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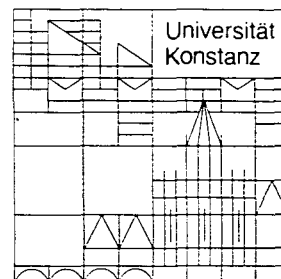
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„Internationalisierung der Wirtschaft“

Diskussionsbeiträge



Juristische
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wissenschaften und Statistik

Werner Smolny

**Dynamic Factor Demand
in a Rationing Model**

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IN A RATIONING MODEL**

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Werner Smolny

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Abstract: A dynamic model of the firm is worked out, which pays special attention to a delayed adjustment of employment, investment, and the production technology. A three-step decision structure is assumed, with short-run adjustment of output, medium-run adjustment of employment, and long-run adjustment of the capital stock and capital-labour substitution. Special attention is paid to dynamic inefficiencies like underutilizations of the capital stock and labour hoarding. Market disequilibrium is introduced by allowing for a sluggish adjustment of wages and prices. The model of the firm is complemented by explicit aggregation over firms. The aggregate model is estimated for the FRG.

Keywords: Disequilibrium, macroeconomic model, aggregation, micro market, dynamic labour demand, investment, rationing model.

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"Dynamic economics is still, in large part, a thing of the future ..."

Marc Nerlove

1 Introduction

Despite some very interesting developments in the field of economic theory and econometrics there is still a gap between theoretical models of economic behaviour and their empirical applications. Most theoretical work is devoted to the analysis of equilibrium situations without saying very much about how this equilibrium is achieved. Econometric practise, however, finds strong support for long lags in economic behaviour and pays special attention to the modelling of dynamic adjustment paths. A good example for the method still used in empirical applications of economic theory is the work of Jorgenson on distributed lags in investment behaviour.¹ In Jorgenson's work, the optimal capital stock is derived from optimal demand for capital services equivalent to those of the pure static case, while actual investment expenditures are estimated in the form of lagged responses to changes in the demand for capital. However, as Nerlove notes,² "...the concept of long run equilibrium towards which adjustment occurs may not be an empirically useful concept in a dynamic context." The sources of dynamic adjustment may also change the properties of the long-run equilibrium and should be carefully considered in theoretical work. The intention of this work is the theoretical elaboration of some properties of disequilibrium adjustment and an empirical investigation of their importance.

Two common but different meanings of the word *(dis)-equilibrium* are currently used in economics.³ The first is connected with the dynamic adjustment of certain variables to long-run stationary values. Equilibrium can be defined as⁴ "...a constellation of selected interrelated variables, so adjusted to one another that no inherent tendency to change prevails in the model which they constitute." Applying this definition to a model of dynamic investment demand, equilibrium refers to a situation in which the actual capital stock coincides with the long-run optimal value derived from firms' net value maximization, or in a dynamic context, the investment rate is equal to its steady state value. Every departure from this point is a disequilibrium.

The other usage of the term is even more accentuated in economics. It is connected to market clearance and defines a situation where supply equals demand. This second definition of equilibrium is closely related to the first; in most economic models, the inequality of supply and demand implies a strong tendency for change. Usually, the "law of supply and demand" is applied, which⁵ "...asserts that price rises when demand exceeds supply and falls in the contrary case." A disequilibrium in terms of the second definition may therefore be seen only as a special case of a disequilibrium in terms of the first definition. However, this does not hold in general, and there are different kinds of models which constitute an equilibrium as a state of rest of a system without requiring permanent market clearing. One important

¹ Jorgenson (1963).

² Nerlove (1972), p. 222.

³ See Benassy (1982), p. 3.

⁴ Machlup (1958), p. 9.

⁵ Arrow (1959), p. 43.

example of equilibrium models without market clearing are the fix-price models of the New Keynesian Macroeconomics. These models focus on the short run, and the determination of wages and prices is left outside of the analysis. Quantity adjustment is assumed to take place before price adjustment and the emphasis is placed on consistent determination of quantities and spillovers between markets. Most work in this approach does not deal explicitly with dynamic adjustment towards equilibrium, instead equilibrium situations are analyzed which allow supply to differ from demand. The other important stream of non market-clearing models consists of those related to the microeconomic foundations of "rigid" wage and price behaviour. While in the fix-price models the rigidity of wages and prices in the short run is simply assumed, and the macroeconomic consequences of this behaviour are analyzed, here the emphasis is put on the sources of rigid wages and prices. Again, much of the analysis is devoted to equilibrium situations,⁶ but in some of the models, special attention is given to the dynamic adjustment of wages and prices.

The model developed here is a disequilibrium model in both meanings of the term. First, it deals explicitly with the dynamic adjustment of the endogenous variables with respect to exogenous disturbances and second, it relies on the fix-price method to allow for non-market-clearing. In contrast to the general equilibrium models of the New Keynesian Macroeconomics, however, the analysis is restricted to the behaviour of firms in disequilibrium situations. The behaviour of households is left outside the analysis. It is intended to incorporate some aspects of dynamic quantity adjustment into the non-market-clearing approach. In the models of the New Keynesian Macroeconomics, a very simple dynamic structure is assumed: immediate adjustment of output and employment combined with an unspecified long-run adjustment of wages, prices, and investment. Evidence from labour economics, however, emphasizes the importance of lags in employment adjustment and of labour as a quasi-fixed factor.⁷ A concept which allows for different adjustment behaviour has been developed within the theory of dynamic factor demand. The idea behind it is simple: the adjustment of factor inputs is not immediate because quick adjustment is more expensive than slow adjustment. However, while the introduction of general adjustment costs leads to dynamic optimization models of the firm, which include as special cases all forms of dynamic adjustment, for the analytical tractability of the problem simplifying assumptions are needed. Thus a simplified model is developed which uses the fix-price method, i.e. wage and price decisions are left outside of the model, but incorporates important aspects of dynamic quantity adjustment. In the model, a fixed adjustment structure is assumed, with short-run adjustment of output, medium-run choice of employment, and long-run decision on the capital stock and capital-labour substitution. It is argued that the most important determinant of adjustment costs is the time used for adjustment, and that the assumption of constant adjustment delays combined with uncertainty allows for a plausible introduction of dynamic aspects while preserving the tractability of the approach. The fixed decision structure permits a rather simple stepwise optimization procedure, and interesting results for investment and employment demand can be derived.

⁶This is explicit, for instance, in the title of Shapiro and Stiglitz' paper "Equilibrium unemployment as a worker discipline device", see Shapiro, Stiglitz (1984).

⁷This term was introduced by Oi (1962).

There are two important outcomes of the approach of dynamic adjustment. The first is the consistent derivation of the foundations of the slow adjustment with respect to changes in the economic environment observed for many economic variables. The second refers to the inefficiencies that are associated with the dynamic adjustment. A firm that cannot adjust the capital stock instantaneously will usually work with an underutilized capital stock or cannot satisfy demand. A similar consideration can be applied to the labour demand. If the firm must choose employment before demand realization, the observed employment level will differ from those required for the actual production. The underutilization of capital and the presence of labour hoarding during recession periods are stylized facts which can be explained by these arguments. The discussion of these two aspects of dynamic quantity adjustment is the main theme of this work.

Another important issue is the consistent aggregation of microeconomic decisions. For instance, on the labour market, some firms cannot realize their labour demand due to an insufficient labour supply, while other firms post no-help-wanted signs due to high wages, a low demand for goods, or insufficient capacities. In consequence, on the aggregate level, unemployment and vacancies usually coexist. Furthermore, it is desirable to determine the importance of the different arguments for the employment level at any period solely by reference to aggregate data. A very interesting aggregation procedure for micro-markets in disequilibrium situations which accounts for these aspects was developed by Lambert.⁸ He started with a Keynesian model, but applied the minimum condition, i.e. the transacted quantity on each market is given by the minimum of supply and demand, only to *micro-markets*. Then, by assuming a lognormal distribution of the micro-markets, he showed that the aggregate transacted quantity can be approximated by a simple CES-type function solely in terms of aggregate supply, aggregate demand, and a mismatch parameter. A related aggregation procedure is applied for this work. The aggregation procedure, which was originally developed for micro-markets in disequilibrium situations, can equally be applied to *firms* in disequilibrium situations, which corresponds exactly to the model derived here. The main behavioural relations can be expressed in terms of minimum or maximum conditions at the firm level. For instance, the optimal labour demand is given by the minimum of the available number of working places and the labour requirement to satisfy goods demand. In turn, actual employment is determined as the minimum of labour supply and demand. These cases are captured by the aggregation procedure, and share of firms facing the different constraints can be determined from aggregate data.

The remainder of the paper is devoted to the empirical investigation of the behavioural relations with an application to the data of the Federal Republic of Germany. The model yields testable hypotheses about output, employment, capital formation, and capital-labour substitution, and in section 3, the results of the estimation are reported.

⁸Lambert (1988).

2 The model

2.1 Assumptions

In this section, a disequilibrium model of the behaviour of the firm is developed. As in the models of the New Keynesian Macroeconomics, the fix-price method is applied. The main features of the model are dynamic factor input adjustments and an explicit aggregation over firms in different goods and labour market situations. Within this framework, the optimal decisions of the firm with respect to output, employment, investment, and capital-labour substitution are derived from microeconomic optimization behaviour. The general outline of the firm's decision is formalized in the following way: each firm faces the problem of maximizing expected value V which is given by

$$V_{t_0} = {}_{t_0} E \int_{t_0}^{\infty} (p_t \cdot YT_t - w_t \cdot LT_t - c_t \cdot K_t) \cdot \exp(-r \cdot t) dt \quad (1)$$

with:

- YT : output
- LT : employment
- K : capital stock
- p : output price
- w : wage rate
- c : usage cost of capital
- r : discounting rate

E is the expectation operator. Each firm produces output with labour and capital as inputs under the constraint of a production function. In the model, there is no allowance for overtime work and inventory adjustment and the analysis is restricted to two input factors, labour and capital, without considering energy or other material inputs.

2.1.1 Wages, prices, and capital costs

Most microeconomic foundations of wage and price rigidities are concerned with the sluggish adjustment of wages to labour supply and demand. This is in line with the standard Keynesian theory relying on fixed nominal wages to allow for the possibility of unemployment. The reasons for this behaviour of wages can be found on the supply side as well as on the demand side of the labour market: in Germany as in most developed countries unions play a considerable role in the wage setting process and the union's wage demand need not coincide with the requirement of equilibrium on the labour market. The process of wage negotiations takes time and wage bargaining usually takes place on a rather aggregate level. Thus it does not necessarily take account of different situations on the microeconomic level. Supply factors can also be important without the existence of unions via the wage setting power of insiders. While the wage demand of the insiders can take account of the situation at the firm level it excludes to a great extent market pressure and the wage dampening effect of high unemployment. Apart from these supply side determinants of rigid wages, it is recognized that firms also have a strong interest in a steady wage behaviour. The importance of firm specific knowledge strongly suggests a long-run perspective in employment relations and reduces the necessity of short-run wage adjustments.

In addition, a cautious adjustment of wages is enforced via negative effects of wage variation on reputation, work effort, and turnover. These theoretical aspects shed some light on the observed behaviour of nominal wages which is characterized by slow adjustment of the aggregate wage level and little variation of the wage structure on the microeconomic level.

Although there is less theoretical foundation for price stickiness, empirical evidence is in favour of more price rigidity as compared with wage rigidity. Price reactions to market disequilibria are more quickly, but in the medium run there is more wage adjustment than price adjustment. Consequently, real wages are weakly procyclical. A reason for this can be the higher elasticity of output supply with respect to adverse demand shocks as compared with the low elasticity of labour supply. Most theories of price stickiness rely on some form of menu costs of changing prices. While direct costs of changing prices as there are costs of reprinting catalogues, of seeking informations, and of making decisions may be seen as not very important, a more appealing introduction of price adjustment costs relies on dynamic demand adjustment. Oligopol theories and theories of customers behaviour emphasize short-run kinked demand curves. The price elasticity of demand is higher (in absolute value) for price increases than for price decreases, thus the short-run demand curve is kinked at the current price. This implies that every firm incurs losses in the short run when changing the price. Higher profits are only achieved in the long-run. Therefore, firms have strong incentives to make a cautious price policy and change prices only if the shifts of the demand curve are expected to be permanent. Temporary demand shocks favour a quantity adjustment against a price adjustment.

The model developed here abstracts completely from an endogenous wage and price setting of firms which, of course, is a restrictive assumption. On the other hand, immediate adjustment of prices and permanent market clearing is equally restrictive and much less consistent with the empirical behaviour of wages and prices. It would be desirable to model dynamic wage and price adjustment explicitly and to derive the optimal pricing rules within the model. However, the model here is designed to capture the dynamic adjustment pathes of *quantities* and it may be argued that the consequences of endogenous price adjustment are less important for the quantity adjustment. Quantity constraints are more relevant for the business cycle than endogenous price setting. Further, while immediate price adjustment and permanent market clearing would alter the results completely, allowing for slow price changes would not alter the qualitative results. It is obvious that wages and prices are endogenous and that the firms change them in the presence of market disequilibria, but the elasticity of prices seems to be so low that it can be neglected for modelling the dynamic adjustment of quantities. This is even more plausible, since the optimal behaviour of the firm depends on relative prices and the variation of the relative prices is much smaller than the variation of nominal wages and prices.

The same exogeneity assumption is applied to capital costs. The user costs of capital consist of real interest rates, depreciation, and the price of investment goods. Financial constraints are not considered, rather it is assumed that firms can finance their capital investment at the current interest rate on the money market. Also, depreciation is assumed independent from the utilization of the capital stock, and the prices of investment goods do not depend on the firm's investment demand.

2.1.2 The concept of micro-markets

The macroeconomic structure of the model relies on the concept of micro-markets introduced by Muellbauer, Malinvaud, Kooiman, and Lambert.⁹ The aggregate goods and labour market can be divided into a number of micro-markets with homogeneous labour and output on each micro-market but limited mobility between micro-markets. This allows for a straightforward introduction of structural imbalances between labour and output supplied and demanded at the aggregate level, while preserving homogeneity at the microeconomic level. The usefulness of this approach becomes evident when a micro-market is defined by a single firm operating on it.¹⁰ Each firm is engaged on one market for its output, one labour market, and one market for capital goods. For example, the output supply of one firm defines total supply on one micro goods market, while output demand is given by the demand for the firm's product. This does not imply any loss of generality since no assumption is necessary about the mobility between firms. A higher degree of mobility between firms reduces structural imbalances or mismatch on a broader concept of micro-markets or on the aggregate goods market, but does not change the general properties of the aggregation procedure. The approach captures also mobility of firms between micro-markets: the engagement of a firm in another regional area is an example for this kind of mobility. One can also allow for firms reducing supply for one output good and using labour and capacities to produce another good. If a firm is defined as a micro unit producing one homogeneous output good, this represents just the formation of a new micro-market.

In addition, it is assumed that each firm is engaged in only one labour market and one market for capital goods. This seemed to be a restrictive assumption but again it does not imply any serious loss in generality. The introduction of heterogeneous labour and capital necessary to produce one output good does not change any of the qualitative results of the model. Furthermore, for aggregate employment and the aggregate market for capital goods, it does not make any difference at all, because mismatch between firms cannot be distinguished from mismatch within the firm. Thus, the assumption just simplifies notation.

Due to these assumptions, homogeneity of output, labour, and capital at the firm level, rigidity of prices in the short run, and the omission of inventories, the minimum condition can be applied to the firm's markets. Output is determined by the minimum of exogenous goods demand and the optimal goods supply, while employment is given by the minimum of exogenous labour supply and the optimal demand for labour of the firm. For the market for capital goods another simplifying assumption is introduced. It is assumed that the firm gets all the investment goods it demands. This assumption has been introduced by Sneesens and Drèze,¹¹ and the reasoning behind it is as follows: if the firms do not get all the investment goods on the domestic market due to strong demand pressure, they can always change to the foreign markets. Effective rationing of investment demand will only be observed for simultaneous constraints on the domestic and the foreign market.¹² Of course, this

⁹ Muellbauer (1978), Malinvaud (1980), Kooiman (1984), and Lambert (1988).

¹⁰ For this approach see Kooiman (1984).

¹¹ Sneesens, Drèze (1986).

¹² For an actual example for this reasoning, see DIW (1990).

is a simplifying assumption. On the other hand, supply constraints for investment goods are probably not the most important determinants of investment activity, which may permit their omission. Furthermore, they will affect at most the timing of investment and less its medium- or long-run level.

2.1.3 The dynamic decision structure

For the following derivations of the model, it is assumed that adjustment costs depend solely on the delay between the decision to change a factor input and the completion of adjustment. In addition, it is assumed that adjustment costs are negligible if firms take account of a factor-specific adjustment delay τ_i , and prohibitive if firms try to adjust faster. This results in constant adjustment delays for labour and the capital stock. This is a simple consideration of adjustment constraints, but it is argued that the combination of constant adjustment delays and uncertainty is a very promising way to introduce a dynamic adjustment in a plausible manner, while at the same time preserving the tractability of the model. Adjustment delays are probably smaller for labour than for capital. The production technology is approximated by a putty-clay production function with short-run limitationality and long-run substitution possibilities for labour and capital. The capital-labour ratio can only be changed with investment, which implies the same adjustment delay for the capital-labour ratio as for investment. Finally, it is argued that output can be adjusted rather quickly to demand changes in the presence of sufficient capacity.

These assumptions allow to reduce the dynamic decision problem of the firm in equation 1 to three static problems. The time structure of the decisions makes it possible to solve the optimization problem sequentially:

1. Short-run adjustment of output with predetermined employment, capital stock, and capital-labour ratio. Output is subject to the minimum condition of supply and demand:¹³

$$YT = \min(YS, YD) \quad (2)$$

YS : goods supply

YD : demand for goods

Output supply is given by a short-run limitational production function:

$$YS = \min \left[\left(\frac{Y}{L} \right) \cdot LT, \left(\frac{Y}{K} \right) \cdot K \right] = \min(Y_{LT}, YC) \quad (3)$$

Y_{LT} : employment constraint

YC : capacities

$\left(\frac{Y}{L} \right)$: productivity of labour

$\left(\frac{Y}{K} \right)$: productivity of capital

The factor productivities are predetermined by the capital-labour ratio and the production function.

¹³The time index is omitted for convenience.

2. Medium-run adjustment of employment with uncertain output and still predetermined capital stock and capital-labour ratio. Employment is determined by the minimum of supply and demand.

$$LT = \min(LD, LS) \quad (4)$$

LD defines labour demand, while LS denotes labour supply.

3. Long-run adjustment of the capital stock and of the capital-labour ratio with uncertain output and employment. It is assumed that the firm can always realize the optimal capital stock. This implies the absence of supply constraints for investment goods and that the firm can also reduce the capital stock (if necessary) in the long run.

2.2 The optimization program of the firm

2.2.1 Output

Without allowance for overtime work and inventory adjustment, the optimal output of the firm can be derived from

$$\max_{YT} V = p \cdot YT - w \cdot LT - c \cdot K \quad (5)$$

under the constraints:

$$\begin{aligned} YT &\leq YD \\ YT &\leq YS \end{aligned}$$

Equation 1 is reduced to the static formulation in equation 5. Demand is exogenous and output supply is predetermined by the employment and investment decisions. This yields:

$$YT^* = \min(YD, YS) \quad (6)$$

Optimal output is determined by the minimum of the demand for goods and the supply constraint of the firm's capacities and employment.

