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**Working Paper**

## Foreign debt and capital accumulátion

Diskussionsbeiträge - Serie II, No. 28

**Provided in Cooperation with:**

Department of Economics, University of Konstanz

*Suggested Citation:* Siebert, Horst (1987) : Foreign debt and capital accumulátion, Diskussionsbeiträge - Serie II, No. 28, Universität Konstanz, Sonderforschungsbereich 178 - Internationalisierung der Wirtschaft, Konstanz

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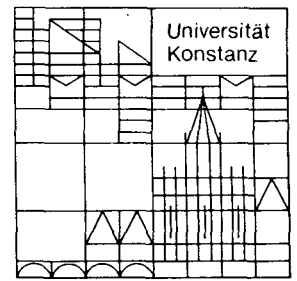
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**Sonderforschungsbereich 178**  
**„Internationalisierung der Wirtschaft“**

Diskussionsbeiträge



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wissenschaften und Statistik

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Horst Siebert

**Foreign Debt and Capital Accumulation**

FOREIGN DEBT AND CAPITAL ACCUMULATION

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Serie II - Nr. 28

Juli 1987

A 9 3304 / 87 Weltwirtschaft  
Kiel  
76

## Foreign Debt and Capital Accumulation

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Capital accumulation is at the core of economic development. An increase in the capital stock allows a country to move upward along its production function or, if new technologies are incorporated into the new capital goods, to shift its production function to a higher level. Historical experience suggests that any process of industrialization must be driven by accumulating capital. This holds for production capital in the industrial base as well as for human capital. Moreover, social overhead capital like infrastructure in transportation is needed to make use of economies of scale and to increase the size of markets. In a closed economy, capital accumulation can only be generated from postponing consumption. In an open economy<sup>1)</sup>, a country may borrow initially, however only at the opportunity costs of paying interest and repaying the debt in future periods.

A country will accumulate capital by postponing consumption if the benefit from not consuming, that is the marginal productivity of capital, exceeds the costs of waiting, that is the time preference rate. When foreign borrowing is introduced, it is worth while to finance the capital stock by debt if the marginal productivity of capital exceeds the interest rate, that is the cost of borrowing. Moreover, if the time preference rate is greater than the interest rate on debt, it is worth while to borrow for consumption and repay the debt later. Compared with capital accumulation from postponing consumption, foreign borrowing will then allow a higher consumption profile in the initial periods and hence a higher present value of consumption; capital accumulation via borrowing clearly is better than through postponement of consumption. However, in future periods the opportunity costs of borrowing become

apparent, and future generations may have different perceptions on the benefits and the opportunity costs of borrowing.

In this paper, we compare the case of foreign borrowing with a scenario in which the country accumulates capital by postponing consumption. We study the optimal level of foreign debt and of the capital stock. We also point out the impact of debt on the time profile of consumption of the debtor country. In the usual two-period analysis (Chipman 1985), the long time of debt repayment and debt service is collapsed into the second period only. In contrast to this approach, we use a continuous time model which allows to analyze the properties of the long time of debt service and debt repayment (Bardhan 1967, Sachs 1984). We show under which conditions borrowing yields a higher present value of welfare relative to the non-borrowing case. Moreover, we discuss some phenomena that may make it more difficult for the debtor country to pay interest and repay debt.

### I. A Model of Borrowing and Accumulation

Consider an economy that maximizes the present value of utility  $W$  from consumption  $C$

$$\omega = \int_0^T e^{-\delta t} W(C) dt \quad (1)$$

In order to simplify the notation, time indices of variables are omitted. In equation 1,  $\delta$  is the country's given time preference rate,  $T$  is terminal time and the period utility function has the usual properties

$$W_C > 0, W_{CC} < 0 \quad \text{and} \quad \eta = - \frac{C W_{CC}}{W_C} > 0 \quad (2)$$

Terminal time  $T$  can be assumed to be given exogenously or it can be determined endogenously. Output  $Q$  is produced with capital  $K$  and labor  $L$  according to a neoclassical production function

$$Q \leq F(K, L) \text{ with } F_j > 0, F_{jj} < 0, F_{ij} > 0 \text{ for } i, j = K, L \quad (3)$$

The labor supply in each period is given and constant over time; also the initial capital stock  $K_0$  is given. The capital stock changes over time according to

$$\dot{K} = I - dK \quad (4)$$

where  $I$  is gross investment and  $d$  is the rate of depreciation. A dot over a variable denotes a change of a variable over time, a hat is the rate of change.

