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The effect of forward markets and currency options on international trade

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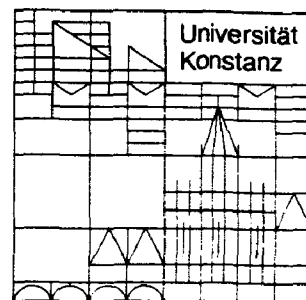
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**Sonderforschungsbereich 178
„Internationalisierung der Wirtschaft“**

Diskussionsbeiträge



**Juristische
Fakultät**

**Fakultät für Wirtschafts-
wissenschaften und Statistik**

**Udo Broll
Jack Wahl**

**The Effect of Forward Markets
and Currency Options
on International Trade**

**THE EFFECT OF FORWARD MARKETS
AND CURRENCY OPTIONS
ON INTERNATIONAL TRADE**

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The Effect of Forward Markets and Currency Options on International Trade

By

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Abstract

This paper presents a model of a competitive risk averse exporting firm under exchange rate uncertainty. If forward market contracts are available neither the distribution parameters of the exchange rate nor the degree of the firm's risk aversion have any impact on the export level. But this separation property does not hold in the case of currency options. It is shown that under some conditions, exports are *larger* under exchange rate uncertainty in the presence of currency options than they are in the so-called certainty equivalent case, and that exports *increase* with volatility of the exchange rate provided that risk aversion is not too high.

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1 Introduction

Fluctuations in the exchange rates of the major industrial countries have substantially increased exchange rate uncertainty (see KRUGMAN (1989)). As a result a variety of hedging techniques have been created or increasingly used by financial markets, for example forward and futures market contracts and currency options. Exchange rate risk also effected international trade as reported by the empirical work of CUSHMAN (1988). The aim of this paper is to study the interaction between exchange rate volatility, different hedging tools and the output decision of an exporting firm.

The recent literature on exporting firms' behaviour under exchange rate uncertainty has focused on the role of forward or futures markets. FEDER, JUST and SCHMITZ (1980), BENNINGA, ELDOR and ZILCHA (1985), KAWAI and ZILCHA (1986), BROLL and ZILCHA (1990) have examined optimum decisions of a risk averse firm that engages in production and hedges exchange risk in forward & futures markets. The main results are the *separation theorem* and the *full hedging theorem*.¹ The separation theorem says that the firm's export decision does not depend on the firm's attitude towards risk or the probability distribution of the random spot exchange rate when forward or futures markets exist. The full hedging theorem states that, if the forward or futures markets are unbiased, the firm completely avoids exchange rate uncertainty by entering into optimum forward or futures contracts.

As a departure from the literature quoted above we assume in our analysis that exporters have access to currency option contracts instead of forward markets. The main feature of the paper is that currency options are explicitly modeled.

A forward contract for foreign exchange calls for delivery of a specified

¹The separation property was first noticed by DANTHINE (1978), HOLTHAUSEN (1979) and KATZ and PAROUSH (1979). For a general equilibrium model of international trade with risk sharing markets see ISHII (1986).

amount of one currency against another at a fixed future date and price. This implies that the contracting partners must fulfill the signed obligation. While forward contracts protect the holder against the risk of adverse movements in exchange rates, they also eliminate the possibility of gaining a profit from favourable movements in the exchange rate. A currency option, by contrast, provides one partner with the choice either to exercise the option or to allow it to expire depending on how the market is going to evaluate the underlying currency.

To illustrate the use of currency options as a hedging alternative to the forward market, let us assume that a domestic exporting firm receives a future payment in foreign currency, for instance in Swiss Francs. If it covers exchange rate uncertainty in the forward market the firm is committed to receive a certain amount of domestic currency. By buying Swiss Franc *put* options with the desired maturity, the exporting firm does not make such a commitment. If the Franc depreciates and the exchange rate falls below the specified exercise or strike price, the exporter will exercise the option. Alternatively, if the exchange rate does not reach the strike price, the firm can let the option expire. The currency option thus allows the exporter, in return for a known premium, to protect the firm against a falling Franc, while allowing the firm to capture capital gains on a rising Franc.

The main rationale of our study is as follows. If exporting firms are risk averse, then exchange rate uncertainty reduces international trade when there are no risk sharing markets or equivalent insurance devices. The reference situation for a comparison between certainty and uncertainty is the so-called *certainty equivalent case*, which implies that the random spot exchange rate \tilde{e} is replaced by the expected exchange rate $E\tilde{e} \equiv \bar{e}$. The optimal output under a certain exchange rate \bar{e} is larger than under a random exchange rate with expected value \bar{e} .

