Broll, Udo; Wahl, Jack E.

Working Paper

Hedging with synthetical forwards and the export decision

Diskussionsbeiträge - Serie II, No. 134

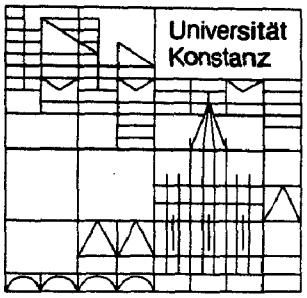
Provided in Cooperation with:
Department of Economics, University of Konstanz

Suggested Citation: Broll, Udo; Wahl, Jack E. (1991) : Hedging with synthetical forwards and the export decision, Diskussionsbeiträge - Serie II, No. 134, Universität Konstanz, Sonderforschungsbereich 178 - Internationalisierung der Wirtschaft, Konstanz

This Version is available at:
https://hdl.handle.net/10419/101794

Sonderforschungsbereich 178
„Internationalisierung der Wirtschaft“

Diskussionsbeiträge



Juristische
Fakultät

Fakultät für Wirtschafts-
wissenschaften und Statistik

Udo Broll
Jack Wahl

Hedging with Synthetical Forwards and the Export Decision

Postfach 5560
D-7750 Konstanz

Serie II — Nr. 134
März 1991

HEDGING WITH SYNTHETICAL FORWARDS AND THE EXPORT DECISION

Udo Broll
Jack Wahl

Serie II - Nr. 134

März 1991

Hedging with Synthetical Forwards and the Export Decision *

Udo Broll and **Jack Wahl**

March 1991

Abstract

The paper focusses on currency options as financial hedging instruments. Since currency forwards imply the well–known separation result, it follows for arbitragefree hedging markets that separation must also hold in option markets if the traded options allow for constructing a synthetical forward contract. Furthermore export revenue is fully hedged by synthetical forwards, if the risk premium in the put price is equal to the risk premium in the call price.

*We would like to thank Günter Franke for helpful discussions. Of course, errors and omissions are ours.

Hedging with Synthetical Forwards and the Export Decision

Introduction

Exchange rates of the major industrial countries have been substantially volatile in the last decade [see Krugman (1989), Franke (1990)]. The hedging policy became more and more a concern of international firms. Therefore hedging instruments have been increasingly used by such firms. Exchange rate risk also affected international trade as reported by Thursby and Thursby (1985) and Cushman (1988). The aim of this paper is to study the interaction between exchange rate risk and the export and hedging decision of an international firm.

The financial hedging technique on which this investigation focusses on is the so-called synthetical forward contract. This contract is a portfolio of European currency put and call options such that a currency forward is duplicated. Hence by using a synthetical forward the hedging policy of the firm will exhibit all the features as if a real forward contract were used.

To illustrate the use of synthetical currency forwards, let us assume that a domestic exporting firm receives a futures payment in foreign currency, in US–Dollars, say. If the firm

hedges against exchange rate risk in the forward market it is committed to receive a certain amount of domestic currency. This situation can also be achieved by purchasing US–Dollar put options and writing US–Dollars call options on the underlying currency, both with equal maturity and striking prices. If the Dollar depreciates and the exchange rate falls below the striking price, the exporter will exercise the put option, whereas the buyer of the call option will then let the call option expire. Alternatively, if the exchange rate does not reach the striking price, the firm will let the put option expire, whereas the buyer of the call option will then exercise the call option implying that the firm is committed to deliver the currency.

The described currency option portfolio thus implies for the exporter that like a real forward sale of foreign exchange a specified amount of one currency against another will be delivered at a fixed future date and price. Hence the synthetical forward contract protects the firm against the risk of adverse movements in exchange rates for which the firm pays the put price. The contract also eliminates the possibility of gaining a profit from favorable movements for which the firm receives the call price [see Cox and Rubinstein (1985)].

