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The Comparative Impact of Integrated Assessment Models' Structures on Optimal Mitigation Policies

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#### Summary

This paper aims at providing a consistent framework to appraise alternative modeling choices that have driven the so-called "when flexibility" controversy since the early 1990s dealing with the optimal timing of mitigation efforts and the Social Cost of Carbon (SCC). The literature has emphasized the critical impact of modeling structures on the optimal climate policy. But, to our knowledge, there has been no contribution trying to estimate the comparative impact of modeling structures within a unified framework. In this paper, we use the Integrated Assessment Model (IAM) RESPONSE to bridge this gap and investigate the structural modeling drivers of differences in climate policy recommendations. RESPONSE is both sufficiently compact to be easily tractable and detailed enough to capture a wide array of modeling choices. Here, we restrict the analysis to the following emblematic modeling choices: the forms of the damage function (quadratic vs. sigmoid) and the abatement cost (with or without inertia), the treatment of uncertainty, and the decision framework (oneshot vs. sequential). We define an original methodology based on an equivalence criterion to carry out a sensitivity analysis over modeling structures in order to estimate their relative impact on two output variables: the optimal SCC and abatement trajectories. This allows us to exhibit three key findings: (i) IAMs with a quadratic damage function are insensitive to changes of other features of the modeling structure, (ii) IAMs involving a non-convex damage function entail contrasting climate strategies, (iii) Precautionary behaviours can only come up in IAMs with non-convexities in damages.

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### The comparative impact of Integrated Assessment Models' structures on optimal mitigation policies

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October 23, 2013

#### Abstract

This paper aims at providing a consistent framework to appraise alternative modeling choices that have driven the so-called "when flexibility" controversy since the early 1990s dealing with the optimal timing of mitigation efforts and the Social Cost of Carbon (SCC).

The literature has emphasized the critical impact of modeling structures on the optimal climate policy. But, to our knowledge, there has been no contribution trying to estimate the comparative impact of modeling structures within a unified framework. In this paper, we use the Integrated Assessment Model (IAM) RESPONSE to bridge this gap and investigate the structural modeling drivers of differences in climate policy recommendations.

RESPONSE is both sufficiently compact to be easily tractable and detailed enough to capture a wide array of modeling choices. Here, we restrict the analysis to the following emblematic modeling choices: the forms of the damage function (quadratic vs. sigmoid) and the abatement cost (with or without inertia), the treatment of uncertainty, and the decision framework (one-shot vs. sequential).

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#### 1 Introduction

While the climate "proof" is no longer a matter of controversy among climate scientists (IPCC, 2007), the climate policy debate remains highly controversial. In a nutshell, the dynamic puzzle arising from a long-standing debate originated in the early 1990s, is about whether we should act strongly now, or gradually, and later.

Some economists promote sharp early abatements as a precautionary measure to prevent potential future catastrophic damages (Stern, 2006), while others argue that it is more economically sound to postpone abatement efforts (following a so-called "policy-ramp") and tolerate higher potential climate risks given that those risks would be better borne by supposedly richer future generations than relatively poor present ones (Nordhaus, 2008).

In order to explain such gap in results, the debate has mainly focused on the parametrization of the Integrated Assessment Models (IAM) used to represent the climate policy debate. In particular, after the release of the Stern (2006) Review, much attention has been paid to the choice of the discount rate (Dasgupta, 2007; Nordhaus, 2007; Weitzman, 2007; Yohe and Tol, 2007) to point out its critical impact on results. Another line of literature tends however to downplay the impact of the discount rate. Sterner and Martin (2007) show that the discounting effect can be offset by the effect of relative prices between a fixed quantity of environmental goods (the quality of the climate) and manufactured reproducible consumption goods. When the former become scarce, then their relative price mecanically goes up and may even compensate for the discounting impact. Hof et al. (2008) demonstrate that scientific uncertainties on climate damage and abatement costs matter as much as value judgements on pure time preference in defining "optimal targets" for climate policies. Following this line of literature, Espagne et al. (2012) show that a broader set of parameters, namely the rate of long term growth, climate sensitivity, the magnitude of climate damage, technological progress must be considered to understand the whole gap in results.

Little has been said however on the discrepancies between modeling structures. By modeling structures we mean the functional forms of the architecture of the model such as the form of the damage and abatement cost functions, the treatment of uncertainty, the choice of the decision framework (one-shot vs. sequential). Surprisingly disregarded in recent years, these modeling structures, which differentiate competing IAMs, were originally vividly debated in the 1990s. Building on the DICE model (Nordhaus, 1994), the so-called "when flexibility" controversy has mainly consisted of gradual refinements of the seminal structure of the DICE model exploring the impact of inertia in abatement costs (Ha-Duong et al., 1997), of non-linearities in damages (Ambrosi et al., 2003; Keller et al., 2004), of introducing uncertainty (Manne and Richels, 1992; Nordhaus, 2008, 2011; De Cian and Massimo, 2012), of irreversibility (Chichilnisky and Heal, 1993; Kolstad, 1996; Ulph and Ulph, 1997; Ha-Duong, 1998; Pindyck, 2000), of learning (Kelly and Kolstad, 1999), and of endogeneous technological change (Goulder and Mathai, 2000). The controversy has been all the more vivid as these choices of modeling structures have significant and non trivial impacts on results.

While the literature emphasized the critical impact of the choice of modeling structures on the optimal climate policy, there has been no contribution, to our knowledge, trying to appraise and compare the relative impact of modeling structures within a unified framework. This paper aims at bridging this gap by investigating the structural modeling drivers of differences in climate policy recommendations and proposing a consistent methodology to compare different modeling structures.

The paper focuses on the impact of structural modeling choices on two output variables of IAMs which are emblematic of the climate debate: the optimal Social Cost of Carbon (SCC) and abatement pathways. Along an optimal path of growth and carbon emission reduction, the SCC is the value equating at each date the discounted sum of the marginal abatement costs with the discounted sum of remaining marginal climate damages (Nordhaus, 2011; Pearce, 2003; Tol, 2008). This optimality rule makes it possible to delineate the efficient border of mitigation efforts.

We point out the pure effect of modeling structures by examining the deviation of the two output variables resulting from a change in the modeling structure. The analysis is carried out with the model RESPONSE (Dumas et al., 2012; Ambrosi et al., 2003). RESPONSE is an IAM which aims at providing a consistent framework to appraise alternative modeling structures used in most existing IAMs. It has thus originally been designed to be a flexible tool, able to adopt different modeling structures and compare results from the modeling frameworks that have driven the "when flexibility" controversy dealing with the optimal timing of mitigation efforts and the optimal trajectories of the SCC. RESPONSE is both sufficiently compact to be easily tractable and detailed enough to be as comprehensive as possible in order to capture a wide array of modeling features. In this paper we restrict the analysis to four emblematic modeling features, namely the forms of the damage function (quadratic vs. sigmoid), of the abatement cost (with or without inertia), the treatment of uncertainty, and the decision framework (one-shot vs. sequential).

One of the incremental value of this work is to propose an original methodology based on an equivalence criterion to carry out a sensitivity analysis over modeling structures in order to estimate their relative impact on optimal mitigation policies. This allows us to exhibit three key findings on IAMs from the RESPONSE family: (i) when they embed a quadratic damage function they are insensitive to changes of other features of the modeling structure, (ii) when they involve a non-convex damage function they entail then contrasting climate strategies, (iii) precautionary behaviours can only come up in IAMs with nonconvexities in damages.

Section 2 introduces the model RESPONSE and the different modeling structures it can take. Section 3 discusses the parametrization of RESPONSE and our methodology for comparing the impact of structural modeling choices. Section 4 uncovers the pure impact of modeling structures through a box plot analysis of the distribution of results. Section 5 sums up the three key findings arising from our analysis. Section 5 concludes.

#### 2 The model RESPONSE

#### 2.1 Storyline of the model

RESPONSE is an IAM that couples a macroeconomic optimal growth model<sup>1</sup> with a simple climate model, following the tradition launched by the seminal DICE model (Nordhaus, 1994).

The optimization program of RESPONSE aims at maximizing an intertemporal social welfare function composed of the consumption of a composite good<sup>2</sup>. Time horizon of the maximization is 2200. Greenhouse gases (GHG) are responsible for temperature increase and thus for climate damages. GHG emissions are a by-product of the production, offset by costly abatement effort. As climate damages negate part of the production, the optimization process consists in allocating the optimal share of the output among consumption, abatement and investment.

In the reference modeling structure of RESPONSE based on DICE (Nordhaus, 2008) and PAGE (Stern, 2006; Hope, 2006), climate damage as well as abatement costs are represented with quadratic functions. This gives a smooth increasing profile to both functions. The program is solved deterministically as no uncertainty on either techno-economic nor climate dynamics is taken into account.

The flexibility of the modeling structure of RESPONSE makes it possible to activate or deactivate some modeling options and thus to re-build step by step the "when flexibility" controversy. It is possible to add an "inertia effect" in the abatement cost function to take into account the impact of the speed of abatement which turns out to be critical in the case of very bad climate outcome that would require rapid change in abatement path. It is also possible to track non-convexity effects in climate damages, replacing the quadratic damage function with a sigmoid one which triggers a jump in damages from a certain level of temperature increase.

