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Working Paper

A general equilibrium model of sovereign borrowing and non-sovereign financial intermediation

Diskussionsbeiträge - Serie II, No. 40

Provided in Cooperation with:

Department of Economics, University of Konstanz

Suggested Citation: Mohr, Ernst (1987) : A general equilibrium model of sovereign borrowing and non-sovereign financial intermediation, Diskussionsbeiträge - Serie II, No. 40, Universität Konstanz, Sonderforschungsbereich 178 - Internationalisierung der Wirtschaft, Konstanz

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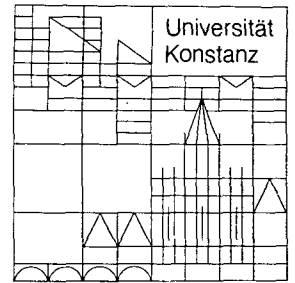
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Sonderforschungsbereich 178
„Internationalisierung der Wirtschaft“

Diskussionsbeiträge



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**A General Equilibrium Model
of Sovereign Borrowing and
Non-Sovereign Financial Intermediation**

A GENERAL EQUILIBRIUM MODEL OF
SOVEREIGN BORROWING AND NON-SOVEREIGN
FINANCIAL INTERMEDIATION

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Serie II - Nr. 40

November 1987

Ag 114/88 *Wirtschaft*
Kiel

Abstract:

In a general equilibrium model, international lending through a non-sovereign financial intermediary (a banking system) to a sovereign borrower is analyzed. Under very pessimistic assumptions, including a principal-agency type of incentive to load future intermediation with certain bankruptcy for the sake of present sovereign debt servicing, sustainable international lending that avoids both, repudiation and bankruptcy is determined. There is a plethora of roll-over and net-transfer steady-states. Compared to financial autarky, steady-state lending is capital depriving. Nevertheless, lending may be welfare improving in the creditor country and the world as a whole.

I. Introduction

Since the emergence of the international debt problem in the early 1980s a variety of theoretical aspects of the sovereign debt issue has been analyzed. [For a recent survey see Eaton, Gersovitz, and Stiglitz (1986).] Although interest has somewhat focussed on the strategic behaviour in the debtor countries [see for example Sachs (1982), Cohen and Sachs (1986), and Mohr (1988a)], the importance of debtor sovereignty for lending countries is well recognized. However, investigations into the supply of sovereign debt have been confined to a partial equilibrium analysis of credit markets [see for example Eaton and Gersovitz (1981), Kletzer (1984), Heffernan (1985), Bulow and Rogoff (1986), and Ozler (1986)]. This paper extends this theory by providing a general equilibrium analysis of sovereign debt to determine its impact on production, savings, capital formation and welfare in the lending countries.

In disaggregating the creditor side to sovereign debt by distinguishing between private depositors and a banking system an important development in international lending is taken into account. Over the decades the importance of commercial banks in supplying international debt has constantly increased, from a share of roughly 3% in the early 1960s to approximately 40% of the 1 trillion US \$ of LDC debt outstanding in 1985 (World Bank (1986)). While the exposure to sovereign lending of the DC banking sector as a whole is still moderate, it is uncomfortably concentrated on the largest money center banks in the US. It is feared that this involvement may

imperil the stability of the financial system in industrialized countries. A related concern raised is that strategic behaviour of debtor countries may cause imprudence on the side of banks, becoming unduely sympathetic towards new and higher credit demands in order to entice repayment of old sovereign debt due today.

The involvement of commercial banks in supplying credits to foreign countries has some important theoretical implications. While the debtor is sovereign, the creditor is not. Banks act as financial intermediaries, passing along funds to the sovereign borrower that have been deposited by domestic households. These deposits must be serviced in full or else a bank must file for bancrupcy. As a financial intermediary, a bank thus acts as a non-sovereign lender to a sovereign borrower.

In this paper, international borrowing with a disaggregated lending side is modeled in an overlapping generations context. Its purpose is to investigate into sustainable international debt relations that avoid both, debt repudiation and bancrupcy of the financial intermediary.

The model may be of interest in particular as its implications suggest some optimism under very pessimistic assumptions. Making it an uphill battle for the optimist a doomsday view of the international debt problem is entertained that features the following characteristics:

1. It is impossible to impose trade or other direct sanctions on the international borrower.

2. A possible credit embargo is discounted by the foreign borrower such that only the present but no future cutoffs from credit markets matter.

3. The lender to the sovereign borrower is a non-sovereign financial intermediary that is forced into bankruptcy if it fails to service deposits.

4. There are no dynastic motives in the lending country, in particular those in charge of financial intermediation have an incentive to steer the financial intermediary into long-run default for the sake of short-run benefits.

Against all these odds it turns out that there is a plethora of steady-states that sustain both, permanent debt servicing by the sovereign borrower and permanent solvency of the financial intermediary. Roll-over as well as net-transfer steady-states are attainable.

Under both types of steady-states there exists an upper bound on sustainable debt per capita in the creditor country. Sovereign lending through a financial intermediary is capital depriving compared to financial autarky. However, even under these pessimistic assumptions it remains possible that some lending steady-states improve welfare in the creditor country and the world as a whole.

The paper is organized as follows: In Section II the basic model is presented. Section III derives the momentary competitive equilibrium. In Section IV it is investigated into existence and properties of net-transfer and roll-over steady-states for a specification of the utility and production functions. The paper concludes with remarks on possible extensions of the model.

II. The Basic Model

A. The Borrowing Country

As this paper focusses on a disaggregated general equilibrium analysis of the creditor side to the international debt problem, the debtor country is modeled only rudimentarily; providing a strategic aspect to international debt servicing as its sole ingredient into the model. Not to belittle the borrowing country's strategic potential I assume that the costs of default are restricted to the loss of future access to international credit markets.¹ Under these circumstances, a sovereign debtor will repudiate at that moment in time when the present value of future net-transfers from the creditor country turns negative [Eaton and Gersovitz (1981), Niehans (1985)].

To capture this default condition in the most simple way possible, it is assumed that the debtor country consists of an infinite sequence of generations which live for one period only without a bequest motive for future generations.² It is further assumed that the borrowing country can default on an individual loan alone, triggering an embargo by the bank involved alone.³ But as banks are assumed to be identical in all respects, including loans outstanding, the individual default vs. servicing conditions may be aggregated over the entire banking sector in the lending country. The repudiation vs. servicing condition then is given by

$$B_t \begin{cases} \geq \\ < \end{cases} (1+r^s_t) \cdot B_{t-1} \quad \Leftrightarrow \quad \begin{cases} \text{no default} \\ \text{default} \end{cases}, \quad (1)$$

where B_t is the total loan obtained from the banking sector in

the lending country in period t . B_{t-1} is the debt inherited from generation $t-1$ with a maturity of one period and r^s_t is the interest rate on sovereign debt. The debtor will never accept negative net-transfers; hence loans, once given, effectively are lost for the creditor country forever. Condition (1) represents in a very simple way the idea that when a debtor's sole deterrent from voluntary default is her concern about the availability of new credits, a loan, in effect, turns into a free lunch. ⁴

If the debtor country defaults on old debt in period t , then she will be penalized by the credit embargo $B_t=0$. ⁵

B. The Creditor Country

Contrary to the borrowing country, the creditor country is represented by two overlapping generations of the Samuelson-Diamond type. ⁶ People are homogeneous within and across generations. Individuals live for two periods, earn income, consume and accumulate wealth when young, retire and dissave previously accumulated wealth when old. In each period t there exists a young generation t of size L_t and an old generation of size L_{t-1} . Growth in the creditor country is represented by a constant one-period proportional rate of growth of generations, n ($1 > n \geq 0$), relating the size of two successive generations by $L_t = (1+n) \cdot L_{t-1}$.

The young generation acts as the financial intermediary. At the beginning of each period it is endowed with a set of unenforceable claims against the sovereign borrower and enforceable obligations towards domestic depositors. I will suggestively call this set the "banking system". Members of a young generation operate the banking system as part of their life cycle utility maximization problem.