The main rationale of our study is as follows. If exporting firms are risk averse, then an increase of the exchange rate risk

reduces their export volume when there are no hedging markets. The benchmark being the so–called *certainty (equivalent) case*, which implies that the random spot exchange rate $\tilde{e}$ is replaced by its expected value $E\{\tilde{e}\} \equiv \bar{e}$. The optimal export under a certain exchange rate $\bar{e}$ will then be larger than the optimal export under a random exchange rate with expectation $\bar{e}$.[1] Hence the question is, how can currency put and call options be used as hedging instruments by international firms to undo the inverse effect of uncertainty upon the export volume?

The paper is organized as follows. First a partial equilibrium model is presented, and the effect of exchange rate risk on exports in the absence of risk sharing markets is briefly analyzed. Then the impact of currency option markets on the firm's export and hedging decision is examined. We show that the well–known separation theorem of forward markets holds, and that the export revenue is fully hedged if put and call options prices incorporate the same risk premia.

[1]This is a special case of a mean preserving spread.

The Export Decision in the Absence of Hedging Markets

Consider a competitive exporting firm under exchange rate risk. Production and exports give rise to a deterministic cost function $C(x)$ denominated in domestic currency, where x represents the export volume. It is assumed that the function C is strictly convex, increasing and twice differentiable, and that the firm always produces a positive amount.

The export decision is made at date 0 and output will be exported and sold at the given foreign currency price, p, that yields risky revenue in domestic currency at date 1. Omitting time subscripts the random profit from exports can be written [Sandmo (1971)][2]

$$\tilde{\Pi} = \tilde{e}px - C(x),$$

where $\tilde{e}$ denotes the random spot rate of foreign exchange at date 1. The exchange rate is defined in domestic currency per unit of foreign currency.

The firm is risk averse with a von Neumann–Morgenstern utility function $U(\cdot)$ and maximizes the expected utility of its profits in domestic currency. Hence, if there are no hedging

[2]Costs $C(x)$ are compounded to date 1.

reduces their export volume when there are no hedging markets. The benchmark being the so–called *certainty (equivalent) case*, which implies that the random spot exchange rate $\tilde{e}$ is replaced by its expected value $E\{\tilde{e}\} \equiv \bar{e}$. The optimal export under a certain exchange rate $\bar{e}$ will then be larger than the optimal export under a random exchange rate with expectation $\bar{e}$.[1] Hence the question is, how can currency put and call options be used as hedging instruments by international firms to undo the inverse effect of uncertainty upon the export volume?

The paper is organized as follows. First a partial equilibrium model is presented, and the effect of exchange rate risk on exports in the absence of risk sharing markets is briefly analyzed. Then the impact of currency option markets on the firm's export and hedging decision is examined. We show that the well–known separation theorem of forward markets holds, and that the export revenue is fully hedged if put and call options prices incorporate the same risk premia.

[1]This is a special case of a mean preserving spread.

The Export Decision in the Absence of Hedging Markets

Consider a competitive exporting firm under exchange rate risk. Production and exports give rise to a deterministic cost function $C(x)$ denominated in domestic currency, where x represents the export volume. It is assumed that the function C is strictly convex, increasing and twice differentiable, and that the firm always produces a positive amount.

The export decision is made at date 0 and output will be exported and sold at the given foreign currency price, p, that yields risky revenue in domestic currency at date 1. Omitting time subscripts the random profit from exports can be written [Sandmo (1971)][2]

$$\tilde{\Pi} = \tilde{e}px - C(x),$$

where $\tilde{e}$ denotes the random spot rate of foreign exchange at date 1. The exchange rate is defined in domestic currency per unit of foreign currency.

