Ognedal, Tone

Working Paper

Morale in the market

Memorandum, No. 04/2014

Provided in Cooperation with:
Department of Economics, University of Oslo

Suggested Citation: Ognedal, Tone (2014) : Morale in the market, Memorandum, No. 04/2014, University of Oslo, Department of Economics, Oslo

This Version is available at:
http://hdl.handle.net/10419/102058

Terms of use:
Documents in EconStor may be saved and copied for your personal and scholarly purposes.
You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.
If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.
M E M O R A N D U M

No 04/2014

Morale in the Market

Tone Ognedal

Department of Economics
University of Oslo
## Last 10 Memoranda

<table>
<thead>
<tr>
<th>No</th>
<th>Authors</th>
<th>Title</th>
</tr>
</thead>
<tbody>
<tr>
<td>03/14</td>
<td>Paolo Giovanni Piacquadio</td>
<td>Intergenerational Egalitarianism</td>
</tr>
<tr>
<td>02/14</td>
<td>Martin Flatø and Andreas Kotsadam</td>
<td>Drought and Gender Bias in Infant Mortality in Sub-Saharan Africa</td>
</tr>
<tr>
<td>01/14</td>
<td>Yngve Willassen</td>
<td>Optimal Migration and Consumption Policies over an Individual’s Random Lifetime</td>
</tr>
<tr>
<td>28/13</td>
<td>Olav Bjerkholt</td>
<td>Promoting Econometrics through Econometrica 1933-37</td>
</tr>
<tr>
<td>27/13</td>
<td>Trygve Haavelmo</td>
<td>Variations on a Theme by Gossen</td>
</tr>
<tr>
<td>26/13</td>
<td>Halvor Mehlum</td>
<td>Samfunnsøkonomiens plass i jussen - Det juridiske fakultets første hundre år.</td>
</tr>
<tr>
<td>25/13</td>
<td>Halvor Mehlum</td>
<td>Samfunnsøkonomiens plass i jussen - Det juridiske fakultets første hundre år.</td>
</tr>
<tr>
<td>24/13</td>
<td>Eric Nævdal and Jon Vislie</td>
<td>Resource Depletion and Capital Accumulation under Catastrophic Risk: Policy Actions against Stochastic Thresholds and Stock Pollution</td>
</tr>
<tr>
<td>23/13</td>
<td>Steinar Holden and Nina Larsson Midthjell</td>
<td>Successful Fiscal Adjustments: Does choice of fiscal instrument matter?</td>
</tr>
<tr>
<td>22/13</td>
<td>Ragnar Nymoen and Victoria Sparrman</td>
<td>Equilibrium Unemployment Dynamics in a Panel of OECD Countries</td>
</tr>
</tbody>
</table>

Previous issues of the memo-series are available in a PDF® format at: http://www.sv.uio.no/econ/english/research/memorandum/
Morale in the Market*

Tone Ognedal, University of Oslo †

Memo 04/2014-v1
(This version February 5, 2014)

Abstract

There is a growing interest in morale as a potential substitute for sanctions, encouraged by experimental evidence that people’s morale affect their economic decisions. I show that while morale may be a substitute for sanctions for each citizen, it is not a substitute in the market. In a model where employed and self-employed differ in their opportunities for tax evasion, I demonstrate that a higher fraction of tax compliant citizens may reduce social surplus and tax revenues. In contrast to sanctions, morale usually differ between individuals and this distorts the ranking of costs among sellers and willingness to pay among consumer. Tax evading sellers crowd out tax compliant sellers with higher productivity. Tax evading buyers crowd out tax compliant buyers with higher willingness to pay. As a result, improved tax morale may lead to less efficient production and exchange. Experiments show how sanctions crowd out morale in some settings. My paper points to the opposite problem in markets: Low sanctions may crowd out morale. While the paper explores the effects of tax morale only, the results apply to a wide range of areas where morale matters for peoples choices in he market, such as environmental and safety regulation.

Keywords: Tax morale, Tax evasion, Norms, Sanctions
JEL classifications: H26,K42, D01

*I am grateful for comments from Vidar Christiansen, Kalle Moene, Fred Schroyen and Gaute Torsvik. This paper is part of the research activities at Oslo Fiscal Group and the ESOP centre, both at the Department of Economics, University of Oslo.

†Department of Economic, University of Oslo, PB 1096 Blindern, 0317 Oslo Norway, e-mail: tone.ognedal@econ.uio.no.
1 Introduction

Morale matters for individual economic decisions. This is now confirmed by a series of experimental studies. Most people are for instance willing to sacrifice some economic gains for more fairness. Since preventing people from violating the law by sanctions alone is expensive, one may wonder whether morale to some extent can substitute for sanctions. This paper claims that it cannot. Even in situations where morale and sanctions are close substitutes for disciplining bad individual market decisions, they are far from close substitutes for disciplining bad market outcomes that reflect the decisions of many individuals.

Below I substantiate this claim by exploring the effects of tax morale in situations where individuals choose between occupations with different opportunities to evade taxes. I focus on a branch of industry, showing how better morale sometimes induce permissive market outcomes. More tax compliant producers can increase the total production cost and more honest consumers can reduce the efficiency of exchange. As a result, improved morale may reduce social surplus and tax incomes.

People differ in their entrepreneurial talent, i.e. in their productivity as self-employed relative to employed. Since tax evasion acts like an implicit tax relief for self-employed only, it increases the private profitability of self-employment relative to employment. As a result, too many people become self-employed. Sanctions, such as expected penalties, reduce the private profitability of self-employment relative to employment, and therefore brings the number of self-employed closer to its optimal level. Morale also brings output closer to its optimal level but it may also distort the allocation of production between sellers and the allocation between buyers.

A key to understand why morale may have this negative effect is the observation that while sanctions are the same for all, morale differ between individuals (Cappelen et al., 2007). Therefore sanctions do not change the ranking of profitability between sellers and the willingness to pay between buyers. The different morale concerns, in contrast, changes the ranking of sellers and of buyers by creating differences in private gains that do not correspond to differences in social gains. Sellers with low productivity and low tax morale may crowd out sellers with higher productivity but also higher tax morale. Customers with low willingness to pay but low tax morale may crowd out customers with higher willingness to pay but lower tax morale. Although higher tax morale may bring the output closer to its optimal level, it may be less efficiently produced.

Survey studies confirm that peoples tax morale differ, both in strength and motivation. A large fraction of citizens say they find tax evasion unjustifiable. This is no surprise, since tax evasion means lying and violating the law, both actions that most people think are wrong. Also, tax evasion is condemned as a sin by many religious authorities. However, as with most moral issues, peoples’ ideals may differ from their

---

1For example, when the World Value Survey 1990 asked whether tax evasion could be justified, 56 percent answered no in Sweden and 70 in Italy.

2For example, the Roman Catholic Church categorizes tax evasion as a sin. The net-site “Christianity.about.com” sites several places in the Bible as evidence for their claim that “So the gospels leave no doubt that Jesus taught his followers not only in words, but by example, to give to the government any
actions: In surveys in Norway (2001 and 2003), almost a third of the 69 percent that find tax evasion unjustifiable say they are willing to evade (Barth et al, 2008). Moreover, peoples’ tax morale is affected by policy variables such as fairness of the tax system (Fortin and Villeval, 2007 and Barth et al, 2013), the use of tax incomes (Alm et al, 1993), the treatment by the tax authorities (Feld and Frey, 2001) and trust in government (Cummings et al, 2009). Since peoples perceptions of fairness and the other policy variables differ, their tax morale also differ.

