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Working Paper

Taxation of income and wealth in a model of endogenous growth

Diskussionsbeiträge - Serie II, No. 227

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Suggested Citation: Grüner, Hans Peter; Heer, Burkhard (1994) : Taxation of income and wealth in a model of endogenous growth, Diskussionsbeiträge - Serie II, No. 227, Universität Konstanz, Sonderforschungsbereich 178 - Internationalisierung der Wirtschaft, Konstanz

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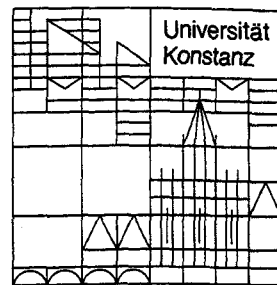
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Sonderforschungsbereich 178
„Internationalisierung der Wirtschaft“

Diskussionsbeiträge



Juristische
Fakultät

Fakultät für Wirtschafts-
wissenschaften und Statistik

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**Taxation of Income and Wealth
in a Model of Endogenous Growth**

**Taxation of Income and Wealth
in a Model of Endogenous Growth**

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Serie II - Nr. 227

Juni 1994

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Abstract

This paper explores the effects of inheritance taxation and income taxation on the growth rate in an endogenous growth model with altruistic parents. Human capital is accumulated according to the standard Lucas specification. The government raises taxes in order to finance a non-productive public good. The public good must be provided in a fixed proportion to output. Pure wealth taxation is proven to deliver a higher steady-state growth rate than any feasible mix of income and wealth taxation.

JEL classification: E62, O11

Key Words: Optimal Taxation, Endogenous Growth, Inheritance Taxation

1. Introduction

The determinants of economic growth, especially the impact of fiscal policy, have received increasing attention in a number of recent contributions such as Jones, Manuelli and Rossi (1993), Rebelo (1991), Rebelo and Stokey (1991), Barro and Sala-I-Martin (1992), Lucas (1990), King and Rebelo (1990) and Barro (1990). These articles analyze the short-run and long-run effects of both taxation and government expenditures on a representative individual's utility and on economic growth. The analysis in these papers, however, is restricted to the taxation of flow variables like consumption, investment or income from various factors of production and to lump-sum taxation. The effects of taxation of stocks of physical capital has so far received little attention. In this paper, we provide an analytical framework to examine the effects of simultaneous taxation of both income flows and stocks of physical capital. The objective of this paper is to determine the effect of the tax structure on the balanced growth rate, the size of the public sector and the capital stock. We also examine the short-run effects of a change of the tax structure on the development of these variables.

In our model, a representative individual lives for one period. The individual is assumed to be altruistic and, thus, leaves a positive amount of physical capital to his heir. Individuals can allocate their lifetime among leisure, education and work. Growth is generated by human capital accumulation. The government raises taxes in order to finance a given amount of a non-productive public good. Production and human capital accumulation are characterized by constant returns to scale. Two different effects have to be taken into account: on one hand, inheritance taxation creates a disincentive to accumulate physical capital. On the

other hand, altruistic parents are lead to invest into human capital for their children. Thus, the steady state capital coefficient of the economy is a decreasing function of the inheritance tax rate. From numerical solutions for the steady-state growth path we derive that the growth rate in the steady state is maximized by the application of pure inheritance taxation as long as the size of the public sector is sufficiently small. For a large public sector, however, income has to be taxed as well. Grüner (1994) shows that in a stationary setting without human capital accumulation, optimal fiscal policy exclusively consists of taxation of flows. Thus, the boundary solutions in the present model are due to the positive effects of a higher tax rate on stocks of physical capital on human capital accumulation.

2. The model

The representative individual lives for one period and leaves bequest to his heir. Bequests are taxed at the rate of τ . Time is allocated between leisure, f_t ; education, h_t ; and work, l_t . Labor-augmenting human capital, H_t , is accumulated at the beginning of the period according to the specification of Lucas (1988):

$$H_t = g(h_t, H_{t-1}) = Ah_t^\alpha H_{t-1}^{1-\alpha} \quad (1)$$

The individual receives income from efficient labor, $H_t l_t$, and from interest payments on the inherited stock of physical capital, K_t . Consumption takes place at the end of the period. The utility of an individual living in period 0 depends on his own utility and the utility of his heir and, thus, preferences can be represented by the following infinite-horizon utility function: Utility is logarithmic and additively separable:

$$U = \sum_{t=0}^{\infty} u_t(c_t, f_t) \beta^t, \quad \beta < 1. \quad (2)$$

$$u_t = \phi_1 \ln c_t + \phi_2 \ln (I - l_t - h_t) \quad (3)$$

In each period, the individual faces a budget and a time constraint:

$$I = l_t + h_t + f_t \quad (4)$$

$$c_t + \frac{K_{t+1}}{1-\tau} = (1-\theta)w_t H_t l_t + (1+(1-\theta)r_t)K_t \quad (5)$$

w_t , r_t , θ , τ denote wage per efficiency unit of labor, interest rate, income tax and inheritance tax, respectively.

The individual maximizes his utility function by the choice of h_t , l_t , H_t and K_{t+1} given the wage rate, interest rate, tax rates and inherited capital. The Lagrangian function corresponding to this problem is:

$$\begin{aligned} \frac{U}{\beta^t} = & \dots + \phi_1 \ln \left[(1-\theta)w_t H_t l_t + (1+(1-\theta)r_t)K_t - \frac{K_{t+1}}{1-\tau} \right] + \phi_2 \ln (I - l_t - h_t) \\ & + \phi_1 \cdot \beta \cdot \ln \left[(1-\theta)w_{t+1} H_{t+1} l_{t+1} + (1+(1-\theta)r_{t+1})K_{t+1} - \frac{K_{t+2}}{1-\tau} \right] \\ & + \phi_2 \cdot \beta \cdot \ln (I - l_{t+1} - h_{t+1}) + \dots \\ & \dots + \lambda_t (H_t - Ah_t H_{t-1} - H_{t-1}) + \lambda_{t+1} (H_{t+1} - Ah_{t+1} H_t - H_t) + \dots \end{aligned} \quad (6)$$