Consumption in any period is equal to production minus gross investment minus interest payment on debt ( $rB$ ) plus borrowing ( $\dot{B}$ )


$$C = Q - rB - I + \dot{B}$$

Debt,  $B$ , and the change in debt,  $\dot{B}$ , are expressed in units of commodity  $Q$ , and  $r$  is the real interest rate assumed to be given. The country is small in the world capital market.

Rearranging, we have

$$\dot{B} = C + I - Q + rB \quad (5)$$

where the right hand side denotes the deficit (or, if negative, the surplus) in the balance of trade and services and the left hand side indicates the balance in the capital account with  $\dot{B} > 0$  standing for capital imports (and  $\dot{B} < 0$  capital exports, i.e. repayment of debt).

In terminal time  $T$ , debt must be repaid or financial assets are accumulated 

$$B(T) \leq 0 \quad (6)$$

Equation 5 is the budget constraint for each period; equation 5 and 6 together represent an intertemporal or dynamic budget constraint (Glick and Khavar 1986).

In terminal time, the capital stock must be nonnegative

$$K(T) \geq 0 \quad (7)$$

Applying the classification of Glick and Khavar (1986), the model describes the problem of an optimizing borrower. Non-optimizing behavior modelled by specific behavioral consumption functions including public choice approaches to government spending or rigidities preventing optimality of consumption and investment over time is not considered. Also, we do not study joint maximization problems of the borrower and the lender or strategic behavior (Sachs 1984). The borrower is assumed to be a small country having no impact on the interest rate of the world capital market; moreover, there is no feed back of the amount of debt on the interest rate (small country assumption) or on additional funds available to the country. Default risk or country risk issues are not considered. Finally, the optimizing model does not explicitly analyze intertemporal market equilibria as in overlapping generation models (Buiter 1981, Schmid 1987).

The Lagrangean function  $Z$  is given by

$$\begin{aligned} Z = & W(C) + \lambda [I - dK] \\ & + \rho [C + I - Q + rB] \\ & + \mu [F(K, L) - Q] \end{aligned} \quad (8)$$

Optimality conditions are

$$\frac{\partial Z}{\partial C} = 0 \quad \Rightarrow \quad W_c = -\rho \quad (8a)$$

$$\frac{\partial Z}{\partial I} = 0 \quad \Rightarrow \quad \lambda = -\rho \quad (8b)$$

$$\frac{\partial Z}{\partial Q} = 0 \quad \Rightarrow \quad \mu = -\rho \quad (8c)$$

$$\dot{\lambda} = \delta \lambda - \frac{\partial Z}{\partial K} \quad \Rightarrow \quad \dot{\lambda} = \delta \lambda + d\lambda - \mu F_K \quad (8d)$$

$$\dot{\rho} = \delta \rho - \frac{\partial Z}{\partial B} \quad \Rightarrow \quad \hat{\rho} = \delta - r \quad (8e)$$

Defining a J-function according to Long and Vousden (1977), we have

$$J = \xi B(T) + \phi K(T) \quad (9)$$

with

$$\xi = e^{-\delta T} \rho(T) \quad (9a)$$

$$\xi B(T) = 0 \quad (9b)$$

$$\phi = e^{-\delta T} \lambda(T) \quad (9c)$$

$$\phi K(T) = 0 \quad (9d)$$

$$e^{-\delta T} H(T) = 0 \quad (9e)$$

## II. The Time Profile of Consumption and Debt

Consumption in any period of time is optimal if the marginal utility of consumption is equal to the shadow price of debt or the user costs of borrowing,  $-\rho$ . Again the user costs of borrowing must be equal to the shadow price of capital accumulation,  $\lambda$ , and the shadow price of output,  $\mu$ .

$$W_c(C) = -\rho = \lambda = \mu \quad (10)$$

Consumption changes over time according to

$$\hat{C} = -\frac{1}{\eta} \hat{\rho} \quad (10a)$$

Assume  $\delta > r$ . Then  $\hat{\rho} = \dot{\rho}/\rho > 0$ , that is  $\dot{\rho} < 0$  and  $-\dot{\rho} > 0$  so that  $\rho$  rises in absolute terms, and consumption is reduced over time.

From equation 8b, we have  $\hat{\lambda} = \hat{\rho}$  so that from equations 8c, 8d and 8e we have



$$r = F_K (K^*) - d \quad (10b)$$

Equation 10b defines the optimal capital stock,  $K^*$ . A capital-poor country<sup>2)</sup>, with  $K_0 < K^*$  and  $F_K (K_0) > F_K (K^*)$ , will accumulate capital. Since equation 10b holds for any period, it must also hold for the first period. This implies that the optimal capital stock must be established right from the start. In the model, there is no restraint on accumulation except the balance of payment. For instance, there is no time involved in accumulating capital. And there are no physical restraints. Thus, accumulation is possible instantaneously in the first period due to borrowing.

