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## Intergenerational redistribution in a small open economy with endogenous fertility

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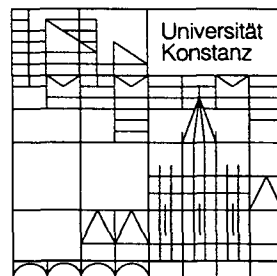
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Martin Kolmar

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# **Intergenerational Redistribution in a Small Open Economy with Endogenous Fertility**

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# Intergenerational Redistribution in a Small Open Economy with Endogenous Fertility\*

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## Abstract

For pay-as-you-go financed pension systems, claims may be calculated according to individual contributions (income) or the number of children of a family. We analyse the optimal structure of these parameters in a model with endogenous fertility. It is shown that for both structural determinants there exists no interior solution of the problem of intragenerational utility maximisation. Thus, pure systems are always welfare maximizing. Furthermore, children-related pension claims induce a fiscal externality that tends to be positive. The determination of the optimal contribution rate shows that the widely accepted Aaron-condition is in general a misleading indicator for the comparison of fully funded and pay-as-you-go financed pension systems.

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# 1 Introduction

The literature on pension systems or intergenerational redistribution with endogenous fertility gives some interesting insights into the function of transfer mechanisms. It is guided by two theoretical questions: First, some authors (for example Samuelson (1975) and Deardorf (1976)) asked for the optimal population growth rate, or the goldenest golden rule for the case of exogenous but variable population-growth paths. This analysis shed light on the fundamental problems due to nonconvexities in the optimisation structure. Schweizer (1995) proved a fundamental structural similarity between Samueson-like models of optimal population and models of optimal club size that rests on the intertemporal interpretation of the Henry-George theorem. As Eckstein and Wolpin (1985) have pointed out, a weakness of this class of models can be seen in the fact that it is difficult to understand how economic policy can be applied in order to achieve the optimal allocation.

Second, the fertility decision of a familiy was included into the microeconomic optimisation problem of a household.<sup>1</sup> This literature treats population growth as an endogenous variable and therefore avoids the weakness mentioned above. There are several attempts to understand intergenerational transfers in this class of models. In a general-equilibrium Cobb-Douglas model, Raut (1990) has shown that the introduction of a actuarially fair pay-as-you-go (PAYG) public-pension system decreases population growth and increases the capital-labor ratio. In the model, the pension structure is exogenous. The question whether actuarially fair systems are optimal remains unsolved. The model by Nishimura and Zhang (1992) differs in the way fertility is motivated. In their model, parents have children because they get gifts (retirement income) from their children when being old. Children are willing to pay these gifts because the old-age consumption of their parents is an argument in their utility function. The intuitive consequence of this setup is that gifts must be positive in a steady-state equilibrium. Otherwise, parents would not have any incentive to invest in children. Perhaps more interestingly, an optimal steady state can in general not be implemented with the introduction of a compulsory pension

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<sup>1</sup>This line of research was originated by Becker (1960), for a recent application to the field of population economics see Razin and Sadka (1995).

system. This follows because children, as a reaction, may change gifts, savings or fertility. A qualitatively similar result can be found in Wigniolle (1995). In a model of two-sided altruism and human-capital accumulation, the introduction of a PAYG is neutral because gifts/bequests are reduced/increased by exactly the same amount. This neutrality result contrasts with the findings of Lapan and Enders (1990) and Wildasin (1990). Getting to the same results as Raut concerning fertility and the capital-labor ratio, Ricardian equivalence fails to hold and an increase in public debt (or PAYG) is welfare reducing. The existence of debt imposes a tax on future generations and therefore on the costs of child rearing if the economy is dynamically efficient. This result rests, however, on the assumption that children are normal goods.

These results should be interpreted in a careful way because the authors, by neglecting the strategic component present in all systems where pensions are calculated on grounds of income. All these models were motivated by the functioning of the “macro”-structure of pension systems. Our framework allows a more detailed investigation of the “micro”-structure of the pension formulae. For example the question whether pension claims should be based on monetary payments during the working life or on the number of children raised can be more systematically analysed. This is the first question addressed in this paper. With exogenous population growth the analysis of structural changes of PAYG-pension systems is restricted to changes in one parameter, namely the contributions to the pension fund. Real-world systems, however, are characterized by more than one variable. If one tries to understand the crucial determinants of intergenerational redistribution, one has to specify the model in a way that all variables of potential economic significance are included. Models of endogenous fertility allow for the introduction of two different types of pay-as-you-go systems, namely a pension formulae that calculates old age benefits according to contributions paid during the working life (IPAYG) and a pension formulae that calculates old age benefits according to the number of children raised (CPAYG). An adequate parametrisation makes it possible to allow for any combination of both principles. The distinction of these systems makes it possible to analyse systems in modern societies which are mainly organised according to the first principle, and systems in rural societies that work on a mainly dynastic basis. The analysis demonstrates that both systems differ

in their economic performance, and that either pure CPAYG or pure IPAYG are preferable.

The optimality of corner solutions carries over to the analysis of changes of the contribution rate for a given pension formulae. This is the second question analysed in the paper. This finding is due to the global nonconvexity of the indirect utility function with respect to changes in the contribution rate and a global monotonicity with respect to the structural parameter. The restriction to a small open economy makes it possible to identify this nonconvexity as a structural determinant of PAYG-systems.

The literature comparing fully funded (FF) and pay-as-you-go (PAYG) financed public-pension systems in small open economies stresses the importance of the Aaron condition (Aaron (1966)) as an empirical measure to decide which system can be expected to lead to a higher long-run welfare. A country with a PAYG system has a higher level of utility than a country with a FF system if the growth rate of total wage income exceeds the interest rate. Endogenising population growth makes one determinant of the growth rate of wage incomes endogenous. We ask whether the Aaron condition is still a good indicator in this case. The answer will turn out to be negative. The Aaron condition remains a good indicator for only small changes of the contribution rate. For discrete changes this condition is insufficient and may even be misleading because it may point to the wrong direction of change. Additionally, for a general class of pension systems the Aaron condition has to be accompanied by a second effect measuring the equilibrium response of population growth due to changes in parameters.