with the necessary conditions:

$$\frac{\partial}{\partial h_t} : \frac{\phi_2}{f_t} = -\lambda_t A h_{t-1}, \quad \frac{\partial}{\partial h_{t+1}} : \frac{\phi_2 \cdot \beta}{f_{t+1}} = -\lambda_{t+1} A h_t \quad (7)$$

$$\frac{\partial}{\partial l_t} : \frac{\phi_1}{c_t} (1-\theta) w_t h_t = \frac{\phi_2}{f_t} \quad (8)$$

$$\frac{\partial}{\partial H_t} : \frac{\phi_1}{c_t} (1-\theta) w_t l_t = -\lambda_t + \lambda_{t+1} (A h_{t+1} + 1) \quad (9)$$

$$\frac{\partial}{\partial K_{t+1}} : \frac{c_{t+1}}{c_t} = \beta (1-\tau) (1 + (1-\theta) r_{t+1}) \quad (10)$$

The production function is Cobb-Douglas:

$$F(K_t, H_t l_t) = K_t^\alpha (H_t l_t)^{1-\alpha} \quad (11)$$

Thus, a competitive equilibrium satisfies:

$$r_t = \alpha \left(\frac{H_t l_t}{K_t} \right)^{1-\alpha} \equiv \alpha k_t^{-(1-\alpha)} \quad (12)$$

$$w_t = (1-\alpha) k_t^\alpha. \quad (13)$$

In the steady state, time devoted to education, leisure and work are constant. Output, consumption and human capital all grow at the common rate g :

$$g \equiv \frac{H_{t+1}}{H_t} = A h + 1 \quad (14)$$

In the steady state, the time spent on education relative to that spent on work rises with the productivity of the human capital accumulation A and falls with the rate of time preference:

$$l = h(1-\beta) + \frac{1-\beta}{A} \quad (15)$$

The equilibrium growth rate g and the equilibrium capital intensity k are revealed in the solution of the following two equations which are derived from the individuals budget constraint and the necessary condition (10):

$$g = \beta(1-\tau) \left(1 + (1-\theta) \alpha k_{t+1}^{-(1-\alpha)} \right) \quad (16)$$

$$g^2 - \left[\left(1 + (1-\theta) \alpha k_t^{-(1-\alpha)} \right) (1-\tau) + \left[\frac{\phi_1}{\phi_2} \frac{2-\beta}{1-\beta} + 1 \right] \cdot (1-\tau) (1-\theta) (1-\alpha) k_t^{-(1-\alpha)} \right] g \\ + \frac{\phi_1}{\phi_2} \cdot (IA+1) \cdot \frac{(1-\theta)(1-\tau)}{1-\beta} \cdot (1-\alpha) k_t^{-(1-\alpha)} = 0. \quad (17)$$

Equation (17) is derived in the appendix. We applied numerical methods to obtain the equilibrium values of growth and capital-labor ratio as a function of the tax rates. This allows for determining the optimal fiscal policy.

3. Optimal Fiscal Policy

We now assume that it is the government's task to provide the public good in fixed proportion to the economy's output level. We define $d \in [0,1]$ as the size of the public sector. The government's objective is to maximize the steady-state growth rate by the choice of the tax rates. The corresponding constraint can be written as:

$$\theta w_r H_r l_t + \theta r_r K_t + \tau \cdot \frac{K_{t+1}}{1-\tau} = d \cdot F(K_t, H_t, l_t) \quad (18)$$

or

$$\theta k_t^\alpha + \frac{\tau}{1-\tau} \cdot g k_t = d f(k_t) = d k_t^\alpha. \quad (19)$$

The growth rate g and the revenue share d will be functions of the two tax rates. For these variables, the results from our numerical analysis can be summarized as follows:

- 1) Up to a certain threshold level d^* for d , it is optimal with regard to the growth rate to finance the public good exclusively by means of inheritance taxation.
- 2) For higher values of d , an interior solution is optimal where the inheritance tax τ should be chosen as high as possible.
- 3) The growth rate g depends positively on the rate of inheritance taxation and does not depend on the income tax rate.
- 4) The capital intensity declines with both θ and τ .

In the steady state, capital per efficiency unit is constant, $k_t = k_{t+1}$. Our simulation of this long-run equilibrium compares different policy mixes (θ, τ) which generate at least the necessary amount of revenues d for the government in order to finance the public good.

With reference to Jones, Manuelli and Rossi, we calibrate our model with the following parameters:

I	ϕ_1/ϕ_2	β	A	α
1,0	2,0	0,95	0,10	0,36

For the two tax rates we have chosen the range $[0,0.9] \times [0,0.9]$. Values for the growth rate and the capital intensity are calculated with the help of MAPLE V. A copy of the program can be made available by the authors on request.

Figure 1 displays an example of the typical behavior of the growth rate inside the set of feasible tax-combinations. The income tax rate does not affect the steady-state growth rate. This can already be seen from equations (16) and (17): in both equations, the term k is always accompanied by the factor $1-\theta$. Thus, an increase in the income tax rate θ in the long-run together with an offsetting decrease in the capital intensity k leaves the growth rate g unchanged. The inheritance tax rate τ , by contrast, has a positive effect on the rate of growth. This can be seen in figure 1. The authors tested this and the other findings for a wide range of parameters A and ϕ . In addition, total differentiation of the equation system (16) and (17) with regard to the inheritance tax rate τ showed a positive dependence of g on τ . A plot of $dg/d\tau$ is attached in the appendix (see figure 5).

Figure 2 displays the size of the government sector as a function of the two tax rates. While it is possible to obtain any public sector size with income taxation, it is impossible with inheritance taxation alone.