What motivates people to pay their taxes belongs to the main issues discussed in the literature on tax compliance Experiments in a general setting indicate that non-selfish motives, such as reciprocity and fairness, matters (Fehr and Fischbacher, 2002, Cappelen et al, 2010). Yet, the effects of appeals to peoples’ tax morale is mixed. In a randomized field experiment, Blumenthal et al (2001) found no effects of letters to taxpayers with morale appeals, while Bott et al (2013) find considerable effects in a similar experiments.\footnote{Both experiments had two types of letters: One stressed the beneficial use of tax revenues, the other appealed to the fact that most taxpayers report their income and wealth correctly.}

The premise of my paper is that morale may constrain tax evasion and I explore what happens in the market when some are tax compliant while others are not.

It is well known that sanctions may crowd out altruism and morale behavior (Gneezy and Rustichini, 2000, Fehr and Rockenbach 2003, Tirole and Benabou 2006). Frey (1997) suggests that extrinsic motivation of expected penalties may crowd out peoples intrinsic motivation to pay taxes. My paper focus on the opposite problem of how lack of sanctions may crowd out agents with intrinsic motivation, and lower the average morale in those markets where morale matters. Reducing the profitability of tax evaders by sanctions, I claim, makes it easier for tax compliant sellers to survive in the market.

Section 2 presents the model and provides some basic intuition for why morale may not increase welfare. Section 3 derives the cost of producing in a market where the sellers’ moral costs differ, while customers have no morale. I show how an increased fraction of tax compliant citizens may increase the cost of producing. In section 4 the cost function is used to derive the effects of tax morale on social surplus and tax revenues. Section 5 explores the effects of tax morale among both buyers and sellers. Section 6 concludes the paper.

2 Morale in the market

Each individual chooses between being a self-employed entrepreneur in the market for a service, or an employee in another market. While the self-employed report their own incomes, the incomes of most employees are reported by their employer. In line with this, I assume that all self-employed have the opportunity to evade taxes, but no employees have this opportunity. All employees earn the same wage rate $w$, exogenous in the model, but the incomes of the self-employed differ because their productivities as entrepreneurs
differ. Each individual supplies one unit of labor.

Let \( a_i \) be the units produced by a self-employed individual, i.e. \( a_i \) is his productivity as self-employed. The price per unit of the service is \( q \), determined by demand equals supply. All incomes are taxed at rate \( t \). With no tax evasion, a persons net income as self-employed is \( (1 - t)q a_i \) and his net income as an employee is \( (1 - t)w \). Maximizing income, he chooses self-employment if and only if \( w/a_i \leq q \), i.e. if his opportunity cost \( w/a_i \) does not exceed the output price in self-employment, \( q \). The opportunity cost is lower the higher his productivity as self-employed, \( a_i \). The tax does not distort the choice between employment and self-employment: Those with productivity above the critical level \( w/q \) become self-employed, independent of the tax rate.

Let \( \lambda \) be the fraction of income that a self-employed hides from the tax authorities, for convenience also referred to as his evasion. With probability \( p(\lambda) \) he is detected and pays a penalty tax rate \( \tau \) on the evaded income. I assume that the probability of detection is increasing and convex in the fraction evaded, i.e. \( p'(\lambda) > 0 \) and \( p''(\lambda) > 0 \).

The expected income of a self-employed that evades a fraction \( \lambda \) is then

\[
y_i(\lambda) = qa_i[(1 - t) + \lambda(t - p(\lambda)\tau)]
\]  

(1)

The last term in the square bracket is the taxes saved from evasion minus the expected penalty, i.e. the expected net gain from tax evasion per dollar earned.

With no moral constraints, the self-employed evades the fraction that maximizes the expected income. The optimal fraction evaded, denoted \( \lambda^* \), is determined by the first order condition

\[
t - p(\lambda^*)\tau - \lambda^* p'(\lambda^*)\tau = 0
\]  

(2)

Clearly, \( \lambda^* \) is a function of \( t \) and \( \tau \), increasing in \( t \) and decreasing in \( \tau \). With no tax morale, all self-employed evade a fraction \( \lambda^* \).

An individual with no tax morale \( i \) chooses to become self-employed if and only if his maximized net expected income from self-employment, \( y_i(\lambda^*) \), exceeds the net income from employment, \( (1 - t)w \). Inserting for \( y_i(\lambda^*) \) from (1) and defining

\[
\alpha = \frac{1 - t}{1 - t + \lambda^*[t - p(\lambda^*)\tau]} < 1
\]  

(3)

we can write the condition for self-employment as:

\[
\alpha(\lambda^*) \frac{w}{a_i} \leq q
\]  

(4)

Recall that the condition for self-employment with no evasion is \( \frac{w}{a_i} \leq q \). Tax evasion lowers the private opportunity cost of self-employment from \( w/a_i \) to \( \alpha w/a_i \). Since the social opportunity cost of self-employment is still \( w/a_i \), tax evasion distorts people’s choice between employment and self-employment. Evasion acts like a tax relief for self-employment only, increasing its private profitability relative to employment.

\(^4\)The crucial assumption is that individuals differ in their relative productivity, i.e. that they have comparative advantage as self-employed or employed.

4
Consider now the case where some of the citizens have a tax morale that constrain their evasion as sellers. A fraction $h$ is "honest" and report their entire income for taxation, while the "dishonest" evade the optimal fraction $\lambda^*$. From above, dishonest citizens become self-employed if $\alpha(\lambda^* w/a_i \leq q$ and honest citizens if $w/a_i \leq q$. Since an honest citizen has higher private opportunity cost of self-employment than a dishonest one with the same productivity, the honest one needs a higher output price to become self-employed.

Let $x$ denote the total supply of output from self-employment, referred to simply as output or supply. Since honest and dishonest citizens face different opportunity costs of self-employment, total supply does not only depends on the output price but also on the fraction of honest citizens. If all citizens are honest ($h = 1$) the total supply of self-employed services is given by

$$S(q) = \int_{w/q}^{\bar{a}} adF(a) \quad (5)$$

where $S'(q) > 0$ for values of $q$ such that $dF(w/q)/dq > 0$. The supply at price $q$ if all sellers are dishonest is the same as if all sellers were honest and faced a price $q/\alpha$. Thus, total supply if all citizens are dishonest ($h = 0$) is given by

$$S(q/\alpha) = \int_{aw/q}^{\bar{a}} adF(a) \quad (6)$$

The total supply when a fraction $h$ is honest and the rest dishonest is can be written as

$$x(q, h) = hS(q) + (1 - h)S(q/\alpha) \quad (7)$$

Supply is increasing in $q$ since $S'(.,.) > 0$, and decreasing in $h$ since $S(q/\alpha) > S(q)$. In the notation of equation (6), $x_q(q, h) > 0$ and $x_h(q; h) < 0$.