For simplicity, I assume that the number of banks is equal to the number of young households in each period. The banking sector can make loans to the sovereign debtor alone. The loan B_{t-1} therefore must be equal to aggregate deposits in period $t-1$. At the moment of transition into the second period of life, generation $t-1$ quits banking for good to become depositors into the banking system that now is taken over compulsorily by generation t . The new banking generation is saddled with unenforceable claims $(1+r^s_t) \cdot B_{t-1}$ as well as enforced liabilities $(1+r^d_t) \cdot B_{t-1}$, where r^d_t is the domestic deposit rate on the deposit B_{t-1} , due in t . Thus at the end of youth generation $t-1$ peters out of being a holder of unenforceable claims against the sovereign debtor to become a holder of enforceable claims against the banking system in period t .

A member of a young generation earns a wage income, w_t , and a banker's income, y_t^B . Let $b_t \equiv B_t/L_t$ be the sovereign loan in period t per capita of generation t .⁷ The banking income is given by

$$y_t^B = b_{t-1} \cdot [d^l_t \cdot (1+r^s_t) - (1+r^d_t)] / (1+n) , \quad (2)$$

where $d^1_t = \begin{cases} 1 \\ 0 \end{cases}$ iff $b_t \begin{cases} \geq \\ < \end{cases} b_{t-1} \cdot (1+r^s_t)/(1+n)$.

Here, d^1_t is a repudiation dummy.

Disposable income when young, y_t , is given by

$$y_t = \sup\{0, w_t + y_t^B\}. \quad (3)$$

A positive income of a member of generation t may go into period t consumption, c^1_t ($0 \leq c^1_t \leq y_t$), or into savings.

There are two stores of wealth. Bank deposits, B_t , that yield r^d_{t+1} , and physical capital, K_{t+1} , that is employed together with labour, L_{t+1} , in a time invariant, well behaved, neoclassical constant returns to scale production function yielding r_{t+1} .

Bank deposits per member of generation t , b_t , and productive capital per capita of generation $t+1$, k_{t+1} , are given by

$$b_t = \alpha_t \cdot (y_t - c^1_t), \quad (4)$$

$$k_{t+1} = (1 - \alpha_t) \cdot (y_t - c^1_t) / (1+n), \quad (5)$$

respectively, where α_t ($0 \leq \alpha_t \leq 1$) represents the portfolio decision of generation t .

Let $f(k_t)$ be output per member of generation t ($f' > 0$, $f'' < 0$). Competitive factor prices are given by

$$r_t = f'(k_t) \quad (6)$$

$$w_t = f(k_t) - k_t \cdot f'(k_t), \quad (7)$$

respectively.

The model is completed by a utility function $U(c^1_t, c^2_t)$ of a young household t , strictly concave in c^1_t and c^2_t , where c^2_t is consumption of a member of generation t in $t+1$ when old. It is

assumed that $\partial U / \partial c^i_t = \infty$ for $c^i_t = 0$ and $\partial U / \partial c^i_t > 0$ for $c^i_t < \infty$, $i=1,2$.

Instead of viewing banking as a second, non-labour related activity of a young household, the economy can be interpreted as consisting of L_t competitive banks with both a foreign and a domestic activity. The production factor L_t then is to be interpreted as a service factor supplied by the banking sector in the production of domestic output. $L_t \cdot w_t$ then is the income of the banking sector, earned from a "non-sovereign" activity. It will turn out that the banking sector's potential of earning "non-sovereign" (domestic) income is of paramount importance to avoiding bankruptcy.

III. The Momentary Competitive Equilibrium

Consider the decision problem of a representative member of generation t of finding optimal values for c^1_t and α_t .

As a first step towards solving this problem it is shown that consumption when old is independent of the decisions of the then young generation. To see this, consider the bankruptcy condition for a bank in $t+1$. As a bank is liable to depositors up to total income, y_{t+1} , and as their claims must be met before new sovereign loans can be granted, debt service $b_t \cdot (1+r^s_{t+1}) / (1+n)$ is not available (in full) for new sovereign loans in case of bankruptcy. Because a bankrupt bank has no means to grant new loans, a rational sovereign

borrower will repudiate debt owed to a bank that is about to default. Hence debt repudiation concurs with a collapse of the banking system.⁸ Therefore the condition for solvency of a bank in $t+1$ is given by

$$w_{t+1} - b_t \cdot (1+r^d_{t+1})/(1+n) \begin{cases} \geq \\ < \end{cases} 0 \quad \Leftrightarrow \quad \begin{cases} \text{solvency} \\ \text{bankruptcy} \end{cases} . \quad (8)$$

Aggregated over all members of generation $t+1$, condition (8) points to the importance for the banking system of generating sufficient non-sovereign earnings if a default is to be avoided. Because in case of bankruptcy in $t+1$, generation t claims the total labour income of generation $t+1$, consumption when old is given by

$$c^2_t = (1+n) \cdot [k_{t+1} + f(k_{t+1})] + \\ + d^2_{t+1} \cdot [(1+r^d_{t+1}) \cdot b_t - (1+n) \cdot w_{t+1}] , \quad (9)$$

where $d^2_{t+1} = \begin{cases} 1 \\ 0 \end{cases}$ iff $w_{t+1} \begin{cases} \geq \\ < \end{cases} b_t \cdot (1+r^d_{t+1})/(1+n)$.

d^2_{t+1} is a bankruptcy dummy. Hence from (9) the young cannot influence consumption of the old.

As a second step towards the optimal choice of (c^1_t, α_t) , note that a member of generation t , which itself is not forced to default on its liabilities against generation $t-1$, will find it optimal to always induce the servicing of sovereign debt. To see this, consider a pure roll-over, i.e. a credit that satisfies $b_t = b_{t-1} \cdot (1+r^s_t)/(1+n)$. Thereby income is increased by b_t , causing neither consumption, c^1_t , nor productive investment, $(1+n) \cdot k_{t+1}$, to decline. However, this additional income, which is generated through a particular form of saving, increases the claims against the next generation's wage income by $b_t \cdot (1+r^d_{t+1})$. Consumption when retired can thus be

increased and it is therefore profitable to at least roll-over old sovereign debt. Hence, if generation t itself is not forced into default, $d^1_t=1$ and income is given by

$$y_t = w_t + b_{t-1} \cdot (r^s_t - r^d_t) / (1+n). \quad (10)$$

The costs for the roll-over, however, are borne by the next generation for it brings about additional claims to a cake of a given size. This is the precise sense in which bankers act imprudently in the context of this paper. The optimal portfolio choice, α_t , therefore is bounded from above and from below by

$$1 \geq \alpha_t \geq \alpha^{\min}(c^1_t) \equiv \frac{\frac{b_{t-1} \cdot (1+r^s_t)}{(1+n)}}{(y_t - c^1_t)}. \quad (11)$$

As a final step towards solving the individual decision problem represent the (reduced form) optimization problem for a member of generation t by:

$$\begin{aligned} & \max_{c^1_t, \alpha_t} U(c^1_t, c^2_t) \\ & \text{subject to (11) and } 0 \leq c^1_t \leq y_t - b_{t-1} \cdot (1+r^s_t) / (1+n), \\ & \text{and given equations (4), (5), (9) and (10).} \end{aligned} \quad \left. \vphantom{\begin{aligned} & \max_{c^1_t, \alpha_t} U(c^1_t, c^2_t) \\ & \text{subject to (11) and } 0 \leq c^1_t \leq y_t - b_{t-1} \cdot (1+r^s_t) / (1+n), \\ & \text{and given equations (4), (5), (9) and (10).} \end{aligned}} \right\} (12)$$

Under conditions of perfect competition, physical capital and bank deposits must yield the same return

$$r_{t+1} = r^d_{t+1} = f'(k_{t+1}). \quad (13)$$

To generate the set of c^1_t and α_t combinations which neither induce repudiation in t nor bankruptcy of banks in $t+1$, reformulate conditions (8) and (11). The RHS inequality of (11) may be written as

$$b_t = \alpha_t \cdot (y_t - c^1_t) \geq \frac{b_{t-1} \cdot (1+r^s_t)}{1+n} \equiv a_1, \quad (14)$$

and condition (8), using (4) and (13), implies

$$b_t = \alpha_t \cdot (y_t - c^1_t) \leq \frac{w_{t+1} \cdot (1+n)}{(1+r_{t+1})} \equiv a_2 . \quad (15)$$

The RHS's of (14) and (15) are exogenous to a competitive member of generation t . For a given income, y_t , conditions (14) and (15) are sketched in Figure 1.