The rest of the paper is organized as follows: In section 2 the individual decision problem and the pension formulae are introduced. In section 3, the individual optimisation problem is solved and the influence of the pension structure on individual behavior is analysed. In section 4 the optimisation problem of the pension carrier is solved. We look first at changes in the pension structure (section 4.2). In a second step, the optimal contribution rate is derived (section 4.3). Section 5 concludes.

## 2 The Model

We analyse a two-stage optimisation problem. In stage two, the households maximise utility by the choice of consumption, savings and children. They take all prices and the structural determinants of a PAYG-financed public-pension system as given. In stage one, the carrier of the public pension system (let's say the government) chooses the structure of the pension system, taking into account the behavior of the households. The country is small relative to the rest of the world, therefore, all factor and goods prices are determined exogenously.

### 2.1 The Decision Problem of a Household

Every household lives for three (overlapping) periods, childhood, working-life and retirement. All households are assumed to have identical preferences.<sup>2</sup> In the first period of life, a child consumes an exogenously given amount of a homogenous consumption good<sup>3</sup> ( $x$ ). In the second period of life, an individual works one unit of time and receives a wage income  $w^t$ . This can be spend for own children<sup>4</sup> ( $1 + n^{t+1}$ ), consumption ( $c_2^t$ ) or savings on capital markets ( $s_2^t$ ). There exists a compulsory PAYG pension system with contributions ( $B^t$ ). In the third period of live, the individual consumes ( $c_3^{t+1}$ ) interest bearing savings ( $(1+r^{t+1})s_2^t$ ) and the pensions paid by the next generation ( $L_i^{t+1}$ ). The number of own children is restricted to some interval  $n^{t+1} \in [-1, \hat{n}]$  specifying minimum and maximum number of children for a family.

With these specifications we get the following (Kuhn-Tucker) Lagrangean of the individual optimisation problem:

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<sup>2</sup>As will become clear in the next sections, they do not act, however, as representative consumers.

<sup>3</sup>In standard models of endogenous fertility (for example Becker (1960)), parents choose the number and the quality of children. We neglect this second variable despite the fact that it may be important for the analysis of some intergenerational effects, for example the demographic consequences of child benefits. In this model,  $x$  can be interpreted as the expenditures necessary for an exogenously given quality of children.

<sup>4</sup>In this notation the population growth rate is simply  $n$ . We neglect matching problems of marriage and assume  $n \in R$  for convenience.



$$\begin{aligned}
\mathcal{L} = & U(c_2^t, c_3^{t+1}, 1 + n^{t+1}) \\
& + \lambda [w_2^t - s_2^t - (1 + n^{t+1})x - B^t - c_2^t] \\
& + \mu [(1 + r^{t+1})s_2^t + L_i^{t+1} - c_3^{t+1}] \\
& + \theta_1 [n^{t+1} + 1] \\
& + \theta_2 [\hat{n} - n^{t+1}]
\end{aligned} \tag{1}$$

The utility function has the usual properties of strict convexity, monotonicity and continuity.

## 2.2 The Pension Formulae

We will now specify the pension system in more detail. Let  $N^t$  be the number of individuals which are in their second period of life at time  $t$ . Every individual pays a fraction  $\tau$  of its income ( $w_2^t$ ) as contribution to the pension system. With this specification, total pension payments are in  $t + 1$ :

$$T = \sum_{j=1}^{N^t} \tau w_2^{t+1} (1 + n_j^{t+1}) = \tau w_2^{t+1} \sum_{j=1}^{N^t} (1 + n_j^{t+1}) \tag{2}$$

We compare three ideal types of pension systems.

- In a *Fully Funded* system, a pension fund collects and invests contributions in the capital market. Therefore, in our model with only one interest rate and unrestricted borrowing and lending of an individual, this is equivalent to private savings. It can therefore be described by a contribution rate  $\tau$  equal to zero.
- The second ideal form of a pension system is the *Pay-As-You-Go* system where individual savings are immediately spent for retired persons. Therefore, payments do affect the total volume of transactions in capital markets. With the assumption of a small open economy, however, this effect has no influence on interest rates and growth. In models of endogenous fertility, two ideal types of PAYG can be distinguished:

- If pension payments are calculated according to the number of children of a household, we have a Children-PAYG (CPAYG).
- If pension payments are calculated according to the income of a household, we have an Income-PAYG (IPAYG).

Most real-world systems in industrialized countries are structured as IPAYGs. Child-benefit systems, analysed in this framework, can be seen as structural changes to a more child-related payment.

More generally, a PAYG is fully described by two parameters, the contribution rate  $\tau \in [0, 1]$  and a parameter  $\alpha \in [0, 1]$  measuring the fraction of pension payments which are calculated according to CPAYG.

With the above specification, per-capita pensions for an IPAYG amount to

$$L_i^{t+1} = \frac{\tau w_2^{t+1}}{N^t} \sum_{j=1}^{N^t} (1 + n_j^{t+1}). \quad (3)$$

For a CPAYG, pension payments are equal to

$$L_i^{t+1} = \tau w_2^{t+1} (1 + n_i^{t+1}). \quad (4)$$

Finally, for a mixed system we get the following formulae for the individual pension payment ( $\bar{n}$  is the average population growth rate of the economy):

$$\begin{aligned} \tilde{L}_i^{t+1} &= \alpha \tau w_2^{t+1} (1 + n_i^{t+1}) + (1 - \alpha) \tau w_2^{t+1} \frac{1}{N^t} \sum_{j=1}^{N^t} (1 + n_j^{t+1}) \\ &= \tau w_2^{t+1} (\alpha (1 + n_i^{t+1}) + (1 - \alpha) (1 + \bar{n}^{t+1})) \end{aligned} \quad (5)$$

In the following, changes in  $\tau$  are called “structural changes of the PAYG”, changes in  $\alpha$  are called “structural changes within the PAYG”.