The firm is risk averse with a von Neumann–Morgenstern utility function $U(\cdot)$ and maximizes the expected utility of its profits in domestic currency. Hence, if there are no hedging

[2]Costs $C(x)$ are compounded to date 1.

markets, then the firm's decision problem reads

$$max_x E\{U(\tilde{\Pi})\}, \tag{1}$$

where E is the expectations operator. Let $C'(x)$ denote marginal costs. Then the interior solution requires that

$$E\{U'[\tilde{e}p - C'(x)]\} = 0. \tag{2}$$

In order to explore the impact of risky exchange rates we use Eq. (2). Since Π increases in e and $U'(\cdot)$ is a decreasing function, we get $Cov\{\tilde{e}, U'(\tilde{\Pi})\} < 0$. We therefore obtain

$$\bar{e}p - C'(x) > 0, \tag{3}$$

where $\bar{e}$ is the expected spot exchange rate at date 1, $\bar{e} \equiv E\{\tilde{e}\}$.

Certainty (equivalent) case. Let us compare the firm's optimal decision under uncertainty with the certainty case, i.e., the uncertain spot rate $\tilde{e}$ is replaced by the certain spot rate $\bar{e}$ [see, e.g., Sandmo (1971), Leland (1972)]. Then from Eq. (3) and the optimality condition for the certainty case we can state

Propositon 1 *(Effect of Uncertainty). If the spot rate of foreign exchange is risky, then the firm's optimal export is lower than its optimal export in the certainty case.*

Proof: Let x_c denote the optimal export level when $\bar{e}$ is the certain spot exchange rate. Since $C'''(x) > 0$ it follows from Eq. (3) and the certainty case that $x_c > x$.

Introducing exchange rate risk causes the well–known effect that the risk averse firm reduces its export volume to deal with the uncertain exchange rate. Suppose that exchange rate risk is measured by the volatility of the exchange rate. Then without hedging markets the firm's optimal export is inversely related to this volatility, other things being equal. Hence a risk averse firm has an incentive to take into account hedging opportunities.

Introducing Synthetical Forwards

The Export Decision

The financial hedging instrument which is underlying the following investigation is the option contract. Such contracts provide the holder with the right to sell (put) or buy (call) the underlying asset at a prefixed striking price and expiration date. We consider a currency option market that offers put and call options for every desired striking price k. For every k there exists an associated put and call price p_o and c_o, respectively, which we assume to be compounded to date 1.

Purchasing a put option gives the exporting firm the right to sell the contracted currency at expiration date 1, which the firm will do, if the striking price exceeds the prevailing spot

rate. Otherwise the firm will just let the put option expire. For this alternative the firm must pay the put price. If the firm finances the purchase of the *put* option contract by writing an appropriate *call* option contract with the same striking price, the firm will have to deliver the currency, if the put is not exercised. For this service the firm receives the call price.

Hence the put price is the maximum amount the firm can lose from the put option contract, and the call price is the maximum amount the firm can gain from the call option contract.[3] Thus the hedge of the firm will be self–financing if and only if the put price and the call price coincide.

Let us introduce some definitions. Be

$z_p \equiv$ the currency European put option contract amount in foreign currency;

$z_c \equiv$ the currency European call option contract amount in foreign currency;

$e \equiv$ the realized exchange rate at date 1;

$(k-e)^+ \equiv max\{0, k-e\}$;

$(e-k)^+ \equiv max\{0, e-k\}$.

The firm purchases (writes) put options if z_p is positive (negative). On the other hand the firm purchases (writes) call op–

[3]If incidentally the exchange rate at date 1 equals the striking price the firm sells the foreign currency in the spot market. Note that we neglect transaction costs, margin requirements, and taxes.

tions if z_c is negative (positive).

Then the decision problem of the international firm is given by:

$$max_{x,z_p,z_c} E\{U(\tilde{\Pi})\} \tag{4}$$

where:

$$\tilde{\Pi} = \tilde{e}px - C(x) + z_p[(k-\tilde{e})^+ - p_o] + z_c[c_o - (\tilde{e}-k)^+].$$

The first order conditions are

$$\begin{aligned} E\{U'(\tilde{\Pi})[\tilde{e}p - C'(x)]\} &= 0, & (5)\\ E\{U'(\tilde{\Pi})[(k-\tilde{e})^+ - p_o]\} &= 0, & (6)\\ E\{U'(\tilde{\Pi})[c_o - (\tilde{e}-k)^+]\} &= 0. & (7) \end{aligned}$$

From these conditions we prove a *separation* result for currency options.