Figure 1 about here

Combinations of α_t and c^1_t below the A' -curve cause debt repudiation by the borrowing country and will therefore not be chosen. Combinations above the A'' -curve imply bankruptcy of the banking sector in period $t+1$.

Figure 1a represents a constellation of parameters, (a_1, a_2) , which will induce bankruptcy (-cum-repudiation) in the next period. The inherited exposure of generation t to sovereign lending is too great to both, protect the selfish interests of the present as well as provide the following generation with productive capital, sufficient to meet the claims of generation t when old. Figure 1b represents a situation in which income maximization in t does not necessarily induce bankruptcy of banks in $t+1$. As we are interested in steady-states that support international lending without debt

repudiation or bank failures, subsequently, only the case $a_1 \leq a_2$, corresponding to Figure 1b, will be discussed.

We can now determine the optimal consumption and portfolio decision, (c^1_t, α_t) . In Appendix 1 it is shown that for $a_1 < a_2$, (c^1_t, α_t) is given by

$$\alpha_t^* \in [a_1/(y_t - c^1_t), a_2/(y_t - c^1_t)] \quad (16)$$

and

$$[\partial U / \partial c^1_t] / [\partial U / \partial c^2_t] = 1 + r_{t+1}, \quad (17)$$

where

$$c^2_t = (1 + r_{t+1}) \cdot (y_t - c^1_t). \quad (18)$$

In an optimum, a member of generation t is indifferent with respect to portfolio composition in the bounds given in (16), because on the margin both assets yield the same return for as long as bankruptcy of the next generation is avoided. In Figure 1b the set of optimal α_t^* is given by the solid line above c^1_t . From the upper bound on α_t^* in (16) the creditor country is represented by generations which, when in command, have an incentive to navigate the banking ship clear of default, as long as this does not conflict with income maximization.

Condition (17) is the familiar interior optimality condition for life cycle consumption [see e.g. Diamond (1965)]. This condition survives here as it is optimal to avoid bankruptcy in the next period if this is possible without thereby incurring an income loss in the present. If banks remain solvent in $t+1$ the marginal return on saving is given by r_{t+1} thus leaving the intertemporal utility tradeoff for a given income unaffected.

It would be premature, however, to conclude at this point that international financial intermediation has no real impact on the lending economy. Although consumption obeys the familiar rule, the engagement in banking affects the income of the young and hence their savings and capital accumulation.

The bankruptcy condition (15) is subjective in the sense that a_2 is taken to be exogenous to a competitive member of generation t , while in fact r_{t+1} and w_{t+1} depend on aggregate investment in physical capital. For a momentary competitive equilibrium to exist today that supports the solvency of banks tomorrow, an optimal consumption and portfolio choice must be such that $a_1 \leq a_2(c^1_t, \alpha_t)$.

We are now ready to investigate into steady-states that sustain a permanent credit relationship between countries without debt repudiation and bank failures.

IV. Steady-States

A steady-state is a sequence of momentary equilibria in which the per capita stock of physical capital in the creditor country and the net-transfer to the borrowing country is constant over time.

Subsequently the analysis of steady-states will be carried out for the following functional specifications of the utility and production functions:

$$U(c^1_t, c^2_t) = \Phi \cdot \ln c^1_t + \Omega \cdot \ln c^2_t \quad (19)$$

where $\Phi + \Omega = 1$, $\Phi > 0$, $\Omega > 0$, Φ, Ω const., and

$$L_t \cdot f(k_t) = K_t^p \cdot L_t^q = L_t \cdot k_t^p, \quad (20)$$

where $p+q=1$, $p>0$, $q>0$, p, q const.. For these specifications, (7), (13), (17) and (18) become

$$w_t = q \cdot k_t^p, \quad (7a)$$

$$r_t = r_t^d = p \cdot k_t^{-q}, \quad (13a)$$

$$c_t^2 / c_t^{1*} = (1+r_{t+1}) \cdot (\Omega / \Phi), \quad (17a)$$

and

$$c_t^2 = (1+r_{t+1}) \cdot \Omega \cdot y_t, \quad (18a)$$

Finally from (17a) and (18a),

$$c_t^{1*} = \Phi \cdot y_t. \quad (21)$$

A. The Autarky Steady-State

As a frame of reference, consider the steady-state under autarky. Under autarky $b_t=0$ at all times. To fix ideas, one might imagine this situation be the result of solving the international debt problem in the lending country by an act of state, wiping clean the international debt chart, by both, forcing the banking sector to write off unenforceable sovereign debt as well as protect it against the claims of domestic depositors.

It is shown in Appendix 2 that the autarky steady-state, distinguished by the superscript "A" is given by

$$b^A = 0 , \quad (22.1)$$

$$k^A = \left[\frac{\Omega \cdot q}{1+n} \right]^{1/q} , \quad (22.2)$$

$$w^A = \left[\left[\frac{\Omega}{1+n} \right]^p \cdot q \right]^{1/q} , \quad (22.3)$$

$$r^A = p \cdot (1+n) / (\Omega \cdot q) \quad (22.4)$$

$$(c^2/c^1)^A = (\Omega/\Phi) \cdot \left[1 + \frac{p \cdot (1+n)}{q \cdot \Omega} \right] , \quad (22.5)$$

$$y^A = w^A . \quad (22.6)$$

B. Lending Steady-States

When investigating into lending steady-states, r^s_t will be kept exogenously fixed and constant through time.⁹ It is straightforward that a lending steady-state only exists for $r^s_t = r^s \leq n$.

Two cases are to be distinguished. For $r^s = n$, the debt servicing condition reduces to $b_t \geq b_{t-1}$, and a constant international lending per capita fixes net-transfers at zero at all times. These steady-states will be referred to as the (pure) roll-over steady-states. For $r^s < n$, a constant international lending per capita implies a positive net-

transfer. These steady-states will be referred to as the net-transfer steady-states.

Net-Transfer Steady-States

Let $r^s_t = r^s < n$ all t . Let $b_t = b > 0$ all t , and define

$$\tau \equiv b \cdot \left[1 - \frac{1+r^s}{1+n} \right] \geq 0 \quad (23)$$

as the net-transfer per period to the borrowing country per capita of the young in the lending country. Steady-states will be distinguished by different values of τ .

The constant endowment with physical capital, $k(\tau)$, implies $r_t = r(\tau)$, $w_t = w(\tau)$ and $y_t = y(\tau)$ all t . Hence from (18a) and (21) it follows that the consumption profile is constant through time, i.e. $c^1_t = c^1(\tau)$ and $c^2_t = c^2(\tau)$ all t .