2.2.2 Employment

The employment decision for time t has to be made at time $t - \tau_L$, which means τ_L periods before demand realization, thus under uncertainty of demand. τ_L is the optimal adjustment delay for labour and is assumed to be constant. Labour supply for time t is known in advance. It is given by current employment, retirement, quits of currently employed workers, and the number of new applicants with matching qualifications and reservation wages. In the model, it is not allowed for wage variation or adjustment of minimum qualification levels, thus labour supply is exogenous for the firm. The capital stock at time t and the capital-labour ratio have to be chosen *before* the employment decision, therefore capacities YC and labour productivity are also fixed. Optimal employment can be derived from:

$$\max_{LT} p \cdot {}_{t-\tau_L}E(YT) - w \cdot LT - c \cdot K + \lambda_{LS} \cdot (LS - LT) \quad (7)$$

$t - \tau_L E(YT)$ is the expected value of output in t , with expectations formed at $t - \tau_L$. The first order condition is given by:

$$p \cdot \frac{\partial_{t-\tau_L} E(YT)}{\partial LT} - w - \lambda_{LS} \stackrel{!}{=} 0 \quad (8)$$

λ_{LS} is the shadow price of the labour supply constraint. The realized value of output is uncertain at the time of the employment decision, but output will be chosen according to equation 6 above. Thus, expected output is given by the expected minimum of supply and demand:

$$t - \tau_L E(YT) = t - \tau_L E[\min(YD, YS)] \quad (9)$$

$$= \int_0^{YS} YD \cdot f_{YD} dYD + YS \cdot \int_{YS}^{\infty} f_{YD} dYD \quad (10)$$

f_{YD} is the probability distribution function (p.d.f.) of demand YD . By substituting output supply, which is determined by the limitational production function, the expected marginal product of employment can be written as:

$$\frac{\partial_{t-\tau_L} E(YT)}{\partial LT} = \frac{\partial_{t-\tau_L} E(YT)}{\partial YS} \cdot \frac{\partial YS}{\partial Y_{LT}} \cdot \frac{\partial Y_{LT}}{\partial LT} \quad (11)$$

The first term describes the expected marginal increase of output from increasing supply:¹⁴

$$\frac{\partial_{t-\tau_L} E(YT)}{\partial YS} = \int_{YS}^{\infty} f_{YD} dYD = 1 - F_{YD}(YS) = \text{prob}(YD > YS) \quad (12)$$

F_{YD} is the cumulative distribution function (c.d.f.) of demand. The marginal product of goods supply is equal to the probability of the supply constrained regime. The second term, the marginal increase in supply by increasing the employment constraint can take only the values 0 or 1:

$$\frac{\partial YS}{\partial Y_{LT}} = 1 - \lambda_{YC} = \begin{cases} 0 & \text{for } YC < Y_{LT} \\ 1 & \text{for } YC > Y_{LT} \end{cases} \quad (13)$$

Increasing employment will increase output supply only if there is capital to employ the additional workers. The third term is the productivity of labour, which is a function of the predetermined optimal capital-labour ratio and the production function:

$$\frac{\partial Y_{LT}}{\partial LT} = \left(\frac{Y}{L} \right) \quad (14)$$

Substituting these derivations back into equation 8, the first order condition can be rewritten as:

$$p \cdot \text{prob}(YD > YS) \cdot (1 - \lambda_{YC}) \cdot \left(\frac{Y}{L} \right) - w - \lambda_{LS} = 0 \quad (15)$$

¹⁴For a continuous random variable x , $\text{prob}(x = x_1) = 0$ for all x_1 , and therefore $\text{prob}(x < x_1) = \text{prob}(x \leq x_1)$. See DeGroot (1987), pp. 102-13.

For the determination of optimal employment three cases can be distinguished:

1. **No constraints:** $\lambda_{YC} = \lambda_{LS} = 0$. The firm is neither constrained by the available labour supply nor by the existing capital stock. Equation 15 reduces to:

$$\underbrace{p \cdot \text{prob}(YD > Y_{L^*})}_{\text{expected marginal returns}} \cdot \underbrace{\left(\frac{Y}{L}\right)}_{\text{marginal costs}} = w \quad (16)$$

The firm chooses employment as to equalize expected marginal returns to marginal wage costs. Defining

$$sl^* = \frac{w}{p} \cdot \left(\frac{L}{Y}\right)$$

where sl^* is the ratio of wage costs to nominal output, i.e. labour's share in output at *full utilization of labour*, equation 16 can be rewritten as:

$$\text{prob}(YD > Y_{L^*}) = sl^* \quad (17)$$

In the optimum, the probability of the supply constrained regime is equal to the full employment labour share. By using this result together with equation 12, optimal employment can be determined from:

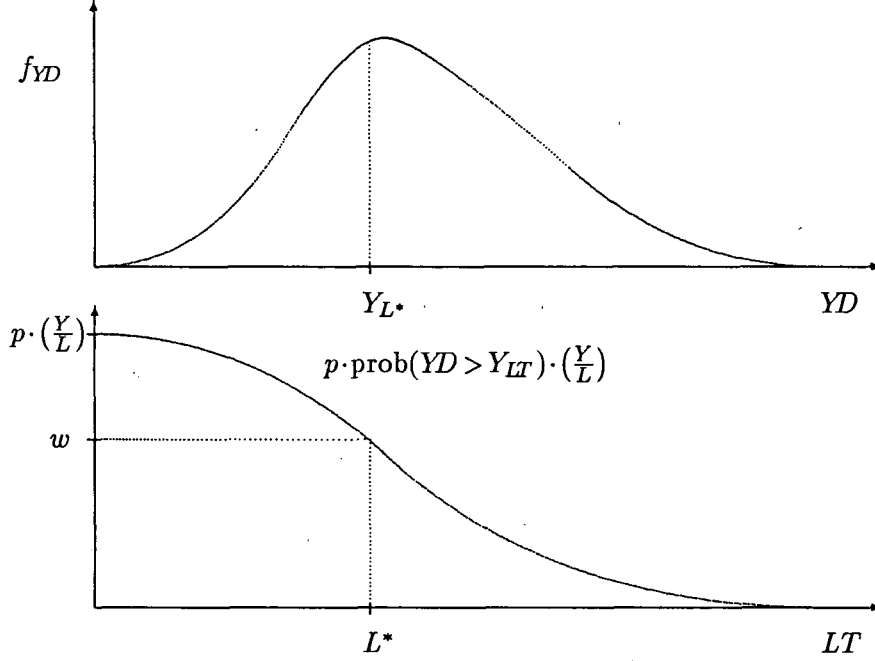
$$L^* = L^* = \left(\frac{L}{Y}\right) \cdot F_{YD}^{-1}(1 - sl^*) \quad (18)$$

F_{YD}^{-1} denotes the inverse c.d.f. of demand YD . Optimal employment is determined by the productivity of labour, the share of labour costs in nominal full employment output, and the parameters of the distribution function of YD . The partial derivative of L^* with respect to the real wage is negative. The effect of a higher productivity of labour on optimal employment is ambiguous: there is an employment increasing effect on profitability sl^* , but there is also a decreasing effect, because less labour is needed to produce a given output. The effect of a higher expected demand on employment is positive, while increased uncertainty, i.e. a higher variance of demand may increase or decrease employment, depending on the value of sl^* and depending on the particular form of the distribution function of demand. For example, if the distribution function is symmetric around the expected value, the critical value of sl^* is 0.5. For $sl^* > 0.5$ optimal employment will decrease with higher uncertainty and vice versa. However, this property cannot be generalized for skewed distributions.

A graphical interpretation of equation 16 is depicted in [figure 1](#). The upper figure is the density function of demand f_{YD} , while the lower figure shows marginal costs and expected marginal return. The marginal costs of an additional unit of labour is constant and equal to the wage rate w . The expected marginal return of labour is equal to $p \cdot \left(\frac{Y}{L}\right)$ for small values L , where $\text{prob}(YD > Y_L)$ is equal to one. The expected marginal return of labour is decreasing for increasing L due to the increasing probability of demand constraints on the goods market. For high values of employment, $\text{prob}(YD > Y_L)$ approaches zero, a unique optimum is therefore assured for $sl^* < 1$.

2. **Capacity constraint:** $\lambda_{LS} = 0$, $\lambda_{YC} = 1$. The firm is rationed by the existing capital stock. This constraint influences optimal employment negatively as no more

Figure 1: Optimal employment



workers will be demanded than can be employed by the (predetermined) capital stock. Optimal employment is determined according to:

$$LT^* = L_{YC} = \left(\frac{L}{Y}\right) \cdot YC = \left(\frac{L}{Y}\right) \cdot \left(\frac{Y}{K}\right) \cdot K \quad (19)$$

Employment is determined by the available number of working places.

3. Labour supply constraint: $\lambda_{LS} \neq 0$, $\lambda_{YC} = 0$. In the final case of insufficient labour supply, the firm has not enough applicants to fill all vacancies. Employment cannot exceed this constraint, and optimal employment is equal to the labour supply:

$$LT^* = LS \quad (20)$$

These three cases can be summarized by the minimum condition for optimal employment:

$$LT^* = \min(L^*, L_{YC}, LS) \quad (21)$$

This formulation of the first order condition has appealing properties, both from an econometric point of view and for the economic interpretation. It will be shown that equation 21 can be estimated easily using observable aggregate data. Further, this equation, together with equation 6 for the optimal output can be used to calculate the optimal degree of labour hoarding. The expected degree of utilization of labour is defined by:

$${}_{t-\tau_L}E(DUL) = \frac{{}_{t-\tau_L}E(L_{YT})}{LT} = \left(\frac{L}{Y}\right) \cdot \frac{{}_{t-\tau_L}E(YT)}{LT}$$

The degree of utilization of labour is given by the amount of labour needed to produce output L_{YT} in relation to actual employment. The utilization of labour varies procyclically, with higher utilization in the presence of positive (unexpected) demand shocks and, hence, a *procyclically* varying measured productivity of labour. This property is in accordance with observed stylized facts and stands in contrast to conventional models of dynamic factor input adjustment, which allow for immediate adjustment of employment and short-run substitution of capital and labour, thus implying an *anticyclical* movement of the productivity of labour. On the other hand, optimal labour hoarding will decrease with less uncertainty of demand and also, less labour hoarding will be observed in the presence of labour supply and capacity constraints.

2.2.3 The investment decision

The corresponding static decision problem for the firm concerning investment is to maximize expected value of profits with respect to the optimal capital stock K and the optimal capital-labour ratio k . The investment decision has to be made with a delay τ_K before output realization and also with a delay $\tau_K - \tau_L$ before employment determination. This is formalized by:

$$\max_{K,k} V = p \cdot {}_{t-\tau_K}E(YT) - w \cdot {}_{t-\tau_K}E(LT) - c \cdot K \quad (22)$$

The expected value depends on expected output and expected employment, and therefore on the density function of YD and LS . In addition, L^* is uncertain at the time of the investment decision. It is influenced by the optimal capital-labour ratio but it also contains a stochastic component in the demand expectation formed at $t - \tau_L$. The investment decision has two components, the stock of capital and the capital-labour ratio, which have to be determined simultaneously.

1. The optimal capital stock: Differentiation of equation 22 with respect to the capital stock K yields the following first order condition:

$$p \cdot \frac{\partial {}_{t-\tau_K}E(YT)}{\partial K} - w \cdot \frac{\partial {}_{t-\tau_K}E(LT)}{\partial K} - c \stackrel{!}{=} 0 \quad (23)$$

The expected value of output can be calculated from:

$${}_{t-\tau_K}E(YT) = {}_{t-\tau_K}E[\min(YD, YS)] \quad (24)$$

It has been shown that employment output Y_{LT} never exceeds capacities YC , therefore output supply YS is equal to the employment constraint:

$$YS = Y_{LT} = \min(Y_{L^*}, Y_{LS}, YC) = \min \left[\left(\frac{Y}{L} \right) \cdot L^*, \left(\frac{Y}{L} \right) \cdot LS, YC \right] \quad (25)$$

The potential supply of goods is determined by employment and labour productivity, while employment is determined from the various constraints discussed above. The expected value of output, with expectations formed at $t - \tau_K$, is given by:

$${}_{t-\tau_K}E(YT) = {}_{t-\tau_K}E[\min(YD, Y_{L^*}, Y_{LS}, YC)] \quad (26)$$

$$\begin{aligned}
 &= YC \cdot \int_{YC}^{\infty} \int_{YC}^{\infty} \int_{YC}^{\infty} f_G dY_{LS} dY_{L^*} dYD \\
 &\quad + \int_0^{YC} \int_{Y_{LS}}^{\infty} \int_{Y_{LS}}^{\infty} Y_{LS} \cdot f_G dY_{L^*} dYD dY_{LS} \\
 &\quad + \int_0^{YC} \int_{Y_{L^*}}^{\infty} \int_{Y_{L^*}}^{\infty} Y_{L^*} \cdot f_G dYD dY_{LS} dY_{L^*} \\
 &\quad + \int_0^{YC} \int_{YD}^{\infty} \int_{YD}^{\infty} YD \cdot f_G dY_{LS} dY_{L^*} dYD
 \end{aligned}$$

f_G is the common trivariate p.d.f. of Y_{LS} , Y_{L^*} , and YD . Partial differentiation with respect to the capital stock yields:¹⁵

$$\begin{aligned}
 \frac{\partial_{t-\tau_K} E(YT)}{\partial K} &= \frac{\partial YC}{\partial K} \cdot \int_{YC}^{\infty} \int_{YC}^{\infty} \int_{YC}^{\infty} f_G dY_{LS} dY_{L^*} dYD \quad (27) \\
 &= [1 - F_G(Y_{LS} = YC, Y_{L^*} = YC, YD = YC)] \cdot \left(\frac{Y}{K}\right) \\
 &= \text{prob}[YC < \min(Y_{LS}, Y_{L^*}, YD)] \cdot \left(\frac{Y}{K}\right)
 \end{aligned}$$

F_G is the common trivariate c.d.f. of YD , Y_{LS} , and Y_{L^*} , and $\left(\frac{Y}{K}\right)$ is the optimal productivity of capital. The marginal product of capital is determined by the probability of the capacity constrained regime on the goods market and the optimal productivity of capital. The expected value of employment is given by:

$$\begin{aligned}
 {}_{t-\tau_K} E(LT) &= L_{YC} \cdot \int_{L_{YC}}^{\infty} \int_{L_{YC}}^{\infty} f_L dLS dL^* \quad (28) \\
 &\quad + \int_0^{L_{YC}} \int_{LS}^{\infty} LS \cdot f_L dL^* dLS \\
 &\quad + \int_0^{L_{YC}} \int_{L^*}^{\infty} L^* \cdot f_L dLS dL^*
 \end{aligned}$$

f_L is the common p.d.f. of LS and L^* . Differentiation with respect to the capital stock yields:

$$\begin{aligned}
 \frac{\partial_{t-\tau_K} E(LT)}{\partial K} &= \frac{\partial L_{YC}}{\partial K} \cdot \int_{L_{YC}}^{\infty} \int_{L_{YC}}^{\infty} f_L dLS dL^* \quad (29) \\
 &= [1 - F_L(LS = L_{YC}, L^* = L_{YC})] \cdot \left(\frac{L}{Y}\right) \cdot \left(\frac{Y}{K}\right) \\
 &= \text{prob}[L_{YC} < \min(LS, L^*)] \cdot \left(\frac{L}{Y}\right) \cdot \left(\frac{Y}{K}\right)
 \end{aligned}$$

F_L is the c.d.f. of LS and L^* . The marginal employment effect of additional capital is determined by the probability of the capital constrained regime *on the labour market*

¹⁵The complete derivations of the model are contained in Smolny (1991).

and the optimal capital-labour ratio. Now equation 23 can be rewritten as:

$$\begin{aligned}
 & p \cdot \text{prob}[\min(YD, Y_{L^*}, Y_{LS}) > YC] \cdot \left(\frac{Y}{K}\right) \\
 & \quad \text{expected returns} \\
 & - w \cdot \text{prob}[\min(LS, L^*) > L_{YC}] \cdot \left(\frac{L}{Y}\right) \cdot \left(\frac{Y}{K}\right) - c = 0 \\
 & \quad \text{expected additional wage costs} \quad \text{capital costs}
 \end{aligned} \tag{30}$$

Three effects of an increased capital stock can be distinguished. The first is the marginal increase in capital costs, which is given by the user cost of capital c . Second, there is the expected increase in labour costs due to a marginal higher capital stock. A higher capital stock increases employment only in the capital constrained regime *on the labour market*. If labour supply is the binding constraint for employment, than a higher capital stock does not influence employment. A similiar argument holds, if unconstrained employment L^* is the minimum of the possible constraints on the labour market. Finally, there is the marginal change in expected return due to a higher capital stock. Employment and the productivity of labour determine also the optimal supply of goods, thus a higher capital stock affects output supply only if it affects employment.¹⁶ This is the first condition for positive returns on capital. Second, higher returns are achieved only, if the firm can sell the product, i.e. if demand exceeds supply. Taken together, a higher capital stock increases output only if capacities are the binding constraints on the goods as well as on the labour market.

Equation 30 has a unique solution $K > 0$ for plausible values of the parameters:¹⁷ for low values of K , both probabilities tend to one, and the left hand side of equation 30 is positive. On the other hand, both probabilities decrease for a increasing capital stock. They tend to 0 for a high value of K , and there are only costs of marginal increases of capital but no returns. An explicit solution of equation 30 is only possible with a simplifying assumption about the difference between these probabilities. One can define

$$a = \text{prob}[\min(LS, L^*) > L_{YC}] - \text{prob}[\min(YD, Y_{L^*}, Y_{LS}) > YC]$$

and a *share of capital at full utilization of capacities* analogously to labour's share in output at full employment:

$$sk^* = \frac{c}{p} \cdot \left(\frac{K}{Y}\right)$$

a is greater than or equal to zero and stems from the marginal reduction of profits due to labour hoarding. It depends, for instance, on the medium-run expectation error of demand, the extend of labour supply constraints, and, of course, it is also influenced by the stock of capital.¹⁸ However, the endogeneity of this term is not

¹⁶Therefore, the probability of the capital constrained regime on the goods market cannot exceed the probability of the capital constrained regime on the labour market.

¹⁷The following constraint must hold: $1 - sl^* - sk^* > 0$. If this condition does not hold, there are never positive profits, and the firm leaves the market. Of course, $YD, LS \geq 0$ holds.