Proposition 2 *(Separation). If currency put and call options are available, the firm's optimal export, x_o, satisfies*

$$C'(x_o) = (k + c_o - p_o)p. \tag{8}$$

The optimal export can be determined independently of the utility function and of the probability distribution of the random spot exchange rate.

Proof: Let $\hat{U}'(\tilde{\Pi}) \equiv U'(\tilde{\Pi})/E\{U'(\tilde{\Pi})\}$, and note that $k - \tilde{e} = (k-\tilde{e})^+ - (\tilde{e}-k)^+$. Then substracting Eq. (7) from (6) it follows

rate. Otherwise the firm will just let the put option expire. For this alternative the firm must pay the put price. If the firm finances the purchase of the *put* option contract by writing an appropriate *call* option contract with the same striking price, the firm will have to deliver the currency, if the put is not exercised. For this service the firm receives the call price.

Hence the put price is the maximum amount the firm can lose from the put option contract, and the call price is the maximum amount the firm can gain from the call option contract.[3] Thus the hedge of the firm will be self–financing if and only if the put price and the call price coincide.

Let us introduce some definitions. Be

$z_p \equiv$ the currency European put option contract amount in foreign currency;

$z_c \equiv$ the currency European call option contract amount in foreign currency;

$e \equiv$ the realized exchange rate at date 1;

$(k-e)^+ \equiv max\{0, k-e\}$;

$(e-k)^+ \equiv max\{0, e-k\}$.

The firm purchases (writes) put options if z_p is positive (negative). On the other hand the firm purchases (writes) call op–

[3]If incidentally the exchange rate at date 1 equals the striking price the firm sells the foreign currency in the spot market. Note that we neglect transaction costs, margin requirements, and taxes.

tions if z_c is negative (positive).

Then the decision problem of the international firm is given by:

$$max_{x,z_p,z_c} E\{U(\tilde{\Pi})\} \tag{4}$$

where:

$$\tilde{\Pi} = \tilde{e}px - C(x) + z_p[(k-\tilde{e})^+ - p_o] + z_c[c_o - (\tilde{e}-k)^+].$$

The first order conditions are

$$E\{U'(\tilde{\Pi})[\tilde{e}p - C'(x)]\} = 0, \tag{5}$$

$$E\{U'(\tilde{\Pi})[(k-\tilde{e})^+ - p_o]\} = 0, \tag{6}$$

$$E\{U'(\tilde{\Pi})[c_o - (\tilde{e}-k)^+]\} = 0. \tag{7}$$

From these conditions we prove a *separation* result for currency options.

Proposition 2 *(Separation). If currency put and call options are available, the firm's optimal export, x_o, satisfies*

$$C'(x_o) = (k + c_o - p_o)p. \tag{8}$$

The optimal export can be determined independently of the utility function and of the probability distribution of the random spot exchange rate.

Proof: Let $\hat{U}'(\tilde{\Pi}) \equiv U'(\tilde{\Pi})/E\{U'(\tilde{\Pi})\}$, and note that $k - \tilde{e} = (k-\tilde{e})^+ - (\tilde{e}-k)^+$. Then substracting Eq. (7) from (6) it follows

after some easy manipulations that $E[\hat{U}'(\tilde{\Pi})\tilde{e}] = k + c_o - p_o$. Combining this result with $E[\hat{U}'(\tilde{\Pi})\tilde{e}]p = C'(x)$ from Eq. (5) implies the claim.