The steady-state values, some properties of which are derived in Appendix 3, are given by:

$$b(\tau) = \tau \cdot (1+n) / (n-r^s) , \quad (24.1)$$

$$k(\tau) = \left[\frac{\left[\frac{\Omega \cdot q}{1+n} \right] - \left[\frac{\Omega \cdot p \cdot \tau}{n-r^s} \right]}{1 + (1-\Omega \cdot r^s) \cdot \tau} \right]^{1/q} , \quad (24.2)$$

$$w(\tau) = q \cdot \left[\frac{\left[\frac{\Omega \cdot q}{1+n} \right] - \left[\frac{\Omega \cdot p \cdot \tau}{n-r} \right]}{1 + (1-\Omega \cdot r^s) \cdot \tau} \right]^{p/q} , \quad (24.3)$$

$$r(\tau) = \frac{p \cdot [1 + (1 - \Omega \cdot r^s) \cdot \tau]}{\left[\frac{\Omega \cdot q}{1+n} \right] - \left[\frac{\Omega \cdot p \cdot \tau}{n-r} \right]}, \quad (24.4)$$

$$c^2(\tau)/c^1(\tau) = (\Omega/\Phi) \cdot [1+r(\tau)] \quad , \quad (24.5)$$

$$y(\tau) = w(\tau) + \frac{[r^s - r(\tau)] \cdot \tau}{n - r^s} \quad . \quad (24.6)$$

For $\tau=0$, the net-transfer steady-state (24) is identical to the autarky steady-state (22). Properties of the continuum of net-transfer steady-states are sketched in Figures 2 and 3.

Figures 2 and 3 about here

Net-transfer steady-states are capital depriving. In the steady-state a larger net-transfer can be sustained only at a lower endowment with physical capital ($dk(\tau)/d\tau < 0$). As $k(\tau)$ decreases, the wage rate and domestic production decline while the interest rate increases with the magnitude of net-transfers.

The intuition behind this capital depriving involvement in sovereign lending is readily given. From an aggregate point of view, wealth, held in the form of bank deposits lent abroad, is

unproductive. However, on a disaggregated level, bank deposits bear a return to each member of an old generation as they constitute an additional claim to part of the wage income of the young. The young are thus left with a smaller fraction of total domestic output for consumption and investment in physical capital. The distortion to life-cycle consumption, arising from the strategic incentive to save in the form of unproductive bank deposits when young, can be seen from (24.5). For $\tau > 0$, $r(\tau) > r^A$ and the consumption profile is distorted towards relative excessive consumption when old and relative austerity when young.

Net-transfers that sustain a steady-state are bounded from above. Substituting (24.1) into the steady-state condition for solvency (15) entails

$$\frac{\tau \cdot [1 + r(\tau)]}{n - r^s} \leq w(\tau) . \quad (25)$$

As the LHS of (25) is strictly increasing and the RHS is strictly decreasing in τ , there exists a unique upper bound for net-transfers, τ_{\max} , implying equality of (25). As debt per capita outstanding is linearly increasing in τ , net-transfer steady-state levels of debt per capita are bounded from above.

Roll-Over Steady-States

Let $r^s = n$ for all t . Then the repayment condition (14) reduces to $b_t \geq b_{t-1}$ so that for a constant debt per capita net-transfers are zero and the world economy remains at the brink of repudiation.

This perpetual debt stalemate, where national output in the lending economy does not dissipate abroad, exists for a variety of debt levels. This multiplicity is represented by a degree of freedom in the choice of $\alpha_t = \alpha$. Roll-over steady-state values as functions of α are given below. (The derivation of some properties is relegated into Appendix (4)).

$$b(\alpha) = \alpha \cdot \Omega \cdot y(\alpha) , \quad (26.1)$$

$$k(\alpha) = \left[\frac{\Omega \cdot (q - \alpha)}{1 + n \cdot (1 - \Omega \cdot \alpha)} \right]^{1/q} , \quad (26.2)$$

$$w(\alpha) = q \cdot \left[\frac{\Omega \cdot (q - \alpha)}{1 + n \cdot (1 - \Omega \cdot \alpha)} \right]^{p/q} , \quad (26.3)$$

$$r(\alpha) = \frac{p \cdot [1 + n \cdot (1 - \Omega \cdot \alpha)]}{\Omega \cdot (q - \alpha)} , \quad (26.4)$$

$$c^2(\alpha)/c^1(\alpha) = (\Omega/\Phi) \cdot [1 + r(\alpha)] , \quad (26.5)$$

$$y(\alpha) = \left[q - \frac{p \cdot \alpha}{1 - \alpha} \right] \cdot \left[\frac{\Omega \cdot (q - \alpha)}{1 + n \cdot (1 - \Omega \cdot \alpha)} \right]^{p/q} \\ + \left[\frac{n \cdot \alpha}{1 - \alpha} \right] \cdot \left[\frac{\Omega \cdot (q - \alpha)}{1 + n \cdot (1 - \Omega \cdot \alpha)} \right]^{1/q} . \quad (26.6)$$

Again, for $\alpha=0$, the roll-over steady-state is identical to the autarky steady-state.

The solvency condition (15) places an upper bound on α . Replacing the variables in (5) by their steady-state values, savings, $y(\alpha) - c^1(\alpha)$, are given by $k(\alpha) \cdot (1+n)/(1-\alpha)$ so that the steady-state value of debt, $b(\alpha)$, in (4) is given by $k(\alpha) \cdot (1+n) \cdot \alpha / (1-\alpha)$. Substituting this expression and $w(\alpha)$ into (15) we obtain the steady-state solvency condition

$$k^q \leq (q - \alpha) / \alpha .$$

Substituting in (26.2) entails

$$1 \left\{ \begin{array}{l} \geq \\ \leq \end{array} \right\} \alpha \cdot \Omega \quad \text{for} \quad \alpha \left\{ \begin{array}{l} < \\ > \end{array} \right\} q .$$

As $\alpha \leq 1$ and $\Omega < 1$, roll-over steady-state values of α are constrained to $0 < \alpha < q$.

Roll-over steady-states are sketched in Figures 4 and 5.

Figures 4 and 5 about here

Like the net-transfer steady-states, roll-over steady-states with a positive level of perpetual debt are capital depriving. Starting from the autarky situation, an increase in

α increases the level of debt outstanding. This debt entails additional claims against the wage income of the young and causes a smaller investment of the young in productive capital. As debt increases, the positive effect on debt of changing the portfolio composition is increasingly offset by a reduction in savings $y(\alpha) - c^1(\alpha) = \Omega \cdot y(\alpha)$, caused by a declining income $y(\alpha)$, forced upon the economy by a vanishing stock of productive capital. There is a maximum of sovereign debt per member of the young generation, b_{\max} , and further increasing the fraction of total wealth, held in the form of bank deposits, decreases savings by so much that debt declines. As α approaches q from below, the productive capital endowment, $k(\alpha)$, and the wage income, $w(\alpha)$, approach zero from above. With vanishing resources to meet the claims of the old, these claims must vanish as well to maintain solvency. Therefore debt outstanding must tend to zero as α approaches q .

C. Welfare

To investigate into welfare effects of both, roll-over and net-transfer steady-states, consider a parameter z , where z either represents α or τ . From (19), (21) and (18a) the steady-state utility is given by

$$U(z) = \Phi \cdot \ln \Phi + \ln y(z) + \Omega \cdot \ln \Omega + \Omega \cdot \ln[1+r(z)] . \quad (27)$$

Upon differentiation with respect to z one obtains

$$\frac{dU(z)}{dz} = \frac{1}{y(z)} \cdot \frac{dy(z)}{dz} + \frac{\Omega}{1+r(z)} \cdot \frac{dr(z)}{dz} . \quad (28)$$

The first term on the RHS of (28) is the marginal income effect of sovereign lending. The sign of this term is ambiguous. The second term is the marginal return on saving effect. It is always positive.

A sufficient condition for a welfare improvement from a marginal increase in z is $dy(z)/dz > 0$. This section is concluded by demonstrating that, compared to the autarky steady-state, some sovereign debt involvement may be welfare improving in the lending country.

From the steady-state value of income, $y(z)$,

$$\frac{dy(z)}{dz} = \frac{dw(z)}{dz} - \frac{b(z)}{1+n} \cdot \frac{dr(z)}{dz} + \frac{r^s - r(z)}{1+n} \cdot \frac{db(z)}{dz} . \quad (29)$$

Equation (29) breaks down the marginal income effect into the part arising from a change in factor prices and the part arising from a changed level of international lending. The first two terms are negative whereas the sign of the last term is ambiguous.

