0.51</td>
<td>0.87</td>
<td>0.44</td>
</tr>
<tr>
<td>$\sigma_I/\sigma_Y$</td>
<td>3.81</td>
<td>3.89</td>
<td>3.42</td>
<td>1.67</td>
</tr>
</tbody>
</table>

Note: FGRU refers to moments reported in FGRU. Correct TA refers to the moments when time aggregation is corrected and structural parameters are kept at the values calibrated in FGRU. Data refers to the data moments obtained from HP-filtered data. Simulations are conducted with 200 repetitions of 96 periods using the FGRU pruning.

predicted moments when time aggregation is corrected but model parameters are kept at the values calibrated by FGRU, and the moments in the data. It can be seen that correct time aggregation implies that FGRU’s calibrated model underpredicts the data moments by a factor of three. As FGRU’s calibration method targets the moments for output, consumption, and investment, this implies that the model needs to be recalibrated.

B. Recalibrating the Model with Correct Time Aggregation

FGRU calibrate their model to monthly frequency and fix most parameters to either standard values in the literature or to match great ratios. Four remaining parameters, i) the standard deviation of TFP shocks $\sigma_x$, ii) the Lawrence J. Christiano, Martin Eichenbaum and Charles L. Evans (2005)-type investment adjustment costs parameter $\phi$, iii) the steady state debt level, $\bar{D}$, measured in output units, and iv) the holding costs of debt, $\Phi_D$, are chosen by a moment matching procedure that minimizes a quadratic form of the distance of the model moments to the data moments. The targets are four moments in quarterly data: i) output volatility, ii) the volatility of consumption relative to output, iii) the relative volatility of investment to output, and iv) the ratio of net exports over output. The net
Table 2—Parameters Obtained by Moment Matching

<table>
<thead>
<tr>
<th></th>
<th>$\Phi_D$</th>
<th>$\bar{D}$</th>
<th>$\phi$</th>
<th>$\sigma_x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recalibration</td>
<td>5.92e-04</td>
<td>18.80</td>
<td>47.84</td>
<td>0.040</td>
</tr>
<tr>
<td>FGRU</td>
<td>1.00e-03</td>
<td>4.00</td>
<td>95.00</td>
<td>0.015</td>
</tr>
</tbody>
</table>

*Note:* first row: parameters obtained by moment matching to reflect the changes detailed in Section I. Second row: parameters obtained by moment matching in FGRU.

Table 3—Targeted Moments of the Recalibrated Corrected Model

<table>
<thead>
<tr>
<th></th>
<th>$\sigma_Y$</th>
<th>$\sigma_C/\sigma_Y$</th>
<th>$\sigma_I/\sigma_Y$</th>
<th>$\tilde{N}X/\tilde{Y}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recalibration</td>
<td>4.77</td>
<td>1.31</td>
<td>3.81</td>
<td>1.78</td>
</tr>
<tr>
<td>Data</td>
<td>4.77</td>
<td>1.31</td>
<td>3.81</td>
<td>1.78</td>
</tr>
<tr>
<td>FGRU</td>
<td>5.30</td>
<td>1.54</td>
<td>3.90</td>
<td>1.75</td>
</tr>
</tbody>
</table>

*Note:* first row: moments obtained from simulating the recalibrated corrected model 200 times for 96 periods using the same pruning, simulation, filtering, and winsorizing scheme as FGRU. Second row: Moments obtained from HP-filtered Argentinean data (1993Q1 - 2004Q3). Third row: moments reported in FGRU.

exports share in output differs from the other three moments as it is not targeted at the ergodic mean (obtained by simulating with random shocks), but at the ergodic mean in the absence of shocks, $\tilde{N}X/\tilde{Y}$, which is obtained by simulating without any shocks until convergence. Our calibration follows the moment matching approach in FGRU and also uses their pruning and simulation scheme together with the same winsorized shocks.

Table 2 reports the resulting parameter estimates, while Table 3 shows the moments of the recalibrated model. Table 3 shows that the moment matching is successful: choosing the four parameters allows to exactly match the four moments. From Table 2 it can be seen that moment matching using the corrected model implies a volatility of TFP shocks that is 2.7 times the volatility in FGRU. As documented in FGRU, TFP shocks alone do not result in a sufficient response of investment and consumption to match their volatility.

5There is also a minor coding issue in the computation of the net exports to output share at the EMAS in FGRU that we correct. See Appendix A.A4 for details.

6See Appendix D for details.

7In the corrected FGRU model, the output volatility was only 1.77 percent compared to 4.77 percent in the data. Given the fixed first stage estimates for the interest rate processes, the required 2.69 fold increase in output volatility is achieved by increasing TFP volatility by almost exactly this amount. In quarterly terms, our new estimates correspond to a TFP shock volatility of 5.5 percent and a TFP volatility of 12.3 percent.
relative to output. Given that the amount of interest volatility is fixed by FGRU’s first stage estimates of the exogenous processes, the transmission mechanism has to adjust. This is achieved by a halving of the investment adjustment and portfolio holding costs, which brings the investment adjustments costs to a more conventional level.\(^8\) The portfolio holding cost parameter is now estimated to be even closer to the value of \(\Phi_D = 4.2e^{-4}\) found in Martín Uribe and Vivian Z. Yue (2006) for a panel of emerging economies. The steady state debt level more than quadruples relative to FGRU. But given the strong non-linearities in the model, this only results in around twice the debt level in the ergodic mean.\(^9\)

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\(^8\)For reference, Christiano, Eichenbaum and Evans (2005) estimate a value of \(\phi = 2.48\) for the US.

\(^9\)In the ergodic mean, this corresponds to an annual debt to GDP ratio of 12 percent compared to 6 percent in FGRU (see Appendix F). According to Carmen M. Reinhart and Kenneth Rogoff (2009), the actual value of Argentinean external debt was 65 percent during the sample considered here.
Figure 1 depicts the IRFs for the recalibrated model together with the original IRFs reported in FGRU for the case of Argentina. As can be seen, a one standard deviation risk shock now leads to a 63 percent larger output drop than originally reported in FGRU. This is mostly driven by a bigger response of consumption and investment due to the deleveraging caused by the now higher foreign debt becoming more risky. The optimal deleveraging is stronger compared to the original FGRU calibration, because of the lower estimated investment adjustment and portfolio holding costs.

