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**Networks of Military  
Alliances, Wars, and  
International  
Trade**

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# Climate Change and Sustainable Development

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### Summary

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**Keywords:** Alliances, Conflict, War, Networks, International Trade, Treaties

**JEL Classification:** D74, D85, F10

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# Networks of Military Alliances, Wars, and International Trade

Matthew O. Jackson and Stephen Nei\*

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## Abstract

We investigate the role of networks of military alliances in preventing or encouraging wars between groups of countries. A country is vulnerable to attack if there is some fully-allied group of countries that can defeat that country and its (remaining) allies based on a function of their collective military strengths. Even with such a demanding notion of vulnerability, we show that there do not exist any networks that are stable against the addition and deletion of alliances. We then show that economic benefits from international trade can provide incentives to form alliances in ways that restore stability and prevent wars. In closing, we briefly examine the historical data on interstate wars and trade, noting that a dramatic (more than ten-fold) drop in the rate of interstate wars since 1960 is paralleled by an unprecedented growth in trade over the same period.

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# 1 Introduction

Wars are caused by undefended wealth. *Ernest Hemingway (repeated by Douglas MacArthur in lobbying to fortify the Philippines in the 1930's*<sup>1</sup>)

The enormous costs of war make it imperative to understand the conditions under which wars are likely to occur, and the ways in which they can be prevented. Although much is known about bilateral conflicts, there is no formal theory of how the structures of multilateral international relationships foster and deter interstate wars. In this paper we introduce a model of networks of military alliances and analyze its predictions.

Arranging multiple alliances to ensure world peace found a famous proponent in Otto von Bismarck and his belief that the European states could be allied in ways that would maintain a peaceful balance of power<sup>2</sup>. The alliances that emerged were stable for some decades, but were ultimately unable to prevent World War I. Indeed, many world conflicts involve multiple countries allied together in defensive and offensive groups, from the shifting alliances of the Peloponnesian and Corinthian wars of ancient Greece to the Axis and Allies of World War II, and so studying the fabric of alliances is necessary for understanding international (in)stability. Moreover, by examining the networks of international alliances over the past two centuries<sup>3</sup>, we see patterns and predictable structures regarding which alliances are formed and which are not. In particular, the structure of the alliance network has changed dramatically over time, as can be seen in Figures 9 through 14. There is a particularly notable shift between pre- and post-World War II. In pre-World War II networks, the cliques are interwoven, often sharing several edges, and with several lines (i.e. a series of  $n$  countries,  $\{c_1, c_2, \dots, c_n\}$ , such that country  $c_i$  has ties with countries  $c_{i-1}$  and  $c_{i+1}$  but no others). In the post-WWII networks, the networks become substantially denser and with alliances that are separated by continent and ideology - with large cliques, corresponding to large geographical areas, which are connected by a few larger states.

To gain insights into networks of alliances and the incidence of wars, we model the incentives of countries to attack each other, to form alliances, and to trade with each other. We begin with a base concept which networks are stable against wars, from a purely military point of view. A group of countries can attack some other country if all members of the attacking coalition are allies with each other. The idea is that alliances represent the necessary means for coordinating military action. A country that is attacked can be defended by its allies. A country is *vulnerable* if there is some coalition of completely allied countries whose collective military strength outweighs that of the country and its remaining allies who are

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<sup>1</sup>See the biography by Bob Considine, source for Chapter 1: Deseret News, Feb 24, 1942.

<sup>2</sup>E.g., see Taylor (1969).

<sup>3</sup>As reported by the Alliance Treaty Obligation and Provisions Project atop.rice.edu.

not in the attacking coalition.<sup>4</sup> There are many networks for which no country is vulnerable, but already this concept rules out structures that are over- or under-connected.<sup>5</sup>

Not having any vulnerable countries does not account for the incentives that countries have to form alliances - so it does not give any idea of which networks are stable. Having a model that endogenizes the network is essential, and so we define a concept of *war-stable* networks that accounts for the incentives of countries to form alliances. To model stability we build upon the concept of pairwise stability of Jackson and Wolinsky (1996), adapting it to the current setting.

A network of alliances is *war-stable* if three conditions are met:

- first, no country is vulnerable to a successful attack by others,
- second, no two countries that are not allied could an alliance that would allow them, as part of some clique, to successfully attack another country, and
- third, each existing alliance serves a purpose - any country that deletes some alliance would end up being vulnerable.

It turns out that there are no war-stable networks, even with this very minimalist definition. The main difficulty is that requiring that every alliance serve some purpose requires that networks have sufficient cliques so that no country can delete an alliance. However, this can only work if each country is part of more than one separate clique, as otherwise it is vulnerable to the rest of the clique. Yet, being part of separate cliques allows a country to delete alliances without becoming vulnerable. The push to economize on alliances conflicts with stability against the formation of new alliances.

This view provides insights into the constantly shifting structures and recurring wars that occur throughout the nineteenth and first half of the twentieth centuries. Wars, however, have greatly subsided in parallel with the huge increase of trade (partly correlated with the introduction of containerized shipping in the 1960s): between 1820 and 1959 each pair of countries averaged .00056 wars per year, while from 1960 to 2000 the average was .00005 wars per year, less than *one tenth* as much.<sup>6</sup> As such, we examine incentives countries have to trade with each other and how those interact with military alliances. Indeed, there has been a rapid increase in global trade since World War II (partly coincident with the growth

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<sup>4</sup>We include technological considerations that may give an advantage to offensive or defensive forces.

<sup>5</sup>We explore two other definitions varying which countries could attack and which can defend, and show that the results also hold for those definitions. Essentially, requiring cliques to attack and any connected countries to defend makes it hardest to defeat a country, and so not having any stable networks for this concept also easily extends to other concepts where we allow non-cliques to attack or require cliques to defend.

<sup>6</sup>Even if one measures this per country rather than per pair of potential combatants, the decrease has been more than threefold.

of container shipping among other stimuli). The empirical relationship between war and trade is an active area of research, with strong suggestions (e.g., Martin et al. (2008)) that network concerns may be important.

Economic considerations lead us to introduce a concept of a network of alliances being *war and trade stable*, which allows countries to form alliances for either economic or military considerations. In the richer model, an alliance allows countries to trade with each other and to coordinate military activities, and so can be formed for either reason. This restores existence of networks of alliances that are stable against the addition or deletion of alliances. Under the war and trade stability definition, there is less of a tendency towards the sparse networks that led to instability and the non-existence of stable networks. Trade provides two helpful incentives: first it provides economic motivations to maintain alliances, and those alliances then have a deterrent effect; and second, it can reduce the incentives of a country to attack another since trade will be disrupted.

We provide some results on the existence and structure of war and trade stable networks of alliances, showing that structures similar to those observed over the past few decades are economically stable under apparently reasonable parameters. We close the paper with some discussion of the potential role that the growth in trade has played in reducing wars over the past half century.

Before proceeding, let us say a few words about how this contributes to the study of war. The literature on war provides many rationales for why wars occur. Our analysis here fits firmly into what has become a “rationalist” tradition based on cost and benefit analyses by rational actors, with roots seen in writings such as Hobbes (1651) *Leviathan*, and has become the foundation for much of the recent international relations literature.<sup>7</sup>

To our knowledge, there are only three previous general models of conflict that examine interactions between multiple agents/countries. The models, by Wagner (1986), Jordan (2006) and Piccione and Rubinstein (2007), are similarly based on a Hobbesian premise that stronger agents can expropriate weaker ones. They do not study networks of alliances and wars, but rather examine the implications of various definitions of equilibrium for the distribution of economic resources. Thus, our analysis is completely different not only in modeling, but also in the basic questions asked and answered.

The complex relationship between trade and conflict is the subject of a growing empirical literature (e.g., Barbieri (1996); Mansfield and Bronson (1997); Martin et al. (2008); Glick and Taylor (2010)). The literature not only has to face challenges of endogeneity and causation, but also of substantial heterogeneity in relationships, as well as geography, and the level of conflict. The various correlations between conflict and trade are complex and quite difficult to interpret, and a model such as ours can provide some structure with which to

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<sup>7</sup>Background can be found in Fearon (1995) and Jackson and Morelli (2011).

interpret some of the more basic findings, as we discuss in detail below.

## 2 The Basic Model

### 2.1 Countries and Networks

There is a set  $N = \{1, \dots, n\}$  of countries.

Countries are linked through alliances, represented by a network of alliances  $g \subset N^2$  with the interpretation that if  $ij \in g$  countries  $i$  and  $j$  are allies.<sup>8</sup>

$N_i(g) = \{j : ij \in g\}$  are the allies of  $i$ .

For a given alliance  $ij \notin g$ , let  $g + ij$  denote the network obtained by adding the alliance  $ij$  to  $g$ . Similarly, given an alliance  $ij \in g$ , let  $g - ij$  denote the network obtained by deleting the alliance  $ij$  from  $g$ .

Let

$$\mathcal{C}(g) = \{C \subset N \mid ij \in g \text{ for all } i, j \in C\}$$

denote the set of all cliques in a network  $g$ : that is the set of all groups of countries such that every pair of countries in the group are allied.

### 2.2 Military Strengths and Wars

Each country  $i \in N$  is endowed with a military strength  $M_i \in \mathbb{R}$ .<sup>9</sup>

For any group of countries  $C \subset N$ , let  $M(C) = \sum_{i \in C} M_i$  be their collective military strength.

If there is a war between sets of countries  $C$  and  $C'$ , with  $C$  being the aggressor, then  $C$  “wins” if  $M(C) > \gamma M(C')$ . The parameter  $\gamma > 0$  is the defensive (if  $\gamma > 1$ ) or offensive (if  $\gamma < 1$ ) advantage in the war.

This modeling of a war outcome based on relative strengths is a sort of contest success function (eg, Skaperdas (1996)), but one that is simplified to be deterministic. Adding random outcomes in our setting would add little insight, and this simple abstraction facilitates our multilateral analysis.

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<sup>8</sup>Here we represent a network by the list of unordered pairs  $ij$  that it comprises. So, for instance, the network  $g = \{12, 23, 45\}$  on a set of countries  $N = \{1, 2, 3, 4, 5\}$  represents situations where country 2 is allied to both 1 and 3, and 4 is allied with 5, and where no other alliances are present.