For $z=0$, that is for the case of international financial autarky, the second term vanishes. The wage effect is smaller the less important capital is for production. In the limit, as the production elasticity of capital, p , goes to zero, $dw(z)/dz$ vanishes.

Suppose $dw(z)/dz$ is neglectible. Then the marginal total income effect of moving from autarky to a lending steady-state solely depends on the volume effect of international banking given by the last term in (29). For $b=0$, $db(z)/dz > 0$ for both, $z=\tau$ and

$z=\alpha$. Hence the sign of $dy(z)/dz$ is the same as the sign of $r^s - r(z) = r^s - r^A$.

From (22.4) $r^A \rightarrow 0$ as $p \rightarrow 0$. Hence for $p \rightarrow 0$, on the margin, income always increases in a lending country, enjoying a positive growth rate, from a replacement of the autarky steady-state by a roll-over steady-state, as in the latter $r^s = n > 0$ and hence $r^s - r^A > 0$. Similarly, in a net-transfer steady-state, where $n > r^s$, it is possible to parametrically fix r^s such that $n > r^s > r^A$ if $n > 0$ and p is small enough.

Thus a moderate involvement in international roll-over or net-transfer steady-state lending may increase income and utility of a representative member of the lending economy. As the net inflow of resources to the borrowing country is always non-negative, a lending steady-state therefore may pareto-dominate the autarky steady-state in the world as a whole.

This, at first glance, may run counter to one's intuition as net-transfers are negative in some of the pareto-better steady-states. The source of welfare improvement transpires, however, if it is borne in mind that the autarky steady-state may be inefficient due to an excessive capital stock.¹⁰ A reduction in that stock caused by sovereign lending may therefore improve welfare.

Income eventually must decrease and be lower than in autarky if z is increased enough because $y(z) \leq w(z)$ for $r(z) > r^s$. It is straightforward from (27) that welfare must be lower than under autarky if $y(z)$ has decreased enough.

V. Extensions

The model has been stripped of many interesting issues in order to isolate and illustrate what are believed to be important features of sovereign debt, features which can be expected to carry through to a variety of models.

The general equilibrium model may be enriched in several directions. The sovereign borrower may be given a richer role in the model to eliminate the degree of freedom in the interest rate on sovereign debt and to investigate into generational conflicts in the borrowing economy. Debt negotiations to find a cooperative solution to sovereign borrowing may be incorporated, since borrower - intermediary interaction is a positive sum game. Finally, introducing a government sector, the role of a lender of last resort for intermediation and cooperation may be analyzed.

Appendix 1:

The optimal consumption and portfolio decision: In (12), the constraint $1 \geq \alpha_t$ and the constraints on c^1_t can be neglected. $\alpha_t = 1$ cannot be optimal, because by (5) and (7) this would entail $k_{t+1} = w_{t+1} = 0$, leaving an investor with claims to an empty shell when old. $c^1_t = 0$ is suboptimal since utility is assumed to have an infinite slope at the origin. Finally, the upper bound on c^1_t is observed if the bounds on α_t are observed. Using (13), from (9)

$$\partial c^2_t / \partial c^1_t = \begin{Bmatrix} -(1+r_{t+1}) \\ -(1-\alpha_t) \cdot (1+r_{t+1}) \end{Bmatrix} \quad \text{if } d^2_{t+1} = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}, \quad (A1)$$

and

$$\partial c^2_t / \partial \alpha_t = \begin{Bmatrix} 0 \\ -(y_t - c^1_t) \cdot (1+r_{t+1}) \end{Bmatrix} \quad \text{if } d^2_{t+1} = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}. \quad (A2)$$

From (A1) and (A2) one obtains upon differentiating the Lagrangian associated with (12)

$$\begin{aligned} \partial U / \partial c^1_t - \partial U / \partial c^2_t \cdot \begin{Bmatrix} (1+r_{t+1}) \\ (1-\alpha_t) \cdot (1+r_{t+1}) \end{Bmatrix} &= -\theta_t \cdot a_1 / (y_t - c^2_t)^2 \\ \text{if } d^2_{t+1} &= \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}, \end{aligned} \quad (A3)$$

and

$$\partial U / \partial c^2_t \cdot \begin{Bmatrix} 0 \\ (y_t - c^1_t) \cdot (1+r_{t+1}) \end{Bmatrix} = \theta_t \quad \text{if } d^2_{t+1} = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}, \quad (A4)$$

where θ_t is the multiplier associated with the lower bound on α_t such that

$$\theta_t \geq 0, \quad \theta_t \cdot [\alpha_t - \alpha^{\min}(c^1_t)] = 0. \quad (A5)$$

Suppose $a_1 < a_2$. Suppose that bankruptcy occurs in $t+1$ so that $d^2_{t+1} = 0$. Then we must have $\alpha_t > \alpha^{\min}(c^1_t)$ and consequently $\theta_t = 0$. For $\theta_t = 0$ and $d^2_{t+1} = 0$, (A4) reduces to

$[\partial U / \partial c^2_t] \cdot (y_t - c^1_t) \cdot (1 + r_{t+1}) = 0$, which is fulfilled only for $c^2_t = \infty$. For a finite endowment with physical capital, c^2_t cannot be infinite. Therefore it cannot be optimal to force banks into bankruptcy in $t+1$ if $a_1 < a_2$. As a roll-over is always profitable and the prevention of default is optimal if $a_1 < a_2$, both must be optimal in the knife-edge case $a_1 = a_2$. Hence the optimal portfolio choice is given by (16).

For $d^2_{t+1} = 1$ it follows from (A4) that $\theta_t = 0$ and hence from (A3) we obtain condition (17). Equation (17) is independent from α_t , because by (6), (7), (13) and $d^2_{t+1} = 1$, (9) reduces to (18).

Appendix 2:

Derivation of the autarky steady-state: From (3), $y^A = w^A$ if $b^A = 0$. Furthermore from (4) $\alpha_t = 0$ for $b^A = 0$. Hence from (5) $k^A = (w^A - c^1_A) / (1+n)$, from which by (7), (21) and $y^A = w^A$, we obtain after collecting terms (22.2). (22.3) - (22.6) may now be derived from ready substitutions of (22.2) into (7a), (13a), (17a), (18a) and (21).

Appendix 3:

Derivation of the net-transfer steady states: (24.1) is straightforward from (23). Substituting (7a), (13a) and (24.1) into (10), disposable income is given by

$$y(\tau) = q \cdot k(\tau)^p + \frac{[r^s - p \cdot k(\tau)^{-q}] \cdot \tau}{n - r^s} \quad (A6)$$

Substituting the steady-state value of saving $k(\tau) \cdot (1+n) / (1-\alpha)$ and (4) into (24.1), we can solve for α to obtain

$$\alpha = \frac{\tau}{(n-r^s) \cdot k(\tau) + \tau} \quad (A7)$$

From (5), upon using (A6), (A7) and (21) we obtain (24.2). Substituting (24.2) back into (A6), (7a), and (13a), (24.3), (24.4) and (24.6) readily follow.