From (6), we can write the supply curve as $q(x, h)$, the price that induces output $x$ when a fraction $h$ of the citizens are honest. The supply curve if all citizens are honest is then $q(x, 1) = S^{-1}(x)$ and the supply curve if no one is honest is $q(x, 0) = \alpha S^{-1}(x)$. Figure 1 illustrates the supply curves for $h = 0$, $h = 1$ and for $0 \leq h' \leq 1$. As $h'$ increases, the curve $q(x, h')$ moves up towards the curve for the case with no evasion. The figure also shows how the price varies with $h$ for a given output $\bar{x}$. The price is $q(\bar{x}, 0)$ if no one is honest, $q(\bar{x}, h)$ if all citizens are honest and $q(\bar{x}, h')$ in between, when a fraction $0 < h' < 1$ is honest.

Let $D(q)$ be the demand for the output from self-employment, where $D'(q) \leq 0$. Demand equal to supply gives

$$x(q; h) = D(q) \quad (8)$$

\[5\]In this section I assume that buyers do not know or do not care about the evasion. In markets where each transaction is small, such as with payments to hairdressers or thrift stores, this is a reasonable assumption.
This determines the price as a function of the fraction of honest citizens, i.e. \( \tilde{q} = \tilde{q}(h) \).
Since \( x_h(q; h) < 0 \) and \( D'(q) < 0 \), the equilibrium price is increasing in \( h \), i.e. \( \tilde{q}'(h) > 0 \).
The equilibrium output is \( x(\tilde{q}(h); h) = D(\tilde{q}(h)) \equiv \hat{x}(h) \). Since \( D'(q) < 0 \) and \( \tilde{q}'(h) > 0 \), \( \hat{x}'(h) < 0 \). Thus, a higher fraction of honest citizens brings the output closer to its optimal level.

With perfectly elastic demand, the effect of tax morale is straightforward: For a given \( q \), tax evasion does not crowd out sellers with tax morale, but simply makes some socially unprofitable sellers survive. As more citizens become honest, socially unprofitable sellers leave the market but no profitable ones. Output moves closer to its optimal level \( \hat{x}(1) \), tax evasion goes down and tax revenue goes up. Moral unambiguously improves allocation.

With perfectly elastic supply, the effect of tax morale is also straightforward. Supply is perfectly elastic if all entrepreneurs have the same productivity \( a \). All those who are willing to evade taxes supply at a price equal to their common opportunity cost \( \alpha(\lambda^*)w/a \).
Since honest entrepreneurs need a higher price to survive, they do not survive as long as there is enough tax evading sellers to satisfy the demand, i.e. as long as demand equals supply at price \( \alpha(\lambda^*)w/a \). A higher fraction of honest sellers has no effect, as they are simply replaced with tax evading sellers. Since the productivity as self-employed is the same for all, however, this replacement does not affect the output or the cost of producing. Thus, morale has no effect in the market.

With downward sloping demand and upward sloping supply curves, however, honest individuals do affect the market outcome. Moreover, the effect is not necessarily desirable. While morale still reduces the distortion from too many self-employed, it creates another distortion: The ranking of the sellers’ private opportunity costs differs from the ranking of their social opportunity costs. This implies that it is no longer those with the highest entrepreneurial productivity who become self-employed. As a result, output is not efficiently produced. In the next section I show how the cost of producing varies with morale.

3 The cost of moral differences

To study the cost of morale in a market, it is useful to derive the market cost function. More specific, I derive the cost of producing \( x \) in a market when a fraction \( h \) of the citizens are honest, and investigate how these costs vary with \( h \).

At the price \( \tilde{q} \), honest sellers supply \( hS(\tilde{q}) \) units and dishonest sellers supply \( hS(\tilde{q}/\alpha) \).
The price that induces output \( \bar{x} \) when a fraction \( h \) is honest is then given by \( \bar{x} = hS(q) + (1 - h)S(q/\alpha) \). The social marginal cost of producing \( \bar{x} \) is the price that induces \( x \) if all citizens are honest, \( q(\bar{x}, 1) = S^{-1}(\bar{x}) \). The cost of producing \( \bar{x} \) in a market where a fraction \( h \) of the citizens are honest and the rest evade a fraction \( \lambda^* \) is then

\[
c(\bar{x}; h) = h \int_0^{S(q(\bar{x}, h))} S^{-1}(x)dx + (1 - h) \int_0^{S(q(\bar{x}, h)/\alpha)} S^{-1}(x)dx \quad (9)
\]

Since \( S^{-1}(S(q)) = q \), the marginal cost of supply from honest producers is \( q \), while the
marginal cost from dishonest producers is \( q/\alpha \).

I can show a non-monotonous relationship between costs and tax morale: The cost of producing a given output is increasing in \( h \) for \( h = 0 \) and decreasing in \( h \) for \( h = 1 \). Production of a given output is efficient if and only if all or none of the citizens are honest, i.e. \( c(x, 0) = c(x, 1) \). For a wide class of supply functions, the cost as a function of \( h \) has only one maximum. For example, this holds with constant elasticity of supply and with any linear supply function. Houthakker (1955) showed that with Pareto-distributed labor use per unit output, \( 1/a_i \), the production function for the industry is a Cobb-Douglas-type, i.e. it has constant elasticity of supply. In the following, I focus on this case. With only one maximum, the cost function has an inverted U-shape and we can conclude as follows:

**Proposition 1** A given output is efficiently produced if and only if all or none of the citizens are honest. The cost of producing a given output is first increasing and then decreasing in the fraction of honest citizens.

The proof is in Appendix A.

Proposition 1 implies that there is a critical level of \( h \), \( \bar{h} \), such that a higher fraction of tax compliant citizens increases total production costs if \( h < \bar{h} \) and reduces total production costs if \( h > \bar{h} \).

If all citizens are honest or no one is honest, output is efficiently produced because the ranking of private and social opportunity cost is the same. If all are honest, social and private opportunity costs are the same and equal to \( w/a_i \). If no one is honest, the private opportunity cost for a self-employed is lower than the social opportunity costs, i.e. \( \alpha w/a_i < w/a_i \), but entrepreneurs with the same social opportunity cost have the same private opportunity cost. This means that output is produced by the most efficient entrepreneurs. When some are honest and some are not, however, the ranking of private and social opportunity costs differ. Among entrepreneurs with the same social opportunity cost the dishonest ones has lower private opportunity cost, which means they are privately more profitable as self-employed. The marginal tax compliant entrepreneur who does not survive may then be replaced by a tax evading entrepreneur with lower productivity.

It follows from proposition 1 that the effect of a higher fraction of honest citizens depends on how large the fraction is to begin with. With higher \( h \), private opportunity costs goes up for those who become honest among the self-employed, and some of them will therefore no longer survive as self-employed. The negative shift in supply leads to a higher out price, which in turn induces entry of new entrepreneurs. If few citizens are honest, few of the entering entrepreneurs have lower social unit costs than the ones they replace, they are just more willing to evade taxes. As a result, total costs of producing a given output goes up. If most citizens are honest, most of the entering entrepreneurs have lower unit costs than the ones they replace. This implies that the cost of producing a given output goes down.

In contrast to improved tax morale, higher sanctions would unambiguously improve welfare. If the expected penalty is increased, the optimal evasion (\( \lambda^* \)) goes down, and
so α goes up. As a result, those who are dishonest evade less and their profitability goes down. Fewer dishonest entrepreneurs survive. Since the marginal dishonest entrepreneur has a higher social revenue as employed and the marginal honest employee has higher social revenue as an entrepreneur, this increases the average productivity among entrepreneurs. As a result, the cost of producing a given output goes down. Total output from self-employment is reduced, which means it is closer to its optimal level. The average morale in the population has not changed, but the average tax morale among the self-employed goes up since more morale individuals are now able to survive as entrepreneurs.