¹⁸It is close to zero for very low or very high values of the capital stock, and achieves a maximum for medium values of the capital stock.

considered to be very important, and for the derivation of the optimal capital stock it is treated as an exogenous variable. Then, equation 30 can be reformulated as

$$\text{prob}[\min(YD, Y_{L^*}, Y_{LS}) > YC] \cdot (1 - sl^*) = sk^* + a \cdot sl^* \quad (31)$$

and optimal capacities stock can be obtained from:

$$YC = F_{Y^a}^{-1} \left(1 - \frac{sk^* + a \cdot sl^*}{1 - sl^*} \right) \quad (32)$$

$F_{Y^a}^{-1}$ is the inverse of the c.d.f. of the distribution of Y^a . The variable Y^a deserves some attention. ${}_{t-\tau_K}E(Y^a)$ is the expected minimum of those constraints which may prevent the firm from full utilization of capacities. Y^a is defined by:

$$Y^a = \min(YD, Y_{LS}, Y_{L^*})$$

Whenever the actual values of these constraints exceed YC , the firm can work with full utilization of capacities and could earn more profits with a higher capital stock. On the other hand, if one of these constraints is less than YC , the firm has excess capacities and the marginal product of capital is zero. Therefore, the decision on the optimal capital stock can be seen as the optimal choice of the probability of being constrained by capacities, and the value of this probability is determined by the ratio of marginal costs and marginal "full employment cash flows".¹⁹

Equation 32 shows very clearly the similarity of the behavioural equations determining employment and capacities. Unconstrained employment L^* is determined by the expected constraint on the goods market and the share of wage costs in value added, and optimal capacities are determined by the expected constraints on the goods and labour market and the "profitability" of capital. Equation 32 determines the optimal capacities and thus also implies the choice of the expected degree of utilization of capital, which is defined by:

$${}_{t-\tau_K}E(DUC) = \frac{{}_{t-\tau_K}E(YT)}{YC}$$

Finally, the optimal capital stock is determined by:

$$\ln K = -\ln \left(\frac{Y}{K} \right) + \ln YC \quad (33)$$

2. The derivation of the optimal capital-labour ratio: The second component of the investment decision concerns the choice of the optimal capital-labour ratio k . The production technology is approximated by a CES-production function with exogenous technical progress and constant returns to scale. Capacities are determined by:

$$Y = \mu \cdot \left\{ \alpha \cdot [\exp(\gamma_K \cdot t) \cdot K]^{-\rho} + (1 - \alpha) \cdot [\exp(\gamma_L \cdot t) \cdot L]^{-\rho} \right\}^{-1/\rho} \quad (34)$$

The rates of labour and capital saving technical progress are given by γ_L and γ_K , respectively, α is the parameter determining the relative production elasticities, and $\rho = 1/\sigma - 1$ with σ being the elasticity of substitution. The partial differentiation

¹⁹See equation 32 above.

of the optimization problem, equation 22 above, with respect to the capital-labour ratio yields the following first order condition:

$$p \cdot \frac{\partial_{t-\tau_K} E(YT)}{\partial k} - w \cdot \frac{\partial_{t-\tau_K} E(LT)}{\partial k} \stackrel{!}{=} 0 \quad (35)$$

The following condition can be derived for the optimal capital-labour ratio:²⁰

$$k^* = \left(\frac{\alpha}{1-\alpha} \right)^\sigma \cdot \exp [(\gamma_L - \gamma_K) \cdot t \cdot (1-\sigma)] \cdot \left(\frac{c}{w + \lambda_{LS}^*} \cdot \frac{t-\tau_K E(DUL)}{t-\tau_K E(DUC)} \right)^\sigma \quad (36)$$

λ_{LS}^* can be interpreted as a “shadow price” of the labour supply constraint.²¹ The slope of the transformation curve is equal to relative factor prices, “corrected” for the expected degrees of utilization of both factors and for the shadow price of the labour supply constraint.²² It can be shown that $\lambda_{LS}^* > 0$, i.e. the optimal capital-labour ratio is always higher in the presence of labour supply constraints. A simple functional form for the factor productivities can be achieved by introducing an assumption about price behaviour. The price setting has been left out of the decision problem of the firm, but it has been argued that real as well as nominal prices are sticky and adjust only slowly to market disequilibria. If real rigidities are more important than nominal rigidities, a dependence of prices on costs can be stated which can be operationalized by a mark-up pricing on expected costs:

$$p = \theta \cdot \frac{w \cdot t-\tau_K E(LT) + c \cdot K}{t-\tau_K E(YT)} \quad (37)$$

Then the optimal productivities of labour and capital can be determined from:²³

$$\left(\frac{Y}{K} \right) = \alpha^{-\sigma} \cdot [\mu \cdot \exp(\gamma_K \cdot t)]^{1-\sigma} \cdot \left(\frac{\theta \cdot c}{p \cdot t-\tau_K E(DUC)} \cdot \frac{1}{1 + \lambda'_{LS} \cdot \theta} \right)^\sigma \quad (38)$$

$$\left(\frac{Y}{L} \right) = (1-\alpha)^{-\sigma} \cdot [\mu \cdot \exp(\gamma_L \cdot t)]^{1-\sigma} \cdot \left(\frac{\theta \cdot w}{p \cdot t-\tau_K E(DUL)} \cdot \frac{1 + \frac{\lambda'_{LS}}{sl}}{1 + \lambda'_{LS} \cdot \theta} \right)^\sigma \quad (39)$$

The factor productivities are proportional to the real factor prices, corrected for the respective expected degree of utilization of the factor and the shadow price of the labour supply constraint.

²⁰ See Smolny (1991).

²¹ $\lambda_{LS}^* = p \cdot \text{prob}_w [Y_{LS} < \min(YC, YD, Y_L \cdot)] \cdot \frac{t-\tau_K E(YT)}{t-\tau_K E(LT)} - w \cdot \text{prob}_w [LS < \min(L_{YC}, L^*)]$

If labour is always in excess supply, the probability terms are zero and therefore $\lambda_{LS}^* = 0$. λ_{LS}^* is the change in expected value when the labour supply constraint increases by one unit. In the absence of labour hoarding, both probabilities are equal and the shadow price of the labour supply constraint is given by the product of these probabilities and the marginal profits of employment $p \cdot (Y/L) - w$.

²² Of course, equation 36 determines the optimal capital-labour ratio not only in terms of the exogenous variables, rather it is a structural form between endogenous variables. λ_{LS}^* , $E(DUC)$, $E(DUL)$ are endogenous in the model.

²³ $\lambda'_{LS} = \text{prob}_w [Y_{LS} < \min(YC, YD, Y_L \cdot)] - sl \cdot \text{prob}_w [LS < \min(L^*, L_{YC})]$, $sl = \frac{w \cdot t-\tau_K E(LT)}{p \cdot t-\tau_K E(YT)}$

Table 1: Regimes on the goods and labour market

goods market	$YD > YS$	$YD < YS$
labour market	$YT = YS$	$YT = YD$
$LT = LS$ $LS < \min(L_{YC}, L^*)$	prob ₁₁ I $YT = Y_{LS}$ repressed inflation	prob ₁₂ II $YT = YD$ underconsumption
$LT = L_{YC}$ $L_{YC} < \min(LS, L^*)$	prob ₂₁ III $YT = YC$ capital shortage	prob ₂₂ IV $YT = YD$ mixed
$LT = L^*$ $L^* < \min(L_{YC}, LS)$	prob ₃₁ V $YT = Y_{L^*}$ mixed	prob ₃₂ VI $YT = YD$ Keynesian unemployment

2.2.4 Regimes on the goods and labour market

In [table 1](#) the different regime constellations of the firm on the goods and labour market are summarized. While in the standard model of the New Keynesian Macroeconomics only three regimes are possible, the non-simultaneity of the output, employment, and capacity decisions introduces the possibility of three further combinations of output and employment constraints. First, if the firm has to decide on employment *before* output realization, this re-introduces the possibility of the underconsumption regime. Underconsumption is characterized by a rationing of the firm on the goods and labour market. If the firm can decide simultaneously on output and employment, only one of the constraints, labour supply or the demand for goods, can be the binding constraint. In the model above, however, the firm is uncertain about goods demand when choosing employment, and there is a positive probability of the demand constrained regime, even if the labour supply is the binding constraint for employment. The other modification of the regime possibilities is related to the non-simultaneity of the employment and investment decision and the assumption of a putty-clay technology. The firm decides on employment after choosing optimal capacities YC . This allows to distinguish two possible sources of demand constraints on the labour market. First, optimal labour demand does not exceed the available number of working places L_{YC} . Second, unconstrained labour demand L^* depends on goods demand expectations and profitability.²⁴ The optimal labour demand can therefore be distinguished, whether it is determined by capacities or by demand expectations. If the probabilities of the regime constellations on the goods and labour market are denoted by prob_{ij} , the following properties hold. First, these probabilities sum up to one:

$$\sum_{ij} \text{prob}_{ij} = 1$$

²⁴It should be emphasized that for plausible values of the parameters of the distribution function of YD , profitability plays only a minor role in determining L^* .

Second, from the first order condition for unconstrained optimal employment it can be derived that:

$$\frac{\text{prob}_{31}}{\text{prob}_{31} + \text{prob}_{32}} - sl^* = 0$$

The conditional probability of the supply constrained regime on the goods market is equal to the share of wage costs in full employment nominal output. Finally, from the first order condition for optimal capacities it follows that:

$$\text{prob}_{21} - sl^* \cdot (\text{prob}_{21} + \text{prob}_{22}) - sk^* = 0$$

The complete set of regime probabilities can be derived from the first order conditions and the parameters of the trivariate distribution function of YD , LS , and Y_L^* . It should be noted that these are the *optimal* probabilities the firm chooses by deciding on capacities YC and labour demand L^* . A situation of equal probabilities of the supply and demand constrained regime, or of equality of supply and demand in expected values, i.e. $E(YD) = YS$, has no special significance in the model and does not define an "equilibrium". The optimal probabilities, which define some kind of equilibrium, are determined by relative factor prices, the parameters of the density function, and the parameters of the production function.

While the heterogeneity of the rationing constellations enriches the interpretation of the regimes, it prevents one from simply carrying over the terms, which are used in the standard Keynesian model to characterize the rationing situations. There is a clear correspondence of the regimes of repressed inflation and underconsumption, but in the context of the model here, the terms Keynesian or classical unemployment should be used with care. Repressed inflation is characterized by excess demand on the goods as well as on the labour market, and an insufficient labour supply is the sole source of this constraint. A higher labour supply would increase employment as well as output. A similar correspondence can be observed for the regime of underconsumption. The firm is rationed on both markets: it cannot realize its labour demand due to labour supply shortages and output is constrained by the demand for goods. This double rationing is possible as a consequence of the non-simultaneity of the output and employment decisions. However, this correspondence does not hold for the regime of classical unemployment. In the standard model of the New Keynesian Macroeconomics this term is used for a situation where the firm can realize its notional trade offers on both markets. Here this holds for the regimes III and IV. In regime III, both output and employment are constrained by the capital stock. However, this capital shortage can be caused by demand expectations, i.e. the sources of the resulting underemployment may be "Keynesian". The standard classification of regimes is even more unsuitable for the regime V. Again the firm can realize its notional trade offers, but the main reason for the low labour demand is the too low expected demand for goods. This regime is characterized as a Keynesian situation on the labour market but a classical situation on the goods market. A similar inaccuracy holds for the regime of Keynesian unemployment: in the regimes IV and VI, insufficient demand constrains output and the firm can realize its labour demand. The sources of the low labour demand are a lack of capacities (regime IV) or the expected demand for goods (regime VI). The latter can therefore be called Keynesian regime for good reasons, but the sources of the underemployment in regime IV can also be factor prices being too high, or even expected labour supply being too

low. However, while the terms “classical unemployment” and “Keynesian unemployment” should be used only with caution, the distinction of the regimes is clear and the greater variety of rationing situations enriches the economic interpretation of the model. The rationing situations can be distinguished even further which is analyzed in the next section.

2.3 Dynamic factor demand

2.3.1 Dynamic adjustment of employment

Employment cannot exceed the labour supply. However, the labour supply consists of those already employed in the firm and job applicants. Therefore it is realistic to allow for a dependence of the current labour supply on the past employment level. A similar dependence on past employment can be stated for the demand for labour. Investments in firm specific human capital, implicit “full employment contracts”, and reputation losses give rise to costs of dismissing workers and tend to restrict the downward adjustment of employment to normal fluctuations, i.e. quits and retirement.²⁵ In addition, the capacity of training entrants may prevent a too rapid growth of employment. A way to introduce these aspects into the employment decision is to assume a constraint on the adjustment *speed* of employment. For labour demand, this can be formalized as:

$$LD_t = \min \left\{ (1 + \delta_t^c) \cdot LT_{t-1}, \max [LD_t^*, (1 - \delta_t^s) \cdot LT_{t-1}] \right\} \quad (40)$$

t is the time index and LD_t^* is the target level of employment to which the firm wants to adjust. The parameter δ_t^c is determined by the capacity of training entrants. The minimum condition restricts the maximum amount of adjustment of employment within one period to δ_t^c percent of the past employment level. The maximum condition implies a limit on the downward adjustment, and δ_t^s is the maximal rate of downward adjustment of employment. δ_t^s can be interpreted as the optimal rate of downward adjustment and a dependence of δ_t^s on the expected persistence of a low labour demand, for instance, is plausible. If the costs of dismissing are prohibitive, than δ_t^s can be identified with the rate of normal separations, i.e. quits and retirement. A similar formalization can be applied to the supply of labour:

$$LS_t = \min [(1 + \delta_t^a) \cdot LT_{t-1}, \overline{LS}_t] \quad (41)$$

Equation 41 reflects a constraint on the maximum rate of applications as well as on the absolute level of labour supply. This implies that the labour supply increases if the firm increases employment, but only until it reaches an exogenous level constraint \overline{LS}_t . It seems to be important to allow for both kinds of constraints. In the short run and during recessions the number of *applications within a time period* restricts employment growth, while in the long run and during boom periods it is plausible that a low *level* of labour supply prevents a higher employment.

There are four restrictions causing employment to differ from the target level of labour demand LD_t^* : first, the level of employment is restricted by the exogenous level constraint on labour supply \overline{LS}_t ; second, the decrease of employment cannot

²⁵ This constraint is important as another explanation of labour hoarding.

exceed maximal (optimal) separations; third, employment increases must be carried out with limited training capacities; finally the number of job applicants within a time period can be binding. The last two restrictions can be combined. Defining $\delta_t^h = \min(\delta_t^a, \delta_t^c)$, optimal employment is determined by:

$$LT_{t_0} = \min \left\{ \overline{LS}_t, (1 + \delta_t^h) \cdot LT_{t-1}, \max [LD_t^*, (1 - \delta_t^s) \cdot LT_{t-1}] \right\} \quad (42)$$

An interesting property of this kind of introduction of adjustment constraints is the simple way to allow for different constraints for upward and downward adjustments of employment. For $\delta_t^s < \delta_t^h$, the downward adjustment is more impeded than the upward adjustment, and $\delta_t^h = \infty$ implies an unconstrained upward adjustment.

Now the analysis is turned to the determinants of the target level of employment LD_t^* . In the dynamic model here, the optimal target level differs from those derived in the static context of the model. LD_t^* again depends on the number of workers that can be employed with the capital stock, L_{YC} . However, it may be profitable to employ more workers than L_{YC} . If the firm is installing capacities at time t_1 , it may be profitable to start earlier with employment adjustment in face of constraints on adjustment speed. The additional wage costs incurred by an earlier adjustment may be more than compensated by the earlier use of the machine. The expected demand for goods is also a determinant for LD_t^* , but once again, the strong minimum condition derived above, equation 21, need not hold. In the face of an expected temporary demand shock, for instance, the employment level may only adjust partially. It should be noted that employment is not only determined by current constraints on adjustment, but depends also on expected future constraints on employment adjustment. The formal decision problem is given by

$$\max_{\rightarrow LT_t} \sum_{t=t_0}^{\infty} E (p_t \cdot YT_t - w_t \cdot LT_t - c_t \cdot K_t) \cdot (1 + r)^{-t} \quad (43)$$

subject to the constraints:

$$\begin{aligned} YT_t &= \min(YD_t, YS_t) \\ YS_t &= \min(YC_t, Y_{LT_t}) \\ LT_t &\leq \overline{LS}_t \\ LT_t &\leq (1 + \delta_t^h) \cdot LT_{t-1} \\ LT_t &\geq (1 - \delta_t^d) \cdot LT_{t-1} \end{aligned}$$

t_0 refers to the period, when the employment level is realized, i.e. the firm has to decide on LT_{t_0} at $t_0 - \tau_L$. The decision problem can be rewritten as:

$$\begin{aligned} \max_{\rightarrow LT_{t_0}} \quad & \sum_{t=t_0}^{\infty} E \left\{ (p_t \cdot YT_t - w_t \cdot LT_t - c_t \cdot K_t) \cdot (1 + r)^{-t} + \lambda_t^{LS} \cdot (LS_t - LT_t) \right. \\ & \left. + \lambda_t^h \cdot [(1 + \delta_t^h) \cdot LT_{t-1} - LT_t] + \lambda_t^s \cdot [LT_t - (1 - \delta_t^s) \cdot LT_{t-1}] \right\} \end{aligned} \quad (44)$$

The first order condition is given by

$$p_{t_0} \cdot \frac{\partial_{t_0-\tau_L} E(YT_{t_0})}{\partial YS_{t_0}} \cdot \frac{\partial YS_{t_0}}{\partial Y_{LT_{t_0}}} \cdot \frac{\partial Y_{LT_{t_0}}}{\partial LT_{t_0}} - w_{t_0} - \lambda_{t_0}^{LS} - \lambda_{t_0}^h + \lambda_{t_0}^s + \lambda_{t_0}^* \stackrel{!}{=} 0 \quad (45)$$

with $\lambda_{t_0}^*$ defined by:

$$\begin{aligned}\lambda_{t_0}^* &= {}_{t_0-\tau_L}E[(1 + \delta_{t_0+1}^h) \cdot \lambda_{t_0+1}^h] - {}_{t_0-\tau_L}E[(1 - \delta_{t_0+1}^s) \cdot \lambda_{t_0+1}^s] \\ &= \lambda_{t_0}^* [LT_{t_0}^*, {}_{t_0-\tau_L}E(LT_{t_0+1}^*, \delta_{t_0+1}^h, \delta_{t_0+1}^s)]\end{aligned}$$

If none of the constraints that may restrict employment at time t_0 is binding, i.e. $\lambda_{t_0}^{LS} = \lambda_{t_0}^h = \lambda_{t_0}^s = 0$, the target level of employment can be determined from:

$$p_{t_0} \cdot \text{prob}(YD_{t_0} > YS_{t_0}) \cdot (1 - \lambda_{t_0}^{YC}) \cdot \left(\frac{Y}{L}\right)_{t_0} - w_{t_0} + \lambda_{t_0}^* = 0 \quad (46)$$

Two cases can be distinguished: first, for $\lambda_{t_0}^{YC} = 0$, i.e. there are sufficient working places in t_0 , it can be derived from equation 46 that:

$$L_{t_0}^* = \left(\frac{Y}{L}\right)_{t_0} \cdot F_{YD_{t_0}}^{-1} \left(1 - \frac{w_{t_0} - \lambda_{t_0}^*}{p_{t_0}} \cdot \left(\frac{L}{Y}\right)_{t_0}\right) \quad (47)$$

In addition to real wages, the productivity of labour, and the parameters of the density function of demand, the optimal employment level depends on expected employment and the expected adjustment constraints in $t_0 + 1$. If the constraint on the upward adjustment is more important, i.e. $\lambda_{t_0}^* > 0$, the optimal employment level is higher than in the reference case, while $\lambda_{t_0}^* < 0$, i.e. more important constraints on the downward adjustment, implies a lower employment level. This can be illustrated by an example: if, for instance, demand is expected to increase in the future (i.e. after t_0), this increases optimal employment already in t_0 , while an expected slowdown in demand reduces employment. A capacity constraint is observed, if $L_{t_0}^*$ from equation 47 exceeds the number of working places $L_{YC_{t_0}}$. In this case, optimal employment is given by:

$$L_{YC_{t_0}}^* = \begin{cases} L_{YC_{t_0}} & \text{if } w_{t_0} > \lambda_{t_0}^* \text{ for } L'_{YC_{t_0}} > L_{YC_{t_0}} \\ L'_{YC_{t_0}} & \text{else} \end{cases} \quad (48)$$

$L'_{YC_{t_0}}$ can be determined from $w_{t_0} = \lambda_{t_0}^*$.

