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By **Kenan Huremovic**, Department of Economics, European University Institute, Italy

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Summary

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Keywords: Network Formation, Structural Balance, Contest

JEL Classification: D85,D74

I am grateful to Fernando Vega-Redondo and Piero Gottardi for their advice. I would also like to thank Andrea Mattozzi, Nicola Pavoni, William H. Sandholm and seminar participants at EUI, SEBS and EEA 2013 Gothenburg

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Address for correspondence:

Kenan Huremovic
Department of Economics
European University Institute
Via Roccettini 9
50014 Fiesole Firenze
Italy
E-mail: kenan.huremovic@eui.eu

Rent Seeking and Power Hierarchies: A Noncooperative Model of Network Formation with Antagonistic Links

Kenan Huremovic*

February 14, 2014

Abstract

Network structure has a significant role in determining the outcomes of many socio-economic relationships, including the antagonistic ones. In this paper we study a situation in which agents, embedded in a network, simultaneously play interrelated bilateral contest games with their neighbours. Spillovers between contests induce complex local and global network effects. We first characterize the equilibrium of a game on arbitrary fixed network. Then we study a dynamic network formation model, introducing a novel but intuitive link formation protocol. As links represent antagonistic relationships, link formation is unilateral while link destruction is bilateral. A complete k-partite network is the unique stable network topology. As a result, the model provides a micro-foundation for the structural balance concept in social psychology, and the main results go in line with theoretical and empirical findings from other disciplines, including international relations, sociology and biology.

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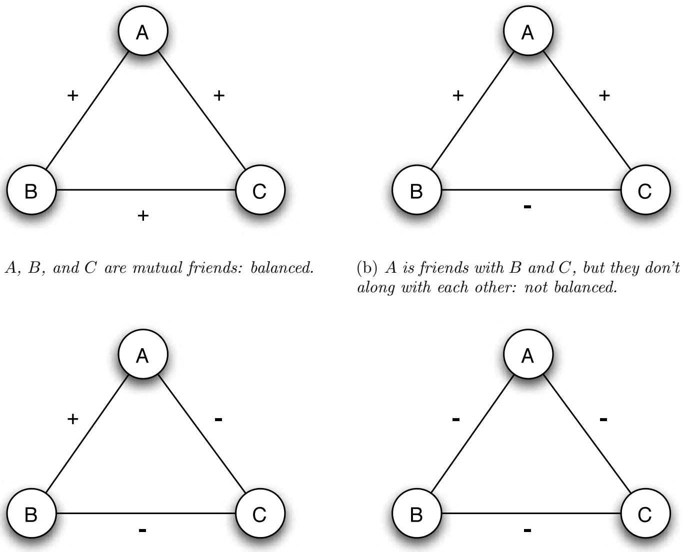
1 Introduction

There are a number of situations where agents are involved in some kind of contest or conflict. In these situations agents can increase the probability of favourable outcome of the process by means of certain costly actions, such as investment in weapons, bribing judges/politicians, hiring lawyers, etc. An agent does not always compete with just one opponent, but rather with several different opponents simultaneously. The contests that an agent is involved into can be, and often are, related (i.e. an agent spends same costly resources for each contest) which creates spillovers.

The environment of interest in language of networks can be described as a network $G = G(N, L)$ in which link $g_{ij} \in L$ between two agents i and j indicate the presence of this type of (negative) relation. We focus on the relations that can be described as (bilateral) contests. Informally, a contest is an interaction in which players exert costly effort in order to extract resources from other player (transferable contest); or receive a larger share of pie to be divided. In the paper we shall focus on the first case, and briefly discuss the second in section 8.

To illustrate a type of interaction we are interested, let us consider a case of patent litigation and antitrust disputes. The U.S. Federal District Courts has registered about 10 000 antitrust and 29 000 patent infringement cases from 2000 to 2010. These types of litigations have consequences for both conflicting parties. The plaintiff argues for forcing another company to refrain from injurious acts and punishments as large as tree times the economic damages sustained. On the other hand, a plaintiff risks being counter-sued and even loosing rights to its intellectual property. The costs of litigation are very high, reaching more that 5 millions USD per lawsuit, excluding damages and royalties (Rea, 2009). The transfers to be paid reach sums which are considerably higher (Sytch and Tatarynowicz, 2013). The firms can be, and usually are, involved in more than one such process at the same time. Other examples include international conflict, patent races, lobbying, Massive Multi-player Role Playing Games(MMORPG), school violence etc.

We first study a model on a fixed network, and then focus on the network formation. In context described above, agents can form both positive links (friendship) and negative links (antagonism, contest, conflict). We focus on the negative links in the paper, and positive links are interpreted as a (self enforcing) commitment by both agents not to engage in a contest. A negative link indicates that agents play a bilateral contest game. Thus the model combines network forma-



(a) *A, B, and C are mutual friends: balanced.*

(b) *A is friends with B and C, but they don't get along with each other: not balanced.*

(c) *A and B are friends with C as a mutual enemy: balanced.*

(d) *A, B, and C are mutual enemies: not balanced.*

Figure 1: Strong structural Balance, (Easley and Kleinberg, 2010)

tion and game on a network. With changes of the network structure, the effort that players devote in each particular contest will in general change. Thus, in the dynamic model of network formation, we study coupled evolution of network topology and play on the network.

Results of the formation model have important implications for the structural balance theory from social psychology. The concept of (strong) structural balance, originated by (Heider, 1946), applies to situations in which relations between agents can be either negative (antagonistic)or positive (friendship). It states that groups of three agents, the only socially and psychologically stable structures are those in which all three agents are friends (all links are positive) or two of them are friends with third as a common enemy (one positive and two negative links). So, a friendship relation is transitive. Figure 1 graphically illustrates Heider's theory.

As it is defined, the structural balance can be seen as a local property of a network. The natural question is what are the global properties of networks that satisfy structural balance. That is, given a complete network, how can we sign links (indicating positive and negative) such that for any triad of nodes in network is structurally balanced. Cartwright-Harary Theorem (Cartwright and Harary, 1956) provides answer to this question. It states that there are two network structures that satisfy structural balance property: (i) all agents are friends (or links are positive) or (ii) agents are divided into two groups, and links within groups are positive and links across groups are negative. In other words, with respect to positive links, a network that satisfies structural balance will be complete network or a network with two components that are cliques. With respect to negative links, it will be either empty or a complete bipartite network.

Extending on Heider's work, (Davis, 1967) argues that in many contexts we may witness a situation in which all links in a triad are negative. To encompass this type of configuration, he proposed the concept of weak structural balance. The implication for the global structure now is an emergence of the additional balanced network structure. With respect to positive links this is a network with more than 2 components, and each component is a clique. With respect to negative links it is a complete k-partite network.

There are number of empirical papers that support (weak) structural balance in the real world networks ((Sytch and Tatarynowicz, 2013), (Szell et al., 2010), (Antal et al. 2006), (Doreian and Krachardt, 2001)). On the theoretical side, there is no micro-founded model that explains the emergence of such networks. The exception is (Hiller, 2012) that provides a network formation model which results with balanced networks. However the interaction between agents is modelled differently, as agents do not make decision how much to invest in negative relations.

This paper provides a micro-founded model of network formation that produces weakly balanced networks as stable networks. Stable networks are always weakly balanced networks (satisfy weak structural balance). The strong structural balance is satisfied in particular cases. It is important to note that the structural balance is a concept concerned only with the sign of links, but does not say anything about the intensities/weights assigned to links. Our model results with signed and weighted network, and thus provides implications beyond structural balance theory.

2 Literature

The paper is related to the several different streams of literature which we review in separate subsections.

2.1 Games on a fixed network

The most often issues that arise when studying games on networks are multiplicity of equilibria (even in very simple games), and intractability of analysis due to complexity of the interaction structure.

One way to deal with these problems is to try to characterize the equilibria for specific classes of games. The representative papers that use this approach include (Ballester et al, 2006). The main results of these papers are based on established relationship between particular measure of centrality of an agent in a network and actions taken in equilibrium. However, the results hold only for games with specific payoff functions.

Another approach is to assume that players have incomplete information about network structure, which can sometimes simplify the analysis. For example (Galeotti et al., 2010) pursues this idea and characterize equilibria in games of strategic substitutes and games of strategic complements.

The closest paper to ours is (Franke and Ozturk, 2009). Section 4 of this paper is parallel to their results on conflict networks, and this section of the paper can be seen as a generalization of their results. However they do not treat the question of network formation at all, which is the central question of this paper.

2.2 Network formation

The main interest of our paper is the model where agents not only decide how much to invest in the bilateral contests but also with whom to get involved in a contest. Thus the paper is related to network formation literature, of which prominent examples are (Jackson and Wolinski, 1996), hereafter JW and (Bala and Goyal, 2000)), hereafter BG.

