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## Stating import prices wrongly: Possibilities of tax and tariff evasion

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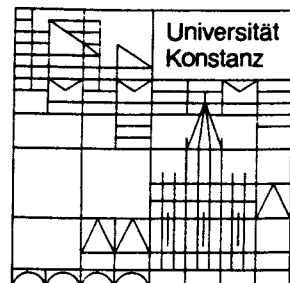
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Juristische  
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wissenschaften und Statistik

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Günther G. Schulze

**Stating Import Prices Wrongly—  
Possibilities of Tax and Tariff Evasion**

**STATING IMPORT PRICES WRONGLY -  
POSSIBILITIES OF TAX AND TARIFF EVASION**

**Günther G. Schulze\***

**Serie II - Nr. 138**

**Mai 1991**

\* I would like to thank Arye Hillman for inspiring me to study this topic and Gerd Ronning, Agnar Sandmo, Guttorm Schjelderup, Hans-Jürgen Vosgerau and Hannelore Weck-Hannemann for very valuable comments on an earlier draft. Most of all I am indebted to Karl-Josef Koch for his constant support and criticism.

## **Abstract**

This paper examines the possibility of a domestic producer evading taxes and duties by misstating the price of an imported intermediate good. In a qualitative response model it is shown that government's revenue maximizing behavior implies a detection function which depends positively on the amount of misdeclaration. As a consequence there exists, for every combination of (linear) import tariff and (linear) income tax, a unique declared price of the imported good which maximizes the expected (true, but not declared) profit. Whether under- or overdeclaration is optimal depends on the relative magnitudes of tax and tariff rates, whereas the amount of misdeclaration is determined by the specification of the detection function and the form of punishment.

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# 1 Introduction

There is a considerable and still growing body of literature dealing with various aspects of the economic approach to crime and punishment pioneered by Becker (1968) and Stigler [Stigler (1970), Becker&Stigler (1974)]<sup>1</sup>. Two major fields of concern have been smuggling and tax evasion which still remain unconnected. The purpose of the paper is to analyze the tradeoff between tax evasion and the evasion of duties on an individual firm's level – a relation which has not been noticed hitherto.

A domestic firm facing a linear ad valorem tariff and a linear income tax imports an intermediate product and declares one import price to both the customs and tax authorities. Authorities audit so as to maximize revenue. As a result, the probability of auditing depends on the declared price of the imported good. The risk-neutral firm in turn maximizes its profits with respect to the declared price, which is equivalent to the minimization of the sum of tax and tariff payments. It turns out that it is optimal for the firm either to *understate* the import price or to *overstate* it – depending on the relative magnitudes of tax and tariff rates. Since taxes are levied on after-tariff profits, underdeclaration reduces tariff liabilities and *simultaneously* increases tax liabilities while for overdeclaration the case is reversed. The extent of misdeclaration hinges on the probability of detection and the penalty.

The fact that overdeclaration may be optimal was not realized by authors dealing with smuggling and related phenomena (except for the case where capital controls were in force).<sup>2</sup> Moreover, the prevailing approach to the

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<sup>1</sup>See also Stern (1978) for a critique on Becker (1968).

<sup>2</sup>See for example Cooper 1974:184. He distinguishes four forms of smuggling, one of

analysis of smuggling is different from the one presented in this paper. Johnson (1972), Bhagwati&Hansen (1973), Sheikh (1974), Pitt (1981), Sheikh (1989), Deardorff&Stolper (1990) and others do not explain the degree of individual smuggling activity, but are concerned with its welfare implications, whereas this paper focusses on the decision-making process of an individual firm. With the exception of Martin&Panagariya (1984) and Norton (1988) the literature on smuggling lacks a rigorous microeconomic treatment of this illegal activity in the sense that the outcome of smuggling is ex ante uncertain and that, consequently, its degree depends upon authorities' law enforcement policy (probability of being audited, punished).<sup>3</sup>

This approach is in line with the tax evasion literature based on the seminal paper of Allingham&Sandmo (1972) in which the determinants and optimal degree of individual tax evasion are analyzed. The only clear-cut relationships are the negative relationships between tax evasion and penalty as well as between the former and the probability of detection. The influences of tax rate and real income are ambiguous – they depend on assumptions about the (absolute and the relative) risk aversion or risk neutrality.<sup>4</sup>

These papers differ from the present work insofar as in this paper the which is *under*-invoicing – the practice we are concerned with.

<sup>3</sup>For example Bhagwati&Hansen (1973) regard smuggling as risk-free though they attribute special resource costs to it. Sheikh (1974) assumes that a firm faces constant risk costs per unit thereby leaving the individual level of smuggling indeterminate. Pitt (1981) does not explicitly model risk either though he introduces a smuggling function that relates the volume of illegal trade to the volume of legal trade.

<sup>4</sup>Inter alia: Srinivasan (1973), Yitzhaki (1974), Pencavel (1979), Fishburn (1981), Koskela (1983). For a survey see Cowell (1985) and Weck-Hannemann&Pommerehne 1989:518-523. A very extensive and excellent analysis of various aspects of evasion including a comprising review of the literature is found in Cowell (1990b).



expected gain from tax evasion is reduced by increased duties; this increase may even exceed the potential gain from tax evasion so that overreporting of taxable income is a rational strategy. The firm chooses one (risky) asset out of a continuum of risky assets in order to maximize the expected (true) profit. This is done by declaring the optimal import price to the authorities which simultaneously determines risk and expected yield since the probability of detection as well as expected (true) net profit are a function of the amount of misdeclaration.

This assumption is crucial for the analysis; it is motivated by the notion that authorities do not know the true import price (otherwise misdeclaration would be unsuccessful a priori), but that they assume a density function describing the probability that the true price takes on different values. The modus of this function coincides with the true price. Hence, the greater the difference between the declared price and the true price, the more promising is scrutiny.

It is a widely accepted tenet of economic theory of politics that bureaucrats seek to maximize their budgets (Niskanen 1971). We share this view and assume that authorities will audit tax declarations and customs documents so as to maximize revenue.<sup>5</sup> In a qualitative response model<sup>6</sup> of authorities' investigation behavior it is shown formally that under rather general

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<sup>5</sup>Quite a few authors have analyzed optimal auditing and punishment under the assumption of a social welfare maximizing government: Kolm (1973), Srinivasan (1973), Fishburn (1979), Kemp&Ng (1979), Polinski&Shavell (1979), Sandmo (1981). Since we consider the economic model of rational utility-maximizing behavior of individuals not to be confined to economic agents but, consequently, also applying to political agents and bureaucrats, we cannot agree with this type of analysis.

<sup>6</sup>I am grateful to Gerd Ronning for inspiring me to study authorities' investigation behavior in a threshold model.

assumptions the probability of detection is a strictly increasing function of the amount of misdeclaration. With the exception of Reinganum&Wilde (1985, 1986) this is in sharp contrast to the majority of the relevant literature. Either a fixed probability of detection is assumed [Inter alia Nayak (1978), Crane&Nourzad (1985), Cowell&Gordon (1988), Gordon (1990)] or a variable detection probability depends – unrealistically – on magnitudes which are unobservable for the authorities, such as true income (Srinivasan 1973) or the ratio of undeclared income to true income (Koskela 1983).<sup>7</sup> In our model the probability of investigation depends on the *signal*, i.e. the declared price that the individual firm gives to the authorities. This has also intuitive appeal: the more “unrealistic” a price seems to be to the authorities, the higher is the probability of detecting evasion activity. Thus, unlike in the relevant body of literature, the probability of detection is not imposed exogenously but results from a maximizing behavior of the government. Moreover, we give special attention to the fact that investigation does not necessarily lead to detection and conviction – a fact that (though commonly known) has not been taken into account so far.

As follows from above, this paper is a positive analysis. We do not deal with welfare implications of tax and tariff evasion. This has been done extensively for smuggling [Bhagwati&Hansen (1973), Sheik (1974, 1989), Pitt (1981), Deardorff&Stolper (1990), among others] as well as for tax evasion [inter alia Weiss (1976), Stiglitz (1980), Yitzhaki (1987), Spicer (1990)]. Our concern is to analyze an avenue for a combined tax and tariff evasion which

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<sup>7</sup>An exception is Allingham&Sandmo (1972) who assume in an extension of their model that authorities have “some ideas about normal incomes in the various professions” (p.331) and audit with a different probability function for each profession, which depends negatively on the declared income. However, this function remains exogenous.

has received no previous attention in the literature: the misdeclaration of import prices. Note that this avenue is of special importance for multinational enterprises. They can misstate the prices for transactions within their hierarchy more easily because often market prices for those activities do not exist, and reference prices are difficult to calculate (the so-called “transfer pricing” problem).

This paper is subject to some qualifications. As in the standard literature on tax evasion, we disregard the interdependence between tax evasion and tax avoidance as pointed out by Seldon (1979), Cross and Shaw (1982), Geeroms&Wilmots (1985), Alm (1988), Alm&McCallin (1990), Cowell (1990a) and others. In our context this makes sense since we are concerned with the principal interdependence of tax and tariff burden and possibility of minimizing their sum. Assume for example that import goods are impossible to hide (e.g. bulk cargos) and the points of entry (harbors, airports) can effectively be controlled so that imports cannot physically be disguised.<sup>8</sup>

Furthermore, this analysis ignores the additional public goods, provided by the government, and financed out of revenues generated by the taxation and levy of duties from the firm under consideration which would enter in its utility function. This is justified on the grounds that a single firm considers overall tax and customs revenue as given, as the number of taxable entities is huge; its contribution to the financing of the public goods from which it

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<sup>8</sup>Moreover, we exclude the possibility that tariff payments could be reduced by misstating the nature of the goods such that a lower/higher tariff rate would apply. This possibility, however, would not alter our basic problem – the tradeoff between tax and tariff liabilities. For a related analysis in the context of tax evasion (income source misreporting for the case of differential taxation) see Yaniv (1990), for smuggling see Cooper (1974).

profits is negligible.<sup>9</sup>

The paper is organized as follows. Section 2 contains a simple basic model that describes sign of misdeclaration: the price of the imported intermediate product is a function of the relative magnitudes of tax and tariff rates. Section 3 derives formally the probability of detection as a function of the declared price in two steps. First, authorities' investigation behavior as a consequence of revenue maximization is analyzed in a qualitative response model. Second, it is discussed how investigation translates into detection and punishment. It is shown that under rather general assumptions the detection function depends positively on the amount of misdeclaration. Subsequently, this detection function is incorporated in an extension of the basic model, thereby making the firm's decision problem more realistic. Two different forms of punishment are looked at: section 4 investigates the impact of a lump-sum penalty, whereas section 5 deals with a form of punishment whose amount is proportional to the attempted evasion of tax and tariff liabilities. Section 6 contains some comparative-static results. Finally, section 7 summarizes and concludes.

## 2 The Basic Model

This simple model runs as follows. The firm under consideration imports an intermediate good  $y$ , which is needed to produce the finished product  $x$ , which in turn is sold in competitive markets at a price  $P$ . W.l.o.g. the input coefficient is set equal to unity. For the sake of simplicity, the cost function

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<sup>9</sup>The relation between public goods provision and tax evasion has been analyzed by Kolm (1973) and Cowell&Gordon (1988).

$K(x)$  (which excludes the cost of the imported input) takes the form:

$$K(x) = kx \quad , \quad k = \text{const.} \quad (1)$$

An ad valorem tariff is levied on imports with the constant rate of  $\tau$  ( $1 > \tau > 0$ ), and income tax is based on the reported gross profit<sup>10</sup> and levied at the fixed rate  $t$  ( $1 > t > 0$ ). This gross profit as well as the amount of tariff liabilities depend on the declared import price  $\tilde{p}$  which is the only choice variable for the importer. It is reasonable to postulate a certain range within which  $\tilde{p}$  must lie. The declared price cannot fall below zero and rise beyond  $P$  – thereby incurring a loss per unit output exceeding the unit cost of production  $k$  plus tariff payments:<sup>11</sup>

$$\tilde{p} \in [0; P] \quad (2)$$

Since prices are fixed and costs, taxes, and duties are linear in quantities in the following, magnitudes are calculated per unit output. The true (but not reported!) profit per unit  $\Pi^t$  is the gross profit per unit from production minus tariff and tax payments. Denoting the true price of the imported intermediate good (which the firm actually paid to the foreign supplier) as  $p^t$  and the gross profit per unit output with  $\Pi^g$  we can write:

$$\begin{aligned} \Pi^t &= P - k - p^t - \tau\tilde{p} - t\Pi^g = \\ &= P - k - p^t - \tau\tilde{p} - t[P - k - \tilde{p}(1 + \tau)] \end{aligned} \quad (3)$$

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<sup>10</sup>Since we are not concerned with the legal form of the enterprise, no distinction is made between corporation and income tax. Moreover, we do not consider tax allowances, loss carryforwards and the like.