### 3 Analysis of the Pension Structure

A rational household with complete information will calculate the effect of its own number of children on its future pension payments. The pension formulae

will therefore be taken into account when maximizing utility.<sup>5</sup> Children have two functions in this model. First, parents derive utility from having them. Second, within the PAYG they transfer income from the working life to the period of retirement. The net costs may thus be either positive or negative. Due to this fact, there might be a corner solution of the optimisation problem with the choice of maximum fertility. Therefore we have to consider two different cases. For an interior solution, the effective costs of a child are such that optimal is below maximum fertility. For a corner solution we get the maximum fertility as optimal behavior.

### 3.1 Interior Solution

In this case we have  $n^{t+1} \in (-1, \hat{n}) \Rightarrow \theta_1 = \theta_2 = 0$ . The first order conditions of the household optimisation problem are:

$$MRS_3^2 = \frac{U_{c_2}}{U_{c_3}} = (1 + r^{t+1}), \quad (6)$$

$$MRS_3^n = \frac{U_n}{U_{c_3}} = \left( (1 + r^{t+1})x - \tau w_2^{t+1} \left( \alpha + (1 - \alpha) \frac{1}{N_t} \right) \right), \quad (7)$$

with  $U_y$  indicating the partial derivative of  $U$  with respect to  $y$ . The direct costs of children,  $(1 + r^{t+1})x$  are reduced by the future pension payments,  $\tau w_2^{t+1}(\alpha + (1 - \alpha))\frac{1}{N_t}$ . The reduction of costs depends on the fraction of payments calculated according to CPAYG.

### 3.2 Corner Solution

In this case, the parameters are such that  $n^{t+1} = \hat{n} \Rightarrow \theta_1 = 0, \theta_2 \geq 0$ . Therefore, maximum fertility is optimal. The relevant first-order conditions for this scenario are as follows:

$$(1 + n^{t+1}) \left( U_n - \lambda x + \mu \tau w_2^{t+1} \left( \alpha + (1 - \alpha) \frac{1}{N_t} \right) - \theta_2 \right) = 0$$

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<sup>5</sup>This structure differs from Peters (1995) who assumes that the pension payment is a parameter in the individual optimisation problem.

$$\begin{aligned} \wedge \quad n^{t+1} &= \hat{n} \\ \Rightarrow \quad U_n - \lambda x + \mu \tau w_2^{t+1} \left( \alpha + (1 - \alpha) \frac{1}{N_t} \right) &= \theta_2 \end{aligned}$$

Substitution of  $\lambda$  and  $\mu$  leads to:

$$U_{c_3} \left[ \frac{U_n}{U_{c_3}} - \left( (1 + r^{t+1})x - \tau w_2^{t+1} \left( \alpha + (1 - \alpha) \frac{1}{N_t} \right) \right) \right] = \theta_2 \quad (8)$$

Figure 1 gives a graphical representation of the above condition for the case of a CPAYG.  $C$  is the aggregate consumption of both periods. For different  $\tau$ , the budget lines rotate through a fixed point  $F$  (see Appendix B). If the effective costs of children are below a critical level, maximum fertility is realized (points A and B). The marginal utility of increasing fertility, which is equal to the direct marginal utility plus the increase in utility due to increased consumption, exceeds the costs of children. This net gain is measured by  $\theta_2 = \frac{\partial V(\hat{n})}{\partial n}$ , with  $V$  being the indirect utility function of the household.

– Figure 1. –

As the comparative static analysis will show, interior solutions for the structural parameters of the PAYG will never be optimal. Therefore we will have to come back to the possibility of corner solution of the household problem if necessary.

We will now analyse the first order conditions for the three special cases mentioned above:

1. Fully Funded system (FF) ( $\tau = 0$ ):

$$MRS^{FF} := \frac{U_n}{U_{c_3}} = (1 + r^{t+1})x \quad (9)$$

2. CPAYG ( $\alpha = 1$ ):

$$MRS^{CPAYG} := \frac{U_n}{U_{c_3}} = ((1 + r^{t+1})x - \tau w_2^{t+1}) \quad (10)$$

3. IPAYG ( $\alpha = 0$ ):

$$MRS^{IPAYG} := \frac{U_n}{U_{c_3}} = \left( (1 + r^{t+1})x - \tau w_2^{t+1} \frac{1}{N_t} \right) \quad (11)$$

One can see that the effective costs of children are smaller for a PAYG than for a FF. This effect decreases for an IPAYG if the population ( $N^t$ ) is getting larger:

**Result 1** *For “large” economies<sup>6</sup>, all relative prices, and therefore marginal rates of substitution, are equal for a fully funded and an IPAYG system.*

**proof:**  $\lim_{N^t \rightarrow \infty} MRS^{IPAYG} = (1 + r^{t+1})x^t = MRS^{FF}$

The result demonstrates that the economically relevant difference between IPAYG and FF is not a change in relative prices as perceived by the individuals. This does, however, not imply that - even for small economies - both systems are equivalent. While a switch from FF to IPAYG implies no substitution effect, it will in general induce an income effect. It can therefore be seen as a tax on future generations. This tax is lump sum even though fertility is endogenous. We will come back to this in the next section.