Finally, RESPONSE enables us to switch from a deterministic to an uncertain model by integrating uncertainty on both climate sensitivity (i.e. on atmospheric temperature increase) and climate damage assuming that both uncertainties are independent. The optimization program can be solved within either a one-shot or a sequential decision framework to appraise the impact of information arrival at different points in time  $t_i$ . At time  $t_i$ , uncertainties about climate sensitivity and damage are resolved<sup>3</sup>. This representation of uncertainty coupled with the inertia effect in abatement cost and non-linearity in climate damages allows us to address the discussion about the relative weight of economic and environmental irreversibilities and explore the concepts of "value of information" (Ambrosi et al., 2003) and "option value" of different types of climate policies (Pindyck, 2000; Ha-Duong, 1998).

<sup>&</sup>lt;sup>1</sup>Much like Ramsey-Cass-Koopmans' models (Ramsey, 1928; Koopmans, 1963; Cass, 1965).

<sup>2</sup>The climate externality does not enter into the utility function. It is only captured by a damage function.

<sup>&</sup>lt;sup>3</sup>Note that RESPONSE does not address the learning issue (Kelly and Kolstad, 1999; Goulder and Mathai, 2000) and thus assumes that information arrives in an exogenous fashion.

#### 2.2 The deterministic model

#### 2.2.1 The representative household

We consider a 2200-horizon discrete-time economy inhabited by a continuum of size  $N_t$  of identical households. These households derive instantaneous utility from consumption of a composite good. A benevolent planner maximizes their intertemporal utility:

$$V_{t_0} = \max_{a_t, C_t} \sum_{t=t_0}^{2200} N_t \frac{1}{(1+\rho)^t} u\left(\frac{C_t}{N_t}\right), \tag{2.1}$$

with  $\rho$  the pure-time preference rate. The model does not endogenise the demographic dynamics, thus the number of households  $N_t$  evolves exogenously.

We use a standard logarithmic utility function as in Stern  $(2006)^4$ :

$$\forall c, u(c) = \log(c) \tag{2.2}$$

This instantaneous utility function has the standard properties: it is increasing, twice differentiable and concave. It furthermore follows the Inada condition  $\lim_{0^+} u' = +\infty$ . The elasticity of intertemporal substitution is constant and equal to 1.

#### 2.2.2 The production side

The economy produces a unique final good  $Y_t$ , from capital  $K_t$  and labour  $L_t$ . The production function is the traditional Cobb-Douglas:

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha} \tag{2.3}$$

The share of capital in revenues is  $\alpha$ .  $L_t$  is an exogenous variable representing the labour force; as there is no unemployment nor work-leisure trade-off, up to a normalization factor, it is equal to the number of households  $N_t$ . The total productivity factor  $A_t$  evolves exogenously.

Depending on consumption and abatement choices, the capital variable  $K_t$  evolves endogenously according to:

$$K_{t+1} = (1 - \delta)K_t + Y_t \left[1 - C_a(a_t, a_{t-1}) - D(\theta_{A,t})\right] - C_t$$
 (2.4)

The depreciation rate of the capital is  $\delta$ . The abatement cost function  $C_a(a_t, a_{t-1})$  depends on the abatement levels at the current period  $a_t$  and possibly of the past period  $a_{t-1}$ , in case of inertia. The damage function  $D(\theta_{A,t})$  varies with the atmospheric temperature increase  $\theta_{A,t}$ . Abatement cost and damages are expressed relatively to total output  $Y_t$ , i.e. in percent of GDP.

Emissions of  $CO_2$  are a by-product of the production, and can be offset by abatement effort  $a_t$ . Thus the total emission level is:

$$E_t = \sigma_t (1 - a_t) Y_t \tag{2.5}$$

<sup>&</sup>lt;sup>4</sup>The logarthmic utility function is a particular case of the constant relative risk aversion utility functions family. In this paper we do not perform sensitivity analysis over the form of the utility function

The carbon intensity of production  $\sigma_t$  is expected to decline progressively thanks to an exogenous technical progress:

$$\sigma_t = \sigma_0 \,\mathrm{e}^{-\psi_t t} \tag{2.6}$$

with  $\psi_t > 0$  that captures the joint impact of technical change and depletion of fossil resources. If the economy grows at rate g, the level of carbon emissions is proportional to  $e^{(g-\psi_t)t}$ . As long as  $g > \psi_t$ , carbon emissions would continue to grow over time. To guarantee that emissions decrease by the end of the century, as predicted by the overwhelming majority of available scenarios (IPCC, 2007),  $\psi_t$  progressively increases so that it can become higher than g as follows, with  $\beta > 0^5$ :

$$\psi_t = \psi_0 e^{-\beta t} + 1.1g(1 - e^{-\beta t}), \tag{2.7}$$

Abatement  $a_t$  is expressed in fraction of emissions cut:

$$0 \le a_t \le 1 \tag{2.8}$$

If  $a_t = 1$ , then emissions become null, if  $a_t = 0$ , then no mitigation efforts are made.

#### 2.2.3 Damage function

Two damage functions are used alternatively in RESPONSE. The first possibility is a quadratic function:

$$D(\theta_{A,t}) = \kappa \theta_{A,t}^2, \tag{2.9}$$

where  $\theta_{A,t}$  is the atmospheric temperature increase at time t. The second possibility is a sigmoid (or logistic) function (Ambrosi et al., 2003):

$$D(\theta_{A,t}) = \kappa \theta_{A,t} + \frac{d}{1 + e^{(\theta_D - \theta_{A,t})/\eta}}$$
(2.10)

This damage function has a linear trend of slope  $\kappa$  with a smooth jump at a temperature threshold  $\theta_D$ . The jump of size d is triggered when atmospheric temperature increase  $\theta_{A,t}$  overshoots the threshold. Non-linearity in damages does not occur abruptly but instead progressively over a range  $\eta$  of temperature increase around  $\theta_D$ .

One single function encapsulates these two forms of damage function:

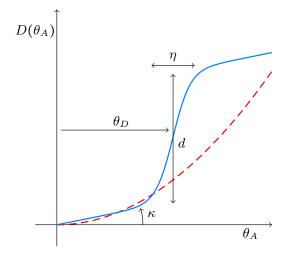
$$D(\theta_{A,t}) = \kappa \theta_{A,t}^{1+\phi} + \frac{d}{1 + e^{(\theta_D - \theta_{A,t})/\eta}}$$
 (2.11)

with  $\phi=1,\,d=0$  correspond to the quadratic case and  $\phi=0,\,d>0$  to the sigmoidal case<sup>6</sup>.

<sup>&</sup>lt;sup>5</sup>The calibration choice of the number 1.1 makes sure that at some far enough point in time t,  $\psi$  is greater than q.

<sup>&</sup>lt;sup>6</sup>Note that in this case the optimization is not a convex problem anymore and may raise computational issues to find the global optimum. To overcome this issue we run the model with multiple draws and compare resulting optima to make sure that the result is not "trapped" in a local optimum

Figure 1: Possible forms of the damage function in RESPONSE



The solid line curve represents the sigmoidal case:  $\theta_D$  is the temperature threshold where the non-linearity occurs,  $\eta$  is the width of the non-linearity phase, d is the size of the jump in damage during the non-linearity phase, and  $\kappa$  is a linear trend of damages. The dotted line curve represents the quadratic case,  $\kappa$  symbolizing the curvature.

#### 2.2.4 Abatement costs

The abatement cost function writes:

$$C_a(a_t, a_{t-1}) = \frac{1}{(1+\gamma)^t} \left( a_t \zeta + (BK - \zeta) \frac{(a_t)^{\nu}}{\nu} + \xi^2 (a_t - a_{t-1})^2 \right)$$
(2.12)

The cost function has two main components: the absolute level of abatement  $\frac{(a_t)^{\nu}}{\nu}$ , with  $\nu$  being a power coefficient, and a path-dependent function that penalizes the speed of decarbonisation  $(a_t - a_{t-1})$ . This abatement cost function allows us to account for an "inertia effect" (when  $\xi \neq 0$ ) which penalizes abatement costs when the speed of abatement increases too rapidly as in Vogt-Schilb et al. (2012).  $\gamma$  is a parameter of exogenous technical progress on abatement technologies, BK stands for the current price of the backstop technology or put in other words the marginal cost when abatement is total.  $\zeta$  is the cost of the first unit of abatement.

#### 2.2.5 The climate module

#### 2.2.6 The three-reservoir climate module

The climate module is described through a three reservoirs linear carbon-cycle model. We use Nordhaus' carbon cycle (Nordhaus and Boyer, 2003), a

linear three-reservoir model (atmosphere, biosphere and surface ocean, and deep ocean). Each reservoir is assumed to be homogeneous in the short run. It is also characterized by a residence time and mixing rates with the two other reservoirs in the long run. Carbon flows between reservoirs depend on constant transfer coefficients. GHG emissions (solely  ${\rm CO_2}$  here) accumulate in the atmosphere and are slowly removed by biosphere and ocean sinks.

