

Viaene, Jean-Marie; Zilcha, Itzhak

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Multiple uncertainty, forward-futures markets and international trade

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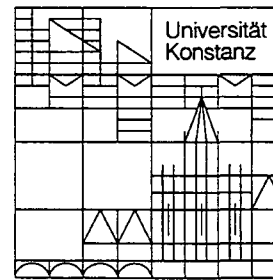
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**Sonderforschungsbereich 178
„Internationalisierung der Wirtschaft“**

Diskussionsbeiträge



Juristische
Fakultät

Fakultät für Wirtschafts-
wissenschaften und Statistik

Jean-Marie Viaene
Itzhak Zilcha

**Multiple Uncertainty,
Forward-Futures Markets and
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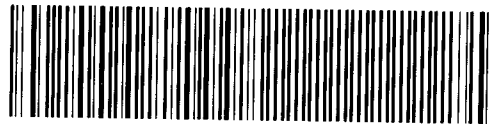
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Itzhak Zilcha

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**Multiple Uncertainty, Forward-Futures Markets
and International Trade***

Jean-Marie Viaene
Erasmus University and Tinbergen Institute Rotterdam

Itzhak Zilcha
The Eitan Berglas School of Economics, Tel-Aviv University

March 1995

Abstract

The optimum behavior of a competitive risk-averse international trader who supplies or demands commodities invoiced in foreign currency is examined when his profits are subject to several forms of risk: production, domestic cost, the exchange rate and the commodity price. The focus of our study is the robustness of the known results regarding the role of forward-futures markets in the presence of cost and output uncertainty. New results on the implications of the framework for the separation and the double hedging theorems are derived. The behavior of the same firm with and without complete markets is compared and conditions are obtained for a domestic price guarantee or a gradual introduction of missing markets to promote the level of international trade (JEL D81, D84, F19, F31)

Key Words: uncertainty, risk aversion, incomplete markets, forward rate, futures rate.

Correspondence to: I. Zilcha, The Eitan Berglas School of Economics, Tel-Aviv University, Ramat Aviv, Tel-Aviv 69978, Israel.

1. Introduction

The uncertainty faced by producers who export their output is composed of several types of risks, some of which can be diversified and some are undiversifiable. Price uncertainty results from the volatility of prices on the international markets and from the wild fluctuations of exchange rates. In the last two decades exchange rate volatility has become a major factor in international trade. These two types of risk can be diversified using financial tools, such as forward-futures markets, which have been expanded significantly in recent years. However, producers face other sources of uncertainty, two of which will be considered in our model explicitly: (a) production uncertainty caused usually by disease, spoilage, weather conditions, strikes, technical breakdown, etc. and (b) cost uncertainty, caused in the use of domestic factors, and their costs (e.g. energy cost, environmental regulation, legal suits etc.).

The literature dealing with the economic role played by the forward and future markets in enhancing production and international trade has concentrated, usually, on one type of uncertainty ignoring other sources of risks that cannot be hedged. In the case of open economy firm with exchange rate risk, such problem has been studied by Ethier (1973), Baron (1976), Kawai (1981), Viaene and Vries (1992) and others. In the closed economy price risk, let us mention Stein (1979), Newbery and Stiglitz (1981), Rolfo (1980), Anderson and Danthine (1983). Kawai and Zilcha (1986) considered the role of forward-futures markets when both commodity price and exchange rate uncertainties exist. However, to our best knowledge, none has analyzed the role of such hedging devices in the presence of output and cost uncertainties. Such undiversifiable risks have a major impact on the results obtained in the

literature. The main purpose of this paper is to examine how robust are the economic implications of forward-futures markets when output and cost uncertainty exist.

Our analysis of the behavior of competitive risk averse exporting firm shows that some well known results, such as the "separation property" and the "full double hedging theorem" fail to hold in many cases. However, there is no symmetry between production uncertainty and cost uncertainty. For example, under some conditions, the full double hedging theorem holds under cost uncertainty but fails in the presence of output uncertainty. We also study the impact of output uncertainty and cost uncertainty on production and trade when currency forward market and commodity futures market exist. Here, the nonsymmetry between these two types of uncertainties is reflected through the significant gap in the assumptions required to derive the results.

The paper is organized as follows. Section 2 discusses a model of the international producer. The firm's optimal behavior is analyzed when both the forward exchange and the commodity futures market exist. Section 3 verifies the implications of the framework for the separation theorem and the full double hedging theorem. Section 4 introduces cost and output uncertainty one by one when both forward-futures markets exist and shows the implication of each situation for the level of international trade. Section 5 verifies the sensitivity of the results to risk aversion. Section 6 deals with the behavior of the same firm with incomplete markets, namely no forward and/or futures markets, and derive conditions under which a domestic price guarantee or the elimination of some uncertainties promote the level of international trade. Finally, Section 7 concludes the paper. An Appendix is added to include proofs that are repetitive or judged too long for the main text.

2. The International Firm's Risk-Bearing Optimum

Consider the problem of a competitive, risk-averse exporting firm that produces one single output to be exported using one domestic primary input. The firm transforms inputs W according to a production function $Q = Q(W, \bar{\varepsilon})$ which is stochastic due to an undiversifiable production shock $\bar{\varepsilon}$, the tilde (-) signifying a random variable. We assume the usual strict concavity conditions for production:

$$(C.1) \quad Q(0, \bar{\varepsilon}) = 0, \quad Q_1(W, \bar{\varepsilon}) > 0, \quad Q_{11}(W, \bar{\varepsilon}) < 0,$$

$$\text{and } Q_2(W, \bar{\varepsilon}) \geq 0 \text{ for all } W \text{ and } \bar{\varepsilon},$$

where the subscripts indicate partial differentiation. Firm's income in local currency is given by $\bar{e}\bar{p}Q(W, \bar{\varepsilon})$ where \bar{e} is the random spot foreign exchange rate and \bar{p} is the random foreign-currency price of the commodity. The production process adopted by the firm gives rise to a stochastic cost function $C = C(W, \bar{\eta})$ where $\bar{\eta}$ is an undiversifiable cost shock. We assume that the cost function is convex, increasing and twice differentiable:

$$(C.2) \quad C_1(W, \bar{\eta}) > 0, \quad C_{11}(W, \bar{\eta}) \geq 0 \text{ for all } W, \bar{\eta}.$$