When *forward markets* for foreign exchange are available, and when the forward exchange rate is equal to the expected exchange rate, then the firm's

optimal export is equal to the certainty equivalent case. In this respect the effects of uncertainty are *neutralized* by a forward contract.²

Suppose that instead of a forward market an option market is available. If the strike price net of the currency option price is equal to the expected exchange rate, then the firm's export is *larger* than in the certainty equivalent case.

With currency options the separation theorem does not hold. Furthermore the volume of international trade increases with exchange rate uncertainty. The more 'volatile' the exchange rate is, the higher is the export level.

We first consider the firm's optimal trade decision under conditions where no risk sharing markets are available. Then we analyse the case of existing forward markets. Alternatively, we then investigate currency options. By comparing these institutions with the certainty equivalent case, we can examine the effect of establishing forward or currency option markets on the level of export production.

The paper is organized as follows. In *Section 2* the model is presented. The effect of uncertainty on exports when no risk sharing markets exist is analysed. Exports are decreasing when uncertainty is introduced. In *Section 3* the impact of establishing forward markets on the firm's output is studied. We show that a separation theorem holds for this exporting firm. In *Section 4* we discuss the optimal hedging policy in the presence of currency options. It is shown that under plausible conditions the separation theorem does not hold. Finally, in *Section 5*, we provide concluding remarks.

2 The Model

Consider a competitive exporting firm under exchange rate uncertainty.

²In our analysis there is no *basis* risk so that there is no difference between forward and futures markets.

The export production gives rise to a deterministic cost function $C(x)$ where x is the export level. It is assumed that the function C is strictly convex, increasing and twice differentiable, and that the firm always produces a positive amount (i.e., $x > 0$).

The production decision is made at time t and the output will be sold at the foreign currency price p^* , that yields uncertain revenue in domestic currency at subsequent time $t + 1$. The random profit from exports can be written in the traditional way (time subscripts omitted)

$$\tilde{\Pi} = \tilde{e} \cdot p^* \cdot x - C(x) \quad ,$$

where \tilde{e} is the random spot exchange rate and p^* the given export price in foreign currency. The exchange rate is defined in domestic currency per unit of foreign currency. First, we investigate the effects of exchange rate uncertainty without any risk sharing markets.

The firm is risk averse with a von Neumann–Morgenstern utility function $U(\cdot)$ and maximizes the expected utility of its domestic currency profits. We take the utility function to be a strictly concave, increasing and twice differentiable function. The firm's decision problem can be written as

$$\max_x E[U(\tilde{\Pi})] \quad , \tag{1}$$

where

$$\tilde{\Pi} = \tilde{e} \cdot p^* \cdot x - C(x)$$

and E is the expectation operator. The interior solution requires the following equation:

$$EU'(\tilde{\Pi})(\tilde{e} \cdot p^* - C'(x)) = 0 \quad . \tag{2}$$

In order to explore the impact of uncertain exchange rates we use eq. (2). Since Π increases in e and $U'(\cdot)$ is a decreasing function, we get $Cov(\tilde{e}, U'(\tilde{\Pi})) < 0$. Hence we obtain

$$\bar{e} \cdot p^* - C'(x) > 0 \quad , \tag{3}$$

where \bar{e} is the expected exchange rate, $E\tilde{e} \equiv \bar{e}$.

Certainty equivalent case. It is common to compare the firm's optimal decision under uncertainty with the certainty equivalent, i.e., with the case where \tilde{e} is replaced by $E\tilde{e} = \bar{e}$ (see for example LELAND (1972)). The certainty equivalent case defines a quantity of exports as a reference for various cases. Therefore we denote the optimal production and export level when \bar{e} is the certain exchange rate by x_c . Then from eq. (3) and the optimality condition for the certainty case, we can state

Propositon 1. (*Effect of Uncertainty*). *The optimal output of the exporting firm under the supposed certain exchange rate \bar{e} is greater than its optimal output under the random exchange rate \tilde{e} .*

Proof. Since $C''(x) > 0$ it is proved from eq. (3) and the certainty case, that $x_c > x$. ||

Introduction of uncertainty causes the exporting firm to reduce its production level in order to reduce the foreign exchange risk.³

3 Forward Markets

Suppose now that perfect forward markets for foreign exchange are available. In this case the firm can realize its optimal hedging $\{e_f, z^*\}$ for foreign currency sale in the forward market, where e_f is the forward exchange rate and z^* the contract amount in foreign currency.