Properties of net-transfer steady-states: From (24.2) $\text{sign } dk(\tau)/d\tau = \text{sign } dR_1(\tau)/d\tau$, where

$$R_1 = \frac{\left[\frac{\Omega \cdot q}{1+n} \right] - \left[\frac{\Omega \cdot p \cdot \tau}{n-r^s} \right]}{1 + (1-\Omega \cdot r^s) \cdot \tau}$$

and

$$\frac{dR_1(\tau)}{d\tau} = - \frac{(1-\Omega \cdot r^s) \cdot \left[\frac{\Omega \cdot q}{1+n} \right] + \left[\frac{\Omega \cdot p}{n-r} \right]}{[1 + (1-\Omega \cdot r^s) \cdot \tau]^2} < 0.$$

From (24.2) $k(\tau) \rightarrow k^A$ for $\tau \rightarrow 0$. Since $dk(\tau)/d\tau < 0$, $dw(\tau)/d\tau < 0$ and $dr(\tau)/d\alpha > 0$ with $w(\tau) \rightarrow w^A$ and $r(\tau) \rightarrow r^A$ as $\tau \rightarrow 0$. Substituting (24.1) in (10) it follows that income is positive if $w(\tau) > \tau \cdot [r(\tau) - r^s]/(n-r^s)$. Comparing the RHS of this condition with the LHS of the solvency condition (25) it follows that $y(\tau)$ and hence $w(\tau)$ and $k(\tau)$ must be positive for $\tau \leq \tau^{\max}$. Properties of (24.6) are discussed in the main body of the paper when welfare implications of sovereign debt are discussed.

Appendix 4:

Derivation of the roll-over steady-states: Substituting (4), (7a), (13a) and (21) into (10), disposable income may be expressed as a function of $k(\alpha)$ and α . Substituting this expression into (5) and (21) we obtain after collecting terms (26.2). Substituting (26.2) back into the expression for

disposable income, (7a) and (13a), (26.3), (26.4) and (26.6) readily follow. From (21) and (4) we obtain (26.1).

Properties of roll-over steady-states: From (26.2) $\text{sign } dk(\alpha)/d\alpha = \text{sign } dR_2(\alpha)/d\alpha$ where

$$R_2(\alpha) = \frac{\Omega \cdot (q - \alpha)}{1 + n \cdot (1 - \Omega \cdot \alpha)},$$

and

$$\frac{dR_2(\alpha)}{d\alpha} = - \frac{\Omega}{1 + n \cdot (1 - \Omega \cdot \alpha)} + \frac{n \cdot \Omega \cdot (q - \alpha)^2}{[1 + n \cdot (1 - \Omega \cdot \alpha)]^2} < 0.$$

$k(\alpha) \rightarrow k^A$ for $\alpha \rightarrow 0$ and $k(\alpha) \rightarrow 0$ for $\alpha \rightarrow q$. As $dk(\alpha)/d\alpha < 0$, $dw(\alpha)/d\alpha < 0$ and $dr(\alpha)/d\alpha > 0$, with $w(\alpha) \rightarrow w^A$ and $r(\alpha) \rightarrow r^A$ as $\alpha \rightarrow 0$. Furthermore $w(\alpha) \rightarrow 0$ and $r(\alpha) \rightarrow \infty$ for $\alpha \rightarrow q$. From (26.5) $d[c^2(\alpha)/c^1(\alpha)]/d\alpha > 0$, $c^2(\alpha)/c^1(\alpha) \rightarrow (c^2/c^1)^A$ for $\alpha \rightarrow 0$ and $c^2(\alpha)/c^1(\alpha) \rightarrow \infty$ for $\alpha \rightarrow q$.

Per capita lending may be expressed in implicit form by

$$H(b, \alpha) = \frac{\alpha}{1 - \alpha} \cdot (1 + n) \cdot k(\alpha) - b = 0.$$

$\text{Sign } db/d\alpha = \text{sign } \partial H/\partial \alpha$ with

$$\partial H/\partial \alpha = \frac{1 + n}{1 - \alpha} \cdot \left[\alpha \cdot \frac{dk(\alpha)}{d\alpha} + k(\alpha) \cdot \frac{1}{1 - \alpha} \right] \begin{cases} \geq \\ < \end{cases} 0.$$

For $\alpha \rightarrow q$, $k(\alpha) \rightarrow 0$ and hence $db(\alpha)/d\alpha < 0$. Furthermore, from $H(b, \alpha) = 0$, $b(\alpha) \rightarrow 0$ for $\alpha \rightarrow q$. For $\alpha \rightarrow 0$, $\partial H/\partial \alpha \rightarrow (1 + n) \cdot k^A > 0$, hence $db(\alpha)/d\alpha > 0$ for 'small' α . From (26.1) and (26.6), $b(\alpha)$ is continuous in α . Since $b(\alpha)$ is increasing for 'small' α and decreasing for 'large' α , $b(\alpha)$ must be maximal for some intermediate value of α . Properties of the disposable income function (26.6) are discussed in the main body of the paper.

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Footnotes

*. This work originated in the "SFB 178", supported by the "Deutsche Forschungsgemeinschaft". Thanks for helpful comments are due to Horst Siebert, Hans-Werner Sinn and participants of a research seminar at the University of Konstanz.

1. Disregarding trade sanctions has other advantages. It is argued that trade sanctions lack a practical importance for a variety of reasons [Niehans (1985)]. On the theoretical side, the (partial) cooperative equilibrium concept that sustains cooperation through sanctions in a noncooperative environment [Friedman (1971)] can be shown to contradict individual rationality in an important class of models [Mohr (1988b)]. The point is that sanctions must be credible in order to be an effective deterrent. In repeated games without learning, where individual rationality should be time invariant, credible sanctions fail to exist under weak assumptions.

2. Alternatively a social planner with concern only for the present may be assumed.

3. This assumption avoids having to deal with the problem of individual free-riding on collective roll-over efforts. I owe this to Horst Siebert.

4. This has several implications. First, the credit demand in each period is infinite for a given interest rate, since from the borrower's point of view a gift cannot be excessive.

Secondly, the borrower is willing to absorb loans of a given size at any rate of interest.

5. In this model the penalty is credible since, as will be shown, repudiation of sovereign debt will coincide with instantaneous bankruptcy of the banking sector in the lending country, leaving it without funds to provide new loans to the borrower.

6. The overlapping generations paradigm has been applied to international lending by Buiter (1981) and Schmid (1987). Contrary to the present paper, they maintain the traditional view of taking international debt servicing enforceable.

7. Throughout, lower case variables with index t denote values per capita of generation t .

8. Compare the statement under Footnote 5 above.

9. Like the sovereign debtor (see Footnote 4 above), generation t , by (18) is indifferent with respect to r^s_{t+1} . Therefore the interest rate on sovereign debt is arbitrary in the model.

10. An inefficient autarky steady-state is characterized by $n > r^A$. In Burbidge (1983) and Carmichael (1982) it is shown that the steady-state is efficient, i.e. $n < r^A$, if there is an intergenerational transfer from old to young. Thus welfare improving lending may be seen as a theoretical possibility that is not of great practical relevance. Furthermore altruistic motives would mitigate the problems arising from sovereign lending through a financial intermediary. Bernheim (1987), however, shows that altruistically motivated intergenerational transfers may disappear if a social planner maximizes a social welfare function. Furthermore, it is the spirit of this paper to view bankers as purely selfish and insensitive to the consequences of their actions for the future stability of the institution they represent. As an immediate consequence of this dynamic principal-agency view of international financial intermediation the model abstracts from altruistic motives.