The subjective probability distributions of \bar{e} , \bar{p} , $\bar{\varepsilon}$ and $\bar{\eta}$ are exogenously given. In addition to choosing production inputs, the firm has up to two options to insure itself against the risks \bar{e} and \bar{p} that it faces. First, if a futures market exists, the firm can sell forward an amount Z of the commodity at the futures price p_f , the transaction adding thus $\bar{e}(p_f - \bar{p})Z$ to the local currency receipts. Second, if a forward exchange market exists, the firm can

sell an amount X of foreign exchange at the current forward rate e_f , bringing $(e_f - \bar{e})X$ to its local currency receipts. Transactions in both forward-futures markets are assumed cost-free and the standard length of forward-futures contracts is assumed to correspond to the production lag. With both forward and futures markets available, firm's profits in domestic currency are expressed as:

$$(1) \quad \bar{\Pi} = \bar{e}\bar{p}Q(W, \bar{\varepsilon}) - C(W, \bar{\eta}) + \bar{e}(p_f - \bar{p})Z + (e_f - \bar{e})X,$$

where the following notation is used:

- $\bar{\Pi}$ = firm's profits in domestic currency units
- \bar{e} = the spot foreign exchange rate one period hence (domestic price of foreign currency)
- e_f = the forward foreign exchange rate (for delivery in one period)
- \bar{p} = the commodity spot price (one period hence) in foreign currency
- p_f = the commodity futures price (for delivery in one period) in foreign currency
- W = the level of primary inputs
- Z = the quantity of the commodity sold (if $Z > 0$) or purchased (if $Z < 0$) in the futures market
- X = the amount of foreign exchange sold (if $X > 0$) or purchased (if $X < 0$) forward
- $\bar{\varepsilon}$ = quantity shock
- $\bar{\eta}$ = cost shock

The firm selects the choice variables (W, X, Z) so as to maximize $EU(\bar{\Pi})$ where E is the mathematical expectations operator, and $U(\cdot)$ is a strictly concave, increasing and differentiable von Neumann-Morgenstern utility function defined over firm's profits in local currency $\bar{\Pi}$. Since $EU(\bar{\Pi})$ is strictly concave in W, X, Z necessary and sufficient conditions for the (unique) optimum (W^*, X^*, Z^*) are:

$$(2) \quad EU'(\bar{\Pi}^*)[\bar{e}\bar{p}Q_1(W^*, \bar{\varepsilon}) - C_1(W^*, \bar{\eta})] = 0,$$

$$(3) \quad EU'(\bar{H}^*)[\bar{e}(p_f - \bar{p})] = 0,$$

$$(4) \quad EU'(\bar{H}^*)[e_f - \bar{e}] = 0,$$

where $U'(\cdot)$ is the marginal utility, an asterisk (*) indicates the optimum levels when markets are complete. In our terminology, not to be confused with Arrow-Debreu's definitions, the term "complete markets" is used for the case where both currency forward and commodity futures markets exist. If one (or both) of these markets is missing the term "incomplete market" is used instead. The first-order conditions (2) to (4) can be rewritten as:

$$(5) \quad E\bar{e}\bar{p}\bar{Q}_1^* - \frac{EU'(\bar{H}^*)C_1(W^*, \bar{\eta})}{EU'(\bar{H}^*)} = \frac{\text{Cov}[U'(\bar{H}^*), \bar{e}\bar{p}Q_1(W^*, \bar{\varepsilon})]}{EU'(\bar{H}^*)},$$

$$(6) \quad E\bar{e}\bar{p} - e_f p_f = - \frac{\text{Cov}[U'(\bar{H}^*), \bar{e}\bar{p}]}{EU'(\bar{H}^*)},$$

$$(7) \quad E\bar{e} - e_f = - \frac{\text{Cov}[U'(\bar{H}^*), \bar{e}]}{EU'(\bar{H}^*)}.$$

The results depend critically upon the cost and output risks and the nature of the covariances. For instance, the currency-forward market would be (individually) unbiased ($E\bar{e} = e_f$) if $\text{Cov}[U'(\bar{H}^*), \bar{e}] = 0$ and the two markets would be simultaneously unbiased as well, if $\text{Cov}[U'(\bar{H}^*), \bar{e}\bar{p}] = 0$.

So far, nothing has been said about the importer's behavior. The latter can be viewed as buying an imported commodity as input for use in the production of a final good that sells on local markets only at a price of unity. This is

equivalent to interpreting $C(W, \bar{\eta})$ as the new production function, $\bar{e}pQ(W, \bar{\varepsilon})$ as the new cost function where the role of $\bar{\varepsilon}$ and $\bar{\eta}$ are reversed. By so doing, the importer's behavior becomes analytically equivalent.

3. Properties of the Optimum with Complete Markets

Our aim is to explore in this framework how, with complete markets, the optimal production level and hedging behavior are affected by the exporter's probability belief and attitudes towards risk. In particular, the interest lies in knowing the extent by which complete markets insulate the level of international trade from the various risks and the role played by hedging in this context. In what follows we shall use the shorthand notation $Q(W)$, $C(W)$ when no uncertainty in production or cost exists.

Full Double Hedging Theorem

Without cost and output uncertainty, the full double hedging theorem states that, under unbiased markets assumption, the optimal forward-futures contracting, derived from equations (5) - (7), is a complete double hedge, i.e., $Z^* = Q(W^*)$ and $X^* = p_f Z^*$ [Kawai and Zilcha (1986)]. Important to the theorem are two basic definitions. The forward exchange and commodity futures markets are called separately unbiased if $E\tilde{\varepsilon} = e_f$, $E\tilde{p} = p_f$. Likewise, the forward-futures markets are called jointly unbiased if $E\tilde{e}\tilde{p} = e_f p_f$. Now let us assume:

(A.1) The forward exchange market is unbiased.

(A.2) The forward-futures markets are jointly unbiased.

A consequence of assumptions (A.1) and (A.2) is that the gains from speculation vanish and it is therefore optimal for the exporter to hedge his trade transactions completely, which renders his profit riskless. In the presence of cost and output uncertainty the following outcome derives.

Proposition 1. If the unbiasedness assumptions (A.1) and (A.2) hold, then: (a) Under cost uncertainty the optimal forward-futures contracting is a full double hedge if the stochastic cost shock $\tilde{\eta}$ is independent of the exchange rate \tilde{e} and of the domestic price of the commodity $\tilde{e}\tilde{p}$; (b) Under output uncertainty the full double hedge does not hold.