The way to model dynamic process of network formation depends strongly on the link formation protocol which is adopted. In JW type models, links are formed bilaterally and destroyed unilaterally, while in BG type models links are formed and destroyed unilaterally. The link formation protocol, of course, depends on the interpretation of links. We propose a model in which the link formation protocol does not coincide with any of the two mentioned above, since the nature of links is fundamentally different. In our model links are formed unilaterally and destroyed bilaterally (only if both agents agree to do so). This is a natural link formation protocol, given that the links represent transfer contest.

For example, to start a war it is enough that one party declares a war (or just attack); while to make peace both parties must commit not o fight.

There is significant connection between the model considered in this paper and other network formation models, as we are interested in the similar questions. However, this paper considers in some dimensions more complex model since agents also make strategic decision on the investment in each link that is created. This makes the paper close to the literature on formation of the weighted networks, but also to the literature that jointly considers the network formation and games on network.

2.3 Formation of weighted network

There is some work done on the formation of weighted networks. (Bloch and Dutta, 2007) consider a model of formation of communication network where agents derive positive benefits from the players with whom they are connected to (both directly and indirectly). In their model, homogeneous agents have some fixed endowment and they need to decide how to allocate this endowment in establishing undirected links (with potentially different capacities) with others. Links can be created unilaterally and the strength of the link is additively separable function of individual investments in the link, and convex in individual investments. The convexity assumption in their model yields an incentive to agents to concentrate their investments in a single link, which in the end makes this model close to the two way flow BG model. (Deroian, 2006) extends this model to the case of directed networks (BG one way flow case).

(Rogers, 2006) discusses two models of the network formation with endogenous link strength, depending on the direction of benefits flow along the links. In 'asking' model (which is close to GB one way flow variant) agents receive benefits trough the links they (unilaterally) create.

These models include the 'partner specific' decision on intensity of the link that is formed. That is, players jointly decide with whom they will be connected and what will be intensity of that connection, which is feature shared with the model considered in this paper. However, the payoff structure and nature of externalities in these models is quite different than in model of network formation considered in this paper.

2.4 Contest games

Informally contest game is defined as follows. There are n players. The players decide (simultaneously or sequentially) on the level of investment in the contest. The investments determine the probability of winning the (endogenous or exogenous) prize according to Contest Success Function (CSF). An example is lobbying where the prize can represent the value of a certain public policy that need to be adopted.

There are two prominent ways to model CSF. The first is to assume that the probability of winning is a function of ratios of efforts, which is introduced by (Tullock,1980) and is approach that we use here. The second assumes that probability of winning is a function of difference between effort levels and is introduced in (Hirschleifer,1987).

A nice, albeit dated, overview of literature can be found in (Crochon, 2007) and (Garfinkel and Skepardas, 2006). In this paper we consider transferable contests as introduced in (Hillman and Riley, 1989) using the variant of Tullock's specification introduced by (Nti, 1997)

An alternative model, which is offered in the appendix, gives a model formulation as colonel Blotto game with Tullock CSF. There is a vast literature on Blotto games and I will not review it here.

3 Bilateral contest game

In this section we introduce the bilateral contest game which will serve as a building block of the model. There are two players, i and j competing over a prize with exogenous size R. In order to increase the probability of wining, players choose a non-negative action (effort, investment). The strategy space is thus given with the set of non-negative real numbers $\mathbb{R}_0^+ := [0, +\infty)$ The effort is transformed into contest specific resource (contest input) by means of function referred here as a technology function. One can think of this function as an analogue to the production function in a classic market setting. Here we assume that technology function $\phi : \mathbb{R}_0^+ \to \mathbb{R}_0^+$ is function that satisfies the following properties:

Assumption 1. *Technology function $\phi : \mathbb{R}_0^+ \to \mathbb{R}_0^+$ is:*

(i) Continuous and twice differentiable

(ii) increasing and (weakly) concave ($\phi' > 0,\ \phi'' \leq 0$)

(iii) $\phi(0) = 0$

The first two assumptions are standard, while the third one states that zero effort implies zero contest input.

The actions determine the probabilities of wining the prize trough contest success function. We choose Tullock ratio form specification of CSF suggested in (Nti, 1997), assuming that the probability that player i when taking action s_{ij} will win the contest against player j is given with:

$$p_{ij} = \frac{\phi(s_{ij})}{\phi(s_{ij}) + \phi(s_{ji}) + r} \tag{1}$$

In (1) $r \in \mathbb{R}_0^+$ determines probability of draw (no player wins the prize). In the paper we shall maintain the assumption that r is small.

Following (Hillman and Riley, 1989) we consider transferable contest game, that is the game in which the prize is transfer from looser to winner. Assuming fixed prize, payoff function of player i is given with

$$\pi_{ij} = p_{ij}R - p_{ji}R - c(s_{ij})$$

where R is a transfer from loser to winner. We assume that (potential) transfer from i to j is the same as a transfer from j to i, although of course in general this does note to be the case. $c : \mathbb{R}_0^+ \to \mathbb{R}_0^+$ is a cost function, and we assume it to be twice continuously differentiable, increasing and convex.

The bilateral contest game has the unique (symmetric) NE equilibrium in the pure strategies, which is interior for r low enough. In this case, the equilibrium strategy of player i is defined with the following implicit function:

$$\phi'(s_{ij}^*)R = (r + 2\phi(s_{ij}^*))c'(s_{ij}^*)$$

4 Game on a fixed network

Let $G = (N, L)$ be (undirected) network with set of nodes N and set of links L. The nodes represent players, and link $g_{ij} \in L$ indicates contest relation between players. Let us also denote the set of neighbours of agent i as N_i, and let $d_i = |N_i|$ denote the degree of node i. Strategy space of a player i is a set $S_i = \mathbb{R}_0^{+d_i}$. A (pure) strategy of player i is d_i touple of levels investments $\mathbf{s}_i = (s_{ij_1}, ..., s_{ij_{d_i}}) \in S_i$. We assume that size of the transfer R is independent of network structure and same for every contest g_{ij} [1], and normalize $R = 1$.

Here we focus on the negative links. As we shall discuss in the section concerned with network formation, the absence of negative links can be interpreted as a commitment not to initiate contest and thus a positive (friendly) link.

The payoff of player i is given with

$$\pi_i(\mathbf{s_i}, \mathbf{s_{-i}}\ G) = \sum_{j \in N_i} \left(\frac{\phi(s_{ij})}{\phi(s_{ij}) + \phi(s_{ji}) + r} - \frac{\phi(s_{ji})}{\phi(s_{ij}) + \phi(s_{ji}) + r} \right) - c(A_i) \qquad (2)$$

where $A_i = \sum_j s_{ij}$ is the overall investment, and $\mathbf{s}_{-i}$ denotes strategies of players other than i. Such specification of cost function generates externalities between the contest that agent i is involved in, making it more interesting to study this model on a network.

It is clear that payoff function π_i is twice differentiable on its domain. Furthermore, the payoff function of player i is concave in $\mathbf{s}_i$. To see this, note that

$$\frac{\partial^2 \pi_i}{\partial {s_{ij}}^2} = \frac{(r + 2\phi(s_{ji}))\left(\phi''(s_{ij})(r + \phi(s_{ij}) + \phi(s_{ji})) - 2\phi'(s_{ij})^2\right)}{(r + \phi(s_{ij}) + \phi(s_{ji}))^3} - c''(A_i) \leq 0$$

$$\frac{\partial^2 \pi_i}{\partial s_{ij} \partial s_{ik}} = c''(A_i) < 0 \ \forall j, k \in N_i$$

Above inequality holds given properties of function ϕ stated in Assumption 1, and since c is strictly convex. Thus, the Hessian H_i of function π_i with respect to $\mathbf{s}_i$ is the sum of diagonal matrix H_{i1} with diagonal elements equal to:

$$\frac{(r + 2\phi(s_{ji}))\left(\phi''(s_{ij})(r + \phi(s_{ij}) + \phi(s_{ji})) - 2\phi'(s_{ij})^2\right)}{(r + \phi(s_{ij}) + \phi(s_{ji}))^3} < 0$$

and matrix H_{i2} which has all the elements equal to $-c''(A_i) < 0$. Matrix $H_{i1}$is negative definite and matrix H_{i2} is negative semidefinite, thus Hessian $H_i = H_{i1} + H_{i2}$ is negative definite.