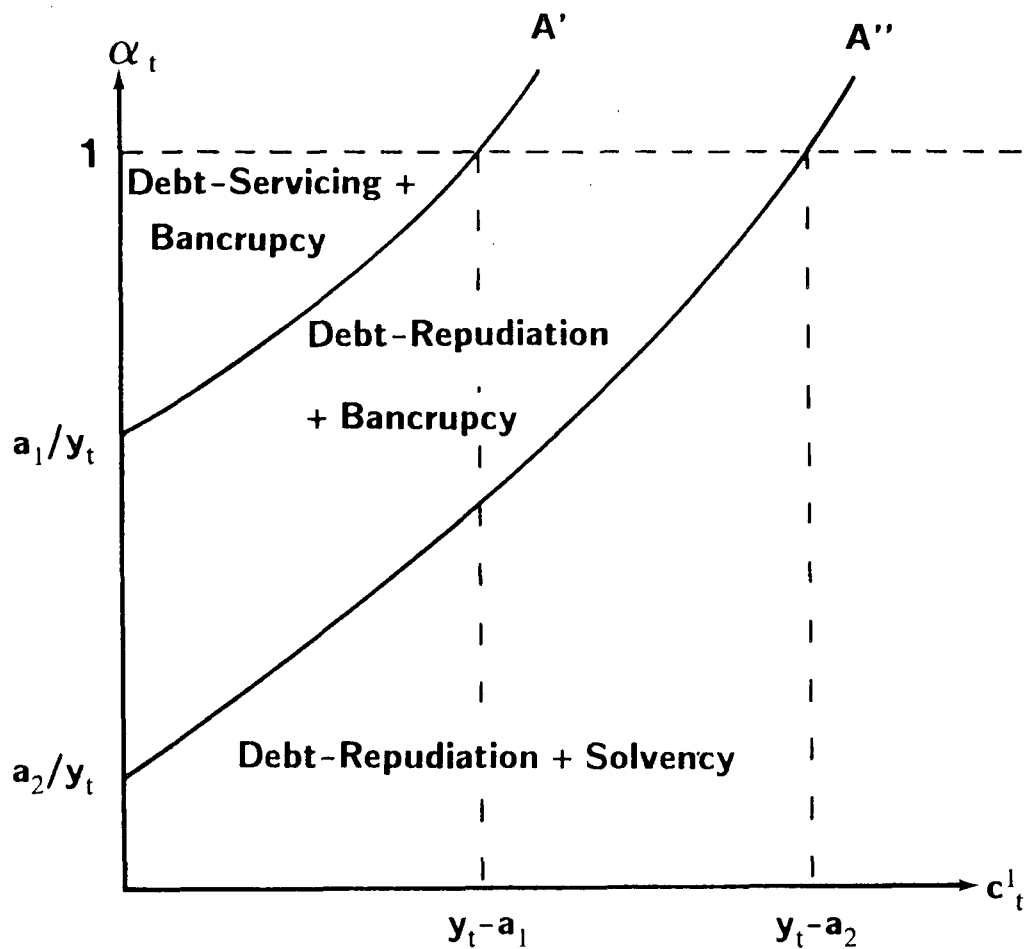


Figure 1a: $a_1 > a_2$

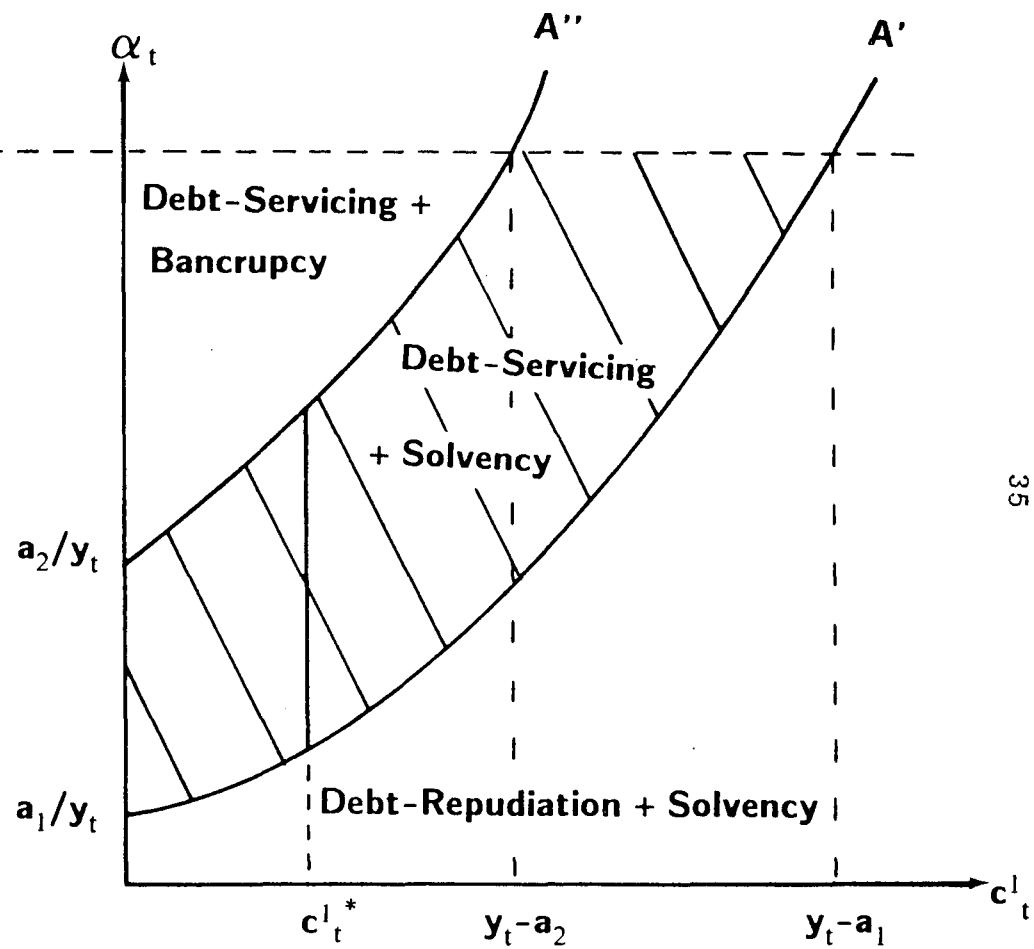


Figure 1b: $a_1 < a_2$

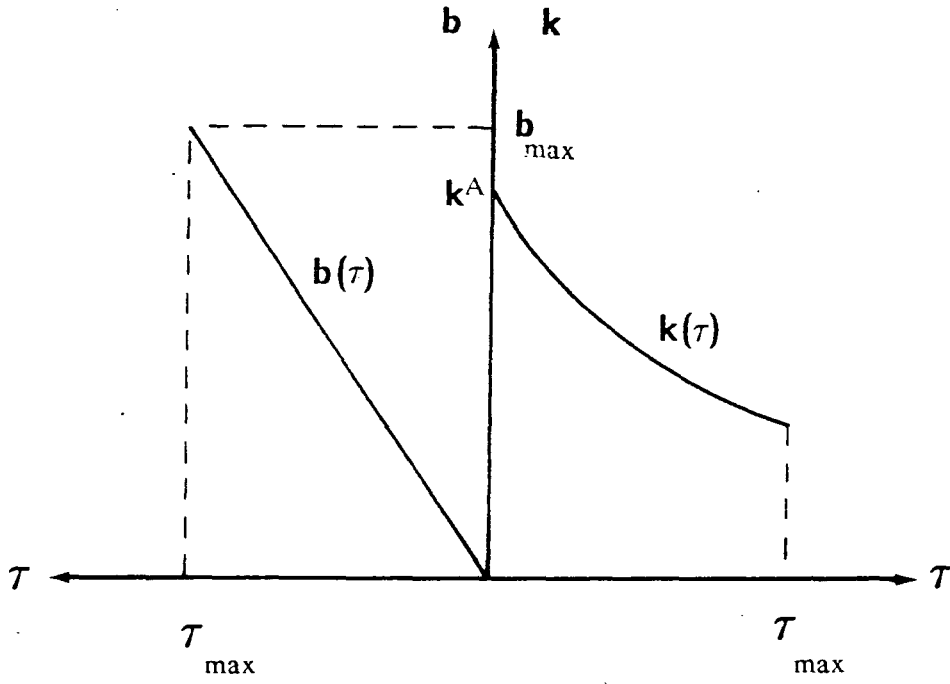
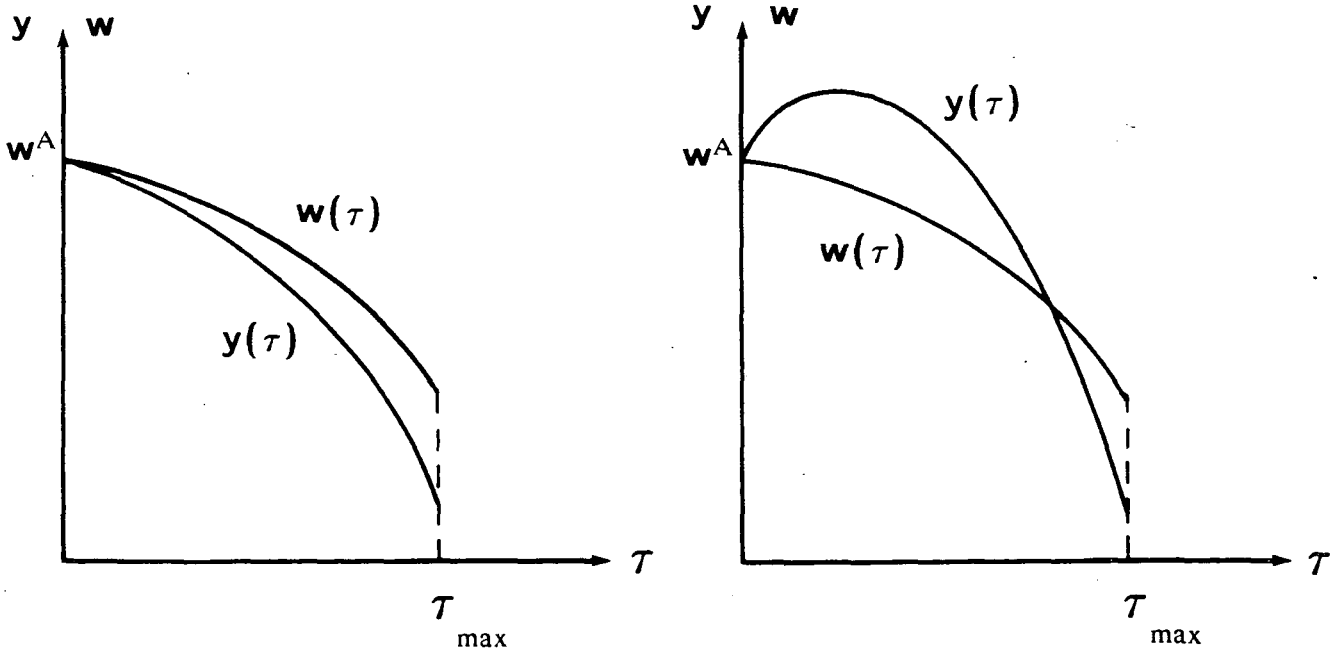