4 The effects of moral on social surplus and tax revenues

In this section, I use the cost function derived in section 3 to demonstrate the ambiguous effects of improved morale on social surplus and tax revenue. Without moral constraints, the output from self-employment is too high, but efficiently produced. I demonstrate that with different moral constraints, output is closer to its optimal level, but inefficiently produced.

4.1 Social surplus

Social surplus can be written as

\[ W = \int_0^{\tilde{x}(h)} D^{-1}(x) dx - c(\tilde{x}(h); h) \]  

(10)

The first term is the willingness to pay for the equilibrium output \( \tilde{x} \), i.e. its value to consumers. The second term is the cost of producing \( \tilde{x} \) when a fraction \( h \) of the citizens are honest while the rest are not. Let \( x(1) \) denote the socially optimal output, i.e. output in the equilibrium where all citizens are honest (\( h = 1 \)). The social surplus can then be rewritten as

\[ W = \int_0^{x(1)} [D^{-1}(x) - S^{-1}(x)] dx + \int_{x(1)}^{\tilde{x}} [D^{-1}(x) - S^{-1}(x)] dx - [c(\tilde{x}; h) - c(\tilde{x}; 1)] \]  

(11)

The first term in (11) is the maximized social surplus, i.e. the willingness to pay for the optimal output \( x(1) \) minus the minimum cost of producing it. The second term is the loss from too high output. Since the customers marginal value of the output is below the marginal social cost for \( x > \tilde{x}(h) \), \( D^{-1}(x) < S^{-1}(x) \) for all units from \( x(1) \) to \( \tilde{x} \). These socially unprofitable units are produced because tax evasion reduces the private opportunity cost of dishonest entrepreneurs below the social opportunity cost. The last term in (11) is the loss from inefficient production of \( \tilde{x} \). From proposition 1, a given output is efficiently produced if and only if all citizens are honest or all of them are
dishonest. This means that \( c(x; h) > c(x; 1) = c(x; 0) \) for \( 0 < h < 1 \). The last term is therefore zero for \( h = 1 \) and \( h = 0 \) and positive for all \( 0 < h < 1 \).

Differentiating \( W \) with respect to \( h \), taking into account that the equilibrium price and output are determined by (7) and (8), gives the following:

\[
dW/dh = [D^{-1}(\tilde{x}) - S^{-1}(\tilde{x})]\tilde{x}'(h) - [c_x(\tilde{x}; h) - c_x(\tilde{x}; 1)]\tilde{x}'(h) - c_h (\tilde{x}; h) \tag{12}
\]

The first term is the effect of lower output when it is efficiently produced. The last two terms show the change in costs as \( h \) goes up: While the second term is the cost effect of lower output for given \( h \), the last term is the effect of increased \( h \) on the cost of producing the equilibrium output \( \tilde{x} \).

In appendix B I show the following

**Proposition 2** A marginal increase in the fraction of honest citizens may reduce social surplus for \( h < \bar{h} \). Social surplus is reduced if the increased cost of production exceeds the welfare gain from lower output. If supply is perfectly elastic on the margin or demand inelastic, the output effect is zero and an increase in \( h \) reduces social surplus for \( h < \bar{h} \).

With demand decreasing and supply increasing in price (\( q \)) a higher fraction of honest citizens have two opposite effects on social welfare. On the one hand, an increase in \( h \) reduces the output from self-employment, which brings it closer to the optimal level. On the other hand, the cost of producing goes up if the initial fraction is below the critical level \( \bar{h} \). With perfectly elastic supply or inelastic demand on the margin, output does not change and so the change in social welfare equals the change in the cost of producing a given output. Following the sign of \( c_h \) from proposition 1, \( dW/dh \) is negative for \( h < \bar{h} \) and positive for \( h > \bar{h} \). I illustrate this case with an example in figure 2 below.

Figure 2 depicts three productivity-types: \( n_1 \) individuals have the highest productivity \( a_1 \), \( n_2 \) have lower productivity \( a_2 \) and there is an unlimited supply of individuals with the lowest productivity \( a_3 \). Social opportunity costs are \( \alpha w/a_i \) for \( i = 1, 2, 3 \). The dotted curve in figure 2 is the supply curve if no sellers evade, with the steps being the opportunity cost of type 1, 2 and 3. The bold curve is the supply curve if all sellers evade.

If all citizens are honest, the equilibrium price is \( \bar{q}(1) \) and the corresponding output \( \bar{x}(1) \). Since the price equals the opportunity cost of a type 3 entrepreneur, i.e. \( \bar{q}(1) = w/a_3 \), some of the entrepreneurs with the lowest productivity survive. Supply is perfectly elastic on the margin. If no citizens are honest, \( h = 0 \), the equilibrium price \( \bar{q}(0) \) equals the private opportunity cost of a type 3 entrepreneur who evades, i.e. \( \bar{q}(0) = \alpha w/a_3 \). Output is then \( \bar{x}(0) \). The shaded area is the social loss from too high output in the equilibrium where no one is honest.

Starting from a situation where no one is honest, consider what happens if a fraction \( h \) of the citizens become honest. The private opportunity cost of honest sellers increases from \( \alpha w/a_i \) to \( w/a_i \), \( i = 1, 2, 3 \). At the price \( \bar{q}(0) \), the supply from type 1 and 2 goes down. However, since there is a perfectly elastic supply of type 3, there is also perfectly
elastic supply of dishonest sellers of type 3. Consequently, the price and output do not change. In the example, the honest sellers with productivity $a_1$ survive but those with productivity $a_2$ do not. Honest sellers with productivity $a_2$ are replaced by dishonest sellers with the lower productivity $a_3$. Social surplus is clearly reduced since the same output is now produced by less productive entrepreneurs. The dotted area shows the welfare loss from higher tax morale in the population. The loss is equal to the increased production cost.

4.2 Tax revenue

The effect of a higher fraction of tax compliant citizens on tax revenues is ambiguous: On the one hand, tax revenue goes up from those who become honest and still survive as entrepreneurs. On the other hand, the total reported income, i.e. the tax base, may go down because of mis-allocation of labor between employment and self-employment. The total reported income is given by

$$R = \tilde{q}[h \int_{w/\tilde{q}}^{\tilde{a}} a dF(a) + (1 - h)(1 - \lambda) \int_{\alpha w/\tilde{q}}^{\tilde{a}} a dF(a)] + tw[hF(w/\tilde{q}) + (1 - h)F(\alpha w/\tilde{q})]$$ (13)

The first two terms are the tax revenues from the self-employed. The honest ones report all incomes for taxation while the dishonest ones report a fraction $1 - \lambda$. The last two terms are the tax revenue from employees, who are unable to cheat on taxes. In appendix A I show the following:

**Proposition 3** A higher fraction of honest citizens may lead to lower tax revenues.

Some of those who become tax compliant do not survive as self-employed, and some of those who exit are replaced by tax evaders with lower productivity. In the new equilibrium, some self-employed would earn a higher gross income as employees and some employees would earn a higher gross incomes as self-employed. This mis-allocation reduces total taxable incomes and thereby also the tax revenues. Thus, when moral constraints differ, moral improvements may not increase tax payments. The intuition is that dishonest but productive entrepreneurs may bring in more taxes than honest, but low productive ones.

