

Worrall, Tim

**Working Paper**

**Debt with potential repudiation**

Diskussionsbeiträge - Serie II, No. 69

**Provided in Cooperation with:**

Department of Economics, University of Konstanz

*Suggested Citation:* Worrall, Tim (1988) : Debt with potential repudiation, Diskussionsbeiträge - Serie II, No. 69, Universität Konstanz, Sonderforschungsbereich 178 - Internationalisierung der Wirtschaft, Konstanz

This Version is available at:

<https://hdl.handle.net/10419/101756>

**Standard-Nutzungsbedingungen:**

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

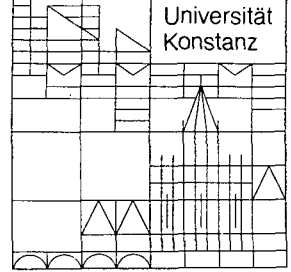
Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

**Terms of use:**

*Documents in EconStor may be saved and copied for your personal and scholarly purposes.*

*You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.*

*If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.*



**Sonderforschungsbereich 178**  
**„Internationalisierung der Wirtschaft“**

Diskussionsbeiträge

Juristische  
Fakultät

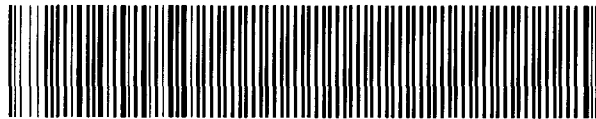
Fakultät für Wirtschafts-  
wissenschaften und Statistik

---

Tim Worrall

**Debt with Potential Repudiation**

c 156645



DEBT WITH POTENTIAL REPUDIATION

by

TIM Worrall

Serie II - Nr. 69

Juni 1988

Ag 3306 / 88 / *Wirtschaft*

# DEBT WITH POTENTIAL REPUDIATION

TIM WORRALL

## ABSTRACT

Lending across national boundaries is different from lending within national boundaries because of the difficulty of imposing legal sanctions. This note examines a simple model of international lending where the borrower can repudiate, without legal sanction, if this is to his advantage. The model has an infinite time horizon and it is assumed the borrower has an i.i.d. income stream. It is shown that, although debt is initially restricted, in the long run consumption is completely stabilised.

## 1. INTRODUCTION

Lending across national boundaries is different from lending within national boundaries because the borrower's assets cannot be sequestered (excepting gunboat diplomacy or if the borrower holds assets in the lender's country) and so collateral requirements cannot usually be enforced. Basically there is no legal sanction against the repudiation of the debt by the borrower. Of course there are a number of possible non-legal sanctions, for example, trade embargoes, diplomatic and political pressure, withdrawal of trade finance and withdrawal of future credit, but the effectiveness of all of these actions is questionable. Certainly there is strong evidence from North Korea, Rhodesia and South Africa that trade sanctions do not work and although Mexico has repudiated its debt in the past it still continues to have access to financial credit markets.

In a recent paper Eaton, Gersovitz and Stiglitz (1986) have stressed the importance of potential repudiation for international debt contracts. They argue that insolvency and illiquidity are not important considerations in the international context (countries cannot go bankrupt) and that a necessary, though of course not sufficient, condition for understanding the current international debt crisis and prescribing public policy is to study simple models of borrowing with potential repudiation. In this note, as in Eaton and Gersovitz (1981), it will be assumed the only sanction against repudiation is the threat of withdrawal of future credit and we will examine its effect upon debt structure. Although this is an extremely crude assumption some new results are derived. In the short run the borrower is constrained

in the amount he can borrow but consumption never decreases and in the long run it is perfectly stabilised.

## 2. MODEL

There are two types of agents: sovereign borrowers and international banks. The borrower has a per-period, strictly concave utility (objective) function,  $u$ , defined on consumption (absorption),  $c$ . It is assumed that  $u(c)$  is twice continuously differentiable and  $\lim_{c \rightarrow 0} u'(c) = -\infty$ . The borrower's income,  $y$ , is i.i.d. over an infinite number of dates,  $t=0, 1, \dots, \infty$ , and has one of  $S$  possible values,  $y_1 < y_2 < \dots < y_S$ , where the income  $y_s$  occurs with probability  $p_s$  and  $\sum_s p_s = 1$ .