<sup>9</sup>Although it would be interesting to endogenize the strengths, for our purposes in this paper we take these as given. For a bilateral model of endogenous military strengths see Jackson and Morelli (2009).

## 2.3 Vulnerable Countries and Networks

We say that a country  $i$  is *vulnerable* at a network  $g$  if there exists  $C \in \mathcal{C}(g)$  such that  $i \notin C$  and  $M(C) > \gamma M(i \cup (N_i(g) \cap C^c))$ .

We say that a country  $j$  is a *potential aggressor* at a network  $g$  if there exists  $i$  and  $C \in \mathcal{C}(g)$  such that  $j \in C$  and  $i \notin C$  and  $M(C) > \gamma M(i \cup (N_i(g) \cap C^c))$ .

Note that a country can be both vulnerable and a potential aggressor at some networks.

In the above definitions, in order for a group of countries to attack, they must be a clique. However, if a clique  $C$  attacks some country  $i$ , then  $i$  can be defended by all of its allies that are not in the attacking clique  $j \in N_i(g) \setminus C$ . The idea behind these definitions is quite simple: alliances are a channel of communication between countries. If two countries are allied, they can coordinate on military operations. The restriction of attacking coalitions to cliques while allowing defense by all neighbors captures the fact that more coordination is needed when attacking, while defense might only require each neighbor only to lend aid to the attacked node. Of course, there are many other possible definitions, and ours serves as a starting point.

In the appendix, we show that our results extend if we change the definition to allow some country and any subset of its allies (not necessarily a clique) to attack, or even if we allow any connected component to attack, and similarly if we require that a country can only be defended by a clique.

The incentives of countries to attack or defend, are embodied in the definitions below. The above definitions just define the technology of war.

Thus, no country is vulnerable at a network  $g$  if for all cliques  $C \in \mathcal{C}(g)$ , and all countries not in the clique,  $i \notin C$ , the clique cannot successfully attack the country:  $M(C) \leq \gamma M(C')$  where  $C' = i \cup (N_i(g) \cap C^c)$  and  $C^c$  is the complement of  $C$ .

If some country is vulnerable, then a clique that can defeat the country and its remaining allies has an incentive to attack and defeat the country. This presumes that the benefits from defeating a country outweigh costs of war. If a country is not vulnerable were to be attacked then it and its allies would be successful in holding off the attackers. Implicit in the definition is that if the country and its allies could be successful in fending off an attack, then they would do so. Essentially, for now, we simply assume that winning a war is desired and losing a war is not. When we explicitly model trade and economics below, we will be more explicit about gains and losses.

## 2.4 Observations and Illustrations of Vulnerability

Before moving on to the main definitions and analysis, we present a few simple observations and illustrations of networks and vulnerability. The results are all straightforward, so we



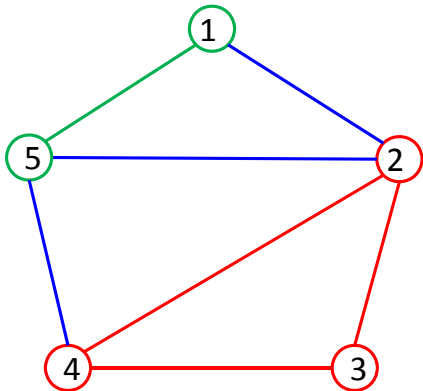


Figure 1: Example of an alliance network

omit proofs.

First, for an illustration of our definition of vulnerability, consider Figure 1.

In this network, country 1 is vulnerable if  $\gamma(M_1 + M_5) < M_2 + M_3 + M_4$ . Countries 2,3,4 form a clique and hence can attack country 1, which then has country 5 to defend it. Country 5 cannot join countries 2, 3, and 4 in attacking country 1 since it is not allied with country 3. In the case where all countries have equal strength, no country is vulnerable if and only if  $\gamma \geq 3$ , since 2,4,5 attacking either 1 or 3 is the most lopsided possible ratio of offensive to defensive strengths.

Let  $\overline{M} = \max_i M_i$  and  $\underline{M} = \min_i M_i$ .

**OBSERVATION 1.** *If  $M(N \setminus \{i\}) > \gamma \underline{M}$  for  $i = \operatorname{argmin} M_i$ , then some country is vulnerable in the complete network. If  $\overline{M} > \gamma \underline{M}$ , then any network which has no vulnerable countries is nonempty and incomplete.*

So, in most settings of interest, requiring that no country be vulnerable implies that networks will be “intermediate.”

**OBSERVATION 2.** *Suppose that  $N$  can be partitioned into cliques  $C_1, \dots, C_m$  with  $m \geq 2$  and such that  $M(C_k) \leq \gamma M(C_{k'})$  for all  $k, k'$ . No country is vulnerable in a network formed of such cliques only if  $M(C_k \setminus \{i\}) < \gamma M_i$  for  $i = \operatorname{argmin}_{j \in C_k} M_j$  for each  $k$ .*

Not having some country be vulnerable within a clique, at a minimum, requires that  $\gamma \geq h - 1$  where  $h$  is the size of the largest clique.

**OBSERVATION 3.** *Let  $d \geq \frac{2\overline{M}}{\underline{M}}$  be an integer. No country is vulnerable in any network such that each country has at least  $d$  alliances and there is 0 clustering (so no cliques of size 3 or more exist).*

Note that for such a network the maximum clique is of size 2 and so the maximum attack strength is  $2\overline{M}$ . Any attacked country has at least  $d - 1$  alliances not in the attacking coalition, so the minimum defending strength is  $d\underline{M}$ , and so by the condition on  $d$ , the attacking strength  $2\overline{M}$  does not exceed  $\gamma$  times the defending strength,  $\gamma d\underline{M}$ .

Note that the above descriptions of networks that have no vulnerabilities are far from exhaustive. For example, suppose that there are just two strengths  $\overline{M}$  and  $\underline{M}$  and an equal number of countries have each strength with  $n \geq 4$ . Then a “circle” network with alternating countries by strength has no vulnerable countries for any  $\gamma \geq 1$ . Or, suppose there are at least two strong and two weak, with an even number of weak. Divide the weak into pairs. Then connect every weak to every strong. Each pair of weak and a strong form a clique, and these are the only cliques. This network has no vulnerable countries for any  $\gamma \geq 1$ . The weak can also be put into cliques of larger size  $k > 2$  as long as  $\frac{(k-1)\underline{M} + \overline{M}}{\underline{M} + \overline{M}} \leq \gamma$ .

## 2.5 War-Stable Networks

We now introduce the concept of *war-stability* that accounts for countries’ incentives to add or delete alliances.

Define a network  $g$  to be *war-stable* if three conditions are met:

- S1** no country is vulnerable at  $g$ ;
- S2**  $\forall ij \notin g$ , no country is vulnerable at  $g + ij$  (no pair of countries have an incentive to add an alliance);
- S3**  $\forall ij \in g$ , both  $i$  and  $j$  are vulnerable at  $g - ij$  (the countries involved both have an incentive to maintain each alliance).

So,  $g$  is war stable if no pair of countries can destabilize the network and there are no superfluous alliances.

Why does the statement that no country is vulnerable at  $g + ij$  translate into no pair of countries have an incentive to add an alliance? Since, no country is vulnerable at  $g$ , then if some country is vulnerable at  $g + ij$  it would have to be that  $i$  and  $j$  are part of the attacking clique, as all other cliques are unchanged, and so their addition of the link must have given a clique that they become part of enough strength to defeat some other country  $k$ . So,  $i$  and

$j$  would have an incentive to add that link since there is then some country that they can conquer.

This definition is similar to that of pairwise stability of Jackson and Wolinsky (1996) in that we consider changes in the network one alliance at a time, and both additions or deletions - requiring two countries to consent to forming an alliance, but only one to breaking an alliance. One can enrich the definition in various directions, by allowing groups of countries to add alliances, countries to delete multiple alliances, payments for forming links, forward-forward looking countries, and so forth, but we focus on this definition here.<sup>10</sup>

### 3 Characterizing War-Stable Networks

For the case of 2 countries, it is direct to check that the only possible stable network is the empty network and it is incentive stable if and only if  $\gamma \geq \bar{M}M$ . Thus, we consider the more interesting case with  $n \geq 3$ .

The following theorem shows that there are no war-stable networks in basic cases of interest, even when all countries have equal strengths.

**THEOREM 1.** *Let  $n \geq 3$  and consider the uniform strength case of  $M_i = \bar{M} \forall i$ . If  $1 \leq \gamma < 2$ , then there are no war-stable networks. If  $\gamma \geq 2$ , then the unique war-stable network is the empty network.*

To get some intuition for the theorem, consider the network pictured in Figure 2.

For the network have no vulnerable countries ([S1]), it must be  $\gamma \geq 1$ , so that, eg, countries 1 and 2 cannot successfully attack country 4 (which would then be defended by 5). In fact, it is easy to check the network pictured has no vulnerable countries if and only if  $\gamma \geq 1$ . Next, note that for no countries to wish to drop an alliance, it must be that  $\gamma < 2$  (so that, for instance, then if country 3 drops its link to country 4, country 3 will be vulnerable to an attack by countries 1 and 2). So, in order to ensure that the network is weakly stable and that no countries wish to drop an alliance, it must be that  $1 \leq \gamma < 2$ . However, this is incompatible with the condition that no two countries want to add an alliance. Consider the deviation of countries 2 and 5 creating an alliance and attacking country 1, as in Figure 3. To prevent this, it would have to be that  $\gamma \geq 2$ , which is incompatible with the condition that no two countries want to delete an alliance, and so the ring network is not war-stable.

This is just one possible network, and the challenge is in showing that this is not just true for a ring network, but is in fact true of *all* nonempty networks when  $\gamma \geq 1$ . For example, the clique of 5 countries in Figure 4 is also unstable. If  $\gamma < 4$  then any 4 countries can defeat

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<sup>10</sup>See Jackson (2008); Bloch and Jackson (2006) for overviews of alternative network formation models.