Equation 10b is illustrated in Figure 1 where  $\tan \alpha$  is  $r + d$ ,  $K^*$  is the optimal capital stock and  $K_0$  is the initial capital stock<sup>3)</sup>.  $K^* - K_0$  represents debt incurred for capital accumulation.

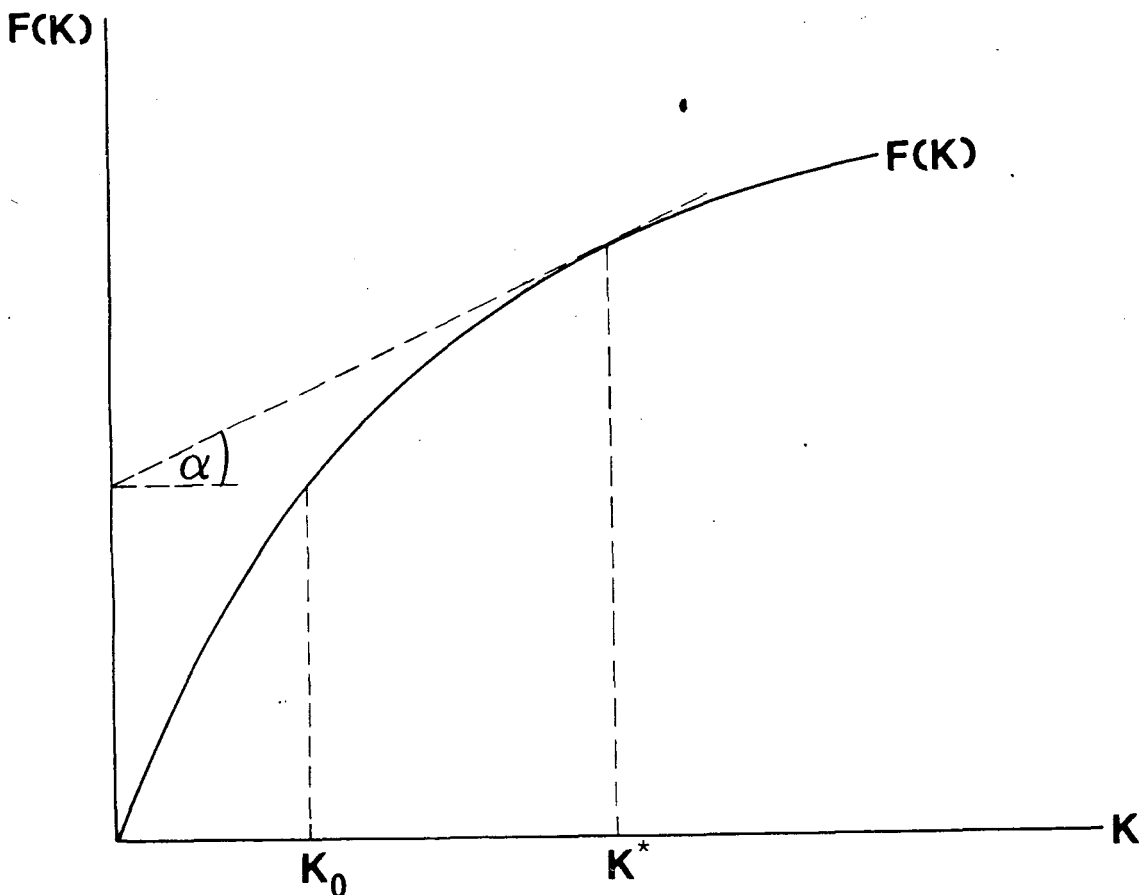


Figure 1

Once the optimal capital stock is reached, there is no net investment. Gross investment is a constant fraction of the constant (optimal) capital stock,  $dK^*$ . Output is completely specified by the production function, and it remains constant.

Equation 5 can now be written as

$$\tilde{Q} = F(K^*, L) - dK^* = C + rB - \dot{B} = C + N \quad (11)$$

where net output  $\tilde{Q}$  on the left hand side is a constant. With consumption falling over time due to equation 10a, the term  $rB - \dot{B} = N$  must rise over time, that is  $\dot{N} = -\dot{C} > 0$  (for  $\delta > r$ ).

With the capital stock being accumulated instantaneously in the first period, borrowing for capital accumulation will occur in the first period. Borrowing for consumption, however, may be stretched over several periods initially. Consider the case, in which the initial capital stock  $K_0$  is identical to the optimal capital stock  $K^*$ . Then, with  $\delta > r$ , borrowing will occur for consumption.