International banks are risk neutral and like the borrowers discount the future by a constant factor  $\alpha$ . They observe the borrower's income at every date, so the amount loaned can be made conditional upon income. They cannot sequester the borrower's assets and the only sanction they have against default is the threat of withdrawal of future credit. It is assumed all banks are aware of any default by a borrower and observe a moratorium on future lending. All debt contracts will be designed so as to take account of the future moratorium against borrowers who default and consequently, in equilibrium, no-one will default. Obviously a complete moratorium is an extreme case, but the analysis could be carried out if it were assumed credit was only withdrawn for a finite number of periods.

### 3. DEBT CONTRACTS

Suppose an international bank agrees to loan a sovereign borrower an amount,  $b$  in the current period in return for a promised payment of  $d$ 's next period if the borrower's income is  $y_s$ . Such a loan is feasible provided

$$b = \alpha E d'_s \quad (F)$$

where  $E$  is the expectations operator taken over income in the next period. In words, a loan is feasible if it is equal to the expected discounted debt-service obligations. Then the bank breaks-even period by period. So (F) is the zero profit condition for one period loans<sup>1</sup>. It is assumed new loans will not be granted unless debt-service obligations have been met.

Let  $z_s = y_s - d_s$  be the net income of the borrower in state  $s$  and let net income next period be  $z'$ . At date  $t=0$ ,  $z_s = y_s$  for each state  $s=1,2,\dots,S$ . At every date consumption is net income plus new loans,  $c = z + b$ . Given  $z$ , and that debt-service obligations have been met, banks will be forced by competition to offer loans and a repayment schedule which maximises the borrower's long run utility. Let  $U(z)$  denote the maximum future expected discounted utility of the borrower taking into account all future loans. If the borrower is not to repudiate his debt, that is if he is not to default on his debt-service obligations, it must be that

$$U(z) \geq u(y_s) + \alpha u(y_*) / (1 - \alpha) \quad (ND)$$

1. Debt contracts of two or more periods can do no better. This is a corollary of the results on long run wage contracts of Thomas and Worrall (1987).

for all  $s=1,2,\dots,S$ , where  $y^*$  is the certainty equivalent income, i.e.  $u(y^*)=Eu(y_s)$ . The no-default condition, (ND), says the borrower must do at least as well by meeting his debt-service obligations as defaulting and consuming only his own income thereafter.

The function  $U(z)$  is defined recursively by the principle of optimality, that is for all  $s=1,2,\dots,S$

$$U(z)=\max_{b, (d'_s)} u(z+b)+\alpha EU(y_s-d'_s) \quad (O)$$

subject to (F) and (ND). Substituting (F) into (O) and letting  $\mu_s$  be the multiplier for (ND) the first order conditions are

$$\alpha u'(z+b)=(\alpha+\mu_s)U'(z'_s) \quad (FO)$$

for all  $s=1,2,\dots,S$ . Together with the envelope condition

$$u'(z+b)=U'(z) \quad (E)$$

they determine the optimal loan and the borrower's consumption. By (E),  $U(z)$  is strictly concave so standard arguments imply the existence of a unique optimum. Note that  $z$  is bounded away from zero,  $y_s \geq d_s$ , so the borrower is never illiquid or insolvent.

Before turning to the results, consider as a benchmark the case where the borrower is completely honest and does not default even if it is to his advantage. Then  $z'_s=z$  in each state  $s=1,2,\dots,S$ . Defining  $y^*=Ey$  and  $y_s(0)$  as the actual income at date  $t=0$ , consumption is completely stabilised at  $c=\alpha y^*+(1-\alpha)y_s(0)$ .

Now consider the case where the borrower would default so the (ND) constraint must be imposed.