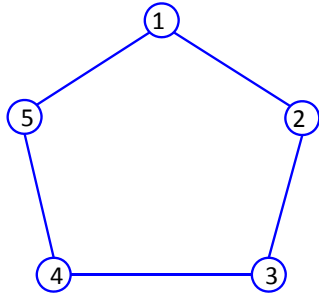


Figure 2: Unstable Ring Network

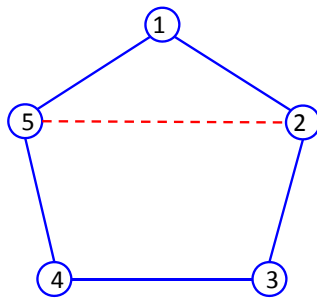


Figure 3: Unstable Ring Network

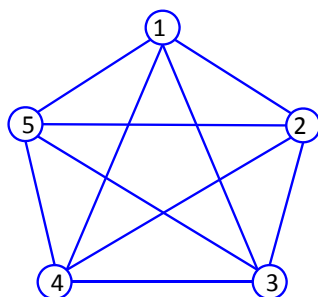


Figure 4: Unstable Clique of 5



Figure 5: Stable Alliance Network

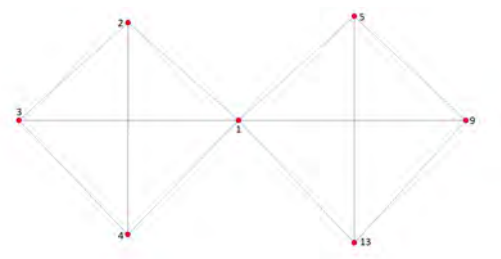


Figure 6: Close-up of Country 1 and Its Allies

the remaining country, while if  $\gamma \geq 4$  then any country could delete an alliance and still not be vulnerable. These are just two examples, but show some of the issues.

The proof is involved and uses a combinatorial pigeonhole argument, showing that certain sorts of clique configurations and imbalances are possible in all nonempty graphs and for all levels of  $\gamma \geq 1$ .

The restriction to  $\gamma \geq 1$  is important. If the offense has a substantial advantage, there are many war-stable networks. At first blush it might be surprising that a world where attackers have an advantage over defenders leads to more stability, but it can be understood as follows. An offensive advantage provides incentives for countries to maintain alliances, as without alliances countries easily become vulnerable. This allows one to build up networks of alliances that are denser. The key to then getting stability is to have each country be involved in several separate cliques, so that no attacking clique is large enough to overcome the country and its other allies.

One such war-stable configuration is pictured in Figure 5.

Figure 6 is a subgraph of Figure 5 focusing on country 1, its neighbors, and the cliques

it is involved in (note that each other node in 6 has 3 other neighbors in the larger graph, with each pair of countries one from the 1-2-3-4 clique and the other from the 1-5-9-13 clique sharing one neighbor in common). The network is isomorphic, so that country 1 is perfectly representative of what is faced by all countries. It can be easily verified that this network has no vulnerable countries if  $\gamma \geq \frac{3}{4}$ . For instance if country 1 were attacked by 2,3,4, it would be defended by 5,9,13 and so there would be 3 attackers and four defenders (counting 1) - and the defenders would win if  $\gamma \geq \frac{3}{4}$ . Any other cliques outside of 1's neighbors would face 7 defenders if they attacked 1, and so would also lose. It is easy to check that this network is stable against the addition of new links, since adding a new link does not increase the size of any clique, it just adds a new pair and pairs cannot win when attacking any country. Thus, [S2] is easily checked. So, it remains to check [S3]. If country 1 drops one of its links, e.g. with country 2, it will be vulnerable if  $\gamma < 1$  (country 1 could be attacked by countries 5, 9, and 13 and only defended by 3 and 4). Since similar arguments can be made for every other node (so that [S3] is satisfied for the graph as a whole), if  $\gamma \in [\frac{3}{4}, 1)$ , the network is stable.

Similar examples can be constructed for even lower  $\gamma$ s, by having countries be part of more separate cliques. For instance having each country be part of 3 separate cliques of size 4 (adding one more clique to the right hand side of Figure 5 and for every country) would lead to a stable network for  $\gamma \in [\frac{3}{7}, \frac{1}{2})$ . By varying the sizes of the cliques and the number of cliques that each country is in involved in, even arbitrarily small  $\gamma$ s nonempty stable networks can be found for large enough  $n$ .

**PROPOSITION 1.** *For any  $\gamma < 1$ , there exists a large enough  $n$  such that there is a nonempty war-stable network in which every country has at least one alliance.*

Proposition 1 provides an interesting contrast to Theorem 1. With an offensive advantage, stable networks exist as the offensive advantage provides incentives for countries to maintain relationships as a deterrent that they might sever in conditions where there is a defensive advantage.

## 4 War and Trade Stable Networks

As we have seen, in cases where there is no offensive advantage, purely military considerations do not lead to stable networks, since countries only maintain alliances if they serve a purpose, which leads to sparse networks, but then those are destabilized by the addition of new alliances. As we now show, accounting for economic incentives associated with gains from trade can restore stability.

A country  $i$  gets a payoff or utility from the network  $g$  given by  $u_i(g)$ . This represents the economic benefits from the trade that occurs in the network  $g$ .

As mentioned in the introduction, a link now represents both a trading relationship and potential to coordinate military activities.<sup>11</sup>

Each country also has economic resources  $E_i(g)$ , that could also depend of the network. In particular, what we include in  $E_i(g)$  are the resources or economic gains that would accrue to conquerors. For example these include natural resources or other potential spoils of war.<sup>12</sup>

We now introduce a concept of vulnerability based on the incentives of countries to attack others when explicitly accounting for the benefits and costs associated with conquering a country.

The resources that are destroyed in war are captured via a parameter  $\delta \in [0, 1]$ , which represents the proportion of economic resources of a country that are gained by others if they conquer the country.

We say that a country  $i$  is *vulnerable despite trade* in a network  $g$  to a clique  $C \in \mathcal{C}(g)$  if  $i \notin C$  and

- $M(C) > \gamma M(i \cup (N_i(g) \cap C^c))$  (i.e.,  $C$  could conquer  $i$ ), and
- $u_j(g - i) + \delta \frac{E_i(g)}{|C|} \geq u_j(g) \forall j \in C$  with some strict inequality: every  $j \in C$  would benefit from conquering  $i$ , given only  $\delta$  of  $i$ 's wealth can be redistributed.

The second item is new to this definition of vulnerability and incorporates three aspects of economic incentives of countries to attack each other:

First,  $\delta$  captures the amount of  $i$ 's resources that are actually gained by the attacking coalition, and so now explicitly includes the costs of war and fraction of the potential spoils that may be gained by attackers. Wars that are likely to be more destructive are less attractive.

Second, the  $\frac{E_i(g)}{|C|}$  represents the potential benefits that  $j$  enjoys from conquering  $i$ . This presumes an equal split of  $i$ 's resources among the members of the attacking clique  $C$ . If a country is poor in natural resources, and much of its economy is built upon nontransferable or difficult to extract human capital, it would tend to have a lower  $E_i$  and would be less attractive.

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<sup>11</sup>Mansfield and Bronson (1997) examine correlations between alliances, trade, and participation in preferential trading agreements over the period of 1960 to 1990. They find that alliances (and participation in a preferential trading agreement) lead to increased bilateral trade, with the effect being considerably larger when the pair of countries have both an alliance and mutual participation in a preferential trading agreement. Interestingly, this relationship differs in the recent 1960-1990 period compared to pre-World War II. Long and Leeds (2006), looking at pre-World War II Europe, finds that trade between allies is only statistically larger than trade between non-allied countries when economic provisions are explicitly mentioned in the alliance. This in fact consistent with our analysis in that opportunities for trade were substantially limited in pre-World War II Europe, and so the economic trade incentives emerge to a much greater extent in the 1950s and thereafter when costs of trade begin to plummet and incomes increase and trade grows significantly. Regardless of the relationships between *explicit* alliances and trade, the open lines of communication are what is essential for this theory.

<sup>12</sup>For an important discussion of the spoils of inter-state wars, see Caselli et al. (2012).

Third, the  $u_j(g-i)$  accounts for the fact that as  $i$  is conquered then the network of trade will adjust. If  $j$  is a trading partner of  $i$ , then  $j$  could lose via the elimination of  $i$ , with  $u_j(g-i) < u_j(g)$ .<sup>13</sup> Note that this effect works both ways: it might also be that a country  $j$  benefits from the elimination of some country  $i$ , for instance if it improves  $j$ 's position in the resulting trade network.

With this framework, we now define a stability notion corresponding to war stability but adding the economic considerations.

Our definition of war and trade stability now incorporates two incentives for adding or deleting alliances. First, countries might add or maintain an alliance because of its military value in either preventing a war or in instigating one, just as with war stability. This is similar to what we considered before, except that countries now consider the economic spoils and consequences of war in deciding whether to take part in an attack. Second, countries add or maintain an alliances for the economic benefits in terms of trade.

Let us now consider the incentives for countries to add an alliance and attack another country.

Starting from a network  $g$ , some alliance  $ij \notin g$  is *war-beneficial* if there exists some  $C \in \mathcal{C}(g+ij)$  with  $i \in C$ ,  $j \in C$  and  $k \notin C$  such  $k$  is vulnerable despite trade to  $C$  at  $g$  and

- $u_j(g+ij-k) + \delta \frac{E_i(g+ij)}{|C|} \geq u_j(g)$ , so,  $j$  would benefit from forming the link and attacking, and
- $u_i(g+ij-k) + \delta \frac{E_i(g+ij)}{|C|} \geq u_i(g)$ , similarly for  $i$ , with one of these inequalities holding strictly.

We say that a network  $g$  is  *$\delta$ -war and trade stable* if three conditions are met:

**ES1** no country is vulnerable despite trade at  $g$ ;

**ES2**  $\forall ij \notin g$ : if  $u_i(g+ij) > u_i(g)$  then  $u_j(g+ij) < u_j(g)$ , and also  $ij$  is not war-beneficial

**ES3**  $\forall ij \in g$  either  $u_i(g-ij) \leq u_i(g)$  or  $i$  is vulnerable despite trade at  $g-ij$ , and similarly for  $j$ .

So,  $g$  is war and trade stable if no country is vulnerable despite trade, if no two countries can add an alliance and both profit either through economic or war means, and either economic or war considerations prevent any country from severing any of its alliances.