<table>
<thead>
<tr>
<th>Table 4—Variance Decomposition: Argentina</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
</tr>
<tr>
<td>------</td>
</tr>
<tr>
<td>$\sigma_y$</td>
</tr>
<tr>
<td>$\sigma_c$</td>
</tr>
<tr>
<td>$\sigma_i$</td>
</tr>
</tbody>
</table>

Note: first column: moments obtained from HP-filtered Argentinean data (1993Q1 - 2004Q3); second column: moments from 200 simulations of the recalibrated model; third column: TFP shocks only; fourth column: without volatility shocks to spread and T-Bill rate; fifth column: only level shocks to the spread and the T-Bill rate; sixth column: without TFP shocks; seventh column: only shocks to the volatility of spreads and the T-Bill rate; eighth column: variance decomposition for volatility shocks only reported in FGRU.

To judge the importance of risk shocks for business cycle moments, it is instructive to consider a variance decomposition. Due to the non-linearity of the model and the resulting interaction of shocks, such a variance decomposition cannot be performed analytically. One way to gauge the relative importance of shocks is simulating the model with only a subset of the shocks. We follow FGRU and consider six cases: i) with all shocks, ii) using only TFP shocks while shutting off both level and volatility shocks to the interest rate in the form of the T-bill rate and the risk spread, iii) using only TFP and level shocks to the interest rate, iv) using only the level shocks to the interest rate, v) level and volatility shocks to the interest rate and no TFP shocks, vi) only volatility shocks. Table 4 shows the results for the recalibrated model. The last column also shows the variance for the “volatility shocks only” case reported in FGRU. Compared to FGRU, the contribution of volatility shocks to the standard deviation of output, consumption, and investment increases by factors of...
2.6, 2.4, and 2 in the recalibrated model, respectively (see last column). Volatility shocks alone account for 10 percent of output volatility and one third of investment volatility in the recalibrated corrected model.

II. The Cyclicality and Volatility of Net Exports

A. Net Exports Keeping the Structural Parameters at the Values Calibrated in FGRU

FGRU compute the quarterly absolute deviation of net exports from the deterministic steady state in their model solution using the national income accounting identity based on quarterly output, consumption, and investment. The formula they end up using is:\textsuperscript{10}

\begin{equation}
NX_t - \overline{NX} = \hat{Y}_t - \hat{C}_t - \hat{I}_t,
\end{equation}

where hats denote percentage deviations from the deterministic steady state and bars denote steady state values, that is:

\begin{equation}
\hat{Y}_t = \frac{Y_t - \bar{Y}}{\bar{Y}}, \quad \hat{C}_t = \frac{C_t - \bar{C}}{\bar{C}}, \quad \hat{I}_t = \frac{I_t - \bar{I}}{\bar{I}}.
\end{equation}

The correct formula should have weighted the percentage deviations of output, consumption, and investment by their respective steady state values:

\begin{equation}
NX_t - \overline{NX} = \bar{Y} \hat{Y}_t - \bar{C} \hat{C}_t - \bar{I} \hat{I}_t.
\end{equation}

Table 5 reports the volatility and cyclicality of net exports reported in FGRU and the volatility and cyclicality of net exports when the time aggregation error and the error in the computation of net exports are corrected but the structural parameters are kept at the values calibrated in FGRU. There are two main differences between the results reported by FGRU and the results in the corrected model.\textsuperscript{11} First, net exports turn from approximately acyclical in FGRU to procyclical in the corrected model. The procyclicality of net exports

\textsuperscript{10}See Appendix A.A3 for details.

\textsuperscript{11}The weighting issue also affects the current account implications after risk shocks reported in Figure 6 of FGRU. An updated version of the figure can be found in Appendix B.
Table 5—Net Exports Keeping the Structural Parameters at the Values Calibrated in FGRU

<table>
<thead>
<tr>
<th></th>
<th>Argentina</th>
<th>Ecuador</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>FGRU</td>
<td>TA</td>
</tr>
<tr>
<td>(\rho_{NX,Y})</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>(\sigma_{NX}/\sigma_Y)</td>
<td>0.48</td>
<td>1.43</td>
</tr>
</tbody>
</table>

Venezuela Brazil

<table>
<thead>
<tr>
<th></th>
<th>FGRU</th>
<th>TA</th>
<th>TA+NX</th>
<th>Data</th>
<th>FGRU</th>
<th>TA</th>
<th>TA+NX</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\rho_{NX,Y})</td>
<td>-0.10</td>
<td>-0.10</td>
<td>0.47</td>
<td>-0.11</td>
<td>0.18</td>
<td>0.17</td>
<td>0.78</td>
<td>-0.26</td>
</tr>
<tr>
<td>(\sigma_{NX}/\sigma_Y)</td>
<td>1.60</td>
<td>13.33</td>
<td>1.87</td>
<td>0.18</td>
<td>0.60</td>
<td>1.95</td>
<td>3.87</td>
<td>0.18</td>
</tr>
</tbody>
</table>

Note: first and fifth column: moments reported in FGRU. Second and sixth column: moments obtained using the FGRU simulation, but correcting the time aggregation (TA). Third and seventh column: moments obtained using the FGRU simulation, but correcting the time aggregation and net export computation (TA+NX). Fourth and eighth column: moments obtained from HP-filtered data. Simulations are conducted with 200 repetitions of 96 periods using the FGRU pruning scheme. For Argentina, the same set of pseudo-random numbers as in FGRU was used, while the simulation for the other countries had to rely on a different pseudo-random number generator seed.

in the corrected model contrasts with mostly countercyclical net exports in the data.\(^{12}\) The reason why net exports become procyclical when the computation of net exports is corrected is that equation (3) puts a relatively larger weight on output fluctuations than equation (1) due to output in steady state being greater than both consumption and investment.\(^{13}\) This increase in the weight of the “output component” of net exports mechanically increases the comovement of net exports with output and hence increases the procyclicality.

The second main difference between the results reported by FGRU and the results in the corrected model is that the corrected model predicts a different volatility of net exports (see Table 5). To understand the source of this difference, it is useful to start with the benchmark case of Argentina. The relative volatility of net exports reported in FGRU is 0.48. Correcting time aggregation, the relative net export volatility increases by a factor of three to 1.43, because the time aggregation error only affects output volatility (FGRU obtain

\(^{12}\) Explaining this behavior has motivated prominent papers arguing for the importance of permanent TFP shocks (e.g. Aguiar and Gopinath, 2007) and financial frictions (e.g. García-Cicco, Pancrazi and Uribe, 2010). Andrea Raffo (2008) has argued that GHH-preferences might be instrumental in getting the correct correlation. Adopting the latter may be a way to improve the model fit of FGRU.

\(^{13}\) In equation (1) all three components entered with an equal weight of 1, while equation (3) in the FGRU-model implies relative weights of 1, 0.84, and 0.13, for output, consumption, and investment, respectively.
the volatility of net exports directly at quarterly frequency). Correcting the computation of net exports leads to a minor further change in relative volatility. As a result, the corrected relative volatility for the benchmark case of Argentina is approximately 3.4 times the value reported in FGRU.