Summarizing these results, optimal employment is determined by

$$LT_t^* = \min \left\{ \overline{LS}_t, (1 + \delta_t^h) \cdot LT_{t-1}, \max [LD_t^*, (1 - \delta_t^s) \cdot LT_{t-1}] \right\} \quad (49)$$

with

$$LD_t^* = \min [L_t^*, \max(L_{YC_t}, L'_{YC_t})] \quad (50)$$

i.e. six cases can be distinguished for the determination of optimal employment. At a first glance, these different regimes are somewhat confusing, however, each of the regimes has a clear economic interpretation and is important for the determination of employment: (1.) the availability of qualified workers can constrain employment, (2.) the growth of employment is restricted by the number of applicants per period or by training capacities, (3.) costs of dismissals can inhibit a too rapid downward adjustment of employment, (4.) labour demand usually will not exceed the number of working places, with the exception of a situation like those specified in equation 48, (5.). Finally, if none of these constraints is binding, employment is determined by expected demand and relative wage costs (6.).

2.3.2 Investment and capital-labour substitution

A similar model with a limited adjustment speed can be developed for the adjustment of the capital stock as well. Assuming irreversibility of investment decisions, i.e. gross investment required to be positive, limits the maximal downward adjustment of the capital stock to depreciation, while supply constraints for investment goods $I_{t_0}^s$ can limit the upward adjustment. This implies that the optimal capital stock in t_0 depends on all future period decisions about the capital stock, employment, and output. While a complete solution seems to be algebraically unfeasible, there are some interesting interpretations of the model. First, the irreversibility of the investment decision probably tends to smooth investment behaviour. Gross investment cannot be negative, and if the constraint is not binding in the current period, the optimal capital stock will be lower than in the standard model. It is immediately plausible that the firm does not adjust completely with respect to, for instance, an expected temporary positive demand shock in t_0 . A smoothing of investment behaviour is also implied by supply constraints of investment goods in t_0 . This smoothing is apparent in the condition $0 \leq I_{t_0} \leq I_{t_0}^s$. Further, consider an expected permanent demand shock for $t_0 + 1$, which increases the desired capital stock from that period onward and which may also increase the probability of future supply constraints for investment goods. This has a positive effect on the optimal capital stock already for t_0 and the adjustment path to the higher capital stock is smoothed.

These considerations about smoothing of adjustment seem to be more important for capital adjustment than for employment adjustment. Due to the longer delays and the stronger irreversibilities in capital formation, the firm has to bear the costs of a wrong investment decision for a longer time. Therefore, the firm must make a cautious, *long-run* investment policy and does not adjust to temporary shocks. A smoothing of investment is further enhanced by a slow adjustment of expectations. The smoothness of the demand for capital is apparent in empirical data, and nonlinear adjustment costs, irreversibility of investment decisions, adjustment delays, and partial adjustment of expectations can account for this behaviour.

A partial adjustment is also plausible for changes in the optimal capital-labour ratio. It can be argued that capital investment is a necessary ingredient for adjustments of the capital-labour ratio, and further, not every small investment allows for any desired change in the factor input ratios. This suggests that the adjustment of the factor proportions will be at least as slow as the adjustment of the capital stock.

Summarizing these arguments, while the basic model of investment decisions with only adjustment delays can give some insight into the main determinants of investment behaviour under uncertainty, it should be extended for several reasons. The most important extension seems to be the introduction of additional non-linearities in the adjustment cost function, which can explain the smooth adjustment paths of capital demand and of the capital-labour ratio.

2.4 The aggregation of micro-markets

In the preceding chapters, the analysis of the model was limited to the behaviour of one firm. It has been shown that different rationing situations on the goods and labour market are possible and that the firm's choice of employment and capacities, together with the properties of the distribution function, determine the *probabilities*

of the regime constellations at the firm level. In the aggregate, however, at every moment in time different firms face different constraints and it is plausible that usually all rationing constellations coexist. This can be illustrated by an example: on the aggregate labour market normally there are both unemployment and vacancies. Even in boom periods some of the unemployed don't find work while in recession periods with high unemployment some firms cannot fill vacancies immediately. The reasons for this can be manifold. The vacancies and the unemployed can be in different regional areas, and there is a limited degree of mobility of firms or workers between regions. Further, the workers may not have the qualifications required for the job, or the worker and the firm disagree about the wage rate and the working conditions of the job. Finally, the process of matching labour supply and labour demand takes time, thus preventing the immediate filling of vacancies. Of course, a similar consideration can be applied to the goods market. While some firms work with underutilized labour or capital, other firms cannot satisfy demand due to labour supply shortages or insufficient capacities. All these cases are captured in the model of the firm with different supply and demand conditions on each firm market and a limited degree of mobility between the micro-markets.

For applied work, however, it is desirable to find an aggregation procedure which takes account of the differences at the firm level, and allows for the derivation of a relation between aggregate quantities transacted, supplied, and demanded on the goods and labour market. In addition, it is of interest to determine the *proportion* of firms in the different regimes. An appealing way to derive this relation is to state a density function for demand and supply on the micro-markets. If the number of firms is large, the densities can be approximated by a continuous density function f_{SD} and the aggregate transacted quantities can be written as

$$LT = N \cdot \int_0^\infty \int_{LD_j}^\infty LD_j \cdot f_{SD} dLS_j dLD_j + N \cdot \int_0^\infty \int_{LS_j}^\infty LS_j \cdot f_{SD} dLD_j dLS_j \quad (51)$$

and analogously for the goods market.²⁶ This relation can be developed further, if the properties of the density function f_{SD} are known. The first way is simply to *assume* a distribution for the micro-markets. Supply and demand on the micro-markets depend on a large number of events, therefore the central limit theorem may be applied and the resulting distribution can be approximated by a normal distribution. However, it is plausible that many economic forces have a *proportional* impact on supply and demand, which would imply that the distribution of micro-markets may be better approximated by a lognormal distribution. In addition, the distribution of many economic variables is skewed, which strengthens the case of the lognormal distribution of micro-markets as opposed to the normal distribution.

A more appealing way than just assuming the distribution is to derive it from the underlying stochastic process. Supply and demand are determined by economic forces, which can be analyzed to derive the properties of the resulting distribution. As a starting point, labour supply on each micro-market, by definition, can be written as:

$$LS_{i,t} = LT_{i,t-1} - \text{separations} + \text{applications}$$

²⁶ The index i refers to micro-markets. LS_j , LD_j refer to the corresponding continuous random variables.

The labour supply at time t is given by the past employment level, minus the number of separations between time t and $t-1$, plus applications. This simple formula allows for some interesting considerations. The first is the dependence of the current labour supply on the past employment level. "Workers who walk out of the factory gate on a Friday afternoon will typically return through the same gate on a Monday morning, if not before."²⁷ At least, if the time period under consideration is short, the main determinant of the labour supply is the past employment level. On the other hand, labour supply changes through time by mobility processes, i.e. quits of currently employed workers, retirement, rejected applications, and new applications. The sources of change again depend on the past level of employment or labour supply. Additional determinants are the wage rate paid by the firm and the working conditions, the aggregate wages and the aggregate unemployment and vacancy rate, and finally the employment chances in the firm, i.e. the relation between the firm's labour demand and supply. It seems plausible that these forces work proportionally to the firm size, i.e. the employment level of the firm. Further, the wage paid by the firm depends on the labour market conditions at the firm level and aggregate wages, prices, and unemployment. Therefore the labour supply can be written as a reduced form by:

$$LS_{i,t} = LS_i(LS_{i,t-\tau}, LD_{i,t-\tau}, \varepsilon_{i,t+1-\tau}^{ls}) \quad (52)$$

$\tau = 1, \dots, t - t_0$ and t_0 is the starting period. The labour supply is determined by past levels of labour supply, the demand for labour in the past, and a vector ε_i^{ls} . This vector includes the exogenous variables and contains also the stochastic elements of the dynamic process. By similar considerations, an equivalent relation for the demand for labour can be derived. The current demand for labour depends on past levels of labour demand, employment, and a vector of exogenous and random variables ε_i^{ld} :

$$LD_{i,t} = LD_i(LD_{i,t-\tau}, LS_{i,t-\tau}, \varepsilon_{i,t+1-\tau}^{ld}) \quad (53)$$

The endogenous variables from the goods market, i.e. capacities and the capital-labour ratio can be substituted by exogenous variables contained in ε_i^{ld} , and employment is substituted by $LT_i = \min(LD_i, LS_i)$.²⁸ Equation 53 contains again a strong autoregressive root due to slow changes of capacities and relative prices. Equations 52 and 53 form a system of higher order nonlinear stochastic difference equations, which is difficult to solve analytically. However, some properties of the process can be derived from a simple first order linear approximation. The proportionality of the effects on the level of the variables suggests a formalization in logarithmic levels of the variables:

$$\log LS_{i,t} = \alpha_1 \cdot \log LS_{i,t-1} - \alpha_2 \cdot \log \left(\frac{LS_{i,t-1}}{LD_{i,t-1}} \right) + \varepsilon_{i,t}^{ls} \quad (54)$$

$$\log LD_{i,t} = \beta_1 \cdot \log LD_{i,t-1} - \beta_2 \cdot \log \left(\frac{LD_{i,t-1}}{LS_{i,t-1}} \right) + \varepsilon_{i,t}^{ld} \quad (55)$$

²⁷Nickell (1986), p. 473. Nickell related this statement to the demand for labour, but, of course, it can be applied to labour supply as well.

²⁸These exogenous variables are the relative prices, expected demand, and may also include productivity shocks.

When considering small time intervals between t and $t-1$, α_1 and β_1 converge to one and the weight of each individual shock becomes small. Therefore the central limit theorem can be applied to derive approximatively a bivariate lognormal distribution for demand and supply on the micro-markets.

However, the stochastic processes analyzed above are still rather crude approximations of the process determining labour supply and demand on the micro-markets. At least, three critical points should be mentioned. The first is the assumed independency of the exogenous shocks over time. It has been stated above that the $\varepsilon_{i,t}^{ls}$ depend, amongst others, on the aggregate unemployment rate and aggregate wages. Both variables exhibit a high degree of persistence, thereby implying a strong dependence of $\varepsilon_{i,t}^{ls}$ on $\varepsilon_{i,t-1}^{ls}$. The same holds for demand shocks $\varepsilon_{i,t}^{ld}$. On the other hand, this does not alter the qualitative properties of the stochastic process. If the autoregressive root of the $\varepsilon_{i,t}$ is less than one, the speed of convergence against the lognormal distribution is weakened but convergence is still preserved. Another critique is concerned with the assumed independency of the shocks between micro-markets and the lack of modelling a feedback from the employment decision of one firm to the shocks of other firms. This can be shown by a simple example: the employment decision of one firm influences the aggregate unemployment rate. In addition, the sum of employment over all micro-markets cannot exceed the aggregate labour supply, which is bounded at least in the short run. This implies a restriction on the sum of shocks which is neglected in the processes analyzed above. A consequence of this restriction is that the variance of labour supply on the micro-markets is bounded when the aggregate labour supply is bounded. Therefore one should bear in mind that the results can be seen only as approximations for a finite time interval and with each firm having only a small market share, so that these feedbacks can be neglected. They cannot be extended to the infinite time case. Finally, the linear approximation of the process may be criticized. It has been shown above that labour demand is determined in a highly nonlinear way by the various minimum conditions and the delayed capacity formation. However, nonlinear difference or differential equations are difficult to analyze and normally an algebraic solution is not feasible. On the other hand, the power of the central limit theorem and the goodness of the normal approximation of other distributions are important arguments in favour of using this distribution. Taken this together with the – plausible – proportionality of the effects from the determining factors gives some confidence in the approximation of the distribution of micro labour markets by a lognormal distribution.

The distribution of micro goods markets can be derived in an analogous manner as the distribution of micro labour markets. A stochastic process, similar to that in equations 54 and 55, can be developed also for the demand and supply for goods on the micro-markets. Then, the process can be analyzed in the same manner as above and yields again approximatively a lognormal distribution for demand and supply on the goods markets. One important difference between the processes determining supply and demand on the labour market and on the goods market should be noted. While the boundedness of aggregate labour supply assures a finite variance of labour supply and probably also for the demand for labour, this is less true for the supply and demand for goods. Therefore, stochastic processes like those above for the supply and demand for goods can serve as a part of the analysis of stochastic growth processes.

The aggregation procedure of micro-markets in different regimes rests on a derivation put forward by Lambert.²⁹ For lognormally distributed micro-markets, a very simple analytical expression for LT can be derived. If the weighted probabilities of the regimes are approximated by a logistic curve,³⁰ it can be shown to yield the following CES-type aggregate function for employment:

$$LT = \left\{ LD^{-\rho} + LS^{-\rho} \right\}^{-1/\rho} \quad (56)$$

ρ serves as a mismatch parameter with:

$$\frac{\partial LT}{\partial \rho} > 0, \quad \lim_{\rho \rightarrow \infty} LT = \min(LD, LS)$$

This aggregation procedure for micro-markets in disequilibrium situations corresponds exactly to the model of the firm developed above. Under plausible assumptions it can be derived that supply and demand on the firm markets are approximately lognormally distributed. This implies that aggregate output and employment can be determined from aggregate supply and demand, and the variance of the logarithmic difference of both variables on the micro level. A CES-function can be used to approximate this relation. It has been derived above that output supply is determined by output at full utilization of labour, while demand is assumed to be exogenous to the firm. Then output can be obtained by summing up over all micro-markets, or with the approximation of the CES-function:

$$YT = \left\{ YD^{-\rho_y} + Y_{LT}^{-\rho_y} \right\}^{-1/\rho_y} \quad (57)$$

Similar considerations can be applied to the labour market. Employment is determined by the minimum of supply and demand, while, in turn, labour demand is given by the minimum of the capacity constraint and demand determined employment. For this case, the CES-property can be applied as well. The distribution of the minimum of two lognormally distributed variables can again closely be approximated by a lognormal distribution and aggregate employment can be determined from:

$$LT = \left\{ LS^{-\rho_m} + LD^{-\rho_m} \right\}^{-1/\rho_m} \quad (58)$$

$$LD = \left\{ L^{*- \rho_f} + L_{YC}^{-\rho_f} \right\}^{-1/\rho_f} \quad (59)$$

This example shows that the aggregation procedure, which was originally designed to capture the case of *micro-markets* in disequilibrium situations, can also be applied to those kind of firm's (or households) behaviour, which can be expressed by minimum conditions. Two CES-functions can be stated, one for the labour market and one for the labour demand of the firm, which can be nested to determine aggregate employment in terms of labour supply, capacity employment, and demand determined employment:

$$LT = \left\{ LS^{-\rho_m} + \left[L^{*- \rho_f} + L_{YC}^{-\rho_f} \right]^{\rho_m/\rho_f} \right\}^{-1/\rho_m} \quad (60)$$

²⁹ Lambert (1988), appendix A. See also Quandt (1988).

³⁰ See Quandt (1982) for the advantages of approximating normal densities by e.g. logistic curves for empirical work.

In addition, for the lognormal distribution demand determined employment at the firm level follows from:

$$\ln L_i^* = \ln \left(\frac{L}{Y} \right)_i + \ln_{t-\tau_L} E(YD_i) - 0.5 \cdot \sigma_{\ln YD_i}^2 + \sigma_{\ln YD_i} \cdot F^{-1}(1 - sl_i^*) \quad (61)$$

F^{-1} is the inverse of the cumulative standard normal distribution function. The aggregation procedure allows an easy transformation of the firm specific values into the corresponding aggregate values. When approximating the distribution of the variables contained in equation 61 by a lognormal distribution, a simple aggregation over firms is possible. For a lognormal distributed variable x_i it holds

$$E(\ln x_i) = \ln E(x_i) - 0.5 \cdot \sigma_{\ln x_i}^2 \quad (62)$$

i.e. the equation containing the aggregate variables has the same structure as for the individual firms. The only difference is a change in the normalizing constant, which is affected by the variance of these variables on the micro-markets.³¹

The aggregation procedure of micro-markets in disequilibrium situations can also be applied to capture the extensions of the model. Equation 49 contains a *maximum* condition but it can be shown in an analogous procedure than those applied for the aggregation of the minimum condition that the expected maximum of two lognormally distributed variables can equally be approximated by a CES-function. The only modification is given by the change in the sign of the mismatch parameter and aggregate employment can be obtained from³²

$$LT_t = \left\{ LS_t^{-\rho_m} + LD_t^{-\rho_m} \right\}^{-1/\rho_m} \quad (63)$$

$$LS_t = \left\{ \overline{LS}_t^{-\rho_h} + [(1 + \delta_t^h) \cdot LT_{t-1}]^{-\rho_h} \right\}^{-1/\rho_h} \quad (64)$$

$$LD_t = \left\{ LD_t^{*\rho_s} + [(1 - \delta_t^s) \cdot LT_{t-1}]^{\rho_s} \right\}^{1/\rho_s} \quad (65)$$

while the target level of labour demand is determined by:

$$LD_t^* = \left\{ L_t^{*- \rho_f} + [L_{YC_t}^{\rho_c} + L'_{YC_t}{}^{\rho_c}]^{-\rho_f/\rho_c} \right\}^{-1/\rho_f} \quad (66)$$

Six regimes can be distinguished at the aggregate level and normally at every moment of time some firms will be in each of the regimes. Even if aggregate employment is increasing, some firms will not participate and face a constraint on the downward adjustment of employment, while in recession periods with high unemployment some firms cannot find the appropriate workers. It should be noted that the δ parameters

³¹For the estimation of the employment equation, F^{-1} can be calculated for all observed values of sl^* . For the inverse of the c.d.f. of the standard normal distribution, no analytical expression exists. However, it can be approximated with nearly any accuracy by certain fractions of higher order polynomials. The formula which is used here for the calculation of $F^{-1}(1 - sl^*)$ is taken from Kinder et al. (1982).

³²This determination of LS_t and LD_t holds only if the constraints on the upward adjustment of labour demand can be neglected. However, employment can always be determined from the nested CES-function.

must not be equal for all firms, the only requirement is the close approximation of the distribution of all variables by a lognormal distribution.