The set of individual first-order conditions can be used to calculate “Marshallian” reaction functions,  $[c_{2i}^t(\tau, \alpha, n_{-i}^{t+1}), c_{3i}^t(\tau, \alpha, n_{-i}^{t+1}), n_i^{t+1}(\tau, \alpha, n_{-i}^{t+1})]$ ,  $i = 1, \dots, N^t$  with  $n_{-i}^{t+1}$  indicating the vector of family growth rates of all households except  $i$ . A Nash equilibrium is a fixed point of the above system. In the following we will focus attention on perfectly symmetric equilibria.<sup>7</sup> This assumption can be justified because all households are equal.<sup>8</sup> We use this assumption because it is not essential for the qualitative nature of our results but makes the mathematics manageable.

## 4 Comparative Statics

We will now turn to the decision problem of the government managing the pension system. We assume that it seeks to maximize the utilitarian welfare of

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<sup>6</sup>An economy is large, if the individual neglects the effects of its own fertility decision for the calculation the population growth rate.

<sup>7</sup>From our assumptions and the existence theorem by Nash (1950) we know that an equilibrium exists.

<sup>8</sup>This assumption is, however, restrictive because even in the case of identical households in terms of utility functions and strategy spaces, asymmetric equilibria might occur.

the living generation by the choice of the contribution rate  $\tau$  and the structural parameter  $\alpha$ . This criterion differs from the criteria most commonly found in the literature. We will therefore give some intuition for its normative status.

## 4.1 Normative Criteria in Population Economics

Any detailed review of population ethics would be far beyond the scope of this paper. Therefore, we must restrict attention to some principle remarks. Normative criteria to evaluate policy changes commonly used in static problems are not well-defined in intertemporal models of endogenous population. For example the Pareto criterion rests on the condition that the number and identity of individuals is unaffected by the choice of policy. Several authors (for example Blackorby and Donaldson (1984), Dasgupta (1994) or Razin and Sadka (1995)) developed different attempts to understand this problem. The differences in their argumentations point to a - so far underestimated - weakness of normative economics based on the concept of methodological individualism, or to be more precise, welfarism: A collective ordering based on individual orderings can not take into consideration the “preferences” of unborn individuals because these preferences do not exist. Any assignment of preferences to potential individuals must be due to the idea of future interests of the generation *currently alive*. In this respect, any problem of “intergenerational” preference aggregation is in fact a problem of intragenerational preference aggregation. It is impossible to assign ethical rights to potential persons that are not developed from the point of view of the present generation. This impossibility has a purely logical status. But if this is so, these considerations must be represented in the individual orderings of alternatives of this generation.<sup>9</sup> Therefore we will restrict attention to the optimal policy for the working generation currently alive.

## 4.2 Structural Changes Within the PAYG

The optimisation problem analysed in section 2.3 gave rise to a perfectly symmetric Nash equilibrium  $[c_2^t(\tau, \alpha), c_3^t(\tau, \alpha), n^{t+1}(\tau, \alpha)]$  and an indirect utility function  $v = V(\tau, \alpha)$ .

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<sup>9</sup>For a detailed derivation of this argument see Kolmar and Stolte (1996).

We will first analyse changes within the PAYG. In order to do so we assume that the government maximises a utilitarian social choice function of individuals<sup>10</sup> living in  $t$ ,  $W(\alpha, \tau) = \sum_{i=1}^{N^t} V_i(\tau, \alpha)$ . The first derivative of  $W$  with respect to  $\alpha$  is:

$$W_\alpha = \sum_{i=1}^{N^t} \mu_i \tau w_{2i}^{t+1} \left( (1 + n_i^{t+1}) - (1 + \bar{n}^{t+1}) + (1 - \alpha) \frac{\partial(1 + n_{-i})}{\partial \alpha} \right) \quad (12)$$

Using the symmetry of the equilibrium, the condition reduces to

$$W_\alpha = \underbrace{\mu \tau w_2^{t+1} (1 - \alpha) (N^t - 1)}_{>0} \frac{\partial(1 + \bar{n})}{\partial \alpha} \quad (13)$$

The total effect on generational utility depends on the equilibrium change of the population growth rate. The calculation of this term can be found in Appendix A. Let  $H$  be the bordered Hessian matrix of the above problem.

**Lemma 1** *In a perfectly symmetric Nash-equilibrium, a change in the pension structure in the direction of a CPAYG increases per-capita indirect utility, and therefore generational welfare, iff the population growth rate increases.*

**Result 2** *For the class of utility functions with zero cross derivatives, an increase in  $\alpha$  increases indirect utility if  $\det(H)$  is positive. If the determinant of  $H$  is negative, an increase in  $\alpha$  reduces indirect utility.*

A sufficient condition for  $\det(H)$  being positive is that  $H$  is definite. This is a usual assumption in economic maximisation problems. If it is accepted, we can conclude that a pure CPAYG can be expected to be welfare maximizing. What is the intuition for this result? In an IPAYG pension system, the future benefits of children are distributed to all individuals of the society whereas the costs of children are borne entirely by the individual family. This fiscal

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<sup>10</sup>We neglect the currently old generation in the formulae because changes in  $\alpha$  leave their retirement income unchanged.

externality is negative for the above class of problems and can be reduced if pension payments are calculated on a more child-oriented basis. This implies that a pure CPAYG is always preferable to a pure IPAYG and any mixed system. If the fiscal externality is positive ( $\det(H)$  is negative), a pure IPAYG can be expected to be welfare maximizing.

Now, if the optimisation problem led to a corner solution initially,  $W_\alpha = 0$  because in this case, maximum fertility will be realized and remains unchanged for at least a small neighborhood of  $\alpha$ . Therefore, 1) changes in the pension structure are locally neutral in a corner solution, and 2) if fertility is exogenous, the structural parameter  $\alpha$  has no influence on economic behavior.

### 4.3 Structural Changes of the PAYG

In the following we will analyse changes in the contribution rate. In the light of the last section we will restrict attention to the two polar cases of IPAYG and CPAYG. Whereas income based pension systems are a prominent form in most modern western countries<sup>11</sup> you will find CPAYG in some rural societies and countries where a large part of the population is excluded from the capital market and a public-pension fund can not be credibly implemented.