From Table 5 it can also be seen that for Ecuador, Venezuela, and Brazil, the difference between the corrected relative volatility of net exports and the relative volatility reported in FGRU is sometimes larger than for the benchmark case of Argentina and sometimes smaller. The reason turns out to be the poor numerical convergence behavior of FGRU’s measure of net export volatility. Because net exports can be negative, instead of using a log-linear approximation, FGRU use the Isabel Correia, Joao C. Neves and Sergio Rebelo (1995)-approximation of the net exports

\[ \overline{N_X}_t \equiv \frac{N_X_t}{\left| \text{mean}(N_X_t) \right|} - 1. \]

This formula takes the percentage deviations from the absolute value of the mean in order to preserve the sign.\(^{14}\) If the mean of net exports is close to 0, this can have drawbacks in short simulations where the mean is imprecisely estimated.

This issue is illustrated in Figure 2. On the vertical axis the figure displays the volatility of net exports relative to the volatility of output for the benchmark case of Argentina (blue solid line), computed with the Correia, Neves and Rebelo (1995)-approximation as in FGRU but correcting the time aggregation error affecting output volatility and the error in the computation of net exports. The horizontal axis displays the number of simulation repetitions over which the relative volatility has been computed. As in FGRU, each repetition is based on a simulation of 96 time periods. The black arrow marks 200 simulations, which is the number of simulation repetitions that FGRU use to obtain the predicted relative volatility of net exports reported in Table 2. The figure suggests the simulated value of \( \sigma_{N_X}/\sigma_Y \) varies substantially with the number of repetitions, even when we go to more than the

\(^{14}\)Although this expression is already in the same units as output deviations from steady state, FGRU additionally divide by a factor of 100. Thus, the relative volatility of net exports, \( \sigma_{n_x}/\sigma_y \) is not 0.39 for Argentinean data and 0.48 for the model as reported in FGRU, but 39 and 48, respectively. Subsequently, we report the values in the form originally reported in FGRU.
**Figure 2. Convergence Behavior of the Relative Net Export Volatility Statistic**

*Note:* relative volatility of net exports to output $\sigma_{NX}/\sigma_Y$ for the case of Argentina. Net exports transformed to percentage deviations using the Correia, Neves and Rebelo (1995)-approximation. The blue solid line shows the mean standard deviation (y-axis) over the up to 10,000 samples (x-axis) of simulating 96 months of data. The black dashed dotted line shows the actual data moments. The data are based on the corrected aggregation and net export computation. The black arrow indicates the value after 200 replications.

200 repetitions used by FGRU.\(^{15}\) Because of this instability, the seed of the pseudo-random number generator may matter a lot.\(^{16}\) As we know the seed FGRU use for Argentina, but not for Ecuador, Venezuela, and Brazil, this numerical instability drives a further wedge between the relative volatility for Ecuador, Venezuela, and Brazil reported in FGRU and the corrected relative volatilities we report in Table 5.

**B. Net Exports in the Recalibrated Corrected Model**

Table 6 reports the volatility and cyclicality of net exports. The recalibrated corrected model continues to predict a positive correlation (0.43) of net exports with output. Regarding the volatility of net exports we find that the recalibrated corrected model overpredicts the relative volatility of net exports to the volatility of output by a factor of 2.8. However, as FGRU measure net exports using the Correia, Neves and Rebelo (1995)-approximation, which we have found to be numerically unstable in Figure 2, this may reflect a numerical issue. In the last column of Table 5 we therefore also examine the volatility of the net exports to output ratio, $NX_t/Y_t$. Due to this ratio being scaled with output, it does not suffer from

\(^{15}\)The convergence problem is driven by the numerator $\sigma_{NX}$, while the denominator $\sigma_Y$ converges quickly. Moreover, the convergence problem only affects the volatility of net exports, but not its correlation with output.

\(^{16}\)This behavior can also be seen in FGRU’s official replication code. Changing the pseudo-random number seed from the 2 they used to 20, leaving aggregation, net export computation, simulation length, and the number of replications unchanged, leads to a tripling of $\sigma_{NX}/\sigma_Y$ from 0.48 to 1.46.
Table 6—Untargeted Moments in the Recalibrated Corrected Model

<table>
<thead>
<tr>
<th></th>
<th>$\rho_{NX,Y}$</th>
<th>$\sigma_{NX}/\sigma_Y$</th>
<th>$\rho_{NX/Y,Y}$</th>
<th>$\sigma_{NX/Y}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recalibration</td>
<td>0.43</td>
<td>1.08</td>
<td>0.41</td>
<td>6.55</td>
</tr>
<tr>
<td>Data</td>
<td>-0.76</td>
<td>0.39</td>
<td>-0.75</td>
<td>3.47</td>
</tr>
<tr>
<td>FGRU</td>
<td>0.05</td>
<td>0.48</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

*Note:* first row: moments obtained from simulating the recalibrated corrected model 200 times for 96 periods using the same pruning, simulation, filtering, and winsorizing scheme as FGRU. Second row: Moments obtained from HP-filtered Argentinean data (1993Q1 - 2004Q3). Third row: moments reported in FGRU, based on time aggregation error and wrong net export computation.

It can be seen that with this measure of the volatility of net exports, the model prediction is closer to the data than with the Correia, Neves and Rebelo (1995)-measure used by FGRU. But the predicted volatility is still almost twice the volatility of the net exports to output ratio in the data. From columns 1 and 3 in Table 6 it can be seen that for the cyclical properties of net exports it does not matter much which measure of net exports is used.

### III. Conclusion

FGRU find that risk shocks have important effects on aggregate variables and might contribute to explaining the current account movements of small open developing economies. We noted an error in the time aggregation of flow variables that results in the calibrated model not matching the targeted data moments. Correcting this error and recalibrating the model to fit the volatilities of output, consumption, and investment, we find that the peak effect of a risk shock on output increases by 63 percent. The business cycle contribution of volatility shocks also increases by more than a factor of two.

We also pointed to a weighting error in the net export computation that leads to reported acyclical net exports when net exports are actually procyclical in the corrected model. We found that net exports continued to be procyclical in the recalibrated model, thus being at odds with one of the most robust stylized facts in emerging markets business cycles.