**REMARK 1.** *If  $u_i(\cdot)$  is constant for all  $i$ , and  $E_i(g) > 0$  for all  $i$  and  $g$ , then war and trade stability reduces to war stability.*

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<sup>13</sup>As Glick and Taylor (2010) documents, the economic loss resulting from trade disruption during wars can be of the same order as more traditional estimates of losses resulting from interstate conflict. This does not even account for the potential loss of trade if a partner is lost altogether.



We say that a network  $g$  is *strongly war and trade stable* if it is *war and trade stable* and this holds regardless of the specification of the  $E_i$ 's.

#### 4.0.1 Results and Examples of War and Trade Stable Networks

Let us examine the set of war and trade stable networks. We begin by identifying a simple sufficient condition.

**PROPOSITION 2.** *Suppose that  $g$  is pairwise stable with respect to  $u$ . Suppose also that no country is vulnerable despite trade at  $g$  or  $g + ij$  for any  $ij \notin g$ . Then  $g$  is war and trade stable.*

This leads to the following corollary.

**COROLLARY 1.** *Suppose that  $g$  is pairwise stable with respect to  $u$ . If no country is vulnerable despite trade at  $g$  or  $g + ij$  for any  $ij \notin g$ , then  $g$  is strongly war and trade stable.*

There are many examples of networks that are war and trade stable but not war stable. The following proposition outlines a whole class of war and trade stable networks, showing that economic considerations restore general existence results.

We consider a canonical case in which  $u_i(g) = f(d_i(g)) - c \cdot d_i(g)$ , where  $d_i(g)$  is the degree of  $i$  and  $f$  is concave, nondecreasing, and that there exists some  $d \leq n - 1$  such that  $f(d) < c \cdot d$ . This is a simple model of gains from trade and costs of having trading relationships, abstracting from heterogeneity in goods and trading partners and inter-dependencies in trading relationships beyond diminishing returns. Let  $d^*$  maximize  $f(d) - c \cdot d$  among nonnegative integers.

**THEOREM 2.** *If  $d^* \geq 4$  then any network in which all countries have  $d^*$  alliances, the largest clique is of size at most  $\lceil \frac{d^*}{2} \rceil$ , and any two cliques intersect in at most one node is strongly war and trade stable for any  $\gamma \geq 1$ .<sup>14</sup>*

Note that Theorem 2 provides an existence result that applies regardless of the structure of the economic values ( $E_i$ 's), the relative costs of wars (captured in  $\delta$ ), or details of the gains from trade beyond the optimal number of trading partners  $d^*$ . As long as there exist enough countries so that each country has  $d^*$  neighbors, without overly large cliques, then there exist strongly war and trade stable networks.

A particularly interesting class of strongly war and trade stable networks is one that is built up from a set of cliques, called quilts.

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<sup>14</sup>With  $d^* = 3$ , the same proposition holds with  $\gamma \geq 3/2$  and with  $d^* = 2$  it then it moves to  $\gamma \geq 2$ .

A network is said to be a *quilt* if all nodes have at least two links and the network can be written as a union of cliques, each of size at least 3, and such that any two cliques share at most one node in common.<sup>15</sup>

**COROLLARY 2.** *If  $d^* \geq 4$  then any quilt in which all countries have  $d^*$  alliances and the largest clique is of size at most  $\lceil \frac{d^*}{2} \rceil$  is strongly war and trade stable for any  $\gamma \geq 1$ .*

Quilts are of interest as we see their underpinnings in, for example, the network in Figure 14, consists of a number of cliques that have some small overlap. These sorts of networks can emerge naturally given geographic biases in benefits from trading with nearby countries, as discussed below.

An example illustrating Corollary 2 appears in Figure 5 in which  $d^* = 6$ , and which is in fact strongly war and trade stable for all  $\gamma \geq \frac{3}{4}$ .

The quilts take advantage of the fact that no clique is large enough to outweigh any country and its remaining allies, and yet the alliances are held in place by trading concerns so that no alliances are deleted. Thus, the quilts leverage the economic incentives to maintain links, thereby overcoming the sparsity issues that can prevent stability.

Another aspect that trade incentives bring is that they reduce incentives for members of a clique to follow through with attacks. For example, there are networks that are war and trade stable, but for which there is a  $ij \notin g$  such that some country is vulnerable despite trade at  $g+ij$ . For instance, let  $d^* = 3$  and the number of nodes be 6. Let  $g$  be formed of two triangles, with each node in one triangle connected to exactly one node in the other triangle. In particular, consider network  $g = \{12, 13, 23, 45, 46, 56, 14, 25, 36\}$ , as pictured in Figure 7. Let  $\gamma < \frac{3}{2}$ . Then, adding another link between the two triangles would make some country vulnerable despite trade at  $g + ij$  (with an attacking coalition consisting of two nodes from one triangle and one node from the other attacking the third node from the first triangle). In particular, if 15 is added, then 125 can defeat 3 who only has ally 6 remaining. However, there are several possible deterrents to this attack. One is that  $\delta$  is low enough so that the gains from conquering 3 are not worth it for 2 who then only has 2 trading partners in the resulting network. Similarly it might be that  $E_3$  is small. Another possibility is that 5 might not wish to have four allies instead of three, and that the resulting gains from conquering country 3 are not worth the cost of an additional ally.

## 5 Concluding Remarks

In the context of the alliances that we have analyzed, trade motives are essential to avoiding wars and sustaining stable networks.

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<sup>15</sup>This definition of quilts differs slightly from that of social quilts introduced by Jackson et al. (2012) in that larger cycles are permitted here.

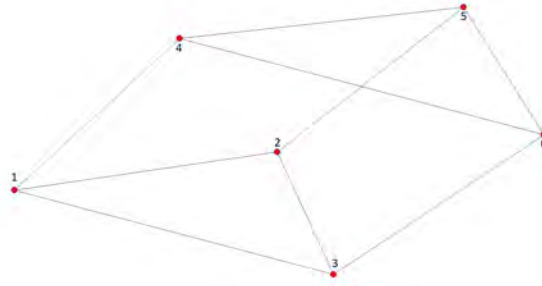


Figure 7: An alliance network

In closing, we briefly discuss some of what is known regarding trade and wars and then comment on some issues for further research.

## 5.1 Empirics of Trade and Wars

### 5.1.1 Trends of Wars and Conflicts

The relationship between trade and wars is a complicated one. One thing that is evident, is that the number of wars per country has decreased dramatically post World War II, and that this decrease comes even though the number of countries has increased - so that there are many more pairs of countries that could be going to war. For example, the average number of wars per pair of countries per year from 1820 to 1959 was .00056 while from 1960 to 2000 it was .00005, less than a tenth of what it was in the previous period.<sup>16</sup> We see this in Figure 8

### 5.1.2 Trade

International trade has had two major periods of growth over the last two centuries, one in the latter part of the nineteenth century and beginning of the twentieth, disrupted by the first world war, and then picking up again after the second world war, recovering to its 1914 levels through the 1960s and then continuing to grow at an increasing rate thereafter. In particular, Estevadeordal et al. (2003) finds that trade per capita grew by more than 1/3 in each decade from 1881 to 1913, while it grew only 3 percent per decade from 1913 to 1937.

<sup>16</sup>This finding is robust to when the cut takes place: from 1850 to 1949 it was .00059 while from 1950 to 2000 it was .00006, from 1850 to 1969 it was .00053 while from 1970 to 2000 it was .00005. If one looks at wars per country instead of per pair of countries, then from 1820 to 1959 it was .012 while from 1960 to 2000 it was .004. One could also include all Militarized Interstate Disputes (MID2-5) instead of just wars (MID5s - involving at least 1000 deaths). In that case, from 1820 to 1959 there are .006 MID2-5s per pair of countries while from 1960 to 2000 there were .003.

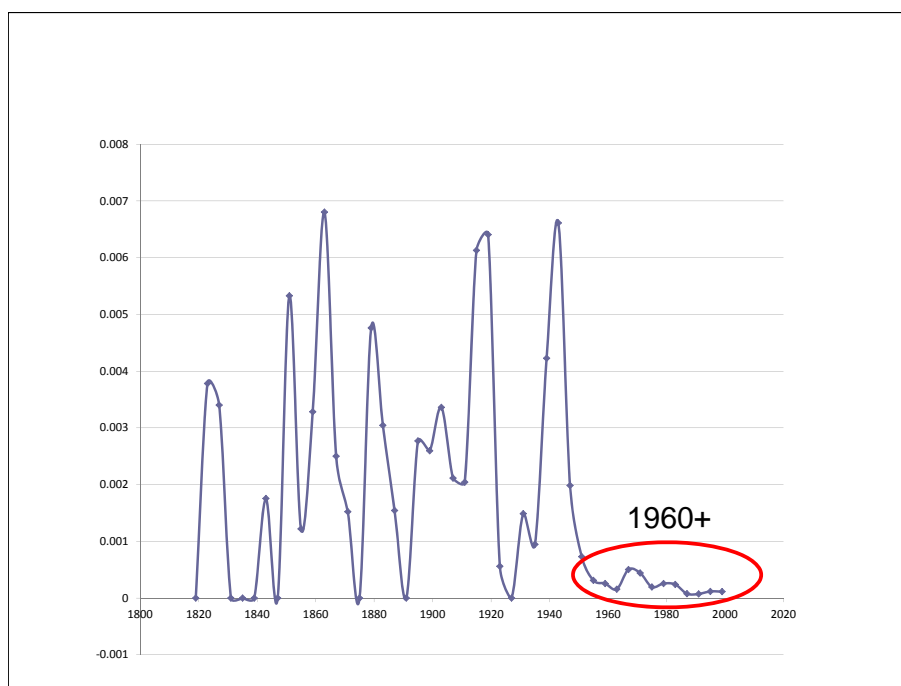


Figure 8: Wars per pair of countries by year, 1820-2000. (COW MID5 data)

Table 1, from Krugman (1995),<sup>17</sup> provides a view of this dynamic.<sup>18</sup>

Table 1: World merchandise exports as percent of GDP: Krugman (1995)

Year	1850	1880	1913	1950	1973	1985	1993	2012
Percent	5.1	9.8	11.9	7.1	11.7	14.5	17.1	25.3

The trade has been further bolstered or accompanied by the advent of container shipping as well as increases in world per capita income. Hummels (2007) looks at the interaction between transportation costs and international trade, while Bernhofen et al. (2013) and Rua (2012) investigate the rise of containerization and its spread through international shipping. The relative correlations between income and trade and transportation costs and trade have been open to some debate. Baier and Bergstrand (2001), looking at trade between OECD countries from the late 1950s through the late 1980s, argues that decreasing transportation

<sup>17</sup>The figure for 2012 is directly from the World Bank indicator (<http://data.worldbank.org/topic/private-sector?display=graph>, December 11, 2013), from which Krugman (1995) quotes the other numbers.