As was already mentioned in (3.2), the main difference between the two systems is the magnitude of the price effect on child rearing. This effect will turn out to be crucial in the following analysis. For a mixed system, the first derivative of  $W$  with respect to  $\tau$  is:

$$W_\tau = \mu N^t \left( (w_2^{t+1}(1 + \bar{n}) - (1 + r^{t+1})w_2^t) + w_2^{t+1}(1 - \alpha) \frac{1}{N^t} \frac{\partial(1 + n)}{\partial \tau} \right) \quad (14)$$

The equation divides into two terms: The first term is the well-known (*local*) Aaron condition. If you expand this part by  $\frac{w_2^t}{w_2^t}$  and let  $\gamma^{t+1}$  be the growth rate of wage payments, it becomes:

$$(1 + \gamma^{t+1})(1 + n_i^*) - (1 + r^{t+1})$$

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<sup>11</sup>In a model with identical individuals and exogenous income, minimum-income systems of the Beveridge type and insurance systems of the Bismarck type can both be identified as IPAYG.



It states that with endogenous fertility and neglecting the second term, a PAYG is individually preferable to a FF, if the dynastic growth factor of income exceeds the interest factor of the capital market.

The second term measures the increase in welfare (or utility because of the assumption of identical behavior and symmetric equilibrium) due to a change in population growth.

We can now compare both pure PAYG systems. For a CPAYG (14) reduces to the (*local*) Aaron condition:

$$W_\tau = \mu N^t (w_2^{t+1}(1 + \bar{n}) - (1 + r^{t+1})w_2^t) \quad (15)$$

On the other hand, for a pure IPAYG we get:

$$W_\tau = \mu N^t \left( (w_2^{t+1}(1 + \bar{n}) - (1 + r^{t+1})w_2^t) + w_2^{t+1} \frac{1}{N^t} \frac{\partial(1 + n)}{\partial \tau} \right) \quad (16)$$

We will first analyse pure CPAYG systems:

**Result 3** *For marginal changes in  $\tau$ , a CPAYG is preferable to a FF system if the individual local Aaron condition is fulfilled.*

This result demonstrates the validity of Aaron-based comparisons in models of exogenous population growth. For discrete changes in  $\tau$ , however, it must be *assumed* that this local condition is globally fulfilled. As the next result will show, this assumption is misleading because the indirect utility function is U-shaped in the space of contribution rates:

**Result 4** *1. The optimisation problem of the government for changes in  $\tau$  has no interior solution. All interior extrema are minima. 2. For plausible parameter values, a CPAYG will always lead to a higher per-capita utility than a FF system.*

The proof of this result can be found in Appendix B. The intuition for the result is straightforward. To fix ideas we will talk about total consumption  $C = c_2^t + \frac{1}{1+r^{t+1}}c_3^{t+1}$  in the following.<sup>12</sup>

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<sup>12</sup>This aggregation is possible because the conditions for the validity of the Hicks-Leontief theorem are fulfilled.

Changing  $\tau$  rotates the budget line of the household through a fixed point  $F$ . Every point different from  $F$  can not be an extremum. Using a revealed-preference argument, at  $F$ , changes in  $\tau$  will increase utility in both directions. Therefore, an interior solution must be a minimum. A similar argument holds if  $F$  lies outside the positive orthant of  $R^3$ . Again, this finding shows that either pure fully funded systems are optimal, or the PAYG should redistribute as much income as possible.

There are two situations for which a child-related pension system as modelled above can be applied:

First, as was already mentioned, this is the relevant form of old-age insurance in rural societies. Our result opposes the widespread belief that the development of a perfect capital market in a credit-constrained economy will reduce fertility. This belief can not be deduced from purely theoretical considerations. On the contrary, a small country may as well increase population growth because it is no longer restricted to an aggregate foreign debt of zero.

Second, a CPAYG can be seen as a model of a representative household taking into account the full demographic effect of changes in parameters. In a small, open economy, the only effect of pension systems is on demographic variables. Factor prices remain unchanged. This makes it possible to increase population growth beyond the golden rule level and, therefore, to create a dynamically inefficient time path. In a closed economy with exogenous population growth, the introduction of a PAYG may be an instrument to make the economy dynamically efficient by reducing the amount of savings on capital markets and therefore, increasing interest rates. For a small open economy, this rationale is reversed. The possibility to increase fertility beyond the golden-rule level without influencing the factor prices creates an incentive to finance consumption with foreign credits without increasing per-capita debt beyond any limit. The security for this debt is the large future population. The introduction of a pension system *creates* the population growth rate necessary to make it preferable to a fully funded system. The Ponzi-like character of this solution can be seen if we look at the individual savings for a contribution rate of  $\tau = 1$ ,  $s_2^t = -(1 + n^{t+1})x - c_2^t$ , which is negative, the country is a net

borrower. This implies that, if the world economy is dynamically efficient, in the limit the country has a positive per-capita stock of foreign debt. But this is a condition for a Ponzi game.

We will now turn attention to pure IPAYG systems:

As can be seen from (16), the local average population-growth rate must be completed by a term measuring the change in utility due to changes in equilibrium population growth. If population growth increases by a marginal unit, every household gets an additional income that is equal to the share of additional future income that is distributed to the household. this increase is evaluated according to  $\mu$ .

Again, for the empirically relevant case of a large economy, this effect becomes negligible. In this case the local Aaron condition holds.

**Result 5** *For an IPAYG, the local average Aaron condition does not hold as a measure for increases in utility. It must be corrected by a term measuring the individual value of an increase in population growth.*

As can be expected and as it is demonstrated in Appendix A, the sign of  $\frac{\partial(1+n)}{\partial\tau}$  can not be deduced without further restrictive assumptions on individual preferences.