A solution for the optimal capital stock can be obtained equally. By using the same procedure as above for the derivation of optimal employment L^* , optimal capacities can be obtained from:

$$\ln YC = (-1/\rho_1) \cdot \ln \left\{ t_{-\tau_K} E(YD)^{-\rho_1} + t_{-\tau_K} E(Y_{LS})^{-\rho_1} + t_{-\tau_K} E(Y_{L^*})^{-\rho_1} \right\} \quad (67)$$

$$-0.5 \cdot \sigma_{\ln Y^a}^2 + \sigma_{\ln Y^a} \cdot F^{-1} \left(1 - \frac{sk^* + \alpha \cdot sl^*}{1 - sl^*} \right)$$

F^{-1} is again the inverse of the c.d.f. of the standard normal distribution and can be calculated for all observed values of the argument.

3 Results

The behaviour of a profit maximizing firm facing different constraints was derived, and an aggregation procedure for firms in different regimes was presented. Now it is tried to evaluate this model with an empirical application for the Federal Republic of Germany. The model is applied to the private sector of the economy. The aggregation procedure allows to investigate the behavioural relations solely by use of aggregate data which simplifies the empirical application and facilitates the interpretation of the results in terms of the business cycle. The model is estimated with quarterly data for the time period 1960-1989. This period includes years of supply as well as demand shortages and the model supplies a consistent framework to investigate the main determinants of output, employment, and investment during the business cycle.

3.1 The recursive structure

The model of the firm consists of four behavioural relations. Output is given by the minimum of supply and demand, and supply, in turn, is determined by employment. Employment depends on the expected demand, the labour supply, capacities, and the past employment level. The investment decision has to be made τ_K periods before demand realization with uncertainty about labour supply and goods demand. It will fix capacities as well as the capital-labour ratio. The optimal capacities depend on profitability and the possible constraints on the goods and labour market and the optimal factor productivities are determined by real factor costs, weighted by the utilization of the factors and the shadow price of the labour supply constraint.

While it is desirable to estimate these equations simultaneously, the non-linearities encountered by this procedure cause serious problems. On the other hand, there is a simple three-step procedure, which circumvents these problems and allows a recursive estimation of the system.³³ In a first step, expectations are estimated by use of a method relying on rational expectations formation. The second step consists of the

³³ A similar estimation procedure has been proposed by Sneesens, Drèze, (1986) and is applied in most of the work contained in Drèze et al. (1990).

determination of the optimal productivities of labour and capital. The observed productivities deviate from the optimal ones by the respective utilization of the factor in question. This implies a relation between actual productivities, expected factor costs, the shadow price of the labour supply constraint, and the utilization of the factors. The expected costs can be estimated, and by using indicators for the degrees of utilization of labour and capital, the productivity equations can be expressed completely in terms of observable variables.

The optimal productivities are necessary ingredients for the calculation of the output and employment series. Output supply can be calculated from employment and the productivity of labour, capacities are calculated from capital productivity and stock of capital, and the supply of labour is given by employment plus the number of unemployed. The assumptions applied in the model contain also an implication about the derivation of excess demand. It has been argued that the firms can always realize their investment demand by switching to the foreign markets in the case of supply constraints on the domestic market. When applying the same assumption also for the households, the government, and the foreign demand, a clear instruction for the calculation of demand is obtained: the excess demand for domestic products is given by those imports, which are caused by supply constraints on the domestic market plus those part of exports, which are *not* carried out due to supply constraints of domestic firms. Total imports M can be distinguished by "structural imports" MD , i.e. those imports, which are caused by relative prices, income, etc., and those imports, which are induced by supply constraints

$$M = MD + M'(\text{excess demand}) \quad (68)$$

with $M' \geq 0$. On the other hand, "export demand" XD exceeds observed exports X because domestic firms cannot deliver due to supply constraints

$$XD = X + X'(\text{excess demand}) \quad (69)$$

with $X' \geq 0$. The trade equations can be estimated with the most important determinants of export demand and structural imports included, and containing also an indicator for the excess demand on the domestic market. This yields an estimate for the excess demand and aggregate demand can be calculated according to:³⁴

$$\hat{YD} = YT + \hat{M}'(\text{excess demand}) + \hat{X}'(\text{excess demand}) \quad (70)$$

Demand is greater than or equal to transactions and the amount of excess demand can be estimated from the spillover to the foreign markets. Again, the expected values must be estimated, and then all series necessary for the estimation of output, employment, and capacity are available. This is the third step of the estimation procedure. Aggregate output and employment can be estimated by the respective minimum conditions and capital formation is estimated by equation 67

3.2 The choice of technique

A first aspect concerns the treatment of expectations within the productivity equations. For the derivation of the theoretical model it has been assumed that wages,

³⁴ A hat stands for an estimated series.

prices, and capital costs are known with certainty. This is a very strong assumption, and it can be shown that it must not be applied. As long as wages, prices, and capital costs are assumed to be *exogenous*, the same results can be achieved with uncertainty, with the exception that the relative prices in equations 38 and 39 must be replaced by their expected values. When applying the hypothesis of rational expectations one can estimate these expectations in a first step and then use these values for the estimation of the productivity equations. The same procedure is applied to the expected degrees of utilization of labour and capital. For the utilization of capital an indicator is given by the business survey series on capacity utilization q for industry published by the ifo-institute.³⁵

$$D\hat{U}C = D\hat{U}C(q) \quad (71)$$

No similar clearcut series is available for the utilization of labour within the firm. However, one can base an indicator on the correlation of the utilizations of labour and capital. The most important source of underutilization are unexpected demand shocks, with employment adjusting faster to those shocks. This is captured by the following specification of $D\hat{U}L$:

$$D\hat{U}L_t = D\hat{U}L_t(\underset{+}{DUC_t}, \underset{-}{DUC_{t-1}}) \quad (72)$$

The significance of these indicators provides also a first test of the assumptions applied for the derivation of the model. Significant underutilization of capital in recession periods accentuates the importance of a delayed capital formation. On the other hand, significant underutilization of labour stresses the role of a delayed adjustment of employment as well as a delayed adjustment of the capital-labour ratio. With an immediate adjustment of employment and a putty-clay technology, the productivity of labour should not be cyclical at all. An immediate adjustment of employment together with short-run substitution possibilities between labour and capital would imply an *anticyclical* productivity of labour. In case of an adverse demand shock the possibility of short-run changes in the capital-labour ratio implies an overshooting of the demand for labour. A procyclical productivity of labour within the firm can occur only in the case of a slow adjustment of employment and the capital-labour ratio. The underutilization of labour and capital is also an indicator of price rigidities. In a model with perfectly flexible prices, firms can always lower prices in case of a negative demand shock. This should increase utilization, and if instead of this firms prefer underutilizations of labour and capital, there is some form of price rigidity. This may be caused by, for instance, low short-run demand elasticities or price adjustment costs. Underutilization can occur only in case of a downward rigidity of prices.

A final aspect is concerned with additional dynamics in the productivity equations. The productivity equations include already one form of dynamics insofar as *expected values* of some of the variables are contained: the firm adjusts the capital-labour ratio with respect to, for instance, an unexpected shift in the relative factor

³⁵It captures only industry, but it is likely that this is highly correlated with the utilization of the whole economy, and most results achieved with this indicator exemplify its reliance. See e.g. Entorf, Franz, König, Smolny (1989), Franz, Smolny (1989), Smolny (1990), Franz, Heidbrink, Scheremet (1990), König, Entorf (1990).

prices only with a delay of length τ_K . However, it has been argued that investment is a necessary ingredient for a change of the technology and further, not every small investment allows every desired change of the capital-labour ratio. A similar argument can be derived from adjustment costs for the capital-labour ratio. This results in adjustment processes, which can be approximated by a partial adjustment mechanism or an error correction model.³⁶

An error correction model is estimated, and the mostly preferred results are depicted in table 2.³⁷ These results rely also on an explicit treatment of expectation formation; in a first step, expectations are estimated by a regression of the respective exogenous variable on a lagged information set. From these regressions, the fitted values are calculated. In a second step, these fitted values are taken as the expected values of the respective variable and are used for the estimation of the productivity equations. For the productivity equations, the lowest standard errors are obtained with expectations formed five or six quarters before the actual change in the productivities. This is about the lag which was expected from other estimations of lags in capacity formation, and it is in line with the theoretical model. Therefore, the procedure applied for modelling expectation formation finds some support from the empirical results. On the other hand, one caveat of this procedure should not be suppressed. For the appropriateness of this proceeding, it is necessary that the adjustment delay is identical for all investment projects. This is, of course, an unrealistic assumption. Rather, it is plausible that this delay varies with respect to the investment project under consideration. In this case, the estimation procedure yields only the most important adjustment delay. However, it has been tried to include different expected values into the productivity equations, but no results superior to those reported in table 2 have been achieved.

The estimated elasticity of substitution is about 0.5. Surprisingly, no significant effects from labour supply constraints on the productivities were found. Capital-labour substitution is a long-run decision, and it can be argued that firms always expect to find workers. However, the sixties and the beginning of the seventies were many years of labour supply shortages, and it is surprising that no effects from these shortages on productivity growth were found. The simple decreasing time trend can "explain" the slowdown of labour productivity growth better than the decreasing gross investment rates and the decreasing importance of labour supply constraints. In addition, no significant effects from the *expected* degrees of utilization on the long-run factor substitution decision were found. This may be due to the difficulty of modelling expected utilization, but may also be caused by the low influence of utilization on *relative* factor costs: both factor costs are influenced by changes in utilization, and both utilizations are highly correlated.

The most important results are related to the actual utilization indicators. The model is build on a slow adjustment of prices, factor inputs, and factor productivities, which should result in significant underutilizations of labour and capital. First, very significant coefficients are found from the utilization indicator on the productivities of labour and capital. Second, the model implies that the effect of utilization should

³⁶See Nickell (1985) for the relation of the error correction model and optimal behaviour of the economic agents in a dynamical environment.

³⁷The estimation method was OLS with coefficient restrictions. The estimations rely also on an explicit treatment of the working time (not reported).

Table 2: The productivity of labour and capital

$\ln \left(\frac{Y}{L} \right)_t = (1 - 0.332 + 0.103 - 0.654 + 0.225) \cdot \ln \left(\frac{Y}{L} \right)_t^*$ $+ \frac{0.332}{(0.07)} \cdot \ln \left(\frac{Y}{L} \right)_{t-1} - \frac{0.103}{(0.05)} \cdot \ln \left(\frac{Y}{L} \right)_{t-2}$ $+ \frac{0.654}{(0.05)} \cdot \ln \left(\frac{Y}{L} \right)_{t-4} - \frac{0.225}{(0.07)} \cdot \ln \left(\frac{Y}{L} \right)_{t-5}$ $\ln \frac{YT_t}{LT_t} = \ln \left(\frac{Y}{L} \right)_t + \frac{0.444}{(0.06)} \cdot \left(\ln q_t - \frac{0.408}{(0.10)} \cdot \ln q_{t-1} \right)$ $\ln \left(\frac{Y}{L} \right)_t^* = \frac{0.510}{(0.16)} \cdot \ln \frac{w_t}{p_t} + (1 - 0.510) \cdot \left(\frac{0.017}{(0.001)} \cdot t - \frac{1.1 \cdot 10^{-4}}{(2.0 \cdot 10^{-5})} \cdot \frac{t^2}{2} \right)$	
SEE: 0.0107 BP(8): 7.3 BP(12): 13.6	

$\ln \left(\frac{Y}{K} \right)_t = (1 - 0.332 + 0.103 - 0.654 + 0.225) \cdot \ln \left(\frac{Y}{K} \right)_t^*$ $+ \frac{0.332}{(0.07)} \cdot \ln \left(\frac{Y}{K} \right)_{t-1} - \frac{0.103}{(0.05)} \cdot \ln \left(\frac{Y}{K} \right)_{t-2}$ $+ \frac{0.654}{(0.05)} \cdot \ln \left(\frac{Y}{K} \right)_{t-4} - \frac{0.225}{(0.07)} \cdot \ln \left(\frac{Y}{K} \right)_{t-5}$ $\ln \frac{YT_t}{K_t} = \ln \left(\frac{Y}{K} \right)_t + \frac{0.509}{(0.04)} \cdot \ln q_t$ $\ln \left(\frac{Y}{K} \right)_t^* = \frac{0.510}{(0.17)} \cdot \ln \frac{p_{i,t}}{p_t} + (1 - 0.510) \cdot \left(\frac{0.0008}{(0.002)} \cdot t - \frac{5.3 \cdot 10^{-5}}{(2.9 \cdot 10^{-5})} \cdot \frac{t^2}{2} \right)$	
SEE: 0.0116 BP(8): 6.1 BP(12): 10.3	

Sample 1962.4-1989.4. *t*-values in parentheses.
Both equations include a constant and seasonal dummies.
Restriction are placed on those coefficient, where no standard errors are reported.
corr.: 0.81

be equal in the short run and in the long run. This restriction is tested and is not rejected by the data. Third, it is expected that the short-run impact of a demand shock on the utilization of labour and capital are of an about equal amount. However, in the medium run, employment can be adjusted faster to the demand shock, and the average utilization of labour should be higher than the utilization of capital. This corresponds very much to the estimated coefficients. A change in the q variable changes the observed productivities of labour and capital by an about equal amount in the first period, while already in the second period the productivity of labour adjusts in the direction of the former level. The estimated utilizations of labour and capital are depicted in figure 2.³⁸ The maximum levels of the degrees of utilization are supplied by the assumption that the observed maximum of the respective utilization

³⁸The data depicted in the figures are seasonally adjusted by constant seasonal factors.

variable corresponds with full utilization.³⁹ Then the degrees of utilization of labour and capital, respectively, can be determined as:

$$\ln \hat{D}\hat{U}_t = 0.509 \cdot (\ln q_t - \ln q^{max}) \quad (73)$$

$$\ln \hat{D}\hat{U}_t = 0.444 \cdot [(\ln q_t - 0.408 \cdot \ln q_{t-1}) - (\ln q_t - 0.408 \cdot \ln q_{t-1})^{max}] \quad (74)$$

The index *max* stands for the observed maximum of the variable. The estimated coefficients imply an average utilization of capital of about 95 percent and an average utilization of labour of about 97 percent. The implied amount of labour hoarding can be seen from figure 3. LT is actual employment and \hat{L}_{YT} is the number of workers which are necessary for the production of output. The degree of utilization of labour corresponds to an average amount of labour hoarding of about 600,000 workers, and in recession periods, labour hoarding exceeds 1,000,000 workers. One can see very clearly the similar movement of both utilization series, but also the faster medium-run adjustment of the utilization of labour. Both series correspond very close to the level of economic activity (this is closely related to \hat{L}_{YT} in the short run), and lead before employment.⁴⁰

What are the implications of these empirical results for the assumptions applied for the derivation of the theoretical model? First, the underutilizations of labour and capital are important phenomena, and should be taken into account in the theoretical analysis. The short-run rigidity of prices, a delayed adjustment of employment and the capital stock, a short-run limitationality of the production technology, and nonlinear adjustment costs for employment, the capital stock, and the capital-labour ratio can contribute to the explanation of these phenomena. Second, no significant effects from labour supply constraints and from *expected* utilizations on the capital-labour substitution decision were found. It is not clear, whether this result is attributable to specification errors of these variables, or to the low influence stemming from these variables, or whether they remark a serious deficiency of the theoretical model. Third, the error correction estimation of the dynamic adjustment process seems to be appropriate for the modelling of the development of the productivities of labour and capital. It should be augmented by an explicit treatment of expectations.

3.3 Calculation of demand – the trade equations

The trade equations are necessary to estimate the excess demand on the goods market, which is used for the estimation of the output, employment, and investment equations. For this purpose, “augmented” import and export demand equations are estimated. Exports and imports are mainly determined by demand factors, but supply factors like supply constraints and delayed adjustment of supply with respect to demand are also introduced.

³⁹ Only the relative level of the utilizations can be determined from the productivity equations, i.e. it can be derived that the utilization of capital was about 10 percent higher in 1970 than in 1982, but it cannot be said, whether the utilization of capital is 100 percent at the beginning of 1970 or, for instance, 99 percent.

⁴⁰ Nearly the complete short-run variability of the productivity of capital can be attributed to changes in utilization. This picture is less pronounced for the productivity of labour, however, even in this case, changes in utilization account to a considerable extend to the short-run variability of the productivity.

Figure 2: The utilization of labour and capital

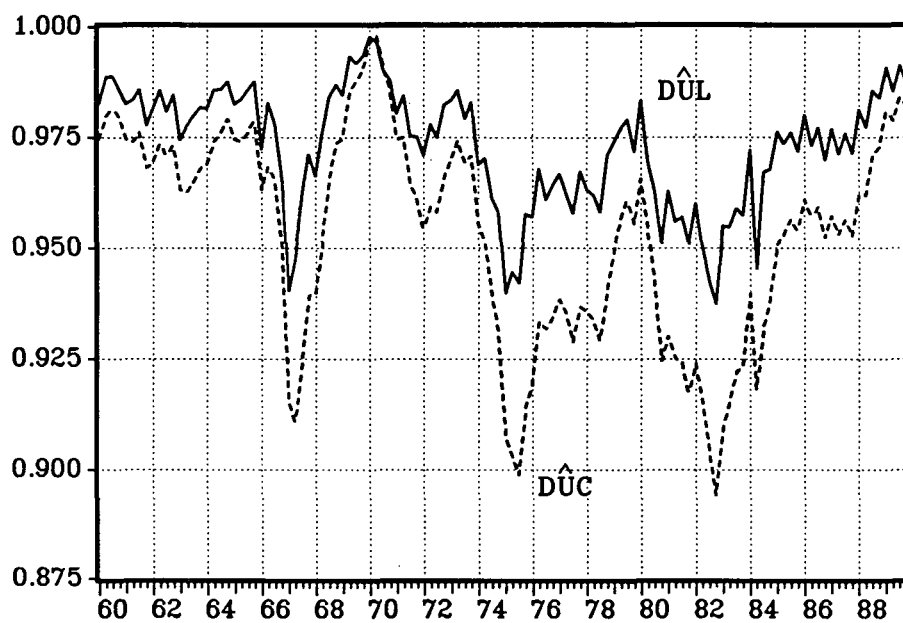
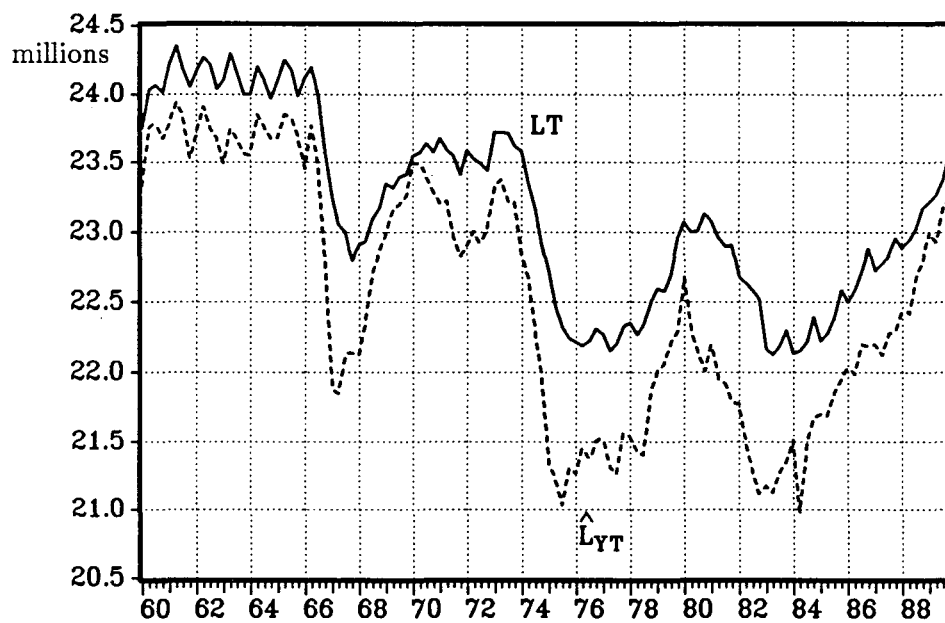