<sup>18</sup>Dean and Sebastia-Barriel (2004) provide an overview of changes in the level of world trade in relation to world output over the course of the 20th century, while Estevadeordal et al. (2003) looks at the period 1870 to 1939.

costs explains 8 percent of the growth in trade, with the lion's-share of the increase (67 percent) correlating with increased incomes. Regardless of the source, trade has increased dramatically over time, and especially post World War II, where it has increased by almost a factor of four.

### 5.1.3 Relations Between Trade and Wars

So, the decrease in wars, is mirrored by an increase in trade. The percentage of trade varies mainly between 5 and 12 percent from 1850 to 1959 and between 12 and 25 percent from 1960 onwards.

These numbers cannot be taken as evidence for the theory. There are many confounding variables in the relationship between trade and wars, so although there was an unprecedented growth in trade post World War II, coincident with an unprecedented drop in the frequency of wars, there was also a cold-war and many technological changes (the advent of nuclear weapons, for instance) as well as an increase in income and wealth levels world-wide, which make it difficult to test the theory directly.<sup>19</sup> Moreover, one could also hypothesize that the absence of war led to the increase in trade instead of the reverse. Thus, although we do see a strong correlation that is in line with what our theory would suggest, there are many confounds which make causation impossible to infer, and there may be multiple forces at work. Nonetheless, the theory provides one possible explanation.

There are many papers that have investigated the empirical relationship between conflict and trade at a more dyadic level, and as one might expect causation and the specifics of the relationships are difficult to disentangle. Indeed, Barbieri (1996) – investigating the period 1870 to 1938 in Europe and including conflicts that fall substantially short of war – find that although low to moderate levels of economic interdependence may be accompanied by a decrease in military conflicts; high levels of economic interdependence can be accompanied by increased incidence of conflicts. This is further nuanced, as Martin et al. (2008) – looking at trade and militarized disputes over the period 1950-2000 – find that an increase in bilateral trade between two countries correlates with a decreased likelihood of these countries entering military dispute with each other, while an increase in one of the country's multilateral trade (i.e. an overall increase in a country's trade share without an increase in the bilateral trade between the two countries) leads to an increased likelihood of war between the pair. The definition of dispute is broader than that of war and could include posturing for bargaining

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<sup>19</sup>The advent of nuclear weapons changes the technology of war, potentially providing a large defensive advantage if a defending country has nuclear weapons, and a large offensive advantage if an attacking country has such weapons and a defender does not. This could amplify gains from trade in parts of the developed world to lead to greater stability there, and could also explain why remaining recent wars tend to involve at least one country without nuclear capabilities. This raises another question of the endogeneity of arms, which is another interesting issue (e.g., see Baliga and Sjöström (2004); Jackson and Morelli (2009)).

purposes, or simply the increase in contact that accompanies trade leading to an increase in minor incidents.

In summary, although the broad data are consistent with the theory, establishing a causal relationship would require finding appropriate exogenous variation that could be used to test the theory, or to enrich the theory with a detailed modeling of the specific gains from trade and war technology. A study of this kind does not yet exist, and is an important one for future research for which our model provides explicit hypotheses.

## 5.2 Other Issues for Further Research

Starting with a purely militaristic model of networks of alliances, we've found that stable networks fail to exist. We then include economic considerations, and show that with sufficient benefits from international trade, stability is restored.

There are two obvious ways in which to enrich the model. First, one can enrich the modeling of trade. There are many ways to introduce heterogeneity, for instance along the lines of Dixit and Stiglitz (1977); or else, capturing the complexity of trade dynamics as discussed in Gowa and Mansfield (2004), Long and Leeds (2006), and Mansfield and Bronson (1997). Second, and relatedly, there is the question of geography. Both trade and war have strong relationships with geography (see, e.g., Eaton and Kortum (2002)), as well as Caselli et al. (2012), who find that from 1945 to 1987 eight six percent of significant international wars were between neighboring states). Geography constrains both the opportunities and benefits from trade and war, and so it has ambiguous effects on stability. Nonetheless, it plays an important role in explaining realized networks of trade and alliances and deserves further attention.

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# Appendix: Proofs

## Proof of Theorem 1:

The conditions on stability can be recast as requirements on the  $\gamma$  parameter. Namely, the first condition that no country be vulnerable, [S1], translates as:

$$\gamma \geq \max_{C \in \mathcal{C}(g)} \left( \max_{i \in C^c} \frac{M(C)}{M(i \cup (N_i(g) \cap C^c))} \right). \quad [\text{S1}] \quad (1)$$

The second condition that no additional link leads to [S2] translates as:

$$\gamma \geq \max_{jk \notin g} \left( \max_{C \in \mathcal{C}(g+jk)} \left( \max_{i \in C^c} \frac{M(C)}{M(i \cup (N_i(g+jk) \cap C^c))} \right) \right). \quad (2)$$

Note that given (1), we need only check (2) with respect to  $C$  such that  $j \in C$  and  $k \in C$ . Thus, we can change the denominator in (2) to be  $M(i \cup (N_i(g) \cap C^c))$ . Therefore, stability implies that

$$\gamma \geq \max_{jk \notin g} \left( \max_{C \in \mathcal{C}(g+jk)} \left( \max_{i \in C^c} \frac{M(C)}{M(i \cup (N_i(g) \cap C^c))} \right) \right). \quad [\text{S2}] \quad (3)$$

The third condition [S3] translates as (providing  $g$  is nonempty):

$$\gamma < \min_{ij \in g} \left( \min \left\{ \max_{C \in \mathcal{C}(g-ij)} \frac{M(C)}{M(i \cup (N_i(g-ij) \cap C^c))}, \max_{C \in \mathcal{C}(g-ij)} \frac{M(C)}{M(j \cup (N_j(g-ij) \cap C^c))} \right\} \right). \quad [\text{S3}] \quad (4)$$

We now specialize to equal strengths.

**CLAIM 1.** *There does not exist a non-empty war stable network with maximum degree less than 3.*

**Proof of Claim 1:** Consider a network with a maximum degree of 1. If  $\gamma < 1$ , then the network must violate [S1], since a (strongest) country in any linked pair can defeat the other country. So, consider the case in which  $\gamma \geq 1$ . Let  $j$  be the weakest country. Let  $i$  either be the ally of  $j$ , or else some other country if  $j$  has no allies. It follows that  $\gamma \geq 2$ , as otherwise  $i$ , together with some country  $k$  different from  $i$  and  $j$  that is either an existing ally of  $i$ 's or by forming a new link  $ik$ , could defeat  $j$ , which would violate [S1] or [S2] respectively. However,  $\gamma \geq 2$  implies that the network cannot be war stable. This is seen as follows. Consider the strongest country  $i$  that has positive degree. Either  $i$  can sever its link violating [S3], or else (given that  $\gamma \geq 2$  and  $i$  is the strongest among those having connections and cliques are at most pairs) it must be that there is some country  $k$  that has no ties that could defeat  $i$  if  $i$  severed its link to its ally  $j$ . However, then by adding  $ik$  they would defeat  $j$  (since  $j$  is no stronger than  $i$  and  $i$  would be defeated by  $k$  when  $k$  is all alone) violating [S3].

Next, consider a network  $g$  with maximum degree two. So there exists a country  $j$  and links  $ij \in g$  and  $jk \in g$ . First, consider the case in which  $ik \in g$ . Then for the network to be war stable it must be that  $\gamma \geq 2$  by consideration of the strongest two countries, say  $i$  and  $j$ , not attacking  $k$  under [S1].

Given that the biggest clique is of size 3 and  $\gamma \geq 2$ , then any country  $i$  with degree 2 could sever one of its links and not be vulnerable (its remaining ally cannot be part of any clique of size more than 2, and any clique of size 3 could not defeat  $i$  and its remaining ally). Thus there is no country with degree 2 if the maximum degree is 2, which is a contradiction.

Thus, it must be that the maximum degree is at least three. ■

**CLAIM 2.** *Consider  $i$  of maximum degree and some  $ij \in g$ . There exists  $C \in \mathcal{C}(g - ij)$  such that*

$$\gamma < \frac{M(C)}{M(i \cup (N_i(g - ij) \cap C^c))} \quad (5)$$

and every such  $C$  satisfies  $C \cap N_i(g - ij) \neq \emptyset$  and  $i \notin C$  and  $j \notin C$ .

**Proof of Claim 2:**

We know from [S3] there exists  $C \in \mathcal{C}(g - ij)$  such that

$$\gamma < \frac{M(C)}{M(i \cup (N_i(g - ij) \cap C^c))}.$$

Suppose that some such  $C$  has  $C \cap N_i(g - ij) = \emptyset$ . This implies that  $|C| > \gamma d_i$ , and since  $\gamma \geq 1$  and  $d_i$  is maximal, this implies that  $|C| = d_i + 1$ . However, this is a contradiction since then all but one member of  $C$  can defeat a remaining member (who necessarily has degree  $d_i$  and thus only has connections to other members of  $C$ ). This follows since  $d_i > \gamma$  given that  $|C| = d_i + 1 > \gamma d_i$  and  $d_i \geq 1$ .