To be able to study network formation, we need to know if the equilibrium strategies on a fixed network are uniquely determined. In this section we prove the uniqueness of the equilibrium on the fixed network (this part is generalization of similar result in (Franke and Ozturk, 2009))

We shall prove two propositions. The first states uniqueness and the second

[1] We use g_{ij} when we talk about link $g_{ij} \in L$ but also when referring to contest between players i and j

that gives conditions for equilibrium to be interior. The first proposition relies on the results from (Rosen, 1965). For the sake of the presentation let us first introduce the following definition:

Definition 1. *A game is n persons concave game if (i) Strategy space of game S is product of closed, convex and bounded subsets of m dimensional Euclidian space, $S = \{S_1 \times S_2 \times ... \times S_n | S_i \subset E^{m_i}\}$ [2] and (ii) payoff function of every player $_2, ..., \mathbf{s}_n)$, and concave in $\mathbf{s}_i \in S_i$, for each fixed value $\mathbf{s}_{-i} \in S_{-i}$*

Let us also introduce function $\sigma : S \times \mathbb{R}_0^{+n} \to \mathbb{R}$ assigned to n persons concave game given with $\sigma(\mathbf{s}, \mathbf{z}) = \sum_{i=1}^{n} z_i \pi_i(\mathbf{s})$. Rosen's results states that:

1. There exists a pure strategy equilibrium of n persons concave game
2. If function σ is diagonally strictly concave for some $\mathbf{z} \geq 0$ then the equilibrium point is unique

Proposition 1. *There exists unique pure strategy Nash equilibrium of contest game on a network.*

Proof. As discussed above, the payoff function of every player i is continuous and concave in $\mathbf{s}_i$. Strategy space of is in general unbounded, but since the transfer R is finite, and cost function c is strictly increasing, there will exit a point $M \in \mathbb{R}$ such that $c(M) > R$. No player will ever wish to exert effort larger than M, and therefore we can bound the strategy space from above, and apply Rosen's result. Thus, considered game is n persons concave game as defined above. Following (Rosen, 1965) there exist pure strategy equilibrium of the game. To prove the uniqueness we will use the following specification of diagonally strictly concave function proposed by (Goodman,1980), that states states that $\sigma(\mathbf{s}, \mathbf{z})$ will be diagonally strictly concave if payoff functions are such that for every player i: (i) $\pi_i(\mathbf{s})$ is strictly concave in $\mathbf{s}_i$, (ii) $\pi_i(\mathbf{s})$ is convex in $\mathbf{s}_{-i}$ and (iii)$\sigma(\mathbf{s}, \mathbf{z})$ is concave in $\mathbf{s}$ for some $\mathbf{z} \geq 0$.

For the game that we are considering we have already shown above that π_i has a negative definite Hessian with respect to $\mathbf{s}_i$

We also have that:

[2] Rosen actually proved more general result when strategy space is 'coupled', that is when $S \subset E^m = E^{m_1} \times E^{m_2} \times ... \times E^{m_n}$ is closed, convex and bounded set. Here we consider special case when strategy space is 'uncoupled'

$$\frac{\partial^2\pi_i}{\partial s_{ji}^2} = \frac{(r+2\phi(s_{ij}))\left(2\phi'(s_{ji})^2 - \phi''(s_{ji})(r+\phi(s_{ij})+\phi(s_{ji}))\right)}{(r+\phi(s_{ij})+\phi(s_{ji}))^3} > 0$$

when there is link g_{ij}. Furthermore, $(\forall g_{jk} \in L : k \neq i)$, $\frac{\partial^2\pi_i}{\partial s_{jk}^2} = 0$ and $\frac{\partial^2\pi_i}{\partial s_{jk}\partial s_{lt}} = 0$ for any other combination of players, j, k, l and t. Thus Hessian of π_i with respect to $\mathbf{s}_{-i}$ is diagonal matrix with all entries positive or zero and therefore positive semi-definite.

To prove concavity of $\sigma(\mathbf{s}, \mathbf{z})$ in $\mathbf{s}$ we choose $\mathbf{r} = \mathbf{1}$. Then:

$$\sigma(\mathbf{s}, \mathbf{1}) = \sum_i \sum_{j \in N_i} \left(\frac{\phi(s_{ij})}{\phi(s_{ij})+\phi(s_{ji})+r} - \frac{\phi(s_{ji})}{\phi(s_{ij})+\phi(s_{ji})+r} R_i - c(A_i) \right) = -\sum_i c(A_i)$$

The later equality holds since every summand $\frac{\phi(s_{ij})}{\phi(s_{ij})+\phi(s_{ji})+r}$ appears exactly once with positive sign (as a part of payoff function π_i) and exactly once with a negative sign (as part of function π_j). Function $-\sum_i c(A_i)$ is strictly concave due to strict convexity of function c. □

Proposition 2. *The equilibrium is interior when $r > 0$ is small enough*

Proof. Consider two arbitrary connected players i and j. Let us first prove that in equilibrium it cannot be $s_{ij} = s_{ji} = 0$ $\forall r > 0$[3]. Assume otherwise. Then the payoff for both players in contest g_{ij} will be 0. Consider the deviation of player i from $s_{ij} = 0$ to $s_{ij} = r$ Now the probability of wining for player i becomes $p_{ij} = \frac{\phi(r)}{\phi(r)+r} = \alpha > 0$ and the probability of loosing will still be 0. This deviation will be profitable as long as $c(\tilde{A}_i) - c(A_i) < \alpha$, where $\tilde{A}_i = A_i + r$ As c is continuous, we can always find such r so that $|c(\tilde{A}_i) - c(A_i)| < \alpha$ when $|\tilde{A}_i - A_i| \leq r$ Therefore, for such r, it cannot be that $s_{ij} = s_{ji} = 0$ in equilibrium.

Let us now prove that for two arbitrary connected players i and j it cannot be that $s_{ij} \neq 0 \wedge s_{ji} = 0$ $\forall r > 0$. Again, suppose this is the case. that is the case, then necessary conditions imply that in the equilibrium we have

$$\frac{\partial \pi_i}{\partial s_{ij}}|_{(s_{ij},0)} = \frac{(r+2\phi(0))\phi'(s_{ij})}{(r+\phi(s_{ij})+\phi(0))^2} - c'(A_i) = \frac{r\phi'(s_{ij})}{(r+\phi(s_{ij}))^2} - c'(A_i) = 0 \qquad (3)$$

[3]We omit * with equilibrium actions in the rest of the proof, but it is clear when s_{ij} denotes action in equilibrium

We can always find r small enough such that (3) cannot hold for any value $s_{ij} > 0$ and $A_i > 0$. Indeed, since the reward is finite and the number of nodes in the network is finite, then A_i must be finite for any node i in arbitrary network. For any cost function $c \in C^2$ satisfying assumptions for we can find $U > 0$ such that $c'(A_i) < U$ for every A_i. But we can always choose $r > 0$ small enough such that $\frac{r\phi'(s_{ij})}{(r+\phi(s_{ij}))^2} > U \ \forall s_{ij} \in [0, M]$, since $\frac{r\phi'(s_{ij})}{(r+\phi(s_{ij}))^2} \to \infty$ when $r \to 0$ for any fixed s_{ij}. □

In what follows, we will assume that r is chosen in such way that interiority of equilibrium is guaranteed. Note also that above results imply that the equilibrium of the game on a fixed network is defined with FOC system of equations.

Consider now two connected players i and j. The first order conditions that characterize their behaviour in a contest g_{ij} are given with:

$$\left(\frac{(r + 2\phi(s_{ji}))\phi'(s_{ij})}{(r + \phi(s_{ij}) + \phi(s_{ji}))^2} - c'(A_i) = 0 \right) \wedge \left(\frac{(r + 2\phi(s_{ij}))\phi'(s_{ji})}{(r + \phi(s_{ij}) + \phi(s_{ji}))^2} - c'(A_j) = 0 \right) \quad (4)$$

From (4) we get:

$$\frac{(r + 2\phi(s_{ji}))\phi'(s_{ij})}{(r + 2\phi(s_{ij}))\phi'(s_{ji})} = \frac{c'(A_i)}{c'(A_j)} \quad (5)$$

As $\phi' > 0$ and $\phi'' \leq 0$ and $c'' > 0$ we have: $A_i > A_j \Leftrightarrow \frac{c'(A_i)}{c'(A_j)} > 1 \Leftrightarrow \frac{(r+2\phi(s_{ji}))\phi'(s_{ij})}{(r+2\phi(s_{ij}))\phi'(s_{ji})} \Leftrightarrow s_{ji} > s_{ij}$ where last equvialence is due to the fact that ϕ is increasing and ϕ' is decreasing function.

This means that in the equilibrium a player with lower total spending will win a contest with the higher probability. This observation reflects the fact that more 'exhausted' (one that spend more resources in equilibrium) player performs worst in an contest, because the additional unit of resources is more costly to him (his marginal costs are higher). Note that it does not necessarily mean that player involved in more contests will have higher total spending in the equilibrium, although the total spending is increasing in number of contests (keeping everything else fixed). It will be rather a player who has many and more intensive contests. Which contest will be more intensive, depends on the global position of players in the network. Identification of the characteristic of a node in network that would determine total spending of that node in the equilibrium proved to be very challenging task. A property of a node that would determine total spending in equilibrium is a nonlinear measure of centrality in a network.