Figure 3 displays the capital per efficient labor unit k of the economy as a function of the two tax rates, the partial derivative being negative. Finally, figure 4 displays the growth rate where the tax income is required to match at least the given size of the public sector d . Evidently, the government can use different tax policies in order to finance their public spendings. However, for a large size of the public sector, choices are restricted and income taxation becomes necessary.

4. Short-run dynamics

In the previous section, we found that, in the long-run, inheritance taxation was superior to income taxation with regard to the growth rate. The examination of steady-state growth paths are, of course, only a first step in the analysis of optimal taxation of income and wealth in a growth model. Switching from one tax policy to the optimal one will entail short-run effects which are likely to adversely affect both utility and growth. Thus, further analysis of the transition dynamics is needed. For this purpose, we used a slightly modified truncated version of the above model. The model can be found in the appendix 2. A positive steady state exists if ϕ_2 , the preference-parameter for leisure, is sufficiently low to allow for investment into human capital. It can be shown that partial derivatives of the steady state growth rate of this economy with respect to the two tax rates have the same signs as in our infinite-horizon model in section 2. Figures 6-9 display simulation results for a change of the tax system. Figure 6 illustrates the time path for consumption in the steady state with $\tau =$

0 and $\theta = 0.5$. Figure 7 presents the case where the inheritance tax rate is changed to 0.1 in period 1. Consumption initially decreases and then subsequently overtakes the consumption level corresponding to the initial steady state. Thus, the earlier generations suffer from an increase in wealth taxation because of the negative effects on the stock of physical capital. Later, the gains from an increase of the stock of human capital become large enough to offset this negative effect and future generations are made better off. Figures 8 and 9 show how this negative effect on consumption can be reduced through a simultaneous and gradual adjustment of the income tax rate. In both cases, τ is increased by 0.125 percent per period, while θ is decreased by 0.125 percent and 0.03125 percent, respectively.

5. Conclusion

The above discussion shows that the integration of wealth taxation into models of optimal taxation with endogenous growth provides new results and interesting implications for fiscal policy. We see that the steady state rate of growth is affected positively by taxes on inherited capital. Inheritance taxation creates an incentive to be benevolent towards one's heir through other channels. The only remaining channel in the present model being human capital transmission, it is not surprising that taxation of physical capital promotes growth in this kind of model. Other models of endogenous growth emphasizing different determinants of growth might, of course, lead to different conclusions.

Two points of future research seem worth mentioning to us. First, the political decision process may be endogenized as well. Such advances have already been made in the field of pure income taxation by Perotti (1990) and Persson and Tabellini (1991). Certain political forces will have an interest in sustaining a certain tax system and will be particularly opposed to the introduction of wealth taxation. Secondly, we only regarded one representative individual. Individuals, however, differ with regard to their endowment of learning ability and inherited physical capital. The model can be extended by examining heterogeneous groups of individuals to study the question of growth and both income and wealth distribution.

Appendix 1

The growth rate g satisfies:

$$g = Ah+1 \Leftrightarrow h = \frac{g-1}{A} \quad (20)$$

From (15), it follows:

$$l = \frac{1-\beta}{A} g \quad (21)$$

and

$$f = I-l-h = I + \frac{1}{A} - \frac{2-\beta}{A} \cdot g \quad (22)$$

(8) can be substituted into the individuals budget constraint (5) together with the above equations and after dividing by $H_l l_i$:

$$\frac{\phi_1}{\phi_2} (1-\theta) w_i \left[\frac{I - \frac{2-\beta}{A} \cdot g + \frac{1}{A}}{\frac{1-\beta}{A} \cdot g} \right] + g \frac{k_i}{1-\tau} = (1-\theta) w_i + (1+(1-\theta)r_i)k_i. \quad (23)$$

Multiplying this expression by $(1-\beta) \cdot g$ yields:

$$\begin{aligned} & \frac{\phi_1}{\phi_2} (1-\theta) w_t (IA - (2-\beta) \cdot g + 1) \\ = & -(1-\beta) \cdot \frac{k_t}{1-\tau} g^2 + (1-\theta) w_t (1-\beta) \cdot g + (1+(1-\theta) r_t) k_t (1-\beta) \cdot g \end{aligned} \quad (24)$$

Dividing by $w_t(1-\theta)$ and rearranging

$$\begin{aligned} & \frac{\phi_1}{\phi_2} (IA - (2-\beta) g + 1) \\ = & -\frac{1-\beta}{1-\theta} \cdot \frac{k_t}{w_t(1-\tau)} g^2 + (1-\beta) \cdot g + (1+(1-\theta) r_t) \frac{k_t}{w_t} \frac{(1-\beta)}{(1-\theta)} \cdot g \end{aligned} \quad (25)$$

$$\begin{aligned} \Leftrightarrow g^2 - & \left[(1+(1-\theta) r_t) (1-\tau) + \frac{\phi_1}{\phi_2} \frac{2-\beta}{1-\beta} \cdot (1-\tau) (1-\theta) \frac{w_t}{k_t} + (1-\tau) (1-\theta) \frac{w_t}{k_t} \right] g \\ & + \frac{\phi_1}{\phi_2} \cdot (IA+1) \cdot \frac{(1-\theta) (1-\tau)}{1-\beta} \cdot \frac{w_t}{k_t} = 0. \end{aligned} \quad (26)$$

Equation (17) follows with (12) and (13) .