The firm chooses x , and therefore also z^* , so as to

$$\max_{x, z^*} E[U(\tilde{\Pi})] \quad , \quad (4)$$

where

$$\tilde{\Pi} = \tilde{e} \cdot p^* \cdot x - C(x) + z^* \cdot (e_f - \tilde{e}) \quad .$$

³Note that Proposition 1 refers to the special case of a mean preserving spread in the exchange rate, the benchmark being a zero spread.

The first order conditions are given by

$$EU'(\tilde{\Pi})(\tilde{e} \cdot p^* - C'(x)) = 0 \quad , \quad (5)$$

$$EU'(\tilde{\Pi})(e_f - \tilde{e}) = 0 \quad . \quad (6)$$

Owing to the assumed strict concavity of the utility function and strict convexity of C , these are also sufficient conditions for a unique maximum.

Now we can state the impact of forward markets on the firm's production and hedging behaviour. From eqs. (5) and (6) we obtain

Proposition 2. (*Strong Separation*). *When forward markets are available, the firm's optimal export is given by*

$$C'(x_f) = e_f \cdot p^* \quad . \quad (7)$$

The optimal export x_f is independent of the utility function and of the probability distribution of the random spot exchange rate.

For this competitive exporting firm a *separation theorem* holds. Proposition 2 claims that the optimal export level is chosen at a point where the marginal cost is equal to the marginal revenue. Except for the costs of production the only parameter which affects the export level is the forward rate e_f and the foreign commodity price. Neither the distribution parameters of the random exchange rate nor the firm's risk aversion have any impact on the quantity of exports.⁴

Suppose the forward rate e_f is unbiased, i.e., e_f is equal to the expected exchange rate \bar{e} . This implies, that the optimal export x_f is equal to the export level in the certainty equivalent case (i.e., $x_f = x_c$). Hence by introducing an unbiased forward market international trade will increase, if there are no hedging markets before.

In the sequel we drop the consideration of forward markets and we assume that only option markets exist.

⁴See KATZ and PAROUSH (1979), KAWAI and ZILCHA (1986), ELDOR and ZILCHA (1987).

4 Currency Option Markets

In general, an option is a financial instrument which provides the holder with the right to sell (put) or buy (call) the underlying asset at a prefixed price and expiration date. When applied to foreign currencies, a put option gives an exporting firm the right to sell the contracted currency at the expiration date. Thereby the firm has the 'option' - as distinct from the obligation under a forward contract - of selling foreign currency at a prearranged strike price. The firm will do so, if the option is in the money. Otherwise the firm will just let the option expire and sell the foreign currency in the spot market. For this privilege the firm must pay some premium. This premium is the maximum amount the buyer can lose from an option contract.⁵

Now we consider the behaviour of our exporting firm in the presence of a currency option contract. The model illustrates the implications for international trade from buying a put option with strike price b and premium (option price) p_o . At a spot exchange rate lower than b , the option will be exercised; with a spot rate equal to or higher than b the option will not be exercised. Hence, the domestic currency gain, per unit of foreign currency, from the option contract can be formalized as $(b - e)^+ \equiv \text{Max}\{0, b - e\}$.

Thus, the firm chooses the export level x , and therefore the option contract amount z^* , by

$$\max_{x, z^*} E[U(\tilde{\Pi})] \quad , \quad (8)$$

where

$$\tilde{\Pi} = \tilde{e} \cdot p^* \cdot x - C(x) + z^* \cdot (b - \tilde{e})^+ - p_o \cdot z^* \quad .$$

The optimal solution requires the equations

$$EU'(\tilde{\Pi})(\tilde{e} \cdot p^* - C'(x)) = 0 \quad , \quad (9)$$

⁵To keep the model simple, we assume that options can be exercised only on their maturity date (European options), see, e.g., SHAPIRO (1989).

$$EU'(\tilde{\Pi})((b - \tilde{e})^+ - p_o) = 0 \quad . \quad (10)$$

We claim the following proposition concerning the optimal quantity x_o to be produced and exported in the presence of currency option contracts.