<sup>11</sup>It is assumed that tax liabilities cannot fall below zero; in other words losses are not even in part born by the government.

The importer's sole objective is to maximize  $\Pi^t$  with respect to  $\tilde{p}$  because all other parameters are given  $(t, \tau, P, k)$ .<sup>12</sup> Since  $\Pi^t$  is a linear function of  $\tilde{p}$ , it does not have a local interior extremum. The optimal price must lie on either boundary of the price interval (2). The solution depends on the relative magnitudes of  $t$  and  $\tau$ . Thus, differentiating (3) with respect to  $\tilde{p}$  yields

$$\partial\Pi^t/\partial\tilde{p} = t(1 + \tau) - \tau \quad (4)$$

It is optimal to overstate the import price to  $\tilde{p} = P$ , if  $\partial\Pi^t/\partial\tilde{p} > 0$ .

$$\partial\Pi^t/\partial\tilde{p} > 0 \quad \iff \quad \tau - t(1 + \tau) < 0 \quad (5)$$

$$\iff \quad t > \frac{\tau}{1 + \tau} \quad (6)$$

The economic interpretation of the above findings can easily be seen from (5) : declare the value of your imports as high as possible if the duty you have to pay  $(\tau\tilde{p})$  falls short of the tax you can save  $(t(1 + \tau)\tilde{p})$ . If the case is reverse, declare  $\tilde{p} = 0$ . If  $\partial\Pi^t/\partial\tilde{p} = 0$ , the situation is indeterminate because tax saving equals additional tariff expenditure for every  $\tilde{p}$ . Obviously, a linear formulation of the problem is not very realistic. Misdeclaration implies the risk of being detected and being subject to penalty with a certain probability. In the subsequent models (section 4 and 5), the trade-off between the amount of penalty, the probability of detection, and the reduction of tariff and tax liabilities will be able to produce inner solutions, so that unreasonable results (like zero-declaration) can be excluded by properly specifying the relevant parameters. To begin with, however, we analyze government's investigation behavior and the possibility of detection and punishment.

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<sup>12</sup>To be sure, there may be other - conflicting - aims such as to enhance pre-tax profits, if shareholders agreed to tie the pay of their managers to the company's profit. But this is irrelevant in this context.

## 3 Investigation and Detection

### 3.1 Informational Structure and Investigation

Economic theory of politics argues that bureaucrats seek to maximize their own discretionary budget (Niskanen 1971). Consequently, taxes should be collected and tax payers should be audited so as to maximize net revenue accruing to the authorities. There is some empirical evidence supporting this hypothesis (Wertz 1979) – as opposed to a social welfare maximizing government as presumed by many authors dealing with tax evasion (inter alia Kolm 1973, Srinivasan 1973, Fishburn 1979, Polinski&Shavell 1979). We share the basic behavior postulate of public choice theory that man is an egoistic, rational, utility maximizer – regardless if he is an economic agent, a politician, or a bureaucrat.<sup>13</sup> Hence we assume that authorities examine tax declarations and customs documents in order to increase their net revenue. This means that they expand their investigation activities as long as the expected marginal amount of detected evasion plus penalty exceeds the marginal cost of investigation.<sup>14</sup>

Tax officials and customs officers do not know the true import price  $p^t$

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<sup>13</sup>For a survey of the public choice theory see, e.g. Mueller (1989).

<sup>14</sup>One could argue that  $\tau$  and  $t$  should also be determined endogenously as a consequence of revenue maximization. In our context these parameters are looked upon as exogenous because we consider the revenue raising arm of the bureaucracy being distinct from the government as an actor in the political market for protectionism and for determining the tax rate, respectively. Though we impose this additional political constraint whereby  $t, \tau$  are predetermined by the legislator, it seems obvious that these policy variables result from a political market process. The political economy of taxation is analyzed by Brennan&Buchanan (1980) and Hettich&Winer (1990); for a survey of the literature on the political economy of protection see Hillman (1989) and Weck–Hannemann (1991).

with certainty (otherwise evasion would never be successful) but they have an (a priori) density function of the latent variable  $\tilde{p}^t$ ,  $q(\tilde{p}^t)$  attaching the true import price the highest probability. (The tilde indicates the random variable – from the standpoint of the authorities.) Clearly this density function depends on the quality of the imported good: the more homogeneous the good is and the better established the world market for it, the lower will be the variance of  $q(\tilde{p}^t)$ .

Let us assume that  $q(\tilde{p}^t)$  has the following properties.

1.  $q(\tilde{p}^t)$  is continuous and defined on  $[0, P]$ .
2.  $q(\tilde{p}^t)$  is twice continuously differentiable and strictly concave for  $\tilde{p}^t \in (\tilde{p}_l^t, \tilde{p}_u^t)$  with  $dq(\tilde{p}^t)/d\tilde{p}^t = q'(\tilde{p}^t) = 0$ . That means that the modulus of the distribution coincides with the actual (true) import price  $p^t$ .
3.  $q(\tilde{p}^t) = 0$  for  $\tilde{p}^t \notin (\tilde{p}_l^t, \tilde{p}_u^t)$ .

$q(\tilde{p}^t)$  is depicted in figure 3.1

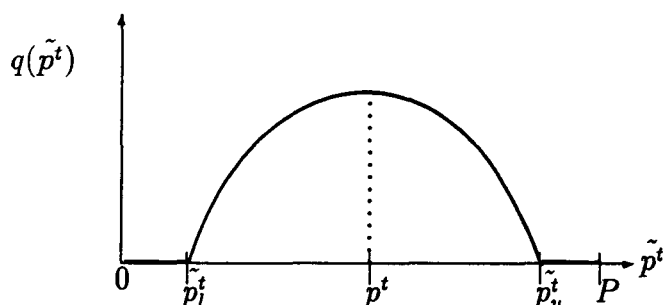


Fig. 3.1: Density function of  $\tilde{p}^t$ .

It is understood that other classes of functions are also conceivable, e.g. bell-shaped functions. One special case would be the normal probability



distribution, however in a truncated version since the domain of the function is bounded. Moreover, symmetry concerning the vertical axis at  $p^t$  (i.e.  $p^t = P - p^t$ ) is not required. From the following analysis it can be deduced that a bell-shaped function  $q(\tilde{p}^t)$  would result in an inverted bell-shaped investigation function.<sup>15</sup> Nevertheless, adopting this class of functions would not change the subsequent analysis fundamentally. Since there is no a priori information about the knowledge of the authorities, and requirements of strict concavity and differentiability of  $q(\tilde{p}^t)$  within  $(\tilde{p}_l^t, \tilde{p}_u^t)$  are rather general, assumption two is justifiable.

The authorities observe the declared price  $\tilde{p}$ . Given this price  $\tilde{p}$ , the extent of misdeclaration  $M = \tilde{p} - \tilde{p}^t$  is also a random variable with the same properties due to the linear transformation of  $q(\tilde{p}^t)$ . If the absolute value of misdeclaration falls short of a critical value  $a$  investigation is not rewarding because the costs of investigation exceed the expected gain of detection. Hence it follows that authorities' probability of investigation must be identical with the probability that  $|M| = |\tilde{p} - \tilde{p}^t| \geq a$ .<sup>16</sup>

The probability that investigation is not rewarding, i.e.  $-a \leq M \leq a$  depends on the the declared price  $\tilde{p}$ . With  $Q(\tilde{p}^t)$  being a primitive of  $q(\tilde{p}^t)$

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<sup>15</sup>This would parallel the standard probit analysis. For further reference see Ronning (1991).

<sup>16</sup>Investigation need not imply that tax and tariff evasion is uncovered with certainty. The economic calculation focusses on expected detection. Consequently,  $a$  is not only determined by the costs of investigation and the tax and tariff rate,  $t$  and  $\tau$ , which link the evaded amount to the extent of misdeclaration; it is also influenced by the probability that investigation produces detection. This probability in turn could depend on the amount of misdeclaration since a huge over- or underdeclaration may be proved more easily than a small one.

this probability is given by

$$\begin{aligned}
\text{prob}(M \leq a) - \text{prob}(M \leq -a) &= \text{prob}(\tilde{p}^t \geq \tilde{p} - a) - \text{prob}(\tilde{p}^t \geq \tilde{p} + a) = \\
&= 1 - \int_{-\infty}^{\tilde{p}-a} q(\tilde{p}^t) d\tilde{p} - 1 + \int_{-\infty}^{\tilde{p}+a} q(\tilde{p}^t) d\tilde{p} \\
&= Q(\tilde{p} + a) - Q(\tilde{p} - a). \tag{7}
\end{aligned}$$

The probability that authorities do investigate is correspondingly

$$i(\tilde{p}) = 1 - Q(\tilde{p} + a) + Q(\tilde{p} - a). \tag{8}$$

For convenience we make a further assumption on  $q(\tilde{p}^t)$ :

4.  $\tilde{p}_l^t \leq a$ , and  $\tilde{p}_u^t \geq P - a$ , i.e. the regions in which  $p^t$  occurs with zero-probability are limited from the bounds of the domain of  $q(\tilde{p}^t)$  by the critical value  $a$ .

Assumption four is not indispensable, it is made to ensure strict convexity of  $i(\tilde{p})$  for  $\tilde{p} \neq p^t$ . It can be shown that for  $\tilde{p} \in [0, P]$

$$i(\tilde{p}) = 1 \quad \text{if} \quad \begin{cases} \tilde{p} \leq \tilde{p}_l^t - a \\ \tilde{p} \geq \tilde{p}_u^t + a \end{cases}$$

In other words if assumption four was removed the probability of investigation would reach unity for some  $\tilde{p} \in (0, P)$ .

The derivation of the investigation function is illustrated in figure 3.2. The vertical difference between the two functions  $1 - Q(\tilde{p} - a)$  and  $1 - Q(\tilde{p} + a)$  represents the probability that for a given  $\tilde{p}$  authorities do *not* investigate (i.e.  $1 - i(\tilde{p})$ ). This strictly concave function is depicted by a solid line. The dotted graph depicts the investigation function  $i(\tilde{p})$ .

From (7), (8) and assumptions (1) - (3) on  $q(\tilde{p}^t)$  it follows that  $i(\tilde{p})$  is a strictly convex function with the vertex  $(p^t, i(p^t))$ .

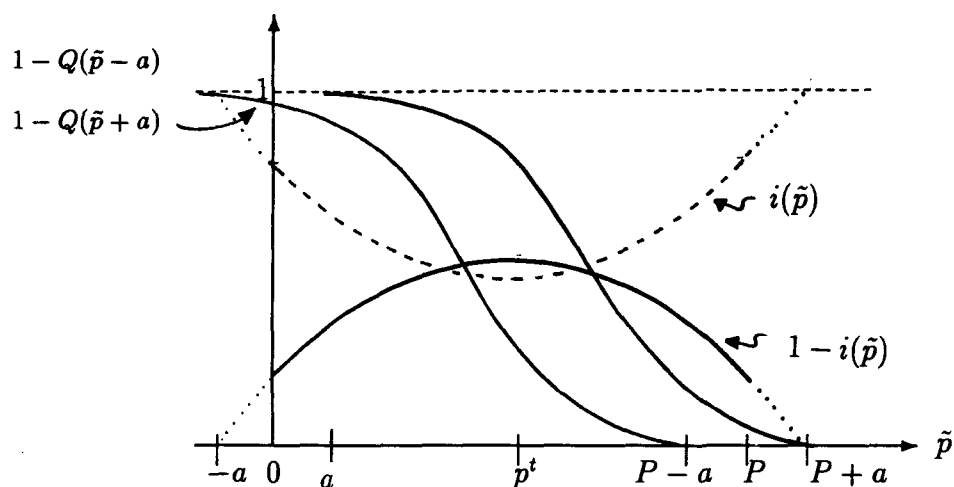


Fig. 3.2: Derivation of the investigation function  $i(\tilde{p})$ .