17 Appendix E documents the better convergence behavior of this measure. Using one long simulation for the Correia, Neves and Rebelo (1995)-approximation instead of averaging over many short ones is no alternative. It does not allow for capturing small sample biases potentially present in the data and, due to the particular pruning scheme and simulation scheme used in FGRU, leads to results that are not comparable to the short simulations. See Appendix D for details.
REFERENCES


Appendix A documents the coding issues in the Matlab implementation of the model simulation in more detail by showing the associated computer code of the Fernández-Villaverde et al. (2011) (FGRU) replication files posted at http://www.aeaweb.org/articles.php?doi=10.1257/aer.101.6.2530. FGRU calibrate their model at monthly frequency, perform a variable substitution to obtain a log-linearization, and use third-order perturbation techniques to simulate the model. We make use of the third order perturbation capacities of Dynare (Stéphane Adjemian, Houtan Bastani, Frédéric Karamé, Michel Juillard, Junior Maih, Ferhat Mihoubi, George Perendia, Johannes Pfeifer, Marco Ratto and Sébastien Villemot, 2011) to simulate the model. Appendix B presents the corrected version of Figure 6 in FGRU. Appendices C and D document the simulation and pruning schemes used for IRF generation and moment computation. Appendix E compares the numerical convergence behavior of the standard deviation of the Correia, Neves and Rebelo (1995)-approximation and of the net export to output ratio. Appendix F compares the deterministic steady state, the EMAS, and the ergodic mean.

A. THE CODING ISSUES IN THE PUBLISHED REPLICATION FILES

A1. VARIABLE SUBSTITUTION AND CALIBRATION TO MONTHLY FREQUENCY

As can be seen in Listing 1, a variable substitution is performed to obtain log-linearized decision rules.

```
Listing 1 - THE MODEL IN emerging.nb

1 func1 = (Exp[c])^-ups - Exp[[Lambda]];  
2 func2 = betabeta*Exp[[Lambda][p] - Exp[[Lambda]]/(1 + Exp[r]) +  
3   Exp[[Lambda]] (dp - ds) dtheta;  
4 func3 = -Exp[pi] +  
5   betabeta ((1 - delta)*Exp[pi] +  
6       alpha Exp[yp]/Exp[kp] Exp[[Lambda][p]]);  
7 func4 = thetheta (Exp[h])^(1

The resulting policy functions are identical up to the 8th digit to the ones derived from Mathematica by FGRU.

Listing 1 and 2 are Mathematica code, all others Matlab.
```
\[
\omega + 1) - (1 - \alpha) \exp[y] \exp[\lambda] + 
\exp[\pi] (1 - \phi/2 (\exp[\text{invest}] / \exp[\text{invest lag}] - 1)^2 - 
\phi \exp[\text{invest}] / 
\exp[\text{invest lag}] (\exp[\text{invest}] / \exp[\text{invest lag}] - 1)) + 
\beta \beta \ast \exp[\pi] \phi (\exp[\text{invest p}] / \exp[\text{invest}] - 1)^2 \exp[\text{inv}est]; 
\]

Listing 2 shows that the decision rules are for a model calibrated to monthly frequency \((\bar{r} = 0.02)\).

Listing 2 - The Model Calibration in **emerging.nb**

```math
\text{parmrule2} = \{ \omega -> 1000, \text{dtheta} -> 0.001, \text{thettheta} -> 1, \text{ups} -> 5, 
\text{delta} -> 0.014, \alpha -> 0.32, \phi -> 95, \text{rlib} -> 0.02, \text{ecap} -> 0, 
\text{rhosigmar} -> 0.94, \text{rhor} -> 0.97, \text{sigmag} -> \log[0.015], 
\text{eeta} -> 0.46, \text{meansigmar} -> -5.71, 
\text{rhosigmatb} -> 0.94, \text{rhotb} -> 0.95, \text{eetb} -> 0.13, \text{rhog} -> 0.95, 
\text{meansigmatb} -> -8.06\}; 
```

A2. Time Aggregation: Moments and IRFs

Listing 3 - Variable Aggregation in **irf_moments.m**

```math
xaux = x_c'; 
xauxtrimestral=zeros(size(xaux,1)/3,3); 

% Transform simulation from monthly to quarterly 
for i=1:3; 
    for j=1:size(xauxtrimestral,1); 
        xauxtrimestral(j,i)=sum(xaux((j-1)*3+1:3*j,i)); 
    end 
end 
```
% c_d order: consump invest output
% Compute net exports using transformation in Correia, Neves,
% Rebelo: Business Cycle in Small Open Economies (European Economic
Review, 1995)
net_exp = xauxtrimestral(:,3) - xauxtrimestral(:,2) - xauxtrimestral
(:,1) ;
net_exp = (net_exp/abs(mean(net_exp)) - 1)/100 ;

% HP filter data
% NOTE: HPF computes HP trend
%       Please use your preferred HP filter in the next lines
for i = 1:3
c_d(:,i) = xauxtrimestral(:,i) - HPF(xauxtrimestral(:,i),1600) ;
end
c_d(:,4) = net_exp - HPF(net_exp,1600) ;

Line 206 of Listing 3 assigns the matrix of simulated control variables at monthly frequency
\( x_c \) (with the first three entries being consumption, investment, and output) into \( x_{aux} \). The
loop in lines 210-214 then aggregates monthly percentage deviations from steady state into
quarterly data for output, consumption, and investment (contained in the columns of \( x_{aux} \)).
Instead of averaging the percentage deviations of these flow variables, they are accumulated.
As lines 226-230 show, these are the quarterly variables that are HP-filtered and used to
compute the moments.

**Listing 4 - IRF aggregation in irf_moments.m**

```matlab
% Unit of time in model is month
% IRFs are accumulated to transform to quarters
figure
for i = 1:nc-3
    if i == 6 % Average quarterly debt
        aux = 100/(bss+xlast(6))*(x_c(i,2:Tplot)-xlast(i)*ones(1,Tplot-1)) ;
    end
    for j = 1:(Tplot-2)/3
```
Listing 4 shows the time aggregation from monthly model IRFs to the quarterly IRFs reported in FGRU. As can be seen in lines 313-319, the stock of debt is first expressed as a percentage deviation from the EMAS (line 314) and then aggregated by taking the mean response over three subsequent quarters: the subsequent three months are summed up (line 316) and then divided by 3 before plotting (line 318). Similarly, lines 321-325 take the interest rate spread $\varepsilon_{r,t}$, average it and multiply it by $12 \times 100 \times 100 = 120,000$ to transform it into annualized basis points.

Line 328 computes the difference between the log of the monthly model variables (output, consumption, investment, and hours) and the EMAS times 100, which has the interpretation of a percentage deviation. Lines 329 to 331 sum up the monthly percentage deviations over the quarter, before line 332 plots them without dividing by three as was the case in line 318. As a result, the percentage deviations from steady state of the flow variables like output,
consumption, investment, and hours are overstated by a factor of three when plotting the IRFs.