RESULT 1: There is a critical state  $c$  such that  $z'_q$  is constant for  $q=1,2,\dots,c$  and  $U(z'_q)=u(y_q)+\alpha u(y^*)/(1-\alpha)$  for  $q=c+1,c+2,\dots,S$ .

PROOF: Let  $U(z'_{c+1})=u(y_{c+1})+\alpha u(y^*)/(1-\alpha)$ . By (ND), for  $q>c+1$ ,  $U(z'_q)>U(z'_{c+1})$ . But then from (FO)  $\mu_q>\mu_{c+1}\geq 0$ . And by assumption  $\mu_q=0$  for  $q<c+1$  so  $U'(z'_q)=u(z+b)$  independent of  $q$  for  $q<c+1$ .

RESULT 2: Let  $c$  be the critical state at date  $t$  and suppose state  $s$  occurs at date  $t$ . Let  $m=\max(c,s)$ . Then  $z'_q=z_s$  for  $q=1,2,\dots,m$ , and  $z'_q=z_q$  for  $q=m+1,m+2,\dots,S$ .

PROOF: Consider  $q=1,2,\dots,m$ , and suppose  $\mu_q>0$ . Then  $z'_q\leq z_q\leq z_s$  by (ND) and Result 1. But from (FO) and (E)  $z_s<z'_q$ , a contradiction. So  $\mu_q=0$  and  $z'_q=z_s$ . Next consider  $q=m+1,m+2,\dots,S$  and suppose  $\mu_q=0$ . Then  $z'_q=z_s$  and by Result 1  $z_q>z_s$ . But then  $z_q>z'_q$  which violates (ND) since  $U(z_q)=u(y_q)+\alpha u(y^*)/(1-\alpha)$  by assumption. So  $\mu_q>0$  and  $z'_q=z_q$ .

Result 1 shows the borrower is constrained by the no-default condition when income is 'high'. This is to be expected, 'high' income states will be associated with net repayments. In 'low' income states net income, and hence consumption, is stabilised.

Result 2 shows how 'high' income states are defined. If the critical state at date  $t$  is  $c$  and state  $s$  occurs then the critical state at date  $t+1$  will be the maximum of  $c$  or  $s$ .

It only remains to determine the critical state at date  $t=0$ . Let  $D_s$  be the maximal repayment it is optimal to have in state  $s$  and let  $Z_s$  be the corresponding net income, that is  $U(Z_s)=u(y_s)+\alpha u(y^*)/(1-\alpha)$ . At date  $t=0$ ,  $z_s=y_s(0)$  so the critical

state,  $c$  at date  $t=0$  when income is  $y_s(0)$  is the lowest state  $c$  such that  $y_s(0) \leq Z_{c+1}$ .

COROLLARY Let  $c'_q$  be next periods consumption in state  $q$ . Then  $c'_q = c_s$  for  $q=1,2,\dots,m$  and  $c'_q = c_q$  for  $q=m+1,m+2,\dots,S$ .

PROOF: From (0),  $b$  is a function of  $z$ . Let  $b_q = b(z_q)$ . Then  $c'_q = z'_q + b(z'_q)$ . So for  $q \leq m$ ,  $c'_q = z_s + b_s = c_s$ , and for  $q \geq m+1$ ,  $c'_q = z_q + b_q = c_q$ .

RESULT 3: Consumption is non-decreasing in income:  $0 \leq dc/dz \leq 1$ .

PROOF: Using Result 2 in (0),  $dc/dz = (1 - \alpha \sum_{q \leq m} p_q)$ .

Result 3 says consumption never decreases. If today's income is higher than yesterday's then consumption increases but by the smallest possible amount to prevent the borrower defaulting. If today's income is lower than yesterday's then consumption remains the same.

In the long run consumption will be perfectly stabilised. Let  $c^*$  be the long run value of consumption and define  $z^* = \max(y_s(0), Z_s)$ .