Proof. Consider the case of cost uncertainty only, output uncertainty being treated by analogy and therefore relegated to the Appendix. Rewrite the profit function (1):

$$(8) \quad \tilde{\Pi} = \tilde{e}\tilde{p}(Q(W) - Z) + \tilde{e}(p_f Z - X) - C(W, \tilde{\eta}) + e_f X.$$

By assumptions (A.1) and (A.2), $e_f = E\tilde{e}$ and $e_f p_f = E\tilde{e}\tilde{p}$. From (5) and (6) these assumptions about unbiasedness imply $\text{Cov} [\tilde{e}\tilde{p}, U'(\tilde{\Pi})] = 0$ and $\text{Cov} [\tilde{e}, U'(\tilde{\Pi})] = 0$. Hence, if $Q(W^*) = Z^*$ and $p_f Z^* = X^*$ is the optimal solution, (8) turns out $\tilde{\Pi} = e_f X^* - C(W^*, \tilde{\eta})$. As a result, the conditions $\text{Cov} [\tilde{e}\tilde{p}, U'(e_f X^* - C(W^*, \tilde{\eta}))] = 0$ and $\text{Cov} [\tilde{e}, U'(e_f X^* - C(W^*, \tilde{\eta}))] = 0$ hold if and only if the distributions of \tilde{e} and of $\tilde{e}\tilde{p}$ are independent of that of $\tilde{\eta}$. Under these two conditions of independence, the full double hedging theorem holds under cost uncertainty.

The general result that seems to come from the first part of Proposition 1 is that independence between $\tilde{\eta}$ and $\tilde{\epsilon}$, $\tilde{\eta}$ and $\tilde{\epsilon}\tilde{p}$, and therefore full double hedging is more likely to result from a cost disturbance that is firm-specific than one that is economy-wide. In contrast, nation-wide cost disturbances, like a major technological advance or an overall rise in domestic factor costs, are shocks that are likely to affect the underlying distribution of $\tilde{\epsilon}$ and $\tilde{\epsilon}\tilde{p}$ and make full double hedging not the optimal contracting. The second part of the proposition could explain the empirical finding that international firms do not hedge completely [Van Nieuwkerk (1979)].

The Separation Theorem

Consider first a world without cost and output uncertainty in (5) to (7). Equation (5) would simplify to $e_f p_f Q_1(W^*) = C_1(W^*)$, which equates the value marginal product of the input to its marginal cost, i.e., the competitive input rental. The optimal demand for the single primary factor and hence, the optimal level of output and trade, would be chosen at that point and would therefore be independent of the distribution of the random variables and of the firm's attitude towards risk. This is the contribution of futures-forward markets and the essence of the separation theorem as stated by Ethier (1973), Danthine (1978), Holthausen (1979), Feder, Just and Schmitz (1980) and others. With cost and output uncertainty we have different results stated in the following proposition.

Proposition 2. In the presence of both forward-futures markets, when cost and output are uncertain, the firm's optimum demand for inputs depends on the utility function and the probability distribution of the random variables. The

separation theorem holds only if both marginal product and marginal cost of input are not random.

Proof. Suppose that the marginal cost of input is not random. The firm's optimum demand for inputs initially given by (5) then becomes:

$$(11) \quad E\bar{e}pQ_1(W^*, \bar{\varepsilon}) = - \frac{\text{Cov}[U'(\bar{H}^*), \bar{e}pQ_1(W^*, \bar{\varepsilon})]}{EU'(\bar{H}^*)} + C_1(W^*).$$

The expected value marginal product is higher than the certain marginal cost as long as $\text{Cov}[U'(\cdot), \bar{e}pQ_1(\cdot)] < 0$. With $Q_1(W^*, \bar{\varepsilon})$ random, W^* does depend on the utility function and the probability distribution of the random variables. With a non-random marginal product, the substitution of (6) into (11) gives $e_f p_f Q_1(W^*) = C_1(W^*)$ and the separation theorem holds.

The theoretical and empirical literature has extensively used two functional forms to specify the supply randomness: (1) a multiplicative risk (e.g., $\bar{\varepsilon}Q(W)$ or $\bar{\varepsilon}W$, $E\bar{\varepsilon} = 1$) and (2) an additive risk (e.g. $W + \bar{\varepsilon}$, $E\bar{\varepsilon} = 0$). When multiplicative, the stochastic supply shocks affect a proportion of the production while, when additive, these are independent of the size of the production. In terms of our model a multiplicative shock satisfies conditions (C.1) with $Q_{12} > 0$, the marginal product remaining random. An additive risk requires instead to make conditions (C.1) less strict by requiring $EQ(0, \bar{\varepsilon}) = 0$, $Q_1 > 0$ and $Q_{11} \leq 0$. It is possible that $Q_{12} = 0$ in which case the marginal product is independent of the production shock and is therefore non random.

Though the focus of the literature has been on stochastic output, the cost shocks can be specified along the same lines. A difference, however, is that

with cost uncertainty a "semi-separation" result is obtained when the random shock is multiplicative.¹

Proposition 3 (Semi-Separation Theorem). Assume that multiplicative cost uncertainty is introduced, and that the unbiasedness assumptions (A.1) and (A.2) about the futures-forward markets hold. The optimal input and trade levels are independent of the joint distribution of (\tilde{e}, \tilde{p}) .

Proof. Let $C(W, \tilde{\eta}) = \tilde{\eta}C(W)$ with $E\tilde{\eta} = 1$. Consider the necessary and sufficient conditions for an optimum in equations (2), (3) and (4). Due to the strict concavity of the maximand there is a unique solution (W^*, Z^*, X^*) . Let W^* be the unique solution to the equation:

$$(12) \quad e_f p_f \frac{Q_1(W^*)}{C_1(W^*)} - 1 = \frac{\text{Cov}[\tilde{\eta}, U'(e_f p_f Q(W^*) - \tilde{\eta} C(W^*))]}{U'(e_f p_f Q(W^*) - \tilde{\eta} C(W^*))}$$

and define:

$$Z^* = Q(W^*) \text{ and } X^* = P_f Z^*$$

In this case, $\tilde{\Pi}^* = e_f X^* - \tilde{\eta} C(W^*)$ and it is easy to verify that equations (2), (3) and (4) hold. Thus this is the optimal solution. Also, it is easy to see from (12) that W^* does not depend upon the joint distribution of (\tilde{e}, \tilde{p}) , although it depends on the utility function. Hence, with multiplicative cost uncertainty, and forward-futures markets, any two firms with identical technologies but different probability beliefs about the exchange rate and price will export an equal amount of output as long as their attitude toward risk is the same.

4. The Effects of Cost and Output Uncertainty

In this section we consider the impact of each type of uncertainty separately, cost uncertainty first and output uncertainty later. Assuming complete markets we study how each randomness affects the level of international trade.