To see this, consider e.g. quarterly output $Y_q$ as the sum of the three monthly outputs $Y_{m,i}$:

(A1) \[ Y_q = Y_{m,1} + Y_{m,2} + Y_{m,3} \, . \]

Performing a log-linearization around the deterministic steady state yields:

(A2) \[ \hat{Y}_q \hat{Y}_q = \hat{Y}_m \hat{Y}_{m1} + \hat{Y}_m \hat{Y}_{m2} + \hat{Y}_m \hat{Y}_{m3} \, , \]

where bars denote steady state values and hats percentage deviations from steady state. Divide by $\hat{Y}_q = 3\hat{Y}_m$ to obtain quarterly percentage deviations:

(A3) \[ \hat{Y}_q = \frac{\hat{Y}_m}{\hat{Y}_q} \hat{Y}_{m1} + \frac{\hat{Y}_m}{\hat{Y}_q} \hat{Y}_{m2} + \frac{\hat{Y}_m}{\hat{Y}_q} \hat{Y}_{m3} = \frac{1}{3} \left( \hat{Y}_{m1} + \hat{Y}_{m2} + \hat{Y}_{m3} \right) \, . \]

Thus, the mean of the percentage deviations from steady state is appropriate, not the sum.

A3. Computing Net Exports

Line 219 of Listing 3 computes net exports as

(A4) \[ NX_t - \overline{NX} = \hat{Y}_t - \hat{C}_t - \hat{I}_t \, , \]

where $\hat{X}_t \approx \frac{X_t - \overline{X}}{X}$ denotes a variable in percentage deviations from steady state obtained from summing up the percentage deviations (line 212 of Listing 3). The correct approximation is

(A5) \[ NX_t - \overline{NX} = Y \hat{Y}_t - \overline{Y} \hat{C}_t - \overline{I} \hat{I}_t \, . \]

Line 220 performs a Correia, Neves and Rebelo (1995)-approximation of the net exports:

(A6) \[ \overline{NX_t} = \frac{NX_t}{\text{mean}(NX_t)} - 1 \, , \]

but additionally divides by 100. For consistency reasons, the same is done when reporting the
empirical standard deviation (see line 36 of Listing 5). This implies that both the empirical
and theoretical moments for net exports are underreported by a factor of 100 in the paper.

Listing 5 - Net Export display in EMPIRICAL MOMENTS.m

disp('Moments Argentina: vol c/vol y vol invt/vol y vol net export/vol y')
[std(cd) std(id) std(nd)/100]/std(yd)

A4. Computing the Net Exports Share

Line 247 of Listing 6 computes the net exports to output share from the national income
accounting identity:

\[(A7)\quad NX_t = Y_t - C_t - I_t = D_t - \frac{D_{t+1}}{1+r_t} + \frac{\Phi D}{2} (D_{t+1} - \bar{D})^2\]

at the EMAS as

\[(A8)\quad \frac{\tilde{N}X}{\bar{Y}} = \frac{\left[\bar{D} + \left(\bar{D} - \tilde{D}\right)\right]^{\frac{r-1}{r}}}{e^{\log(\bar{Y}) + (\log(\bar{Y}) - \log(Y))}} ,\]

where the respective deviations of the EMAS from the deterministic steady state are stored
in xlast. But in the EMAS \(\bar{D} \neq \tilde{D}\). Thus, the adjustment cost term in equation (A7) is not
zero. As a consequence, the permanent portfolio holdings costs paid at the EMAS are not
accounted for when computing the net exports required to finance the debt stock. For the
original calibration this coding issue is inconsequential due to the low debt adjustment costs.
But when recalibrating the model, the debt holding costs need to be taken into account as
one cannot know a priori if they are substantial.

Listing 6 - Net-Export Share Calibration in IRF MOMENTS.m

% 2.2.1 Compute moments in Table 7, Column M1

disp('Ratio net exports/output')
((bss+xlast(6))*(irate-1)/irate)/exp(adyss+xlast(3)))
B. Figure FGRU6: IRFs Debt/Output, Current Account, Net Exports

Figure 6 in FGRU, reproduced here as Figure B1 due to non-availability of replication codes, depicts the responses of the debt to output ratio, the current account, and net exports to a risk shock.

![Debt/output, Current account, Net exports](image)

**Figure B1. IRFs Debt/Output, Current Account, Net Exports**

*Note: Reproduced Figure 6 from Fernández-Villaverde et al. (2011), p. 2553.*

FGRU report the debt to output ratio IRF in that figure not in percentage deviations from the EMAS, but as the absolute value. Figure B2 displays that the debt to output ratio dropped from about 0.293 by about 2.9 percentage points to 0.263 after 11 periods. But this is inconsistent with the FGRU-IRFs in Figure C1, which show that debt dropped by 3.873 percent while output dropped by 0.1907 percent. Thus, the new debt-to-output-ratio should be up to first order \((1 - 0.03873)D / ((1 - 0.001907)Y) \approx 0.963D / Y\), i.e. it should drop by about 3.7 percent (not percentage points). The net export IRF is affected by from the incorrect weighting used in their computation as shown in section II. Regarding the current account, FGRU state that it is in “percentage points of [its] ergodic mean”. But this is not possible for it is defined as \(CA_t = D_t - D_{t-1}\) and thus has ergodic mean zero. Thus, it is unclear what the lower left panel of Figure B2 depicts.

The left column of Figure B2 shows the corrected version of Figure 6 in FGRU that uses output to normalize net exports and the current account, giving them the interpretation of
being in output units. As can be seen, in the corrected model with the original calibration the current account implications of risk shocks are rather muted.

The right column of Figure B2 displays the IRFs from the recalibrated corrected model quantifying the central “debt reduction mechanism”. After a one standard deviation risk shock, the debt to quarterly output ratio falls by about 6.5 percentage points after three years, while current account and net exports increase by about 1 percent of output on impact in order to finance the deleveraging.
C. IRFs at the Ergodic Mean

C1. IRF Generation

The use of higher-order perturbation techniques to solve the model implies that the model solution is not linear anymore. Thus, the IRFs will depend on both the sequence of future shocks, $u_t$, and the point in the state space at which the IRFs are started, i.e. the past history of shocks, $\Omega_t$. To circumvent this problem, Gary Koop, M. Hashem Pesaran and Simon M. Potter (1996) suggested the concept of Generalized Impulse Response Functions (GIRFs) that e.g. allow considering “representative” IRFs at the ergodic mean. The GIRF at time $t + n$ after a shock $u_t$ is given by

\[(C1) \quad GIRF_n (u_t, \Omega_{t-1}) = \mathbb{E}[Y_{t+n}|u_t, \Omega_{t-1}] - \mathbb{E}[Y_{t+n}|\Omega_{t-1}],\]

that is, given a point in the state space, the future shock realizations are averaged out.