In the example illustrated in figure 2, the change in tax revenue when a fraction $h$ becomes honest is

$$th[\lambda n_1 a_1 - n_2 w(a_2/a_3 - 1)]$$ (14)

The first term is the increased tax revenues from the now honest entrepreneurs with productivity $a_1$. The second term is the lost tax revenues because the taxable incomes goes down when honest entrepreneurs with productivity $a_2$ are replaced with the less productive and dishonest entrepreneurs with productivity $a_3$. The number of type 3 entrepreneurs necessary to replace those who leave is $hn_2/a_3$. If the tax loss due to lower taxable incomes exceeds the increased tax payment from the honest entrepreneurs who survive, tax revenue goes down.
5 Tax morale among both buyers and sellers

Consider now the case where there are honest people among both buyers and sellers. A fraction $m$ of potential buyers are honest, and a fraction $h$ of potential sellers. Each customer either buys one unit of the output or does not buy. The reservation price, i.e., the willingness to pay for a unit, differ between them. I assume that an honest customer can make sure that his payment is reported. The sellers are responsible for paying the taxes, and face a probability of being detected and penalized for income that they do not report.

With honest customers, it matters whether the probability of detection depends on the fraction evaded for each customer or only on the total fraction evaded. This may differ between markets. In markets where each payment is large, such as the construction market, it may be a lot easier to hide a fraction of the payment from each customer than to hide the entire payment from a fraction of the customers. In such markets, it is reasonable to assume that the probability of detection depends on the fraction evaded for each customer. In markets where each payment is small, such as in the market for haircuts, the seller may easily hide the entire payment from some customers and report the payments from others in full. It is reasonable to assume that detection is based on the total fraction evaded, not evasion from each customer’s payment. I discuss both cases, starting with the one where the probability of detection is increasing in the fraction evaded from each customer, such that the seller needs to report some income from each customer.

5.1 Detection based on fraction evaded from each unit

Consider a market where the probability of detection is an increasing, convex function of the fraction evaded from each customer’s payment. It is then optimal for a dishonest seller to spread his evasion between customers. To have an interesting problem, I focus on the case where at least some potential sellers are dishonest, i.e., $h < 1$. When a seller deals with a dishonest customer, he hides a fraction $\lambda^*$, determined by (1). When he meets an honest customer, however, he must report all incomes. Thus, for a given price, dealing with an honest customer gives a lower net revenue than a dishonest one.

In equilibrium, the sellers must be indifferent between the two types of customers, which means honest customers are charged a higher price. Let $q$ be the price paid by honest customers and $q^d$ the price charged from the dishonest ones. The seller is indifferent if and only if $q(1-t) = q^d[1 - t + \lambda^*(t - p(\lambda^*)\tau)]$. Recalling from section 2 that $(1-t)/[(1-t) + \lambda^*(t - p(\lambda)\tau)] \equiv \alpha$, the price charged from dishonest customers equals $\alpha q$. For any $h \geq 0$ I can then show the following:

**Proposition 4** If a fraction $m$ of potential buyers are honest, where $0 < m < 1$, the allocation of production between sellers is efficient but allocation of output between buyers is inefficient. Exchange is efficient if all customers are honest ($m = 1$) or if they

---

6In many countries, it is likely that the seller report the customers pay via a bank, since bank transactions are reported to the tax authorities.
are all dishonest \((m = 0)\). Starting from a situation where no customers are honest, a higher fraction of honest customers reduces the efficiency of exchange. Starting from a situation where all customers are honest, a lower fraction of honest customers reduces the efficiency of exchange.

The proof is in appendix C.

The reason why allocation of production between sellers is optimal for \(0 < m < 1\) is that both honest and dishonest sellers face the net marginal revenue \((1 - t)q\) on all customers. With equal marginal revenue for all sellers, the allocation of production between them is efficient. Total supply is \(S(q)\), as given by (5). Honest and dishonest buyers face different prices, however, and so the allocation between them is inefficient. Dishonest customers pay a lower price than the honest ones. This implies that the marginal dishonest buyer will have a lower value of the output than the marginal discouraged, honest buyer.

The result for inefficient allocation between customers is parallel to the result for inefficient allocation between sellers, proposition(1): If all customers were honest, those with the highest willingness to pay for the output end up buying it. If all customers were dishonest, the demand is higher for any given price \(q\), but it is still those with the highest willingness to pay that end up with the output. When some customers are honest and some dishonest, output is inefficiently allocated between them because they face different prices. Dishonest customers face lower prices than the honest ones. In appendix C I demonstrate how the distortions change with \(m\) by deriving the social value of a given output \(\bar{x}\) as a function of \(m\). The social value, \(v(x, h)\), is the total willingness to pay for the output among those who end up buying it.

A higher fraction of honest buyers raises the price \(q\) since the sellers costs go up. As a result, some customers no longer buy. Starting from a situation where all customers are dishonest, those who replace the discouraged buyers have lower willingness to pay, but they pay less since they accept that some of their payments is not reported. Consequently, the total value social value of output goes down. Starting from a situation where almost all customers are honest, the discouraged buyers are replaced by buyers with higher willingness to pay. Consequently, the social value of output goes up when \(m\) goes up. If the value-function has one maximum only, as with constant elasticity of demand, there is a critical value of \(m\), \(\bar{m}\), such that the value is decreasing in \(m\) for \(m < \bar{m}\) and increasing in \(m\) for \(m \geq \bar{m}\).

The effects on social surplus is also parallel to the case with honesty among sellers only: A higher fraction of honest customers have two opposite effects on social surplus. One the one hand, social surplus goes up when output is reduced, since output is too high when some customers accept evasion. On the other hand, it leads to inefficient allocation of output among those who buy. If output does not change, for example because supply is inelastic, the effect on social surplus is determined by the effect on allocation among buyers. Starting from a situation where \(m = 0\), i.e. where all customers are dishonest, a higher fraction of honest customers reduces social surplus if supply is inelastic.
5.2 Detection based on total fraction evaded

Consider now the case where the probability of detection depends on the total evasion, not evasion in each project. The expected income of the entrepreneur does not depend on how his evasion is allocated between different customers. He may hide the entire income from some customers and report the entire income from others. I show that in this case, the fraction of honest buyers must be above a critical level before it has any effect in the market.

**Proposition 5** For each \( h \) there is a critical fraction of honest buyers, \( \hat{m}(h) \), such that for \( m \leq \hat{m}(h) \) output is inefficiently produced, but efficiently allocated between buyers. A marginal increase in \( m \) has no effect on allocation. For \( \hat{m}(h) < m < 1 \), output is efficiently produced, but inefficiently allocated between buyers. A marginal increase in \( m \) changes allocation. The critical fraction of honest buyers is increasing in \( h \), i.e. \( \hat{m}'(h) > 0 \).

The proof is in appendix C.