Cost Uncertainty Only

Throughout this section we assume separate and joint unbiasedness as given by (A.1) and (A.2). Consider random cost $C(W, \tilde{\eta}) = \tilde{\eta}C(W)$, $\tilde{\eta}$ being independent of \tilde{e} and \tilde{p} and the production function being nonrandom. Denote the optimum in this case by W^* , X^* , Z^* . Initially, consider the benchmark case where \tilde{e} and \tilde{p} are random and $\eta = 1$, i.e., the "certainty equivalent" cost. Denote by $(\bar{W}, \bar{X}, \bar{Z})$ the optimum in this case. It is easy to verify that under the unbiasedness assumptions the optimum, under uncertain cost, is given by:

$$Z^* = Q(W^*) \quad X^* = P_f Z^*$$

where W^* is uniquely determined by the equation:

$$(13) \quad E[\tilde{e}\tilde{p} Q_1(W^*) - C_1(W^*, \tilde{\eta})] U'(e_f p_f Q(W^*) - C(W^*, \tilde{\eta})) = 0$$

Now we prove that,

Proposition 4. Under assumptions (A.1)-(A.2) introducing uncertainty about cost results in a lower level of international trade, i.e., $W^* < \bar{W}$.

Proof. Let $C(W, \tilde{\eta}) = \tilde{\eta} C(W)$. We have shown that full double hedging holds in this case. Equation (13) can therefore be rewritten as:

$$E_{e,p}\{E_{\tilde{\eta}} [\tilde{e}pQ_1(W^*) - \tilde{\eta}C_1(W^*)] U'(e_f p_f Q(W^*) - \tilde{\eta}C(W^*))\} = 0.$$

This equation can be written as follows:

$$E_{\tilde{\eta}} [(e_f p_f Q_1(W^*) - \tilde{\eta}C_1(W^*)) U'(e_f p_f Q(W^*) - \tilde{\eta}C(W^*))] = 0$$

or

$$e_f p_f Q_1(W^*) E_{\tilde{\eta}} U'(e_f p_f Q(W^*) - \tilde{\eta}C(W^*)) = C_1(W^*) E_{\tilde{\eta}} \tilde{\eta} U'(e_f p_f Q(W^*) - \tilde{\eta}C(W^*))$$

which implies that ($E\tilde{\eta} = 1$),

$$e_f p_f Q_1(W^*) = C_1(W^*) + \text{Cov} \left(\tilde{\eta}, \frac{U'(e_f p_f Q(W^*) - \tilde{\eta}C(W^*))}{EU'(e_f p_f Q(W^*) - \tilde{\eta}C(W^*))} \right).$$

Since $U'(e_f p_f Q(W^*) - \tilde{\eta}C(W^*))$ is an increasing function of $\tilde{\eta}$ the covariance is positive. Therefore, $e_f p_f Q_1(W^*) > C_1(W^*)$. Hence we obtain that

$$(14) \quad [e_f p_f Q_1(W^*) - C_1(W^*)] U'(e_f p_f Q(W^*) - C(W^*)) > 0.$$

On the other hand in the benchmark case we have (see Kawai and Zilcha (1986)):

$$\bar{Z} = Q(\bar{W}), \quad \bar{X} = P_f \bar{Z}, \quad \Pi = e_f \bar{X} - C(\bar{W})$$

where \bar{W} is given by:

$$(15) \quad E_{e,p}\{[\tilde{e}pQ_1(\bar{W}) - C_1(\bar{W})] U'(e_f p_f Q(\bar{W}) - C(\bar{W}))\} = 0$$

or

$$(15') \quad [e_f p_f Q_1(\bar{W}) - C_1(\bar{W})] U'(e_f p_f Q(\bar{W}) - C(\bar{W})) = 0$$

But the function $U(e_f p_f Q(W) - C(W))$ is a strictly concave function in W due to our assumption that $Q(\cdot)$ is strictly concave while the cost function $C(\cdot)$ is a convex function of W . Thus from (14) and (15') we obtain that $W^* < \bar{W}$. Namely, the output for export is lower when the cost is random. Due to risk aversion low production levels, and therefore revenues, are the response in order to lower losses that would be incurred under unfavourable states of nature.

Production Uncertainty Only

The study of the impact of production uncertainty on output, in the presence of complete markets, requires some additional assumptions on the utility function, due to the fact that the full double hedging theorem does not hold in this case.

$$(A.3) \quad [U'(x) - xU''(x)] \text{ is strictly convex in } x.$$

$$(A.4) \quad \text{The absolute risk aversion } -U''(x)/U'(x) \text{ is convex in } x.$$

Assumptions (A.3) and (A.4) hold, for example, for quadratic utility functions and for constant relative risk aversion utility $U(x) = x^{1-\gamma}/(1-\gamma)$ for $1 > \gamma \geq 0$ and $U(x) = \ln x$ for $\gamma = 1$. Assume now that $\eta = 1$ while $\tilde{\epsilon}$ is random with $E\tilde{\epsilon} =$

1. Denote by $(\bar{W}, \bar{X}, \bar{Z})$ the optimum when $\varepsilon = 1$, i.e., the "certainty equivalent" production. Now we prove:

Proposition 5. Assume that (A.3) and (A.4) hold and that $Q(W, \tilde{\varepsilon}) = \tilde{\varepsilon}Q(W)$. Uncertainty about production results in (a) lower output (and hence lower trade), i.e., $W^* < \bar{W}$ and (b) lower hedging levels, i.e. $X^* < \bar{X}$ and $Z^* < \bar{Z}$.

The proof of this proposition is relegated to the Appendix. Thus increasing output risk has similar effect on the trade level to that of increased cost uncertainty. However, note that we do not assume unbiasedness about markets in this case to derive the results in Proposition 5 (unlike the cost uncertainty case). On the other hand, to derive Proposition 4 we assume that the utility exhibits risk aversion only.

5. The Degree of Absolute Risk Aversion

Given our model let us explore the effect of increasing risk aversion upon the production level in the presence of complete markets. This boils down to comparing the optimal trade levels of two exporting firms, which differ only in their attitude towards risk. We shall take $\tilde{\varepsilon}$ and $\tilde{\eta}$ to be independent of $(\tilde{\varepsilon}, \tilde{p})$. To simplify the proof let us show separately the cases of cost uncertainty and output uncertainty.

Proposition 6. Consider a competitive exporting firm facing price and exchange rate uncertainty. Increasing risk aversion results in: (a) decreasing output in the presence of production uncertainty if $Q_{12} > 0$ but increasing

output if $Q_{12} < 0$; (b) decreasing output in the presence of cost uncertainty if $C_{12} > 0$ but increasing output if $C_{12} < 0$.