Finding such a measure, although interesting, is a very complex task.

The equation (5) gives us an additional interesting result. Each link g_{ij} has a two actions assigned to it s_{ij} and s_{ji}, and we can interpret them as a weights assigned to directed links $i \to j$ and $j \to i$ respectively. Then in equilibrium, for any cycle (path starting and ending at the same node), we have [4] (we omit *):

$$\frac{\phi(s_{12})}{\phi(s_{21})}\frac{\phi(s_{23})}{\phi(s_{32})}...\frac{\phi(s_{n-1n})}{\phi(s_{nn-1})}\frac{\phi(s_{n1})}{\phi(s_{1n})} = \frac{c'(A_2)}{c'(A_1)}\frac{c'(A_3)}{c'(A_2)}...\frac{c'(A_n)}{c'(A_{n-1})}\frac{c'(A_1)}{c'(A_n)} = 1$$

that is:

$$\phi(s_{12})\phi(s_{23})...\phi(s_{n-1n}) = \phi(s_{nn-1})\phi(s_{n-1n-1})....\phi(s_{21}) \tag{6}$$

That is product of contest inputs assigned to some cycle is the same in both directions.

5 Network formation

The fact that a player with a higher total spending in equilibrium looses in expectation from a player with lower equilibrium spending, gives some hints on how agents behave when contest are determined endogenously. But one must note that the results from previous part are ex-post, and cannot be directly used in a network formation model. This is because the fact that $A_i^* < A_j^*$ in the equilibrium on network G does not imply that we will still have $A_i^* < A_j^*$ in the equilibrium on network $G + g_{ij}$ (where $+$ denotes addition of the link g_{ij} to the network). When a link g_{ij} is created, players i and j will, in general, change their efforts in all other contests that they are involved into. This will, furthermore, result with changes in equilibrium actions of all opponents of i and j in all of their contests; all according to the system of nonlinear equations defined with (4). Given the general structure of the network, one can see why the effects of a link creation, which is a 'fundamental' action of network formation game, are in general case very hard to completely characterize. An example of the global effects that addition of link causes in a very simple network is given in Appendix B.

Because of the complex spillover effects we shall assume that agents are not able to fully take into account these effects when making decision to create or

[4] letting $r \to 0$, but analogue result holds for any r as long as the equilibrium is interior

severe a link. Informally, we assume that when deciding on creating or destroying link, agents do not take into the account the complex adjustment in actions that will occur in all other contests, given the change of the network topology. Instead, they assume that all other actions in the network will remain constant when making this decision. If the action is to create the link, the assumption is that equilibrium efforts of that particular contest game will be according to the NE of the bilateral contest game discussed above, keeping all other actions in the network fixed.

We believe that the bounded rationality assumption here is more realistic, and it makes the analysis more tractable.

In what follows we assume that r is sufficiently small, so that the equilibrium of the game on fixed network is always interior. We shall also assume that ϕ is identity mapping. However, all results hold when ϕ has a general specification from the previous section.

We consider two coupled dynamics processes. The first, which happens on the 'slow' scale, governs the evolution of network topology. The second, on much faster scale, is what we call the action adjustment process. It is the process that describes how actions of players adjust to the new NE when network changes. The reason for the second process is to be consistent with the assumption of bounded rationality that we made in the network formation process.

Let us now be more precise. Time is indexed with $t \in \mathbb{N} \cup \{0\}$. In period $t = 0$ an arbitrary contest network $G(N, L)$ is given. [5] We say that network G is in the *actions equilibrium* when all players play the equilibrium strategy of a contest game on a fixed network described above.

Definition 2 (Actions equilibrium)**.** *A network $G(N, L)$ is in actions equilibrium if all actions s_{ij} and s_{ji} assigned to every link $g_{ij} \in L$ are part of equilibrium of a game on a fixed network.*

Given the definition we can describe the dynamics process that we consider:

For every period t:

(i) At the beginning of period t the network from $t - 1$ is in the actions equilibrium

[5] Due to 'zero sum like' nature of the game, the empty network will always be a stable in our model. In order to describe the dynamic process that leads to the non-empty stable networks we assume that, because of some non modelled mutation or a tremble the initial conditions are given with the non-empty arbitrary network

(ii) Random player i is chosen and she updates her links according to the link formation protocol, resulting with network G_{t+1}

(iii) Second dynamic process (on the fast scale) starts, in which all agents update their strategies according to the process formally described in the section 6 (better reply dynamics), until the actions equilibrium is reached

Steps (ii) and (iii) deserve some further explanation. First let us define the link formation protocol.

Definition 3 (Link formation protocol)**.** *A link g_{ij} will be formed if player i or j decide to form it. A link g_{ij} will be destroyed if both i and j agree to destroy it.*

This means that the link formation is unilateral and the link destruction is a bilateral action. It is the only natural to define a link formation protocol for the antagonistic (purely competitive) relations in this fashion. A decision to start a contest (i.e. war, litigation) is unilateral by nature, and the 'attacked' player, weather she decides to fight back or not, cannot change that. To make a peace it is necessary that both parties agree to do it. This is the first paper to our knowledge that considers such link formation protocol.

We assume that in each period t a random player can update his linking strategy according to the link formation protocol defined above. Given this, we define the stability concept, named Myopically stable network as follows:

Definition 4 (Myopically stable network)**.** *A network $G = G(N, L)$ is Myopically stable network if for any player i and any two (possibly empty) sets of nodes $A \subset N$ and $B \subset N$.*

$$\pi_i(G + \{g_{ij}\}_{j \in A} - \{g_{ij}\}_{j \in B}) > \pi_i(G) \Rightarrow (\exists j \in B) : \pi_j(G - g_{ij}) < \pi_j(G)$$

$$\pi_i(G + \{g_{ij}\}_{j \in A}) < \pi_i(G)$$

This definition assumes that no player will wish to change her linking strategy - destroy or create links. The possibility of replacing a link is essential for the results, however it does not matter if a player can only replace one or more of his links or destroy/create one or more links at the same time. The results will (qualitatively) hold for example if we would consider a process in which an agent in a single period can only crate a link, destroy a link or replace a link. That is if we would consider the following definition of stability:

Definition 5 (Myopically stable network - alternative)**.** *A network* $G = G(N, L)$ *is Myopically stable network if the following conditions hold:*

$$\pi_i(G - g_{ij}) > \pi_i(G) \Rightarrow \pi_j(G - g_{ij}) < \pi_j(G) \ (\forall i, j \in N)$$

$$\pi_i(G + g_{ik} - g_{ij}) > \pi_i(G) \Rightarrow \pi_j(G - g_{ij}) < \pi_j(G) \ (\forall i, j, k \in N)$$

$$\pi_i(G + g_{ij}) < \pi_i(G) \ (\forall i \in N)$$

Let us now clarify what do we exactly mean when say that agents update the connections myopically. When deciding on his connections agent i knows the total spending of all players in the existing network. The effort levels (s_{ij}, s_{ji})[6] assigned to newly formed link are determined as the solution of a bilateral contest game, keeping all other actions in the network fixed. To fix ideas, consider a case of quadratic cost function $c(x) = \frac{1}{2}x^2$. When link g_{ij} is created the corresponding actions $s_{ij}$and s_{ji} are determined as an equilibrium actions of a bilateral contest game between players i and j keeping the spending of these two players fixed in all other contest.

$$\frac{2s_{ji} - r}{(s_{ij} + s_{ji} + r)^2} = (A_i + s_{ij}) \quad \wedge \quad \frac{2s_{ij} - r}{(s_{ij} + s_{ji} + r)^2} = (A_j + s_{ji})$$

Solution of this system is given with:

$$s_{ij} \;=\; \frac{2 + A'_i\left(A'_i + A'_j - \sqrt{4 + (A'_i + A'_j)^2}\right)}{2\sqrt{4 + (A'_i + A'_j)^2}} > 0 \qquad (7)$$

and symmetric for s_{ji}, where $A'_i = A_i - r/2$. Player i will whish to form link when :

$$\frac{s_{ij} - s_{ji}}{(s_{ij} + s_{ji} + r)} + A_i^2 - (A_i + s_{ij})^2 > 0 \qquad (8)$$

and (s_{ij}, s_{ji}) are determined with (7), and analogously for player j.