To key to understand why honest buyers have no effect when \( m \) is below a critical level, is that part of the output is reported even when no buyers are honest: The threat of sanctions induce the dishonest sellers to report a fraction \( 1 - \lambda \) of their income, and honest sellers report their entire income. Consider what happens if a fraction \( m \) of the potential buyers become honest. If their demand is lower than the fraction of output that the sellers would report even without honest buyers, their honesty is costless. A sellers that meets and honest customer, his payment becomes a part of the income that would be reported in any case. A marginal increase in the fraction of honest customers only leads to a reshuffling of customers: A larger fraction of the payments from dishonest buyers are not reported to make room for the honest buyers on the reported account. 7

When the honest buyers can be satisfied at no cost to the sellers, honest and dishonest buyers face the same price, which means the allocation of output between them is efficient. The net price differ between the honest and dishonest sellers, however. Consequently, the output is inefficiently produced. The equilibrium when \( m < \hat{m} \) is the same as in an economy where a fraction \( h \) of potential sellers are honest but no buyers, explored in section 2, 3 and 4. The higher the fraction of honest sellers, \( h \), the higher is the fraction of output that is reported by sellers even without honest buyers, and the higher is the fraction of honest buyers needed to make a difference.

If \( m > \hat{m} \), honest customers matter because their demand for reported services then exceeds supply, which means they are costly to sellers on the margin. As a result, the sellers charges honest customers a higher price. In equilibrium the sellers must be indifferent between honest and a dishonest buyers. If \( q \) is the price to honest buyers, the

---

7With market frictions, i.e. if it is costly to reshuffle customers such that the honest ones are satisfied, honest customers may have an effect even in when the total demand for reported services is lower than the sellers optimal reported incomes. A market with tax evasion and frictions, but without tax morale among agents, is discussed in Strand (2005).
price to honest buyers must be \( q(1 - t) \). Since dishonest consumers pay a lower price than the honest ones, dishonest consumers with low valuation may crowd out honest consumers with higher valuation. Since honest and dishonest sellers now face the same net price \((1 - t)q\) from all customers, output will be produced by the most productive sellers. While production is efficient, exchange is distorted. For \( m > \hat{m} \), the analysis of (5.1) and proposition 5 applies.

It follows from proposition 4 and 5 is that allocation is never inefficient among both sellers and buyers. If output between buyers is inefficient, production between sellers is efficient, and if production is inefficient, allocation between buyers is efficient. Allocation of output between buyers is inefficient when they face different prices to make the sellers indifferent between honest and dishonest buyers on the margin. To be indifferent, the prices must be such that sellers get the same revenue from honest as from dishonest buyers. This in turn means the marginal revenue from self-employment is the same for both honest and dishonest sellers, and so production is efficient. If allocation between sellers is inefficient, they face different net revenues because some evade and some do not. To make buyers indifferent, they must face the same price, which means output is efficiently allocated between them.

### 6 Concluding remarks

I have shown that even if morale is a substitute for sanctions for each individual, this is not the case in the market. The reason is that moral constraints creates differences in private costs and gains between people even if their social costs and gains are the same. A fundamental condition for efficient markets is that it is the most productive sellers that produce the output and the buyers with highest willingness to pay who end up buying it. With differences in morale, this efficiency condition is not met. Some experiments have shown that sanctions may crowd out moral behavior. My result shows that in markets, sanctions may improve morale. For example, increased sanctions for tax evasion improves morale among sellers because it is possible for tax compliant sellers to survive.

Morale creates distortions only if it differs between individuals. Social norms, defined as "group-held beliefs about how members should behave in a given context" (Wikipedia), are unambiguously desirable. The reason is that a common norm do not change the ranking between sellers and buyers: Sellers who are most profitable without norms are also the most profitable with norms that constrain their evasion. However, most norms are not followed by all - and this creates distortions. A key result in the paper is that the effect of an increase in the fraction who follows the norm depends on how many already follows it. If most citizens are tax compliant, an increase in the fraction of tax compliant citizens unambiguously reduces distortions. If few citizens are tax compliant, however, more tax compliant people may actually increase distortions.

In some cases, there is a market for intermediaries who can relieve people of their moral responsibility. For example, if buyers have a clean conscience as long as they are
not personally involved in tax evasion, there may be a market for intermediaries that hire evaders. An honest customer may feel obliged to pay all taxes if he hires carpenters to build a house, but he may not feel bad about buying a house from an intermediary even if he knows that the intermediary hires tax evading carpenters. Such schemes are well known within the construction sector, where several layers of contractors is common. An honest entrepreneur may escape the cost of his high morale by selling the firm if his productivity can be transferred to the new owner. This is the case if the productivity is not a quality of him as an owner, but rather embedded in the firm he starts up, as with a technical invention. Since a business has higher value for a tax evader than one who is tax compliant, high-productive but tax compliant citizens would start up businesses but sell them to tax evaders. If the market for intermediaries that take the moral blame is efficient, morale clearly has little or no effect on the market outcome.

The model has implications for how to measure morale. In a large random-audit study Kleven et al (2011) find that while evasion is negligible among employees, who are not able to evade, it is substantial among self-employed, who are able to. They conclude that tax compliance is high because most people are unable to cheat on taxes, not because they are unwilling to. This conclusion about peoples tax morale may be too pessimistic if the opportunity to evade affects the choice between self-employment and employment. We would expect the average tax morale among the self-employed to be lower than in the rest of the population, since low tax morale is an advantage in self-employment but not in employment. In the extreme, evading taxes may be necessary to survive as self-employed. Thus, the observed tax morale among self-employed may underestimate the average morale among citizens. Similarly, experiments may overestimate the role of morale in the market, because the most morally constrained agents do not survive in markets where their moral matters.

In my framework there is a clear distinction between those who have the opportunity to evade, the self-employed, and those who have not, the employed. Moreover, self-employment is assumed to be a one-man business. The reality is of course not as simple. Some entrepreneurs run small firms with a few employees, and may agree to not report all wage incomes. Also, employees in large firms with no opportunity to evade wage incomes may do shadow work as "moonlighting". Tax evasion may therefore affect the choice between running a large, taxpaying firm or a smaller one where it is easier to evade. It may also affect the workers’ choice between jobs in large firms, with little opportunity to evade, and small firms with more opportunities. However, none of these extensions of the model changes the main result; that tax evasion distorts peoples choices of occupation and that differences in tax morale may increase rather than decrease these distortions. More realism in the model would easily distract from the main message without adding new insight.

---

For most people, acting morally means not being actively involved in something they consider unethical. Fewer feel morally obliged to make sure that the products or services they buy are produced in an ethical way.

These problems are discussed in Ognedal (2014)
References


Bott, K., A. W. Cappelen, B. Tungodden and E. Ø. Sørensen, 2013. You’ve got mail: A randomized field experiment on tax evasion, working paper.