Again we complete the argument with the analysis for the case of corner solutions. As was mentioned above, for a corner solution, fertility is unchanged for small changes in the parameter ( $\tau$ ). Therefore,  $W_\tau = N^t\mu((1+n)w_2^{t+1} - (1+r)w_2^t)$ . The Aaron condition holds for any  $\alpha \in [0, 1]$ , we are back to the standard result of the public pension literature with exogenous population growth.

## 5 Summary

What are the main findings and implications of the above analysis? First, introducing endogenous fertility allows us to get deeper into the micro structure of PAYG systems. The determination of pension payments - child or income

related - make a difference in the behavior of economic agents, and thus the performance of the pension system. The existence of a substitution effect present in CPAYG systems induces a net fiscal externality that can be expected to be positive. Calculating the optimal mixture between both principles holding the contribution rate constant shows that only pure systems are optimal. A pure CPAYG is always preferable to any other system if the bordered Hessian matrix meets the condition of definiteness. This is due to two counterbalancing fiscal externalities present in the IPAYG. The costs of bringing up a child are reduced if a PAYG exists. This reduction is itself reduced in an IPAYG. If the first externality is positive, a pure CPAYG should be implemented. This effect holds even though IPAYG does not change relative prices compared to a fully funded system.

Asking for the optimal contribution rate of a PAYG we open the discussion whether PAYG or fully funded systems lead to a higher utility level. The main finding is that the commonly used Aaron condition is a misleading indicator for a comparison. For a CPAYG this criterion holds only as a local condition. Furthermore, there exists no interior solution for the determination of an optimal contribution rate. Again, either purely funded or purely pay-as-you-go financed systems are optimal. For a large number of economically meaningful parameter values, a fully funded systems always leads to a smaller level of utility. For an IPAYG, the local Aaron condition must be expanded by a term measuring the change in income due to a change in equilibrium population growth.

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## A Comparative Statics

In this appendix we calculate the change in population growth  $n$  according to a change in  $\alpha$  and  $\tau$ . If possible we suppress time indices. The first order conditions of the maximisation problem of an individual household ( $FOC^i$ ) are for an interior solution:

$$\begin{aligned}
 u_2^i - \lambda^i &= 0 \\
 u_3^i - \mu^i &= 0 \\
 u_n^i - \lambda^i x + \mu^i \tau w_2^{t+1} \left( \alpha + \frac{1}{N}(1 - \alpha) \right) &= 0 \\
 -\lambda^i + (1 + r)\mu^i &= 0 \\
 (1 - \tau)w_2^t - s^i - (1 + n^i)x - c_2^i &= 0 \\
 (1 + r)s^i + \tau w_2^{t+1} \left( \alpha + \frac{1}{N^t}(1 - \alpha) + (1 - \alpha)\bar{n}^{-i} \right) - c_3^i &= 0
 \end{aligned}$$

The Nash equilibrium is defined as a vector  $[c_2, c_3, n]$  for which:

$$\begin{aligned}
 FOC^1 &= 0 \\
 FOC^2 &= 0 \\
 &\dots \\
 FOC^{N^t} &= 0
 \end{aligned} \tag{A.1}$$

We assume the existence of a unique and symmetric equilibrium. The comparative static system for a change of the Nash-equilibrium according to changes in  $\zeta = \alpha, \tau$  is:

$$\begin{aligned}
 FOC_{c_2^1}^1 dc_2^1 + FOC_{c_3^1}^1 dc_3^1 + FOC_{n^1}^1 dn^1 + FOC_{n^{-1}}^1 dn^{-1} &= -FOC_{\zeta}^1 d\zeta \\
 FOC_{c_2^2}^2 dc_2^2 + FOC_{c_3^2}^2 dc_3^2 + FOC_{n^2}^2 dn^2 + FOC_{n^{-2}}^2 dn^{-2} &= -FOC_{\zeta}^2 d\zeta \\
 &\dots \\
 FOC_{c_2^N}^N dc_2^N + FOC_{c_3^N}^N dc_3^N + FOC_{n^N}^N dn^N + FOC_{n^{-N}}^N dn^{-N} &= -FOC_{\zeta}^N d\zeta
 \end{aligned} \tag{A.2}$$

with  $FOC_x^i$  being the derivative of the first order system of individual  $i$  with respect to  $x$ . Using the symmetry of the Nash equilibrium, the system can be substantially simplified:

$$FOC_{c_2}dc_2 + FOC_{c_3}dc_3 + (FOC_n + FOC_{n-i})dn = FOC_\zeta d\zeta \quad (A.3)$$

Calculating the above system for a representative individual and changes in  $\alpha$  and  $\tau$ , we get:

$$[H] \begin{bmatrix} \frac{\partial c_2^t}{\partial \alpha} \\ \frac{\partial c_3^{t+1}}{\partial \alpha} \\ \frac{\partial n^{t+1}}{\partial \alpha} \\ \frac{\partial s_2^t}{\partial \alpha} \\ \frac{\partial \lambda}{\partial \alpha} \\ \frac{\partial \mu}{\partial \alpha} \end{bmatrix} = [B_\alpha] = [B_\tau] \quad (A.4)$$

With

$$[H] = \begin{bmatrix} U_{22} & 0 & 0 & 0 & -1 & 0 \\ 0 & U_{33} & 0 & 0 & 0 & -1 \\ 0 & 0 & U_{nn} & 0 & -x & \tau w_2^{t+1} \left( \alpha + \frac{1-\alpha}{N^t} \right) \\ 0 & 0 & 0 & 0 & -1 & (1 + r^{t+1}) \\ -1 & 0 & -x & -1 & 0 & 0 \\ 0 & -1 & \tau w_2^{t+1} & (1 + r^{t+1}) & 0 & 0 \end{bmatrix}$$

$$[B_\alpha] = \begin{bmatrix} 0 \\ 0 \\ -\mu \tau w_2^{t+1} \left( 1 - \frac{1}{N^t} \right) \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad [B_\tau] = \begin{bmatrix} 0 \\ 0 \\ \mu w_2^{t+1} \left( \alpha + \frac{1-\alpha}{N^t} \right) \\ 0 \\ w_2^t \\ w_2^{t+1}(1 + n) \end{bmatrix}$$

for changes in  $\alpha$  and  $\tau$  respectively. The partial derivative of  $n$  with respect to  $\zeta$  is:

$$\frac{\partial n^{t+1}}{\partial \zeta} = \frac{\det(Hn)}{\det(H)} \quad (A.5)$$