On the other hand, existing link ij will be destroyed if both players agree to destroy it, that is when $\pi_i(\mathbf{s}_i, \mathbf{s}_{-i}, G - g_{ij}) > \pi_i(\mathbf{s}_i, \mathbf{s}_{-i}, G)$ and $\pi_j(\mathbf{s}_j, \mathbf{s}_{-j}, G - g_{ij})) > \pi_i(\mathbf{s}_j, \mathbf{s}_{-j}, G))$. This will be the case when:

$$A_i^2 - (A_i - s_{ij})^2 - \frac{s_{ij} - s_{ji}}{(s_{ij} + s_{ji} + r)} \geq 0 \wedge A_j^2 - (A_j - s_{ji})^2 - \frac{s_{ji} - s_{ij}}{(s_{ij} + s_{ji} + r)} \geq 0$$

[6]We omit time index t

A decision to destroy a link is, again, done assuming that all other actions in the network will remain fixed. Creation and destruction of more links simultaneously is defined analogously. We additionally assume that player will create link only if it is strictly beneficial to do so. If a player is indifferent between keeping or destroying link, link will be destroyed. So, player prefers to have less links. This could be justified by saying that there is some infinitesimal fixed cost associated to maintaining the link, and easily included in the model. This tie breaking rule does not affect the results.

If after some period t^* no player wish to destroy or create link we say that process has reached the steady state. Thus a network is stable if no player can myopically improve his payoff by changing his linking strategy.

Consider a network G which is in a actions equilibrium. We can sort the nodes in increasing order with respect to their total spending ($A_1 < A_2 < ... < A_K$), $K \leq n$ where K is the number of different total spending levels in a network. Note that we use A_i to denote both total spending of player i and the $i - th$ smallest level of total spending in network. From the context it will be always clear what A_i stands for. Recall also that the equation (5) implies that in any bilateral contest node that has a larger overall spending looses in expectation.

Denote with $\mathcal{A}_i$ the class of nodes that have total spending A_i. Let $K \leq n$ denotes the number of classes in network G. When a player $i \in N$ has a total spending A_i we denote that as $i \in \mathcal{A}_i$. We say that a player i has a control over link g_{ij} if it is beneficial for player j to destroy a link g_{ij}. Thus, when a player i is in control over a link it is completely up to him will the link be destroyed.

If $A_i > A_j$ in the actions equilibrium we will say that player j is stronger than player i or that player i is weaker than player j and will refer to A_i as a strength of player i. It is clear that when i is stronger than j then i controls link g_{ij}. Furthermore, both players i and j shall have control over link g_{ij} if this link is not beneficial for them. A link g_{ij} is said to be beneficial for player i if a creation of this link (if it does not exist) makes player i better off and if a destruction of this link (if it does exist) makes player i worst off.

In what follows we provide a characterization of stable networks. We proceed by stating and proving series of propositions and lemmas. Abusing the notation let $\pi_i(s^*_{ij}, g_{ij}) = \frac{s^*_{ij} - s^*_{ji}}{(s^*_{ij} + s^*_{ji} + r)} - c(A^*_i)$ denote the equilibrium payoff of player a from link g_{ab} in actions equilibrium. Then the following holds:

Proposition 3. *Let $a \in \mathcal{A}_i$, $b \in \mathcal{A}_j$, $c \in \mathcal{A}_k$ and $i < j < k$. Then $s^*_{ab} > s^*_{ac}$, $s^*_{ba} > s^*_{ca}$ and furthermore $\pi_a(s^*_{ab}, g_{ab}) < \pi_a(s^*_{ac}, g_{ac})$*

Proof. Recall that FOC that determine s^*_{ab} is given with. Recall that FOC for any a, b contest are given as:

$$\frac{2s^*_{ab} + r}{(s^*_{ab} + s^*_{ba} + r)^2} = c'(A^*_b) \text{ and } \frac{2s^*_{ba} + r}{(s^*_{ab} + s^*_{ba} + r)^2} = c'(A^*_a) \tag{9}$$

Expressing s^*_{ab} and s^*_{ba} from (9) we get that, in equilibrium:

$$s^*_{ab} = \frac{2c'(A^*_b)}{(c'(A^*_a) + c'(A^*_b))^2} - \frac{r}{2},\ s^*_{ba} = \frac{2c'(A^*_a)}{(c'(A^*_a) + c'(A^*_b))^2} - \frac{r}{2} \tag{10}$$

The function $f(x, y) = \frac{2c'(x)}{(c'(y)+c'(x))^2} - \frac{r}{2}$ is strictly decreasing in x as long as $x > y$ and strictly increasing when $x < y$. f is always strictly decreasing in y

$$\frac{\partial f}{\partial x} = \frac{2\left(-c'(x) + c'(y)\right)c''(x)}{\left(c'(x) + c'(y)\right)^3} \lesseqgtr 0 \text{ when } x \gtreqless y \text{ and } \frac{\partial f}{\partial y} = -\frac{4c'(x)c''(y)}{\left(c'(x) + c'(y)\right)^3} < 0$$

This, together with (10) and $A^*_a < A^*_b < A^*_c$ implies that $s^*_{ab} > s^*_{ac}$ and $s^*_{ba} > s^*_{ca}$.

To prove that $\pi_a(s^*_{ab}, g_{ab}) < \pi_a(s^*_{ac}, g_{ac})$ we use (10) and (after some algebra) get:

$$\pi_a(s^*_{ab}, g_{ab}) = \frac{s^*_{ab} - s^*_{ba}}{(s^*_{ab} + s^*_{ba} + r)} = 1 - \frac{2c'(A_a)}{c'(A_a) + c'(A_b)}$$

It is clear that π_a is strictly increasing in A_b due to strict convexity of function c. Thus, $A^*_c > A^*_b > A^*_a \implies \pi_a(s^*_{ac}, g_{ac}) > \pi_a(s^*_{ab}, g_{ab})$ □

The previous proposition implies that the contest between two players who are more equal in strength is more costly. A strong player spends less when compete with weaker player and has highest payoff from that contest. The results of this claim illustrates the incentive that strong player has to compete with the weakest player, given that the transfer for every contest is the same. This effect is self-reinforcing in the sense that it makes the weak player even weaker, and thus more probable target for other strong players. For the sake of the exposition let us state the following definition.

Definition 6. *Player $a \in \mathcal{A}_i$ is an attacker (winner) if has has all of his links with players from family of classes $\overline{\mathcal{A}}_i = \{\mathcal{A}_j | j > i\}$. Player $a \in \mathcal{A}_i$ is mixed type if there exist players b and c such that $g_{ab}, g_{ac} \in G$ and $A_b > A_a > A_c$. Player $a \in \mathcal{A}_i$ is victim(looser) if he has all of his links with players from classes $\underline{\mathcal{A}}_i = \{\mathcal{A}_j | j < i\}$*

It is clear than every player a must be one of these types. Note also that in a stable network all attackers must have positive payoff. If this is not true for some attacker a then, since he controls all of his links, he could profitably deviate destroying his links.

Lemma 1. *Let $a \in \mathcal{A}$ and $\mathcal{A}$ is the class of attackers. Let b and c be two nodes in the network such that $A_b^* \leq A_c^*$, $g_{ab} \in G$ and $g_{ac} \notin G$. Then the deviation of player a such that he replaces contest g_{ab} with g_{ac} is payoff improving.*

Proof. From (10) we have that s_{ac} in case of the deviation is given with:

$$s_{ac}^* = \frac{2c'(A_c^* + s_{ca}^*)}{(c'(A_a^* - s_{ab}^* + s_{ac}^*) + c'(A_c^* + s_{ca}^*))^2} - \frac{r}{2}$$

and we can write.

$$s_{ab}^* = \frac{2c'(A_b^* - s_{ba}^* + s_{ba}^*)}{(c'(A_a^* - s_{ab}^* + s_{ab}^*) + c'(A_b^* - s_{ba}^* + s_{ba}^*))^2} - \frac{r}{2}$$

Because of the interiority of the equilibrium, $A_c^* + s_{ca}^* > A_c^* \geq A_b^* > A_b^* - s_{ba}^*$. Since $A_c^* + s_{ca} > A_b^*$ the proposition 3 implies that this deviation is profitable. □

From the previous lemma the we have directly get.

Corollary 1. *If in stable network player $a \in \mathcal{A}_i$ has a link with player $b \in \mathcal{A}_j$ then she has a link with every player $c \in \mathcal{A}_{j+k}$ $k = 1, 2, ..K - j$*

Proof. Assume not. If link g_{ab} is not profitable for player a then, as noted before, it is not profitable for player b. Then link g_{ab} cannot be part of a stable network. So it must be that link g_{ab} is profitable for player a. Let $c \in \mathcal{A}_{j+k}$ be a node such that link g_{ac} does not exist. Then, from the Lemma 1 the deviation of player a such that she destroys link g_{ab} and creates link g_{ac} will be profitable. □

Lemma 2. *A stable network must be connected if not empty*

Proof. Suppose not [7]. Then there are at least two components. Choose two arbitrary components from the network and denote them with C_1 and C_2. Let two players with the highest total spending in these components as $h_1 \in \mathcal{A}_{c_1}$ and $h_2 \in \mathcal{A}_{c_2}$. Assume, without lost of generality, that $A_{c_1} \geq A_{c_2}$. Then, for any player in that attacks player h_2 (and there must be at least one) it is profitable to attack player h_1 instead. □

From now on we always talk about connected network.