Appendix 2 The OLG-model

We used the following OLG model for the simulation of the transition dynamics. Individuals live for two periods. In the first period, they allocate time 1 between leisure f and education h . Human capital H is built up according to the Lucas' specification. In the second period, individuals supply their labor inelastically and receive wage w per efficiency unit of labor. Parents are altruistic and leave their heirs bequests at the amount of b . Bequests are taxed at the rate of τ , income from labor and from interest are taxed at the rate of θ . An individual derives utility from 2nd period consumption c , leisure f , and bequests to his heir. The population is assumed to be constant. The individual's problem in period t is

$$\begin{aligned} \max_{h, b_{t+1}} u &= u(c_{t+1}, f, (1-\tau) b_{t+1}) \\ &= \phi_1 \ln c_{t+1} + \phi_2 \ln(I-h_t) + \phi_3 \ln(1-\tau) b_{t+1} \end{aligned} \quad (27)$$

subject to

$$c_{t+1} + b_{t+1} = (1-\theta) w_{t+1} H_{t+1} + (1 + (1-\theta) r_{t+1}) (1-\tau) b_t \quad (28)$$

$$H_{t+1} = Ah_t H_t + H_t \quad (29)$$

This leads to an expression for u, to be maximized over h and b:

$$\begin{aligned} u &= \phi_1 \ln[(1-\theta) w_{t+1} (Ah_t H_t + H_t) + (1 + (1-\theta) r_{t+1}) (1-\tau) b_t - b_{t+1}] \\ &\quad + \phi_2 \ln(I-h_t) + \phi_3 \ln(1-\tau) b_{t+1} \end{aligned} \quad (30)$$

and necessary conditions

$$\frac{\partial u}{\partial h_t} : \frac{\phi_1}{c_{t+1}} (1-\theta) w_{t+1} \cdot AH_t = \frac{\phi_2}{I-h_t} \quad (31)$$

$$\frac{\partial u}{\partial b_{t+1}} : \frac{\phi_1}{c_{t+1}} = \frac{\phi_3}{b_{t+1}} \quad (32)$$

The production function is Cobb-Douglas,

$$F(K_t, H_t) = K_t^\alpha (H_t I)^{1-\alpha}, \quad (33)$$

$$f(k_t) = k_t^\alpha, \quad k_t \equiv \frac{K_t}{H_t} \quad (34)$$

and wage and interest rate are determined by

$$w_t = (1-\alpha) k_t^\alpha, \quad (35)$$

$$r_t = \alpha k_t^{-(1-\alpha)} \quad (36)$$

Investment in human capital in period t can be calculated from (28),(31),(32),(35) and (36)

$$h_t = 1-\delta \cdot \left(h_t + \frac{1}{A} \right) \left[1 + \frac{(1-\tau)(k_{t+1}^{1-\alpha} + (1-\theta)\alpha)}{(1-\theta)(1-\alpha)} \right] \quad (37)$$

The capital stock in period t+2 can be obtained from (28) and (32), using $K_{t+2} = b_{t+1}(1-\tau)$:

$$K_{t+2} = \frac{1-\tau}{\left[1 + \frac{\phi_1}{\phi_3} \right]} \left[(1-\theta)(1-\alpha)k_{t+1}^\alpha H_{t+1} + \left(1 + (1-\theta)\alpha k_{t+1}^{-(1-\alpha)} \right) K_{t+1} \right] \quad (38)$$

Equations (37) and (38), together with (29), describe the dynamics of h, H and K. The conditions for the steady state can be obtained from (37) and dividing by H_{t+1} :

$$\frac{Ah+1}{1-\tau} \cdot \left[1 + \frac{\phi_1}{\phi_3} \right] = 1 + (1-\theta)k^{-(1-\alpha)} \quad (39)$$

and

$$h = 1-\delta \cdot \left(h + \frac{1}{A} \right) \left[1 + \frac{(1-\tau)(k^{1-\alpha} + (1-\theta)\alpha)}{(1-\theta)(1-\alpha)} \right]. \quad (40)$$

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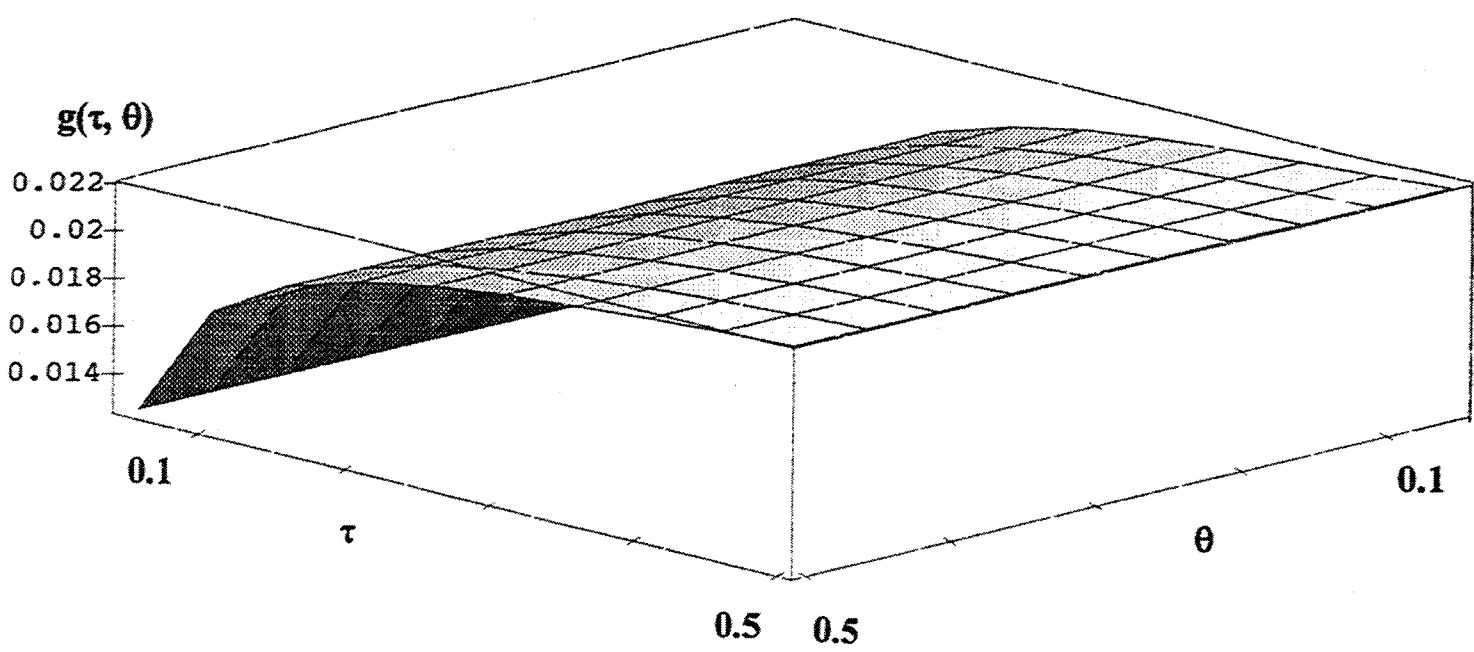


Figure 1: the growth rate as a function of the tax rates.