### 3.2 Detection

An investigation does not bring a certain conviction. First, evasion must be discovered. This is the more likely the greater the difference between the true and the stated price is, because small over- or underinvoicing can be explained by (unobservable) price fluctuations, special discounts or surcharges and the like. Even if in the course of an investigation authorities find a reference price for the imported product on the world market they may falsely trace back a small price differential to legal transaction-specific costs or benefits. This probability of misinterpretation, however, will diminish with rising price differential and the deal will become more suspicious.

Second, in order to prove the evasion and to convict the importer, authorities have to collect evidence with probative force and to charge him. Then, proceedings against him must be instituted. Scrutiny and legal proceedings incur costs so that it makes sense to dismiss the case if the expected gain (evaded tax and tariff liabilities plus penalty multiplied with the probability of condemnation) falls short of the costs.<sup>17</sup> Obviously the expected gain rises with an increasing price differential.

Hence, the probability that an investigation leads to a conviction  $l(\tilde{p} - p^t)$  is an increasing function of the price differential. It is zero for  $\tilde{p} = p^t$  and assumed to be a nonnegative and strictly convex function of  $(\tilde{p} - p^t)$ . Connecting multiplicatively  $i(\tilde{p})$  and  $l(\tilde{p} - p^t)$  results in a function  $f$  that describes the probability that for a stated import price  $\tilde{p}$  tax and customs declarations will be scrutinized, evasion will be detected, and the importer will be sentenced. This “detection function”  $f$  is relevant for the importer’s calculations. As follows from the above analysis  $f$  has the following properties.

$$f = f(\tilde{p} - p^t) \tag{9}$$

and being a probability,

$$0 \leq f \leq 1 . \tag{10}$$

Clearly,

$$f(0) = 0 . \tag{11}$$

Furthermore,  $f$  is twice continuously differentiable and a strictly convex func-

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<sup>17</sup>Under German law a case can be dismissed on its merits according to §398 AO (Abgabenordnung, i.e. German Fiscal Code which is the basic tax law incorporating substantive and procedural provisions common to all special tax legislation).

tion of  $\tilde{p} - p^t$ . This can be expressed as

$$\begin{array}{ccc} & > & > \\ \partial f / \partial \tilde{p} \stackrel{\text{def}}{=} f' = 0 & \text{iff} & \tilde{p} = p^t . \\ & < & < \end{array} \quad (12)$$

$$\partial^2 f / \partial \tilde{p}^2 \stackrel{\text{def}}{=} f''(\tilde{p} - p^t) > 0 \quad (13)$$

Different assumptions about  $l(\tilde{p} - p^t)$  are conceivable. We have assumed that the probability of detection and condemnation  $f$  is *strictly convex* with minimum zero at  $\tilde{p} = p^t$ , i.e.  $f(0) = 0$ . Alternatively, one could assume that also marginal evasion would lead to discovery and condemnation with a probability distinctly greater than zero. For example if every investigation would lead to discovery and condemnation for  $\tilde{p} \neq p^t$  the resulting probability distribution for condemnation would coincide with  $i(\tilde{p})$  for  $\tilde{p} \neq p^t$  and would have a discontinuity at  $\tilde{p} = p^t$  since correct declarations cannot be punished.<sup>18</sup>

Nevertheless, the qualitative results of the model – the existence of an unique optimal point of misdeclaration – is not altered for those classes of detection functions. However, critical values of the lump-sum penalty and the rate of penalty, respectively, exist above which misdeclaration will not take place at all. This is shown in detail in appendix C.

It is the importer's economic calculation we turn to now. We assume that he is aware of the authorities' detection function as well as the penalty he has to face if detected. The case of a lump-sum punishment is analyzed next.

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<sup>18</sup>In appendix C the classes of functions  $l_i(\tilde{p} - p^t)$  are shown that generate a detection function  $\phi(\tilde{p} - p^t)$  – instead of  $f$  –, the continuous extension of which is a strictly convex function that takes on its positive valued minimum at  $\tilde{p} = p^t$  whereas  $\phi(p^t) = 0$ .

## 4 Detection and Penalty: Model I

When misdeclaration is brought to light, a lump-sum penalty  $S$  ( $S \geq 0$ ) is imposed, tax and tariff being based on the true price  $p^t$ . The importer is assumed to be risk neutral in the relevant range so that the expected value of (dis-) utility stemming from tax and tariff payments can be expressed by the expected value of  $\Pi^t$ ,  $E(\Pi^t)$ . In addition the authorities of the importer's country may not co-operate with the authorities of the exporting country. Hence, (3) is modified as follows:

$$E(\Pi^t) = P - k - p^t - t[P - k - \tilde{p}(1-f)(1+\tau) - p^t f(1+\tau)] - Sf - \tau[(1-f)\tilde{p} + p^t f] \quad (14)$$

Defining

$$z \stackrel{\text{def}}{=} t(1+\tau) - \tau \quad (15)$$

and - for convenience

$$A \stackrel{\text{def}}{=} (P - k)(1 - t) - p^t \quad (16)$$

equation (14) can be simplified to

$$E(\Pi^t) = A + (p^t - \tilde{p})zf + z\tilde{p} - Sf. \quad (17)$$

Calculating the first derivative and simplifying the notation we obtain:

$$\partial E(\Pi^t) / \partial \tilde{p} \stackrel{\text{def}}{=} \Pi' = (p^t - \tilde{p})f'z + (1-f)z - f'S \quad (18)$$

and the first order condition for a maximum yields:

$$\tilde{p}^* = p^t + \frac{1-f}{f'} - \frac{S}{z}. \quad (19)$$

Calculating the second derivative from (18) (with simplified notation)

$$\partial^2 E(\Pi^t) / \partial \tilde{p}^2 \stackrel{\text{def}}{=} \Pi'' = (p^t - \tilde{p})z f'' - 2f'z - Sf'' \quad (20)$$

and substituting  $\tilde{p}^*$  from (19) for  $\tilde{p}$  we can write

$$\Pi(\tilde{p}^*)'' = -\frac{(1-f)f''}{f'}z - 2f'z. \quad (21)$$

How is the  $E(\Pi^t(\tilde{p}))$  sloped? Are there any relevant ranges to be identified, in which  $\tilde{p}^*$  must lie? Three cases are distinguished.

**I.  $z > 0$  i.e.,  $t(1+\tau) > \tau$  (tax burden exceeds tariff burden)**

Taking the properties of  $f$  into account it is seen from (21) for  $z > 0$

$$\begin{aligned} \Pi^t(\tilde{p}^*)'' < 0 & \quad \text{for} \quad \tilde{p} > p^t \quad \text{and} \\ \Pi^t(\tilde{p}^*)'' > 0 & \quad \text{for} \quad \tilde{p} < p^t \end{aligned} \quad (22)$$

In other words, in case the tax burden dominates the tariff burden an interior solution which maximizes  $E(\Pi^t)$  – if existing – is found only in the range of overdeclaration. The qualitative shape of  $E(\Pi^t(\tilde{p}))$  depicts as shown in figure 4.1. For a detailed qualitative analysis see appendix A.

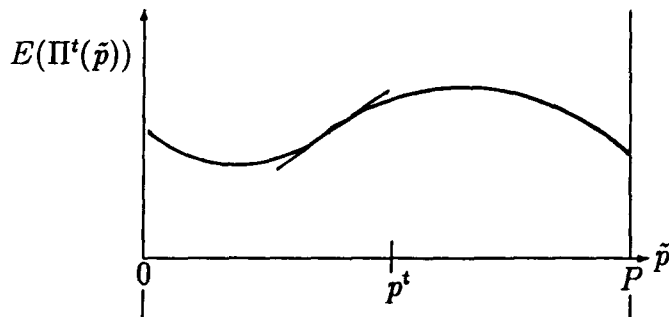


Fig. 4.1: Expected true profit function for  $z > 0$ .

So far, the global maximum could either be a boundary or an interior solution. Without specifying the respective parameters and the concrete

detection function  $f$ , it is impossible to know whether  $E(\Pi^t)$  reaches a local maximum or a local minimum in the interior of the interval  $[0; P]$ . First we examine whether the boundary solution  $\tilde{p} = 0$  may establish a global minimum. For this purpose the expected tax and tariff payments for  $\tilde{p} = 0$  and  $\tilde{p} = p^t$  are compared.

$$\begin{aligned} E(\Pi(p^t)) &= P - k - p^t - t(P - k - p^t(1 + \tau)) + \tau p^t = \\ &= A + z p^t \end{aligned} \quad (23)$$

$$E(\Pi^t(0)) = A + z f(-p^t) p^t - S f(-p^t) \quad (24)$$

The differential expected payment to the state amounts to

$$E(\Pi^t(p^t)) - E(\Pi^t(0)) = z p^t (1 - f(-p^t)) + S f(-p^t) > 0 \quad (25)$$

because  $z > 0$  and because of the properties of  $f$ . Consequently, this boundary "solution" cannot establish a global minimum, and for the case of  $z > 0$  only overdeclaring is considered.<sup>19</sup> This finding is compatible with economic intuition since overstating makes sense only if tax saving is greater than additional tariff burden - the case  $z > 0$ .

It remains to be examined whether  $\tilde{p}^* > p^t$  from (19) is an interior solution (i.e.  $\tilde{p}^* \in (p^t; P)$ ). This is valid iff the sign of  $\Pi'$  changes between  $p^t$  and  $P$  (mean value theorem). From (18) it is calculated:

$$\Pi(p^t)' = +z > 0 \quad (26)$$

$$\Pi(P)' = [(p^t - P)z + S]f(P - p^t)' + (1 - f(P - p^t))z \quad (27)$$

$$\Pi(P)' > 0 \iff f(P - p^t)' > 1 - f(P - p^t)' \{(P - p^t) + S/z\} \quad (28)$$

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<sup>19</sup>Stating the true value is suboptimal since  $\Pi'(p^t) = +z > 0$ . Cf. (18).



An interior solution is guaranteed if we assume that the probability of detection equals unity when the importer declares the price of his intermediate good to be as high as his finished product (Cf. (12)). How realistic this assumption (which is sufficient but not necessary for an interior solution!) is depends on the fraction of the intermediate good of the total value, i.e.  $p^t/P$ . If it is well below unity it seems justifiable to assume  $f(P - p^t) = 1$ .<sup>20</sup>

For  $z > 0$  some possible  $E(\Pi^t)$  and corresponding  $\Pi^t$  curves are shown (in the relevant range) in figure 4.2.

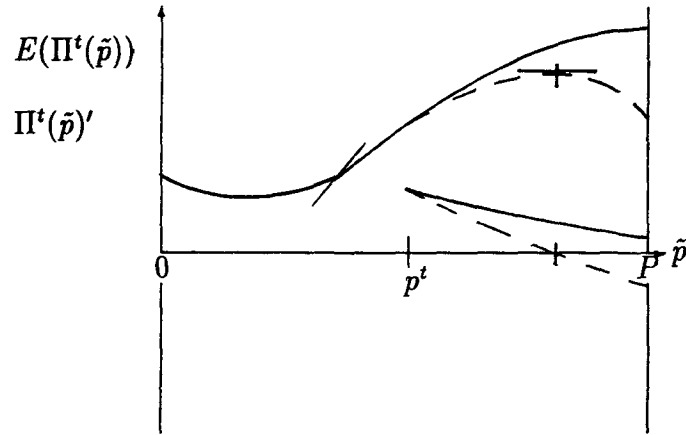


Fig. 4.2: Interior and boundary maximum of  $E(\Pi^t(\tilde{p}))$  for  $z > 0$ .