Proposition 3. (*Weak Separation*). *Assume that only currency option markets are available. Then the firm's optimal production is determined by the following equation (with $\hat{U}'(\tilde{\Pi}) \equiv U'(\tilde{\Pi})/EU'(\tilde{\Pi})$):*

$$C'(x_o) = (b - p_o) \cdot p^* + E[\hat{U}'(\tilde{\Pi})(\tilde{e} - b)^+] \cdot p^* \quad . \quad (11)$$

In general, the degree of the dependence of optimal export x_o upon the utility function or upon the probability distribution of the random spot exchange rate is determined by the level of risk aversion.

Proof. From eqs. (9) and (10) we get

$$p^* \cdot E\hat{U}'(\tilde{\Pi})\tilde{e} = C'(x_o) \quad , \quad (12)$$

$$E\hat{U}'(\tilde{\Pi})(b - \tilde{e})^+ = p_o \quad . \quad (13)$$

Multiplying eq. (13) by p^* , adding the result to eq. (12), and rearranging terms we obtain eq. (11) by using the relationship $(b - e)^+ = (b - e) + (e - b)^+.$

Proposition 3 shows that the firm's output for export is affected by its probability beliefs and the degree of its risk aversion.

The RHS of eq. (11) reveals that two effects are present: an effect similar to the forward market case (represented by the first summation item) and a 'distortion' which comes from the specific feature of the currency option. The impact of this strictly positive distortion upon the export can be illustrated as follows.

Suppose that the firm's probability beliefs are such that the strike price net of the option price $(b - p_o)$ is equal to the expected exchange rate \bar{e} . Then optimal export of the firm with currency option markets, x_o , turns out to be greater than in the certainty equivalent case (i.e., $x_o > x_c$). The intuition

for this result is that the option allows the firm to participate in favourable changes of the exchange rate (i.e., $\tilde{e} > b$) while protecting it against unfavourable movements. Hence this participation feature of the option produces the positive impact on international trade.

Let us now investigate the implications of the weak separation property of the currency option. We concentrate the analysis on the market's completeness, the firm's risk aversion, and the exchange rate's volatility.

Complete Market. Consider the binomial case with two states of nature, i.e., the future spot exchange rate can assume two values only. Then the production level is determined by technology, both exchange rate values, the strike price, the premium and the export price. Hence the firm's export is independent of its utility function and of its probability assessments w.r.t. the occurrence of the states of nature. In other words, the firm's optimal export decision can be separated from its hedging decision. This is summarized in the following

Corollary 1. *With currency options the strong separation theorem holds if and only if the market is complete.*

The *proof* is provided in the Appendix.

Risk aversion. Because the strong separation property does not hold in the presence of currency options when there are more than two states of nature (\equiv incomplete market), we demonstrate the impact of increasing risk aversion upon the level of production and exports. Consider two exporting firms A and B with identical technologies. Since the firms' profits are monotonically increasing in e , the more risk averse firm will produce less. Denoting x_A and x_B the optimal export of firm A and B , respectively, we claim

Corollary 2. *Suppose that currency options only are available as a hedging instrument in an incomplete market setting. Then, if firm A is more risk averse than firm B , the optimal export of firm A is smaller than the optimal export of firm B (i.e., $x_A < x_B$).*

The *proof* is shown in the Appendix.

Volatility. The option argument (see FRANKE (1986)) considers exports as an opportunity for (risk neutral) firms to profit from international price differences. This 'export option' is sometimes exercised, sometimes not, depending on the exchange rate level. An increase in exchange rate volatility implies that potential international price differences are increasing so that the option becomes more valuable. Hence the volume of international trade *increases* with exchange rate volatility.

Introducing risk aversion reduces the impact of optional behaviour. In general the net effect of increased volatility becomes ambiguous.

In our model, the answer to the question whether the degree of risk aversion or the use of the currency option is dominant in determining the effect of exchange rate volatility on international trade, can be expressed as follows.

Corollary 3. *Suppose an incomplete market setting. Then a higher 'volatility' in the spot rate of foreign exchange will increase exports provided that risk aversion is not too high.*

The *proof* is given in the Appendix (part (a)).

Corollary 3 shows that under some conditions higher exchange rate volatility may stimulate international trade when currency option markets exist. Furthermore the following sufficient condition is implied: if the marginal income elasticity of profit, induced by a marginal change in the spot rate of foreign exchange, is smaller than the proportional risk tolerance, then export volume will increase with exchange rate volatility. (see part (b) of the proof of Corollary 3).