Figure 3: Labour hoarding



The most important hypothesis which is tested with these equations is the significance of a spillover from domestic supply constraints on the trade flows. In the presence of these constraints, imports should be higher than in a corresponding situation without constraints. Domestic firms and households switch to the foreign markets to bypass the constraint. A similar argument can be applied to exports. Supply constraints and delivery lags of German firms impede also foreign demand. The significance of these spillovers is a further indicator of price rigidities on the goods market. In a world with flexible prices, supply constraints should be reflected completely in prices. A firm that cannot deliver should always increase prices and no separate effect of quantity constraints should be found. Concerning exports, supply shortages of German firms should be reflected completely in the export price, and for imports, they should be reflected by the domestic price level. An additional spillover from quantity constraints can occur only in the presence of price rigidities. An indicator for supply constraints can be derived from the theoretical structure of the model. The share of supply constrained firms is given by:

$$\text{prob}_w(YD_i > Y_{LT_i}) = \left(\frac{Y_{LT}}{Y_T} \right)^{-\rho_y} \quad (75)$$

The relation between output transacted and output supply is estimated by the productivity equations and is equal to the degree of utilization of labour. Only those firms cannot satisfy demand which work with a fully utilized labour force, and a high degree of utilization of labour seems to be the best indicator of supply shortages on the domestic goods market. These arguments must also be applied to the foreign market. Supply constraints anywhere in the world induce a spillover on trade and thereby increase also the demand for German products. This effect is captured by a weighted average of the degrees of utilization of capacities in the most important trading partners of the FRG. Beside these considerations about supply constraints, the specification of the trade equations follows fairly standard versions with income and relative prices as the main explanatory variables. Export demand, i.e. the demand of foreigners for German products, depends on world trade and the relative prices are defined by the relation of German export prices and the weighted average of the export prices of the main competitors:

$$X = X \left(\begin{array}{cccc} \frac{p_x}{p_{wt}} & WT & q^a & D\dot{U}L \\ - & + & + & - \end{array} \right) \quad (76)$$

WT : world trade

p_x : export price

p_{wt} : price of world trade

q^a : world utilization

The import equation is derived in an analogous way and is specified for imports excluding raw materials and intermediates. The German demand for foreign products depends on income, relative prices, and again the rationing indicators. The relative price variable is defined by the relation between the import price and the output deflator, and the same rationing indicators are used as in the case of exports. A supply constraint on the domestic market is expected to increase import demand, while German demand for foreign products may be constrained on the foreign markets.

This is especially important, because it introduces the possibility of simultaneous constraints on the domestic as well as on the foreign markets, and therefore that German firms cannot realize their investment demand. Taken together, the import equation can be written as

$$M = M \left(\begin{matrix} Y, & \frac{p_m}{p}, & q^a, & D\dot{U}L \\ + & - & - & + \end{matrix} \right) \quad (77)$$

p_m : price of imports

For both trade equations, i.e. exports as well as imports, a dynamic adjustment with respect to changes in the exogenous variables is plausible. The export and import equation are both estimated by an error correction model to allow for these effects.

The results are contained in table 3. The long-run elasticity of imports with respect to domestic income is about 2, accounting for the increasing share of imports in income, and the short-run impact of a change in income is considerably below this value. Both coefficients are significantly different from zero, and especially the long-run coefficient is very stable regardless of the specification. Surprisingly, the short-run impact of a change in the relative prices is about equal to the long-run effect, which implies a very quick adjustment. The error correction coefficient is significantly different from zero which indicates the existence of a long-run solution.

The more important results for the theoretical model concern the effects from the rationing indicators. The domestic rationing indicator is significant in the short run as well as in the long run. The estimated influence from foreign supply constraints on German imports is less pronounced, however, the long-run effect from this variable is significant. Therefore, the hypothesis of excess demand which is not immediately removed by price increases is confirmed. The results also indicate the presence of effective rationing barriers for the demand of firms, households, and the government. The German demand for foreign products competes with the demand of other countries on the world market. The negative coefficient estimated for the foreign degree of utilization implies lower actual imports in case of high utilization in the trading partner countries of the FRG. In addition, the short-run impact of domestic rationing on imports is lower than the long-run effect. This implies that an excess demand on the domestic market spills over to imports, for instance, firms try to bypass supply constraints by changing to the foreign market. However, the change to the foreign market requires time, and in the short run, firms cannot realize their investment demand. This must be taken into account for the calculation of the excess demand on the domestic market. The results imply that the excess demand is higher than the actual spillover to the foreign market; it must be calculated from the completed adjustment, i.e. from the long-run coefficient of the domestic rationing variable.

Before turning to these calculations, the results of the estimation of the export equation are reported. Here the opposite effects for the rationing indicators are expected. An excess demand on the world markets should increase the demand for German products, while supply constraints on the domestic market should hinder exports. It is started with the results of the "standard" determinants of exports. The estimated coefficients imply a slow adjustment of German exports with respect to world trade, and the long-run elasticity is very well determined and is not significantly different from one. The long-run elasticity of exports with respect to relative

Table 3: Trade**Imports:**

$$\begin{aligned} \Delta \ln M = & \frac{0.416}{(0.13)} \cdot \Delta \ln YT_t - \frac{0.834}{(0.16)} \cdot \Delta \ln \left(\frac{p^m}{p} \right)_t + \frac{1.452}{(0.42)} \Delta \ln \hat{DUL}_t - \frac{0.006}{(0.01)} \cdot \Delta q_t^a \\ & - \frac{0.248}{(0.08)} \cdot \Delta \ln M_{t-1} + \frac{0.354}{(0.19)} \cdot \Delta \ln \left(\frac{p^m}{p} \right)_{t-1} - \frac{0.872}{(0.42)} \cdot \Delta \ln \hat{DUL}_{t-1} \\ & - \frac{0.322}{(0.06)} \left(\ln M_{t-1} - \frac{1.986}{(0.12)} \cdot \ln YT_{t-1} + \frac{0.784}{(0.29)} \cdot \ln \left(\frac{p^m}{p} \right)_{t-1} - \frac{4.049}{(1.4)} \cdot \ln \hat{DUL}_{t-1} + \frac{0.083}{(0.02)} \cdot q_{t-1}^a \right) \end{aligned}$$

SEE: 0.0276 BP(8): 4.1 BP(12): 9.0

Sample 1962.1-1988.4.

Exports:

$$\begin{aligned} \Delta \ln X = & \frac{0.260}{(0.11)} \cdot \Delta \ln WT_t - \frac{0.315}{(0.10)} \cdot \Delta \ln \left(\frac{p^x}{p} \right)_t + \frac{0.018}{(0.008)} \cdot \Delta q_t^a \\ & - \frac{0.406}{(0.10)} \cdot (\Delta \ln x_{t-1} - 0.018^* \cdot \Delta q_{t-1}^a) - \frac{0.278}{(0.10)} \cdot (\Delta \ln x_{t-2} - 0.018^* \cdot \Delta q_{t-2}^a) \\ & - \frac{0.147}{(0.09)} \cdot (\Delta \ln x_{t-3} - 0.018^* \cdot \Delta q_{t-3}^a) \\ & - \frac{0.290}{(0.06)} \left(\ln X_{t-1} - \frac{1.004}{(0.02)} \cdot \ln WT_{t-1} + \frac{0.698}{(0.13)} \cdot \ln \left(\frac{p^x}{p} \right)_{t-1} + \frac{1.817}{(0.81)} \cdot \ln \hat{DUL}_{t-1} - 0.018^* \cdot q_{t-1}^a \right) \end{aligned}$$

SEE: 0.0260 BP(8): 2.1 BP(12): 5.7

Sample 1962.1-1987.4.

Standard errors in parentheses. All equations include a constant and seasonal dummies.

*Restricted coefficient.

prices is about 0.7, while the short-run impact is about 0.3. The error correction term is significantly different from zero and higher than for imports. The estimated coefficients of the domestic rationing indicators are lower than those obtained for imports; no short-run impact was found and the long-run coefficient is considerably lower. However, this result is in accordance with the implications of the theoretical model. In case of supply constraints on the domestic market, the spillover to imports reflects the whole effect stemming from firms' investment demand, households' consumption demand, and the governmental demand changing to the foreign market, and these demand components are considerably higher than the export demand. More surprisingly is the lower effect of the foreign rationing variable on German exports. For the same reason as above, a higher coefficient than for imports is expected. However, the respective coefficient is lower. On the other hand, this result can be affected by the high multicollinearity of foreign demand WT , and foreign rationing. In addition, the indicator of foreign rationing must be seen as a somewhat crude approximation, and the result may also be due to the specification.

These results together provide evidence for the relevance of supply constraints, unsatisfied demand, and a spillover from these constraints on international trade. They are also ingredients for the estimation of the output, employment, and invest-

ment equation. Structural imports are calculated according to⁴¹

$$\ln \hat{M}D_t = \ln M_t - 4.049 \cdot (\ln \hat{D}UL_t - \ln \hat{D}UL^{min}) \quad (78)$$

where $\hat{D}UL^{min}$ is the observed minimum of $\hat{D}UL_t$. The export demand is calculated according to

$$\hat{X}D_t = X_t(\hat{D}UL_t = \hat{D}UL^{min}) \quad (79)$$

which captures the current effect of supply constraints on exports. Finally, the demand for domestic products is calculated according to:

$$\hat{Y}D_t = YT_t + (\hat{X}D_t - X_t) + (M_t - \hat{M}D_t) \quad (80)$$

3.4 Estimation of the minimum conditions

3.4.1 Determination of output

The discussion of the empirical specification of the output equation can be brief. In the model, output is determined by the minimum of supply and demand on the micro-markets, and aggregation yields the CES-transaction function. Output supply Y_{LT} is given by the employment constraint of the short-run limitational production function, and aggregate demand can be derived from the spillovers in the trade equations. No additional dynamics should be necessary for the explanation of the transacted quantity on the goods market.

The output equation is estimated by nonlinear least squares, and the results are contained in [table 4](#). From the theoretical model it is expected that the error term enters the equation multiplicatively,⁴² therefore, the estimation of the output equation is carried out in logs. The mismatch parameter is reported in terms of $1/\rho_y$ and specified by a linear and quadratic time trend.⁴³ The results revealed for the mismatch imply a structural rate, i.e. a rate of excess demand and supply at $YD = Y_{LT}$, at the beginning of the observation period of about 2 percent which increases to about 3.5 percent in 1989.⁴⁴ A troublesome results concerns the Box-Pierce statistic which tests for autocorrelation of the residuals. The estimate reveals a high and significant autocorrelation. Autocorrelation often indicates a misspecification of the equation. There is some structure contained in the residuals which is not explained by the model. In fact, there are some possible sources for a wrong specification of the output equation. Only a crude specification for the mismatch on the goods market

⁴¹The minimum level of excess demand is supplied by the assumption that there is no excess demand in the period of the lowest value of the rationing indicator. Similarly to the case of the utilizations of labour and capital, only the relative effects of supply constraints on imports and exports can be determined from these equations. It can be stated, for instance, that the spillover in 1979 exceeds the spillover in 1975 by a certain amount. However, it cannot be said, whether the spillover in 1975.1, this is the period with the lowest value of the excess demand, is zero, or whether there is some excess demand even in this period.

⁴²The derivation of the distribution of firms is built upon the law of proportional effects, and this property should also be applied for the specification of the error term.

⁴³This parameter depends on the uncertainty of demand at the time of the employment decision, adjustment barriers on employment, the correlation between labour supply and goods demand constraints, but also on price rigidities on the goods market. No convincing indicator was found to capture these effects.

⁴⁴The structural rate can be calculated approximatively as: $sr \approx (2/3) \cdot (1/\rho_y)$. See Smolny (1991).

Table 4: Output

$YT = \left\{ \hat{Y}_{LT}^{-\rho_y} + \hat{YD}^{-\rho_y} \right\}^{-1/\rho_y}, \quad 1/\rho_y = \begin{matrix} 0.029 & + & 0.0005 \cdot t & + & -4 \cdot 10^{-6} \cdot \frac{t^2}{2} \\ (0.003) & & (0.0001) & & (2 \cdot 10^{-6}) \end{matrix}$		
AR(1): 0.474 (0.09)	AR(2): 0.411 (0.09)	AR(3): -0.200 (0.09)
SEE: 0.0023 BP(8): 21.6		
Sample 1960.1–1989.4. Standard errors in parentheses. The estimation is carried out in logs. The equations include seasonal dummies, but no constant.		
AR(n): coefficient of autocorrelation of order n .		

has been used. It can well be argued that the mismatch is affected by the economic situation, with more mismatch in recession periods than in boom periods due to a more easy adjustment of employment in boom periods. On the other hand, it can be argued that these aspects are not very important for the determination of output. The output equation explains most of the variance of output and the remaining errors are very low. Therefore, it is proceeded with a simple correction for autocorrelation by the estimation of an autoregressive process for the residuals.⁴⁵

In figure 4, the results for the regime shares are depicted. They are calculated according to:

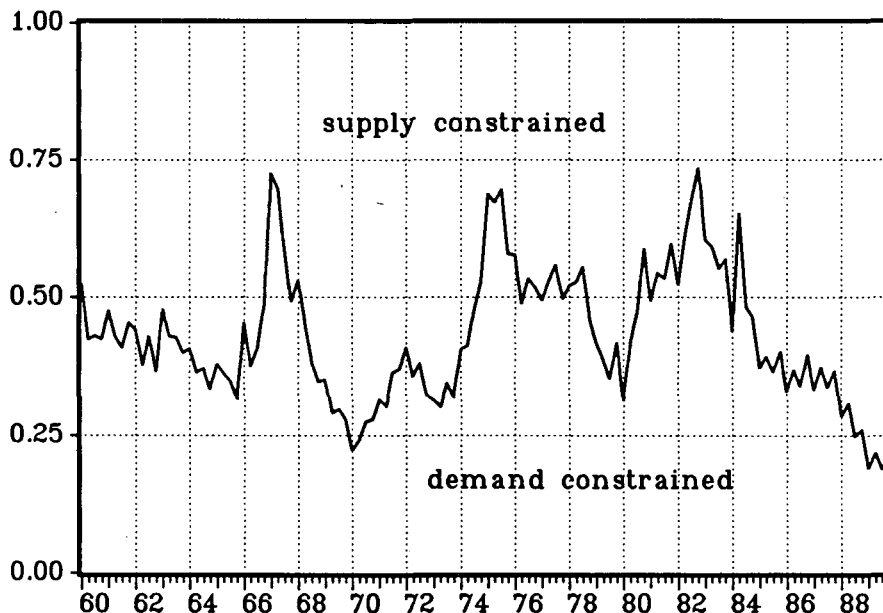
$$\hat{\text{prob}}_w(YD_i < YS_i) = \left(\frac{\hat{YD}}{\hat{YT}} \right)^{-\rho_y} \quad \text{demand constrained} \quad (81)$$

$$\hat{\text{prob}}_w(YS_i < YD_i) = \left(\frac{\hat{YS}}{\hat{YT}} \right)^{-\rho_y} \quad \text{supply constrained} \quad (82)$$

It can be seen that in the recession periods 1966/67, 1974/75, and 1982 about 70 percent of firms are constrained by insufficient demand, while in boom periods this share amounts to less than half of this value. On the other hand, if the argument is reversed, even in the recession periods 30 percent of the firms work with a fully utilized labour force, and in the boom periods, this share did not increase above 80 percent. This result can be seen as evidence for a rather quick adjustment of employment with respect to demand changes. In the medium run, the firm can realize the optimal probability of demand constraints on the goods market by employment adjustment. The result obtained for the average regime probability is also in accordance with the implications of the theoretical model. In the model, the optimal probability of the supply constrained regime is mainly determined by the share of labour costs in value added, and the estimated average regime share corresponds about to this value.

⁴⁵Some autocorrelation remains even after introducing the *AR*-terms into the equation. It was tried to remove this autocorrelation by introducing additional *AR*-terms, however, the equations became unstable.

Figure 4: Regimes on the goods market



3.4.2 Labour demand and employment

The basic model of employment determination is concluded by the minimum condition for employment. Employment is determined either by labour supply or by labour demand, with labour demand determined either by the available number of working places or by “demand determined” employment L^* . Capital-labour substitution is assumed to be possible only along with capital investment, therefore the only short- or medium-run influence from wages on employment takes place via the share of labour costs and L^* . Two mechanisms of dynamic adjustment of labour demand (and supply) with respect to equilibrium values are tested. The first corresponds to equations 49 and 50 and implies a lower and upper constraint on the adjustment speed of actual employment with respect to the target level. Alternatively, a more standard specification of the dynamic adjustment of employment is tested, which relies on nonlinear adjustment costs instead of adjustment constraints. The adjustment path implied by nonlinear adjustment costs can be approximated by a partial adjustment mechanism. In every period, the labour demand adjusts by a certain percentage in the direction of the target employment level:

$$LD_t = \lambda \cdot LD_t^* + (1 - \lambda) \cdot LT_{t-1} \quad (83)$$

Actual employment is again determined by supply and demand, and the target level of employment is determined by the available number of working places and expected demand. This combination of minimum conditions and a partial adjustment mechanism seems to be another promising root for the specification of dynamic adjustment processes as compared with a sole specification in terms of minimum and maximum conditions. Compared with a standard partial adjustment mechanism for employment, it has the advantage that the restriction $LT \leq LS$ is implied. On the other

hand, compared with a sole specification in terms of constraints, it has the advantage of less nonlinearities, and fewer coefficients must be estimated. The estimation of the productivity and trade equations allows to calculate all series required for the estimation of the employment equation:

$$\begin{aligned} \text{capacity employment} \quad \hat{L}_{YC} &= \left(\frac{\hat{L}}{\hat{Y}}\right) \cdot \left(\frac{\hat{Y}}{\hat{K}}\right) \cdot K \\ \text{demand determined employment} \quad \hat{L}_{YD} &= \left(\frac{\hat{L}}{\hat{Y}}\right) \cdot \hat{YD} \\ \text{labour supply} \quad \bar{L}S &= IT + U \end{aligned}$$

These series are seen as one of the most important outcomes of the approach, therefore they will be discussed in some detail. In figures 5 and 6 they are depicted together with actual employment IT . It is started with some considerations about the labour supply. The labour supply was treated as an exogenous variable for the derivation of the basic model, but the endogeneity of the labour supply is taken into account for the derivation of the dynamic adjustment of employment. From figure 5, it can be seen that the labour supply is an endogenous variable also on the aggregate level. The labour supply decreased during recessions and increased during boom periods. There are two factors accounting for this:⁴⁶ first, the dependence of international factor mobility on the employment situation in the FRG and second, the inverse relation between the unemployment rate and the participation rates of workers in Germany. From the figure one can also see the distinct development of unemployment. There was virtually no unemployment in the sixties until 1973, apart from the short recession in 1966/67. Then the number of unemployed people increased to about one million. Despite the partial recovery of the employment level in the late seventies, the unemployment rate remained high due to the sharply increasing labour supply. The mild decrease of the unemployment rate was terminated abruptly by the recession at the beginning of the eighties and the number of unemployed increased to more than two million people. Since then, it remained rather stable and decreased only slowly since 1986 despite the enormous increase in employment since 1983.