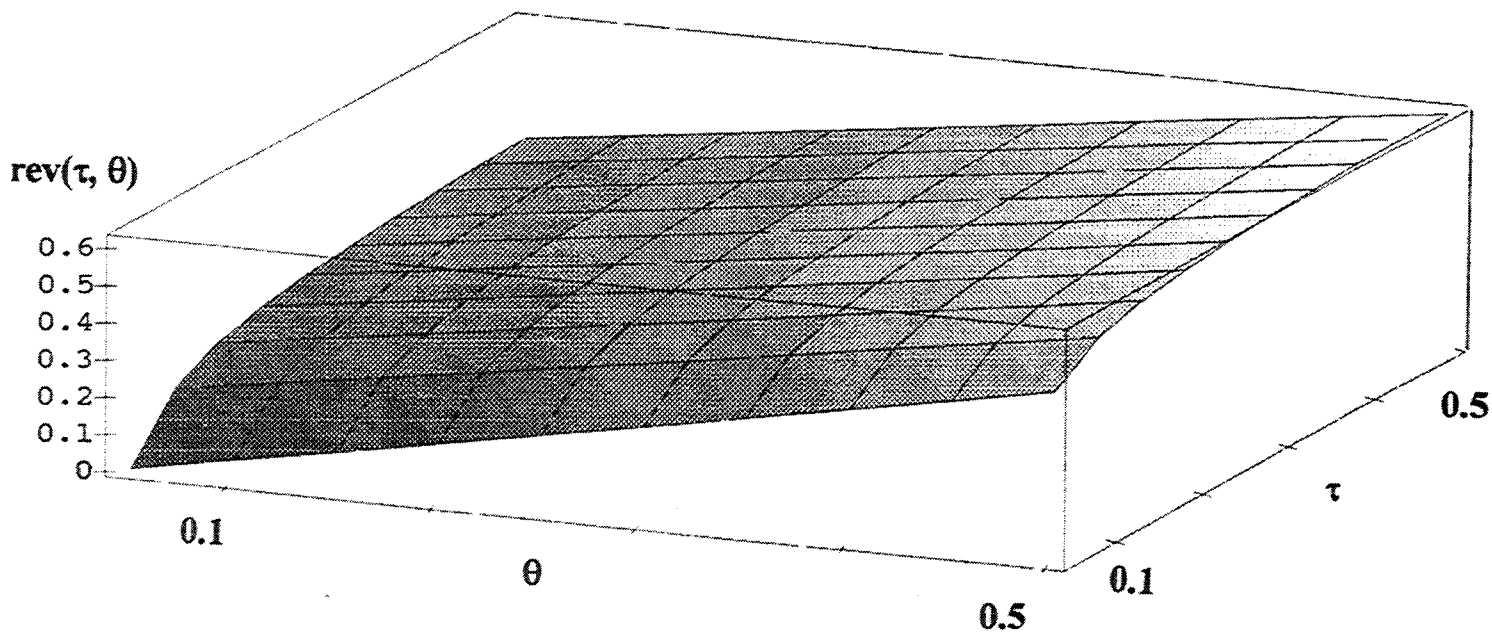


Figure 2: the size of the government sector as a function of the two tax rates.

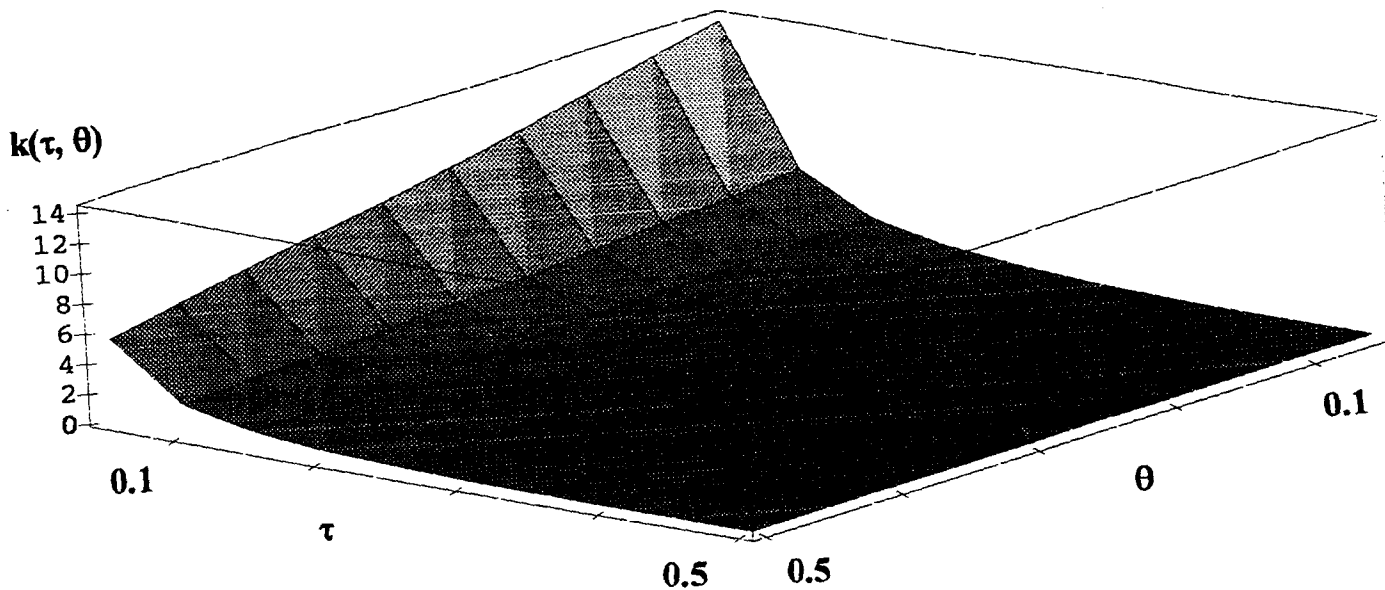


Figure 3: the capital per efficient labor unit k of the economy in dependence of the two tax rates.