**II.  $z < 0$**  i.e.,  $t(1 + \tau) < \tau$  (tax burden falls short of tariff burden)

<sup>20</sup>A necessary and sufficient condition for an interior solution can also be calculated from the first order condition (19) and the above finding

$$p^t > \bar{p}^* > P$$

resulting in:

$$1 - (S/z)f(\bar{p}^* - p^t)' - (P - p^t)f(\bar{p}^* - p^t)' < f(\bar{p}^* - p^t) < 1 - (S/z)f(\bar{p}^* - p^t)'.$$

The line of arguments is analogous to case I. From (21) and the properties of  $f$  it follows that for  $z < 0$

$$\begin{aligned} \Pi(\tilde{p}^*)'' &> 0 & \text{for } \tilde{p} > p^t & \text{and} \\ \Pi(\tilde{p}^*)'' &< 0 & \text{for } \tilde{p} < p^t \end{aligned} \quad (29)$$

Hence, in the case that tax burden falls short of the tariff burden an interior solution which maximizes the function  $E(\Pi^t)$  – if existing – is found only in the range of underdeclaration. As previously, the possibility that  $\Pi'$  becomes positive for some  $p^t < \tilde{p} < P$  cannot be excluded<sup>21</sup> so that it has to be studied whether the boundary value  $E(\Pi^t(P))$  may establish a global maximum. This is done subsequently by recalling (23) and calculating (17) for  $\tilde{p} = P$ :

$$E(\Pi^t(P)) - E(\Pi^t(p^t)) = [1 - f(P - p^t)]z[P - p^t] - Sf(P - p^t) < 0 \quad (30)$$

The expected result is obtained: if the tariff burden dominates the tax burden ( $z < 0$ ), then only underdeclaring is of concern.

In figure 4.3 some possible slopes of  $E(\Pi^t)$  and  $\Pi'$  are shown for  $z < 0$ .<sup>22</sup>

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<sup>21</sup>For  $z < 0$  the inflection point lies above the true price;  $\tilde{p}^t > p^t$ . Thereafter,  $\Pi'$  is continuously increasing and may become negative in the range considered.

<sup>22</sup>The line of argument is, mutatis mutandis, analogous to the one given in appendix A. For that reason an additional qualitative analysis of the shape of  $E(\Pi^t)$  seems to be dispensable.

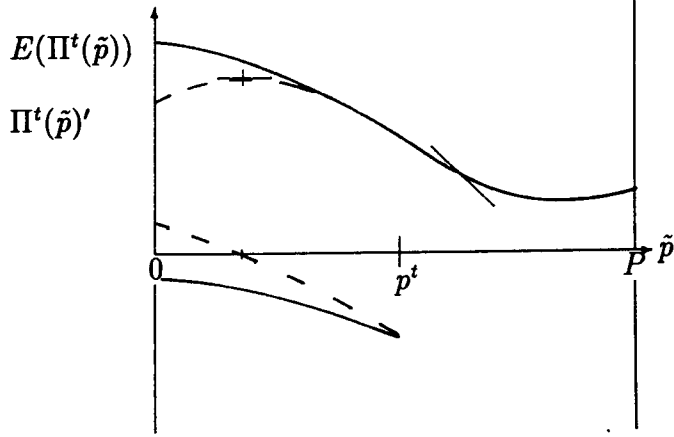


Fig. 4.3: Interior and boundary maximum of  $E(\Pi^t(\tilde{p}))$  for  $z < 0$ .

As previously,  $\tilde{p}^*$  is an interior solution if the sign of the first derivative changes from  $\Pi(0)'$  to  $\Pi(p^t)'$ . From (18)  $\Pi'(p^t) = z < 0$ . Hence, the condition for the solution to lie within  $(0; p^t)$  is

$$\Pi(0)' = zf(0 - p^t)'p^t + (1 - f(0 - p^t))z - Sf(0 - p^t)' \stackrel{!}{>} 0 \quad (31)$$

$$\implies f > 1 + f'(p^t - S/z) \quad (32)$$

Since the term in parentheses is positive and  $f'$  is negative in the range under consideration it is sufficient for an interior solution to exist that we assume  $f(-p^t) = 1$  ( since  $f(-p^t)' < 0$ , see (12) ). In other words, the importer cannot conceal his imports. Whether this assumption is reasonable depends on the character of the goods exchanged (e.g. bulk goods versus small-scale mixed cargo) and the efficiency of the surveilling authorities.<sup>23</sup>

<sup>23</sup>As for the case  $z > 0$  a necessary and sufficient condition can be calculated from the first order condition (19) and the above finding that  $\tilde{p}^* \in (0; p^t)$  :

$$0 < \tilde{p}^* < p^t$$

$$\iff 0 < p^t + (1 - f(\tilde{p}^* - p^t))/f(\tilde{p}^* - p^t)' - S/z < p^t$$

$$1 + p^t f(\tilde{p}^* - p^t)' - (S/z) f(\tilde{p}^* - p^t)' > f(\tilde{p}^* - p^t) > 1 - (S/z) f(\tilde{p}^* - p^t)' .$$

III.  $z = 0$  i.e.  $t(1 + \tau) = \tau$  (tax burden equals tariff burden)

From (17) and the definition of  $A$  follows

$$E(\Pi^t) = A - Sf \quad (33)$$

As  $A$  is independent of  $\tilde{p}$ ,  $f$  must be minimized with respect to  $\tilde{p}$ . From (9) - (13) it follows that  $\tilde{p}^* = p^t$  and hence profits sum up to  $\Pi = A$  since the tax reduction that is due to the costs of the imported intermediate goods is fully offset by the tariff levied on these goods. For this special case,  $E(\Pi)$  is sloped as shown in figure 4.4. Note that  $\Pi' = -Sf'$ .

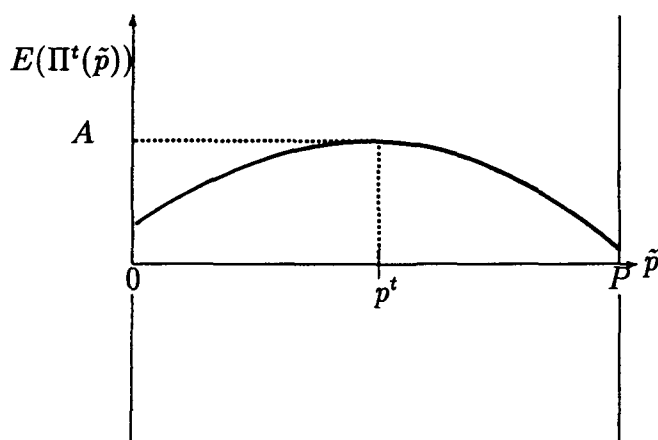


Fig. 4.4: Expected true profit function for  $z = 0$ .

Summarizing the above findings we can conclude: if and only if the tax burden exceeds the tariff burden it is optimal to overstate the value of imported intermediate goods, whereas if and only if the case is reversed understating makes sense. The amount of overall tax and tariff evasion is dependent on the relation between tax and tariff rates ( $z$ ), the probability of being detected ( $f$ ) and the magnitude of the (lump-sum) penalty ( $S$ ). Honesty is profitable only if tax burden equals tariff burden.

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May these results be altered if we investigate a different form of punishment? In the following section, the penalty depends on the amount of tax and tariff liabilities which were attempted to be evaded.

## 5 Detection and Penalty: Model II

Though the analysis of lump-sum punishment is elucidative from a theoretical standpoint, it is rather unrealistic that an evader should be penalized irrespectively of the severity of his offence. For that reason the above model is modified such that the penalty is tied to the (net) amount of taxes and tariffs which were unsuccessfully attempted to be evaded. This kind of punishment is chosen because income tax evasion is punished this way in the United States of America, Israel (Yitzhaki 1974) and the Federal Republic of Germany<sup>24</sup> as well as in other countries. As can easily be verified the evasion sought ( $TE$ ) amounts to

$$TE = (\tilde{p} - p^t)z \quad (34)$$

so that in the case of  $TE > 0$  the penalty ( $SP$ ) equals

$$SP = sz(\tilde{p} - p^t) \quad (35)$$

with  $s$  being the nonnegative and constant rate of penalty (surcharge). If  $TE < 0$  (i.e. the importer has paid too many taxes and tariffs) we reasonably assume that no reward is made by the authorities for his stupid misdeclaration so that  $SP$  cannot become negative. Modifying (17) we can now write:

$$E(\Pi^t(\tilde{p})) = A + zf(p^t - \tilde{p}) + z\tilde{p} - \alpha sz(\tilde{p} - p^t)f \quad (36)$$

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<sup>24</sup>§§71, 370 AO (i.e. Abgabenordnung, German Fiscal Code).

$$\alpha = \begin{cases} 1 & \text{iff } z(\tilde{p} - p^t) > 0 \\ 0 & \text{otherwise} \end{cases} \quad (37)$$

whereas the first derivative  $\Pi'$  is altered as follows:

$$\Pi' = (p^t - \tilde{p})z f'(1 + \alpha s) + (1 - f)z - \alpha s z f. \quad (38)$$

Hence the first order condition can be written as

$$\tilde{p}^* = p^t + \frac{1 - f(\tilde{p}^* - p^t)(1 + \alpha s)}{f(\tilde{p}^* - p^t)'(1 + \alpha s)}. \quad (39)$$

Obviously, no  $\tilde{p}$  can establish an extremum for which  $f(\tilde{p} - p^t) \geq 1/(1 + \alpha s)$ . From the calculation of the second derivative of  $E(\Pi^t)$

$$\Pi'' = (p^t - \tilde{p})z f''(1 + \alpha s) - 2z f'(1 + \alpha s) \quad (40)$$

it follows that for  $z > 0$

$$\Pi'' > 0 \iff \tilde{p} < p^t \quad (41)$$

whereas for  $z < 0$  the case is reverse. For  $z > 0$  the function  $E(\Pi^t)$  plots in principle as shown in figure 5.1. As in model I, however, without further information it is unknown whether local extrema exist in the interior of the interval  $[0; P]$ .

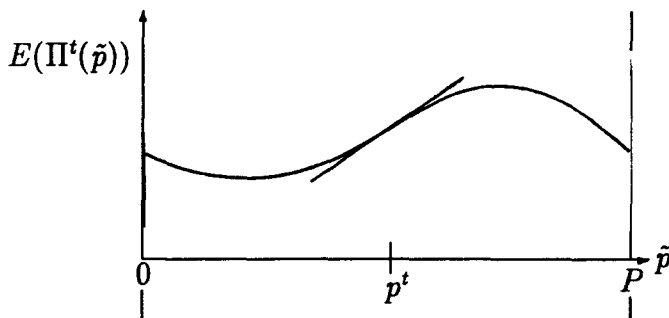


Fig. 5.1: Expected true profit function for  $z > 0$ .

Qualitatively this figure does not differ from figure 1 of model I with the exception that the inflection point coincides with the true price  $p^t$ . For a more detailed analysis of  $E(\Pi^t)$  of model II see appendix B.

As in the previous section we exclude that  $E(\Pi^t)$  takes on its maximal value at  $\tilde{p} = 0$  for  $z > 0$  and  $\tilde{p} = P$  for  $z < 0$ , respectively.

$z > 0$ . From (36)<sup>25</sup>

$$E(\Pi^t(0)) - E(\Pi^t(p^t)) = -zp^t(1 - f) \leq 0. \quad (42)$$

Equality holds iff the probability of being detected equals unity, and so overall tax and tariff liabilities are based on the true import price – obviously, this misdeclaration does not make any sense at all. By analogy we calculate for  $z < 0$ :<sup>26</sup>

$$E(\Pi^t(P)) - E(\Pi^t(p^t)) = (P - p^t)(1 - f)z \leq 0 \quad (43)$$

Equality holds iff  $f(P - p^t) = 1$ . Apparently this case can also be excluded from further consideration. These findings correspond with those of section 2 and 4: understating makes sense if and only if tariff burden exceeds tax burden. If the case is reverse, then overdeclaration is fruitful. If  $z = 0$ , it does not matter which price is declared because – in contrast to model I – misdeclaration as such is not punished. In this model the penalty is based on the sum of taxes and tariffs which were sought to be evaded. For  $z = 0$  this sum is always zero irrespectively of the price declared.

The condition for an interior solution is investigated next. For  $z > 0$   $\Pi(p^t)'$  is compared with  $\Pi(P)'$  according to the mean value theorem. From (12) and (37) it follows that

$$\Pi(p^t)' = z > 0 \quad (44)$$

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<sup>25</sup>For  $\tilde{p} = 0$   $\alpha = 0$  since  $-p^t z < 0$ .