The dynamics of carbon flows is given by:

$$\begin{pmatrix} A_{t+1} \\ B_{t+1} \\ O_{t+1} \end{pmatrix} = \mathbf{C}_{trans} \begin{pmatrix} A_t \\ B_t \\ O_t \end{pmatrix} + \begin{pmatrix} E_t \\ 0 \\ 0 \end{pmatrix}$$
 (2.13)

 $A_t$  represents the carbon stock of the atmosphere at time t,  $B_t$  the carbon stock of the upper ocean and biosphere at time t and  $O_t$  the carbon stock of deep ocean at time t;  $\mathbf{C}_{trans}$  is the transfer coefficient matrix. As there is no direct exchange between atmosphere and deep ocean,  $c_{AO} = c_{OA} = 0$ .

In spite of the well-known limitations of this simplified carbon cycle model (Archer and Brovkin, 2008; Archer et al., 2009; Friedlingstein et al., 2006; Tol, 2009; Gitz et al., 2003), it makes it possible to provide policy-relevant information regarding  $\rm CO_2$  atmospheric concentration evolution which could not be delivered by a simple carbon budget rule.

#### 2.2.7 The two-box temperature module

The temperature module resembles Schneider and Thompson's two-box model (Schneider and Thompson, 1981) and builds on Ambrosi et al. (2003). Two equations are used to describe global mean temperature variation since pre-industrial times in response to additional GHG forcing. More precisely, the model describes the modification of the thermal equilibrium between the atmosphere and surface ocean in response to anthropogenic greenhouse effects.

The radiative forcing equation at time t is given by:

$$F(A_t) = F_{2x} \log_2(A_t/A_{PI})$$
 (2.14)

where  $F_{2x}$  is the instantaneous radiative forcing for a doubling of pre-industrial concentration; and  $A_{PI}$  is the atmospheric stock at pre-industrial times.

The temperature equation is given by:

$$\begin{pmatrix}
\theta_{A,t+1} \\
\theta_{O,t+1}
\end{pmatrix} = \begin{pmatrix}
1 - \sigma_1(F_{2x}/\vartheta_{2x} + \sigma_2) & \sigma_1\sigma_2 \\
\sigma_3 & 1 - \sigma_3
\end{pmatrix} \cdot \begin{pmatrix}
\theta_{A,t} \\
\theta_{O,t}
\end{pmatrix} + \begin{pmatrix}
\sigma_1F(A_t) \\
0
\end{pmatrix} (2.15)$$

where  $\theta_{A,t}$  and  $\theta_{O,t}$  are, respectively, global mean atmospheric and sea surface temperature increases from pre-industrial times (in Kelvin);  $\sigma_1$ ,  $\sigma_2$ , and  $\sigma_3$  are transfer coefficients, and  $\vartheta_{2x}$  is the climate sensitivity, i.e. the ultimate temperature increase due to a doubling of pre-industrial level of atmospheric GHG concentration.

#### 2.3 The model with uncertainty

#### 2.3.1 The representation of uncertainties

Moving from the deterministic case to the uncertain case aims at taking into account current limitations of human knowledge about climate change.

Even though the two most recent Intergovernmental Panel on Climate Change reports, and the Stern Review have already brought the "climate proof", all kinds of controversies are far from resolved, especially on the value of climate sensitivity and the extent of climate damages. Then, instead of single values, scientific results in the field rather provide ranges of reasonable values along with levels of confidence. As no decisive scientific argument has been brought so far to pick one state of the world rather than another, there are different competing beliefs in the climate debate about which state of the world will occur.

To encompass the entire range of scientific uncertainties about climate damage, we assume that there are different possible states  $\omega$  of nature, different with respect to climate sensitivity  $\vartheta_{2x}^{\omega}$  and the form of the damage function:

$$D^{\omega}(\theta_{A,t}) = \kappa^{\omega} \theta_{A,t}^{1+\phi} + \frac{d^{\omega}}{1 + e^{(\theta_D^{\omega} - \theta_{A,t})/\eta}}$$
(2.16)

As climate change is basically a non-reproducible event, subjective distribution of probabilities are given over the possible states of the world considering that climate sensitivity and climate damage are independent. These distribution of probabilities account for the different competing beliefs in the climate debate and RESPONSE can be run for each belief.

We assume that there is a period  $t_i$  at which information about the true state of the world arrives. Then, in  $t_i + 1$  people adapt their behavior to the new information. They accelerate abatement in the case of "bad news" or relax their efforts in the case of "good news". The question each stakeholder must consider then becomes: what is the good trade-off between the economic risk of rapid abatement now against the corresponding risk of delay (Lecocq et al., 1998)?

Such modeling of uncertainty makes it possible to appraise whether taking into account both kinds of uncertainties affects the solution by inducing more conservative (i.e. precautionary) decisions. This is particularly interesting when the damage function is not simply increasing but also non-linear, as it is the case with the sigmoid damage function.

A two-step analysis is conducted that consists in solving the program recursively. The intertemporal optimization program is divided between two subprograms, after and before the information arrival date  $t_i$  respectively. Note that we can also account for the case with deep uncertainty when there is no resolution of information, if we take  $t_i = \infty$ .

#### 2.3.2 After uncertainty is resolved

At time  $t_i + 1$  when the true state of nature  $\omega$  is known, that is, the climate sensitivity  $\vartheta_{2x}^{\omega}$  and the damage function  $D^{\omega}$  are known, the intertemporal maximization program is the same as in the deterministic case we investigated previously. Variables corresponding to the solution of this program will be denoted by an upper script  $\omega$ .

When we compute the discounted utility along the solution, we get the welfare  $V_{t_{i+1}}(\omega)$  for each revealed state of nature  $\omega$  at  $t_i$ .

#### 2.3.3 Before uncertainty is resolved

Before information arrival on the true states of nature at the end of period  $t_i$ , the objective function to maximize writes<sup>7</sup>:

$$W_{t_0} = \max_{\overline{a}_t, \overline{K}_t} \mathbb{E} \left[ \sum_{t=t_0}^{t=t_i} \frac{1}{(1+\rho)^t} u(C_t^{\omega}) + V_{t_i+1}(\omega) \right].$$
 (2.17)

The variables following the same trajectory in all states of nature before  $t_i$  are overlined. This is the case for capital and abatement variables  $\overline{K}_t$  and  $\overline{a}_t$ , and thus also for production  $\overline{Y}_t$ , emissions  $\overline{E}_t$  and carbon stocks  $\overline{A}_t$ ,  $\overline{B}_t$ ,  $\overline{O}_t$ .

The other variables which depend on the state of nature  $\omega$  are written with an upper script. This is the case for the temperatures  $\theta_{A,t}^{\omega}$ ,  $\theta_{O,t}^{\omega}$  because their evolution depends on the unknown climate sensitivity  $\theta_{2x}^{\omega}$ . The damage function  $D^{\omega}$  also depends on the state of nature. So does the consumption  $C^{\omega}$  by equation (2.4). This implicitly means that different damages across different states of nature only affect consumption level and not the investment. If the consumption were also set at a fixed level, then the observation of either the investment or the capital would immediately lead to the observation of the true state of nature. This is why we make the hypothesis that the consumption cannot be observed per se, but instead that only the sum of the consumption and the damage can be quantified. This modeling choice is debatable but we guess that it is easier to observe the level of investment than the consumption one and therefore that it makes sense to consider the capital variable as the control variable.

The resolution of the first-order conditions and the analytical calculation of the social cost of carbon are solved in appendix.

## 3 A methodology to carry out a sensitivity analysis over modeling structures

#### 3.1 The reference case

#### 3.1.1 The reference modeling structure

The reference modeling structure of RESPONSE is close to the seminal DICE model. This structure involves:

- a quadratic damage function (i.e.  $\phi = 1$ , d = 0)
- no inertia in the abatement cost function (i.e.  $\xi = 0$ )
- a perfectly certain environment, both regarding climate damage and climate sensitivity

This structure is considered as the reference case, against which all structural comparisons are made hereafter.

<sup>&</sup>lt;sup>7</sup>Here and in the rest of the document,  $\mathbb{E}$  stands for the expectation operator:  $\mathbb{E}[f] = \sum_{\omega} p(\omega) f(\omega)$ .

Table 1: The four variables accounting for the differences in worldviews

Annual growth rate	$g \in \{0.01, 0.017, 0.023, 0.03\}$
Pure time preference	$\rho \in \{0.001, 0.01, 0.02, 0.03\}$
Abatement cost in 2005	$BK = \$1,200/\text{tCO}_2 \text{ with four an-}$
	nual rates of decrease $\gamma$ tied to four
	initial marginal costs $\zeta$ . $(\gamma, \zeta) \in$
	$\{(0.0025,0),(0.019,76),(0.036,153),(0.052,229)\}$
Climate sensitivity	$\vartheta_{2x} \in \{2; 2.67; 3.33; 4\}$

#### 3.1.2 A set of worldviews

We run RESPONSE in its reference structure with 256 scenarios resulting from a sensitivity analysis over four variables, namely the growth rate, abatement costs, pure time preference, and climate sensitivity taking four values within "reasonable" ranges provided by IPCC (2007). The calibration of those variables remains highly controversial in the literature and eventually results from a *subjective* choice within *objective* ranges provided by most advanced research. This is why those scenarios represent as many types of what we call a "worldview". We summarize the ranges of values chosen for the different worldviews in table 1.