Initially we have

$$\tilde{Q} < C \Leftrightarrow N < 0 \Leftrightarrow rB - \dot{B} < 0 \quad (11a)$$

and thus  $r < \dot{B}/B$ . This implies, that debt increases in the initial period(s). The increase in debt is larger than the interest payment so that interest is paid through new debt.

Over time, consumption is reduced, and  $N$  is increased, that is  $N$  becomes smaller in absolute terms. Eventually, we must have

$$\tilde{Q} = C \Leftrightarrow N = 0 \Leftrightarrow rB = \dot{B} \quad (11b)$$

so that  $r = \dot{B}/B$  and debt rises with the interest rate. Due to the continuing reduction in consumption,  $N$  continues to rise, now becoming positive.

$$\tilde{Q} > C \Leftrightarrow N > 0 \Leftrightarrow rB - \dot{B} > 0 \quad (11c)$$

and  $r > \dot{B}/B$ . The increase in debt becomes smaller, and eventually in period  $t'$  we have

$$\dot{B} = 0 \Leftrightarrow \tilde{Q} = C + rB \quad (11d)$$

so that net output is sufficient for consumption and interest payment. From  $t'$  on,  $B$  must fall so that  $\dot{B} < 0$ .

Note that we have from equation 11

$$\ddot{B} = r\dot{B} + \dot{C} < 0 \quad \text{for } t > t' \quad (11e)$$

so that  $B$  is a concave function and does not fall asymptotically to zero. In terminal time, the amount repaid is not infinitesimally small.

The consumption profile is steered by the shadow price  $\rho$ . We have  $\hat{\rho} = \hat{\lambda}$  and for  $\delta > r$  consumption is reduced over time. This decline in consumption is engeneered by the change in the shadow price  $\rho$  with the initial  $\rho_0$  determining the initial

level of consumption. With  $\delta > r$  it is worthwhile to borrow initially in order to reach a high consumption level.

The shadow price  $\rho_0$  in the initial period plays a strategic role in determining the initial level of consumption. Start from the case in which the initially given capital stock is equal to the optimal capital stock and then reduce the initially given stock parametrically. The country has to borrow more for capital accumulation thus incurring a higher debt. Consequently,  $\rho_0$  must rise in absolute terms allowing a lower level of consumption initially and a lower consumption profile over time.

Assume a fixed finite planning horizon  $T < \infty$ . Then the transversality conditions 9a and b together with a rising  $-\rho$  and thus a positive  $-\rho(T) > 0$  imply that  $B(T) = 0$ . Debt has to be zero in  $T$ . As a matter of fact, the shadow price  $-\rho(T)$  in terminal time and the equation of motion of this shadow price determine the implicit value of borrowing in the initial period and thus specify the initial level of consumption and its time profile.

With  $\lambda(T) > 0$ , the capital stock must be zero in finite terminal time due to the transversality condition 9d. Due to the optimality condition 10b, the capital stock remains constant up to terminal time  $T$ . But due to the monotony in the consumption profile a discontinuity in the consumption profile in  $T$  is ruled out. The capital stock cannot be used up for consumption in  $T$ . It is used to repay debt. For finite terminal time, the capital stock and debt are zero.

Extending  $T$  further and further into the future, this result will hold. The shadow price  $-\rho(T)$  continues to drive the

system. With  $B$  falling over time,  $rB$  will fall, but debt repayment will rise over time.

When  $T$  is determined endogenously, and if  $T < \infty$ , the transversality condition 9e simplifies to<sup>4)</sup>

$$e^{-\delta T} \{W(C) - W_C C\} = 0. \Leftrightarrow W_C = W(C)/C \quad (12)$$

This implies that  $C(T)=0$ . Condition 12 can only be reached in finite time if  $W_C(0) < \infty$ . With the usual assumption that the marginal utility of consumption is infinite with  $C$  approaching zero,  $C(T) = 0$  cannot be reached in finite time<sup>5)</sup>.

Figure 2 spells out the long-run balance of payments implications of foreign borrowing. Consumption  $C_b$  declines<sup>6)</sup> due to a relatively high time preference rate, that is  $\delta > r$ .