Now the relationship between volatility, risk aversion and international trade will be illustrated in Figure 1.

Consider eq. (11) and denote x_{min} the output which arises when the strike price is (almost) equal to the maximum spot rate of foreign exchange. Given the risk neutral firm R_0 eq. (11) shows that the optimal export x is

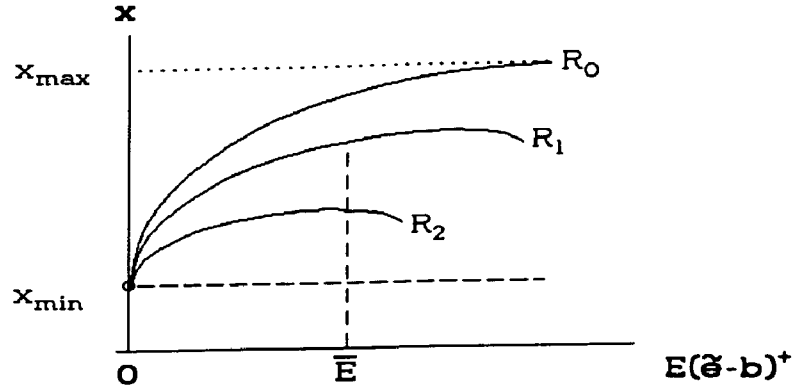


Figure 1: Volatility, Risk Aversion and International Trade

an increasing and concave function of the upper partial mean $E(\tilde{\epsilon} - b)^+$. This result includes the observation that the price of the put increases with volatility (see GARMAN and KOHLHAGEN (1983)). Of course, the output is bounded.

Now consider the risk averse firms R_1 and R_2 (with $R_1 < R_2$). Applying Corollary 3 it follows that an increase of the upper partial mean beyond \bar{E} will lead to an increase in the optimal export only for the less risk averse firm R_1 . Hence there exists a maximum export level for a given degree of a firm's risk aversion.

5 Currency Option Contracting: An Example

This section illustrates an example of the firm's optimal decision on export x and currency option contract z^* under a specific mean-variance function. Let us assume that the firm maximizes $E(\hat{\Pi}) - \alpha V(\hat{\Pi})$, where α is positive and V is the variance operator.

The optimal contract amount z^* is given by

$$z^* = s + h \cdot p^* x \quad , \quad (14)$$

where

$$\begin{aligned} s &= \frac{E(b - \tilde{e})^+ - p_o}{2\alpha V(b - \tilde{e})^+} \quad , \\ h &= \frac{Cov[\tilde{e}, -(b - \tilde{e})^+]}{V(b - \tilde{e})^+} \quad . \end{aligned}$$

Hence the option demand is a linear function of export revenue. Here $h \cdot p^* x$ represents the pure hedging demand, and s denotes the pure speculative demand. Note that the hedge ratio h is greater than one, and that the pure speculative demand disappears if and only if the option price is 'fair', i.e., equal to the lower partial mean. Even in the latter case there will be more than full hedging as opposed to the unbiased forward market case.

The optimality condition for the export level reads

$$C'(x_o) = (b - p_o)p^* + \{E(\tilde{e} - b)^+ - 2\alpha K\}p^* \quad , \quad (15)$$

where

$$K = Cov[(\tilde{e} - b)^+, (b - \tilde{e})^+] \cdot z^* + Cov[(\tilde{e} - b)^+, \tilde{e}] \cdot p^* x_o \quad .$$

First, it is obvious from eq. (15) that our example confirms the proposition that the strong separation does not hold in the presence of currency options (Proposition 3). Second, an increase in risk aversion, which is a function of α , induces a lower level of export (Corollary 2). Third, the interaction between international trade and the volatility of the spot rate of foreign exchange (here measured by the covariance terms) depends on the degree of risk aversion (Corollary 3).

6 Concluding Remarks

In the preceding analysis we examined the optimal behaviour of a competitive risk averse firm that exports commodities invoiced in foreign currency.

A risk averse trading firm takes hedging opportunities into account. We distinguish between no risk sharing markets, forward markets and currency option markets.

Under exchange rate uncertainty, the firm determines the level of exports and the extent of forward foreign exchange or foreign currency options. Specifically, in the absence of risk sharing markets the firm reduces its business risk by lowering exports.

If forward markets exist, then the risk elements affect only the level of hedging but not the quantity exported. In the case of currency options the exporter has the opportunity to protect against a rising domestic currency and to cash in on a falling domestic currency. Under some conditions the volume of exports is larger with currency option markets than in the so-called certainty equivalent case.