The Appendix contains the main arguments of the proof of part (a). As the proof of part (b) is very similar, it has been deleted. This proposition suggests new insights about the way in which risk aversion affects exporting firms' trade level. The result shows that the monotonicity of the marginal product in $\tilde{\epsilon}$ and the marginal cost in $\tilde{\eta}$ matters in determining the effect of increasing risk aversion on production. The presence of undiversifiable risks creates the possibility of trade to increase as risk aversion increases for exporting firms whose marginal product and marginal cost of input are random and negatively affected by these risks. Note that if both marginal product and marginal cost are not random, it is clear from Proposition 2 that firms' export volumes are, among others, independent of the utility function.

6. Incomplete Trading Regimes

In this section we shall consider cases where some of the risk-sharing markets are missing. If $\tilde{\epsilon}$ and \tilde{p} , or one of them, becomes an undiversifiable risk because of missing organized markets, the set of choice variables available to the exporter reduces to (W, X) if the futures market is missing ($Z \equiv 0$), to (W, Z) if the forward exchange market is missing ($X \equiv 0$), or to (W) if both are missing ($X=Z \equiv 0$). Each of the last three situations is characterized as incomplete market in comparison to the situation under (1). Our aim in this section is to study the trade effects of the gradual introduction of missing markets. As there are numerous situations possible, it is important to limit

the number of issues. With developing and transition economies in the background, this section limits itself to the comparison of no market at all to the benchmark case of complete markets. Later we consider the case of one missing market, the other being introduced.

The next two propositions consider the elimination of the domestic price uncertainty when the firm is first facing random output and then random cost. For the case of random output, we need to consider $\text{Cov}(\tilde{e}\tilde{p}, \tilde{\varepsilon})$ which turns out to be important (see Proposition 7). If individual output shocks are uncorrelated across firms, they will approximately cancel on the aggregate by the law of large numbers, and it is legitimate to assume $\text{Cov}(\tilde{e}\tilde{p}, \tilde{\varepsilon}) = 0$. However, if correlated across firms, the shocks will have economy-wide repercussions and it is likely that domestic price and output shocks will be negatively correlated, i.e., $\text{Cov}(\tilde{e}\tilde{p}, \tilde{\varepsilon}) < 0$. For the purpose of next proposition assume also:

(A.5) The relative measure of risk aversion $R_r(x)$ satisfies: $R_r(x) \leq 1$ for all $x > 0$.

Proposition 7. Assume that (A.3) and (A.5) hold, and that the firm is producing under output uncertainty only (besides exchange rate and price uncertainty). Eliminating the domestic price uncertainty results in higher production level if $\text{Cov}(\tilde{e}\tilde{p}, \tilde{\varepsilon}) \leq 0$.

Note that one can construct an example where the result is reversed if $\text{Cov}(\tilde{e}\tilde{p}, \tilde{\varepsilon}) > 0$ and large.

Proof: Denote by W^* the optimal level of input when all \tilde{e} , \tilde{p} , $\tilde{\varepsilon}$ are random. To simplify the proof let $Q(W, \tilde{\varepsilon}) = \tilde{\varepsilon}Q(W)$. W^* is determined by the equation:

$$E[(\tilde{e}\tilde{p}\tilde{\varepsilon}Q_1(W^*) - C_1(W^*)) U'(\tilde{e}\tilde{p}\tilde{\varepsilon}Q(W^*) - C(W^*))] = 0$$

Since (A.3) holds we obtain from this equation the following inequality (Note that $xU'(x) - U'(x)$ is strictly concave):

$$(16) \quad E_{\varepsilon} \left\{ [E(\tilde{e}\tilde{p}|\varepsilon) \tilde{\varepsilon}Q_1(W^*) - C_1(W^*)] U'(E(\tilde{e}\tilde{p}|\varepsilon) \tilde{\varepsilon}Q(W^*) - C(W^*)) \right\} > 0.$$

Now let us note that due to $\text{Cov}_{\varepsilon}(E(\tilde{e}\tilde{p}|\varepsilon), \tilde{\varepsilon}) \leq 0$ and $E\tilde{\varepsilon} = 1$ we derive that,

$$E\tilde{e}\tilde{p}\tilde{\varepsilon} = E_{\varepsilon} \left\{ E(\tilde{e}\tilde{p}|\varepsilon) \tilde{\varepsilon} \right\} = E_{\varepsilon}[E(\tilde{e}\tilde{p}|\tilde{\varepsilon})] + \text{Cov}_{\varepsilon}(E(\tilde{e}\tilde{p}|\tilde{\varepsilon}), \tilde{\varepsilon}) \leq E_{\varepsilon}[E(\tilde{e}\tilde{p}|\tilde{\varepsilon})] = e_f p_f.$$

Claim: Under (A.5) the function $xU'(x) - U'(x)$ is increasing in x .

To proof the claim let us differentiate this function:

$$\frac{d}{dx} [xU'(x) - U'(x)] = U'(x) + xU''(x) - U''(x) > U'(x) + xU''(x) \geq 0,$$

since $-xU''(x)/U'(x) \leq 1$ by our assumption. Now let us replace $E[\tilde{e}\tilde{p}|\varepsilon]$ in inequality (16) by $E_{\varepsilon}[E(\tilde{e}\tilde{p}|\tilde{\varepsilon})] = e_f p_f$, i.e., the certain domestic price of the commodity. By doing that, the inequality sign remains valid due to two facts: First, since $xU'(x) - U'(x)$ is strictly concave in $E(\tilde{e}\tilde{p}|\tilde{\varepsilon})$, the averaging of $\tilde{\varepsilon}$ will only increase the left hand side of (16). Secondly, this function is strictly increasing in $E(\tilde{e}\tilde{p}) = e_f p_f$, thus again we have only increased the left hand side. Thus we obtain:

$$(17) \quad E_{\varepsilon} \left\{ (e_f p_f \tilde{\varepsilon} Q_1(W^*) - C_1(W^*)) U' (e_f p_f \tilde{\varepsilon} Q(W^*) - C(W^*)) \right\} > 0$$

When the firm chooses optimal \bar{W} under certain price $e_f p_f$, it is the solution of:

$$(18) \quad E_{\varepsilon} \left\{ (e_f p_f \tilde{\varepsilon} Q_1(\bar{W}) - C_1(\bar{W})) U' (e_f p_f \tilde{\varepsilon} Q(\bar{W}) - C(\bar{W})) \right\} = 0$$

Since $EU(e_f p_f \tilde{\varepsilon} Q(W) - C(W))$ is strictly concave in W from equations (17) and (18) we derive that $\bar{W} > W^*$.