The separation result with respect to an export decision is well–known under forward markets [see for example Danthine (1978), Holthausen (1979), Katz and Paroush (1979), Kawai and Zilcha (1986), Paroush and Wolf (1989)]. Our result follows because we implicitly consider a forward contract hedge: The hedging is realized by building a portfolio of currency put and call options such that we obtain a *synthetical* forward contract [see Cox and Rubinstein (1985)]. If a newly–written real forward contract were to be considered, then by the put–call parity relationship for European options its forward rate, e_f, would have to be $e_f = k + c_o - p_o$. Otherwise hedging markets would not be arbitragefree.

The Hedging Decision

Let us now consider the optimal hedging policy of the international firm. The result is summarized in

Proposition 3 *(Hedging): The firm completely hedges its risky export revenue in the options market via a synthetical forward contract if and only if the risk premium in the put price is equal to the risk premium in the call price. If the put option risk*

premium is higher (lower) than the call option risk premium, then the firm will over(under)hedge its risky export revenue.

Proof: The proof proceeds in three steps: Firstly, the derivation of a risk premium relationship, secondly, a continuity relationship, and thirdly, the application of these relationships with respect to the optimal hedge.

(i) Let $\lambda_p \equiv E(k-\tilde{e})^+ - p_o$ denote the expected risk premium in the put price, and let $\lambda_c \equiv E(\tilde{e}-k)^+ - c_o$ denote the expected risk premium in the call price. From Eq. (5) we get $\frac{C'(x)}{p} = E\{\hat{U}'(\tilde{\Pi})\tilde{e}\} = E\{\tilde{e}\} + Cov\{\hat{U}'(\tilde{\Pi}), \tilde{e}\}$. Combining this result with Eq. (8) we deduce $c_o - p_o + k - E\{\tilde{e}\} = Cov\{\hat{U}'(\tilde{\Pi}), \tilde{e}\}$. Using the fact that $k - E\{\tilde{e}\} = E(k-\tilde{e})^+ - E(\tilde{e}-k)^+$ and applying the risk premium notation we therefore have

$$\lambda_p - \lambda_c = Cov\{\hat{U}'(\tilde{\Pi}), \tilde{e}\}. \tag{9}$$

This covariance is zero for all probability beliefs if and only if $\tilde{\Pi}$ is independent of $\tilde{e}$.

(ii) Consider $\tilde{\Pi} = a + b\tilde{e}$. Then with the stricly decreasing function of $\tilde{\Pi}$, $U'(\tilde{\Pi})$, it follows that $sgn(Cov\{U'(\tilde{\Pi}), \tilde{e}\}) = -sgn(b) = sgn(Cov\{\hat{U}'(\tilde{\Pi}), \tilde{e}\})$, since $U'(\tilde{\Pi}) > 0$.

(iii) Given a synthetical *forward* contract, $z_p = z_c$ must hold in the optimum.[4] Then the profit function may be written as

$$\tilde{\Pi} = -C(x) + z_p(k + c_o - p_o) + (px - z_p)\tilde{e}. \qquad (10)$$

Combining Eqs.(9) and (10) it follows that $\lambda_p = \lambda_c$ implies and is implied by a zero covariance and, therefore implies and is implied by a full hedge of the export revenue, i.e., $px = z_p$. The claim follows by the continuity argument of part (ii).

Note that the condition of zero risk premia in the options markets, i.e., 'fair' option prices are sufficient but not necessary for a full hedge to be optimal. Furthermore if in both markets the prevailing risk premia are positive, then *over*hedge is still possible if $\lambda_p > \lambda_c$.[5] The intuition runs as follows: A speculative position of an overhedge occurs, if the advantage (disadvantage) of a positive (negative) risk premium in the put price is greater (smaller) than the disadvantage (advantage) of a positive (negative) risk premium in the call. The argument

[4]Suppose $z_p \neq z_c$. Extend the profit's definition of Eq. (4) by $z_p[c_o - (\tilde{e} - k)^+]$. Then the firm's profit is given by the following non–linear function of the exchange rate: $\tilde{\Pi} = -C(x)+z_p(k+c_o-p_o)+(px-z_p)\tilde{e}+(z_c-z_p)[c_o-(\tilde{e}-k)^+]$. Therefore hedging with *forwards*, be they synthetical or real, cannot be optimal [see Franke, Stapleton, and Subrahmanyam (1991)].