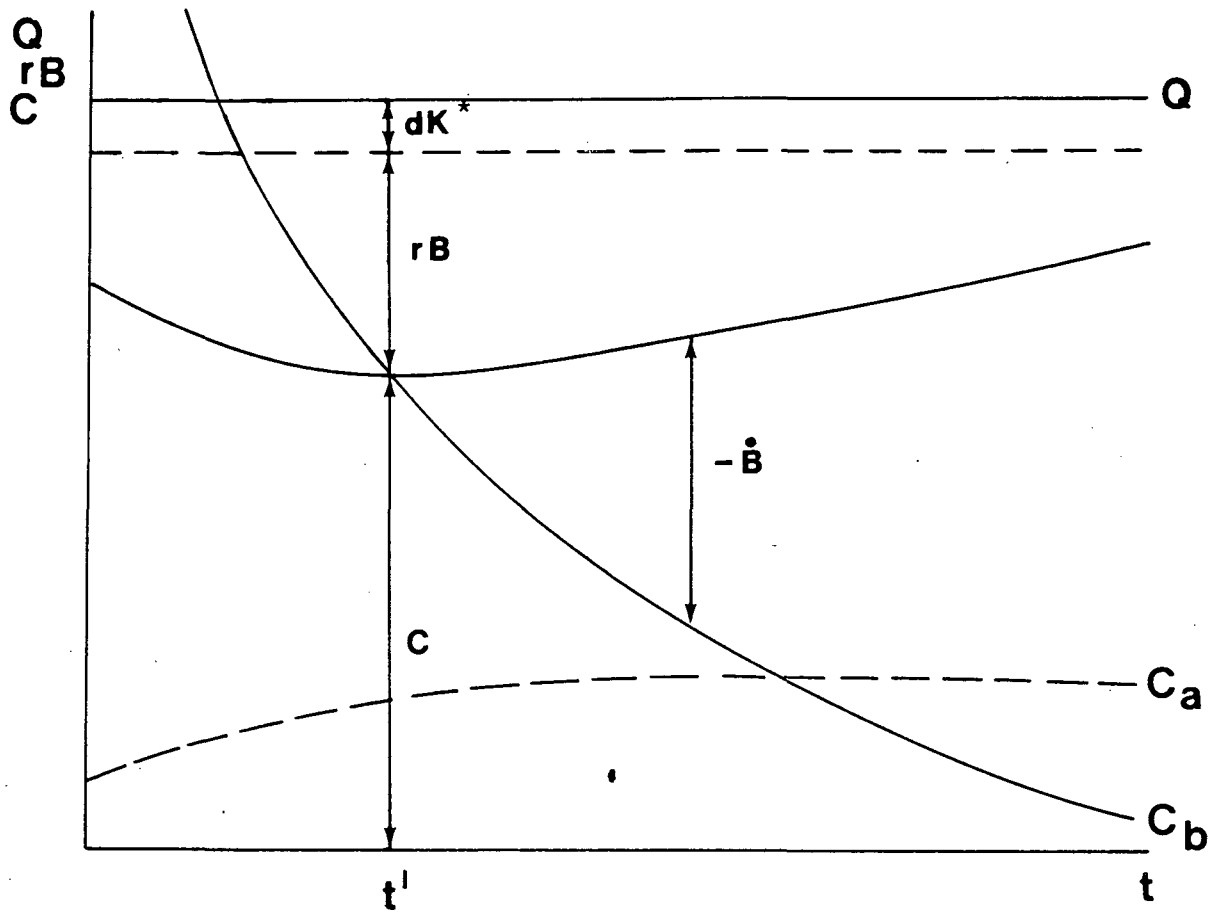


Figure 2

Capital accumulation occurs instantaneously in the first period; and debt rises for some periods to finance consumption. Borrowing is reduced, and it stops in  $t'$ . From then on, debt is repaid and an increasingly larger part of output is being used up for interest payment and the repayment of debt.

The term  $N = rB - \dot{B}$  in equation 11, that is interest payment and repayment of debt, can also be interpreted as the trade balance. Prior to  $t'$ ,  $N < 0$  is the import deficit; after  $t'$ ,  $N > 0$  denotes the export surplus. Since the term  $N$  rises over time the country has to reduce its import deficit over time prior to  $t'$ ; after  $t'$ , the country has to generate a rising export surplus over time. The model thus specifies the time profile of the balance of trade being driven by the shadow price of debt.

The country will not borrow for consumption if the interest rate is higher than the time preference rate, i.e.  $r > \delta$ . Then  $\hat{p} < 0$ , and consumption increases over time. This implies a high  $p$  in absolute terms in the initial situation in order to keep initial consumption down. If we have  $F_K(K_0) - d > r > \delta$ , capital will be accumulated because the benefit of accumulating capital outweighs its opportunity costs. The accumulation occurs instantaneously due to the condition 10b. This implies borrowing for capital accumulation<sup>7)</sup>. Due to the low consumption profile initially, the country actively contributes to the accumulation of capital by postponing consumption. This consumption profile reduces the amount to be borrowed.