That way, we can observe a large set of trajectories (for each structure and each worldview) of two output variables: the optimal SCC and the abatement path. In fact, we focus on these two variables because they are emblematic of the ambition of any mitigation policy. The abatement path expresses the timing of investment efforts in mitigation projects, while the SCC symbolizes the price a society is willing to pay to curb climate change at each period of time.

Sensitivity analyses over what we call "worldviews" are crucial for the sake of the stability of policy recommendations. In addition, we apply in this paper similar analyses to the choice of modeling structures, the impact of which has never been estimated, to our knowledge, within a unified modeling framework. We provide here a methodological tool to examine the sensitivity of climate policy recommendations to modeling structures and shed light on a wide diversity of responses to the climate challenge which may be forgotten or undetectable in a more restrictive modeling framework.

#### 3.2 The different modeling structures

Starting from the reference case, we explore 15 other modeling structures listed in table 2. The structures result from:

- a combination of two forms of damage function, labelled "q" for quadratic and "s" for sigmoid, with two forms of abatement function, labelled "in" when inertia is integrated and "no" when not
- the integration or not of uncertainty
- two dates of information arrival (2050 and 2150) in the uncertain cases.

Table 2: Reference and structural modeling choices

Structures	Climate damage	Climate sensitivity	Abatement costs
Functional forms uncertainty	Quadratic Sigmoid certain uncert certain uncert	certain uncert	no in- inertia ertia
$t_{info}$	2050 2150 2050 2150		
qc-tc-no qc-tu-no-2050 qc-tu-no-2150 qu-tc-no-2050 qu-tc-no-2150 sc-tc-no su-tc-no-2050 su-tc-no-2150		✓	\ \ \ \ \ \ \
qc-tc-in qc-tu-in-2050 qc-tu-in-2150 qu-tc-in-2050 qu-tc-in-2150 sc-tc-in su-tc-in-2050 su-tc-in-2150		✓ ✓ ✓ ✓ ✓ ✓ ✓ ✓ ✓ ✓ ✓ ✓ ✓ ✓ ✓ ✓ ✓ ✓ ✓	

When climate sensitivity is uncertain (respectively certain), the corresponding modeling structure is labelled "tu" (respectively "tc"). When uncertainty is on the form of the sigmoid damage function (respectively, the form of the quadratic damage function), the corresponding modeling structure si labelled "su" (respectively "qu"). In the certain case those two modeling structures become "sc" and "qc".

Moving from the reference modeling structure to another modeling structure and then appraising the "pure effect" of this structural change on the results is not straightforward. Our approach aims at comparing modeling structures which a priori entail contrasting optimal timing for climate policies.

We need to make sure that we only measure the effect of the structural change in the model and neutralize the impact of the parametrization of the different modeling structures. It will be confusing indeed to directly compare results (in terms of optimal SCC and abatement) from the modeling structures "qc" and "qu" whithout making sure that differences do not come from the calibration of the damage function (uncertainty on the curvature of the quadratic function may increase the expected value of this curvature and then increase the value of damage) but only from the integration per se of uncertainty in the damage function. Making the comparison possible and meaningful requires then an equivalence criterion between modeling structures.

#### 3.3 The equivalence criterion

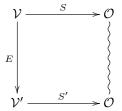
To better understand the problem underlying the comparison of structures let us take the following example.

If to a given quadratic damage function we compare a sigmoid function with substantial higher damages (picking a high d for example), then we could not say that differences in output are only the effects of a different shape of damage function. It could equally result from the fact that the damages are higher ceteris paribus and that this accounts for the differences in outcome. This is the main problem we find in most of the literature dealing with the "when flexibility" when it comes to compare the relative impacts arising from different modeling structures. The natural tendency was to add a "structure" to an existing model and compare the outcome with the one without that "structure", which to our opinion is not the appropriate comparison point. We suggest here a methodology to overcome that difficulty and estimate the comparative impact of a structural choice in the timing of mitigation policies. This is why we need an equivalence criterion to cancel out the effect of differences in damage.

Hereafter we give the generalization of this example in a more formalized fashion.

Let us denote S a modeling structure. The modeling structure is defined by a combination of structural relations and a variables space. We consider a modeling structure as a map  $S: \mathcal{V} \to \mathcal{O}$  relating the variables space  $\mathcal{V}$  to the output space  $\mathcal{O}$  composed of model's results (such as trajectories of abatement and SCC).

Now, take a second modeling structure S'. We need to give the full description of the modeling structure, that is  $S': \mathcal{V}' \to \mathcal{O}'$ . S' relates the variables space  $\mathcal{V}'$  to the output space  $\mathcal{O}'$ . To make sure that output spaces are comparable we assume that  $\mathcal{O} = \mathcal{O}'$ , i.e.  $\mathcal{O}$  is composed of the same elements as  $\mathcal{O}'$  (trajectories of abatement and SCC). However, in general, variables space are different. How can we compare the output  $S(x), x \in \mathcal{V}$  with  $S'(y), y \in \mathcal{V}'$  given that  $\mathcal{V} \neq \mathcal{V}'$ ? Indeed the difference between the outputs can come from the difference in modeling structures as well as from the difference in variables x and y. We thus need to relate a point x to a particular point y(x), if we want to isolate the pure effect of the modeling structure. The comparison between two outputs would be meaningless, unless we can link somehow a certain point in the variable space  $\mathcal{V}$  to an "equivalent" point in  $\mathcal{V}'$ . We thus need an equivalence map E from  $\mathcal{V}$  to  $\mathcal{V}'$  as sketched in the following chart:



Then we can compare the modeling structures by comparing the effect on the variables space  $\mathcal{V}$  of S and  $S' \circ E$ .

To build the equivalence map between two variables spaces, let us go back to the example of the equivalence map from the variables space of the reference quadratic certain case to the variables space of the sigmoid certain case without inertia (from "qc-tc-no" to "sc-tc-no" with the notations of Table 2). A point in the variables space is a given set of variables. In the quadratic certain case these variables are  $(g, \rho, \gamma, \vartheta_{2x}, \kappa_Q)$  whereas in the sigmoid certain, these are  $(g, \rho, \gamma, \vartheta_{2x}, \kappa_S, \eta, d, \theta_D)$  (growth rate, pure-time preference, technical progress on abatement cost, climate sensitivity, slope of the linear part of damage function, width of the jump in damage, size of the jump, temperature threshold). Some variables, like growth, pure time preference rate technical progress on abatement cost, climate sensitivity have the same meaning in both spaces, and thus are identically related. The equivalence map is thus the identity on these components. In this case, our equivalence map is tantamount to a relation between the parameter of the quadratic damage function  $\kappa_Q$  and the parameters of the sigmoid damage function  $(\kappa_S, \eta, d, \theta_D)$ .

Our equivalence criterion satisfies the following definition:

A modeling structure S is equivalent to a modeling structure S' when non discounted cumulated climate damage entailed by S and S' along the baseline (*i.e.* when no abatement is undertaken) till 2100 are equal.

In the case discussed above, we implement the equivalence criterion as follows: we set  $\kappa_S$ ,  $\eta$  and d, and adjust the temperature threshold  $\theta_D^8$  such that the sum of damages in the baseline scenario of the modeling structure "sc" is equal to the sum of damages in the baseline scenario of the reference modeling structure.

We acknowledge that the choice of the equivalence criterion is somehow arbitrary. The following points delves into the reasons for our choice.

- First, the equivalence criterion is computed along a baseline scenario as we do not want to embark the effect of the optimization in the equivalence, as the residual cumulated damage (after optimal abatement) would do, but concentrate on the effect of the modeling structure per se.
- Second, we make the equivalence criterion rely on the damages only, because CO<sub>2</sub> emissions and temperature increase feedback on the economy only through climate damages. As climate policies are proportionate to the expected climate change outcome, if climate damage were not kept equal between different modeling structures it would be hardly possible to disentangle the pure impact of S from the impact of the magnitude of climate damage.

 $<sup>^8</sup>$  The choice of adjusting  $\theta_D$  is somehow arbitrary as the three other parameters can be seen as equally good candidates. Still we believe that the temperature threshold of the damage function has a greater policy relevance than the magnitude of the damage d since the climate policy debate has always been more focused on temperature targets, in particular the 2°C target, than on the size of the damages which remains highly controversial. Adjusting on  $\kappa_S$  or  $\eta$  would have brought unclear interpretation as they are two technical parameters of the sigmoid damage function.

<sup>&</sup>lt;sup>9</sup>To compare the quadratic certain case with the quadratic case with climate sensitivity uncertainty, we set the support of the probability distribution of climate sensitivities and look for the probabilities so that the expected sum of damage in the baseline scenario of both modeling structures are equal. We then impose that the probabilities have a certain law, depending on only one parameter, and solve for this parameter to satisfy the equivalence criterion. The equivalence map for other comparisons always relies on the same equivalence criterion. Details are available upon request.

• Third, the reasons for not discounting the sum of damages is justified by our objective to put emphasis on the different optimal timing for climate policies expressed by our modeling structures. In particular, the quadratic damage function entails relatively low damages in the short term and very high ones in the long term as soon as temperature increase is significant; while on the contrary, the sigmoid form of the damage function induces high damages for relatively lower temperatures (in the medium term). Discounting the equivalence criterion would distort this differential timing effect and embark the proper effect of the discount parameter in the comparison.