With currency options as an hedging instrument the separation theorem does not hold generally, i.e., distribution and utility elements affect the level of production and thus the volume of international trade. An increase in risk aversion implies lower exports. On the other hand, it is demonstrated, that higher volatility of the exchange rate may stimulate higher exports, provided that the firm's attitude towards risk is not too high.

Our paper extends the results in the literature by demonstrating the importance of establishing currency option markets for trading firms. The impact of such markets on international trade differs from that obtained for exporting firms which have access to forward or futures markets only.

Appendix

In the appendix, we will prove Corollaries 1, 2, and 3.

Proof of Corollary 1. Let

$$\mathbf{A} \equiv \begin{pmatrix} e(1) \cdot p^* - C'(x) & e(2) \cdot p^* - C'(x) \\ (b - e(1))^+ - p_o & (b - e(2))^+ - p_o \end{pmatrix},$$

and

$$\mathbf{y} \equiv \begin{pmatrix} f(1) \cdot U'(\Pi(1)) \\ f(2) \cdot U'(\Pi(2)) \end{pmatrix},$$

where $e(\theta)$ denotes the spot rate of foreign exchange if state θ occurs, $f(\theta)$ defines the positive probability of the occurrence of state θ , and $\Pi(\theta)$ denotes the firm's profit in state θ ($\theta = 1, 2$). Suppose w.l.o.g. that $e(1) > e(2)$. Then, of course, $e(1) > b > e(2)$.

With two states of nature eqs. (9) and (10) imply the following linear system:

$$\mathbf{A} \cdot \mathbf{y} = \mathbf{0}.$$

Hence \mathbf{y} has a nontrivial solution ($\mathbf{y} \neq \mathbf{0}$) iff \mathbf{A} is singular ($\det \mathbf{A} = 0$). A vanishing determinant is equivalent to

$$C'(x) = \frac{e(1)d(2) - e(2)d(1)}{d(2) - d(1)} \cdot p^*$$

where $d(\theta) = (b - e(\theta))^+ - p_o$ ($\theta = 1, 2$). It follows that $C'(x)$ and, therefore, x is independent of the utility function and the probability assessments. ||

Proof of Corollary 2. Let firm A be more risk averse than firm B with utility functions $U_A(\cdot)$ and $U_B(\cdot)$, respectively. Since A is more risk averse than B , we have $U_A(\cdot) = F(U_B(\cdot))$, where $F' > 0$ and $F'' < 0$ (see PRATT (1964)).⁶

⁶The proof is adapted from KAWAI and ZILCHA (1986).

Using eq. (11), a necessary condition for optimal export of firm A is given by

$$C'(x_A) = (b - p_o) \cdot p^* + E[\hat{U}'_A(\tilde{\Pi}_A)(\tilde{e} - b)^+] \cdot p^* \quad .$$

This condition can be rewritten as

$$0 = E[F'(U_B(\tilde{\Pi}_A))U'_B(\tilde{\Pi}_A) \cdot \tilde{\omega}_A] \quad , \quad (16)$$

where

$$\tilde{\omega}_A \equiv (\tilde{e} - b)^+ + b - p_o - \frac{C'(x_A)}{p^*} \quad .$$

From eq. (16) we get

$$E_{\omega_A > 0}[F'(U_B(\tilde{\Pi}_A))U'_B(\tilde{\Pi}_A)\tilde{\omega}_A] = -E_{\omega_A \leq 0}[F'(U_B(\tilde{\Pi}_A))U'_B(\tilde{\Pi}_A)\tilde{\omega}_A] \quad . \quad (17)$$

Since $U'_B(\cdot) > 0$ and $F''(\cdot) < 0$ we obtain

$$Sup_{\omega_A > 0} F'(U_B(\tilde{\Pi}_A)) \leq Inf_{\omega_A \leq 0} F'(U_B(\tilde{\Pi}_A)) \quad . \quad (18)$$

We use eqs. (17) and (18) to derive

$$E_{\omega_A > 0}[U'_B(\tilde{\Pi}_A)\tilde{\omega}_A] > -E_{\omega_A \leq 0}[U'_B(\tilde{\Pi}_A)\tilde{\omega}_A]$$

which implies

$$E[U'_B(\tilde{\Pi}_A)\tilde{\omega}_A] > 0 \quad . \quad (19)$$

Hence the export of firm B must increase in order to fulfill the necessary condition for maximizing $E[U_B(\tilde{\Pi}_B)]$, i.e., $x_A < x_B$. ||