### III. Capital Accumulation without Borrowing

Why does a country borrow in the first place? Can it improve its welfare by not only relying on capital accumulation from internal saving? If we study capital accumulation in a closed economy, equation 5 simplifies to  $C=Q-I$ , that is, consumption

is given by output minus gross investment, and the equation of motion for the capital stock changes into

$$\dot{K} = Q - C - dK \quad (13)$$

The Lagrangean then is given by

$$\begin{aligned} Z = W(C) + \lambda [Q - C - dK] \\ + \mu [F(K, L) - Q] \end{aligned} \quad (14).$$

with the optimality conditions

$$\begin{aligned} \frac{\partial Z}{\partial C} = 0 & \Rightarrow W_C = \lambda \\ \frac{\partial Z}{\partial Q} = 0 & \Rightarrow \lambda = \mu \\ \dot{\lambda} = \delta \lambda - \frac{\partial Z}{\partial K} & \Rightarrow \hat{\lambda} = \delta + d - F_K \end{aligned}$$

For a capital poor country, that is  $F_K(K_0) > \delta + d$ ,  $\lambda$  falls, that is consumption increases over time. The country has to put a high penalty on consumption initially, allowing more consumption with capital being accumulated.

The optimal capital stock now has to be interpreted as the steady state capital stock being reached in the long run when

$$\delta + d = F_K(\tilde{K}^*).$$

Without borrowing, the capital-poor country will accumulate a smaller capital stock if we assume  $\delta > r$ . Now the time preference rate (assumed to be identical in the borrowing and in the non-borrowing case) determines the long run capital stock. Moreover, the capital-poor country cannot accumulate the (smaller) capital stock instantaneously as in the borrowing case, but it needs time. Finally, it must start at a lower consumption level initially, using a higher penalty on consumption in order to induce internal capital accumulation. Its consumption path ( $C_a$  in Figure 2) is on a lower level for quite a few of the earlier periods.



The advantage of borrowing is to allow a higher present value of welfare from consumption<sup>8)</sup>. The higher time profile of consumption in the initial periods outweighs the reduced consumption possibilities of future periods.

#### IV. What can go Wrong?

Borrowing implies restraints for future periods in terms of giving up goods (or generating a balance of trade surplus) in order to pay interest and repay debt. In our model of optimal borrowing, the opportunity costs of borrowing are, of course, taken into account. A country will only incur a debt, if the marginal benefit from a unit borrowed will be equal to the user cost of borrowing.

In the real world, quite a few phenomena can affect this intertemporal optimality condition so that the initially chosen time profiles of consumption and debt repayment will no longer be followed. Including such phenomena into our framework allows a richer analysis.

A high time preference rate increases the divergence between the time profile of consumption in the borrowing and the non-borrowing case. A high discount rate is the expression of the country's impatience, and impatience may be caused by a high population pressure, by high aspiration levels and by political demands of voters or groups of society. Such an impatience implies a low weight on user costs, and it thus allows a high level of consumption initially, that is a relatively low incentive for capital accumulation. In the borrowing case,  $-\rho$  has to rise more quickly over time. Of course, the burden for future periods will be increased. In the case of internal capital accumulation, a smaller capital stock will be accumulated. The  $C_a$ -curve will shift upwards for initial periods, but the consumption level will be lower later on.<sup>9)</sup>

A low stock of capital initially increases the amount to be borrowed, and a higher borrowing implies a higher penalty on consumption both initially and in later periods. The country will be on a lower consumption profile.

In the optimization model, the marginal productivity of capital is known, but in reality, it is an expected variable. If expectations on the marginal productivity of capital do not materialize as in Poland in the seventies, the country is stuck with interest and principal payments as shown in Figure 2, but it cannot generate the output  $Q$ . Thus, either the consumption path must be suppressed or debt service will be curtailed. Of course, the strategy of capital accumulation as a means for economic development is not too promising if the country's technology is characterized by a production function on a low(er) level (than in Figure 2) suggesting only a small accumulation opportunity.

A similar argument relates to the terms of trade. In the model, the country borrows in real terms and repays in real terms, so that the terms of trade (in an intertemporal sense) are set equal to unity remaining constant over time. The analysis can be made richer by assuming that the country borrows foreign currency and by introducing a price for its export commodity  $Q$  in terms of foreign currency. If this price is reduced over time, lower foreign currency earnings are available to finance debt service ( $rB - B$ ).

In the model the amount borrowed is used instantaneously for accumulating capital (and for consumption). This transfer of debt into physical capital may be interrupted at several instances, and funds borrowed may be used for consumption. This occurs for instance when the time preference rate increases. Funds intended for capital accumulation may even be used for capital exports. In these cases, not enough real output is generated in the future necessary for the debt service.

If contrary to the model, the real rate of interest does not remain constant, for instance if the country has borrowed at a floating rate, the costs of borrowing rise increasing the interest burden, but the country does not produce a sufficiently high output to service the debt.