Proposition 8. Consider an exporting firm with random cost independent of the exchange rate and price and assume that (A.3) holds. Eliminating the domestic price uncertainty, by introducing unbiased "joint" futures market, will increase the output of the firm.

Proof. The optimal input used by the firm in the presence of all uncertainties W^* is given by:

$$(19) \quad E_{\eta} \{ E_{\varepsilon p} [(\tilde{\varepsilon} \tilde{p} Q_1(W^*) - \tilde{\eta} C_1(W^*)) U' (\tilde{\varepsilon} \tilde{p} Q(W^*) - \tilde{\eta} C(W^*))] \} = 0$$

By assumption (A.3) the function $xU'(x) - U'(x)$ is strictly concave, inserting the expectation with respect to the joint distribution of $(\tilde{\varepsilon}, \tilde{p})$ inside (19) yields (taking $E\tilde{\varepsilon}\tilde{p} = e_f p_f$):

$$(20) \quad E_{\eta} \{ (e_f p_f Q_1(W^*) - \tilde{\eta} C_1(W^*)) U' (e_f p_f Q(W^*) - \tilde{\eta} C(W^*)) \} > 0.$$

On the other hand when there is only cost uncertainty, i.e., taking the price

as $e_f p_f$, the optimum \hat{W} is given by the equation:

$$(21) \quad E_{\tilde{\eta}} \{ (e_f p_f Q_1(\hat{W}) - \tilde{\eta} C_1(\hat{W})) U' (e_f p_f Q(\hat{W}) - \tilde{\eta} C(\hat{W})) \} = 0.$$

However, the maximand $EU[e_f p_f Q(\hat{W}) - \tilde{\eta} C(\hat{W})]$ is a strictly concave function in W . This implies that $W^* < \hat{W}$, which proves that without domestic price uncertainty the output is larger.

Propositions 7 and 8 give a justification for the introduction of complete markets as a policy to promote international trade. It should be noted, however, that any other policy that guarantees to producers a price in domestic currency equivalent to $e_f p_f$ would achieve the same level of output and trade.

So far we have shown that removing the domestic price uncertainty altogether enhances production. Would it be true also if we remove "part" of the uncertainty in $\tilde{e}p$? Or, if only one market exists is it true that introducing the missing market will increase output? We believe that the answer to these questions is positive but let us show the result for one case: only unbiased commodity futures exist and we introduce the market for currency.

Proposition 9. Consider an exporting firm under cost uncertainty (as well as exchange rate and price). Assume that only unbiased commodity futures market is available. Eliminating exchange rate uncertainty (e.g., by introducing unbiased currency forward market) will increase the production level of the exporting firm.

Note that only risk aversion assumption is required for this proposition. We shall bring the main arguments of the proof in the Appendix. The economic relevance of the result is to give support to the generally alleged view that, in the presence of exchange rate uncertainty, the introduction of an insurance market like the forward exchange market increases the level of international trade.

7. Concluding Remarks

This paper considers risk involved decisions of an international firm facing multiple risks. As is often the case, traders when evaluating their profits do not face isolated risks but a product of them. Among these risks, some are diversifiable like price and exchange rate uncertainty and some are not like random cost and production shocks.

The existing literature has dealt with the optimal behavior of a risk averse international firm facing diversifiable risks at great length. The main contribution of this paper is to verify the robustness of the existing results with respect to adding cost and output uncertainty. Many existing results fail in this respect as we have indicated. However, an important result is robust: introducing unbiased forward-futures markets increases the volume of trade above the level when no organized market exists.

An important message we obtain from our analysis is the nonsymmetry between randomness in production and randomness in cost. Aspects of this nonsymmetry involve not only differences in the assumptions needed throughout the paper

for each case, but also in the very basic properties that one can obtain for each type of uncertainty. The full double hedging theorem holds, under some conditions, for the cost uncertainty model while it fails completely in the output uncertainty case. Also the separation theorem is partially valid for cost uncertainty only. In many instances under cost uncertainty results were obtained when unbiasedness assumptions were necessary; while in the output uncertainty case usually more restrictive assumptions about the utility function were needed, but without the unbiasedness assumption. One economic consequence of this fact is the following. Importers are usually grasped as the reversed image of exporters but this is even more true for our framework. Consider the multiplicative cost uncertainty case. This is, in fact, output uncertainty for the importer of foreign inputs; but, in this case the optimum output of the importer does not depend upon the joint distribution of the exchange rate and the commodity price. No such result exists for the exporter as we know from Propositions 2 and 3. This is a consequence of the nonsymmetry of these uncertainties.

Footnote

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1. A multiplicative shock is defined in our context as the product of a function of the shock with a function of the inputs (separably multiplicative). An additive shock is the sum of a function of the shock and of a function of the inputs (separably additive).

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Appendix

Proof of Proposition 1 (Output Uncertainty)

With output uncertainty only, the profit function (1) evaluated at the optimum is:

$$\tilde{\Pi}^* = \tilde{e}\tilde{p}(Q^* - Z^*) + \tilde{e}(p_f Z^* - X^*) - C(W^*) + e_f X^* + \tilde{e}\tilde{p}(Q(W^*, \tilde{\varepsilon}) - Q^*)$$

where we have added and subtracted $Q^* \equiv Q(W^*)$. Let $e_f = E\tilde{e}$ and $e_f p_f = E\tilde{e}\tilde{p}$. From (5) and (6), unbiasedness implies $\text{cov}[\tilde{e}\tilde{p}, U'(\tilde{\Pi})] = 0$ and $\text{cov}[\tilde{e}, U'(\tilde{\Pi})] = 0$. Hence, if $Q^* = Z^*$ and $p_f Z^* = X^*$, we obtain $\text{cov}[\tilde{e}\tilde{p}, U'(e_f X^* - C(W^*) + \tilde{e}\tilde{p}(Q(W^*, \tilde{\varepsilon}) - Q^*))] = 0$ which is impossible. Thus the full double hedging theorem under market unbiasedness does not hold under output uncertainty.

Proof of Proposition 5

Before the proof of Proposition 5 it is necessary to show the following Lemma:

Lemma. Under assumptions (A.3) and (A.4) about the utility, the function

$$H(A) = \frac{E_{\tilde{\varepsilon}} U'(A\tilde{\varepsilon} + B)}{U'(A + B)} \text{ is increasing in } A \text{ on } [0, \infty].$$

Note that $H(0) = 1$ and due to the convexity of U' we have $H(A) > 1$ for $A > 0$.