In contrast, FGRU also condition on future shocks by setting them to 0 when generating their IRFs and start the IRFs at the EMAS. Denote the future realization of shocks with $\Omega_{fut}^t$. FGRU effectively use the definition

\[(C2) \quad IRF_n (\nu_t, \Omega_t) = \mathbb{E}[Y_{t+n}|u_t, \Omega_{t-1} = \{\ldots, 0\}, \Omega_{fut}^{t+1} = \{0, \ldots\}] - \mathbb{E}[Y_{t+n}|0, \Omega_{t-1} = \{\ldots, 0\}, \Omega_{fut}^{t+1} = \{0, \ldots\}],\]

where the expected values can be dropped as everything is deterministic.

This choice of computing the IRFs at the EMAS has two important implications. First, computing the non-linear IRFs not as the expected difference in responses as in (C1) but also conditioning on future shocks and setting them to 0, only allows capturing part of the economic effects of risk shocks. To see this, inspect the particular pruning algorithm\(^{20}\) used

\(^{20}\) As first noted in Jinill Kim, Sunghyun Kim, Ernst Schaumburg and Christopher A. Sims (2008), higher order perturbation solutions tend to explode due to the accumulation of terms of increasing order. For example, in a second order approximated solution, the quadratic term at time $t$ will be raised to the power of two in the quadratic term at $t + 1$, thus resulting in a quartic term, which will become a term of order 8 at $t + 2$ and so on. As a solution, Kim et al. (2008) proposed “pruning” all terms of higher order, i.e. computing the quadratic term at $t + 1$ by only squaring the first-order term from time $t$. This procedure, however, is not easily generalized to third order as there are several potential ways of pruning.
in FGRU for IRF-generation.\textsuperscript{21} Consider a generic model solution of the form

\begin{equation}
\begin{aligned}
x_t &= g \left( x_{t-1}^{\text{states}}, u_t, \sigma \right),
\end{aligned}
\end{equation}

where $x_t$ is an $n_x \times 1$ vector of endogenous variables, $x_{t-1}^{\text{states}}$ is the vector of states contained in $x_t$,\textsuperscript{22} $u_t$ is an $n_u \times 1$ vector of mean zero disturbances, and $\sigma$ is the perturbation parameter. Denote partial derivatives with subscripts. The pruned third order solution for the endogenous variables’ deviations from their steady state, $\hat{x}_t^{3rd} = x_t^{3rd} - \bar{x}$, used by FGRU, is computed from the recursion

\begin{equation}
\begin{aligned}
\hat{x}_t^{3rd} &= g_x \hat{x}_{t-1}^{3rd, \text{states}} + g_u u_t \\
&+ \frac{1}{2} \left[ g_{xx} \left( \hat{x}_{t-1}^{1st, \text{states}} \otimes \hat{x}_{t-1}^{1st, \text{states}} \right) + 2 g_{xu} \left( \hat{x}_{t-1}^{1st, \text{states}} \otimes u_t \right) + g_{uu} \left( u_t \otimes u_t \right) + g_{\sigma \sigma} \sigma^2 \right] \\
&+ \frac{1}{6} \left[ g_{xxx} \left( \hat{x}_{t-1}^{1st, \text{states}} \otimes \hat{x}_{t-1}^{1st, \text{states}} \otimes \hat{x}_{t-1}^{1st, \text{states}} \right) + g_{xuu} \left( u_t \otimes u_t \otimes u_t \right) \\
&+ 3 g_{xxx} \sigma^2 \hat{x}_{t-1}^{1st, \text{states}} + 3 g_{xuu} \sigma^2 u_t \right]
\end{aligned}
\end{equation}

\begin{equation}
\hat{x}_t^{1st} = g_x \hat{x}_{t-1}^{1st, \text{states}} + g_u u_t.
\end{equation}

That is, all higher order terms are based on the first-order terms.\textsuperscript{23} The recursion in equations (C4)-(C5) is completed by an initial condition\textsuperscript{24} of:

\begin{equation}
\hat{x}_0^{3rd} = \tilde{x} - \bar{x}
\end{equation}

\begin{equation}
\hat{x}_0^{1st} = 0.
\end{equation}

Because $\hat{x}_0^{1st} = 0$ and all higher order terms in equation (C4) are based on it, the effect

\textsuperscript{21}The IRF-pruning scheme differs from the scheme used for simulations, see Appendix D.

\textsuperscript{22}We use the Dynare notation that stacks the state transition and observation equations (see Adjemian et al., 2011).

\textsuperscript{23}This choice results in an inferior performance compared to e.g. the pruning scheme by Martin M. Andreasen, Jesús Fernández-Villaverde and Juan F. Rubio-Ramírez (2013) that augments the state space to keep track of first to third order terms and uses the Kronecker product of the first and second order terms to compute the third order term (see Hong Lan and Alexander Meyer-Gohde, 2013b, for more details).

\textsuperscript{24}As shown in Lan and Meyer-Gohde (2013b) there are infinitely many different past shock realizations that can lead to being at a particular point in the state-space at time 0, all of them associated with particular values for $\hat{x}_0^{3rd}$ and $\hat{x}_0^{1st}$. Equations (C6) to (C7) are consistent with the EMAS in that one particular shock combination giving rise to these values is the total absence of past shocks.
of the initial condition $\bar{x} - \bar{x}$ will mostly be neglected. Equation (C5) effectively is a first-order policy function, which is known not to react to risk shocks, except for the state $\sigma_{t-1}$. Considering (C4), this and the conditioning on all shocks being 0 $\forall t + i$, $i > 0$ implies that, in the terminology of Hong Lan and Alexander Meyer-Gohde (2013a), only the “risk adjustment channel” is present (via the constant term $1/2 \times g_{\sigma \sigma} \times \sigma^2$ and the time-varying risk-adjustment $1/2 \times g_{u \sigma \sigma} \times \sigma^2 \times u_t$ in period $t$ where $u_t \neq 0$). But the difference in “amplification effects” introduced by (risk) shocks and embedded in the other higher order terms is totally absent. Thus, the difference in the interaction between the location in the state space and future shocks, introduced by the risk shock, is not captured.

Second, the IRFs are computed at a particular point in the pruned state space where agents factor in the uncertainty of the system, but where there has been an infinite absence of shocks. Due to the absence of shocks and thus of “amplification effects” embedded in the higher order terms, agents will dare to incur a relatively high amount of debt. As shown in Table F1, the difference between the EMAS and the unconditional mean amounts to 20 percent.\(^{25}\)

\textit{C2. IRF Generation}

Figure C1 compares the responses after a one-standard deviation interest rate risk shock reported in FGRU (red dashed lines) with the responses when the time aggregation error is corrected (blue solid lines). It can be seen that correcting the time aggregation error mechanically results in the size of the shock response dropping to one third of the value reported in FGRU. For example, instead of dropping by 0.19 percent, output falls by a 0.06 percent at its trough.