<sup>26</sup>Cf. (36):  $\alpha = 0$  since  $(P - p^t)z < 0$ .

Hence,

$$\begin{aligned} \Pi(P)' &= z[(p^t - P)f(P - p^t)'(1 + s) + \\ &\quad + (1 - f(P - p^t)) - sf(P - p^t)] \stackrel{!}{<} 0 \end{aligned} \quad (45)$$

$$\implies f(P - p^t) \stackrel{!}{>} 1/(1 + s) - (P - p^t)f(P - p^t)' \quad (46)$$

As a sufficient, but *not* necessary condition an interior solution is obtained if  $f(P - p^t)$  equals unity because  $f(P - p^t)' > 0$ , owing to the strict convexity of  $f$ , and  $s$  being nonnegative.<sup>27</sup>

Does  $\Pi(0)' > 0$  hold for  $z < 0$ ? (Note that  $\Pi(p^t)' < 0$  for  $z < 0$ .)

$$\Pi(0)' = z[p^t f(0 - p^t)'(1 + s) + 1 - f(-p^t)(1 + s)] \stackrel{!}{>} 0 \quad (47)$$

$$\implies f(-p^t) \stackrel{!}{>} 1/(1 + s) + p^t f(-p^t)' \quad (48)$$

Also in this model  $f(-p^t) = 1$  is a sufficient condition for the existence of an interior solution.

## 6 Comparative Static Results

In this section we shall examine how the optimal price  $\tilde{p}^*$  depends on the parameters  $s$ ,  $S$ , and  $f(\tilde{p} - p^t)$  of the models I and II. To begin with the influence of the lump-sum penalty  $S$  on the solution of the maximizing problem is investigated.<sup>28</sup> Restating the first order condition of model I, eq. (19)

$$[(p^t - \tilde{p}) - S/z]f' + 1 - f = 0 \quad (49)$$

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<sup>27</sup>This coheres with the result of model I.

<sup>28</sup>We exclude the trivial case of  $z = 0$  from further examination. Thus, denominators are always non-zero.



shows that the term  $[(p^t - \tilde{p}) - S/z]f'$  is less than or equal to zero since  $1 - f \geq 0$ . In other words for  $z > 0$  ( $z < 0$ ) it follows that the term in brackets is negative (positive) due to the properties of  $f'$ .

Calculating the total differential of (49) and solving it for  $d\tilde{p}/dS$  yields:

$$\frac{d\tilde{p}}{dS} = \frac{1/z f'}{f''[(p^t - \tilde{p}) - S/z] - 2f'} \quad (50)$$

From the above considerations it follows that

$$\text{sign} \left( \frac{d\tilde{p}}{dS} \right) = - \text{sign}(z) \quad .$$

Raising the lump-sum penalty reduces the amount of (optimal) misdeclaration and vice versa.

We study the influence of the rate of penalty  $s$  (model II) next. Recalling the first order condition (39) we obtain after some simple manipulations:

$$0 = (1 + \alpha s)[(p^t - \tilde{p})f' - f] + 1 \quad (51)$$

Since the term in parentheses is obviously positive, the term in brackets is negative. The total differential of the first order condition (51) is solved with respect to  $d\tilde{p}/ds$ :

$$\frac{d\tilde{p}}{ds} = \frac{\alpha(f - (p^t - \tilde{p})f')}{(p^t - \tilde{p})f''(1 + \alpha s) - 2f'(1 + \alpha s)} \quad (52)$$

Disregarding the trivial case in which  $\alpha$  equals zero we see that the numerator is unambiguously positive, since it is the negative value of the term in brackets of equation (51) multiplied by  $\alpha$ . The sign of the denominator is negative in the case of  $z > 0$  and positive for negative  $z$ .<sup>29</sup>

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<sup>29</sup>For  $z > 0$  only overstatement makes sense and hence  $(p^t - \tilde{p}) < 0$  while  $f'$  and  $f''$  are positive (Eq. (12) - (13)). For  $z < 0$  analogous.

Also in this model we obtain the negative relationship between misdeclaration and penalty. This is very much in line with the results in the (tax evasion) literature<sup>30</sup> and economic intuition: other things being equal, higher penalties lower expected profits from illegal activities.

This may raise the question why the government does not eliminate costlessly the problem of tax and tariff evasion by maximal punishment. Though the government could not eliminate evasion totally since  $\Pi(p^t)' = +z > 0$  (see (26)) it could reduce it to an arbitrarily small amount by threatening potential evaders with the loss of their total wealth or life. If these penalties deter effectively they will almost never have to be applied so that the authorities do not “even have to incur the small cost of paying the wages of the hangman” (Cowell 1985:181). We do not want to argue along these lines though there are also economic counterarguments (see Cowell 1985:180-182 and the literature quoted). We regard maximal punishment of evasion as contradicting the basic principles of law and thus incompatible with the legal system in all Western countries so that ethical considerations alone constitute a ceiling for the surcharge.<sup>31</sup>

For given  $\bar{p}$  the probability of detection will increase ceteris paribus either if the costs of investigation decrease or if the expected gain of detection increases, i.e. if  $|z|$  rises. In the notation of section 3.1 that means that the critical value  $a$  decreases (see page 12). In order to provide insights in the

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<sup>30</sup>Inter alia Allingham&Sandmo 1972: 330.

<sup>31</sup>See also Stern (1978) and Cowell (1990b, chs 7, 8) for further discussion. Individuals might evade taxes and tariffs by mistake. This possibility let extreme penalties become unjustifiable. Moreover, maximal punishment also for relative small offenses reduces effective deterrence for major offenses – “one might as well be hung for a sheep as for a lamb”, Cowell 1990b: 150.

way in which  $\tilde{p}^*$  depends on the probability of detection we substitute

$$e(\kappa, \tilde{p} - p^t) = f(\tilde{p} - p^t) + \kappa\eta(\tilde{p} - p^t)$$

for  $f$ . The function  $\eta$  is arbitrary but fixed and has the same properties as  $f$  (strict convexity in  $(\tilde{p} - p^t)$  and minimum at  $\tilde{p} = p^t$  with  $\eta(0) = 0$ ). The parameter  $\kappa$  is a (nonnegative) scalar such that  $e(\kappa, \tilde{p} - p^t) \in [0, 1]$ . Thus,  $e(\kappa, \tilde{p} - p^t)$  establishes a one parameter family of functions with the same geometrical properties as  $f$ .

We confine our analysis to model I. (As the reader may easily verify, analogous results are obtained for model II.)

The first order condition (49) is altered to

$$[(p^t - \tilde{p}) - S/z](f' + \kappa\eta') + 1 - (f + \kappa\eta) = 0. \quad (53)$$

Due to  $1 - (f + \kappa\eta) \geq 0$  and (12) and the respective properties of  $\eta$  the term in brackets is nonnegative (nonpositive) for  $z < 0$  ( $z > 0$ ).

We analyze the impact of a variation of  $\kappa$  on the optimal declared price. An increase (decrease) in  $\kappa$  implies that for every  $\tilde{p} \neq p^t$  the value of  $e$  is increased (decreased),  $\partial e / \partial \kappa = \eta$ . Totally differentiating (53) with respect to  $\tilde{p}$  and  $\kappa$  yields

$$\frac{d\tilde{p}}{d\kappa} = \frac{-[(p^t - \tilde{p}) - S/z]\eta' + \eta}{[(p^t - \tilde{p}) - S/z](f'' + \kappa\eta'') - 2(f' + \kappa\eta')}. \quad (54)$$

From (53) and the properties of  $f$  and  $\eta$  it is seen that the numerator is always nonnegative while the denominator is positive (negative) for  $z < 0$  ( $z > 0$ ). Thus,

$$\text{sign} \left( \frac{d\tilde{p}}{d\kappa} \right) = -\text{sign}(z).$$

The optimal declared price shifts towards the true price if the probability of detection is increased. This negative relationship between the probability of

detection and the extent of misdeclaration is in accordance with the familiar results of the literature which are quite plausible for the same reasoning as noted above.

## 7 Concluding Remarks

The misdeclaration of import prices of intermediate products has opposite effects on the tax and the tariff liabilities. In this paper we have shown that for every combination of (linear) import tariff and (linear) income tax there is one definite optimal point of misdeclaration, assuming that the probability of detection and punishment depends positively on the amount of misdeclaration and that the respective function is convex. This assumption is justified on grounds of the authorities' revenue maximizing behavior as shown in a qualitative response model.

The sign of misdeclaration depends solely on the relative magnitudes of tax and tariff rates, whereas the amount of under- or overdeclaration is (additionally) determined by the concrete detection function and form of punishment (inter alia base and rate of penalty). The higher the rate of penalty or the amount of a lump-sum punishment is, the smaller is the misdeclaration. The extent of evasion is jointly determined by the four parameters mentioned. If the detection function is discontinuous, i.e. the probability of detection and punishment is well above zero for all declared prices except for the very true price (for which it is naturally zero), then a threshold value for the surcharge or the lump-sum penalty exists above which no evasion takes place at all.

However, these results have to be taken with a grain of salt. In this paper only economic variables have been considered. Empirical studies, however,

provide strong indications that also psychological and sociological variables should enter the models as determinants of evasion (e.g. Geeroms&Wilmots (1985) for an extensive discussion and Cowell 1985:178-180 for a survey of the literature). People are willing to pay rent for a roof over their head; the better the home, the more they are willing to pay – up to a certain point: the taxpayer is willing to pay taxes for the provision of public goods – as long as he feels he is getting his money worth. Moreover, if the taxpayer's beliefs coincide with those of “the powers that be” then he is less inclined to evade taxes and tariffs.<sup>32</sup>

Hence, in order to tackle the problem of evasion, politicians should bear in mind the advice of the grand seigneur of politics, Marcus Tullius Cicero:

*“Sin qua necessitas hujus muneris alicui rei publicae obvenerit,  
... danda erit opera, ut omnes intelligant, si salvi esse velint,  
necessitati esse parendum.”<sup>33</sup>*

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<sup>32</sup>The influence of equity considerations and the belief in existing laws and institutions on the attitude towards tax evasion is analyzed in Baldry (1987), Geeroms&Wilmots (1985), Spicer (1986), Spicer&Becker (1980), among others. Weigel et al. (1987) elaborate a social psychological model of tax evasion behavior.

<sup>33</sup>*“If a government must resort to extraordinary taxes, it must take care that everybody comes to the understanding that he has to bow to the inevitable, if they do not want to perish altogether.”*, Marcus Tullius Cicero, *De Officiis*, Lib.II, Cap.XXI; own translation.

# A p p e n d i c e s

## A Qualitative Analysis of $E(\Pi(\tilde{p}))$ of Model I

We confine the analysis to case of  $z > 0$ . Results apply mutatis mutandis for  $z < 0$ .

Recalling (17)

$$E(\Pi^t) = A + (p^t - \tilde{p})zf + z\tilde{p} - Sf \quad (55)$$

it is seen that this function is the sum of three functions

$$E(\Pi(\tilde{p})) = g_1(\tilde{p}) + g_2(\tilde{p}) + g_3(\tilde{p}) \quad (56)$$

with

$$g_1(\tilde{p}) := (p^t - \tilde{p})f(\tilde{p} - p^t)z \quad (57)$$

$$g_2(\tilde{p}) := -Sf(\tilde{p} - p^t) \quad (58)$$

$$g_3(\tilde{p}) := A + z\tilde{p}. \quad (59)$$

Since  $z$  is positive  $g_1$  is positive, downward sloping and convex for  $\tilde{p} < p^t$ , has an inflection point at  $\tilde{p} = p^t$  for which  $g_1(p^t) = g_1(p^t)' = 0$  and is negative and concave afterwards ( $\tilde{p} > p^t$ )<sup>34</sup>. This can easily be seen by differentiating  $g_1(\tilde{p})$  and is shown in figure A1.

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<sup>34</sup>see (11) - (13) .