The most striking characteristic of demand determined employment is the high variance over the business cycle. During recession periods, it lies far beyond the employment level, while in boom periods it increases faster than employment. This gives also a hint to labour hoarding. The employment level that is necessary to produce output is always less than or equal to L_{YD} and $L_{YD} < IT$ implies labour hoarding.⁴⁷ On the other hand, during boom periods, demand determined employment increases faster than employment. These distinct developments already indicate the importance of adjustment constraints for employment. Referring solely to this figure, employment adjusts only slowly with respect to demand during the upswing and during the downswing. On the other hand, the development of capacity employment is smoother than actual employment. The recessions 1966/67 and 1974 are less pronounced in \hat{L}_{YC} than in employment. In addition, \hat{L}_{YC} lags behind employment which indicates the slower adjustment of capacities with respect to demand.

⁴⁶The endogeneity of the aggregate labour supply is not analyzed in this work. Some important determinants of the labour supply in the FRG are analyzed by Franz (1981), Franz, Smolny (1990), and Smolny (1990).

⁴⁷See also figure 3.

Figure 5: Employment series I: LT , L_{YC} , LS

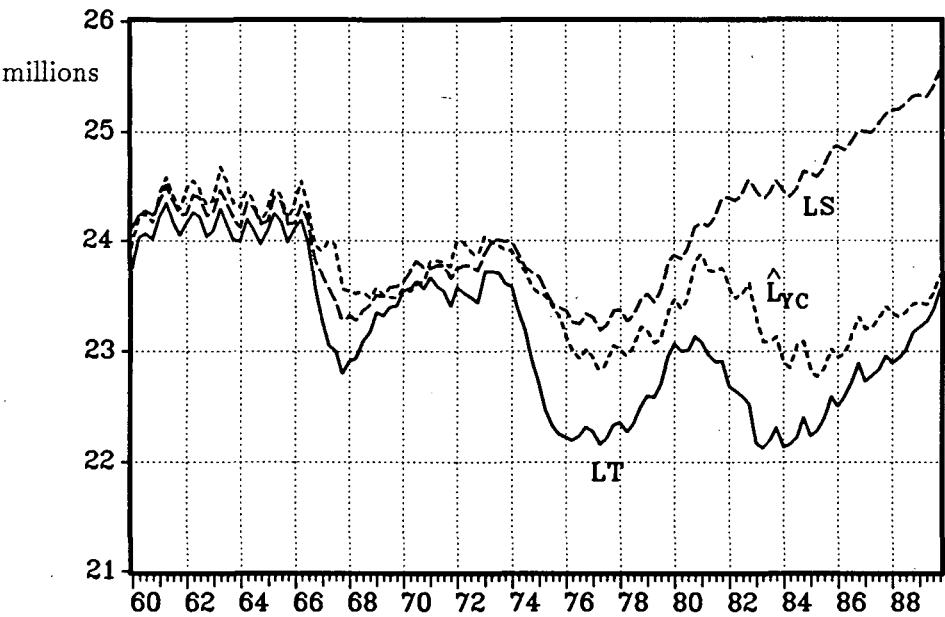
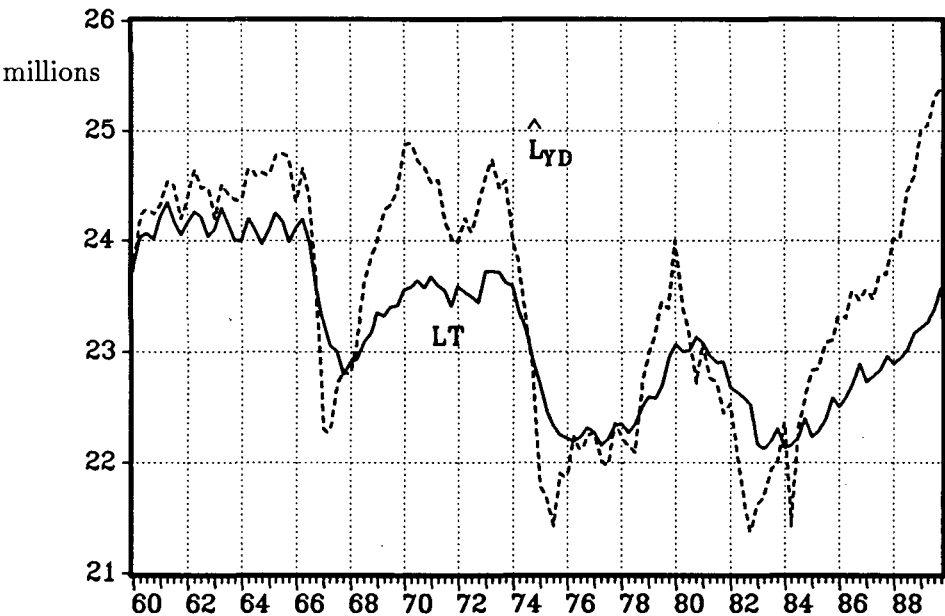


Figure 6: Employment series II: LT , L_{YD}



Taken together, these figures already draw a rather detailed picture of the economic situation in the FRG. Until 1966, an equilibrium situation can be stated. The labour supply was slightly below capacity employment, goods demand about equals capacities, and the unemployment rate stays at about one percent. In addition, employment and the degrees of utilization of labour and capital remain fairly stable.⁴⁸ This picture changes sharply with the recession in 1966. Demand determined employment decreased and the unemployment figures increased to above 500,000 people, despite the remigration of many guest workers. Capacities adjusted downward and in the first quarter in 1967, gross investment was nearly 20 percent below the corresponding level of the preceding year. However, the recession was only short-termed and demand increased again until 1970, brought about mainly by higher exports caused by the undervaluation of the Deutsche Mark and high growth rates of the world economy. The labour supply and capacities adjusted only slowly, and in 1970, the shortages of capital and labour supply are the main factors restraining a higher growth rate of the economy. The following slowdown of demand in 1971 had hardly consequences for employment and investment, and the economy boomed when the first oil price shock hit the German economy. In addition, high inflation rates at the beginning of the seventies, caused by the enormous wage push in 1970, and increases in the monetary growth in course of the breakdown of the Bretton-Woods exchange rate system induced the Deutsche Bundesbank to switch to a restrictive policy, and short-run interest rates exceeded 14 percent. These high interest rates reduced investment and consumption demand, and exports declined in consequence of the slowdown of world demand. In 1975, the unemployment figure exceeded one million and the utilizations of labour and capital decreased to very low levels. The partial recovery since then was terminated with the second oil price shock. Again high inflation rates induced a restrictive monetary policy.⁴⁹ Between 1979 and 1981, the money supply remained below the minimum of the target set by the Deutsche Bundesbank and interest rates were high. In consequence, investment and consumption decreased in real terms. Furthermore, the fiscal authorities changed to a restrictive course and in 1983, the unemployment figure exceeded two millions. Since then, the economy switched on a path of sustained growth and the figures indicate that a higher employment growth at the end of the eighties is mainly impeded by the slow adjustment of capacities.

In table 5, the estimation results of the employment equation are reported. In the first rows, the dynamic adjustment of employment is specified solely in terms of minimum and maximum conditions. In the last rows, the CES-function of the basic model is augmented by a partial adjustment mechanism as depicted in table 5. It is started with some general results.

Only one mismatch parameter could be determined, any attempts to estimate different mismatch parameters for the different minimum and maximum conditions result in ρ -parameters not significantly different from each other. Usually, the standard errors were higher when allowing for different mismatch parameters and sometimes, the equations did not converge. In addition, some versions allowed for trends in the mismatch parameter and it was tried to capture "market mismatch" by indica-

⁴⁸ See also figure 2.

⁴⁹ Another reason for the restrictive monetary policy was a deficit in the trade balance and a devaluation of the Deutsche Mark.

Table 5: Employment

$LT = \{LS_t^{-\rho} + \hat{LD}_t^{-\rho}\}^{-1/\rho}$		
CES		
$LS_t = \{\overline{LS}_t^{-\rho} + [(1 + \frac{0.0072}{(0.002)}) \cdot LT_{t-1}]^{-\rho}\}^{-1/\rho}$		
$\hat{LD}_t = \{\hat{LD}_t^{*\rho} + [(1 - \frac{0.0037}{(0.002)}) \cdot LT_{t-1}]^{\rho}\}^{1/\rho}$		
$\hat{LD}_t^* = \{(\alpha_1 \cdot \hat{L}_{YD_t})^{-\rho} + (\alpha_2 \cdot \hat{L}_{YC_t})^{-\rho}\}^{-1/\rho}$		
$1/\rho = \frac{0.0136}{(0.002)}, \quad \alpha_1 = \alpha_2 = \frac{1.020}{(0.004)}$		
AR(4): $\frac{0.602}{(0.07)}$	SEE: 0.0034	BP(8): 28.1
Partial adjustment		
$LS_t = \overline{LS}_t$		
$\hat{LD}_t = \frac{0.190}{(0.019)} \cdot \hat{LD}_t^* + (1 - \frac{0.190}{(0.019)}) \cdot LT_{t-1}$		
$\hat{LD}_t^* = \{\hat{L}_{YD_t}^{-\rho} + \hat{L}_{YC_t}^{-\rho}\}^{-1/\rho}$		
$1/\rho = \frac{0.0097}{(0.002)}$		
AR(4): $\frac{0.630}{(0.07)}$	SEE: 0.0032	BP(8): 33.2
Data sample 1960.1 1989.4. The estimation sample is shortend to allow for lags. Standard errors in parentheses. The estimation is carried out in logs. All equations include seasonal dummies, but no constant.		

tors like the share of long-term unemployed, but no significant results were obtained. Better results were obtained by a unique and constant mismatch parameter. This result stands in some contrast to former estimates obtained from the estimation of the CES-function for the FRG,⁵⁰ however, it should be noted that one very important source of mismatch, i.e. adjustment constraints for employment, is taken explicitly into account here.⁵¹ The variability of employment is higher in the seventies and eighties than in the sixties, therefore adjustment constraints are more important in this period.⁵² Second, no significant effect from the share of wage costs in value added on L^* was found. Only small effects are expected, therefore, the result is not surprising and it was proceeded without this variable. Third, the dynamic model

⁵⁰See e.g. Entorf, Franz, König, Smolny (1990), Franz, Heidbrink, Scheremet (1990), and Franz, Smolny (1990).

⁵¹The result stands also in contrast to the estimates obtained by Smolny (1990), which allowed also for a dynamic adjustment. However, in this study, annual data are used, therefore the results cannot be compared directly.

⁵²The result is in accordance with those obtained by Franz (1989b), where different sources of rigidities on the labour market are analyzed. Most indicators of rigidities remained fairly stable in the seventies and eighties.

of employment adjustment implies a dependence of the current labour demand on expected future employment changes. In the presence of adjustment constraints, the actual labour demand depends positively on expected employment changes. This was modelled by introducing ${}_tE(\Delta IT_{t+1})$ into the specification of the target level of employment LD_t^* . However, this term did not contribute significantly to the explanation of employment. Fourth, demand determined employment L_t^* depends on the expected demand for goods, with expectations formed at time $t - \tau_L$. It was tried to estimate this lag within the employment equation by the same approach as applied for the estimation of the productivity equation. However, no satisfactory results were obtained. In general, the results did not change very much with regard to the expectation variable used. This result may indicate that some adjustment of employment takes place already within the quarter, but may also indicate the difficulties of modelling short-term expectations. It does *not* indicate that adjustment constraints of employment are of minor importance. This can be seen from the results contained in table 5.

In the first version, the CES-specification of the dynamic adjustment is employed. The results for the adjustment coefficients are somewhat ambiguous. The estimated coefficient for the upward adjustment δ^h is about 0.007 which implies that the firms, on average, cannot increase their labour force by more than 0.7 percent per quarter. The coefficient is rather stable in the different specifications tested and seems to be plausible. It can be seen from figure 5 and 6 that the maximal adjustment speed of the upward adjustment is about in this dimension. However, the estimated coefficient δ^s implies that the downward adjustment is impeded much more. The maximal downward adjustment is estimated with only 0.3 percent per quarter. In addition, the coefficient is not very well determined and changed in different versions; the estimate is seen as not very reliable.

It is also tested for correcting constants for the maximum utilizations of labour and capital, respectively, and the minimum excess demand. For instance, $\alpha_1 > 1$ indicates positive excess demand at the minimum level of DUL . Similar for the second parameter, $\alpha_2 > 1$ indicates $DUC^{max} < 1$. The correcting constants are significant, but do not contribute very much to the explanation. The approximation of the minimum and maximum levels of DUL and DUC by the observed minima and maxima may be rather good. In this case, the estimation of the correcting constants is not necessary. However, this concerns only the question, whether the coefficients are different from one. These coefficients also test, whether the corresponding variable contributes *at all* to the explanation of employment. For instance, $\alpha_1 = 0$ would indicate that demand plays no role for the determination of employment. A similar argument holds for the adjustment parameters δ^s and δ^h . If the downward (upward) adjustment is not constrained at all, this should result in an estimate $\delta^s = 1$ ($\delta^h = \infty$). The results indicate that all variables contribute very significantly to the explanation of employment. Demand as well as capacities are important, and the adjustment of employment is impeded upwards as well as downwards.

Table 5 reports also the results obtained from a partial adjustment specification of the labour demand.⁵³ The results achieved with this combination of minimum conditions and the usual partial adjustment mechanism are very encouraging. Only

⁵³ A similar combination of the CES-function and the partial adjustment mechanism was used by Sneessens, Drèze (1986).

two coefficients are estimated and the standard error of the equation is below 0.5 percent.⁵⁴ The correcting coefficients α_1 and α_2 are not significantly different from one, therefore they are restricted for the reported versions.⁵⁵ The estimated dynamic adjustment is similar to those obtained from the pure CES-specification, the adjustment coefficient λ is about 0.2.⁵⁶

What are the implications of these results for the assumptions applied for the derivation of the model of the firm? First, no significant effects from wages on the short-run employment decision were found. From the theoretical model only a low influence is expected, therefore this result is not in very much contrast to the model. Real wage costs enter the employment determination mainly in the long run via capital-labour substitution and via capital formation. Second, the dynamic adjustment of employment is very pronounced. Allowing for a dynamic adjustment yielded a remarkable better explanation of actual employment as compared with a static equation and the estimated adjustment is rather slow. The partial adjustment model yielded an estimate of 20 percent adjustment per quarter, while the CES-approach yielded the result that the maximum adjustment per quarter is below one percent. This is in accordance with the model and can explain the considerable amount of labour hoarding during recession periods. Third, two different approaches for the modelling of the dynamic adjustment were tested: a constrained adjustment, and a partial adjustment mechanism for the labour demand. The results indicate that it is very difficult to discriminate between these approaches solely from aggregate data.⁵⁷ Both approaches yield a close approximation of the dynamic adjustment of aggregate employment.

One advantage of the CES-approach of employment determination is that it allows to calculate the share of firms facing the different constraints for employment determination. These shares are equal to the elasticities of aggregate employment with respect to the aggregate value of the respective constraint and can serve an important function for the evaluation of policy instruments; only those firms will increase employment in case of an increase in demand, which are in the demand constrained regime. Therefore, an expansionary demand policy affects only those firms' employment. The regime shares are calculated from the second employment equation in table 5, and are depicted in figure 7. The share of labour supply constrained firms can be calculated from:

$$\hat{\text{prob}}_w(LS_i < LD_i) = \left(\frac{LS}{LT} \right)^{-\rho} \quad (84)$$

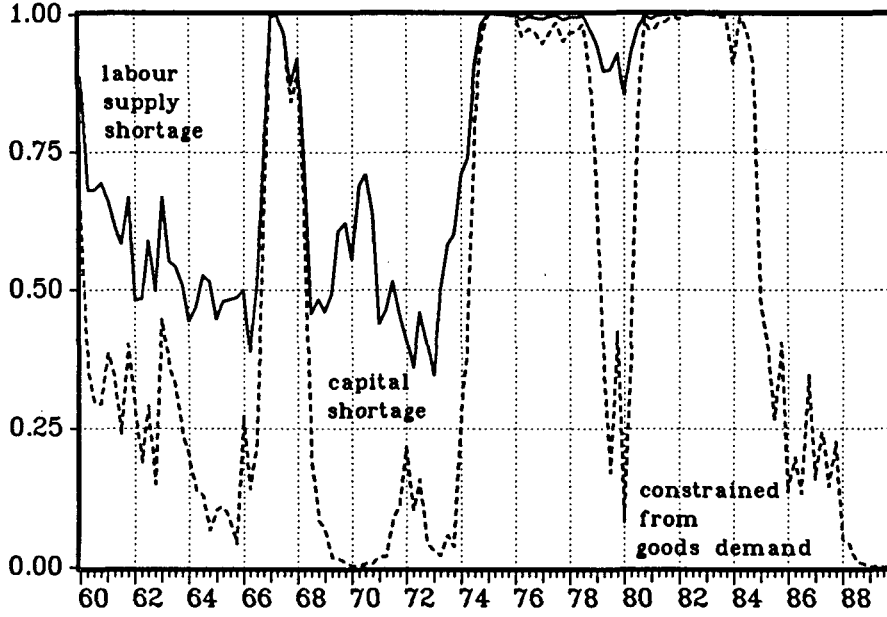
⁵⁴ In addition, three coefficients of the seasonal dummies are estimated.

⁵⁵ All equations reveal significant autocorrelation of fourth order, which was corrected by an autoregressive process in the preferred versions. Some autocorrelation remains which may be due to measurement errors of utilization and to errors in the specification of mismatch.

⁵⁶ This result is in accordance with those obtained from, for instance, Flaig, Steiner (1989) with an error correction model of employment adjustment. However, these results are only partly comparable. In their model, employment is determined by output and relative factor prices, while in the model here, the causation is reversed: output is determined by employment and demand, while employment is determined by the labour supply, goods demand, capacities, and the productivity of labour. A slightly faster adjustment is obtained from Franz, König (1986) for employment measured in hours.

⁵⁷ A similar result was obtained by Hamermesh (1989, 1990). In this study, it could not be distinguished between convex and concave adjustment costs solely from aggregate data. Both approaches lead to a similar partial adjustment of aggregate employment.