Proof of the Lemma. Differentiate $H(A)$ for a given fixed B ,

$$H'(A) = \frac{E\tilde{\varepsilon}U''(A\tilde{\varepsilon}+B)U'(A+B) - U''(A+B)EU'(A\tilde{\varepsilon}+B)}{U'(A+B)^2} =$$

$$= - E \left[\tilde{\varepsilon} \frac{-U''(A\tilde{\varepsilon}+B)}{U'(A+B)} \right] + \frac{-U''(A+B)}{U'(A+B)} E \left[\frac{U'(A\tilde{\varepsilon}+B)}{U'(A+B)} \right] >$$

$$- E \frac{-U''(A\tilde{\varepsilon}+B)}{U'(A+B)} + \frac{-U''(A+B)}{U'(A+B)} E \left[\frac{U'(A\tilde{\varepsilon}+B)}{U'(A+B)} \right]$$

Since $\text{Cov}[\tilde{\varepsilon}, \frac{-U''(\tilde{\varepsilon}A+B)}{U'(A+B)}] < 0$ due to a decreasing absolute risk aversion. Now,

$$H'(A) > \frac{-1}{U'} E \left[\frac{-U''(A\tilde{\varepsilon}+B)}{U'(A\tilde{\varepsilon}+B)} \frac{U'(A\tilde{\varepsilon}+B)}{1} - \frac{-U''(A+B)}{U'(A+B)} \frac{U'(A\tilde{\varepsilon}+B)}{1} \right]$$

$$= \frac{1}{U'} E \left[U'(A\tilde{\varepsilon}+B) \left[-\frac{U''(A+B)}{U'(A+B)} + \frac{-U''(A\tilde{\varepsilon}+B)}{U'(A\tilde{\varepsilon}+B)} \right] \right]$$

$$= \frac{1}{U'} EU'(\cdot) \left[E \left[-\frac{U''(A\tilde{\varepsilon}+B)}{U'(A\tilde{\varepsilon}+B)} \right] - \frac{-U''(A+B)}{U'(A+B)} \right]$$

$$- \text{Cov} \left[U'(A\tilde{\varepsilon}+B), \frac{-U''(A\tilde{\varepsilon}+B)}{U'(A\tilde{\varepsilon}+B)} \right] > 0.$$

Since the covariance is negative due to assumption (A.4) and also

$$E \left[\frac{-U''(A\tilde{\varepsilon}+B)}{U'(A\tilde{\varepsilon}+B)} \right] > \frac{-U''(A+B)}{U'(A+B)}. \text{ Thus } H'(A) > 0 \text{ for } A > 0.$$

Turning to the proof of Proposition 5, consider equation (2) that can be rewritten in this case as:

$$E_{e,p} \{ E_{\varepsilon} [\tilde{\varepsilon} p \tilde{\varepsilon} Q_1(W^*) - C_1(W^*)] U'(\tilde{\varepsilon} p \tilde{\varepsilon} Q(W^*) - \tilde{\varepsilon} p Z^* \\ + \tilde{\varepsilon}(p_f Z^* - X^*) - C(W^*) + e_f X^*) \} = 0.$$

The term inside the expectation with respect to $\tilde{\varepsilon}$'s distribution is a concave function in $\tilde{\varepsilon}$ for each given values of (\tilde{e}, \tilde{p}) . This stems from our assumption that $xU'(x) - U'(x)$ is a concave function of x . Thus inserting the expectation with respect to $\tilde{\varepsilon}$ inside this expression will yield:

$$(1') \quad E_{e,p}\{[\tilde{e}\tilde{p}Q_1(W^*) - C_1(W^*)] U'(\tilde{e}\tilde{p}Q_1(W^*) - \tilde{e}\tilde{p}Z^* + \tilde{e}(p_f Z^* - X^*) - C(W^*) + e_f X^*)\} > 0.$$

But this type of argument can be applied to eqs (3) and (4) in the main text as well.

Consider now equation (4). Define the event $M = \{e | e \geq e_f\}$. Rewrite equation (4) as follows:

$$E_{p|e} \left\{ \int_{-M} \frac{E_{\varepsilon} U'(\tilde{\Pi})}{U'(E_{\varepsilon} \tilde{\Pi})} U'(E_{\varepsilon} \tilde{\Pi})(e_f - \tilde{e}) \right\} =$$

$$= E_{p|e} \left\{ \int_M \frac{E_{\varepsilon} U'(\tilde{\Pi})}{U'(E_{\varepsilon} \tilde{\Pi})} U'(E_{\varepsilon} \tilde{\Pi})(\tilde{e} - e_f) \right\}$$

By the above claim for each $e \in M$ and $\hat{e} \in -M$ we have (note that ep increases in e):

$$\frac{E_{\varepsilon} U'(\tilde{\Pi}(e))}{U'(E_{\varepsilon} \tilde{\Pi}(e))} > \frac{E_{\varepsilon} U'(\tilde{\Pi}(\hat{e}))}{U'(E_{\varepsilon} \tilde{\Pi}(\hat{e}))}.$$

Thus from the above equation we derive:

$$E_{p|e} \left\{ \int_{-M}^{\infty} U'(E_{\varepsilon} \tilde{\Pi})(e_f - \tilde{e}) \right\} > E_{p|e} \left\{ \int_M^{\infty} U'(E_{\varepsilon} \tilde{\Pi})(\tilde{e} - e_f) \right\}.$$

In other words we obtained:

$$(2') \quad E[U'(E_{\varepsilon} \tilde{\Pi})(e_f - \tilde{e})] > 0.$$

Similarly we can show that

$$(3') \quad E\{U'(E_{\varepsilon} \tilde{\Pi})[\tilde{e}(p_f - \tilde{p})]\} > 0.$$

But the function

$$EU[\tilde{e}pQ(W) - C(W) + \tilde{e}(p_f - \tilde{p})Z + (e_f - \tilde{e})X]$$

is strictly concave in W , Z , X . All the inequalities (1'), (2') and (3') hold as equalities for \bar{W} , \bar{Z} , \bar{X} (since it is the optimum for the benchmark case). Due to the concavity of this maximand we conclude that $\bar{W} > W^*$, $\bar{Z} > Z^*$ and $\bar{X} > X^*$.

Proof of Part (a) of Proposition 6.