Our methodology is not intended to solve all the complexities of comparing modeling structures. For example, we have found no sensible equivalence criterion for abatement costs to relate structures with or without inertia. For the sake of clarity and policy relevance of the results we think it still makes sense to compare modeling structures with and without inertia even though we do not apply an accurate equivalence criterion between them.

Beyond this attempt to compare modeling structures in a consistent framework, further research is needed to refine our equivalence criterion and single out more systematically the comparative impact of modeling structure when climate policy debates involve differences in modeling frameworks.

#### 4 Uncovering the impact of modeling stuctures on the distribution of results

#### 4.1 Descriptive statistics

To distinguish the effects of modeling structures from the impacts of the parameters composing the worldviews on the optimal SCC and abatement trajectories we look at the distribution of results for different modeling structures. We use boxplot analyses which give a synthetic representation of the distribution of results, to pinpoint key features of the distribution of results for each given modeling structure at a given date. Figures 2, 3 and 4 offer vivid snapshots of the distribution of results in the short (2020), medium (2050) and long (2100) terms, and make it easy to visually compare the effect of the different modeling structures over time.

The Boxplot analysis reads as follows:

- the first and third quartiles delineate the box and a horizontal bar spliting the box indicates the median value of the variable of interest (abatement and SCC),
- the whiskers gather a fraction (here 90%) of the population of worldviews for each given structural choice.

<sup>&</sup>lt;sup>10</sup>A natural candidate could have been the cumulated abatement costs. But as inertia in abatement costs is modeled as an extra-cost, it is likely that equal cumulative costs imply lower abatement over the whole period in cases with inertia. Then the equivalence criterion would directly make a difference on the timing of abatement. A criterion such as cumulated undiscounted costs to meet a carbon budget would offset this drawback but is more difficult to handle.

#### 4.2 The impact of the functional form of climate damage

Figure 2 shows that the functional form of the damage function has a strong impact on the abatement path and the SCC in the short, medium, and long term. The general effect of changing the damage function from a quadratic form to a sigmoid one is to lower the values of both abatement levels and the SCC. Median values of the SCC for instance resulting from the quadratic case are almost twice as high as those resulting from the sigmoid case.

Another striking result is that abatement remains very low during a long period in the sigmoid case as the median level of abatement is still null in 2050 while it almost reaches 30% in the quadratic case. Yet, this difference significantly shrinks in the long term as abatement levels in 2100 are similar in both cases.

This can be explained by the fact that damages in the sigmoid case are not distributed over time in the same fashion as in the quadratic case. Indeed in the former case they occur suddenly when temperature increase  $\theta_{A,t}$  exceeds the temperature threshold  $\theta_D$  (see equation 2.10). The threshold effect is triggered in the short to mid term when climate sensitivity is high and far in the future when climate sensitivity is low. While in the quadratic case, climate damages follow a smooth ramp since the beginning of the period. Given the equivalence criterion cumulated damages are equal in both cases. However, as high level of damages mostly occur in the future in the sigmoid case it may be then less costly to postpone the beginning of mitigation efforts due to the effect of discounting (whatever the level of the discount rate) and then strongly increase abatement in order to catch up abatement levels of the quadratic case in the long run.

#### 4.3 The impact of uncertainty

Figure 3 shows that uncertainty on the quadratic functional form (i.e. on the parameter  $\kappa$ ) and uncertainty on climate sensitivity impact neither the abatement path, nor the SCC. This result points out the remarkable stability of the reference modeling structure.

On the contrary, figure 4 points out that uncertainty on the temperature threshold  $\theta_D^{\omega}$  expands the distribution of abatement in the short term, strongly increases median abatement in the mid term and makes median abatement converge on the same level as in the case without uncertainty in the long run.

The date of information arrival  $t_i$  about the true state of the world  $\omega$  also has a strong impact. The later information arrives the higher and the more scattered the results.

This increase in abatement effort due to uncertainty may be interpreted as the effect of a precautionary behaviour. This precautionary behaviour is particularly striking in 2050 when mean abatement level turns out to be around 20% while it is still null in the certain case. Regarding the SCC, uncertainty also impacts the results by pushing up the median and expanding the 25-75 percentiles ranges.

#### 4.4 The impact of inertia

It is first worth recalling that as was previously discussed in section 2 we do not use any equivalence criterion to compare modeling structures with and

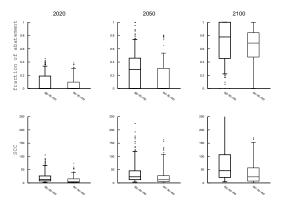


Figure 2: The impact of the damage function on both the abatement path and the SCC. The boxes gather the 25 to 75 percentiles while the whiskers gather 90% of the worldviews. Points that appear beyond the whiskers are considered as outliers

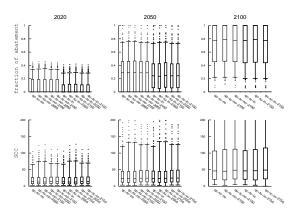


Figure 3: The impact of inertia and uncertainty on both the abatement path and the SCC when the damage function has a quadratic form  $\frac{1}{2}$ 

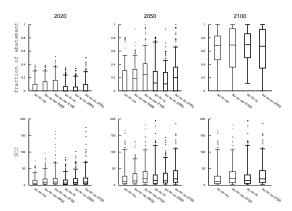


Figure 4: The impact of inertia and uncertainty on both the abatement path and the SCC when the damage function has a sigmoid form  $\frac{1}{2}$ 

without inertia.

While inertia in abatement cost has almost no impact on the SCC, in any structural cases, and at any time horizon, it significantly impacts the abatement paths. In all but one case, the overall effect of inertia is to lower abatement levels, compared with the cases without inertia. For example, in 2020, the third quartile in the reference case is at almost 20%, while it only reaches 10% with inertia. Note that in 2020, the median level of abatement is null for all modeling structures. This downward effect is mostly observed in the short term (2020) and tends to vanish with time as a catching up process of abatement paths between modeling structures without inertia and those with inertia seems to occur in the mid and long runs. This is due to the fact that inertia increases the cost of rapid short term abatement and makes it optimal to postpone abatement in the future when it will be relatively less costly due to technological progress and the discounting effect.

This effect of inertia occurs for all structures but the one with sigmoid climate damage where inertia leads to a significant increase of the median level of abatement in 2050. Inertia makes it optimal to smooth the abatement path since fast increase in abatement after 2050 would become too costly. In 2100 this effect is no longer noticeable as abatement levels get pretty similar in all modeling structures. When combined with uncertainty this effect does not happen as uncertainty by itself has already triggered a precautionary behaviour pushing up abatement in the mid run.

#### 5 Three key findings

Our methodology to perform a sensitivity analysis over IAM structures based on an equivalence criterion has allowed us to disentangle the impacts of core modeling features from those of worldviews parametrization that are more commonly addressed in usual sensitivity analyses. It also brings qualitative information about the form of the results, and thus the type of policy recommendations that one can expect to come up from a given modeling structure whatever the worldviews retained to run the simulations. This useful information is summed up in the three following "key findings" on the behaviours of IAMs from the RESPONSE family.

#### Key finding 1: RESPONSE with a quadratic damage function is insensitive to changes of other features of the modeling structure

It is quite astonishing that modeling changes operated on the reference quadratic damage function barely impact the distribution of results. Neither uncertainty on the curvature of the quadratic function, nor uncertainty on climate sensitivity, nor inertia in abatement costs make a significant difference on results. It turns out that modeling structure incorporating a quadratic damage function are the most robust ones to changes. Such results may suggest that refinements of the reference structure has no incremental value and may explain why commonly used IAMs keep taking this structure, originally designed by Nordhaus with the DICE model. As IAMs based on this modeling structure are likely to find out that smooth increasing optimal trajectories of abatement and SCC are almost insensitive to uncertainty and inertia, then the sources of significant differences

in results can only come from differences in worldviews (i.e. parametrization).

## Key finding 2: When RESPONSE embeds a sigmoid damage function then it entails contrasting climate strategies

The integration of non-convexities in the climate damage function makes RESPONSE produce equivocal results. It entails indeed significantly different results contingent upon the integration or not of uncertainty and inertia. Without inertia and uncertainty, the sigmoid function tends to postpone mitigation efforts and reduce the SCC in comparison to the reference case. With inertia and/or uncertainty, strong increase in abatement in the mid term as well as higher SCC turn out to be the optimal outcomes. Therefore structures with a sigmoid damage function are sensitive to the other features of the modeling structure and the results may be as driven by these choices of modeling features as by differences in worldviews.

#### Key finding 3: A precautionary behaviour only comes up in RE-SPONSE when non-convexities in damages are considered

We notice a significant difference between the sigmoid certain case without inertia and the sigmoid uncertain case. Such difference accounts for a precautionary effect. Even though the quadratic reference structure leads to higher level of abatements than the sigmoid cases all over the period, it does not make happen any precautionary behaviour since no difference happens when uncertainty is taken into account. Hence, the only IAMs which can deal with precaution may be those with non convexities in their core modeling structure.