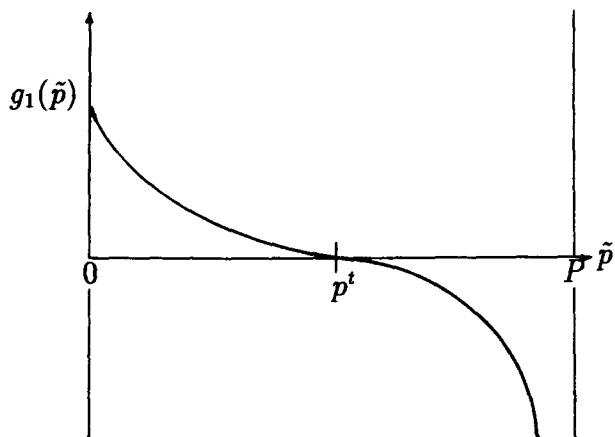


Fig. A1

From  $S > 0$  it follows that  $g_2$  has the shape shown in figure A2.

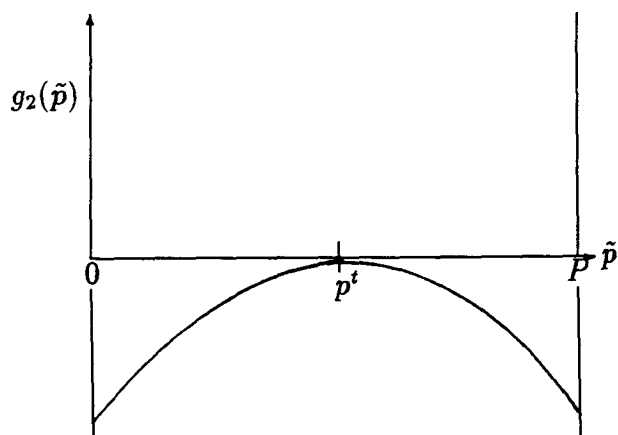


Fig. A2

The sum  $g_1 + g_2 := g_{12}$  has in principle the following shape:<sup>35</sup>

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<sup>35</sup>The term "in principle" means it is not certain that  $g_{12}$  reaches the interior local maximum, since information about the values of the parameters and the concrete specification of  $f$  is lacking. This remark is, mutatis mutandis, valid for the whole analysis of

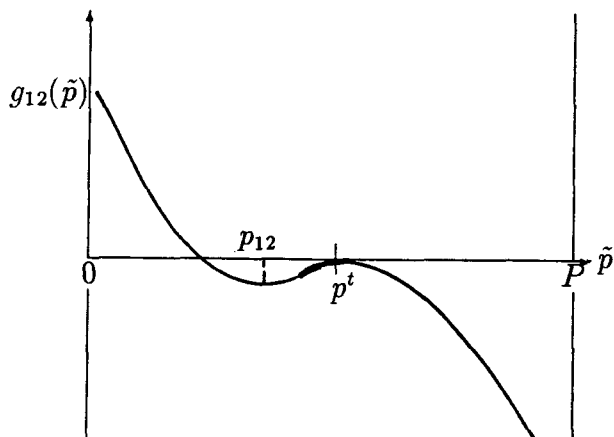


Fig. A3

The argument runs as follows. As the sum of two concave functions is again a concave function,  $g_{12}$  is concave for  $\tilde{p} > p^t$ . Moreover,  $g_{12}(p^t) = 0$  since  $g_1(p^t) = g_2(p^t) = 0$ . The slope of  $g_{12}(\tilde{p})$  for  $\tilde{p} < p^t$  is analyzed by differentiating  $g_{12}(\tilde{p})$ .

$$g_{12}(\tilde{p}) = [(p^t - \tilde{p}) - S/z]zf(\tilde{p} - p^t) \quad (60)$$

Differentiating  $g_{12}$  and setting it equal to zero we obtain

$$g_{12}(\tilde{p})' = [(p^t - \tilde{p}) - S/z]zf' - zf \stackrel{!}{=} 0. \quad (61)$$

It is immediately seen that  $\tilde{p} = p^t$  solves (61) and establishes a local maximum because the second derivative

$$g_{12}(\tilde{p})'' = [(p^t - \tilde{p}) - S/z]zf'' - 2zf' \quad (62)$$

calculated for  $\tilde{p} = p^t$  is equal to

$$g_{12}(p^t)'' = -Sf'' < 0 \quad (63)$$

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appendix A.



which is positive owing to the strict convexity of  $f$ .

(61) can be rewritten as

$$\tilde{p} = p^t - S/z - f(\tilde{p} - p^t)/f(\tilde{p} - p^t)' \quad (\text{F.O.C.}) . \quad (64)$$

From the properties of  $f$  and  $f'$  it is seen that no  $\tilde{p} > p^t$  can solve this equation but that there may exist an extremum ( $p_{12}$  in fig. A3) for  $\tilde{p} < p^t$ . Setting the second derivative (62) equal to zero we obtain the equation for an inflection point,  $\tilde{p}^i$ .

$$\tilde{p}^i = p^t - S/z - 2f(\tilde{p}^i - p^t)' / f(\tilde{p}^i - p^t)'' \quad (65)$$

If  $f$  is assumed to be three times continuously differentiable with  $f''' > 0$  it follows that  $-f'/f''$  is a positive and monotonically decreasing function for  $\tilde{p} < p^t$  and becomes zero at  $\tilde{p} = p^t$  thanks to the strict convexity of  $f$ . Hence  $g_{12}$  has exactly one inflection point  $\tilde{p}^i$  (for which  $\tilde{p}^i < p^t$ ) and thus the extremum – if existing – is a maximum.<sup>36</sup> Equation (65) is illustrated in figure A4.

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<sup>36</sup>Fig. A4 elucidates the fact that the inflection point always lies in the range for which  $E(\Pi^t)$  is defined – in contrast to the local interior extrema.

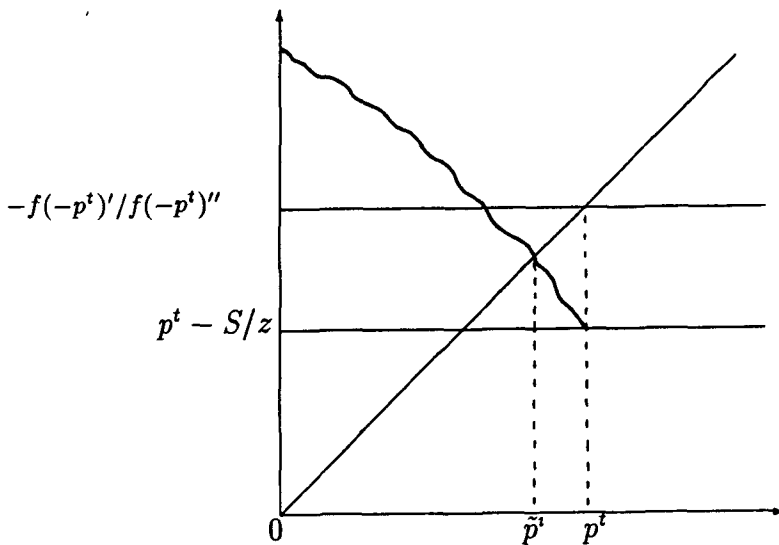


Fig. A4

If the assumption  $f''' > 0$  is dropped we cannot exclude the existence of multiple extrema of  $g_{12}$  in the interval  $[p^t - S/z; p^t]$ <sup>37</sup> but this will not alter the result that  $E(\Pi^t(\tilde{p}))$  has one local extremum at most for  $\tilde{p} < p^t$  - which is a maximum (see (22)). For further analysis it is assumed that  $f''' > 0$ .

Finally,  $g_3$ , which is linear in  $\tilde{p}$ , is added to  $g_{12}$ . Obviously this does not shift the inflection point as the second derivative remains unchanged. The local interior extrema, however - supposed they exist - move outwards since the first derivative increases by  $z$  (compared with (61)). This is shown in figure A5.

<sup>37</sup>Before  $g_{12}$  becomes positive ( $g_{12} = 0$  for  $\tilde{p} = p^t - S/z$ ) the function is strictly convex as seen from  $g_{12}''$  and therefore cannot have an extremum.

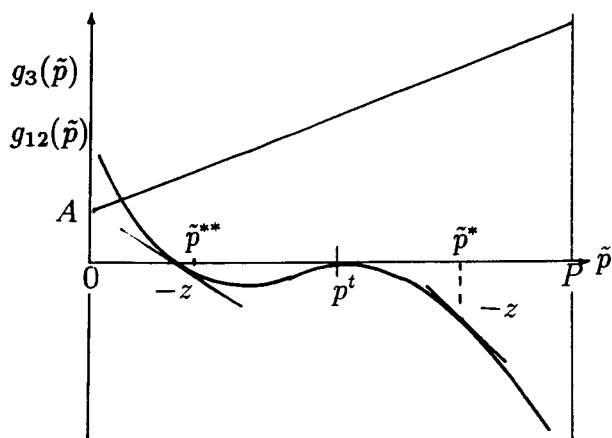


Fig. A5 a

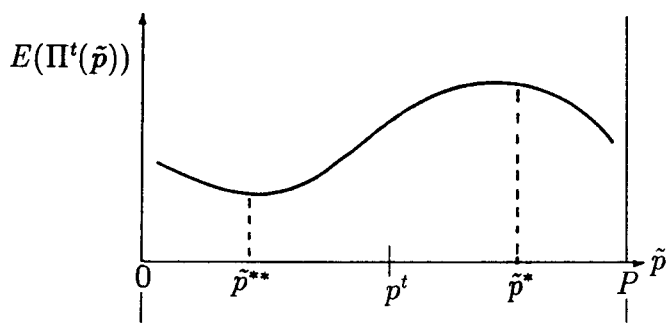


Fig. A5 b

The necessary condition for an extremum of  $E(\Pi^t(\tilde{p}))$  is that  $g'_{12} = -g'_3$ . Due to the linearity of  $g_3$  in  $\tilde{p}$  its first derivative is constant and equals  $z$ . Consider the concave part of  $g_{12}$  first (i.e. the interval  $[\tilde{p}^t; P]$ ). Because of  $g''_{12} > 0$ ,  $g'_{12}$  decreases strictly monotonically starting from a positive value at  $\tilde{p}^t$ , equals zero at  $\tilde{p} = p^t$  (see above, (61)) and may reach  $-z$  at some price

$\tilde{p}^* > p^t$ . Since the addition of a concave (convex) function and a linear one creates a concave (convex) function, it is clear that  $E(\Pi^t(\tilde{p}^*))$  establishes a minimum which is unique owing to strict concavity of  $g_{12}$  in  $[\tilde{p}^i; P]$ .

Lowering  $\tilde{p}$  beginning with  $\tilde{p}^i$  the first derivative of  $g_{12}$  decreases strictly monotonically thanks to the convexity of  $g_{12}$  in  $[0; \tilde{p}^i]$ . At  $p_{12}$ ,  $g'_{12}$  equals zero and may reach at most one  $\tilde{p}^{**} < p_{12}$  for which  $g_{12}(\tilde{p}^{**})' = -z = -g_3(\tilde{p}^{**})'$ . Thus  $E(\Pi^t(\tilde{p}^{**}))$  is a local minimum. Hence,  $E(\Pi^t(\tilde{p}))$  has the shape shown in figure A5b. From the above considerations it is clear that the existence of a local maximum does not depend on a penalty function. This penalty function only shifts the point of optimal misdeclaration  $\tilde{p}^*$  towards the “true” price  $p^t$ .

Q.E.D.

## B Qualitative Analysis of $E(\Pi^t(\tilde{p}))$ of Model II

We restrict our analysis to  $z > 0$ . The function (see (36))

$$E(\Pi^t(\tilde{p})) = A + z\tilde{p} + (p^t - \tilde{p})zf - \alpha s(\tilde{p} - p^t)zf \quad (66)$$

can be decomposed into three additively connected functions, two of which are identical with  $g_1$  and  $g_3$ , respectively.<sup>38</sup> These functions are defined as follows.

$$h_1 := g_1(\tilde{p}) = (p^t - \tilde{p})zf(\tilde{p} - p^t) \quad (67)$$

$$h_2 := -\alpha s f(\tilde{p} - p^t)z(\tilde{p} - p^t) \quad (68)$$

with  $\alpha = 1$  iff  $z(\tilde{p} - p^t) > 0$ ,  $\alpha = 0$  otherwise.