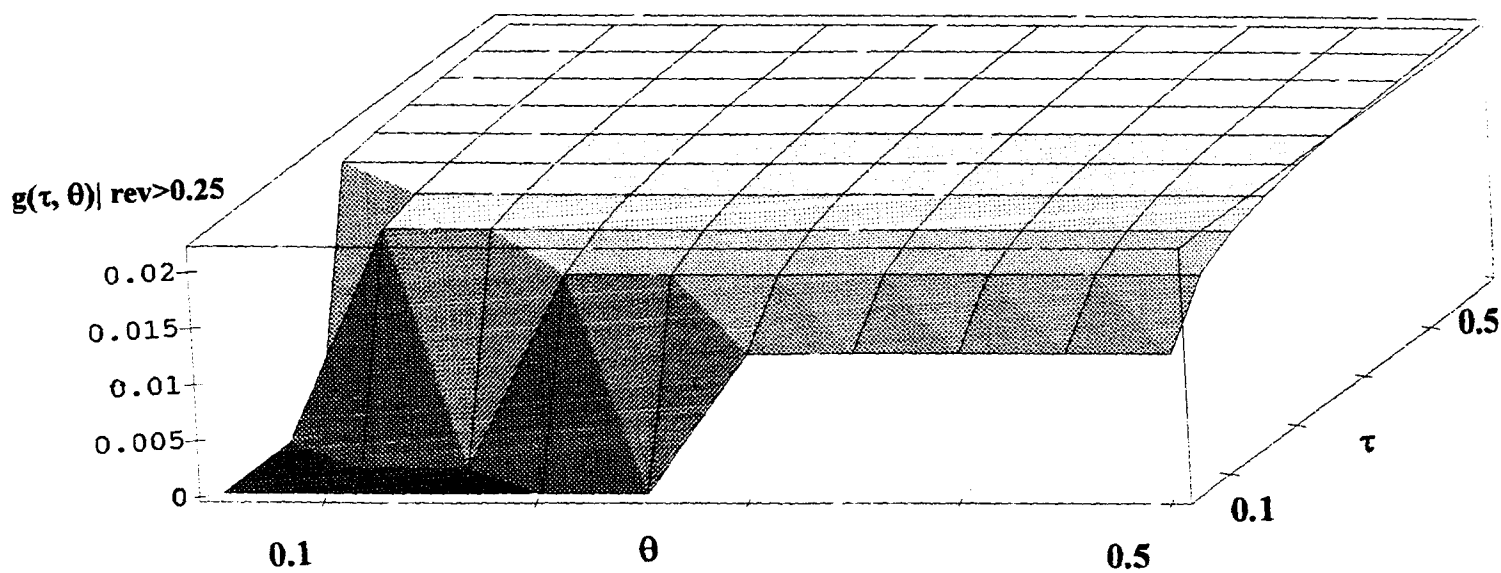


Figure 4: the growth rate where the tax income is required to match at least the given size of the public sector $d = 0.25$.