## 6 Conclusion: The puzzling choice of a modeling structure

We have proposed in this paper an innovative method to carry out a sensitivity analysis over emblematic choices of modeling features, namely the form of the damage and abatement cost functions, the treatment of uncertainty within the unified modeling framework of RESPONSE. We show that a given set of worldviews leads to very different distributions of results, and therefore different snapshots of the climate debate, contingent upon the modeling structure used. This clearly indicates that traditional sensitivity analysis over core parameters of the model only cannot be sufficient to cover the whole spectrum of possible climate policies. By distinguishing the combined impact of alternative modeling structures our methodology brings additional information into the climate debate.

Our method essentially makes it possible to avoid restricting artificially ranges of SCC or disregarding some possible climate policies because the outcome of different, though legitimate, structural modeling choices is not considered. The basic idea is then to exhibit, as far as possible, the greatest spectrum of possible optimal climate strategies.

At the end of the process we are not able however to exhibit ranges of SCC or abatement level that would be more "true" than those which have been computed so far. Regardless of the values of the results, our methodology aims at

pointing out phenomena that could have been forgotten or simply undetectable in a standard approach. This is the case for the non-intuitive impact of inertia in abatement costs combined with sigmoid climate damages. That way, this method makes a consistent dialogue possible between modeling frameworks and indicates the modeling drivers we should focus on to better understand differences in climate policy recommendations.

#### Appendix

#### A First-Order Conditions Resolution

In this part, our calculations follow the two-step resolution method already described in part 2.3. We distinguish before and after uncetainty is resolved.

#### A.1 After uncertainty is resolved

After uncertainty is resolved, we know the state of nature  $\omega$ . The Lagrangian writes:

$$\mathbf{L}^{\omega} = \sum_{t=t_{i}+1}^{\infty} N_{t} \frac{1}{(1+\rho)^{t}} u\left(\frac{C_{t}^{\omega}}{N_{t}}\right)$$

$$+ \sum_{t=t_{i}+1}^{\infty} (\lambda_{A,t}^{\omega}, \lambda_{B,t}^{\omega}, \lambda_{O,t}^{\omega}) \begin{pmatrix} A_{t+1}^{\omega} - (c_{AA}A_{t}^{\omega} + c_{AB}B_{t}^{\omega} + (1-a_{t}^{\omega})\sigma_{t}Y_{t}^{\omega}) \\ B_{t+1}^{\omega} - (c_{BA}A_{t}^{\omega} + c_{BB}B_{t}^{\omega} + c_{BO}O_{t}^{\omega}) \end{pmatrix}$$

$$+ \sum_{t=t_{i}+1}^{\infty} (\nu_{A,t}^{\omega}, \nu_{O,t}^{\omega}) \begin{pmatrix} \theta_{A,t+1}^{\omega} - ((1-\sigma_{1}(\frac{F_{2x}}{\theta_{2x}^{\omega}} + \sigma_{2}))\theta_{A,t}^{\omega} + \sigma_{1}\sigma_{2}\theta_{O,t}^{\omega} + \sigma_{1}F(A_{t}^{\omega})) \\ \theta_{O,t+1}^{\omega} - (\sigma_{3}\theta_{A,t}^{\omega} + (1-\sigma_{3})\theta_{O,t}^{\omega}) \end{pmatrix}$$

$$+ \sum_{t=t_{i}+1}^{\infty} \mu_{t}^{\omega} \left( -K_{t+1}^{\omega} + (1-\delta)K_{t}^{\omega} + Y_{t}^{\omega} \left[ 1 - C_{a}(a_{t}^{\omega}, a_{t-1}^{\omega}) - D^{\omega}(\theta_{A,t}^{\omega}) \right] - C_{t}^{\omega} \right)$$

$$+ \sum_{t=t_{i}+1}^{\infty} \overline{\tau}_{t}^{\omega} \cdot (1-a_{t}) + \underline{\tau}_{t}^{\omega} \cdot a_{t}$$

The Lagrange multiplier attached to the capital constraint (2.4) is  $\mu_t^\omega$ ; the Lagrange multipliers attached to the carbon cycle dynamics constraints (2.13) are  $\lambda_{A,t}^\omega$ ,  $\lambda_{B,t}^\omega$ , and  $\lambda_{O,t}^\omega$ . The Lagrange multipliers attached to the temperature constraints (2.15) are  $\nu_{A,t}^\omega$  and  $\nu_{O,t}^\omega$ . The Lagrange multipliers attached to inequality constraints (2.8) are  $\overline{\tau}_t^\omega$  and  $\underline{\tau}_t^\omega$ .

At the beginning of the program, stock variables are inherited from the past, i.e. from the maximization program under uncertainty. Some do not depend on the state of nature:

$$A_{t_{i}+1}^{\omega} = \overline{A}_{t_{i}+1}, \ B_{t_{i}+1}^{\omega} = \overline{B}_{t_{i}+1}, \ O_{t_{i}+1}^{\omega} = \overline{O}_{t_{i}+1}, \ K_{t_{i}+1}^{\omega} = \overline{K}_{t_{i}+1} \tag{A.2}$$

By convention,  $a_{t_i}^{\omega} = \overline{a}_{t_i}$ .

We calculate the first-order conditions with respect to the two fluxes variables:  $C_t^{\omega}$  and  $a_t^{\omega}$ , and to the six stock variables:  $K_t^{\omega}$ ,  $A_t^{\omega}$ ,  $B_t^{\omega}$ ,  $O_t^{\omega}$ ,  $\theta_{A,t}^{\omega}$ , and  $\theta_{O,t}^{\omega}$ . Recall also that stock variables at  $t = t_i + 1$  are initial conditions, so we cannot derive first-order conditions for them at this stage. We get:

• For consumption,  $\forall t \geq t_i + 1$ :

$$\frac{\partial \mathbf{L}^{\omega}}{\partial C_{t}^{\omega}} = 0 \quad \Leftrightarrow \\
\mu_{t}^{\omega} = u' \left( \frac{C_{t}^{\omega}}{N_{t}} \right) \frac{1}{(1+\rho)^{t}} \tag{A.3}$$

Then  $\mu_t^{\omega}$  is the discounted marginal utility.

• For the abatement capacity,  $\forall t \geq t_i + 1$ :

$$\begin{split} \frac{\partial \mathbf{L}^{\omega}}{\partial a_{t}^{\omega}} &= 0 \quad \Leftrightarrow \\ \lambda_{A,t}^{\omega} \sigma_{t} &= \mu_{t}^{\omega} \partial_{1} C_{a}(a_{t}^{\omega}, a_{t-1}^{\omega}) + \mu_{t+1}^{\omega} \partial_{2} C_{a}(a_{t+1}^{\omega}, a_{t}^{\omega}) \frac{Y_{t+1}^{\omega}}{Y_{t}^{\omega}} + \frac{\overline{\tau}_{t}^{\omega} - \underline{\tau}_{t}^{\omega}}{Y_{t}^{\omega}} \end{split} \tag{A.4}$$

For  $t=t_i+1$ , recall the conventional notation that  $a^\omega_{t_i}=\overline{a}_{t_i}$ . Recall that  $\overline{\tau}^\omega_t>0$  only when  $a^\omega_t=1$ , and  $\underline{\tau}^\omega_t>0$  only when  $a^\omega_t=0$ .

• For capital,  $\forall t \geq t_i + 2$ :

$$\frac{\partial \mathbf{L}^{\omega}}{\partial K_{t}^{\omega}} = 0 \quad \Leftrightarrow \\
\partial_{K} Y_{t}^{\omega} \left( 1 - \frac{\lambda_{A,t}^{\omega} (1 - a_{t}^{\omega}) \sigma_{t}}{\mu_{t}^{\omega}} - C_{a}(a_{t}^{\omega}, a_{t-1}^{\omega}) - D^{\omega}(\theta_{A,t}^{\omega}) \right) \\
= (1 + \rho) \frac{u' \left( \frac{C_{t-1}^{\omega}}{N_{t-1}} \right)}{u' \left( \frac{C_{t}^{\omega}}{N_{t}} \right)} - (1 - \delta)$$
(A.5)

• For the carbon stocks,  $\forall t \geq t_i + 2$ :

$$\frac{\partial \mathbf{L}^{\omega}}{\partial A_{t}^{\omega}} = 0 \quad \Leftrightarrow 
\lambda_{A,t-1}^{\omega} = \lambda_{A,t}^{\omega} c_{AA} + \lambda_{B,t}^{\omega} c_{BA} + \nu_{A,t}^{\omega} \sigma_{1} F'(A_{t}^{\omega}) \tag{A.6}$$

$$\begin{split} \frac{\partial \mathbf{L}^{\omega}}{\partial B_{t}^{\omega}} &= 0 \quad \Leftrightarrow \\ \lambda_{B,t-1}^{\omega} &= \lambda_{A,t}^{\omega} c_{AB} + \lambda_{B,t}^{\omega} c_{BB} + \lambda_{O,t}^{\omega} c_{OB} \end{split} \tag{A.7}$$

$$\begin{split} \frac{\partial \mathbf{L}^{\omega}}{\partial O_{t}^{\omega}} &= 0 \quad \Leftrightarrow \\ \lambda_{O,t-1}^{\omega} &= \lambda_{B,t}^{\omega} c_{BO} + \lambda_{O,t}^{\omega} c_{OO} \end{split} \tag{A.8}$$

• For the temperatures,  $\forall t \geq t_i + 2$ :

$$\frac{\partial \mathbf{L}^{\omega}}{\partial \theta_{A,t}^{\omega}} = 0 \quad \Leftrightarrow \\
\nu_{A,t-1}^{\omega} = \quad \nu_{A,t}^{\omega} \left( 1 - \sigma_1 \left( \frac{F_{2x}}{\vartheta_{2x}^{\omega}} + \sigma_2 \right) \right) \\
+ \nu_{O,t}^{\omega} \sigma_3 + \mu_t^{\omega} \partial_{\theta} D^{\omega} (\theta_{A,t}^{\omega}) Y_t^{\omega}$$
(A.9)

$$\frac{\partial \mathbf{L}^{\omega}}{\partial \theta_{O,t}^{\omega}} = 0 \quad \Leftrightarrow \\
\nu_{O,t-1}^{\omega} = \nu_{A,t}^{\omega} \sigma_{1} \sigma_{2} + \nu_{O,t}^{\omega} (1 - \sigma_{3}) \tag{A.10}$$

#### A.2 Before uncertainty is resolved

The Lagrangian of the maximization program then equals the expectation over the possible states of nature of the sum of the objective function and a cluster of dynamic equations.