\(^{25}\)An alternative would be to compute GIRFs at the true ergodic mean using the methods proposed in Andreasen, Fernández-Villaverde and Rubio-Ramírez (2013).
Figure C1. Comparison of Quarterly IRFs for Different Aggregation Schemes.

Note: IRFs to a one-standard deviation shock to interest rate risk premium uncertainty. Blue solid line: correct aggregation by averaging percentage deviations of monthly flow variables; red dashed line: aggregation by summing up monthly percentage deviations of flow variables.
D. Starting Simulations at the Ergodic Mean in the Absence of Shocks

The simulations conducted in FGRU use a different pruning scheme than the IRF-generation. Denote the time periods of the simulations with \( t = 1, \ldots, 96 \), the simulation repetition with \( i = 1, \ldots, 200 \), and a generic variable with \( y_{t,i} \).

1) At time \( t = 1 \)
   - set the third order term of the states \( x_{1,i}^{3rd,states} \) to the EMAS and the non-state elements of \( x_{1,i}^{3rd} \) to the deterministic steady state
   - set \( u_{2,1}^{1st} = 0 \)

   If \( i = 1 \)
   - set the first-order term \( x_{1,i}^{1st} \) to the deterministic steady state
   - set the shock term used in the first-order term to \( u_{2,i}^{1st} = 0 \)
   - draw a random shock vector \( u_{2,1} \)

   else if \( i \neq 1 \)
   - set the first-order state term \( x_{1,i}^{1st,states} \) to \( x_{96,i-1}^{1st,states} \)
   - set the shock term used in the first-order term to \( u_{2,i}^{1st} = u_{97,i-1}^{1st} \)
   - set \( u_{2,i} = u_{2,1} \)

2) for \( t = 2 \) to \( 96 \):
   - Use the unpruned state space representation to compute the time \( t \) values of the exogenous state variables
   - To compute the time \( t \) values of the endogenous states, use the recursion

   \[
   \hat{x}_{t,i}^{3rd} = g_x \hat{x}_{t-1,i}^{3rd,states} + g_u u_{t,i}^{1st} \\
   + \frac{1}{2} \left[ g_{xx} (\hat{x}_{t-1,i}^{1st,states} \otimes \hat{x}_{t-1,i}^{1st,states}) + 2 g_{xu} (\hat{x}_{t-1,i}^{1st,states} \otimes u_{t,i}^{1st}) + g_{uu} (u_{t,i}^{1st} \otimes u_{t,i}^{1st}) + g_{\sigma \sigma} \sigma^2 \right]
   \]

   (D1)

   \[
   + \frac{1}{6} \left[ g_{xxx} (\hat{x}_{t-1,i}^{1st,states} \otimes \hat{x}_{t-1,i}^{1st,states} \otimes \hat{x}_{t-1,i}^{1st,states}) + g_{xuu} (u_{t,i}^{1st} \otimes u_{t,i}^{1st} \otimes u_{t,i}^{1st}) + g_{\sigma \sigma} \sigma^2 \right]
   \]

   (D2)

   \[
   \hat{x}_{t,i}^{1st} = g_x \hat{x}_{t-1,i}^{1st,states} + g_u u_{t,i}^{1st}
   \]
• Draw a random shock vector $u_{t+1,i}$
• Set $u_{t+1,i}^{1st} = u_{t+1,i}$
• Use $\hat{x}_{t,3}^{3rd}$ as the simulated variable

Four things are noteworthy. First, the simulations for the exogenous laws of motion for TFP, the T-bill rate, the country risk premium, and the two volatility processes do not use pruning. They are instead based on iterating the full third-order approximated policy function forward. This seems to pose no practical problems in the simulations we conducted as we encountered no explosive behavior. But using the full higher-order polynomial approximation to the true stationary exogenous law of motion implies that the stability properties of the underlying policy function are not necessarily inherited (see e.g. Wouter J. Den Haan and Joris De Wind, 2012). Thus, the exogenous laws of motion may suffer from exactly the problem for which using a pruning algorithm was advocated. Second, the actual simulations only start at time $t = 2$, because for $t = 1$ the endogenous variables are assumed to be at the deterministic steady state. Nevertheless, this first time point with zero deviations from steady state is included in the 96 time periods used to compute simulated moments. As the simulated system will on average transition to the ergodic mean, this introduces an initial jump from $t = 1$ to $t = 2$, which even the subsequent HP-filtering will not completely smooth out. Third, for the first actual simulation period, i.e. $t = 2$, the simulated first and third order terms are based on different structural shocks, $u_{t,i}^{1st}$ and $u_{2,i}$, respectively. Hence, agents in the model are assumed to react to two different shock realizations at the same time. Fourth, the first shock $u_{2,i}$ at $t = 2$ is always equal to the one of the first simulation, i.e. $u_{2,1}$.

One important implication of this particular simulation scheme is that due to starting at the EMAS for the third order term and then hitting the equilibrium system with shocks, the simulations will slowly transition to the ergodic distribution. As the simulations are always restarted at this point after 96 periods and there is no burnin, most draws will not yet come from the ergodic distribution. Put differently, the moments from 10,000 simulations of 96 periods and the ones from one simulation of 960,000 periods considerably differ, as shown in Table D1.
### Table D1—Second Moments of Long vs. Short Simulations

<table>
<thead>
<tr>
<th></th>
<th>Argentina</th>
<th></th>
<th>Ecuador</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Short Sim.</td>
<td>Long Sim.</td>
<td>Data</td>
<td>Short Sim.</td>
</tr>
<tr>
<td>$\sigma_Y$</td>
<td>4.77</td>
<td>1.78</td>
<td>2.06</td>
<td>2.46</td>
<td>0.73</td>
</tr>
<tr>
<td>$\sigma_C/\sigma_Y$</td>
<td>1.31</td>
<td>1.52</td>
<td>1.72</td>
<td>2.48</td>
<td>2.22</td>
</tr>
<tr>
<td>$\sigma_I/\sigma_Y$</td>
<td>3.81</td>
<td>4.06</td>
<td>5.56</td>
<td>9.32</td>
<td>9.89</td>
</tr>
<tr>
<td>$\sigma_{NX}/\sigma_Y$</td>
<td>0.39</td>
<td>4.72</td>
<td>5.80</td>
<td>0.65</td>
<td>2.41</td>
</tr>
<tr>
<td>$\rho_{NX,Y}$</td>
<td>-0.76</td>
<td>0.41</td>
<td>0.38</td>
<td>-0.60</td>
<td>0.24</td>
</tr>
<tr>
<td>$\bar{NX}/\bar{Y}$</td>
<td>1.78</td>
<td>1.75</td>
<td>1.75</td>
<td>3.86</td>
<td>3.95</td>
</tr>
</tbody>
</table>

|                | Venezuela |             | Brazil  |             |            |
|                |           | Short Sim.  | Long Sim.| Data        | Short Sim. | Long Sim.  |
| $\sigma_Y$     | 4.72      | 1.51        | 1.70    | 4.64        | 1.50       | 1.67       |
| $\sigma_C/\sigma_Y$ | 0.87      | 0.51        | 0.51    | 1.10        | 0.45       | 0.46       |
| $\sigma_I/\sigma_Y$ | 3.42      | 3.94        | 4.57    | 1.65        | 1.73       | 2.16       |
| $\sigma_{NX}/\sigma_Y$ | 0.18      | 0.34        | 0.31    | 0.23        | 6.23       | 2.00       |
| $\rho_{NX,Y}$  | -0.11     | 0.45        | 0.37    | -0.26       | 0.77       | 0.71       |
| $\bar{NX}/\bar{Y}$ | 4.07      | 4.14        | 4.14    | 0.10        | 0.52       | 0.52       |