$\frac{dg}{dt}, A=1$

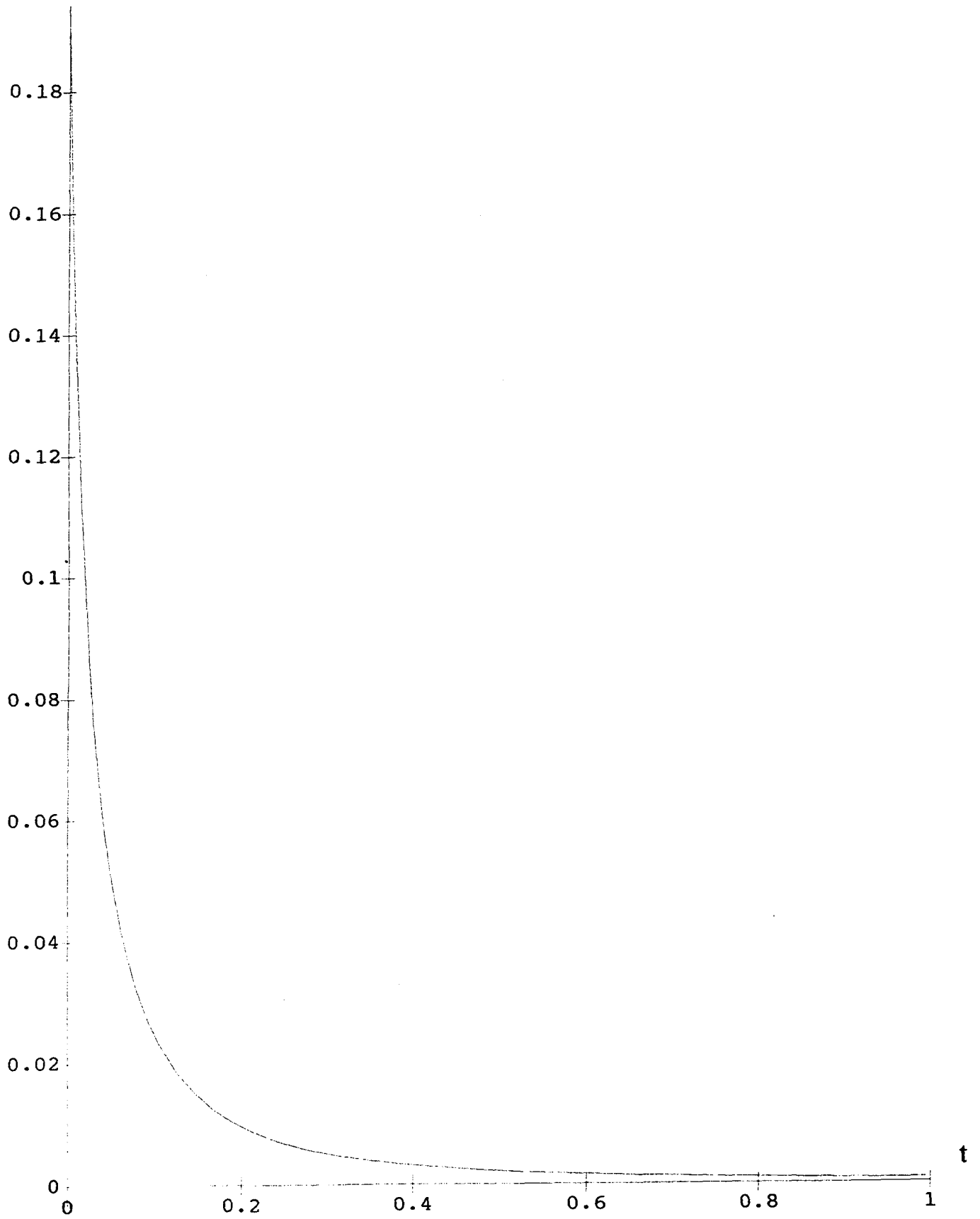


Figure 5: $dg/d\tau$

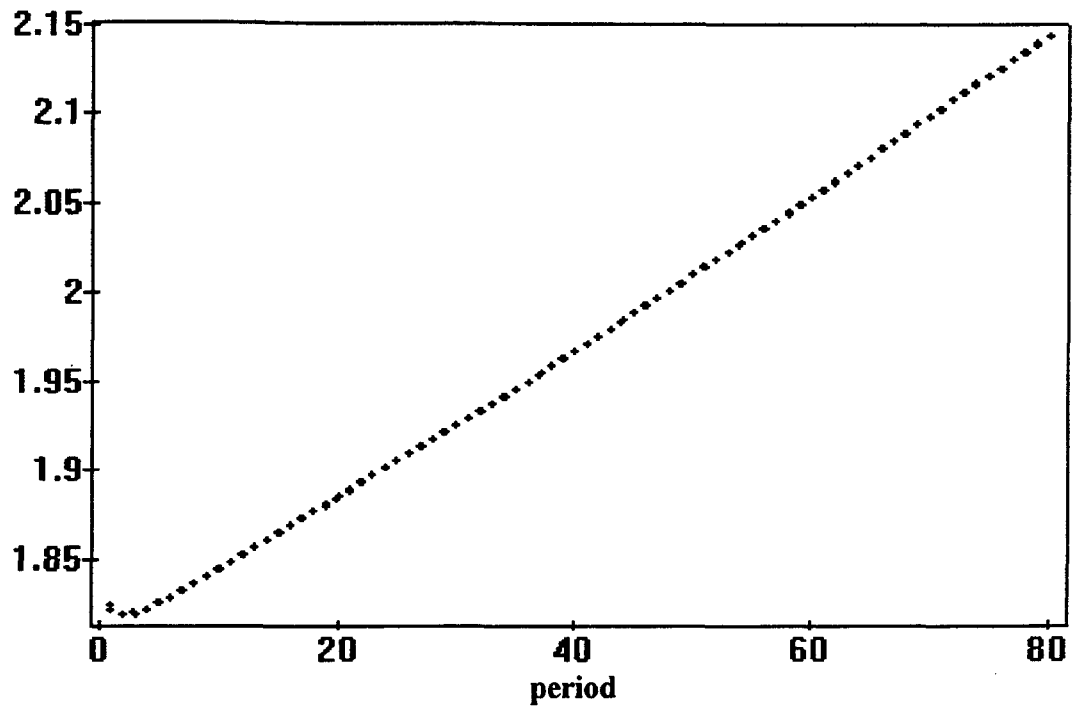


Figure 6: the steady state values of consumption and tax revenues for $\theta=0.5$ and $\tau=0$.