The Lagrangian writes:

$$\mathbf{L}_{u} = \sum_{t=t_{0}}^{t=t_{i}} \mathbb{E}\left[\frac{1}{(1+\rho)^{t}} u(C_{t}^{\omega}, S_{t}^{\omega}) + V(\omega)\right]$$

$$+ \sum_{t=t_{0}}^{t_{i}} (\lambda_{A,t}, \lambda_{B,t}, \lambda_{O,t}) \begin{pmatrix} \overline{A}_{t+1} - (c_{AA}\overline{A}_{t} + c_{AB}\overline{B}_{t} + (1 - \overline{a}_{t})\sigma_{t}\overline{Y}_{t}) \\ \overline{B}_{t+1} - (c_{BA}\overline{A}_{t} + c_{BB}\overline{B}_{t} + c_{BO}\overline{O}_{t}) \end{pmatrix}$$

$$+ \sum_{t=t_{0}}^{t=t_{i}} \mathbb{E}\left[ (\nu_{A,t}^{\omega}, \nu_{O,t}^{\omega}) \begin{pmatrix} \theta_{A,t+1}^{\omega} - ((1 - \sigma_{1}(\frac{F_{2x}}{\theta_{2x}^{\omega}} + \sigma_{2}))\theta_{A,t}^{\omega} + \sigma_{1}\sigma_{2}\theta_{O,t}^{\omega} + \sigma_{1}F(\overline{A}_{t})) \\ \theta_{O,t+1}^{\omega} - (\sigma_{3}\theta_{A,t}^{\omega} + (1 - \sigma_{3})\theta_{O,t}^{\omega}) \end{pmatrix} \right]$$

$$+ \sum_{t=t_{0}}^{t=t_{i}} \mathbb{E}\left[ \mu_{t}^{\omega} (-\overline{K}_{t+1} + (1 - \delta)\overline{K}_{t} + Y_{t}^{\omega} \left[ 1 - C_{a}(a_{t}^{\omega}, a_{t-1}^{\omega}) - D^{\omega}(\theta_{A,t}^{\omega}) \right] - C_{t}^{\omega}) \right]$$

$$+ \sum_{t=t_{0}}^{t_{i}} \overline{\tau}_{t} \cdot (1 - \overline{a}_{t}) + \underline{\tau}_{t} \cdot \overline{a}_{t}$$
(A.12)

We calculate the first-order conditions with respect to all endogenous variables: fluxes variables  $C_t^{\omega}$  and  $\overline{a}_t$ , and stock variables  $\overline{K}_t$ ,  $\overline{A}_t$ ,  $\overline{B}_t$ ,  $\overline{O}_t$ ,  $\theta_{A,t}^{\omega}$ , and  $\theta_{O,t}^{\omega}$ . The derivation will be specific for stock variables at  $t_i + 1$ , and for the flux variable  $\overline{a}_{t_i}$  due to the inertia in abatement cost, for we have to take into account their impact on  $V(\omega)$ . We get:

• For consumption,  $\forall t \leq t_i$ :

$$\frac{\partial \mathbf{L}^{\omega}}{\partial C_{t}^{\omega}} = 0 \quad \Leftrightarrow \\
\mu_{t}^{\omega} = u' \left( \frac{C_{t}^{\omega}}{N_{t}} \right) \frac{1}{(1+\rho)^{t}} \tag{A.14}$$

• For the abatement capacity,  $\forall t < t_i$ :

$$\frac{\partial \mathbf{L}_{u}}{\partial \overline{a}_{t}} = 0 \quad \Leftrightarrow \\
\lambda_{A,t}\sigma_{t} = \quad \mathbb{E}\left[\mu_{t}^{\omega}\right] \partial_{1}C_{a}(\overline{a}_{t}, \overline{a}_{t-1}) \\
+ \mathbb{E}\left[\mu_{t+1}^{\omega}\right] \partial_{2}C_{a}(\overline{a}_{t+1}, \overline{a}_{t}) \frac{\overline{Y}_{t+1}}{\overline{Y}_{t}} + \frac{\overline{\tau}_{t} - \underline{\tau}_{t}}{\overline{Y}_{t}} \tag{A.15}$$

For  $t = t_i$ , actions, which are decided at the beginning of the period, cannot take into account the information that arrives in this period:

$$\frac{\partial \mathbf{L}_{u}}{\partial \overline{a}_{t_{i}}} = 0 \quad \Leftrightarrow \\
\lambda_{A,t_{i}} \sigma_{t_{i}} = \quad \mathbb{E} \left[ \mu_{t_{i}}^{\omega} \right] \partial_{1} C_{a}(\overline{a}_{t_{i}}, \overline{a}_{t_{i}-1}) \\
+ \mathbb{E} \left[ \mu_{t_{i}+1}^{\omega} \partial_{2} C_{a}(a_{t_{i}+1}^{\omega}, \overline{a}_{t_{i}}) \right] \frac{\overline{Y}_{t_{i}+1}}{\overline{Y}_{t_{i}}} + \frac{\overline{\tau}_{t_{i}} - \underline{\tau}_{t_{i}}}{\overline{Y}_{t_{i}}} \tag{A.16}$$

because  $\frac{\partial V(\omega)}{\partial \overline{a}_{t_i}} = \frac{\partial L^{\omega}}{\partial \overline{a}_{t_i}} = \mu_{t_i+1}^{\omega} \partial_2 C_a(a_{t_i+1}^{\omega}, \overline{a}_{t_i}) \overline{Y}_{t_i+1}.$ 

• For capital,  $\forall t \leq t_i$ :

$$\frac{\partial \mathbf{L}_{u}}{\partial \overline{K}_{t}} = 0 \quad \Leftrightarrow \\
\partial_{K} Y_{t} \left( 1 - \frac{\lambda_{A,t} (1 - \overline{a}_{t}) \sigma_{t}}{\mathbb{E}[\mu_{t}^{\omega}]} - C_{a}(\overline{a}_{t}, \overline{a}_{t-1}) - \frac{\mathbb{E}[\mu_{t}^{\omega} D^{\omega}(\theta_{A,t})]}{\mathbb{E}[\mu_{t}^{\omega}]} \right) \\
= \frac{\mathbb{E}[\mu_{t-1}^{\omega}]}{\mathbb{E}[\mu_{\omega}^{\omega}]} - (1 - \delta)$$
(A.17)

For  $t = t_i + 1$ ,

$$\frac{\partial \mathbf{L}_{u}}{\partial \overline{K}_{t_{i+1}}} = 0 \quad \Leftrightarrow \\
\partial_{K} Y_{t_{i+1}} \left( 1 - \frac{\mathbb{E} \left[ \lambda_{A,t_{i+1}}^{\omega} (1 - a_{t_{i+1}}^{\omega}) \sigma_{t_{i+1}} \right]}{\mathbb{E} \left[ \mu_{t_{i+1}}^{\omega} \right]} - \frac{\mathbb{E} \left[ \mu_{t_{i+1}}^{\omega} C_{a} (a_{t_{i+1}}^{\omega}, \overline{a}_{t_{i}}) \right]}{\mathbb{E} \left[ \mu_{t_{i+1}}^{\omega} \right]} - \frac{\mathbb{E} \left[ \mu_{t_{i+1}}^{\omega} D^{\omega} (\theta_{A,t_{i+1}}) \right]}{\mathbb{E} \left[ \mu_{t_{i+1}}^{\omega} \right]} \right) \quad (A.18)$$

$$= \frac{\mathbb{E} \left[ \mu_{t_{i}}^{\omega} \right]}{\mathbb{E} \left[ \mu_{t_{i+1}}^{\omega} \right]} - (1 - \delta)$$

because 
$$\frac{\partial V(\omega)}{\partial \overline{K}_{t_{i+1}}} = \frac{\partial L^{\omega}}{\partial \overline{K}_{t_{i+1}}} = \mu_{t_i+1}^{\omega} (1 - \delta + \partial_K Y_{t_i+1} (1 - C_a(a_{t_i+1}^{\omega}, \overline{a}_{t_i}) - D^{\omega}(\theta_{A, t_i+1}))) - \lambda_{A, t_i+1}^{\omega} (1 - a_{t_i+1}^{\omega}) \sigma_{t_i+1} \partial_K Y_{t_i+1}.$$