A Proof of proposition 1

Recall that the social cost of producing an output $x$ in a market where a fraction $1-h$ evade taxes, given by (9), is:

$$c(x; h) = h \int_0^{S(q(x,h))} S^{-1}(x) dx + (1-h) \int_0^{S(q(x,h)/\alpha)} S^{-1}(x) dx$$

where $q(x,h)$ is the price that induces supply $x$ when a fraction $h$ of potential suppliers is honest, determined by (6), i.e. by $hS(q) + (1-h)S(q/\alpha) = x$. The partial derivative of $q$ with respect to $h$ is

$$q_h(x; h) = \frac{S(q/\alpha) - S(q)}{hS'(q) + (1-h)S'(q/\alpha)/\alpha}$$

To find how a higher fraction of honest citizens affect the cost of producing a given output $\bar{x}$, we take the partial derivative of $c(\bar{x}; h)$ with respect to $h$. Using $q_h(x; h)$ and
rearranging gives us
\[
c_h(\bar{x}; h) = - \int_{S(q(\bar{x}), h)}^{S(q(\bar{x}), h)/\alpha} S^{-1}(x)dx + [S(q(\bar{x}), h)/\alpha - S(q(\bar{x}), h)]q(\bar{x}, h)b(q(\bar{x}, h); h) \tag{A.3}
\]
where
\[
b(q; h) = \frac{hS'(q) + (1 - h)S'(q/\alpha)/\alpha^2}{hS'(q) + (1 - h)S'(q/\alpha)/\alpha} \tag{A.4}
\]

If \(h = 0\), \(b(q, h) = 1/\alpha\) and so the last term in (A.3) equals the rectangle \([S(q/\alpha) - S(q)]q/\alpha\). This rectangle is larger than the integral in (A.3): Integrating under \(S^{-1}\) from \(S(q)\) to \(S(q/\alpha)\), the value goes from \(q(\bar{x}, 0)\) to \(q(\bar{x}, 0)/\alpha\). Consequently, \(c_u(x, 0) > 0\), i.e. the cost of producing \(x\) goes up. If \(h = 1\), \(b(q; h) = 1\) and so the last term in (A.3) is the rectangle \([S(q/\alpha) - S(q)]q\). This rectangle is smaller than the integral, since the value of \(S^{-1}(x)\) in the integral goes from \(q\) to \(q/\alpha\). Consequently, \(c_h(x, h) < 0\) for \(h = 1\). Thus, starting from a situation where no citizens are honest, making some of them honest increases the cost of producing. Starting from a situation where all citizens are honest, making some of them dishonest increases the cost of producing.

Let \(\bar{h}\) be the value of \(h\) that makes \(c_h(x, h) = 0\). If \(c_{hh}(x, \bar{h}) < 0\), the cost function has only one maximum, for \(\bar{h}\). This condition holds for example for constant elasticity of supply: With a constant elasticity supply function, \(S(q) = q^\alpha\), the cost function is
\[
c(\bar{x}, h) = h \int_0^{q^\alpha} x^{1/s} + (1 - h) \int_0^{(q/\alpha)^s} x^{1/s} \tag{A.5}
\]
Integrating gives
\[
c(\bar{x}, h) = \frac{s}{1 + s} (q(\bar{x}, h))^{s+1} h + (1 - h)/\alpha \tag{A.6}
\]
From \(\bar{x} = hq^s + (1 - h)(q/\alpha)^s\) we derive
\[
q(\bar{x}, h) = \left[\frac{\bar{x}}{h + (1 - h)/\alpha^s}\right]^{1/s} \tag{A.7}
\]
The cost function (A.6) can then be rewritten as
\[
c(\bar{x}, h) = \frac{s}{1 + s} \left[\frac{\bar{x}}{h + (1 - h)/\alpha^s}\right]^{s+1} h + (1 - h)/\alpha \tag{A.8}
\]
Differentiating with respect to \(h\) yields
\[
c_h(\bar{x}, h) = \frac{\bar{x}^\gamma}{\gamma} \left[\frac{1 - h}{\alpha^s}\right]^{-\gamma} \left[\frac{1 - h}{\alpha^s}\right](1 - \frac{1}{\alpha}) - \frac{s + 1}{s} \left[\frac{1 - h}{\alpha}\right](1 - \frac{1}{\alpha^s}) \tag{A.9}
\]
where \(\gamma = \frac{s+1}{s}\). Define \(A(h) = \frac{\bar{x}^\gamma}{\gamma} \left[\frac{1 - h}{\alpha^s}\right]^{-\gamma} \gamma\left[\frac{1 - h}{\alpha}\right](1 - \frac{1}{\alpha^s})\left(\frac{1}{\alpha^s}\right)\), such that (A.9) can be rewritten as \(c_h(\bar{x}, h) = A(h)B(h)\). Since \(A(h) > 0\), \(c_h = 0\)
if \( B(h) = 0 \). Differentiating \( c_h \) with respect to \( h \) gives \( c_{bh} = A'(h)B(h) + A(h)B'(h) \). Since \( B(h) = 0 \) for \( c_h = 0 \), \( c_{bh} \leq 0 \) for \( c_h = 0 \) if \( B'(h) < 0 \), which is easily verified. Thus, with constant elasticity of supply, the cost function has one maximum, which implies that \( c_h(x, h) > 0 \) for all \( h < \bar{h} \) and \( c_h(x, h) < 0 \) for all \( h > \bar{h} \).

\section{Proof of proposition 2 and 3}

\textbf{Proof of proposition 2}

Differentiating social surplus, as given by (11) with respect to \( h \) yields

\[
dW/dh = \left[ D^{-1}(\bar{x}) - S^{-1}(\bar{x}) \right] \bar{x}'(h) + \left[ c_x(\bar{x}; 1) - c_x(\bar{x}; h) \right] \bar{x}'(h) - c_b(\bar{x}, h)
\]

(B.1)

If supply is perfectly elastic on the margin, output does not change as \( h \) goes up, i.e. \( \bar{x}'(h) = 0 \). This makes the first two terms equal to zero, and so the sign of \( dW/dh \) depends only on the sign of \( c_h(\bar{x}, h) \). From proposition 1, \( c_h(\bar{x}, h) > 0 \) for \( h = 0 \) and \( c_h(\bar{x}, h) < 0 \) for \( h = 1 \). Consequently, welfare is decreasing in \( h \) for \( h = 0 \) and increasing in \( h \) for \( h = 1 \)

\( \). If \( c_h(\bar{x}, h) > 0 \) for all \( 0 < h < \bar{h} \) and \( c_h(\bar{x}, h) < 0 \) for all \( \bar{h} < h < 1 \), welfare is decreasing in \( h \) for \( h < \bar{h} \) and increasing in \( h \) for \( h > \bar{h} \).

If supply is decreasing in \( h \) on the margin, i.e. \( \bar{x}'(h) < 0 \), the first term is positive for \( h < 1 \) and zero for \( h = 1 \). The second term can be rewritten as \( [q(\bar{x}, 1) - q(\bar{x}, h)b(\bar{q}, h)]\bar{x}_h \), where \( b(\bar{q}, h) \) is given by (A.4) and \( q(x, h) \) is derived from \( hS(q) + (1 - h)S(q/\alpha) = x \).

The second term is therefore zero for \( h = 0 \) and \( h = 1 \) since \( b(q, 0) = 1/\alpha \), \( q(x, 0) = q(x, 1)/\alpha \) and \( b(q, 1) = 1 \). For \( 0 < h < 1 \) the sign is ambiguous since \( b(\bar{q}, h) \geq 1 \) but \( q(\bar{x}, h) < q(\bar{x}, 1) \). From proposition 1, the third term, \( c_h(\bar{x}, h) \), is positive for \( h = 0 \) and negative for \( h = 1 \). It follows that for \( h = 1 \), \( dW/dh > 0 \), since the first term is positive, the second zero and \( c_h(\bar{x}, h) < 0 \). For \( h = 0 \), the first term is positive, the second zero and \( c_h(\bar{x}, h) > 0 \) and so the sign of \( dW/dh \) is ambiguous. For \( 0 < h < 1 \), the first term is positive, but we cannot sign the last two terms in the general case.