With  $H$  being the first matrix on the left-hand side of the above system and  $Hn$  being the same matrix where the third row is replaced by the vector on the right-hand side of the system. The determinant of  $H$  is:



$$\begin{aligned} \det(H) = & (w_2^{t+1}(w_2^{t+1}(\tau^2\alpha N + (1-\alpha)) - (1+r)\tau x((1-\alpha) + (1+\alpha)N)) \\ & + (1+r)^2 N x^2) \frac{u_{22}u_{33}}{N} + u_{22}u_{nn} + (1+r)^2 u_{33}u_{nn} \end{aligned} \quad (\text{A.6})$$

Unfortunately, the sign of  $\det(H)$  can not be determined unambiguously. The second and third terms are positive whereas the first term may be positive or negative. For  $H$  being definite, however,  $\det(H)$  is positive.

The determinant of  $Kn$  for variations in  $\alpha$  is:

$$\det(Kn)_\alpha = -((1+r)^2 U_{33} + U_{22}) \tau w_2^{t+1} \frac{N^t - 1}{N_t} > 0 \quad (\text{A.7})$$

Therefore, for  $H$  being definite or in all other cases where  $\det(H) > 0$  we get:

$$\frac{\partial(1+n^{t+1})}{\partial\alpha} = \frac{\det(Kn)}{\det(H)} > 0, \quad (\text{A.8})$$

increasing  $\alpha$  increases the population growth rate. In all other cases, the opposite result holds.

By the same principles as above we get for changes in  $\tau$ :

$$\frac{\partial n^{t+1}}{\partial\tau} = \frac{\det(Kn)}{\det(H)} \quad (\text{A.9})$$

with

$$\begin{aligned} \det(Kn)_\tau = & \frac{(1+r)u_{33}^2}{N} (((1+n)w_2^{t+1} + (1+r)w_2^t)(w_2^{t+1}\tau\alpha - (1+r)x) \\ & + w_2^{t+1}\tau(1+n)(w_2^{t+1}(1-\alpha) + (1+r)w_2^t)) \\ & + \frac{\alpha N + (1-\alpha)}{N} (w_2^{t+1}\mu(u_{33}(1+r)^2 + u_{22})) \end{aligned} \quad (\text{A.10})$$

This determinant may be either positive or negative implying that we can not determine the sign of  $\frac{\partial(1+n)}{\partial\tau}$ .<sup>13</sup> The second and first terms are positive whereas the first term may be negative. The higher  $\tau$  and  $\alpha$ , the more likely will this term be positive.

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<sup>13</sup>For  $u$  being quasi-linear in  $c_2$  or  $c_3$  the determinant must be negative because the income effect is absorbed by the linear term.

## B Derivation of $W_\tau$ for an IPAYG.

First we calculate the contribution rate for which  $W_\tau$  is zero:  $\mu(w_2^{t+1}(1 + n_i^{t+1}) - (1 + r^{t+1})w_2^t) = 0$ . For  $\mu > 0$  we get  $(1 + n^{t+1}) = \frac{(1 + r^{t+1})w_2^t}{w_2^{t+1}}$ . This condition defines a straight line in  $R_+^3$  through which all budget planes defined for different  $\tau$  must rotate. We can look at the intertemporal budget constraint of a household working in  $t$ :

$$\underbrace{(1 + r^{t+1})c_2^t + c_3^{t+1}}_{=C} + \underbrace{((1 + r^{t+1})x^t - \tau w_2^{t+1})}_{=P}(1 + n^{t+1}) = \underbrace{(1 + r^{t+1})(1 - \tau)w_2^t}_{=M}$$

Changes in  $\tau$  have two different effects: The effective costs of children ( $P$ ) and the disposable income ( $M$ ) change. One can show that every budget plane rotates through a point defined by<sup>14</sup>:

$$(1 + n^f) = \frac{w_2^t}{w_2^{t+1}}(1 + r^{t+1}) \quad , \quad C^f = (1 + r^{t+1})w_2^t \left(1 - \frac{(1 + r^{t+1})x^t}{w_2^{t+1}}\right) \quad (\text{B.11})$$

In other words, the FOC is zero in the point of rotation. We will now check the SOC at that point:

$$\frac{\partial^2 W}{\partial \tau^2} = N^t \mu \frac{\partial(1 + n)}{\partial \tau} w_2^{t+1} \quad (\text{B.12})$$

The sign of this condition depends on the derivative of  $n$  with respect to  $\tau$ .