• For the atmospheric carbon multiplier,  $\forall t \leq t_i$ , the atmospheric carbon multiplier dynamic reads:

$$\frac{\partial \mathbf{L}_{u}}{\partial \overline{A}_{t}} = 0 \quad \Leftrightarrow 
\lambda_{A,t-1} = \lambda_{A,t} c_{AA} + \lambda_{B,t} c_{BA} + \mathbb{E} \left[ \nu_{A,t}^{\omega} \sigma_{1} F'(\overline{A}_{t}) \right]$$
(A.19)

$$\begin{split} \frac{\partial \mathbf{L}_{u}}{\partial \overline{B}_{t}} &= 0 \quad \Leftrightarrow \\ \lambda_{B,t-1} &= \lambda_{A,t} c_{AB} + \lambda_{B,t} c_{BB} + \lambda_{O,t} c_{OB} \end{split} \tag{A.20}$$

$$\begin{split} \frac{\partial \mathbf{L}_{u}}{\partial \overline{O}_{t}} &= 0 \quad \Leftrightarrow \\ \lambda_{O,t-1} &= \lambda_{B,t} c_{BO} + \lambda_{O,t} c_{OO} \end{split} \tag{A.21}$$

For  $t = t_i + 1$ ,

$$\begin{split} \frac{\partial \mathbf{L}_{u}}{\partial \overline{A}_{t_{i}+1}} &= 0 \quad \Leftrightarrow \\ \lambda_{A,t_{i}} &= \mathbb{E} \left[ \lambda_{A,t_{i}+1}^{\omega} c_{AA} + \lambda_{B,t_{i}+1}^{\omega} c_{BA} + \nu_{A,t_{i}+1}^{\omega} \sigma_{1} F'(\overline{A}_{t_{i}+1}) \right] \end{split} \tag{A.22}$$

$$\frac{\partial \mathbf{L}_{u}}{\partial \overline{B}_{t_{i}+1}} = 0 \quad \Leftrightarrow \\
\lambda_{B,t_{i}} = \mathbb{E} \left[ \lambda_{A,t_{i}+1}^{\omega} c_{AB} + \lambda_{B,t_{i}+1}^{\omega} c_{BB} + \lambda_{O,t_{i}+1}^{\omega} c_{OB} \right] \tag{A.23}$$

$$\begin{split} \frac{\partial \mathbf{L}_{u}}{\partial \overline{O}_{t_{i}+1}} &= 0 \quad \Leftrightarrow \\ \lambda_{O,t_{i}} &= \mathbb{E} \left[ \lambda_{B,t_{i}+1}^{\omega} c_{BO} + \lambda_{O,t_{i}+1}^{\omega} c_{OO} \right] \end{split} \tag{A.24}$$

• For the temperature,  $\forall t \leq t_i + 1$ :

$$\frac{\partial \mathbf{L}_{u}}{\partial \theta_{A,t}^{\omega}} = 0 \quad \Leftrightarrow \\
\nu_{A,t-1}^{\omega} = \quad \nu_{A,t}^{\omega} \left( 1 - \sigma_{1} \left( \frac{F_{2x}}{\vartheta_{2x}^{\omega}} + \sigma_{2} \right) \right) \\
+ \nu_{O,t}^{\omega} \sigma_{3} + \mu_{t}^{\omega} \partial_{\theta} D^{\omega} (\theta_{A,t}^{\omega}) Y_{t}^{\omega}$$
(A.25)

$$\frac{\partial \mathbf{L}_{u}}{\partial \theta_{O,t}^{\omega}} = 0 \quad \Leftrightarrow \\
\nu_{O,t-1}^{\omega} = \nu_{A,t}^{\omega} \sigma_{1} \sigma_{2} + \nu_{O,t}^{\omega} (1 - \sigma_{3}) \tag{A.26}$$

#### B The Social Cost of Carbon

#### **B.1** Theoretical Definition

The social cost of carbon (SCC) is "the additional damage caused by an additional ton of carbon emissions. In a dynamic framework, it is the discounted value of the change in the utility of consumption denominated in terms of current consumption" (Nordhaus, 2008).

#### **B.2** Definition in RESPONSE

At time t, if there is an additional ton of carbon in the atmosphere, this will increase  $A_{t+1}$  by a unit and thus decrease the welfare from t+1. This variation of welfare is captured by  $\partial W_{t+1}/\partial A_{t+1} = -\lambda_{A,t+1}c_{AA} - \lambda_{B,t+1}c_{BA} - \nu_{A,t+1}\sigma_1 F'(A_{t+1})$ . Once on an optimal path, this is equal, thanks to (A.6) or (A.19), to  $-\lambda_{A,t}$ . So, on an optimal path, the social cost of carbon is also related to the abatement cost thanks to (A.4) or (A.15). The SCC has to be counted in current utility units.

More precisely, the equations for SCC at the different stages of the model are given below.

After uncertainty is resolved  $(t \ge t_i + 1)$ , for each state of the world  $\omega$ , the SCC is:

$$SCC_{t}^{\omega} = \frac{\lambda_{A,t}^{\omega}}{\mu_{t}^{\omega}}$$

$$= \frac{1}{\sigma_{t}} \left( \partial_{1} C_{a}(a_{t}^{\omega}, a_{t-1}^{\omega}) + \frac{\mu_{t+1}^{\omega}}{\mu_{t}^{\omega}} \frac{Y_{t+1}^{\omega}}{Y_{t}^{\omega}} \partial_{2} C_{a}(a_{t+1}^{\omega}, a_{t}^{\omega}) + \frac{\overline{\tau}_{t}^{\omega} - \underline{\tau}_{t}^{\omega}}{\mu_{t}^{\omega} Y_{t}^{\omega}} \right)$$
(B.1)

For  $t = t_i + 1$ , this formula is rewritten as:

$$SCC_{t_{i}+1}^{\omega} = \frac{1}{\sigma_{t_{i}+1}} \left( \partial_{1} C_{a}(a_{t_{i}+1}^{\omega}, \overline{a}_{t_{i}}) + \frac{\mu_{t_{i}+2}^{\omega}}{\mu_{t_{i}+1}^{\omega}} \frac{Y_{t_{i}+2}^{\omega}}{\overline{Y}_{t_{i}+1}} \partial_{2} C_{a}(a_{t_{i}+2}^{\omega}, a_{t_{i}+1}^{\omega}) + \frac{\overline{\tau}_{t_{i}+1}^{\omega} - \underline{\tau}_{t_{i}+1}^{\omega}}{\mu_{t_{i}+1}^{\omega} \overline{Y}_{t_{i}+1}} \right)$$
(B.2)

Before uncertainty is resolved,  $\forall t \leq t_i$  the SCC is:

$$SCC_{t} = \frac{\lambda_{A,t}}{\mathbb{E}[\mu_{t}^{\omega}]}$$

$$= \frac{1}{\sigma_{t}} \left( \partial_{1}C_{a}(\overline{a}_{t}, \overline{a}_{t-1}) + \frac{\mathbb{E}[\mu_{t+1}^{\omega} \partial_{2}C_{a}(a_{t+1}^{\omega}, \overline{a}_{t})]}{\mathbb{E}[\mu_{t}^{\omega}]} \frac{\overline{Y}_{t+1}}{\overline{Y}_{t}} + \frac{\overline{\tau}_{t} - \underline{\tau}_{t}}{\mathbb{E}[\mu_{t}^{\omega}]\overline{Y}_{t}} \right)$$
(B.3)

For  $t \le t_i - 1$ , the formula simplifies to:

$$SCC_{t} = \frac{1}{\sigma_{t}} \left( \partial_{1} C_{a}(\overline{a}_{t}, \overline{a}_{t-1}) + \frac{\mathbb{E}[\mu_{t+1}^{\omega}]}{\mathbb{E}[\mu_{t}^{\omega}]} \frac{\overline{Y}_{t+1}}{\overline{Y}_{t}} \partial_{2} C_{a}(\overline{a}_{t+1}, \overline{a}_{t}) + \frac{\overline{\tau}_{t} - \underline{\tau}_{t}}{\mathbb{E}[\mu_{t}^{\omega}] \overline{Y}_{t}} \right)$$
(B.4)

Comparing the formula in the uncertain case with the certain case, we can give an interpretation of the uncertain social cost in terms of the social cost

if uncertainty had been resolved. The uncertain social cost corresponds to the mean of social cost in the different scenarios, averaged by utility in these scenarios (prices in different scenarios are not comparable, only utility units can be added, this is done by using  $\mu_t^\omega$ ). So if we defined  $SCC_t^\omega$  as previously (see the formulas after uncertainty is resolved), this interpretation of the social cost is tantamount to the formula:  $SCC_t = \frac{\mathbb{E}[\mu_t^\omega SCC_t^\omega]}{\mathbb{E}[\mu_t^\omega]}$  before uncertainty is resolved.

#### **B.3** Computation

To get the value of  $SCC_t^\omega = \frac{\lambda_{A,t}^\omega}{\mu_t^\omega}$  we use the shadow prices associated to the concentration dynamics for  $\lambda_{A,t}^\omega$  and the capital dynamics for  $\mu_t^\omega$  computed by GAMS.

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