\textbf{Proof of proposition 3}

The total reported income, i.e. the tax base, is given by (13). Differentiating the reported income with respect to \( h \) and rearranging yields

\[
dR/dh = \left[ I + qS'(q/\alpha)/\lambda \right] \bar{q}'(h) + \bar{q} \left[ S'(\bar{q}) - (1 - \lambda)S(\bar{q}/\alpha) \right] + q(1 - h)S'(\bar{q}/\alpha)(1 - 1/\alpha)/\alpha q'(h)
\]

(B.2)

where \( I = hS(q) + (1 - h)(1 - \lambda)S(q/\alpha) \), the reported output from self-employment. The first term in (B.2) is the increase in reported income due to higher output price for the self-employed services. The second term is the direct change in reported income from self-employment. This may be negative since the supply from the dishonest sellers is larger than from the honest ones, and there are now also a lower fraction of the dishonest. The last term is negative since \( \alpha < 1 \). It is the income loss from distorted allocation when \( q \) goes up. A higher \( q \) induces more dishonest people to choose self-employment, but their social revenue was higher as employed. The marginal dishonest self-employed
would produce a higher social income as employed, but his net private income is highest as self-employed. If \( \tilde{q}'(h) = 0 \), as in the example where supply is perfectly elastic on the margin, the first and third term in (B.2) is zero. The sign of \( dR/dh \) then depends on the second term, which may be negative.

C Proof of proposition 4 and 5

Proof of proposition 4

Among the honest customers, the buyers are those who are willing to pay at least \( q \). Among the dishonest, the buyers are those who are willing to pay at least \( \alpha q \). Thus, the demand is \( mD(q) \) from honest buyers and \( (1 - m)D((1 - t)q) \) from dishonest buyers. Since the willingness to pay is the area under the demand curve for honest customers, the total social value of the output is

\[
v(x, m) = h \int_{0}^{D(q)} D^{-1}(x)dx + (1 - m) \int_{0}^{D(\alpha q)} D^{-1}(x)dx \tag{C.1}
\]

where \( q = q(x, m) \) is the price that induces demand \( x \) when a fraction \( m \) of consumers are honest, i.e. it is determined by \( x = mD(q) + (1 - m)D(\alpha q) \). Since total demand is decreasing in \( q \) and \( m \), \( q(x, m) \) is decreasing in \( x \) and \( m \). To find how a higher fraction of honest customers affect the total value to the buyers of a given output \( \bar{x} \), I take the partial derivative of \( v(x, m) \) with respect to \( m \). Using \( q_m(x, m) \), derived from the condition \( x = mD(q) + (1 - m)D(\alpha q) \), and rearranging gives us

\[
v_m(\bar{x}; m) = - \int_{D(q(\bar{x}, m))}^{D(\alpha q(\bar{x}, m))} D^{-1}(x)dx + [D(\alpha q(\bar{x}, m)) - D(q)]q(\bar{x}, m)e(q(\bar{x}, m); m) \tag{C.2}
\]

where

\[
e(q; m) = \frac{mSD(q) + (1 - m)D'(\alpha q)\alpha^2}{mD'(q) + (1 - m)D'(\alpha q)\alpha} \tag{C.3}
\]

If \( m = 0 \), \( e(q, m) = \alpha \) and so the last term in (C.2) equals the rectangle \([D(\alpha q) - D(q)]q\alpha\). Since this rectangle is smaller than the integral in (C.2), \( v_m(x, 0) < 0 \). If \( m = 1 \), \( e(q; m) = 1 \) and so the last term in (C.1) is the rectangle \([D(\alpha q) - D(q)]q\). Since this rectangle is larger than the integral, \( v_m(x, m) > 0 \) for \( m = 1 \). Thus, starting from a situation with no honest consumers, increasing the fraction of honest consumers, \( m \), reduces the total value of output to buyers. Starting from a situation where all consumers are honest, reducing \( m \) reduces the total value to buyers.

Let \( \bar{m} \) be the value of \( m \) that makes \( v_m(x, m) = 0 \). If \( v_{mm}(x, \bar{m}) > 0 \), the value function has only one minimum, which means that \( v_m(x, m) < 0 \) for all \( m < \bar{m} \) and \( v_m(x, m) > 0 \) for all \( m > \bar{m} \).

Proposition 5

Let \( \tilde{q} \) be the equilibrium price if there are no honest customers and a fraction \( h \) of honest
sellers. $\tilde{q}$ is determined by (7) and (8), i.e. by

$$D(\tilde{q}) = hS(\tilde{q}) + (1 - h)S(\tilde{q}/\alpha) \quad (C.4)$$

Assume now that a fraction $m$ of the customers become honest. At price $\tilde{q}$ the demand for reported output is $mD(\tilde{q})$. The supply of reported output is the entire supply from honest producers plus the fraction of the supply that the dishonest sellers find it optimal to report, i.e. $hS(\tilde{q}) + (1 - h)(1 - \lambda)S(\tilde{q}/\alpha)$. If the demand for reported services is lower than or equal to the supply at $\tilde{q}$, the honest buyers can be satisfied at no extra cost to the seller. This is the case if

$$mD(\tilde{q}) \leq hS(\tilde{q}) + (1 - h)(1 - \lambda)S(\tilde{q}/\alpha) \quad (C.5)$$

Using (C.4), this can be rewritten as

$$m \leq 1 - \frac{\lambda(1 - h)S(\tilde{q}/\alpha)}{\tilde{x}} \equiv \hat{m} \quad (C.6)$$

where $\tilde{x}$ is the equilibrium output at price $\tilde{q}$. The critical fraction $\hat{m}$ is the fraction of the output that the sellers find optimal to report even when no honest buyers. Since honest customers is satisfies at no cost if $m \leq \hat{m}$, they face the same price $\tilde{q}$ as dishonest customers, which implies that the allocation of output between them is efficient. The sellers face the same gains and costs as with $m = 0$ and $h > 0$, which is inefficient, as shown in section 3.

If $m > \hat{m}$, the demand for reported output exceed the supply at price $\tilde{q}$ for all. To be indifferent between honest and dishonest customers, the sellers charge them different prices. If $q$ is the price paid by an honest customer and $q^d$ the price paid by a dishonest one, sellers are indifferent if and only if $q(1 - t)q^d$. Since all sellers get the same net revenue, $(1 - t)q$ from all customers, the production is optimally allocated between sellers. Since honest and dishonest customers face different prices, the allocation of output between them is inefficient. How the inefficiency varies with $m$ can be analyzed as in the proof of proposition 4.

21
\( q(x, 1) = S^{-1}(x) \)

\( q(x, 0) = \alpha S^{-1}(x) \)

\( S\left(\frac{q'}{\alpha}\right) \)

\( S(q') \)

\( x \)

\( q(\pi, 1) \)

\( q(\pi, h') \equiv q' \)

\( q(\pi, 0) \)

\( q = (x, h') \)

\( q(x, 0) = \alpha S^{-1}(x) \)

\( Figure 1 \)