

Broll, Udo; Zilcha, Itzhak

Working Paper

International production, investments and borrowing with exchange rate risk and futures markets

Diskussionsbeiträge - Serie II, No. 148

Provided in Cooperation with:

Department of Economics, University of Konstanz

Suggested Citation: Broll, Udo; Zilcha, Itzhak (1991) : International production, investments and borrowing with exchange rate risk and futures markets, Diskussionsbeiträge - Serie II, No. 148, Universität Konstanz, Sonderforschungsbereich 178 - Internationalisierung der Wirtschaft, Konstanz

This Version is available at:

<https://hdl.handle.net/10419/101716>

Standard-Nutzungsbedingungen:

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

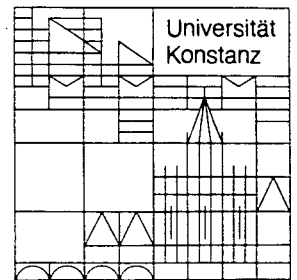
Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

Terms of use:

Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.



Sonderforschungsbereich 178
„Internationalisierung der Wirtschaft“

Diskussionsbeiträge

Juristische
Fakultät

Fakultät für Wirtschafts-
wissenschaften und Statistik

Udo Broll
Itzhak Zilcha

**International Production,
Investments and Borrowing
with Exchange Rate Risk
and Futures Markets**

30. AUG. 1991 Weltwirtschaft
Kiel

Postfach 5560
D-7750 Konstanz

Serie II — Nr. 148
August 1991

**INTERNATIONAL PRODUCTION, INVESTMENTS AND BORROWING
WITH EXCHANGE RATE RISK AND FUTURES MARKETS**

Udo Broll
Itzhak Zilcha

Serie II - Nr. 148

August 1991

International Production, Investments and Borrowing with Exchange Rate Risk and Futures Markets

By

Udo Broll *and* Itzhak Zilcha

Department of Economics, University of Konstanz, FRG

Department of Economics, University of Tel-Aviv, Israel

Abstract

In this paper we consider a multinational firm under exchange rate risk in a multiperiod model. We analyze the impact of exchange rate uncertainty and the use of currency futures on the risk-averse firm's decisions about home and foreign production. Without any markets for hedging an increase in exchange rate risk lowers foreign investments and output. However, when futures markets exist, the "separation property" holds. Introducing another source of uncertainty, such as the interest rate, which is nondiversifiable, affects the production in both countries, i.e., the separation property does not hold. However, we show that the adverse effect of the missing financial market (to hedge against this additional risk) disappears when international borrowing is available.

JEL: 411, 431

Keywords: international production, investments, exchange rate risk, futures markets.

International Production, Investments and Borrowing with Exchange Rate Risk and Futures Markets

1 Introduction

In recent years multinational firms became the most important actors in international trade and international capital movements. Their involvement in multi-country production has grown significantly and there is evidence indicating that multinational firms are responsible for a significant share of foreign direct investments (see, for example, SHAPIRO (1991)).

We shall study the economic behavior of a risk-averse firm producing in two countries facing random exchange rate and (later) random domestic interest rate as well. Since capital investments are usually made long before production takes place and since the allocation of outlays and incomes over time is important to the firm we shall use a two-period framework although production takes place in the second period only. We assume, in most parts of this paper, that the firm finances its capital investments by its own resources and by borrowing in the domestic capital market. Later we examine the effect of the availability of international borrowing.

Due to the high volatility of exchange rates in the last two decades each firm involved in a multi-country production and exports experiences a considerable level of uncertainty regarding its future profit from international trade and foreign direct investments.¹ A risk-averse multinational firm will respond to exchange rate risks by adjusting its production decisions and by taking hedging opportunities into account. These considerations have led to

¹See e.g. CUSHMAN (1988), KRUGMAN (1989), FRANKE (1991).

the intensified use of risk sharing markets such as currency futures markets as well as currency options markets.

Recently there have been many contributions to the theory of exporting firms under stochastic exchange rates where the effects and the role of hedging markets have been analyzed.² One important result of these studies is that in the presence of currency futures markets the exporting firm will choose its output as if it were in a position of certainty. Namely, output is determined regardless of the distribution of the random exchange rate and regardless of its own attitude towards risk. This property is known as the "separation property" and was first proved by DANTHINE (1978) for a competitive firm under price uncertainty.