Thus, any  $C \in \mathcal{C}(g - ij)$  satisfying (8) must satisfy  $C \cap N_i(g - ij) \neq \emptyset$ . The fact that  $i \notin C$  is by definition, and that  $j \notin C$  is that otherwise we would violate [S1] (as  $C$  would defeat  $i$  in the network  $g$  with  $ij$  present). ■

**CLAIM 3.** *Consider  $i$  of maximum degree and some  $ij \in g$ . Consider any  $C \in \mathcal{C}(g - ij)$  such that*

$$\gamma < \frac{M(C)}{M(i \cup (N_i(g - ij) \cap C^c))}.$$

It follows that

$$C \subset N_i(g)$$

and that

$$|C| = \lceil \frac{\gamma d_i}{1 + \gamma} \rceil.$$

Moreover, for any  $C \in \mathcal{C}(g)$  with  $i \notin C$ ,

$$|(C \cap N_i(g))| \leq \lceil \frac{\gamma d_i}{1 + \gamma} \rceil.$$

**Proof of Claim 3:**

Let  $x = |C \cap N_i(g)|$ , and let  $y = |C \cap N_i(g)^c|$  be the number of members of  $C$  who are not connected to  $i$ . Then  $\gamma < \frac{M(C)}{M(i \cup (N_i(g-i) \cap C^c))}$  implies that

$$x + y > \gamma(1 + d_i - 1 - x) = \gamma(d_i - x). \quad (6)$$

Let  $k \in C \cap N_i(g)$  (by Claim 2). [S1] implies that the remaining members of  $C$  cannot defeat  $k$  and so:

$$x + y - 1 \leq \gamma(d_k + 1 - x - y).$$

The fact that  $d_k \leq d_i$  ( $i$  is of maximal degree) and the two above inequalities imply that

$$\gamma(d_i - x) - 1 < \gamma(d_i + 1 - x - y),$$

or  $\gamma(y - 1) < 1$ . Given that  $\gamma \geq 1$  and  $y$  is an integer,  $\gamma(y - 1) < 1$  implies that  $y \leq 1$ . Now, let us argue that  $y = 0$ . Suppose to the contrary that  $y = 1$ . Let  $k'$  be the node in  $C \cap N_i(g)^c$ . Consider the network  $g + ik'$ , and the clique of  $C' = (C \setminus \{k\}) \cup \{i\}$ , and note that  $|C'| = |C|$ . By (6) with  $y = 1$  and  $x = |C| - 1$ , we know that  $|C| > \gamma(d_i - (|C| - 1))$ . But then, since  $i$  has maximal degree and  $|C'| = |C|$ , it follows that  $|C'| > \gamma(d_k - |C'| + 1)$ . However, this contradicts [S2], since then  $i$  and  $k'$  can form a link and the resulting clique  $C'$  defeats  $k$ .

Next, using (6) and  $y = 0$  it then follows that

$$x(1 + \gamma) > \gamma d_i$$

or

$$x > \frac{\gamma d_i}{1 + \gamma}.$$

Given that  $\gamma \geq 1$  and  $x$  is an integer, this implies that

$$x = |C| \geq \lceil \frac{\gamma d_i}{1 + \gamma} \rceil. \quad (7)$$

To see the last part of the claim, let  $z = |(C \cap N_i(g)) \setminus \{i\}|$ . By [S1] (with  $C$  not defeating  $i$ ):

$$z \leq |C| \leq \gamma(d_i + 1 - z)$$

and so

$$z \leq \frac{\gamma(d_i + 1)}{1 + \gamma},$$

which, given that  $z$  is an integer, implies that

$$z = |(C \cap N_i(g)) \setminus \{i\}| \leq \lceil \frac{\gamma d_i}{1 + \gamma} \rceil.$$

as claimed.

The second part of the claim then follows from the last part of the claim and (7). ■

**CLAIM 4.** *Consider  $i$  of maximal degree and  $ij \in g$ . It must be that*

$$N_i(g) \setminus \{j\} \neq N_j(g) \setminus \{i\}.$$

**Proof of Claim 4:**

Consider  $i$  of maximum degree and some  $ij \in g$ . By Claim 2, there exists  $C \in \mathcal{C}(g - ij)$  such that

$$\gamma < \frac{M(C)}{M(i \cup (N_i(g - ij) \cap C^c))} \quad (8)$$

and every such  $C$  satisfies  $C \cap N_i(g - ij) \neq \emptyset$  and  $i \notin C$  and  $j \notin C$ .

By Claim 3,  $C \subset N_i(g - ij)$ . If  $N_i(g) \setminus \{j\} = N_j(g) \setminus \{i\}$ , then  $C \cup \{j\}$  is also a clique. But then  $|C \cup \{j\}| > |C|$ , and we violate the last part of Claim 3. ■

**CLAIM 5.** *There are no nonempty war stable networks.*

**Proof of Claim 5:**

Let  $i$  be of maximum degree. In order to satisfy [S3], it must be that for each  $j \in N_i(g)$  there exists  $C_j \in \mathcal{C}(g)$  such that

$$\gamma < \frac{M(C)}{M(i \cup (N_i(g - ij) \cap C^c))}.$$

Moreover, it follows from Claim 2 that each  $C_j$  can be taken to lie in  $\mathcal{C}(g - ij)$ . From Claim 3, each such  $C_j$  is such that  $C_j \subset N_i(g)$  and  $|C_j| = \lceil \frac{\gamma d_i}{1 + \gamma} \rceil > \frac{d_i}{2}$ .

Moreover, by Claim 3 it must be that for each  $j' \in N_i(g)$ ,  $\cup_{C_j \ni j'} C_j \neq N_i(g)$ . This follows since  $i$  is of maximum degree and otherwise this would imply that  $N_{j'}(g) \setminus \{i\} = N_i \setminus \{j'\}$ , contradicting Claim 4.

Finding such sets  $C_j$  for each  $j$  in  $N_i(g)$  becomes the following combinatorics problem: create subsets  $\{C_1, C_2, \dots, C_S\}$  of a set  $M = \{1, 2, \dots, d_i\}$  of  $d_i$  elements (the neighbors of  $i$ ) such that:

1.  $\forall C_s, |C_s| = x > \frac{d_i}{2}$ ,

2.  $\forall j \in M, \exists C_s$  such that  $j \notin C_s$ ,
3.  $\forall j \in M, \cup_{C_s \ni j} C_s \neq M$ , and
4.  $\nexists D \subset M$  such that  $\forall \{k, j\} \subset D, \exists C_s$  such that  $\{k, j\} \subset C_s$  and  $|D| > x$ .

4 follows from Claim 3 as otherwise  $D$  would be a clique of size larger than  $x = \lceil \frac{\gamma d_i}{1+\gamma} \rceil$ .

We now show that such a collection of subsets is not possible. To do this, we start with just the set  $C_1$  and see what implications hold as we consider each additional  $C_s$ , ultimately reaching a contradiction. For reference, we introduce the three new series of sets:  $\{W_s\}_{s=1}^S, \{Y_s\}_{s=1}^S$ , and  $\{Z_s\}_{s=1}^S$ .  $W_s$  are the set of elements of  $M$  which have been in at least  $\{C_1, \dots, C_s\}$  (i.e.  $W_s = \cup_{i=1}^s C_s$ ).  $Y_s$  are the set of elements of  $M$  which have been in all of  $\{C_1, \dots, C_s\}$  (i.e.  $Y_s = \cap_{i=1}^s C_s$ ).  $Z_s$  are the set of elements of  $M$  which have been in none of  $\{C_1, \dots, C_s\}$  (i.e.  $Z_s = M \setminus W_s$ ).

Let us now complete the proof. Note that, if a set of subsets  $\{C_1, \dots, C_S\}$  satisfying 1-4 existed, then  $Y_S = \emptyset$  follows from point 2 since every element of  $M$  has some  $C_s$  that doesn't contain it. Note also that with each additional  $C_s$ ,  $W_s$  (weakly) grows larger while  $Y_s$  and  $Z_s$  (weakly) grow smaller.

To complete the proof we show that

$$|Y_{s-1} \setminus Y_s| \leq |Z_{s-1} \setminus Z_s|$$

and that

$$|Y_1| > |Z_1|.$$

Together these imply that  $Y_S \neq \emptyset$ , which is then a contradiction.

We start with  $Y_1 = W_1 = C_1$ . Thus,  $|Y_1| = |W_1| = x > d_i - x = |Z_1|$  since  $x > \frac{d_i}{2}$ . So, let us show that  $|Y_{s-1} \setminus Y_s| \leq |Z_{s-1} \setminus Z_s|$ . At each subsequent addition of a  $C_s$ , either  $C_s \cap Y_{s-1} = Y_{s-1}$  or  $C_s \cap Y_{s-1} \subsetneq Y_{s-1}$ . In the first case, the result follows directly since then by definition  $Y_{s+1} = Y_s$  and  $0 \leq |Z_{s-1} \setminus Z_s|$ . So consider the second case. In the second case, we show that  $|Y_{s-1}| - |Y_{s-1} \cap C_s| \leq |Z_{s-1} \setminus Z_s|$ . Let  $A = Y_{s-1} \setminus Y_s$  be the set such that  $j \notin C_s$  for each  $j \in A$ . We show that  $C_s$  must contain as many elements of  $Z_{s-1}$  which haven't been in any  $C_{s'}$  to date as there are elements of  $A$ . To see this, suppose it weren't true. That is, suppose  $|C_s \cap Z_{s-1}| < |A|$ . Then, we would have set  $D = (C_s \setminus Z_{s-1}) \cup A$  of size at least  $x + 1$  that would contradict 4. To see this note that  $C_s$  has by assumption  $x$  members; by excluding  $C_s$ 's intersection with  $Z_{s-1}$ , we are excluding at most  $|A| - 1$  members of  $C_s$ , and adding in the  $|A|$  elements of  $A$ , which all lie in  $C_1$  and each one of which must also lie in some  $C_{s'}$  together with any other member of  $Y_{s-1}$ , which leads to a set of size at least  $x + 1$  satisfying the restrictions of point 4. The contradiction establishes the impossibility of satisfying the combinatorics problem and thus the claim. ■

We have thus shown that there exist no nonempty war stable networks when  $\gamma \geq 1$ . The final part of the theorem, that there are no war stable networks if  $\gamma < 2$ , then follows from the fact that the empty network fails to satisfy [S2] if  $\gamma < 2$  (but satisfies it if  $\gamma \geq 2$ ). This follows from considering the deviation from the empty network of two nodes allying and attacking a third. (More generally the condition is  $\gamma \geq \frac{M_1+M_2}{M_n}$ ). ■

**Proof of Theorem 2:** We apply Corollary 1.

First, note that any network in which each country has  $d^*$  alliances is pairwise stable.