[7] We omit *, but it is clear from context that we are considering the equilibrium strategies

Lemma 3. *If in a stable network two players belong to the same class of attackers $\mathcal{A}$ than they have the same neighborhood*

Proof. Consider two nodes $a, b \in \mathcal{A}$. Let us first prove that they must have equal degree. Suppose that this is not true, so suppose, W.L.O.G., that $d_b > d_a$ where d_i denotes degree of a node i. Let N_i denote neighborhood of player i. It cannot be that $N_a \subset N_b$ because then the total spending of a and b could not be equal (they would not belong to the same class). If $N_a = N_b$ the proof is completed, if not there must be some node $h \in N_a \backslash N_b$ and some node $k \in N_b \backslash N_a$. Suppose, W.L.O.G., that $A_k \geq A_h$. Than it would be better for player a to replace link g_{ah} with link g_{ak} according to the Lemma 1. This is profitable deviation which is contradiction to the assumption that network is stable. So it must be $d_a = d_b$.

Let us now prove that there must be $N_a = N_b$. Again, assume this is not true. This means that we can find two nodes $h \in N_a \backslash N_b$ $k \in N_b \backslash N_a$ such that, W.L.O.G., $A_k \geq A_h$. But then it would be better for player a to replace link g_{ah} with link g_{ak} according to the Lemma 1. Thus, network G cannot be stable. The assumption that $N_a \neq N_b$ led us to a contradiction and thus must be rejected. □

Since all attackers in the same class have the same neighborhood it must be they have the same payoff in a stable network. Next Lemma shows that there can be only one class of the winners (attackers) in equilibrium.

Lemma 4. *There is only one class of attackers in a stable network*

Proof. We again use the proof by contradiction. Suppose there are two different classes of attackers and denote them with $\mathcal{A}_1$ and $\mathcal{A}_2$ and let $A_2 > A_1$. Since players in $\mathcal{A}_1$ and $\mathcal{A}_2$ are attackers they have control over all of their links. Since Lemma 3 implies that all members of a same class of attackers have the same neighborhood, we restrict our attention to the representative nodes $a \in \mathcal{A}_1$ and $b \in \mathcal{A}_2$. Let us first prove that it must be $\pi_a = \pi_b$. Assume this is not the case. Then it must be that $N_a \neq N_b$. Since $A_2 > A_1$ there are two possible situations that we need to consider.

(i) $N_a \subset N_b$ then then if $\pi_a > \pi_b$ player b could mimic player a (as he is attacker), and if $\pi_b > \pi_a$ the opposite will hold.[8]

(ii) $N_a \not\subset N_b \implies (\exists k \in N_a \backslash N_b \wedge \exists h \in N_b \backslash N_a)$. But then, if $A_k \geq A_h$ Lemma 1 implies that b has a profitable deviation, and if not, same Lemma implies that a has a profitable deviation.

[8]Recall that we assume that when a player is payoff indifferent between two actions he prefers to have less links.

We have proved that in stable network it must be $\pi_a = \pi_b$. Since $A_2 > A_1$ then it must be that $d_b > d_a$ or that the distribution of total spending $a's$ and $b's$ opponents is different. We show that in both cases there is possible deviation which makes one of the players better off.

Let us first consider the case when $d_b > d_a$. If $N_a \subset N_b$ we have (i) from above. So there must exist nodes $k \in N_a \backslash N_b$ and $h \in N_b \backslash N_a$. If $A_k \geq A_h$ then player b would be better off by replacing contest g_{bd} with g_{bc}. If not, player a can make analogue profitable deviation.

If $d_a = d_b$ then, since $A_2 > A_1$, the strengths (total equilibrium spending) of $a's$ opponents are different than strength of the $b's$ opponents. Let q be the strongest node from $(N_a \cup N_b) \backslash (N_a \cap N_b) \neq \emptyset$. If link g_{aq} exists, then it is profitable for a to switch from q to any node in the set $N_b \backslash N_a$. If g_{bq} exists, then deviation is switching from q to some node in $N_a \backslash N_b$, and the proof is completed. □

Note that previous Lemma and corollary implies that members of (unique) class of attackers are connected to all other nodes in the network. This is due to the fact that class $\mathcal{A}_2$ must be a class of mixed types or losers. In either case, previous lemma together with the fact that two players from a same class cannot be connected in a stable network implies that all members of $\mathcal{A}_1$ and $\mathcal{A}_2$ are connected.

Let us now say something about mixed types in stable network.

Lemma 5. *In stable network all members of all existing mixed type classes $\mathcal{A}$ are connected to all other nodes in the network except nodes belonging to their class.*

Proof. If there are only two classes of nodes in network $\mathcal{A}_1$ and $\mathcal{A}_2$ then there are no mixed types. Suppose there are more than two classes in the network. Consider first the strongest mixed type class ($\mathcal{A}_2$). A node $m \in \mathcal{A}_2$ must be connected to all of the nodes in the class of winners $\mathcal{A}_1$. This is because as a mixed type m must be connected with at least one stronger player, which must be a winner because of the choice of m. Lemma 4 implies then that m must be connected to all players from the class $\mathcal{A}_1$. Let us now prove that all members of the class $\mathcal{A}_2$ have the same neighborhood. Suppose not. Let $\{m_1, m_2\} \subset \mathcal{A}_2 \wedge N_{m_1} \neq N_{m_2}$. We have $(\mathcal{A}_1 \subset N_{m_1} \wedge \mathcal{A}_1 \subset N_{m_2}) \implies ((N_{m_1}/N_{m_2}) \cup (N_{m_2}/N_{m_1})) \cap \mathcal{A}_1 = \emptyset$. Thus if they differ, neighborhoods of m_1 and m_2 must differ only in the part where m_1 and m_2 have control over their link. It cannot be $N_{m_1} \subset N_{m_2} \vee N_{m_2} \subset N_{m_1}$ because than it cannot be $A_{m_1} = A_{m_2}$. Consider two nodes, $k \in N_{m_1} \backslash N_{m_2}$ and $l \in N_{m_2} \backslash N_{m_1}$. Note that sets $N_{m_1} \backslash N_{m_2}$ and $N_{m_2} \backslash N_{m_1}$ cannot be empty. If $A_k \geq A_l$ then m_2 has a profitable deviation switching from $g_{m_2 l}$ to $g_{m_2 k}$. If not, than m_1 has analogue profitable deviation.

Let $\mathcal{A}_3$ be the third strongest class in the network. If this is the weakest class (if $K = 3$) then, by definition, all players from $m \in \mathcal{A}_2$ must be connected to some of the players of $\mathcal{A}_3$, because otherwise they would not be mixed types. Note that if player $i \in \mathcal{A}_3$ is connected to some player from class $\mathcal{A}_2$ that he is connected to all players from class $\mathcal{A}_2$ since we have showed that all members of class $\mathcal{A}_2$ have the same neighborhood. If there exist some player $j \in \mathcal{A}_3$ who is not connected to a player from $\mathcal{A}_2$ than he is connected only to players from $\mathcal{A}_1$ but than it cannot be $A_i = A_j$, that is i and j cannot belong to the same class. Thus, if $K = 3$ the claim holds.

If not than $\mathcal{A}_3$ is a mixed type class. Corollary 1 implies that all members of $\mathcal{A}_1$ must be connected to all members of $\mathcal{A}_3$ since they are connected to all the members of $\mathcal{A}_2$ and $A_2 < A_3$. Suppose that there does not exist link g_{ij} such that $i \in \mathcal{A}_2$ and $j \in \mathcal{A}_3$. Since all players from $\mathcal{A}_2$ have the same neighborhood there are no any links between members of class $\mathcal{A}_2$ and $\mathcal{A}_3$. This means that players from $\mathcal{A}_3$ loose only in contest with players from $\mathcal{A}_1$, so they have control over all of their links except those that connect them to players $\mathcal{A}_1$. Furthermore, $A_2 < A_3 \Longrightarrow N_i \neq N_j$. As before, we consider first case when $\pi_i \neq \pi_j$.

(i) $N_i \subset N_j$ then j can destroy links towards all players N_j/N_i and have same payoff as i (if $\pi_i \geq \pi_j$), or player i can create links to all players in N_j/N_i (if $\pi_i < \pi_j$)

(ii) $N_i \not\subset N_j \implies (\exists k \in N_i \backslash N_j \wedge \exists h \in N_j \backslash N_h)$. But then, if $A_k \geq A_h$ Lemma 1 implies that j has a profitable deviation, and if not, same Lemma implies that i has a profitable deviation.

If $\pi_i = \pi_j$ since $A_2 > A_1$ then it must be that $d_j > d_i$ or that the distribution of total spending $i's$ and $j's$ opponents is different. We show that in both cases there is possible deviation which makes one of the players better off.

Let us first consider the case when $d_i > d_j$. If $N_i \subset N_j$ we have (i) from above. If not we have analogue of (ii).