PROPOSITION 1: In the long run  $c^* = \alpha y^* + (1-\alpha)z^*$ . There exists an  $\alpha^*$  and an  $\alpha_* < \alpha^*$  such that for  $\alpha > \alpha^*$ ,  $z^* = y_s$ ,  $s=1,2,\dots,S$ , and for  $\alpha < \alpha_*$ ,  $z^* = Z_s$ .

PROOF: Since state  $S$  must occur in finite time, the long run value of net income is  $z^*$ . So by (F),  $c^* = \alpha y^* + (1-\alpha)z^*$ . From (0),  $u(\alpha y^* + (1-\alpha)Z_s) = \alpha u(y^*) + (1-\alpha)u(y_s)$ . So for  $\alpha$  sufficiently small,  $Z_s \geq y_{s-1} \geq y_{s-2} \geq \dots \geq y_1$ . As  $u(c)$  is concave,  $y_s - Z_s > \alpha(y^* - y^*) / (1-\alpha) > 0$ . So for  $\alpha$  sufficiently large,  $Z_s \leq y_1 \leq y_2 \leq \dots \leq y_s$ .

For  $\alpha$  sufficiently large the no-default constraints have no impact. However for  $\alpha < \alpha^*$ , at least for some initial income levels, long run consumption will be higher than it would have been without the no-default constraints. Of course if  $\alpha$  is relatively low this long run consumption is discounted quite a lot and in the short run consumption will be lower.

As an example consider the stochastic model of Eaton and Gersovitz (1981) in which income is either  $1+\sigma$  or  $1-\sigma$  with equal probability. Let  $p=y^*-y_*$  be the risk premium. If the borrower has constant absolute risk aversion with a coefficient of  $A$  and if  $\sigma$  is small,  $p \approx \frac{1}{2}A\sigma^2$ . Define  $r$  by  $u(\alpha y^* + (1-\alpha)y_2 - r) = \alpha u(y^*) + (1-\alpha)u(y_2)$ . Then  $r \approx \frac{1}{2}\alpha(1-\alpha)A(p+\sigma)^2$  and  $Z_2 = (1+\sigma) - (r+\alpha p)/(1-\alpha)$ . Therefore  $Z_2$  is decreasing in  $\alpha$  and  $A$  but may be increasing or decreasing in  $\sigma$ . When either  $\alpha$  or  $A$  is high the borrower is willing to pay a lot for insurance and so consumption is stabilised at a relatively low level. But the effect of an increase in income variability is ambiguous because although an increase in  $\sigma$  makes the borrower more willing to pay for stability it makes its attainment more difficult.

#### 4. CONCLUSION

Since the model is so simple it is worth sketching out some of its deficiencies and possible extensions. There are a number of interesting problems not taken into account. The lender is assumed to be able to observe the borrower's income, so there is symmetric information. A natural extension would be to assume there is asymmetric information and that the bank cannot observe

income. The bank is also assumed to know the total indebtedness of the borrower but this is probably unrealistic (for example, the federal government of Nigeria had difficulty in keeping track of the borrowings of its own member states). There is also no role in the model for renegotiating or rescheduling of debt because no new information ever becomes available. Neither has the denomination of the debt been considered, everything is in real terms, whereas most debt is denominated in dollars to prevent the borrowers servicing their debt by increasing their inflation rates. Nor is there any place for capital in the model. A useful extension might be along the lines of Kletzer (1984) who has foreign capital as a distinct factor of production in the domestic economy. It is hoped that the model presented here may be of some help in examining these more exacting issues.

#### REFERENCES

- Eaton, J. and M. Gersovitz (1981), 'Debt with potential repudiation: theoretical and empirical analysis', *Review of Economic Studies*, 48, p289-309.
- Eaton, J., Gersovitz, M. and J.E. Stiglitz (1986), 'The pure theory of country risk', *European Economic Review*, 30, p481-513
- Kletzer, K.M. (1984), 'Asymmetries of information and LDC borrowing with sovereign risk', *Economic Journal*, 22, p287-307.
- Thomas, J. and T. Worrall (1988), 'Self-enforcing wage contracts', *Review of Economic Studies*, *forthcoming*.