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<sup>38</sup>See (57), (59).

$$h_3 := g_3 = A + z\tilde{p} \quad (69)$$

and

$$E(\Pi^t(\tilde{p})) = h_1(\tilde{p}) + h_2(\tilde{p}) + h_3(\tilde{p}) \quad (70)$$

Seeing that arguments run parallel with those of appendix A only differences to the ideas given there are demonstrated subsequently. For complete argumentation the reader is referred to appendix A. Because  $z > 0$ ,  $h_2(\tilde{p})$  equals zero for  $\tilde{p} \leq p^t$  (i.e.  $\alpha = 0$ ). For  $\tilde{p} > p^t$   $h_2(\tilde{p})$  is a positive, concave and downward sloping function since

$$h_2(\tilde{p}) = \alpha s h_1(\tilde{p})$$

with  $\alpha = 1$  for  $\tilde{p} > p^t$  and  $s > 0$ . Function  $h_2$  is shown in figure B1.

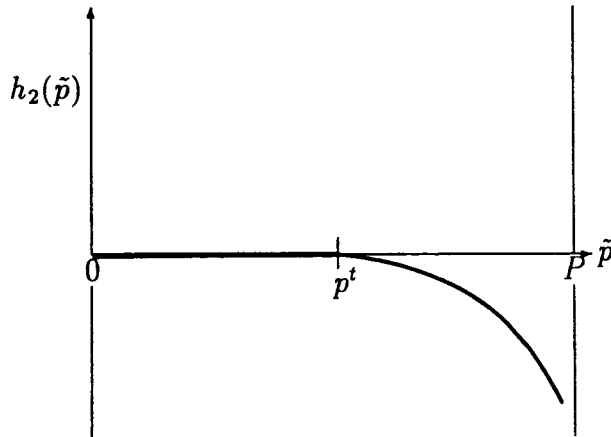


Fig. B1

The sum function  $h_1 + h_2 := h_{12}$  is immediately derived from figures A1 and B1 since the sum of two concave functions is again a concave function and  $s$  is positive. Note that the inflection point  $(p^t; h_{12}(p^t))$  is not altered because  $h_2$  is strictly concave for  $\tilde{p} \in (p^t; P]$  and zero otherwise. Function  $h_{12}$  plots as demonstrated in figure B2.

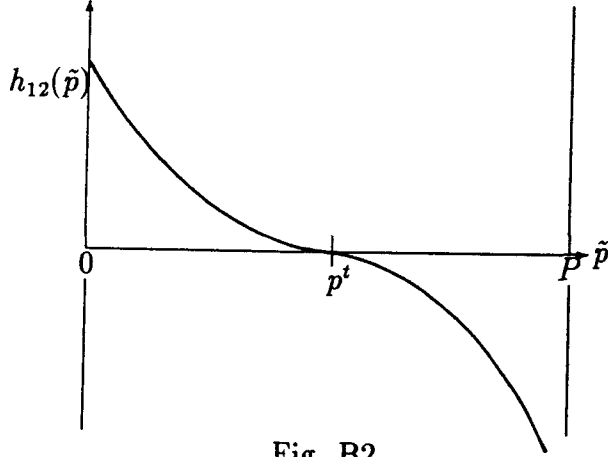


Fig. B2

This can also be calculated by differentiating

$$h_{12}(\tilde{p}) = (1 + \alpha s)h_1(\tilde{p}) = (1 + \alpha s)(p^t - \tilde{p})zf(\tilde{p} - p^t). \quad (71)$$

By adding the linear function  $h_3$  to  $h_{12}$  the shape of the function is transformed from a monotonically decreasing function into a function that resembles a cubic one. As in model I, the inflection point remains unshifted (in model II at  $\tilde{p}^i = p^t$ ). Since the strictly concave and the strictly convex range remain unaltered, the resulting function  $E(\Pi^t(\tilde{p}))$  has at most one interior minimum (for  $\tilde{p}^{**} < p^t$ ) and at most one interior maximum (for  $\tilde{p}^* > p^t$ ), respectively, for which  $h_{12}(\tilde{p}^{**})' = h_{12}(\tilde{p}^*)' = -z = -h_3(\tilde{p})'$ .<sup>39</sup>

Q.E.D.

This is illustrated in figure B3.

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<sup>39</sup>For a more detailed analogous argument see appendix A.

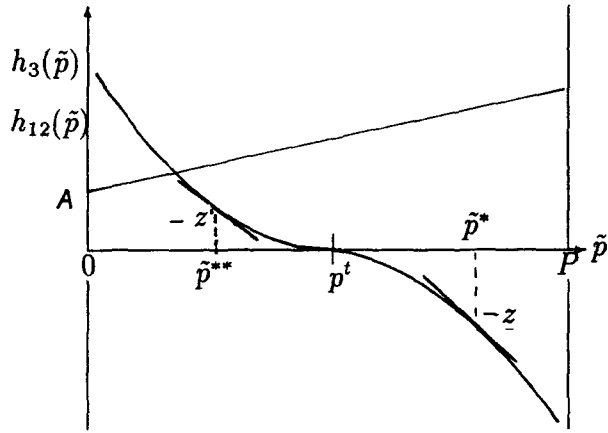


Fig. B3 a

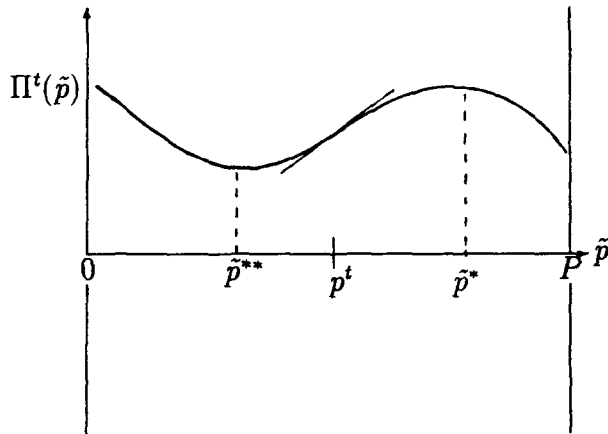


Fig. B3 b

## C Analysis of a Discontinuous Detection Function

### C.1 Derivation of Discontinuous Detection Functions

Recall authorities' investigation function  $i(\tilde{p})$  as derived in section 3.1. It is strictly convex and takes on its minimum at  $\tilde{p} = p^t$ . In general,  $i(p^t) > 0$ . Moreover,  $i(\tilde{p}) \in (0, 1) \forall \tilde{p} \in (0, P)$  and  $i(\tilde{p}) \in [0, 1] \forall \tilde{p} \in [0, P]$ . The investigation function is depicted in figure C1.

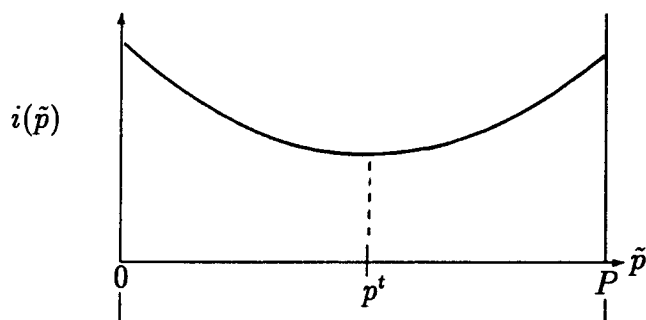


Fig. C1

Now assume that once the authorities decided to investigate they can no longer be deceived. Evasion will be discovered and the importer will be convicted with certainty, i.e.  $l_1(\tilde{p} - p^t) = 1 \forall \tilde{p} \neq p^t$  and  $l_1(0) = 0$ .<sup>40</sup> This is relevant if evasion is easy to discover (though not necessarily at low cost) and the principle of legality<sup>41</sup> applies both to the executive and the judiciary.

<sup>40</sup>Note that the function  $l_1(\tilde{p} - p^t)$  describes the probability that investigation leads to conviction. We have substituted  $l_1(\tilde{p} - p^t)$  for  $l(\tilde{p} - p^t)$  of page 15.

<sup>41</sup>The principle of legality as opposed to the principle of opportunity says that the



Then, the detection function  $f$  is replaced by  $\phi(\tilde{p} - p^t)$ .

$$\phi(\tilde{p} - p^t) = \begin{cases} \phi_1(\tilde{p}) & \text{iff } \tilde{p} \neq p^t \\ 0 & \text{iff } \tilde{p} = p^t \end{cases} \quad (72)$$

with  $\phi_1(\tilde{p}) = i(\tilde{p})$  (in the case of  $l_1(\tilde{p} - p^t) = 1 \forall \tilde{p} \neq p^t$ ). The same qualitative result ( $\phi_1(\tilde{p})$  is a strictly convex function that takes on its minimum at  $\tilde{p} = p^t$ ) is also obtained if  $l_1$  is a constant positive function less than unity for all  $\tilde{p} \neq p^t$ . This is likewise valid for all functions  $l_2(\tilde{p} - p^t)$  for which  $l_2(\tilde{p} - p^t) = \lambda(\tilde{p} - p^t)$  iff  $\tilde{p} \neq p^t$  and  $l_2(\tilde{p} - p^t) = 0$  iff  $\tilde{p} = p^t$  with  $\lambda(\tilde{p} - p^t)$  having the following properties. It is a strictly convex function of  $\tilde{p} - p^t$ ; in contrast to  $l(\tilde{p} - p^t)$ , however, its minimum has a non-zero value, i.e.  $\lambda(0) \neq 0$ . This results also in a discontinuity of the detection function at  $\tilde{p} = p^t$ . Since this is very easily shown we skip the proof.

Subsequently we analyze the different influence of a discontinuous detection function  $\phi$  as described by (72) –with  $\phi_1$  being a strictly convex function with its minimum at  $\tilde{p} = p^t$  but not necessarily identical with  $i(\tilde{p})$ – on the results of the models with a lump-sum penalty as well as with proportional penalty. Since these lines of reasoning follow those set out in appendices A and B, we focus on differences in the analysis and skip arguments where they parallel those already given. Again, it is assumed that  $z > 0$ . Results apply *mutatis mutandis* for  $z < 0$ . We turn to model I next.

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executive or the judiciary have to prosecute and to institute criminal proceedings; discretionary decisions are not possible. Under German law this principle is laid down in §152 StPO (Strafprozeßordnung, i.e. the German code of criminal procedure).

## C.2 Modification of Model I

The expected true profit is altered from (55) to :

$$E(\Pi^t) = A + (p^t - \tilde{p})z\phi + z\tilde{p} - S\phi \quad (73)$$

which correspondingly is the sum of the three functions given below.

$$\gamma_1(\tilde{p}) := (p^t - \tilde{p})\phi(\tilde{p} - p^t)z \quad (74)$$

$$\gamma_2(\tilde{p}) := -S\phi(\tilde{p} - p^t) \quad (75)$$

$$\gamma_3(\tilde{p}) := A + z\tilde{p} \quad (76)$$

The function  $\gamma_1$  has the same geometrical properties as  $g_1$  with the exception that the point of intersection with the abscissa at  $\tilde{p} = p^t$  – an inflection point like  $(p^t, g(p^t))$  – is not a saddle point. The factor  $(p^t - \tilde{p})$  ensures that the discontinuity of  $\phi$  cannot have an effect. Substituting the continuous extension  $\phi_1$  for  $\phi$  and differentiating the resulting  $\gamma_1^{ce}$  at  $\tilde{p} = p^t$  yields  $\gamma_1^{ce}(p^t)' = -\phi_1 z < 0$ .  $\gamma_2$  is a nonpositive discontinuous function as shown in figure C2. This is derived from the definition of  $\phi$  in (72) and the fact that  $S > 0$ .