Figure 7: Regimes on the labour market



The dynamic adjustment of employment causes some difficulties for the calculation and the interpretation of the shares of the capacity and demand constrained firms. In the static model, those firms that are not constrained by the labour supply choose employment either according to capacities, L_{YC_i} , or according to goods demand, L_{YD_i} . This does not hold for the dynamic model of employment adjustment. Here, actual employment of those firms can be higher or lower than the minimum of L_{YD_i} and L_{YC_i} , depending on whether the past employment level was higher or lower than the minimum level LD_i^* .⁵⁸ The firms are *adjusting towards* LD_i^* . However, one can distinguish between those firms adjusting towards L_{YD_i} and those firms adjusting towards L_{YC_i} . These regime shares can be calculated according to:

$$\hat{\text{prob}}_w(LD_i < LS_i \wedge L_{YC_i} < L_{YD_i}) = \left[1 - \left(\frac{LS}{LT} \right)^{-\rho} \right] \cdot \left(\frac{\hat{L}_{YC}}{\hat{LD}^*} \right)^{-\rho} \quad (85)$$

$$\hat{\text{prob}}_w(LD_i < LS_i \wedge L_{YD_i} < L_{YC_i}) = \left[1 - \left(\frac{LS}{LT} \right)^{-\rho} \right] \cdot \left(\frac{\hat{L}_{YD}}{\hat{LD}^*} \right)^{-\rho} \quad (86)$$

This is a rather different interpretation of the regime shares as compared with the static model. In general, these regime shares provide a similar picture of the situation prevailing on the labour market as the employment series; the regime shares are calculated from these series. However, the regime shares allow for a closer look at the relative importance of the regimes. During the sixties until 1973, the labour supply was the most important binding constraint for employment. On average, nearly 50 percent of firms are constrained by an insufficient labour supply. This was

⁵⁸ Of course, employment cannot exceed the labour supply.

interrupted only by the short recession in 1966/67 and by the capital shortage in 1970. The picture changed dramatically with the first oil price shock in 1973/74. Since then, the labour supply was never again an important constraint during the observation period. This gives also a hint to wage rigidities. Until 1973, the labour supply was an important constraint which was not removed by wage increases, while in the second half of the seventies, the high unemployment did not cause sufficient wage decreases.

In the second half of the seventies, the demand for goods was the most important constraint for employment. At this stage, another comment concerning the dynamic adjustment of employment is necessary. It has been already noted above that the dynamic adjustment is the more important, the more rapid are the changes in the desired employment level. The *pure* CES-approach of employment adjustment allows a more detailed look at the importance of these constraints. In general, the regimes calculated from those equations draw a very similar picture of the situation on the labour market than those reported. However, two further regimes can be distinguished. From these versions, one can calculate the shares of firms that are constrained in their upward adjustment of employment, and the share of firms that are constrained in their downward adjustment. This is very important for the interpretation of the situation prevailing in 1979 and in the second half of the eighties. Figure 7 gives the impression that in 1979 the capital stock was the binding constraint for employment for most firms. On the other hand, the degree of utilization of capital was lower in 1979 than in the sixties, and far beyond the maximum values observed in 1970.⁵⁹ This inconsistency can be removed when looking at the exact definition of the regimes. "Capital constrained" implies that $L_{YC_i} < L_{YD_i}$, LS_i . It does *not* imply that the capital stock is really the actual binding constraint for employment. This difference is very important for the interpretation of the situation prevailing 1979 and 1985 until 1988. During these periods, employment was increasing very fast but stayed always rather much below capacity employment \hat{L}_{YC} . Capacity utilization was not very high, actual employment was mainly constrained by the *past employment level*: a large share of firms was increasing employment with maximal speed. The CES-approach of employment adjustment allows to calculate this share, and while it was not very high for most of the observation period, in 1979 and since 1985 it has been dominant.⁶⁰ This underlines the importance of the dynamic adjustment and should be taken into account for the interpretation of the regime shares. In 1979 and in 1985–1987, capital was not really the binding constraint for employment, but the "capacities" of hiring and training entrants or the number of applicants per period. In those years, employment was neither constrained very much by the capital stock, nor by goods demand, and nor by the labour supply, but a main reason for the persistence of unemployment was the constraint on employment growth.

⁵⁹ See figure 2.

⁶⁰ Similar, the downward adjustment regime was not very high for most of the observation period, but was dominant in 1966, 1974, and 1982. However, this result is not very clear due to the poor determination of the respective coefficient.

3.5 The demand for capital

"Time-series properties of investment, output and the cost of capital do not appear to be consistent with well-established theories of investment. The best predictor of investment is found to be its own past history." This somewhat resignative fact is drawn from a recent comprehensive study of investment behaviour of the OECD.⁶¹ This assessment of the contribution of economic theory to the explanation of investment behaviour is probably too pessimistic. However, the quotation illustrates two important problems associated with empirical investment functions. The first is connected with the difficulty to find a stable empirical relation between actual investment expenditures and the determinants expected from theoretical models. Second, it emphasizes the importance of a careful modelling of the dynamic adjustment, which is implicit in the statement that current investment is best predicted by past investment.

Before turning to the estimation results of the investment function, the relation of the model here and other models of investment behaviour is discussed. In most models of investment behaviour, the optimal capital stock is affected by capital-labour substitution and therefore, some kind of relative price variable. Second, it depends on an activity variable, which is interpreted as "demand" in Keynesian models, and as the optimal output level in neoclassical models. Third, profits are introduced into the decision of the optimal capital stock by allowing for credit market failures, which may drive a wedge between market interest rates and internal interest rates, or which may place a bound on the external borrowing. In addition, empirical investment models allow for a slow adjustment of the capital stock with respect to optimal values. These aspects are also important in the model here. However, the differences of the model here are worth to be noted. The substitution effect does not differ very much from most models, therefore it must not be discussed. An important difference is introduced by the activity variable Y^a , which is derived in the model here. While it depends on demand, and therefore has a similarity to traditional Keynesian models, it is also affected by labour supply constraints. Especially for the situation in Germany in the sixties and at the beginning of the seventies, one can argue very well that the availability of sufficient labour was an important determinant of the optimal capital stock. If the capital-labour ratio can be adjusted only very slowly, and the results of the estimation of the productivity equation confirm this assumption, the optimal capital stock is bounded by the labour supply. On the other hand, there is also an important difference to neoclassical models. In neoclassical models, output summarizes the optimal choice of the firm and is therefore an endogenous variable. It should be replaced by the corresponding exogenous variables for the estimation of the investment equation. Output is also an endogenous variable for the firm in the model here. It is determined by supply and demand on the goods market, while supply, in turn, depends on capacities. Therefore it is replaced by goods demand and labour supply constrained output, which do not depend on the capital stock at the firm level. Finally, the profitability variable has a rather different interpretation in the model here. It has nothing to do with financial constraints, rather it has been assumed that the firm can finance investment at the current interest rate on the money market. Profitability affects the optimal capital

⁶¹ Ford, Poret (1990), p. ii.

stock of the firm via the optimal probability of being capacity constrained in its output and employment decision. Therefore, profitability is the main determinant of the optimal degree of the utilization of capital.

The results of the estimation of the investment equations are contained in table 6. The variable Y^a is calculated from the respective CES-function depending on demand and the labour supply constrained output level. However, the relevant ρ -parameter could not be determined from the estimation results of the output and employment equation. Therefore, another CES-function was estimated which relates output to capacities, demand, and labour supply constrained output. The resulting ρ -parameter was used for the calculation of Y^a . It was also tried to estimate the parameter within the investment function, but no reliable results were obtained. The same procedure was applied for the calculation of α contained in the profitability indicator fsk . For the exogenous variables Y^a and fsk , expectations have been calculated by the procedure outlined above for the productivity equations. The variables were estimated on a lagged information set, and the fitted values of these equations are used as the expected values of the respective variables, with expectations formed with those lag. It has been mentioned that probably all of these expected values play a role for the investment decision of the firms. Different investment projects are carried out with a different delay, and by this procedure, only the relative importance of these delays can be determined. The results reveal a relative minimum of the standard errors at a rather short lag and another minimum at a lag of about two years. These delays correspond very close to the expected delays in capital formation. While some investment projects can be started without very much planning delays, others can be carried out only with considerable delays. However, they indicate also the loss of information incurred by aggregation. While perhaps from disaggregated data a more distinct result can be obtained, the analysis of aggregate investment data allows only an estimation of the most important lags. Therefore, aggregate investment was separated into structures and equipment, and the same procedure was applied. Of course, this is only a small disaggregation, but the results indicate another problem. The estimated delays are very similar for structures and equipment, which indicates the simultaneity of investment decisions. This can be seen from the estimation results of the investment equation. Net investment is estimated on the most important expectation variables, the lagged level, and past changes of the capital stock.⁶²

Very significant effects from the activity variable on investment in equipment were found, and an about equal weight of short-term and long-term expectations is revealed. The effect of profitability on the optimal capital stock is not very well determined, but the estimated coefficient in the equation explaining investments in equipment is about in the range expected from the theoretical model. The coefficient can be interpreted as the average expectation error of Y^a . The corresponding coefficient is not significant in the equation explaining investments in structures. The poor determination of the coefficient may be attributed to the difficulty of measuring profitability. The most striking result is the very slow adjustment of the capital stock

⁶²It should be noted that no significant effect from capital-labour substitution on investment was found, therefore the development of capital productivity was approximated by a time trend. Capital productivity was introduced by the corresponding relative price variable and alternatively, by the fitted values of the productivity equation. Capital-labour substitution probably affects the capital stock with even longer lags.

Table 6: Investment

Equipment:			
$\Delta \ln K_t^a$	=	$ \begin{aligned} & -0.0213 \cdot [\ln K_{t-1}^a - 0.619 \cdot_{t-2} E(\ln \hat{Y}_t^a) - 0.615 \cdot_{t-7} E(\ln \hat{Y}_t^a) \\ & \quad (0.004) \quad (0.17) \quad (0.20) \\ & - 0.088 \cdot_{t-2} E(\hat{f}sk_t) - 0.088^* \cdot_{t-7} E(\hat{f}sk_t)] + 5.4 \cdot 10^{-6} \cdot t \\ & \quad (0.05) \quad (2 \cdot 10^{-5}) \\ & + 0.462 \cdot \Delta K_{t-1}^a + 0.061 \cdot \Delta K_{t-2}^a + 0.240 \cdot \Delta K_{t-3}^a \\ & \quad (0.11) \quad (0.11) \quad (0.11) \\ & + 0.178 \cdot \Delta K_{t-4}^a - 0.267 \cdot \Delta K_{t-5}^a \\ & \quad (0.11) \quad (0.09) \end{aligned} $	
SEE ^{a)} :	0.0852	BP(8): 7.2	BP(12): 13.1
structures			
$\Delta \ln K_t^b$	=	$ \begin{aligned} & -0.010 \cdot [\ln K_{t-1}^b - 0.374 \cdot_{t-3} E(\ln \hat{Y}_t^a) - 0.370 \cdot_{t-8} E(\ln \hat{Y}_t^a) \\ & \quad (0.003) \quad (0.19) \quad (0.21) \\ & - 0.028 \cdot_{t-3} E(\hat{f}sk_t) - 0.028^* \cdot_{t-8} E(\hat{f}sk_t)] + 3.4 \cdot 10^{-5} \cdot t \\ & \quad (0.05) \quad (1.5 \cdot 10^{-5}) \\ & + 0.661 \cdot \Delta \ln K_{t-1}^b + 0.006 \cdot \Delta \ln K_{t-2}^b - 0.091 \cdot \Delta \ln K_{t-3}^b \\ & \quad (0.08) \quad (0.07) \quad (0.07) \\ & + 0.589 \cdot \Delta \ln K_{t-4}^b - 0.423 \cdot \Delta \ln K_{t-5}^b \\ & \quad (0.06) \quad (0.08) \end{aligned} $	
SEE ^{a)} :	0.0434	BP(8): 3.9	BP(12): 4.4
$\Delta \ln K_t^b$	=	$ \begin{aligned} & -0.045 \cdot (\ln K_{t-1}^b - 0.562 \cdot \ln K_{t-1}^a) + 0.129 \cdot \Delta \ln K_{t-1}^a + 0.0002 \cdot t \\ & \quad (0.02) \quad (0.02) \quad (0.03) \quad (8 \cdot 10^{-5}) \\ & + 0.444 \cdot \Delta \ln K_{t-1}^b - 0.044 \cdot \Delta \ln K_{t-2}^b - 0.091 \cdot \Delta \ln K_{t-3}^b \\ & \quad (0.09) \quad (0.07) \quad (0.06) \\ & + 0.499 \cdot \Delta \ln K_{t-4}^b - 0.399 \cdot \Delta \ln K_{t-5}^b \\ & \quad (0.06) \quad (0.07) \end{aligned} $	
SEE ^{a)} :	0.0390	BP(8): 5.9	BP(12): 6.1
Sample 1964.1-1989.4. Standard errors in parentheses.			
All equations include a constant and seasonal dummies.			
^{a)} The reported standard errors of the equations are multiplied with 100.			
* Restricted coefficient.			
$\hat{f}sk = F^{-1} \left(1 - \frac{sk^* + \hat{\alpha} \cdot sl^*}{1 - sl^*} \right)$			

with respect to optimal values. For equipment, the adjustment amounts to about two percent of the difference between actual and optimal values per quarter and for structures, the estimated adjustment coefficient is only half of this value. Investment rates adjust only very slowly with respect to Y^a , the most important determinant of investment in the short run is past investment.

What are the reasons for the extreme slow adjustment of the capital stock. It may be argued that the slow adjustment of the capital stock is caused by a slow adjustment of expectations. If the expectations about the constraints on the goods and labour market change only slowly, then investment should also change slowly. However, this cannot be the sole explanation of the slow adjustment of the capital stock. The argument must be complemented by "technical" aspects of the adjustment process. The capital stock *cannot* be adjusted in short time periods. Downward adjustments of the capital stock can nearly exclusively be carried out by depreciation, and a desired increase of capacities can be carried out only with long planning, production, and installation lags. An expansion of capacities often requires new factory buildings and

new equipment can be only installed together with new structures. This introduces an interdependency into the investment decision. While new equipment can be installed rather quickly into existing buildings, the adjustment will be much slower when buildings are not available. In this case, investment decisions affect investment expenditures over a long period, which can explain why actual investment expenditures depend on past investment expenditures. This interdependence is the subject of the last investment equation contained in table 6. In this equation, investment in structures are related to past investments in structures and equipment. The result is a better explanation than in the equation above relating investments in structures to profitability and Y^a . Therefore, it can be concluded that investments in equipment are determined by profitability, the expected constraints on the goods and labour market, and past investments. Investments in structures can be determined solely by past investments in structures and equipment without an explicit effect from Y^a or profitability.

4 Conclusion

In this paper, a disequilibrium model of firms' behaviour has been developed. Disequilibrium analysis has been practiced in both meanings of the term. A dynamic model of the firm has been worked out, which pays special attention to a delayed adjustment of employment, investment, and the production technology. Market disequilibrium is introduced by allowing for a sluggish adjustment of wages and prices with respect to disequilibrium situations. A consistent model of the short-run, medium-run, and long-run adjustment of the firm is developed, however, the adjustment of wages and prices is not introduced into the decision of the firm. There are three reasons which may justify this omission. First, the assumption of an immediate adjustment of wages and prices is restrictive and unrealistic. Second, a satisfactory treatment of a dynamic adjustment of wages and prices would have complicated the analysis beyond analytical tractability. Third, while the endogenous adjustment of wages and prices must be at the center of any theory of inflation and income distribution, it is not seen as very important for the quantity adjustment. This does not mean that wages and prices are not important, rather the modifications that are introduced by an *endogenous* price setting are seen as unessential for the quantity adjustment. In addition, in the presence of a centralized wage bargaining, the wage rate can be seen as a more or less exogenous variable for the individual firm. Therefore, the fix-price method is applied for the derivation of the model. This proceeding can be seen as an approximation which is the less appropriate, the more quick is the wage and price adjustment, and the empirical adjustment seems to be rather slow.

If prices do not clear the markets at any moment of time, than supply will usually differ from demand, and the transacted quantity is given by the minimum of both. The main objects of the work are the investigation of the dynamic adjustment of quantities and the analysis of the resulting inefficiencies and spillovers. An excess supply on the goods market, which is not immediately removed by price or quantity adjustments implies underutilization of labour and capital, an excess demand creates a spillover to the international markets, and excess supply on the labour market is unemployment. The slow adjustment of quantities increases the persistence of these disequilibria and is seen as an important component for the explanation of the

business cycle.

The dynamic model of the firm is supplemented by an explicit aggregation procedure for firms in disequilibrium situations. At any moment of time, different firms face different constraints on the goods and labour market, and from proportional dynamic adjustment processes a lognormal distribution of firms' supplies and demands can be derived. For this case, the aggregate transacted quantity can be closely approximated by an explicit functional relation depending on aggregate supply, aggregate demand, and a mismatch parameter. The mismatch parameter is determined by the correlation of supply and demand on the micro-markets. The aggregation procedure allows an easy transformation of the firm specific variables into aggregate quantities, and the model can be tested solely by using aggregate data.

The results of the estimation of the model generally confirm the assumptions applied for the derivation of the model. Significant underutilizations of labour and capital were found, which indicate a slow adjustment of employment, the capital stock, and the production technology, and indicate also a downward rigidity of prices. An upward rigidity of prices is revealed by the results of the trade equations. Supply constraints on the domestic and foreign markets contribute significantly to the explanation of the trade flows, which strengthens the case of quantity adjustments against price adjustments. In general, there are more rigidities for the quantity adjustment found than expected. For employment, it takes more than two quarters before half of the adjustment is carried out, and the adjustment of the capital stock and the capital-labour ratio is much slower. The short-run effect of relative prices on the determination of quantities is low. Relative prices affect output and employment via capital-labour substitution and via investment in the long run, a short-run effect from wages on employment is not revealed.

An important outcome of the approach are the different measures of disequilibrium on the goods and labour market. A measure of the short-run excess supply on the goods market is provided by the utilization of labour, while the medium-run supply conditions are determined by the labour supply constrained output level and capacities. On the labour market, "Keynesian" labour demand and capacity employment can be determined in addition to the labour supply. The employment series reveal the importance of demand for the medium-run determination of employment. Demand is the driving force of employment changes, and capacities adjust slowly with respect to demand. In the short run, employment growth can be limited by adjustment constraints for employment. These aspects provide a partial explanation for the persistence of high unemployment in Germany in the eighties. At the beginning of the eighties, the demand breakdown in course of the second oil price shock reduced employment. After the recovery of demand in 1984, the employment growth was mainly impeded by adjustment constraints for employment until the end of the eighties, when the slow adjustment of capacities constrained employment. A significant increase of structural unemployment in the usual static sense is not revealed by the estimates. The only kind of mismatch which has increased over the observation period are the adjustment constraints for employment that are more important in periods of rapid changes of demand than in "equilibrium" situations like the sixties.

Another measure of disequilibrium is provided by the shares of firms in the different regimes on the goods and labour market. The regimes on the goods market reflect mainly the adjustment constraints of employment and can be used as short-

run tension indicators. The labour market regimes, on the other hand, display the more fundamental medium-run and long-run constraints for employment and output. They are equal to the respective elasticities of employment with respect to supply, demand, and capacities, and can serve an important function for policy evaluation. A demand policy affects employment only in those firms, where demand is the binding constraint for employment and is ineffective in a situation of capital shortage. In this case, employment policy should focus on investment and capital-labour substitution. The regime shares provide the composition of constraints and can increase the efficiency of policy measures.

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