[5]This is different from the result in real forward markets [see Benninga, Eldor, and Zilcha (1985), Broll and Wahl (1991)].

for an underhedge position follows analogously.[6] Hence only the relative magnitude of the risk premia with respect to the put and call prices is important to determine the optimal hedging policy of the firm. Note that the put and call prices may be identical although the implied risk premia are not.

[6]We only mention the ambiguous case of $sgn(\lambda_p) = sgn(\lambda_c)$. The sign of $\lambda_p - \lambda_c$ is unambiguous if $sgn(\lambda_p) = -sgn(\lambda_c)$.

Summary

The financial hedging instrument which is underlying our investigation is the currency option contract. Suppose that there are no arbitrage opportunities in the hedging markets. Then selling a newly–written real forward contract would be equivalent to a portfolio consisting of one purchased European put option on the underlying currency and one written European call option on the underlying currency, both with expiration at date 1 equal to the delivery date of the forward contract, and both with a common striking price equal to the forward rate. Since forward markets imply separation it follows that this property also holds in option markets if the traded options allow for constructing a synthetical forward contract. Furthermore full hedging of the export revenue by such synthetical forwards occurs if and only if the risk premia implied in the currency put and call options prices are identical.

Bibliography

Benninga, S., R. Eldor and I. Zilcha (1985): "Optimal International Hedging and Output Policies in Commodity and Currency Forward Markets," *Journal of International Money and Finance*, 4:537–552.

Broll, U. and J.E. Wahl (1991): "International Investments and Exchange Rate Risk," *Working Paper* no.131 (Serie II), University of Konstanz.

Cox, J.C. and M. Rubinstein (1985): *Options Markets*, Englewood Cliffs, N.J. (Prentice–Hall).

Cushman, D.O. (1988): "U.S Bilateral Trade Flows and Exchange Risk during the Floating Period," *Journal of International Economics* 24:317–330.

Danthine, J.P. (1978): "Information, Futures Market, and Stabilizing Speculation," *Journal of Economic Theory*, 17:79–98.

Franke, G. (1990): "Exchange Rate Volatility and International Trading Strategy," *Journal of International Money and Finance* (forthcoming).

Franke, G., R. Stapleton, and M. Subrahmanyam (1991): "The Optimality of Non–Linear Sharing Rules," *Working Paper*, University of Konstanz.

Holthausen, D.M. (1979): "Hedging and the Competitive Firm Under Price Uncertainty," *American Economic Review*, 69:989-995.

Katz, E. and J. Paroush (1979): "The Effect of Forward Markets on Exporting Firms," *Economic Letters*, 4:272–274.

Kawai, M. and I. Zilcha (1986): "International Trade with Forward–Futures Markets under Exchange Rate and Price Uncertainty," *Journal of International Economics*, 20:83-98.

Krugman, P.R. (1989): *Exchange-Rate Instability*, Cambridge, London (MIT-Press).

Leland, H.E. (1972): "Theory of the Firm facing Uncertain Demand," *American Economic Review* , 62:278–291.

Paroush, J. and A. Wolf (1989): "Production and Hedging Decisions in the Presence of Basis Risk," *The Journal of Futures Markets*, 9:547–563.

Sandmo, A. (1971): "On the Theory of the Competitive Firm Under Price Uncertainty," *American Economic Review*, 61:65–73.

Thursby, M.C. and J. G. Thursby (1985): "The Uncertainty Effects of Floating Exchange Rates. Empirical Evidence on International Trade Flows," in S.W. Arndt, R.J. Sweeny, and Th. Willet, Eds., *Exchange Rates, Trade and the U.S. Economy,* Cambridge, MASS., 153–165.

Udo Broll and Jack Wahl
University of Konstanz and Stuttgart
Federal Republic of Germany