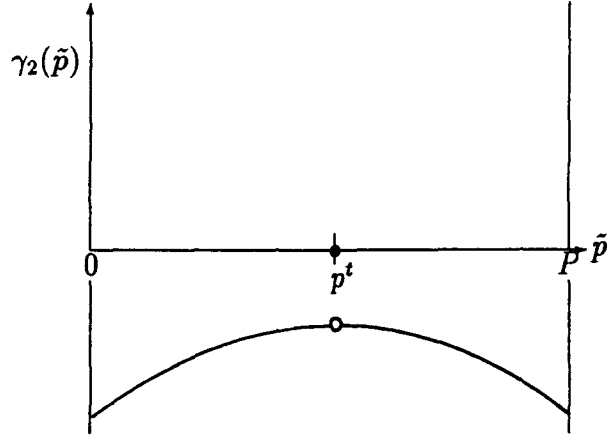


Fig. C2

The sum  $\gamma_1 + \gamma_2 := \gamma_{12}$  has the following properties. It is negative and concave for  $\tilde{p} > p^t$  since it is the sum of two negative and concave functions. At  $\tilde{p} = p^t$ , it has a discontinuity as  $\gamma_{12}(p^t)$  equals zero. The continuous extension of this function at the point of discontinuity, i.e.  $\gamma_{12}^{cc}(p^t)$  is concave. For the same reason as in appendix A there is a unique inflection point  $\tilde{p}^i < p^t$  provided that  $\phi_1''' > 0$ . (Cf. page 36.) For  $\tilde{p} \neq p^t$ ,

$$\tilde{p}^i = p^t - S/z - 2\phi'/\phi'' \quad (77)$$

However, it is not sure that  $\gamma_{12}$  reaches local interior extrema. An interior maximum is reached if the tangent at the inflection point has a positive sign (since  $\gamma_{12}^{cc}(p^t)' = -z\phi_1 < 0$ ). This depends on the properties of  $\phi$  at  $\tilde{p}^i$ :

$$\gamma_{12}(\tilde{p}^i)' > 0 \iff 2[\phi(\tilde{p}^i - p^t)]^2 > \phi(\tilde{p}^i - p^t)'' \phi(\tilde{p}^i - p^t).$$

Whether this inequalities hold cannot be determined a priori. But it is obvious that in both cases, the existence of at most one interior local maximum<sup>42</sup>

<sup>42</sup>Even if a local maximum exists it is still unclear whether a local minimum exists within the domain of the function, see appendix A.

as well as its absence, can occur. On the one hand,  $f(\tilde{p} - p^t)$  establishes a boundary case of  $\phi(\tilde{p} - p^t)$  though we have not included it in the definition of  $\phi$ . Hence, if  $\phi$  is very “similar”<sup>43</sup> to  $f$ , the function  $\gamma_{12}$  will have a local maximum for a  $p^m$  for which  $\tilde{p}^i < p^m < p^t$ . On the other hand, if the penalty  $S$  is arbitrarily small  $\gamma_{12}$  will resemble  $\gamma_1$  and as a consequence will not have any interior extremum.<sup>44</sup>

The expected profit function can be derived now. First we consider the continuous extension  $\phi_1$  instead of  $\phi$ . Arguments run parallel to those given on page 35. This is obvious if  $\gamma_{12}$  has a local interior extremum and applies also if it has not. Since the analysis of the former case is almost identical to appendix A we confine ourselves to the latter case depicted in figure C3.

The first derivative of  $\gamma_{12}^{cc}$  rises as  $\tilde{p}$  is increased from zero to  $p^t$  and amounts to  $-z\phi_1(p^t) < 0$  at the true price. Afterwards,  $\gamma'_{12}$  decreases as  $\tilde{p}$  is raised. Due to  $\phi_1(p^t) < 1$ <sup>45</sup> and hence  $\gamma_{12}(p^t)' > -z$ , and the fact that  $\gamma'_{12}$  takes on its maximum at the true price, there may be at most two prices  $\tilde{p}^* > p^t$  and  $\tilde{p}^{**} < p^t$  for which  $\gamma'_{12} = -\gamma'_3 = -z$  leading to an interior maximum and an interior minimum of  $E(\Pi^t(\tilde{p}))$ , respectively. (For a more detailed analysis see appendix A.)

We analyze the point of discontinuity next. Stating the true import price is preferable to evasion if  $E(\Pi^t(p^t)) > E(\Pi^t(\tilde{p}^*))$ . Note that in this context  $\tilde{p}^*$  indicates the global maximum of  $E(\Pi^t(\tilde{p})) \forall \tilde{p} \neq p^t$ ; if no interior maximum

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<sup>43</sup>More precisely, “similar” means that the continuous extension of the detection function is close to zero at the point of discontinuity:  $\phi_1(0) = \varepsilon$  with  $\varepsilon > 0$  and less than an arbitrarily small value  $o$ .

<sup>44</sup>The inflection point, however, will be shifted to the left.

<sup>45</sup>If  $\phi_1(0) = 1$  convexity of  $\phi_1 \in [0, 1]$  would not be satisfied. Moreover, no evasion would take place – our problem would disappear.

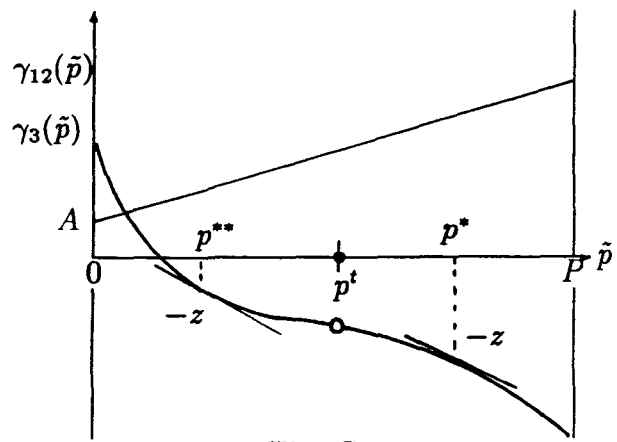


Fig. C3 a

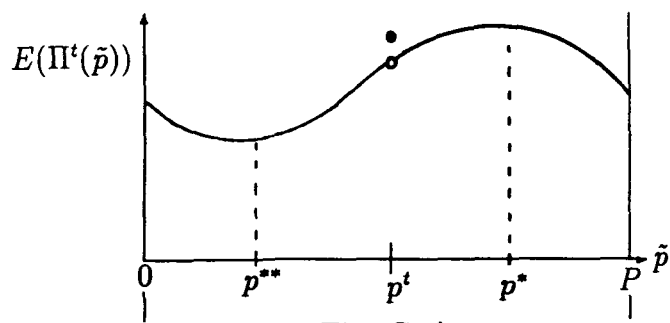


Fig. C3 b

exists  $\tilde{p}^*$  takes on the maximal boundary value. Recalling (17), (19), and (23) and substituing  $\phi$  for  $f$  we can calculate

$$E(\Pi^t(p^t)) - E(\Pi^t(\tilde{p}^*)) = -\frac{[1 - \phi(\tilde{p}^* - p^t)]^2}{\phi(\tilde{p}^* - p^t)'} z + S \quad (78)$$

In other words no evasion takes place if and only if the lump-sum penalty exceeds a critical value  $S^c$  given by

$$S^c = \frac{[1 - \phi(\tilde{p}^* - p^t)]^2}{\phi(\tilde{p}^* - p^t)'} z \quad (79)$$

### C.3 Modification of Model II

For the second model  $E(\Pi^t(\tilde{p}))$  is decomposed into the following functions.

$$\delta_1 := (p^t - \tilde{p})z\phi(\tilde{p} - p^t) = \gamma_1(\tilde{p}) \quad (80)$$

$$\delta_2 := -\alpha s\phi(\tilde{p} - p^t)z(\tilde{p} - p^t) \quad (81)$$

with  $\alpha = 1$  iff  $z(\tilde{p} - p^t) > 0$ ,  $\alpha = 0$  otherwise.

$$\delta_3 := A + z\tilde{p} = \gamma_3 \quad (82)$$

In this model we need not replace  $\phi(\tilde{p} - p^t)$  by its continuous extension  $\phi_1(\tilde{p} - p^t)$  for analytical convenience, because  $\delta_1(p^t) = \delta_2(p^t) = 0$  would be obtained also if  $\phi_1(0) \neq 0$  was substituted for  $\phi(0)$ . The first derivative of the function  $\delta_{12} := \delta_1 + \delta_2$  is given by

$$\delta'_{12} = (1 + \alpha s)[(p^t - \tilde{p})\phi'_1 z - z\phi_1] \quad (83)$$

$\delta'_{12}$  is not differentiable at  $\tilde{p} = p^t$  since

$$\lim_{\epsilon \rightarrow 0} \delta'_{12}(p^t - \epsilon) = -z\phi_1 \neq -z\phi_1(1 + s) = \lim_{\epsilon \rightarrow 0} \delta'_{12}(p^t + \epsilon)$$

and is negative otherwise.  $E(\Pi^t(\tilde{p}))$  is correspondingly derived as shown in appendix B. Note, however, that  $E(\Pi^t(\tilde{p}))$  is continuous but not differentiable at  $\tilde{p} = p^t$ . This is due to a positive rate of penalty  $s$  together with  $\phi_1(0) \neq 0$  (which in turn leads to  $\delta_1(p^t)' \neq 0$ ).

After having replaced  $\phi$  by its continuous extension  $\phi_1$  we calculate from the modified first derivative of  $E(\Pi^t(\tilde{p}))$ , i.e.  $\Pi^{ce'}(\tilde{p})$  (which is not defined for  $\tilde{p} = p^t$ )

$$\Pi^{ce'} = (1 + \alpha s)[(p^t - \tilde{p})\phi_1'z - z\phi_1] + z$$

the following limit:

$$\lim_{\epsilon \rightarrow 0} \Pi^{ce'}(p^t - \epsilon) = (1 - \phi_1(0))z$$

which is positive since  $\phi_1(0) < 0$  (otherwise  $\phi_1$  could not be convex in  $[0, P]$ ). As follows from above misdeclaration is not rewarding if  $\lim_{\epsilon \rightarrow 0} \Pi^{ce'}(p^t + \epsilon) < 0$ . Thus, from its calculation ( $\alpha = 1$ )

$$\lim_{\epsilon \rightarrow 0} \Pi^{ce'}(p^t + \epsilon) = (1 - \phi_1(0) - s\phi_1(0))z$$

we see that there exists a critical value for the rate of penalty,

$$s^c = \frac{1 - \phi_1(0)}{\phi_1(0)}.$$

If  $s$  exceeds  $s^c$  the profit function  $E(\Pi^t(\tilde{p}))$  takes on its maximum value at  $\tilde{p} = p^t$ , then import prices are correctly declared. In other words,

$$\lim_{\epsilon \rightarrow 0} \Pi^{ce'}(p^t + \epsilon) < 0 \iff s > s^c.$$

Q.E.D.

For  $s < s^c$  the derivation of the profit function is depicted in Fig. C4 a and C4 b.

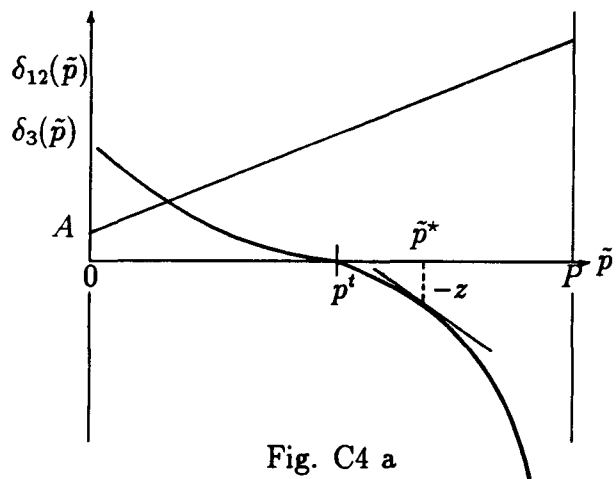


Fig. C4 a

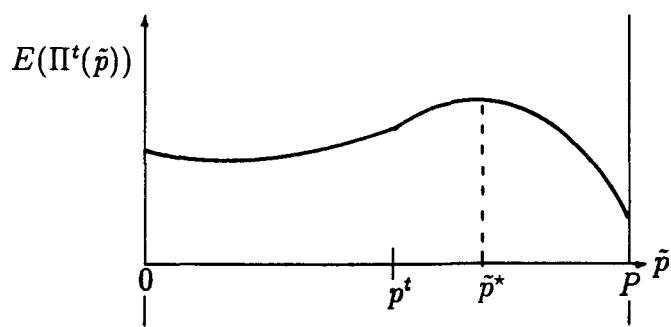


Fig. C4 b



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