Equity capital is an alternative to foreign borrowing. Of course, equity capital implies dividend payments, and in the seventies developing countries did not want to see this transfer occur. But at the same time, dividend payments have the advantage of not implying a constant obligation to pay. They allow more flexibility with respect to some of the unfavorable events discussed above. With equity capital, the country can shift the risk of unfavorable events to the foreign firm. Note, however, that foreign firms will anticipate these risks, and if the risks are sufficiently high, the country may not be able to attract foreign direct investment. Moreover, in addition to economic risks, the foreign firms will consider political risks such as changes in taxation, regulation and property rights.

Finally, the phenomena that make it more difficult for the debtor country to pay interest and repay debt may lead to a breach of contracts (Sachs 1984). As is illustrated from Figure 2, with the passage of time, the utility derived from borrowing will be reduced and the opportunity costs will rise. At a moment of time  $t$  sufficiently far away from the initial period the burden of borrowing,  $rB - \dot{B}$ , may seem high whereas utility derived from consumption may appear low. The political process tends to have a short memory, and in the perceptions of the debtor country the advantage of the time profile  $C_b$  over  $C_a$  is forgotten.

## Footnotes

\* This paper was stimulated by discussions in the course "International Resource Management" of the Kiel Advanced Studies Program. I acknowledge comments from the participants as well as from G. Franke, W. Krelle, E. Mohr, H.H. Nachtkamp, M. Rauscher and I. Walter.

1) On the case in which an open economy transforms its initially given resource stock into capital compare Siebert (1985), Chap. VI.

2) A capital rich country with  $K_0 > K^*$  will reduce its capital stock if it can be used up for consumption.

3) The steady state properties with respect to the optimal capital stock can also be illustrated with the traditional C-K-phase diagram. The implications of the model can also be analyzed by the overall optimality condition

$$\delta - \hat{W}_c = r = F_K(K) - d$$

which follows from 8a - e. The consumption rate of discount  $\delta - \hat{W}_c$  must be equal to the interest rate which must be equal to the net productivity of capital. The consumption profile over time can easily be derived from this overall optimality condition. For instance, let  $\delta > r$ . Then  $\hat{W}_c > 0$  which from  $\dot{W}_c = W_{cc} \dot{C}$  implies  $\dot{C} < 0$ . The above condition can also be interpreted as a market equilibrium condition where the left hand side models the saving behavior of households and the right hand side represents the investment behavior of firms. An intertemporal equilibrium can be characterized by a time path of the interest rate driving the consumption and the investment decisions. I owe this view to H.H. Nachtkamp.

4) The transversality condition for optimal  $T$  is

$$e^{-\delta T} \{W(C) + \rho (dK + C - Q + rB)\} = 0$$

which together with conditions 8a, 9b and 9d implies 12.

5) Allowing terminal time to approach infinity, the transversality condition for optimal time  $T$  is

$$\lim_{T \rightarrow \infty} \{e^{-\delta T} [W(C) - CW_c] + re^{-\delta T} \rho(T)B(T) + \rho_0 e^{-rT} (dK - Q)\} = 0$$

With  $\rho_0 = e^{-(\delta-r)T} \rho(T)$ . The third term approaches zero since  $dK - Q$  is a constant. Moreover, we have  $W/C > W_c$  for  $C > 0$  in the case of a concave utility function. Since  $CW_c > 0$ , we have

$$W > W - CW_c > 0.$$

For  $C \rightarrow 0$ ,  $W \rightarrow 0$  and thus  $W - CW_c \rightarrow 0$ . Therefore the transversality condition is

$$\lim_{T \rightarrow \infty} e^{-\delta T} H(T) = \lim_{T \rightarrow \infty} e^{-\delta T} (T)rB(T) = 0$$

which implies

$$r \rho_0 \lim_{T \rightarrow \infty} e^{-rT} B(T) = 0$$

This can only be satisfied if the term converges so we must have  $\hat{B} < r$ . This is consistent with  $B(T) = 0$ , but for  $T \rightarrow \infty$  we may also have  $B(T) > 0$ . With  $T \rightarrow \infty$ , the penalty  $\rho(T) < 0$  loses its bite. Note that the transversality condition is consistent with  $W_c(0) \rightarrow \infty$ .

For  $T \rightarrow \infty$ , the transversality condition for the capital stock is

$$\lim_{T \rightarrow \infty} \lambda_0 e^{-rT} K(T) = 0$$

Equation 10b continues to require a constant optimal capital stock so that  $K(T) = K^*(T) > 0$  is consistent with the transversality condition. For  $T \rightarrow \infty$ , the capital stock is not used to repay debt.