Next, let us check that no country is vulnerable despite trade at such a network. The largest clique is of size at most  $\lceil \frac{d^*}{2} \rceil + 1$ . A largest clique cannot defeat any country outside of the clique as that country and its allies have at least  $(d^* + 1)$  countries and  $\gamma \geq 1$ . Some subset of a clique cannot defeat any member of the clique, since any country within the clique then has at least  $\lfloor \frac{d^*}{2} \rfloor$  neighbors outside of the clique, and so the remaining (at most)  $\lceil \frac{d^*}{2} \rceil$  members of the clique cannot defeat the country and its remaining neighbors since  $\lceil \frac{d^*}{2} \rceil \leq \lfloor \frac{d^*}{2} \rfloor + 1$  and  $\gamma \geq 1$ .

Finally, let us check that even with the addition of any link the resulting network has no countries that are vulnerable despite trade. Note that either (i) the addition of a link does not change the clique structure, in which case no new countries are vulnerable despite trade; or (ii) it can result in at most a new clique of size three. This latter limitation on the size of the new clique follows since any two cliques have at most one country in common and so a pair of unlinked countries can share at most one neighbor in common. It then easily follows that the resulting network has no countries vulnerable despite trade by similar arguments to those above. ■

## Appendix: Alternative Definition of War and Trade Stability

Let us specialize to the case of  $\delta = 1$ , for ease of exposition. An alternate possibility in defining war and trade stability:

Define a network  $g$  to be *economically stable\** if three conditions are met:

- $g$  has no countries vulnerable despite trade;
- $\forall ij \notin g$ :
  - if  $u_i(g + ij) \geq u_i(g)$  and  $u_j(g + ij) \geq u_j(g)$  (with at least one strict), then  $g + ij$  has no countries vulnerable despite trade and the resulting takeover would leave either  $i$  or  $j$  worse off than the starting  $g$ , and

- for any  $C \in \mathcal{C}(g + ij)$  with  $i \in C$ ,  $j \in C$  and  $k \notin C$  and such that  $M(C) > \gamma M(k \cup (N_k(g + ij) \cap C^c))$ , either:
  - \*  $u_j(g + ij - k) + \frac{E_i(g+ij)}{|C|} \leq \max[u_j(g), u_j(g + ij)]$  (so,  $j$  would rather either stay at  $g$  or stay at  $g + ij$  than to form the link and follow through with an attack),
  - \*  $u_k(g + ij - k) + \frac{E_i(g+ij)}{|C|} \leq \max[u_i(g), u_i(g + ij)]$ , or
  - \*  $\exists h \in C$  such that  $u_h(g + ij - k) + \frac{E_i(g+ij)}{|C|} \leq u_h(g + ij)$
- $\forall ij \in g$  either  $u_i(g - ij) \leq u_i(g)$  or  $i$  is vulnerable despite trade at  $g - ij$ , and similarly for  $j$ .

Note that economically stable\* is implied by war and trade stability. To see this note that  $g$  having no countries vulnerable despite trade and  $g + ij$  having countries vulnerable despite trade implies that  $i$  and  $j$  must both benefit relative to  $g + ij$  from the attack on some  $k$ , and since  $u_i(g + ij) \geq u_i(g)$  and  $u_j(g + ij) \geq u_j(g)$  then they benefit also relative to the original graph. This would contradict the next part of the condition. Thus, the  $u_i(g + ij) \geq u_i(g)$  and  $u_j(g + ij) \geq u_j(g)$  (with at least one strict) cannot be satisfied.

Likewise, war and trade stability is implied by economically stability\*. To see this, consider the one case where the definitions may vary, when there is an  $ij \notin g$  such that  $u_i(g + ij) \geq u_i(g)$  and  $u_j(g + ij) \geq u_j(g)$  with at least one strict. ES\* then requires that  $g + ij$  have countries vulnerable despite trade and the resulting takeover would leave either  $i$  or  $j$  worse off than they were at  $g$ . However, since we have that  $g$  has no countries vulnerable despite trade, it must be that the coalition in  $g + ij$  that is making  $g + ij$  have countries vulnerable despite trade includes both  $i$  and  $j$ . But this implies that both  $i$  and  $j$  are (weakly) better off after the takeover than they are at  $g + ij$ , and so by transitivity, than they are at  $g$ . Since we thus never have the conclusion of the “if..., then...” condition, taking the contrapositive we have that ES\* requires that  $\forall ij \notin g$ , it is not the case that both  $u_i(g + ij) \geq u_i(g)$  and  $u_j(g + ij) \geq u_j(g)$  with at least one strict. That is,  $\forall ij \notin g$ , if  $u_i(g + ij) > u_i(g)$ , then  $u_j(g + ij) < u_j(g)$ . So  $g$  is ES.

## Appendix: Snapshots of Networks of Alliances: 1815 to 2000





Figure 9: Network of Alliances, 1815, red for multilateral alliance, grey for bilateral alliance, green for both

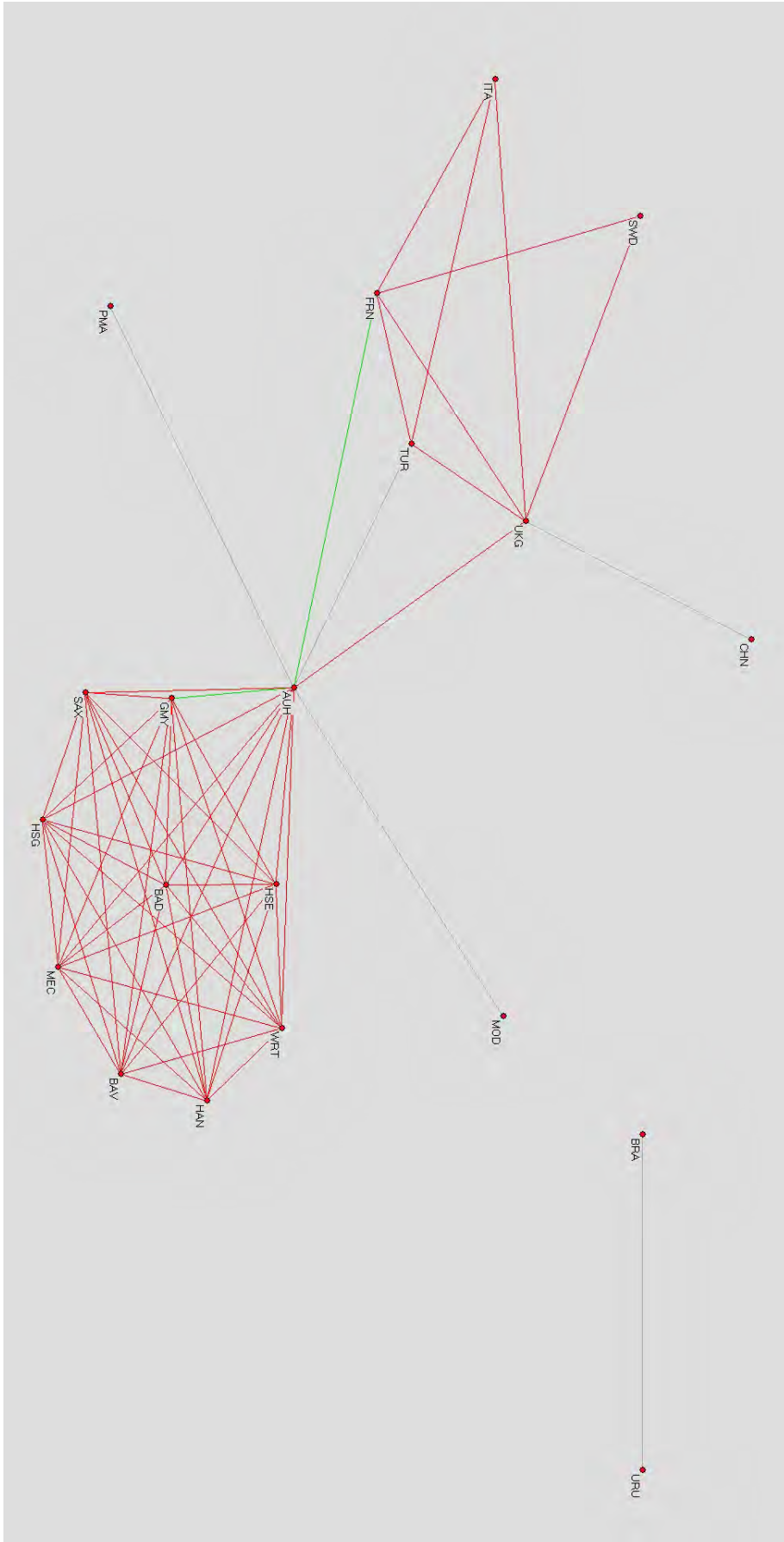


Figure 10: Network of Alliances, 1855, red for multilateral alliance, grey for bilateral alliance, green for both