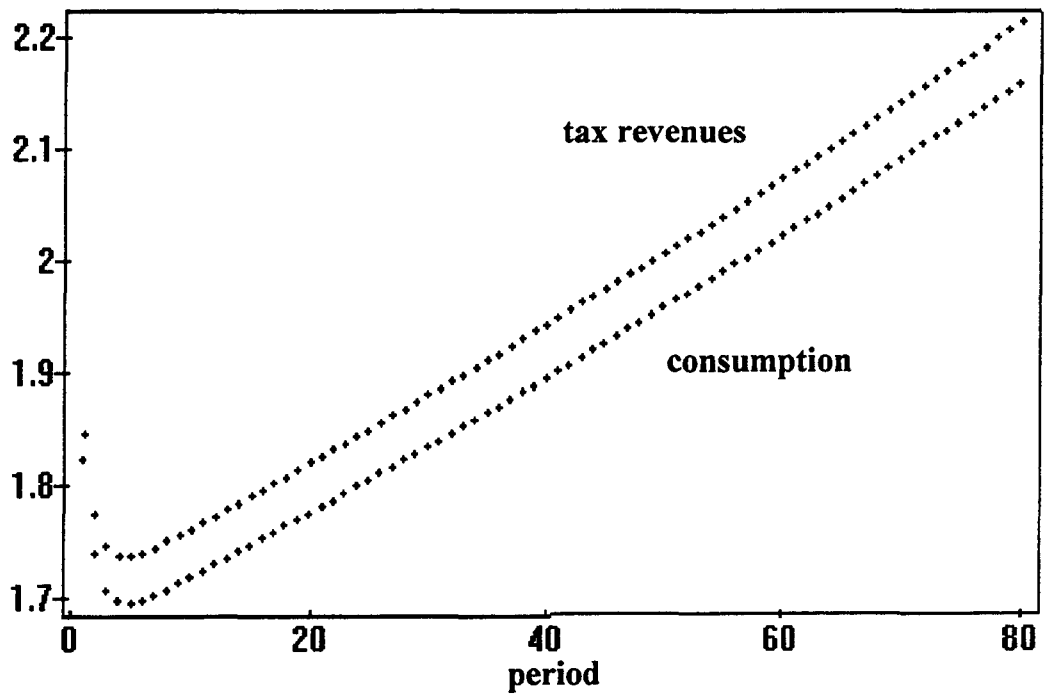


Figure 7: adjustment to the new steady state after an increase of τ to the value of 10%.

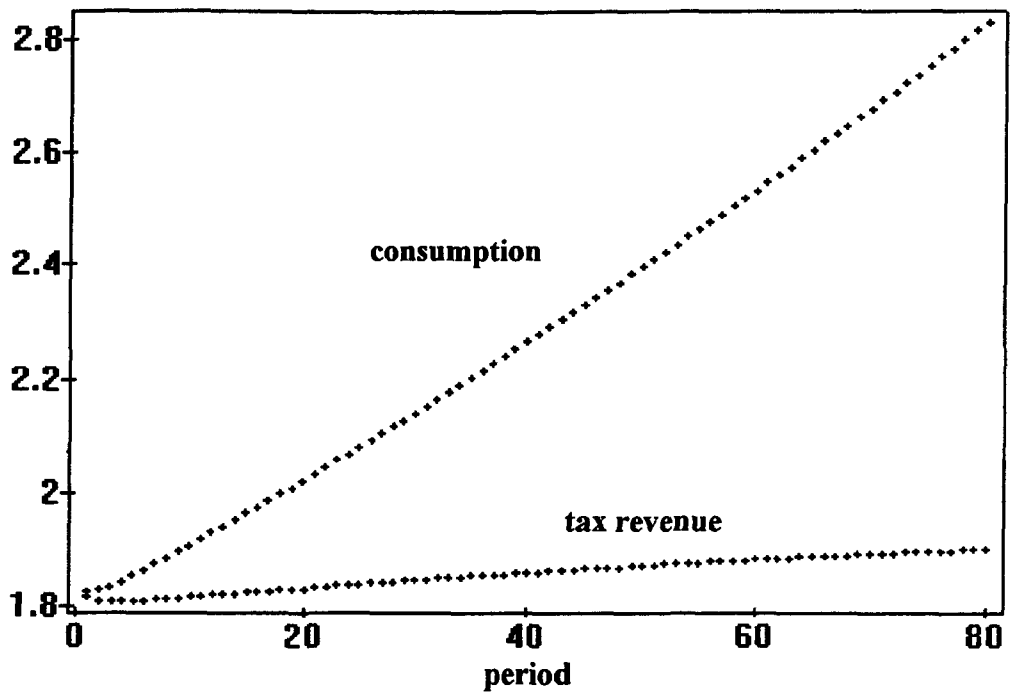


Figure 8: consumption and tax revenues where income tax is decreased by 0.125% per period and inheritance tax is increased by the same amount.

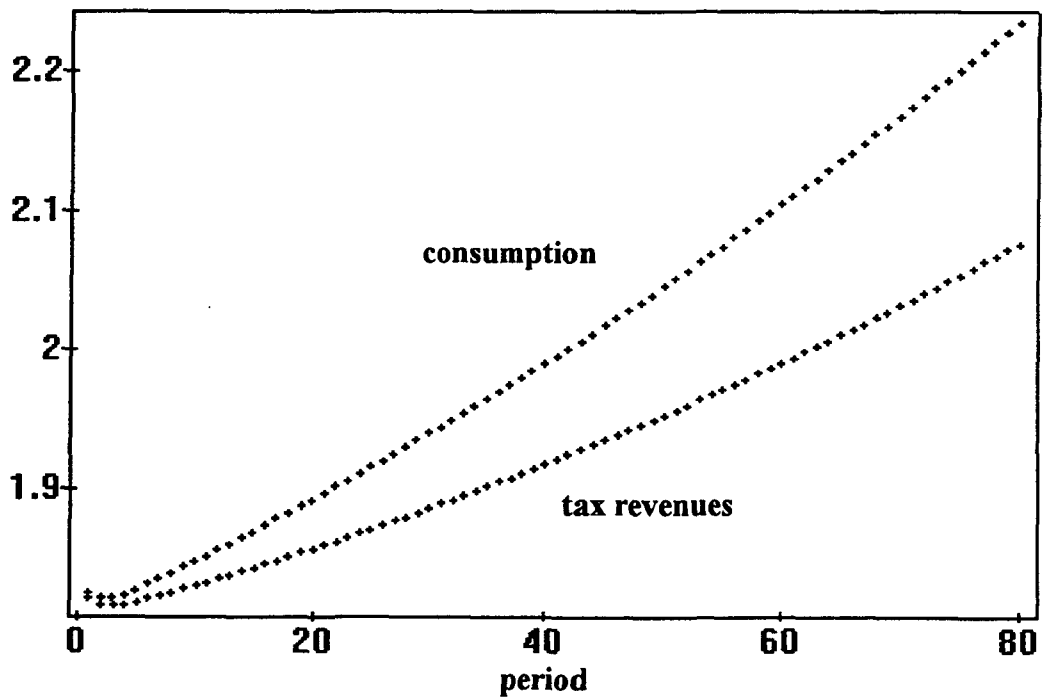


Figure 9: consumption and tax revenues where income tax is decreased by 0.03125% per period and inheritance tax is increased by 0.125% per period.