Figure 2

Figure 3a: $r^A \geq n > r^S$ Figure 3b: $r^A < r^S < n$ and p 'small'

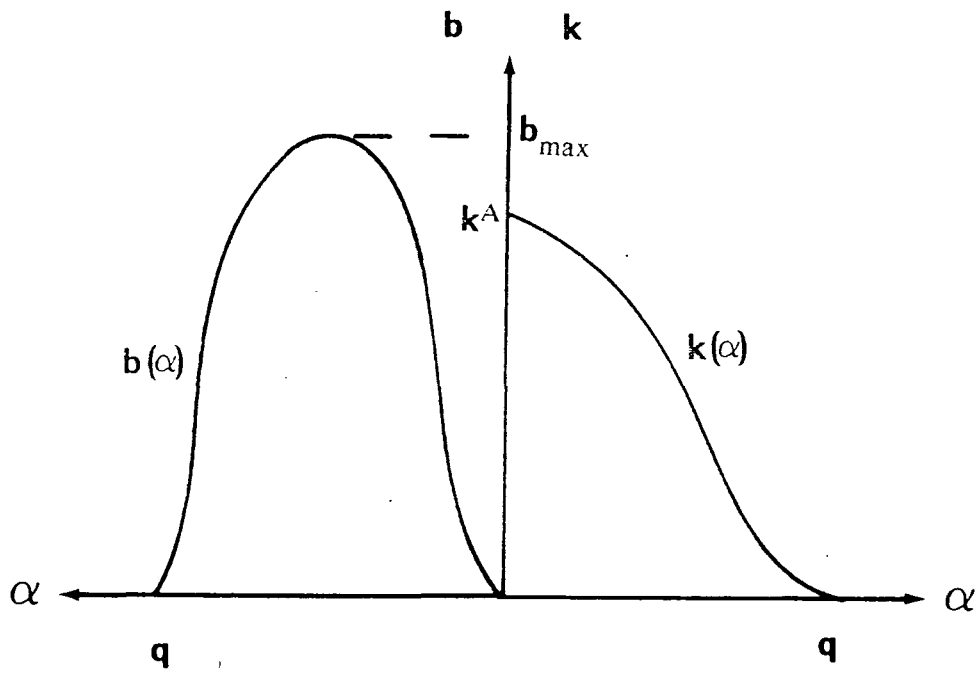


Figure 4

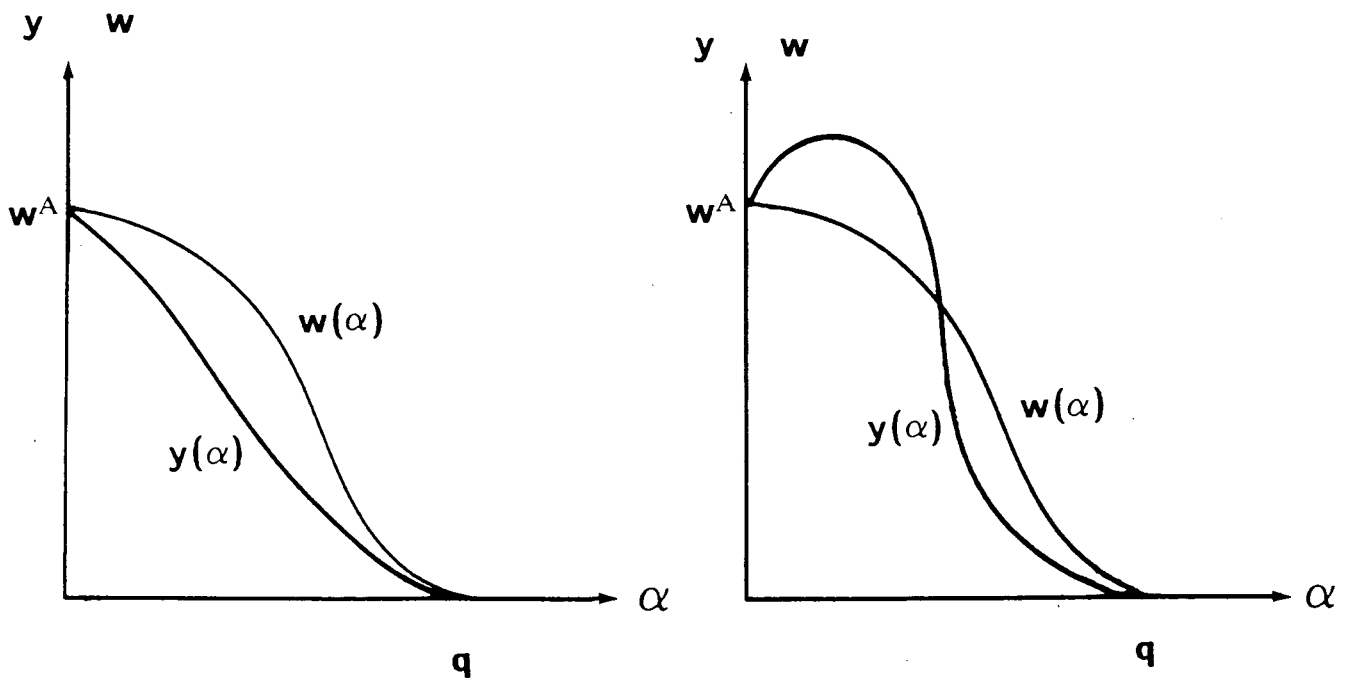


Figure 5a: $r^A \geq n = r^S$

Figure 5b: $r^A < r^S = n$ and p 'small'