6) A sufficient condition for a convex function  $C(t)$  is  $\dot{C} < 0$  and  $\ddot{C} > 0$ . From equation 8a we have

$$\dot{C} = \frac{W_c}{W_{cc}} (\delta - r)$$

and

$$\ddot{C} = (\delta - r) \frac{W^2_{ccc} - W_c W_{cccc}}{W^2_{cc}} \dot{C}$$

For  $\delta > r$ , we have

$$\frac{W_c W_{cccc}}{W_{cc}^2} > 1 \quad \Leftrightarrow \quad \ddot{C} > 0$$

This condition is satisfied, for instance, for an isoelastic utility function.

7) In this case we have from equation 11 that

$$C < \tilde{Q} \quad \Leftrightarrow \quad N > 0 \quad \Leftrightarrow \quad r > \dot{B}/B \quad (i)$$

because the consumption level is low initially. From 11 we also have  $\dot{N} = -\dot{C}$  and consequently

$$\ddot{B} = r\dot{B} + \dot{C} \quad (ii)$$

We can rule out with  $\dot{B} = 0$  because this implies

$$\tilde{Q} = C + rB$$

Over time  $C$  rises, so that with  $\tilde{Q}$  constant,  $B$  has to fall which is impossible for  $\dot{B} = 0$ .

We can also rule out  $\dot{B} > 0$ . Then ii implies  $\ddot{B} > 0$  for rising  $C$ . Debt would rise progressively over time, but the transversality condition implies  $B(T) = 0$ . Consequently  $\dot{B} < 0$ .

Note that this case does not reach a steady state for consumption due to the repayment of debt.  $\rho$  falls in absolute value over time so that consumption can be increased from period to period.

8) A lower present value of welfare in the borrowing case (for  $\delta > r$ ) relative to the non-borrowing case clearly would contradict optimality because in the borrowing case the country can decide not to borrow. If it decides to borrow as it will for  $\delta > r$ , borrowing increases its welfare.

9) Note that if we reduce the time preference rate parametrically and if we make it sufficiently low, we may have  $\delta < r$ , and the time profile of consumption in the borrowing case, i.e. the  $C_b$ -curve, will have a similar shape as the  $C_a$ -curve.  $\hat{\rho} < 0$ , and consumption increases. From  $\hat{\rho} = \hat{\lambda}$  we have  $\delta - r = \delta + d - F_K(K^*)$  which implies  $\delta + d > F_K(K_0)$  so that the country has become a capital-rich country.

References

- Bardhan, P.K. (1967), Optimum Foreign Borrowing, in: K. Shell (Ed.), Essays in Theory of Optimal Economic Growth, Cambridge, MA., 117-128.
- Buiter, W.H., Time Preference and International Lending and Borrowing in an Overlapping-Generations Model; Journal of Political Economy 1981, 89, 769-797.
- Chipman, J.S. (1985), A Two-Period Model of International Trade and Payments, Working Paper No. 194, Konstanz.
- Glick, R. and Khavar, H.J (1986), The Costs and Benefits of Foreign Borrowing: A Survey of Multi-Period Models, Journal of Development Studies, 22, 279-299.
- International Monetary Fund (1986), International Capital Markets. Developments and Prospects, Washington, D.C.
- Long, N.V. and Vousden, N. (1977), Optimal Control Theorems, in: Pitchford, J.A. and Tumorsky, S.J. (Ed.), Application of Control Theory to Economic Analysis, Amsterdam, 11-34.
- Niehans, J. (1985), Internationale Kredite mit undurchsetzbaren Forderungen, in A. Gutowski (Ed.), Die internationale Schuldenkrise, Schriften des Vereins für Socialpolitik, N.F. Bd. 155, Berlin, 151-179.
- Sachs, J. (1984), Theoretical Issues in International Borrowing, Princeton Studies in International Finance, No. 54.
- Schmid, M. (1987), External Debt and the Wealth of Nations with Overlapping Generations, Working Paper, Hamburg.



Schneider, H. (1984), Optimales Wachstum und Auslandsverschuldung, ein Diskussionsbeitrag, Zürich, mimeo.

Siebert, H. (1985), Economics of the Resource-Exporting Country, Intertemporal Theory of Supply and Trade, Greenwich.

Summary

In an open economy, a country may borrow in order to accumulate capital. The paper compares the case of foreign borrowing with a scenario in which a country accumulates capital internally by postponing consumption. The impact of borrowing on the time profile of consumption and of debt is analyzed. The model also specifies the time profile of the balance of trade. The opportunity costs of borrowing are studied and conditions are specified in which it is worthwhile to borrow. In the context of the model, some phenomena are discussed that may make it more difficult for the debtor country to pay interest and repay debt and that may be an inducement to breach the credit contract.