Proof of Corollary 3. (a). Let us define B as the set of all states of the world where the firm will not exercise the put option, i.e.,

$$B = \{e \mid e \geq b\} \quad .$$

Assume $|B| > 2$ and consider $e(\theta')$ with $\theta' \in B$. Furthermore rearrange eq. (11) such that we get

$$0 = E[U'(\tilde{\Pi}) \cdot \{(\tilde{e} - b)^+ + b - p_o - \frac{C'(x)}{p^*}\}] \quad . \quad (20)$$

By differentiating eq. (20) implicitly we obtain (with definition $\tilde{\omega} \equiv (\tilde{e} - b)^+ + b - p_o - \frac{C'(x)}{p^*}$ and $f(\theta') \equiv$ the probability of the occurrence of state θ')

$$\frac{dx}{de(\theta')} = - \frac{f(\theta') \cdot U''(\Pi(\theta')) \frac{\partial \Pi(\theta')}{\partial e(\theta')} \cdot \omega(\theta') + f(\theta') \cdot U'(\Pi(\theta')) \frac{\partial \omega(\theta')}{\partial e(\theta')}}{\{E[U''(\tilde{\Pi}) \cdot \frac{\partial \tilde{\Pi}}{\partial x} \cdot \tilde{\omega}] - \frac{C''(x)}{p^*} \cdot E[U'(\tilde{\Pi})]\}} \quad (21)$$

Since in the optimum $\text{sign} \frac{\partial \Pi(\theta)}{\partial x} = \text{sign}[\omega(\theta)]$ for all θ and $f(\theta') > 0$ we have

$$\text{sign}\left(\frac{dx}{de(\theta')}\right) = \text{sign}\left[U''(\Pi(\theta')) \cdot \frac{\partial \Pi(\theta')}{\partial e(\theta')} \omega(\theta') + U'(\Pi(\theta')) \frac{\partial \omega(\theta')}{\partial e(\theta')}\right] ,$$

from which we deduce

$$\frac{dx}{de(\theta')} > 0 \quad \text{iff} \quad U''(\Pi(\theta')) \cdot \frac{\partial \Pi(\theta')}{\partial e(\theta')} \omega(\theta') > -U'(\Pi(\theta')) \frac{\partial \omega(\theta')}{\partial e(\theta')} .$$

This relationship can be used (with the definition of R as the coefficient of absolute risk aversion, i.e., $R \equiv -U''(\cdot)/U'(\cdot)$ and the envelope result $\frac{\partial \Pi(\theta')}{\partial e(\theta')} = p^* x$) to get

$$\frac{dx}{de(\theta')} > 0 \quad \text{iff} \quad R(\Pi(\theta')) < \frac{\frac{\partial \omega(\theta')}{\partial e(\theta')}}{p^* \cdot x \cdot \omega(\theta')} \quad (22)$$

In order to fulfill the condition, that an increase in the volatility of the exchange rate increases (decreases) international trade, the degree of absolute risk aversion must be lower (greater) than the RHS of eq. (22). Note that the RHS of eq. (22) is positive, of course, otherwise risk aversion does not matter. Hence we have to compare the level of risk tolerance in state θ' (i.e., $1/R(\Pi(\theta'))$) with the export profit (denominated in domestic currency) from optimal production and hedging in state θ' .

(b). Define the coefficient of relative risk aversion $P \equiv R \cdot \Pi > 0$, the exchange rate elasticity of profit $\varepsilon_\Pi \equiv \frac{\partial \Pi}{\partial e} \cdot \frac{e}{\Pi}$ and the exchange rate elasticity of marginal income $\varepsilon_\omega \equiv \frac{\partial \omega}{\partial e} \cdot \frac{e}{\omega}$. Then eq. (22) implies

$$P(\Pi(\theta')) \cdot \varepsilon_{\Pi}(\theta') < \varepsilon_{\omega}(\theta') \quad .$$

Hence eq. (22) is fulfilled if $\varepsilon_{\Pi}(\theta')/\varepsilon_{\omega}(\theta') < 1/P(\Pi(\theta'))$, that is, the exchange rate elasticity of profit over the exchange rate elasticity of marginal income is smaller than the proportional risk tolerance.||

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