*Note:* first and fourth column: moments obtained from HP-filtered data. Second and fifth column: moments of the FGRU model with corrected aggregation and net export computation, based on 10,000 replications of 96 periods. Third and sixth column: moments of the FGRU model with corrected aggregation and net export computation, based on 1 replication of 960,000 periods.
E. Convergence Behavior of the Net Exports to Output Ratio

Figure E1. Convergence Behavior of Different Net Export Volatility Statistics in the Recalibrated Model

Note: Top panel: relative volatility of net exports to output $\sigma_{NX}/\sigma_Y$. Net exports transformed to percentage deviations using the Correia, Neves and Rebelo (1995)-approximation. Bottom panel: standard deviation of the net exports to output ratio $\sigma_{NX/Y}$. The blue solid line shows the mean standard deviation (y-axis) over the up to 10,000 samples (x-axis) of simulating 96 months of data. The black dashed dotted line shows the actual data moments. The data are based on the corrected aggregation and net export computation. The black arrow indicates the value after 200 replications.
F. Steady State, EMAS, and Ergodic Mean

Table F1—Steady State, EMAS, and Ergodic Mean: FGRU Calibration

<table>
<thead>
<tr>
<th></th>
<th>Argentina</th>
<th></th>
<th>Ecuador</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Steady State</td>
<td>EMAS</td>
<td>Erg. Mean</td>
<td>Steady State</td>
</tr>
<tr>
<td>D</td>
<td>4.000</td>
<td>2.551</td>
<td>2.090</td>
<td>13.000</td>
</tr>
<tr>
<td>C</td>
<td>0.878</td>
<td>0.888</td>
<td>0.905</td>
<td>0.945</td>
</tr>
<tr>
<td>H</td>
<td>-0.004</td>
<td>-0.004</td>
<td>-0.004</td>
<td>-0.004</td>
</tr>
<tr>
<td>Y</td>
<td>1.051</td>
<td>1.049</td>
<td>1.056</td>
<td>1.196</td>
</tr>
<tr>
<td>I</td>
<td>-0.975</td>
<td>-0.982</td>
<td>-0.969</td>
<td>-0.523</td>
</tr>
<tr>
<td>NX/Y</td>
<td>0.027</td>
<td>0.018</td>
<td>0.005</td>
<td>0.043</td>
</tr>
<tr>
<td>CA</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Venezuela</th>
<th></th>
<th>Brazil</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Steady State</td>
<td>EMAS</td>
<td>Erg. Mean</td>
<td>Steady State</td>
</tr>
<tr>
<td>D</td>
<td>22.000</td>
<td>21.422</td>
<td>21.445</td>
<td>3.000</td>
</tr>
<tr>
<td>K</td>
<td>4.002</td>
<td>4.009</td>
<td>4.010</td>
<td>4.001</td>
</tr>
<tr>
<td>C</td>
<td>0.982</td>
<td>0.985</td>
<td>0.985</td>
<td>1.030</td>
</tr>
<tr>
<td>H</td>
<td>-0.004</td>
<td>-0.004</td>
<td>-0.004</td>
<td>-0.004</td>
</tr>
<tr>
<td>Y</td>
<td>1.278</td>
<td>1.280</td>
<td>1.280</td>
<td>1.278</td>
</tr>
<tr>
<td>I</td>
<td>-0.267</td>
<td>-0.260</td>
<td>-0.265</td>
<td>-0.267</td>
</tr>
<tr>
<td>NX/Y</td>
<td>0.043</td>
<td>0.041</td>
<td>0.041</td>
<td>0.006</td>
</tr>
<tr>
<td>CA</td>
<td>0.000</td>
<td>0.000</td>
<td>-0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Note: first column: deterministic steady state, second column: ergodic mean in the absence of shocks (EMAS); third column: theoretical mean based on the third-order pruned state space of Andreasen, Fernández-Villaverde and Rubio-Ramírez (2013). D, NX/Y, and CA are reported in levels, while all other variables are in logs. The model is at monthly frequency.

Table F1 implies that $D/Y_{annual} = 2.09/(12 \times exp(1.056)) \approx 0.0606$. In the recalibrated model, $D/Y_{annual} = 4.251/(12 \times exp(1.106)) \approx 0.1172$. 
### Table F2—Steady State, EMAS, and Ergodic Mean: Recalibration

<table>
<thead>
<tr>
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<th>Argentina</th>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Steady State</td>
<td>EMAS</td>
<td>Ergodic Mean</td>
</tr>
<tr>
<td>$D$</td>
<td>18.802</td>
<td>3.595</td>
<td>4.251</td>
</tr>
<tr>
<td>$K$</td>
<td>3.294</td>
<td>3.461</td>
<td>3.463</td>
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<tr>
<td>$C$</td>
<td>0.750</td>
<td>0.933</td>
<td>0.919</td>
</tr>
<tr>
<td>$H$</td>
<td>-0.003</td>
<td>-0.004</td>
<td>-0.004</td>
</tr>
<tr>
<td>$Y$</td>
<td>1.052</td>
<td>1.105</td>
<td>1.106</td>
</tr>
<tr>
<td>$I$</td>
<td>-0.975</td>
<td>-0.807</td>
<td>-0.860</td>
</tr>
<tr>
<td>$NX/Y$</td>
<td>0.129</td>
<td>0.018</td>
<td>0.022</td>
</tr>
<tr>
<td>$CA$</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

*Note:* first column: deterministic steady state, second column: ergodic mean in the absence of shocks (EMAS); third column: theoretical mean based on the third-order pruned state space of Andreasen, Fernández-Villaverde and Rubio-Ramírez (2013). $D$, $NX/Y$, and $CA$ are reported in levels, while all other variables are in logs. The model is at monthly frequency.