To prove part (a) let us assume that \tilde{e} is random while $\eta = 1$. Thus, let two firms be identical except to their von-Neumann Morgenstern utility functions U and V . Let $V(\Pi) = h(U(\Pi))$ for all Π , where $h' > 0$ and $h'' < 0$; namely, the firm with V is more risk averse than the firm with U . Let the optimum input

levels be W_v^* and W_u^* , i.e., the following equations hold, where Π_v^* and Π_u^* are the corresponding optimal profits:

$$(4') \quad E[\tilde{e}pQ_1(W_u^*, \tilde{\varepsilon}) - C_1(W_u^*)] U'(\tilde{\Pi}_u^*) = 0$$

$$(5') \quad E[\tilde{e}pQ_1(W_v^*, \tilde{\varepsilon}) - C_1(W_v^*)] h'(U(\tilde{\Pi}_v^*))U'(\tilde{\Pi}_v^*) = 0$$

Under our assumptions $Q(W, \tilde{\varepsilon})$ and $Q_1(W, \tilde{\varepsilon})$ are monotone increasing in $\tilde{\varepsilon}$. For each fixed values of e, p define $\hat{\varepsilon}$ by $epQ_1(W_v^*, \hat{\varepsilon}) - C_1(W_v^*) = 0$. Equation (5') can be rewritten as:

$$(6') \quad E_{e,p} \left\{ \int_{\varepsilon \geq \hat{\varepsilon}} [epQ_1(W_v^*, \varepsilon) - C_1(W_v^*)] h'(U(\tilde{\Pi}_v^*))U'(\tilde{\Pi}_v^*) \right. \\ \left. - \int_{\varepsilon < \hat{\varepsilon}} [-epQ_1(W_v^*, \varepsilon) + C_1(W_v^*)] h'(U(\tilde{\Pi}_v^*))U'(\tilde{\Pi}_v^*) \right\} = 0.$$

Note that, by Proposition 3, $\tilde{\Pi}_v^* = \tilde{e}pQ(W_v^*, \tilde{\varepsilon}) - C(W_v^*)$. Also, that for $\varepsilon < \hat{\varepsilon}$, it implies that $epQ_1(W_v^*, \varepsilon) - C_1(W_v^*) < 0$; $\tilde{\Pi}_v^*$ is increasing in ε . Hence for any $\varepsilon' > \hat{\varepsilon}$ and $\varepsilon'' < \hat{\varepsilon}$ we have (for any e, p given):

$$\tilde{\Pi}_v(\varepsilon') = epQ(W_v^*, \varepsilon') - C(W_v^*) > epQ(W_v^*, \varepsilon'') - C(W_v^*) = \tilde{\Pi}_v(\varepsilon'').$$

Therefore, for any $\varepsilon' > \hat{\varepsilon}$ and $\varepsilon'' < \hat{\varepsilon}$ we have, $h'(U(\tilde{\Pi}_v(\varepsilon'))) < h'(U(\tilde{\Pi}_v(\varepsilon'')))$.

Hence it is easy to verify from equation (6') that:

$$(7') \quad E_{e,p} \left\{ \int_{\varepsilon \geq \hat{\varepsilon}} [epQ_1(W_v^*, \varepsilon) - C_1(W_v^*)] U'(\tilde{\Pi}_v^*) \right. \\ \left. - \int_{\varepsilon < \hat{\varepsilon}} [-epQ_1(W_v^*, \varepsilon) + C_1(W_v^*)] U'(\tilde{\Pi}_v^*) \right\} > 0.$$

Namely, we have demonstrated that (we are using the independence of \tilde{e} , \tilde{p} from the random variable $\tilde{\varepsilon}$):

$$(8') \quad E[\tilde{e}\tilde{p}Q_1(W_v^*, \tilde{\varepsilon}) - C_1(W_v^*)]U'(\tilde{\Pi}_v^*) > 0.$$

Since the maximand $EU[\tilde{e}\tilde{p}Q(W, \tilde{\varepsilon}) - C(W)]$ is strictly concave in W comparing (4') and (8') we conclude that $W_v^* < W_u^*$. Note that when $Q_{12} < 0$ the inequality in (7') and (8') is reversed.

Proof of Proposition 9.

To simplify our proof let $C(W, \tilde{\eta}) = \tilde{\eta} C(W)$, $E\tilde{\eta} = 1$. The profit function $\tilde{\Pi}$, given the optimum W^* , Z^* is

$$\tilde{\Pi} = \tilde{e}\tilde{p}(Q(W^* - Z^*) - \tilde{\eta} C(W^*)) + \tilde{e}p_f Z^* .$$

The following two necessary and sufficient conditions for the optimum W^* , Z^* are:

$$E[\tilde{e}\tilde{p}Q_1(W^*) - \tilde{\eta} C_1(W^*)]U'(\tilde{\Pi}) = 0$$

$$E[(\tilde{e}p_f - \tilde{e}\tilde{p}) U'(\tilde{\Pi})] = 0$$

Therefore,

$$E[(\tilde{e}p_f Q_1(W^*) - \tilde{\eta} C_1(W^*))U'(\tilde{\Pi})] = 0$$

This equation implies that,

$$p_f Q_1(W^*) E[\tilde{e} U'(\tilde{\Pi})] = C_1(W^*) E\tilde{\eta} U'(\tilde{\Pi})$$

Hence,

$$(9') \quad p_f Q_1(W^*) E\tilde{e} U'(\tilde{\pi}) + p_f Q_1(W^*) \text{Cov}(\tilde{e}, U'(\tilde{\Pi})) = \\ C_1(W^*) EU'(\tilde{\Pi}) + \text{Cov}(\tilde{\eta}, U'(\tilde{\Pi})).$$

Since the profits $\tilde{\Pi}$, denominated in domestic currency, are monotone increasing in \tilde{e} but monotone decreasing in $\tilde{\eta}$, and due to the risk aversion assumption, U' is a decreasing function of $\tilde{\Pi}$ we obtain that:

$$\text{Cov}(\tilde{e}, U'(\tilde{\Pi})) < 0, \quad \text{Cov}(\tilde{\eta}, U'(\tilde{\Pi})) \geq 0.$$

Thus from equation (9') we derive that ($e_f = E\tilde{e}$)

$$e_f p_f Q_1(W^*) = C_1(W^*).$$

Now, without exchange rate uncertainty or in the presence of unbiased currency forward market (in both cases we have the full double hedging theorem hold), the optimal output \hat{W} is given by:

$$e_f p_f Q_1(\hat{W}) = C_1(\hat{W}).$$

Since $e_f p_f Q(W) - C(W)$ is strictly concave we obtain that $W^* < \hat{W}$, namely, the production increases when the exchange rate uncertainty is removed.