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<sup>14</sup>For an effective price for children of 0 we get  $C = (1 - \tau)(1 + r^{t+1})w_2^t$  and  $\tau = \frac{(1 + r^{t+1})x^t}{w_2^{t+1}}$ . Thus,  $C^f = (1 + r^{t+1})w_2^t \left(1 - \frac{(1 + r^{t+1})x^t}{w_2^{t+1}}\right)$ . If there is any fixed point, one coordinate is defined by this consumption vector. For an arbitrary  $\tau$  we must get for  $k$ :

$$k = \frac{(1 + r^{t+1})(1 - \tau)w_2^t - (1 + r^{t+1})w_2^t + \frac{((1 + r^{t+1}))^2 x^t w_2^t}{w_2^{t+1}}}{(1 + r^{t+1})x^t - \tau w_2^{t+1}}, \text{ and after some simplifications}$$

$$k^f = \frac{w_2^t}{w_2^{t+1}}(1 + r^{t+1}).$$

Using the Slutsky decomposition we get:<sup>15</sup>

$$\begin{aligned}
\frac{\partial(1+n^{t+1})}{\partial\tau} &= \frac{\partial(1+n^{t+1})}{\partial P} \frac{\partial P}{\partial\tau} + \frac{\partial(1+n^{t+1})}{\partial M} \frac{\partial M}{\partial\tau} \\
&= -w_2^{t+1} \left[ \frac{\partial(1+n^{t+1})}{\partial P} \right]_{\bar{u}} \\
&\quad + \left( \underbrace{(1+n^{t+1})w_2^{t+1} - w_2^t(1+r^{t+1})}_{=0} \right) \frac{\partial(1+n^{t+1})}{\partial M}.
\end{aligned}$$

Because the effect of substitution is always non-positive, every interior extremum must be a minimum.

Consequently, either  $\tau = 0$  or  $\tau = 1$  must be the maxima of the utility function.  $(1+n^f)$  always lies inside the positive orthant of the  $(C, (1+n))$ -space.  $C^f$  is inside the positive orthant if  $w_2^{t+1} > (1+r^{t+1})x^t$ , the future direct income of one person exceeds the discounted direct costs of bringing up this person. Let this condition be fulfilled. Let  $C^{(1+n=0)}$  be the maximum consumption for a given  $\tau$ . If  $C^{(1+n=0)} < C^f$ , the projection of the budget plane is positively sloped. This condition can be rearranged:

$$(1+r^{t+1})(1-\tau)w_2^t < (1+r^{t+1})w_2^t \left( 1 - \frac{(1+r^{t+1})x^t}{w_2^{t+1}} \right) \quad (\text{B.13})$$

$$\Leftrightarrow \tau > x^t \frac{(1+r^{t+1})}{w_2^{t+1}} < 1 \quad (\text{B.14})$$

Therefore, for  $\tau = 1$  the budget line has a positive slope. A sufficient condition for  $\tau = 1$  being optimal can be derived if the budget restriction for both,  $\tau = 0$  and  $\tau = 1$  are compared:

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<sup>15</sup>The general form of the Slutsky equation is for  $x_i$  being an element of  $\{c_2^t, c_3^{t+1}, n^{t+1}\}$ :

$$\frac{\partial x_i}{\partial\tau} = \frac{\partial x_i}{\partial M} \left( \frac{\partial M}{\partial\tau} - \sum_{j=1}^3 x_j \frac{\partial p_j}{\partial\tau} \right) + \sum_{j=1}^3 \left[ \frac{\partial x_i}{\partial p_j} \right]_{\bar{u}} \frac{\partial p_j}{\partial\tau}$$

This expression simplifies to:

$$\frac{\partial x_i}{\partial\tau} = \frac{n-r}{1+r} \frac{\partial x_i}{\partial M} - \frac{w_2}{1+r} \left[ \frac{\partial x_i}{\partial P} \right]_{\bar{u}}$$

$$\hat{A} > (1 + r^{t+1}) \frac{x}{w_2^t} (\hat{n} - n^0) \quad (\text{B.15})$$

With  $(1 + n^0)$  being the number of children for  $\tau = 0$  and  $\hat{A}$  being the Aaron condition evaluated at the maximum population growth.

Figure 1:

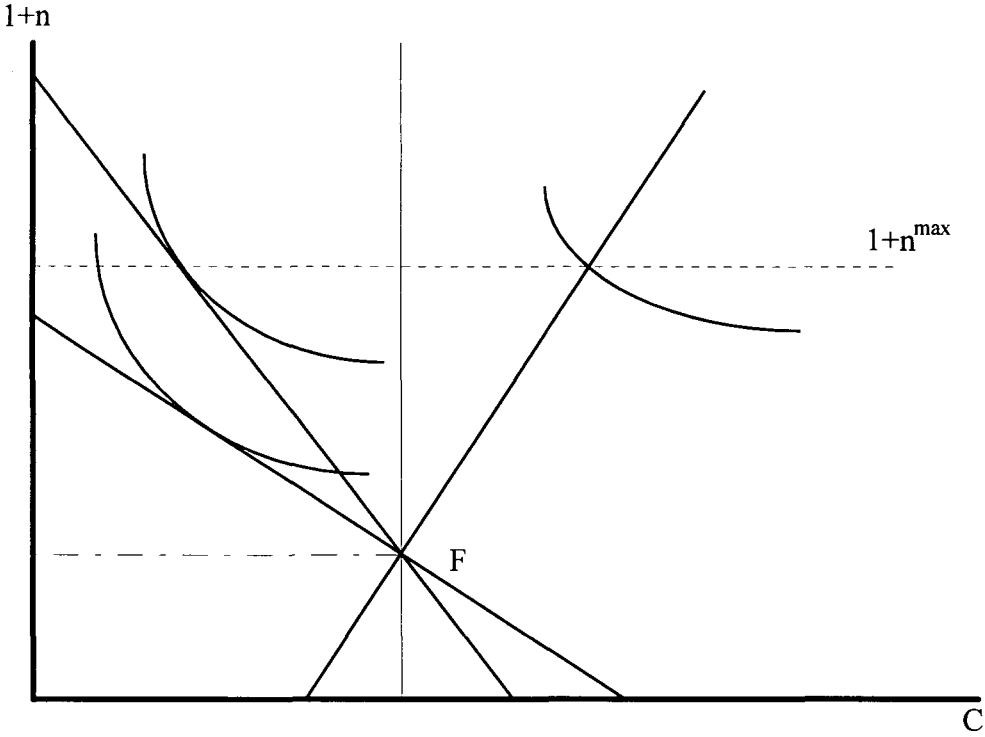


Figure 2:

