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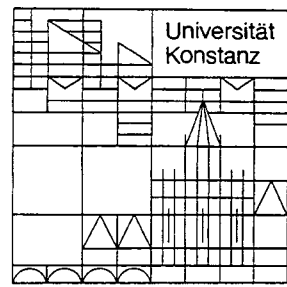
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Diskussionsbeiträge



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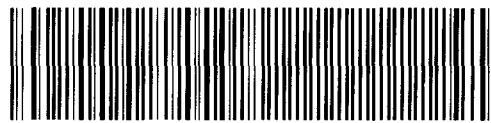
**Public Goods and Commercial Policy
in More or Less Populous Economies**

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Abstract

A global analysis of the welfare effect of changes in tariffs is undertaken in multihousehold economy with any finite number of private and pure (nonexclusive and nonrivalrous) public goods. The sensitivity of the results to various methods of financing the production of public goods is examined in some detail. The concept of the extent of underproduction of public goods in more or less populous economies is made precise and then applied to the analysis of the welfare effects of tariff changes. A globally valid necessary and a sufficient condition for welfare improving tariff changes under different methods of financing are derived and interpreted.

Introduction

In comparing the welfare effects of tariff changes in "advanced" and developing countries in the presence of government produced public goods there seem to be two relatively clear cut differences. Firstly, many developing countries are considerably more populous than many advanced countries and secondly, whilst the production of public goods is financed in developing countries primarily from tariff revenue, the financing in advanced countries relies mainly on commodity or income taxation revenue.

The received literature on commercial policy in the presence of public goods does not address either of these issues.¹ The same applies to the general literature on commercial policy in the presence of commodity or income taxes.² The production of public goods is assumed to be financed either only from tariff revenue or from foreign aid. More importantly, the authors also assume that the economies are one household economies. This entails not surprisingly, that the size of the economy in terms of the number of households does not play any part in the analysis.

Assuming one household economies in models with public goods one loses a unique feature of public goods. Apparently, if they are underproduced they are more underproduced in more populous economies and correspondingly if they are overproduced they are also more overproduced in more populous economies. This is an interesting feature of public goods which, subject to certain assumptions, applies not only to public goods produced from private contributions but also where the financing is undertaken by the government. It is reflected in what may be called the magnification effect of resource reallocations between the private and public goods sectors. There exists only one very special case when this magnification effect can be ignored: when the relevant social shadow prices associated with all the public goods are zero.

In this context it should be noted that use of the term underproduction of public goods, per se, does not imply free riding. Free riding is a term which, strictly speaking, is appropriate only if one allows for the production of public

¹see e.g. Abe 1990, 1991, 1993; Feehan 1988 or Hatzipanayotou and Michael 1993.

²see e.g. Turunen-Red and Woodland 1992.

goods to be financed at least partly from private contributions. In principle the latter is not ruled out in this paper and in the above mentioned references. To rationalize this one could assume that governmental contributions have just crowded out all private contributions.

The subject matter of this paper is relatively narrow: a welfare analysis of the effects of tariff changes with particular emphasis upon the role of the size of the economy in terms of the number of households in affecting the results. However our main message applies very widely. Indeed it applies to the whole field of public economics provided only that one or more goods have locally or globally at least some characteristics of public goods, i.e.: nonexcludability and nonrivalrousness.

Another and possibly equally important issue addressed in this paper is that the welfare effects of tariff changes could be very sensitive to the choice of the method employed in the financing of public good production. One of the so called stylized facts of many developing countries is that tariff revenue (as well as foreign aid) plays a much more significant role than in advanced countries. It therefore appears interesting to ask whether and under which conditions the results on welfare improving tariff changes are or are not sensitive to the method of financing of public goods. It can be shown that the extent of the underproduction of public goods as reflected in the appropriately defined social shadow prices depends both upon the method of financing as well as the size of the population. Tariff changes generally change the allocation of resources not only between private and public goods (because they change the financing constraint) but also between the various public goods, some of which may be much more underproduced than others. Making welfare comparisons between more or less populous economies and different financing methods of public goods is not an easy task. But we shall argue that it is possible in certain special circumstances.

The paper is structured as follows: In the following part (Part II) the model and its assumptions are explained and some useful duality relationships derived. In part III a necessary condition for tariff changes to be welfare improving in more or less populous economies (employing various methods of financing the production of public goods) is proven and interpreted. Part IV derives and interprets a sufficient condition for welfare improving changes in tariffs. As is well known there is no standard approach to welfare improvements in multihousehold economies. We shall rely mainly on the work of Diewert et alii 1989 or 1991 or Turunen-Red and Woodland 1988, 1991 or

1993, and appeal also to modern political economy approaches.

In the conclusions the main results of the paper are summarized and directions for further research indicated.

II

The model, its assumptions and some duality relationships

Part II consists of four sections. In section A the model and its main assumptions are explained. In section B, the main duality tool which is used to model the demand side of the economy is introduced and several of its main properties discussed. In section C our attention turns to the modelling of the supply side. Finally in section D the focus is on the derivation and interpretation of the relevant social shadow prices with different methods of financing the production of public goods. This is an important task because the meaning attached to the underproduction of public goods can be made precise only by means of appropriately defined shadow prices. All the results of part II are fundamental to the analysis in parts III and IV.