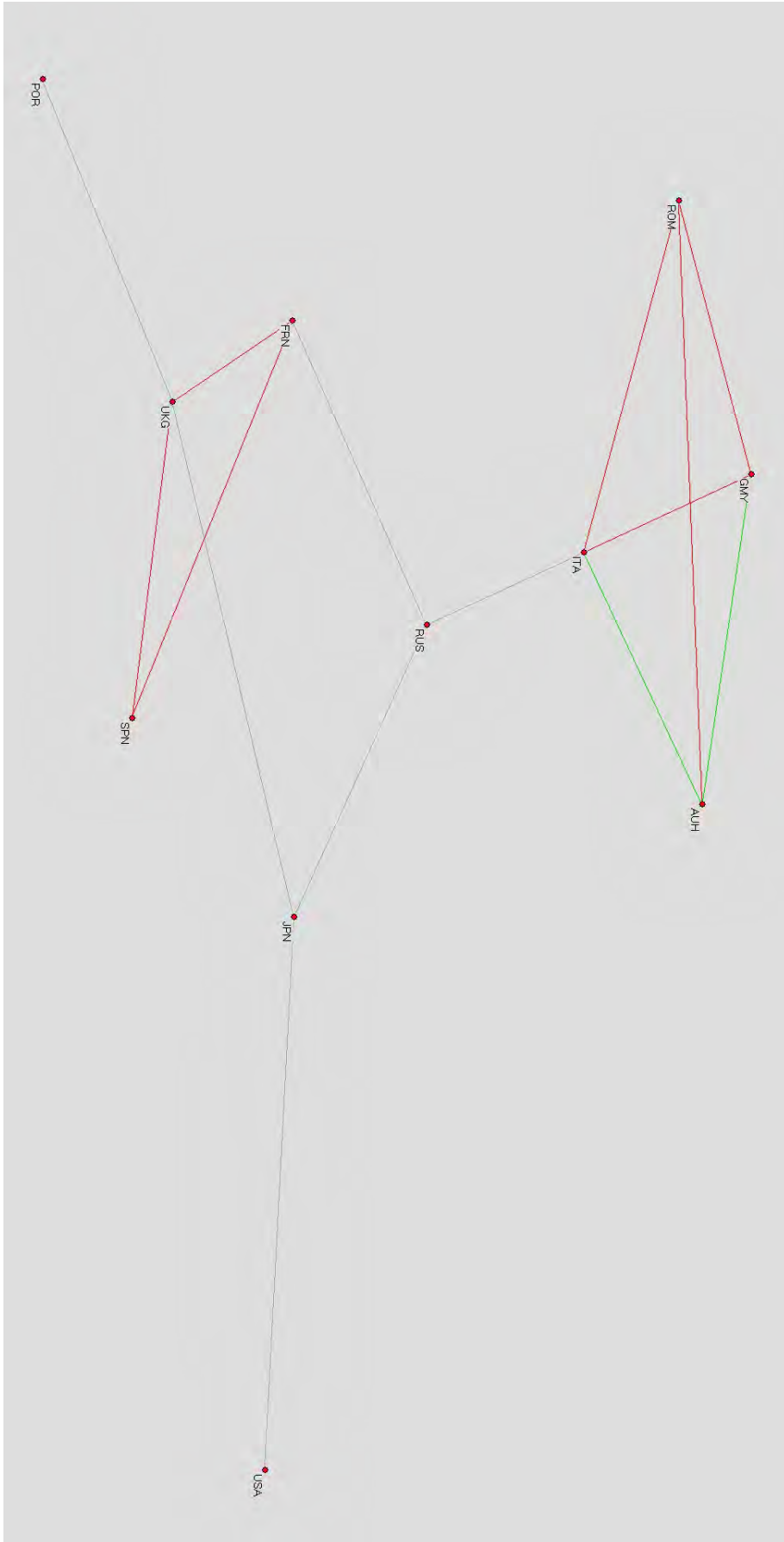


Figure 11: Network of Alliances, 1910, red for multilateral alliance, grey for bilateral alliance, green for both

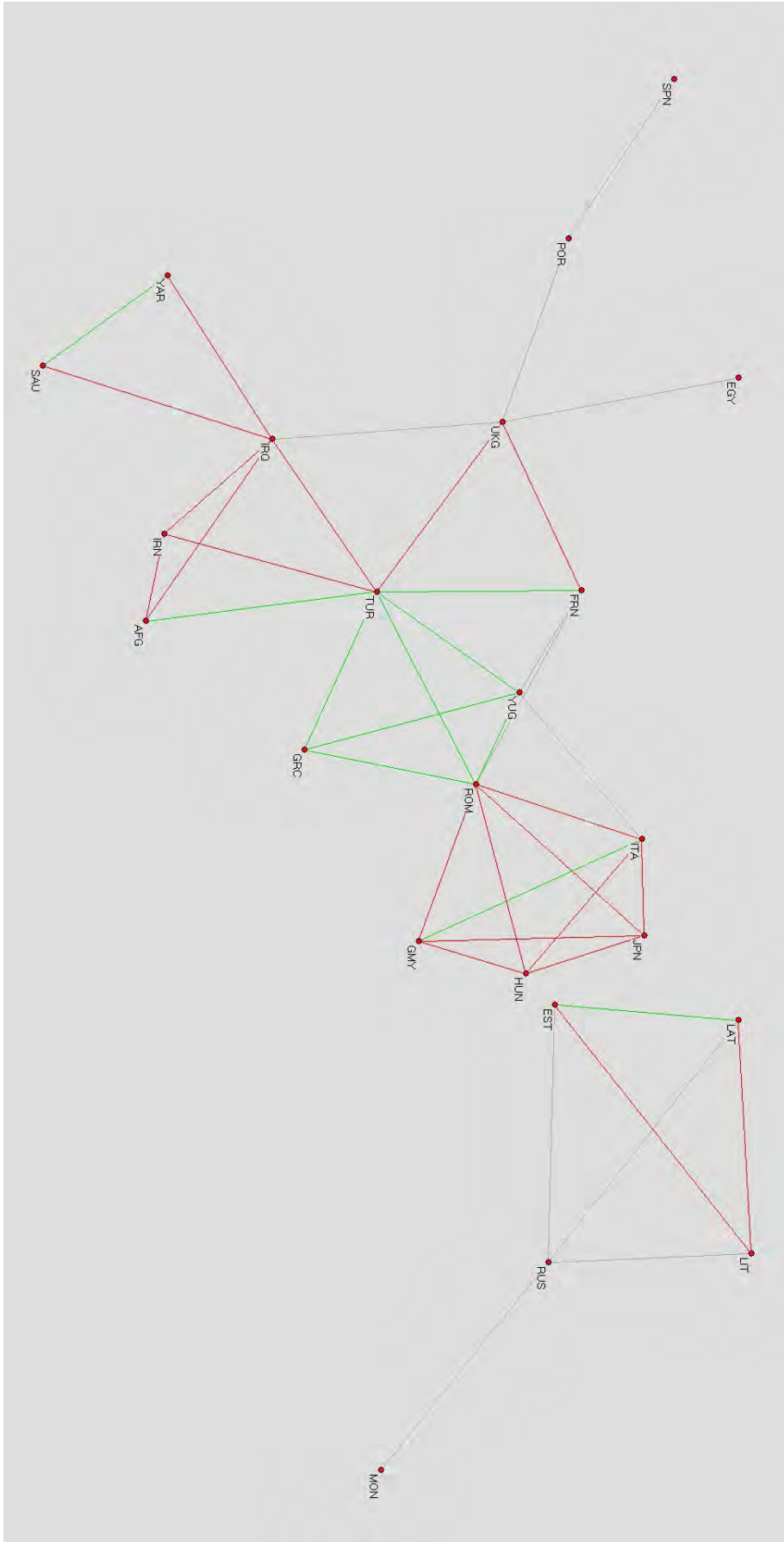


Figure 12: Network of Alliances, 1940, red for multilateral alliance, grey for bilateral alliance, green for both



Figure 13: Network of Alliances, 1960, red for multilateral alliance, grey for bilateral alliance, green for both

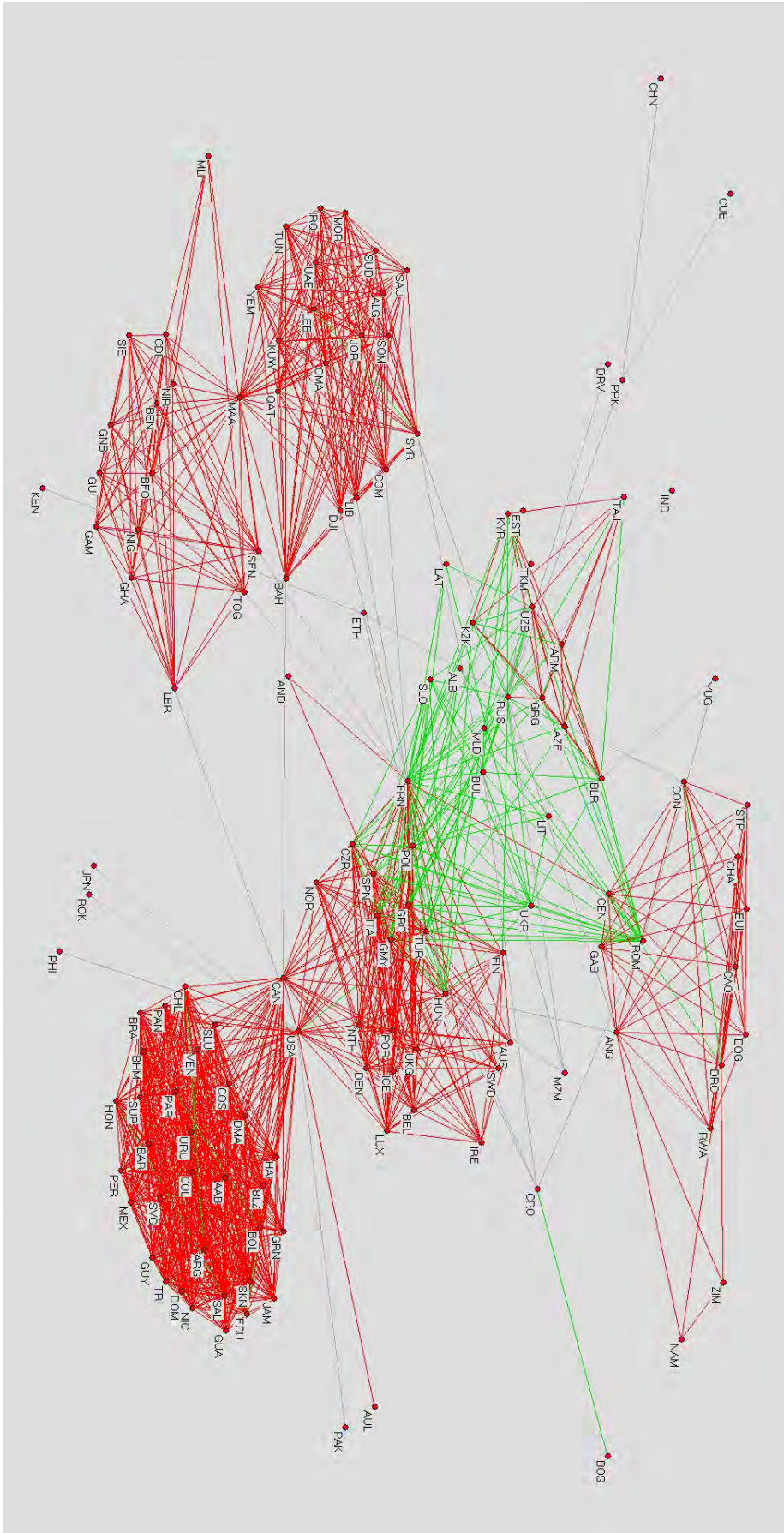


Figure 14: Network of Alliances, 2000, red for multilateral alliance, grey for bilateral alliance, green for both

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