If $d_i = d_j$ then, since $A_2 > A_1$, the strengths (total equilibrium spending) of $i's$ opponents are different than strength of the $j's$ opponents. Let q be the strongest node from $(N_a \cup N_b) \backslash (N_a \cap N_b) \neq \emptyset$. If link g_{iq} exists, then it is profitable for i to switch from q to any node in the set $N_j \backslash N_i$. If g_{jq} exists, then deviation is switching from q to some node in $N_i \backslash N_j$. We have showed that it cannot be that there are no links between $\mathcal{A}_2$ and $\mathcal{A}_3$, thus every player from $\mathcal{A}_2$ is connected to every player from $\mathcal{A}_3$.

Proceeding in the same manner we can show that all players from $\mathcal{A}_k$ must be connected to all players from $\mathcal{A}_{k+1}$. Since number of nodes is finite, number of classes is finite and this procedure reaches $\mathcal{A}_K$ in finite number of steps[9]. $\square$

Corollary 2. *There is only one class of victims and all victims have same neighborhood*

Let us say something about size of partitions in the stable network. Let $|\mathcal{A}_k|$ denote number of nodes that belong to class $\mathcal{A}_k$, the following lemma holds.

Lemma 6. $|\mathcal{A}_k| > |\mathcal{A}_{k+1}|\ \forall k \in \{1, ..., K\}$

Proof. Suppose not. Note that FOC imply that $s^*_{ij} = s^*_{ih}\ \forall \{i, j, h\} \in N \wedge \{j, h\} \in \mathcal{A}_l$. If $|\mathcal{A}_k| < |\mathcal{A}_{k+1}|$ Lemma (previous) implies that $A_k = \sum_{i \neq k} |\mathcal{A}_i| s_{ki}$ and $A_{k+1} = \sum_{i \neq k+1} |\mathcal{A}_i| s_{k'i}$ for any two nodes $k \in \mathcal{A}_k$ and $k' \in \mathcal{A}_{k+1}$. Recall that s^*_{ij} is strictly decreasing in A^*_i which implies that $s^*_{kj} > s^*_{k'j}\ \ \forall j \in \{1, .., K\} \backslash \{k, k'\}$. Also, $A_k < A_{k+1} \implies s_{kk'} > s_{k'k}$. But then $|\mathcal{A}_k| < |\mathcal{A}_{k+1}| \implies (A_k = \sum_{i \neq k} |\mathcal{A}_i| s_{ki} > A_{k+1} = \sum_{i \neq k+1} |\mathcal{A}_i| s_{k'i})$, contradiction! It must be $|\mathcal{A}_k| > |\mathcal{A}_{k+1}|$ $\square$

It is clear that $|\mathcal{A}_k| > |\mathcal{A}_{k+1}|$ is not sufficient condition for stability of network. The difference $|\mathcal{A}_k| > |\mathcal{A}_{k+1}|$ must be large enough so that members of the stronger class do not find it payoff improving to delete links with members of the weaker class. Previous Lemmas imply the following proposition, which is the main result of this section.

Proposition 4. *Stable network is either empty network or complete k-partite with partitions of different sizes. The payoff of members is increasing in size of the partition and total spending per node is decreasing with the size of partition that she belongs to.*

Complete k-partite network is the only network topology that satisfies weak structural balance property, as discussed before. Note that if cost function is too steep, or the transfer size is too small, the only stably network would be complete bipartite network. The complete bipartite network (with respect to negative links) is the only network topology that satisfies strong structural balance property.

[9] If n is not finite the claim is easily proved using mathematical induction

Not all complete k-partite networks will be stable. In order for them to be stable, no player must have an incentive to create or destroy a link. As only links that can be crated are between players from the same partition, no player will wish to create link. This is because a link g_{ij} between players i and j such that $A_i = A_j = A$ cannot be profitable (they will exert the same effort in the equilibrium and thus win and loose contest with the same probability, and since the effort is costly, have a negative net payoff from contest g_{ij}). No player will wish to destroy a link if all links bring a positive payoff to a winner. Combining equilibrium conditions for players i and j we get that in equilibrium [10]

$$\left(s_{ij} = \frac{c'(A_j)}{c'(A_i)} s_{ji} \wedge \frac{2 s_{ji}}{(s_{ij} + s_{ji})^2} = c'(A_i) \right) \Rightarrow s_{ij} = \frac{2c'(A_j)}{(c'(A_i) + c'(A_j))^2} \tag{11}$$

Using (11) equality we can express sufficient conditions for stability of the network in terms of the total spending in the eqilibrium, that is we have that a complete k-partite network will be stable when for any contest g_{ij} we have:

$$2\frac{c'(A_j) - c'(A_i)}{c'(A_j) + c'(A_i)} > c(A_i) - c\left(\frac{2c'(A_j)}{(c'(A_i) + c'(A_j))^2} \right)$$

Let us consider a particular example of complete bipartite network. Note that in this case (due to symmetry) agents will play the same strategy in every contest g_{ij} they are involved in. Then all the contest g_{ij} will result with positive net payoff iff members of the larger partition have a (total) positive payoff in equilibrium. Denote the two partitions with X and Y, and sizes of partitions with x and y respectively, and let $x > y$. Then total efforts of members of two partitions can be written as $A_X = y s_X$ and $A_Y = x s_Y$, where $s_i,\ i \in \{X, Y\}$ is the equilibrium effort level in each particular contest of members of partition i. Using (11) we get that:

$$\pi_X(s_X, s_Y) > 0 \Leftrightarrow y\frac{c'(A_Y) - c'(A_X)}{c'(A_Y) + c'(A_X)} - c(A_X) > 0 \Leftrightarrow \frac{c'(A_Y)}{c'(A_X)} > \frac{y + c(A_X)}{y - c(A_Y)} \tag{12}$$

With cost function $c(x) = \frac{1}{2}x^2$, $s_X = \sqrt{\frac{\sqrt{x}}{\sqrt{y}\left(\sqrt{x}+\sqrt{y}\right)^2}}$

Payoff of an agent from partition X is then:

$$\pi_X(s_X, s_Y) = b\frac{s_X - s_Y}{s_X + s_Y} - (b s_X)^2 = \frac{x\left(x - y - \sqrt{xy}\right)}{\sqrt{x} + \sqrt{y}} \tag{13}$$

[10]To simplify calculations we consider the case $r \to 0$

and from here

$$\pi_X(s_X, s_Y) > 0 \Leftrightarrow x > y\left(\frac{3+\sqrt{5}}{2}\right) \tag{14}$$

We have proved the following proposition:

Proposition 5. *For $c(x) = \frac{1}{2}x^2$ and $\phi(x) = x$ a complete bipartite network will be stable when $x > y\left(\frac{3+\sqrt{5}}{2}\right)$ where x and y are sizes of partitions.*

Payoff of players in the larger partition will be increasing in the number of members of own partition, and increasing in the number of players in the smaller partition for $b \leq \frac{b}{6}\left(14 + \sqrt[3]{1475 + 8\sqrt{41}} + \sqrt[3]{1475 - 8\sqrt{41}}\right) \approx 6.07b$, and decreasing otherwise. There are two effects on payoff of members of larger partition when increasing the number of players in the smaller partition. The first one is that the contests become more costly, as the members of smaller partition become 'stronger'. The second effect is that there are more opportunities to extract rents. Depending on which effect dominates, payoff of an agent from larger partition will increase or decease with the size of partition.

6 Action adjustment process

As we have discussed in the section 3, after network structure is changed, players update their strategies in a myopic way until the actions equilibrium on the new networks is reached.In this section we describe this process and prove the it's global stability property. The actions adjustment process is defined as follows:

$$\frac{d\mathbf{s}_i}{dt} = \alpha \nabla_i \pi_i(\mathbf{s}),\ \alpha > 0,\ i = 1, .., n \tag{15}$$

where $\pi_i(\mathbf{s}) = \pi_i(\mathbf{s}_1, \mathbf{s}_2, ..., \mathbf{s}_i, ...\mathbf{s}_n)$ and $\nabla_i \pi_i(\mathbf{s}) = \begin{pmatrix} \frac{\partial \pi_i}{\partial s_{i1}} & \frac{\partial \pi_i}{\partial s_{i1}} & ... & \frac{\partial \pi_i}{\partial s_{id_i}} \end{pmatrix}$ is gradient of the payoff function with respect to own strategy. It is clear that Nash equilibrium is steady state of this dynamics. We prove in what follows that NE is globally asymptotically stable state of this dynamic system. Let us define function $J : \prod_i [0, M]^{d_i} \rightarrow \prod_i [0, M]^{d_i}$ with:

$$J(\mathbf{s}) = \begin{pmatrix} \nabla_1 \pi_1(\mathbf{s}) \\ \nabla_2 \pi_2(\mathbf{s}) \\ ... \\ \nabla_n \pi_n(\mathbf{s}) \end{pmatrix} \tag{16}$$

Let us define matrix G as the Jacobian of matrix J with respect to $\mathbf{s}$. We can write system (16) in more compact form

$$\dot{\mathbf{s}} = \alpha J(\mathbf{s}) \tag{17}$$

To prove global stability we need to show that rate of change of $||J|| = JJ'$ is always negative (and equal to 0 in equilibrium). So let us check $\frac{d}{dt}||J||$. We get:

$$\frac{d}{dt}JJ' = (G\dot{\mathbf{s}})'J + J'G\dot{\mathbf{s}} = (J'G'J + J'GJ) = J'(G' + G)J$$

where G is he Jacobian of matrix J with respect to $\mathbf{s}$. As proved in (Goodman, 1982) the conditions (i)-(iii) discussed in the proof of Proposition 1 imply that $(G' + G)$ is negative definite. This implies that $\frac{d}{dt}JJ' < 0$ which is what we need to prove.