(A)

A perfectly competitive economy in which any number of private and public goods are produced is assumed. To focus on the distortions associated with public goods and the various taxes which are used to finance their production it is also assumed that all private goods are traded and subject to export/import taxes and commodity taxes or subsidies. Furthermore changes in world market prices, if they occur, at all, are exogenous. There are any finite number of households which differ with respect to tastes and incomes. Public goods are produced using cost minimising techniques of production and financed from one or more forms of governmental revenue (which includes foreign aid). Any possible private contributions to the production of public goods have been crowded out by government financing.

Income taxes are treated here like commodity taxes because in the standard models one may regard labour like a commodity.

Private as well as public goods are produced using constant returns to scale and strictly quasiconcave production functions. Perfectly competitive factor markets exist and all firms producing private goods maximise profits. The outputs of public goods are (subject to constraints) chosen by the government. The constraints are technological (the production possibility set) and financial (the amount of governmental revenue available to finance their production).

Changes in trade taxes or subsidies, if they occur, are taken to be exogenous. In the subsequent parts III and IV a necessary and a sufficient condition for welfare improving changes in trade taxes or subsidies are derived using, in contrast to the received literature, global techniques. This means that we assume that the changes in trade taxes or subsidies are large enough to make a local analysis inapplicable. As we shall see, in the global analysis, a necessary and a separate sufficient condition for welfare improving changes in trade taxes or subsidies emerge whilst in the local (calculus) analysis these two conditions coincide.

(B)

To model the demand side of the economy we make use of the following private expenditure function of household h :

$$E^h = E^h(p, G, u^h) \quad \text{all } h \quad (1)$$

where: p denotes the vector of prices of private goods faced by households, G the vector of public goods and u^h the utility level of household h .

The function $E^h(\cdot)$ is the same as the one used in Abe 1992 or Hatzipanayotou and Michael 1993, except that it is defined over more than one public good.

For our purposes the following three properties of the function $E^h(\cdot)$ are relevant:

1. It is concave in the vector p and its derivatives with respect to p are the Hicksian (constrained) demand functions for private goods.

2. The partial derivatives E_G^h are all negative and represent the marginal willingness of household h to pay for the various public goods. They will be referred to as demand shadow prices of household h .
3. The functions $E^h(\cdot)$ are convex in the vector G .

Whilst properties (1) and (2) are well known from the literature it appears that property (3) to date neither has been stated nor proven. To prove property (3) the following three sets are defined:

$$\begin{aligned} S^0(x^h) &\equiv \{U^h(x^h, G^0) \geq u^h, \quad x^h \geq 0\}, \\ S^1(x^h) &\equiv \{U^h(x^h, G^1) \geq u^h, \quad x^h \geq 0\}, \\ S^\theta(x^h) &\equiv \{U^h[x^h, \theta G^0 + (1 - \theta)G^1] \geq u^h, \quad 0 \leq \theta \leq 1, \quad x^h \geq 0\}. \end{aligned}$$

From the strict quasiconcavity of $U^h(\cdot)$ in G , which is assumed it follows that:

$$S^0(x^h), S^1(x^h) \subset S^\theta(x^h)$$

hence:

$$\begin{aligned} E^h(p, G^0, u^h) &> E^h[p, \theta G^0 + (1 - \theta)G^1, u^h] \quad \text{and} \\ E^h(p, G^1, u^h) &> E^h[p, \theta G^0 + (1 - \theta)G^1, u^h] \end{aligned}$$

and therefore:

$$\theta E^h(p, G^0, u^h) + (1 - \theta)E^h(p, G^1, u^h) > E^h[p, \theta G^0 + (1 - \theta)G^1, u^h] \quad \text{Q.E.D.}$$

To conclude this section it should be emphasized that throughout the paper it is assumed that all households treat the outputs of all public goods parametrically. Households in their decentralized decision making are therefore guided by price and quantity signals both of which they treat parametrically.

(C)

From the assumption that all private and public goods are produced with constant returns to scale and strictly quasiconcave production functions it

follows that the set of outputs of private goods and public goods defined by the following weak inequality is convex:

$$g(y, G, V) \leq 0 \quad y, G \geq 0 \quad (2)$$

where: y denotes the output vector of private goods, G the output vector of public goods and V the vector of economy factor endowments.

Since the outputs of public goods are chosen by the government and these choices treated parametrically by firms producing private goods it follows at once that the output choices of firms producing private goods can be modelled using the following revenue function for private goods:

$$R = R(q, G, V) \quad (3)$$

where: q denotes the vector of prices of private goods faced by firms. This is the same function as used by Abe 1991 or Hatzipanayotou and Michael 1993 except that it allows for many public goods.

The following three properties are relevant for our purposes:

1. The partial derivatives: R_q (if they exist) are the supply functions or production graphs of private goods. The derivatives exist if the number of factors is equal to the number of private goods and the Jacobian determinant of their cost functions is nonvanishing.
2. The partial derivatives R_G denote the so called supply shadow prices of public goods. They are all negative.
3. The function $R(\cdot)$ is concave in G .

Whilst properties (1) and (2) are well known it appears that property (3) to date neither has been stated nor proven. A proof of property (3) follows.

Define the following three sets:

$$\begin{aligned} S^0(y) &\equiv \{g(y, G^0, V) \geq 0, \quad y \leq 0\}, \\ S^1(y) &\equiv \{g(y, G^1, V) \geq 0, \quad y \leq 0\}, \\ S^\theta(y) &\equiv \{g[y, \theta G^0 + (1 - \theta)G^1, V] \leq 0, \quad 0 \leq \theta \leq 1, \quad y \geq 0\}. \end{aligned}$$

Under the stated assumptions the production possibility set is strictly convex. It therefore follows that:

$$S^0(y), S^1(y) \subset S^\theta(y)$$

hence: $R(q, G^0, V) < R[q, \theta G^0 + (1 - \theta)G^1, V]$ all θ and
 $R(q, G^1, V) < R[q, \theta G^0 + (1 - \theta)G^1, V]$ all θ therefore:

$$R[q, \theta G^0 + (1 - \theta)G^1, V] > \theta R(q, G^0, V) + (1 - \theta)R(q, G^1, V) \quad \text{Q.E.D.}$$

As will be shown in parts III and IV the convexity of the private expenditure functions and the concavity of the private revenue function in the outputs of public goods are powerful (global) properties which play a key role in the derivation of global results.

(D)

The purpose of this subsection is to derive social shadow prices of public goods which take into account various methods of financing the production of public goods. As mentioned before with regard to public goods it is imperative to distinguish between private and social shadow prices if the economies are multihousehold economies. Furthermore in deriving social shadow prices it is important to consider various methods of financing the production of public goods. Whether public goods are over or underproduced can be decided only in the light of the appropriately defined social shadow prices. In what follows social shadow prices are defined for the following four cases: (a) no financing constraint, (b) financing from foreign aid only, (c) from foreign aid and trade tax revenue and (d) from commodity tax revenue. In cases (c) and (d) it is assumed that any tax revenue which is not used for the production of public goods is returned to the private sector by means of reductions in commodity taxes or by means of commodity subsidies.

Case (a) In this case there are no commodity taxes, tariffs or foreign aid. The production of public goods is financed by charging Lindahl prices to households. To derive social shadow prices in this case it is convenient to make use of the following multihousehold trade expenditure function (see Lloyd and Schweinberger 1988):

$$B = \sum_h E^h(q^*, G, u^h) - R(q^*, G, V) \quad (4)$$

where q^* stands for the vector of world market prices of private goods.

A welfare improving change in the outputs of public goods is defined as a change in the vector G such that:

$$dB = B_G dG < 0 \quad (5)$$

for given prices q^* and given u^h , all h .

If the G 's change and (5) is satisfied then the net cost (compensated excess expenditure) of sustaining the given utility vector fall. From sections (A), (B) and (C) of part II it follows that:

$$B_G = \sum_h \lambda^{hd} - \lambda^s = \lambda \quad (6)$$

where: λ^{hd} denotes the demand shadow price vector of household h for public goods, λ^s the vector of supply shadow prices and λ the vector of social shadow prices.

If an element of the vector λ is negative (positive) then that public good is understood to be underproduced (overproduced). Borrowing terminology from Mirlees 1976, the social shadow prices λ may be referred to as PARETIAN social shadow prices. If there is no financing constraint the government would of course increase the production of public goods until $\lambda = 0$ (for all public goods) assuming interior solutions exist. If the outputs of the G 's were rationed in the sense that all the λ 's are negative then it is easy to see that an increase in all the G 's will, for given λ^s (supply shadow prices of public goods) entail a bigger reduction in B if there are more households (and all households are unsatiated with respect to all public goods). In this sense then public goods are more underproduced (if they are underproduced at all) in more populous economies.

Whether and in which sense public goods are more underproduced in more populous economies if the production is subject to specific financing constraints will be discussed further below.

To conclude this subsection it should be noted that the objective function, expression (4), is globally convex in G because it has been shown in (B) and (C) above that expenditure functions for private goods are convex and the revenue function concave in G . From this follows that a minimum exists.

Also note that to solve for the equilibrium u^h , all h , and the optimal vector of public goods one has to consider the expenditure/income equality for each household (having specified household endowments with factors) in

addition to the set of optimality conditions, $B_G = 0$. This remark applies to all the results stated subsequently in parts II, III and IV.

Case (b) Let all the public goods be produced from foreign aid as e.g. in Hatzipanayotou and Michael 1993. The government minimises the multi-household trade expenditure function $B(q^*, G, V, u^h)$ subject to $-\lambda^s G = T$, where the vector of supply shadow prices $-\lambda^s$ is positive and constant because under constant returns to scale the per unit cost of production of public goods depends only upon factor prices (which are determined by the given prices p).

Assuming that all the solutions are interior and that the constraint is effective we have:

$$L_G = \lambda + \gamma_1 \lambda^s = 0 \quad (7)$$

where L is the relevant Lagrangean function and γ_1 , the Lagrangean multiplier associated with the financing constraint. Since λ^s and γ_1 , have to be negative it follows at once that in equilibrium $\lambda < 0$, i.e.: public goods are underproduction depending upon the cost of production of public goods λ^s and the value of γ_1 .

An important question now arises. In which precise sense are public goods more underproduced in more populous economies if we have a financing constraint such as foreign aid?

Assume two economies which are exactly the same with regard to tastes, factor endowments and technology. Product and factor prices are also the same. However they differ with respect to the number of households. Furthermore let the more populous economy contain as a proper subset the households of the less populous economy.

Let T_1 be the amount of foreign aid given to these two economies.

Assume that T_1 increases by a small amount in both economies and that it is binding before and after the increase. Why is it that γ_1 is more negative in the more populous economy? An increase in T_1 clearly leads to an increase in the outputs of one or more public goods in the more and in the less populous economy (for given λ^s). However it can be shown that the welfare increase has to be greater in the more populous than in the less populous economy. To see this, assume that the utility levels of all the additional households in the more populous economy are kept fixed by lump sum taxes as the outputs of public goods are increased. Clearly, these additional households can substitute public for private goods and hence keeping their utilities

unchanged the government can raise revenue which it can distribute to the other households. The utility levels of these other households therefore have increased by more in the more than in the less populous economy. That is why γ_1 is more negative (in absolute terms greater) in the more than in the less populous economy. In this sense public goods are more underproduced in more populous economies (*ceteris paribus*) in the presence of a financing constraint associated with foreign aid.

Case (c) In a typical developing country there are commodity taxes and tariffs as well as foreign aid. The production of public goods is financed from foreign aid and tariff revenue. In this case the government chooses a vector of public goods which minimises $B(p, q, G, V, u^h)$ subject to:

$$-\lambda^s G = (q - q^*) \left[\sum_h x^h(p, G, wv^h) - y(q, G, V) \right] + T_1$$

where: q stands for the vector of prices of private goods faced by firms and p the vector faced by households, $x^h(\cdot)$ the Marshallian demand functions for private goods by household h , v^h the primary factor endowments of household h , $\sum_h v^h = V$ and $y(\cdot)$ the supply functions for private goods. Again assuming that the solutions are interior we have:

$$L_G = \lambda + \gamma_2 \left[\lambda^s + (q - q^*) \left(\sum_h x_G^h - y_G \right) \right] = 0. \quad (8)$$

The increase in the output of any one public good can be trade creating or trade reducing. In general it would appear, borrowing terminology from Hicks, see Hicks 1936, that public and private good can be Q substitutes or complements on the demand and supply side. It is therefore impossible to conclude generally that one or more public goods will be less or more underproduced (even *ceteris paribus*) when tariff revenue and foreign aid are used to finance the production of public goods rather than foreign aid alone.

To obtain a more definite conclusion one may postulate a condition which is a public goods analogue of the HATTA normality condition (see Hatta 1977): The indirect effect of an increase in any one G via tariff revenue, i.e.: $(q - q^*)(\sum_h x_G^h - y_G)$ can reinforce or counteract the direct effect, λ^s , but it can never overturn it.

This condition is satisfied if the increase in the output of every public good is in fact on average trade reducing in the sense that: $(q - q^*)(\sum_h x_G^h - y_G) < 0$ all G , or more generally if and only if: $(q - q^*)(\sum_h x_G^h - y_G) + \lambda^s \leq 0$ all G .

It seems reasonable to assume this. Since without it there may not be a binding financial constraint in equilibrium. The constraint set may lose its boundedness and therefore compactness. The optimality conditions (8) can then be interpreted in terms of the sum of the demand shadow prices (over the households) being equal to the economic supply shadow prices, where the latter are defined as $-\lambda^s(1 + \gamma_2) - (q - q^*)(\sum_h x_G^h - y_G)$. For an analogous interpretation see e.g. Atkinson and Stiglitz 1980 p. 490 ff.

From these considerations it follows that one cannot draw general conclusions with regard to the extent of the underproduction of public goods as reflected in the social shadow prices λ in economies with different methods of financing the production of public goods.

Comparing, for example, financing by aid alone on the one hand and by tariff revenue and aid on the other hand one can only conclude that if all public goods are underproduced with pure aid financing, they will also be underproduced with aid and tariff financing if the above mentioned analogue of HATTA normality holds.

Similar conclusions apply if we want to compare the extent of underproduction in more or less populous economies given the method of financing the public goods. The reasoning is exactly the same as under case (b) except that it must be remembered that the tariff revenue effect: $(q - q^*)(\sum_h x_G^h - y_G)$ generally is different in more or less populous economies.

LEMMA: Assume that in the less populous economy public goods are underproduced because the HATTA normality condition as defined above applies (i.e. in equilibrium $\lambda < 0$ for all public goods). Then public goods are more underproduced in the more populous economy if:

$$(q - q^*) \left[\sum_{H_1+1}^{H_2} x_G^h - y^h \right] < 0 \quad \text{all } G \quad (9)$$

where: H_1 denotes the number of households in the less and H_2 the number of households in the more populous economy.

As before it is understood that $H_1 \subset H_2$. Expression (9) can be made operational by employing a technique developed in Lloyd and Schweinberger

1988. The outputs of an economy are inputed to households by decomposing the aggregate revenue function into household revenue functions.

Expression (9) says that the all the additional households are on average reducing their imputed imports of goods which are relatively highly taxed if the output of any one public goods rises. The lemma follows directly from expression (8), if expression (9) is satisfied and the two economies to be compared have the same factor prices and hence supply shadow prices.

Case (d): In advanced countries public goods are financed from commodity tax rather than tariff revenue. In this case the financing constraint assumes the form:

$$-\lambda^s G = (p - q) \sum_h x^h(p, G, wv^h).$$

The optimality conditions (assuming interior solutions) are:

$$\lambda + \gamma_3 \left[\lambda^s + (p - q) \sum_h x_G^h \right] = 0. \quad (10)$$

All the comments made under case (c) apply here (with appropriate qualification). In particular if the redefined Hatta normality condition holds there is underproduction of all public goods. Also public goods are more underproduced in more populous economies if: $\sum_{H_1+1}^{H_2} (p - q)x_G^h < 0$, i.e.: the increase in the output of any one public good reduces on average the consumption of private goods with relatively high commodity taxes.

III

A necessary condition for welfare improving changes in tariffs in developing and advanced countries

It will be recalled that in a developing country the production of public goods is assumed to be financed from tariff revenue and foreign aid. In the advanced country, on the other hand, the financing is from commodity taxation revenue. In what follows the case of the developing country is covered first.

In contrast to the received literature our aim is to derive in this section a globally valid necessary condition for welfare improving changes in tariff rates.

Since it has been shown before that the restricted expenditure functions are convex in the outputs of public goods and because a Taylor's expansion globally understates the true value of a convex function we have:

$$E^h(p^0, G^1, u^{h_0}) \geq E^h(p^0, G^0, u^{h_0}) + \lambda^{h_0}(G^1 - G^0) \quad (11)$$

where λ^{h_0} denotes the vector of demand shadow prices of household h (the marginal willingness to pay, see King 1986).

Summing over the households we have:

$$\sum_h E^h(p^0, G^1, u^{h_0}) \geq \sum_h E^h(p^0, G^0, u^{h_0}) + \sum_h \lambda^{h_0}(G^1 - G^0). \quad (12)$$

In a developing country all tariff revenue and foreign aid are used for the production of public goods. Net commodity tax revenue is zero. Therefore the private expenditure income equality assumes the following form:

$$\sum_h E^h(p^0, G^0, u^{h_0}) = R(q^0, G^0). \quad (13)$$

Making use of (13), one can rewrite (12) as:

$$\sum_h E^h(p^0, G^1, u^{h_0}) \geq R(q^0, G^0) + \sum_h \lambda^{h_0}(G^1 - G^0). \quad (14)$$

Next, deduct from both sides $\sum_h E^h(p^0, G^1, u^{h_0})$. Expression (14) becomes:

$$\begin{aligned} \sum_h E^h(p^0, G^1, u^{h_0}) - \sum_h E^h(p^0, G^1, u^{h_1}) &\geq R(q^0, G^0) + \dots \\ \dots - \sum_h E^h(p^0, G^1, u^{h_1}) + \sum_h \lambda^{h_0}(G^1 - G^0). \end{aligned} \quad (15)$$

From the concavity of $E^h(\cdot)$ in the prices p :

$$\sum_h E^h(p^0, G^1, u^{h_1}) \leq \sum_h E^h(p^1, G^1, u^{h_1}) + (p^0 - p^1)x^1 \quad (16)$$

where x^1 denotes the consumption vector of private goods in the new equilibrium.

In the new equilibrium:

$$\sum_h E^h(p^1, G^1, u^{h_1}) = R(q^1, G^1). \quad (17)$$

Substituting (17) into (16) and then rewriting (15) in the light of (16) one obtains:

$$\begin{aligned} \sum_h E^h(p^0, G^1, u^{h_0}) - \sum_h E^h(p^0, G^1, u^{h_1}) &\geq R(q^0, G^0) - R(q^1, G^1) \\ &\quad - (p^0 - p^1)x^1 + \sum_h \lambda^{h_0}(G^1 - G^0). \end{aligned} \quad (18)$$

Now write $R(q^0, G^0) - R(q^1, G^1) = R(q^0, G^0) - R(q^0, G^1) + R(q^0, G^1) - R(q^1, G^1)$.

From the convexity of $R(\cdot)$ in p :

$$R(q^0, G^1) \geq R(q^1, G^1) + (q^0 - q^1)y^1 \quad (19)$$

where y^1 denotes the output vector of private goods in the new equilibrium. Since it has been shown in part II that $R(\cdot)$ is concave in G we have:

$$R(q^0, G^1) \leq R(q^0, G^0) + \lambda^{s_0}(G^1 - G^0) \quad (20)$$

where λ^{s_0} denotes the supply shadow price vector of public goods.

Multiplying the last expression by minus one and adding expression (18) and (20) we arrive at:

$$R(q^0, G^0) - R(q^1, G^1) \geq (q^0 - q^1)y^1 + \lambda^{s0}(G^1 - G^0). \quad (21)$$

Making use of the expression (21), one may finally rewrite expression (18) as follows:

$$LHS \geq (q^0 - q^1)y^1 - (p^0 - p^1)x^1 + \left(\sum_h \lambda^{h0} - \lambda^{s0} \right) (G^1 - G^0) \quad (22)$$

where: $(\sum_h \lambda^{h0} - \lambda^{s0}) = \lambda^0$, the vector of PARETIAN social shadow prices of public goods (see part II above).

In evaluating expression (22), the following definition is useful:

Definition A: there are welfare gains from changes in the tariffs if either the changes in the tariffs imply a Pareto- improvement, i.e.: $u^{h1} \geq u^{h0}$ all h , or the gainers are able to compensate the losers and still gain.

Modern political economy approaches, see e.g. Grossman and Helpman 1993 make use of the concept of contribution functions. If the L.H.S. of expression (18) is negative it is clear that there exists a set of contribution functions such that the gainers can compensate the losers, make a contribution to the government and still gain.

In this context it must be noted that the aim of political economy approaches and second best welfare theory are, of course, very different. Whilst political economy approaches purport to explain economic policies second best welfare theory attempts to derive policy recommendations. As emerges from expression (18) there exists however a close relationship between them: if contribution functions are used as in Grossman and Helpman op. cit. proposition I of part III may be regarded as a necessary condition for the given tariff changes to be actually carried out. On the other hand, expression (22) of part III and expression (30) of part IV may be regarded as a necessary and a sufficient condition respectively for the given tariff changes to be recommended. This seems to establish an important new linkage between political economy and standard second best welfare approaches where in the former the government plays the role of an agency of gainers to compensate the losers thus ensuring that the policy is actually adopted.

Expression (22) contains on the R.H.S. three relevant effects of tariff changes: (1) a form of a Paasche output price index [if $(q^0 - q^1)y^1 < 0$

this index improves], (2) a form of a Paasche consumption price index [if $(p^0 - p^1)x^1 > 0$ this index improves] and (3) an externality or g effect (using the terminology of Abe 1992). Making use of these definitions we are in a position to state proposition I.

Proposition I

Let the changes in the tariff rates in a developing country entail a welfare increase in the sense of definition A. Furthermore let the production of public goods be financed only from tariff revenue and foreign aid. Then the change in an appropriately defined Paasche output price index plus the change in an appropriately defined Paasche consumption price index plus the change in the value of outputs of public goods at the (negative) PARETIAN shadow prices in the original equilibrium is negative.

Proposition I follows directly from expression (22) and the assumption that the changes in the tariffs are welfare improving in the sense of definition A. In this case the L.H.S. of expression (22) is negative because all expenditure functions are increasing functions of u^h .

Q.E.D.

To gain more in-sight into the economic meaning of proposition I it is convenient to focus on two special cases.

Assume first that there are no commodity taxes or subsidies and world market prices change but the small developing country uses only made to measure tariffs so that the domestic prices of all private goods remain unchanged. In this case all price effects of tariff changes are zero. What remains is (what Abe 1992 calls the g effect) the externality effect associated with the production of public goods. The tariff revenue effect is entirely subsumed in the externality effect because all of tariff revenue is used in the financing of the production of public goods.

In this case clearly, a necessary condition for welfare improving changes in tariffs is that on average the outputs of public goods with high (low) Paretian social shadow prices (in the original equilibrium) rise (fall).

To gain insights into the role of the size of population in determining welfare gains or losses, assume that the original equilibrium was generated as follows. There existed an equilibrium in which public goods were produced from voluntary contributions instead of tariff revenue. The government changed

tariff rates such that tariff revenue increased and by a well known reasoning (see Bergstrom et alii (1986)) this crowded out private contributions. It is a fact that in a COURNOT-NASH equilibrium with private contributions (where all households contribute) the demand shadow price for each public good is equal for all households and equal to the supply shadow price. If the tariff rates are such that the voluntary contributions of all the households are just crowded out, the equality between all demand shadow prices (of all households) and the equality between demand and supply shadow prices still holds. This follows from a consistent application of what is referred to as the neutrality or invariance property.

In this case, one can rewrite expression (22) as:

$$LHS \geq (H - 1)\lambda^{d0}(G^1 - G^0) \quad (23)$$

where λ^{d0} is the vector of demand shadow prices and H the number of households.

Expression (23) highlights what we have referred to as the magnification effect in the introduction. Assume that there are welfare gains in a very populous economy then the L.H.S. of expression (22) is negative and therefore the R.H.S. has to be negative too. In a very populous economy the sign of the R.H.S. is likely to be dominated by the externality effect. This conclusion which can be made much stronger if we make the assumptions stated in part II reflects the greater extent of the underproduction of public goods in more populous economies. As shown in part II it holds even if demand shadow prices are not equalised between households and are not equal to supply shadow prices.

In fact, subject to the assumptions of part II one can show that if two economies differ only with respect to the number of households there always exists a populous enough economy such that assuming welfare gains entails that the value of output of public goods at the original Paretian social shadow prices must have risen. This seems an important message for many developing countries with large populations. To the best of our knowledge it has never been made in the received literature.

To conclude this evaluation of expression (22) and proposition I the readers attention is drawn to another potentially useful interpretation. The R.H.S. of expression (22) can be regarded as a measurable lower bound of the welfare gain as described by the L.H.S.. This can be seen if the expression (22) is multiplied by minus one.

Turning to the case of the advanced country we note that only minor changes are required to derive a result corresponding to expression (22). If the advanced country is a donor of foreign aid and finances the production of public goods only from commodity (income) taxes, expression (13) now assumes the following form:

$$\sum_h E^h(p^0, G^0; u^{h_0}) = R(q^0, G^0) - T_1^0$$

where: tariff revenue is distributed to the private sector via consumption subsidies and T_1^0 stands for the amount of foreign aid given.

Similarly, expression (17) has to be amended too. Taking into account these changes expression (22) becomes:

$$LHS \geq (q^0 - q^1)y^1 - (p^0 - p^1)x^1 + \Delta T_1 + \lambda^0(G^1 - G^0) \quad (24)$$

Surprisingly, if we set ΔT_1 equal to zero, the resulting necessary condition for welfare improving changes in tariffs seems to be the same in the advanced and developing countries. However this apparent conclusion would be wrong. As is clear from part II the values of the Paretian shadow prices λ depend on the size of the economy in terms of the number of households and the method of financing public goods. Developing and advanced countries differ in both respects. However as explained in part II, subject to certain assumptions, see especially the LEMMA of part II, one can compare Paretian shadow prices and hence the extent of underproduction in more and less populous economies for a given method of financing.

The following three observations conclude part III:

(1) Implicit in the analysis presented above are general results concerning the relationship between the gains from trade and the law of comparative advantage in private goods if public goods are produced from commodity tax revenue in the autarkic and free trade equilibria. In particular it is shown that the law of comparative advantage is not a necessary condition for trade gains if the value of output of public goods rises at the original Paretian social shadow prices.

(2) Even if free trade prices are the same as autarkic prices there may be gains from trade (see Kemp and Schweinberger 1991) because welfare superior equilibria may be possible under free trade if there are welfare increasing changes in the quantity signals (and the associated shadow prices).

(3) A free trade equilibrium generally is not Pareto efficient.

A sufficient condition for welfare improving changes in tariffs in advanced and developing countries

As in part III we first analyse the case of a developing country which uses only tariff revenue and foreign aid to finance the production of public goods.

From the convexity of $E^h(\cdot)$ in G (all h):

$$\sum_h E^h(p^1, G^0, u^{h_1}) \geq \sum_h E^h(p^1, G^1, u^{h_1}) + \sum_h \lambda^{h_1} (G^0 - G^1). \quad (25)$$

Setting $\sum_h E^h(p^1, G^1, u^{h_1}) = R(q^1, G^1)$ because the developing country finances the production of public goods from foreign aid (T_1) and tariff revenue (T_2) and commodity tax revenue is assumed to be handed back to the private sector by means of changes in consumption taxes or subsidies and deducting from both sides of (25) $\sum_h E^h(p^1, G^0, u^{h_0})$ we obtain:

$$\begin{aligned} L.H.S. = \sum_h E^h(p^1, G^0, u^{h_1}) - \sum_h E^h(p^1, G^0, u^{h_0}) &\geq R(q^1, G^1) \dots \\ &- \sum_h E^h(p^1, G^0, u^{h_0}) + \sum_h \lambda^{h_1} (G^0 - G^1). \end{aligned} \quad (26)$$

From the concavity of $E^h(\cdot)$ in p (all h) and setting $\sum_h E^h(p^0, G^0, u^{h_0})$ equal to $R(q^0, G^0)$ we derive:

$$\sum_h E^h(p^1, G^0, u^{h_0}) \leq R(q^0, G^0) + (p^1 - p^0)x^0. \quad (27)$$

In the light of expression (27) one can rewrite expression (26) as follows:

$$\begin{aligned} L.H.S. &\geq R(q^1, G^1) - R(q^0, G^0) - (p^1 - p^0)x^0 \dots \\ &\quad + \sum_h \lambda^{h_1} (G^0 - G^1). \end{aligned} \quad (28)$$

Now $R(q^1, G^1) - R(q^0, G^0) = R(q^1, G^1) - R(q^1, G^0) + R(q^1, G^0) - R(q^0, G^0)$.

From the concavity of $R(\cdot)$ in G (see part II):

$$R(q^1, G^0) \leq R(q^1, G^1) + \lambda^{s1}(G^0 - G^1)$$

and from the convexity of $R(\cdot)$ in p :

$$R(q^1, G^0) \geq R(q^0, G^0) + (q^1 - q^0)y^0.$$

Multiplying the last inequality by minus one and adding the last two inequalities one arrives at:

$$R(q^1, G^1) - R(q^0, G^0) \geq (q^1 - q^0)y^0 - \lambda^{s1}(G^0 - G^1). \quad (29)$$

Expression (28) can be rewritten in the light of expression (29) as follows:

$$L.H.S. \geq (q^1 - q^0)y^0 - (p^1 - p^0)x^0 + \lambda^1(G^1 - G^0). \quad (30)$$

where: $\lambda^1 = \sum_h \lambda^{h1} - \lambda^{s1} < 0$ from part II.

Again we can distinguish three effects of changes in tariffs: (a) the change in a Laspeyer output price index [the index improves if: $(q^1 - q^0)y^0 > 0$], (b) the change in a Laspeyer consumption price index [the index improves if: $(p^1 - p^0)x^0 < 0$] and (c) an externality effect. Comparing expressions (22) and (30) we note that the price effects (in Abe's terminology, see Abe 1992, the p effect) and the externality effect are defined differently. This highlights an important difference between local and global analysis. Whilst locally necessary and sufficient conditions for welfare improvement generally coincide, they differ in the global analysis.

Going back to expression (26) we define the tariff changes to be welfare improving if the L.H.S. of expression (31) is positive. To justify this definition of a welfare improvement one may appeal to productivity indices in multi-household economics as can be found in Diewert et alii (1989) or Turunen-Red and Woodland (1989, 91, 93). Alternatively, using the approach of modern political economy (see e.g. Helpman and Grossman 1993) it may be shown that if the L.H.S. of expression (26) or (31) is positive the gainers may make contributions to the government and use the services of the government to compensate the losers yet they still gain.

Definition B:

There are welfare gains from the changes in tariffs if and only if the gainers are able to compensate the losers, make a contribution to the government and still gain.

The result of expression (30) is now formalised as Proposition II.

Proposition II

Let a small developing country finance the production of public goods from tariff revenue and foreign aid. There are welfare gains from changes in tariffs if the change in the appropriately defined Laspeyer output price index plus the change in the appropriately defined Laspeyer consumption price index plus the change in the value of public goods produced at the Paretian shadow prices (in the new equilibrium) is positive.

Proposition II follows directly from expression (30) by setting the R.H.S. equal to a positive number.

The key message of proposition II and expression (30) is again that the number of household is likely to have a major influence on the welfare effects of tariff changes. Subject to the assumptions of part II, see the LEMMA and the modified form of Hatta normality, public goods are more underproduced in more populous economies. Therefore generally if one compares economies which are identical except for the number of households one may show that there generally exists a populous enough economy such that the sign of the externality effect dominates the sign of the R.H.S. of expressions (31), (32) or (33).

Of course $\lambda^1(G^1 - G^0)$ may well be positive yet $\lambda^0(G^1 - G^0)$ negative or the converse. Even if both expressions are positive one may be much larger than the other. This reflects the fact that the extent of the underproduction of the various goods may be very different in the original and in the new equilibrium. To see this consider expression (8) or (9) of part II. If the production costs of public goods rise or the effect of increases in the outputs of public goods on tariff or commodity tax revenue is negative and the outputs of one or more public goods have risen as a result of the changes in tariffs then the extent of the underproduction as reflected in the vector λ generally will have risen too. The point is that the underproduction of public goods is not only a function of the number of households but also the method of financing and the quantities of public goods produced as well as all the prices.

In the derivation of proposition I and II it has been assumed that the government has no net tax revenue apart from the tax revenue which is used for the production of public goods. If desired an alternative assumption could be made. All net tax revenue which is not used for the production of public

goods is transferred lump sum fashion to the private sector. In this case commodity taxes need not change when tariff rates change. It is easy to show that this entails that instead of obtaining a Paasche or Laspeyer output and consumption price index in expression (22) and (30) respectively, the relevant index is a Paasche or Laspeyer import price index of the private sector. In addition to this there is a governmental revenue effect (concerning governmental revenue which is not used for the production of public goods). It will be obvious that subject to these formal changes all the results concerning the importance of the size of the economy and the method of financing in influencing the extent of underproduction of public goods as reflected in the Paretian shadow prices of public goods still apply.

Conclusions

Having made precise the concept of the extent of the underproduction of public goods in more or less populous economies by means of appropriately defined shadow prices (see part II) we proceeded to the derivation of a globally valid necessary and a sufficient condition for welfare improving changes in tariffs. In this derivation we allowed for the financing of the production of public goods from tariff revenue and foreign aid (as in developing countries) or from commodity (income) tax revenue as in advanced countries.

The extent of the underproduction of public goods depends not only upon the size of the population but also on the financing method employed and in fact on all the variables of the model. However subject to certain assumptions (such as analogues of the HATTA normality condition) it is possible to put up a precise argument as to why public goods are more underproduced in more populous economies.

The extent of the underproduction of public goods as reflected in the relevant shadow prices plays a key (and hitherto ignored) role in all the results of parts III and IV. Generally speaking, the welfare effects of tariff changes can be decomposed into price effects regarding the production sector and price effects regarding the household (consumption) sector and an externality effect. In making *ceteris paribus* comparisons of economies which differ in the number of households it has been shown that there always exist populous enough economies such that the extent of underproduction as reflected in the externality effect dominates the other two effects.

If all governmental revenue which is not used for the production of public goods is distributed to the private sector via commodity subsidies, the results which emerge for developing and advanced countries appear formally to be the same (even though developing countries and advanced countries finance the production of public goods differently). However, the appropriately defined social shadow prices of public goods are likely to be very different because of differences in their sizes and the methods of financing. Hence the conditions for welfare improving tariff changes generally are very different too.

In the present paper the assumption of perfect competition in all markets

for private goods has been sustained throughout. On the other hand there exists a fairly extensive literature on the welfare effects of tariff changes under imperfect competition and/or increasing returns to scale. It can be shown that the modelling of public goods essentially involves a conceptually analogous approach as the modelling of imperfect competition or increasing returns to scale, namely the modelling of price and quantity signals. Having defined social shadow prices associated with the quantity signals it should be possible to construct a general theory of commercial policy which includes on the one hand public goods and on the other increasing returns to scale and/or imperfect competition as special cases. The present paper is intended as a first step in this direction.

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