Thus if every player adjusts his action according adjustment process in (17) we have that the process converges. The process (16) can be made discrete without loosing the convergence properties. The discussion from above proves the following proposition:

Proposition 6. *The action adjustment process is globally asymptotically stable*

7 Efficiency

It is easy to show that the unique network that maximize total utility of society is the empty network. This is direct consequence of transferable nature of contest game as the all rent seeking effort is wasteful. Indeed, total payoff that society obtains from network G can be expressed as:

$$\begin{aligned} U(G) &= \sum_i \pi_i(\mathbf{s}_i, \mathbf{s}_{-i}; G) = \\ &= \sum_i \sum_{j \in N_i} \left[\frac{\phi(s_{ij})}{\phi(s_{ij}) + \phi(s_{ji}) + r} - \frac{\phi(s_{ji})}{\phi(s_{ij}) + \phi(s_{ji}) + r} - c(A_i) \right] = -\sum_i c(A_i) \end{aligned} \tag{18}$$

From (18) we have:

Proposition 7. *The efficient network is empty network*

8 Final remarks

A player will be stronger in equilibrium if his enemies are weaker(recall that we refer to the total spending of a node in the equilibrium as strength of a player.). The enemies a player (his first neighbours) will be weaker if they have stronger opponents, which are, apart from player i, the second neighbours of player i, and so on. That is, strong odd order neighbours (can be reached in path of odd length) will make an agent weaker, while the opposite is true for even order neighbours. The strength of a node i is thus endogenous, and intuitively can be understand using the thought experiment from above. If this effect was linear, the strength of a node would be a global linear centrality measure (like Katz-Bonacich centrality) but with negative decay factor β. In this model, although the logic is similar, the strength of a node is a non-linear centrality of a node. Characterizing this measure is, for this and similar models, a challenging task for a future.

Replacing transferable contest with a 'classic' contest (i.e. one in which players compete to get a larger share of a pie) would not change anything in terms of existence and uniqueness results for a game on a fixed network. However, as two types of contest have a different interpretation, the link formation protocol in the formation model needs to be adjusted. Now it is not clear why should link destruction be a bilateral decision, if a pie exist independently of contests. Innovation contest/patent race is a situation which more natural to model in this way (Baye and Hoppe, 2003). This approach could be naturally extended to hypergraphs. For example, consider a situation in which there are n firms and m markets (possible contests) in which firms can innovate. Then a linking strategy of a firm would be to decide in which of these m contests to participate, creating a hyperlink to other participants in these contests. The results for existence and uniqueness will for a fixed hypernetwork will hold if we specify a contest success function for market k as:

$$p_{ik} = \frac{\phi(s_{ik})}{\sum_{j\in(N^k)} \phi(s_{jk})}$$

where N^k is set of players competing in contest k, p_{ik} is the probability with which i wins the contest k and rest of the notation is analogue.

The extension that we are currently working on is modelling positive links explicitly, elaborating more on the role of the positive links. The preliminary results indicate that if the effect of positive links is reducing marginal costs of effort in contest, the qualitative results of the paper will not change.

Appendix A

An alternative formulation

Suppose that instead of a general convex cost function, we have the Blotto type game. That is, each player is endowed with equal amount of resources (time) and the strategy is how to distribute the resources on different contest. Note that this also define 'cost' function to be convex, as resources are free up to some point and then prohibitively costly.

So, suppose for simplicity $\sum_{j \in N_i} s_{ij} = 1 \ \forall i, j$. Keeping the same CSF we also have that existence, uniqueness and interiority guaranteed by the (Rosen, 1965) result. Let λ_i denotes Lagrange multiplier associated to the budget constraint for agent i. Then FOC read (setting $r = 0$ for simplicity):

Consider now two connected players i and j. The first order conditions that characterize their behaviour in a contest g_{ij} are given with (again assuming r is small so the equilibrium is interior):

$$
\begin{aligned}
\frac{(r + 2\phi(s^*_{ji}))\phi'(s^*_{ij})}{(r + \phi(s^*_{ij}) + \phi(s^*_{ji}))^2} - \lambda_i &= 0 \\
\frac{(r + 2\phi(s^*_{ij}))\phi'(s_{ji})^*}{(r + \phi(s^*_{ij}) + \phi(s^*_{ji}))^2} - \lambda_j &= 0 \\
\sum_{k \in N_i} s^*_{ik} &= 1 \\
\sum_{k \in N_j} s^*_{jk} &= 1
\end{aligned}
$$

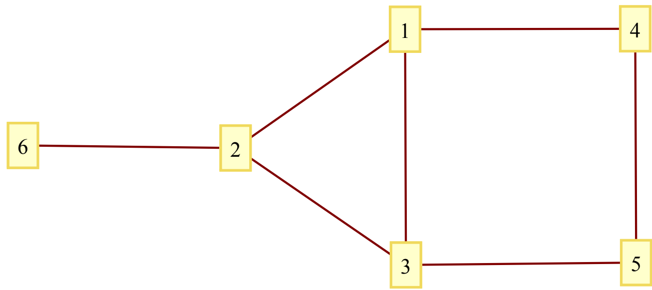
and from here, we get:

$$
\frac{(r + 2\phi(s^*_{ji}))\phi'(s^*_{ij})}{(r + 2\phi(s^*_{ij}))\phi'(s^*_{ji})} = \frac{\lambda_i}{\lambda_j} \tag{19}
$$

Thus role of λ_i is analogous to the role of A^*_i. Higher A^*_i implied higher marginal cost of additional unit of effort, and λ_i is the shadow price of the resource for player i in this formulation.

Appendix B

Let us consider the following example to illustrate the complexity of global ef-



fects in the network.

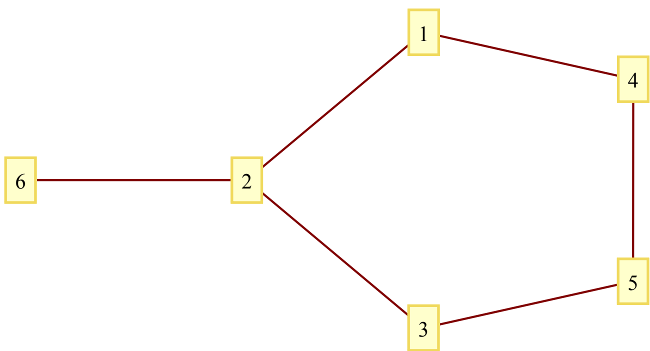
Calculating the equilibrium action (using the action adjustment proceess algorithm based on the results from section()) we get that matrix of the equilbrium efforts S is given with:

$$S = \begin{pmatrix} 0 & 0.289 & 0.289 & 0.286 & 0 & 0 \\ 0.292 & 0 & 0.292 & 0 & 0 & 0.269 \\ 0.289 & 0.289 & 0 & 0 & 0.286 & 0 \\ 0.350 & 0 & 0 & 0 & 0.354 & 0 \\ 0 & 0 & 0.350 & 0.354 & 0 & 0 \\ 0 & 0.479 & 0 & 0 & 0 & 0 \end{pmatrix}$$

And the assigned payoffs are:

$$\pi = (-0.854, -0.999, -0.854, -0.395, -0.395, 0.050)$$

Deleting link g_{13} we get a network with



and

$$\bar{S} = \begin{pmatrix} 0 & 0.351 & 0 & 0.354 & 0 & 0 \\ 0.290 & 0 & 0.290 & 0 & 0 & 0.270 \\ 0 & 0.351 & 0 & 0 & 0.354 & 0 \\ 0.353 & 0 & 0 & 0 & 0.353 & 0 \\ 0 & 0 & 0.353 & 0.353 & 0 & 0 \\ 0 & 0.480 & 0 & 0 & 0 & 0 \end{pmatrix}$$

and payoffs

$$\pi = (-0.402, -1.193, -0.402, -0.501, -0.501, 0.048)$$

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