In this paper we analyze in a two-period model the economic implications of currency futures markets on a risk-averse firm producing at home and abroad and selling abroad. This multinational firm chooses the levels of capital investments as well as loan contracting before the exchange rate is known. All these decisions must be made at an earlier stage called date 0. Production (in both places) takes place in date 1, while the decisions about labor inputs and contracting in the currency futures market (whenever it exists) are made at the beginning of period 1 before the end-of-period exchange rate becomes known.

In *section 2* we present the model and analyze the effects of exchange rate uncertainty on production. In *section 3* we prove that foreign direct investment and trade decrease with higher volatility of the exchange rate if risk sharing markets do not exist. *Section 4* deals with international production decisions in the presence of currency futures markets. It is shown that a Separation theorem holds, i.e., foreign and domestic investment and production are determined independently of the firm's attitude towards risk and of the distribution of the random exchange rate. In *section 5* we show that if the

²See, for example, BENNINGA, ELDOR and ZILCHA (1985), ELDOR and ZILCHA (1987), BROLL and ZILCHA (1991).

interest rate is a random variable then the separation property does not hold. Later, in *section 6* we relax the restriction that the firm can borrow only in the home country. As a result, it is shown that: (a) When both interest rates are nonrandom the Separation property holds even if currency futures markets are missing. Namely, international borrowing can substitute for the missing futures markets in our model and thus restore the Separation property. (b) If the domestic interest rate is random then in the presence of currency futures and international borrowing the Separation property holds. Thus, once again we see that allowing the firm to borrow abroad plays an important role in achieving better conditions for production under multiple-sources of uncertainty.

2 Effects of Exchange Rate Risk

Suppose that a horizontally integrated multinational firm produces in two countries and sells only abroad. The firm has firm-specific advantages that enable it to operate as a monopolist in the foreign country (see HELPMAN (1984), CALDERON-ROSSELL (1985), ITAGAKI (1991)). The multinational firm must choose the capital investments K and K^* in the first period, or the planning period, while the labor applied in each country, L and L^* , is determined at the outset of the second period, when production takes place but *before* the exchange rate is known.³

Sales take place in the foreign country only, hence the realization of the random exchange rate \tilde{e}_1 determines the total revenue in domestic currency. All capital, domestic or foreign, is financed at home, so foreign capital acquisitions constitute foreign direct investments.⁴ We assume perfect capital markets, i.e., the multinational firm can either borrow or lend at the prevailing interest rate. The firm is risk-averse and maximizes expected utility of

³An asterisk * indicates a foreign variable.

⁴Note that international borrowing is introduced in *Section 6* only.

profits denominated in domestic currency. Denote by U its von Neumann–Morgenstern utility function for income at each period and assume that $U' > 0$, $U'' < 0$. The firm is a price taker in the inputs markets and denote by w, w^* and r, r^* the corresponding wages and capital prices. The firm has some monopoly power in the foreign market where it sells this product and let $R(X + X^*)$ be the total revenue function where X, X^* are the home and foreign production levels. We assume that R is continuously differentiable and concave.

The production processes at home and abroad are described by

$$X = F(K, L) \quad \text{and} \quad X^* = F^*(K^*, L^*).$$

We assume that the production functions exhibit constant returns to scale (i.e., homogeneous of degree one), concave and satisfy $F_{KL} > 0$, $F_{K^*L^*}^* > 0$. Let the exchange rate at $t = 0$ be e_0 and at $t = 1$ it is a random variable \tilde{e}_1 with a known distribution. Currently this is the only source of uncertainty. Denote by I the firm's initial wealth, with B the loans made by the firm, at domestic banks, at date 0; i the domestic interest rate and δ the subjective time preference of the firm. The firm's incomes at dates 0 and 1 are given by:

$$\begin{aligned} W_0 &= I - e_0 r^* K^* - rK + B \\ \tilde{W}_1 &= [R(X + X^*) - w^* L^*] \tilde{e}_1 - wL - (1 + i)B. \end{aligned}$$

The firm chooses input levels and loan contracting by maximizing the sum of expected utilities:

$$\text{Max}_{(B, K^*, K, L^*, L)} [U(W_0) + \delta EU(\tilde{W}_1)]. \quad (1)$$

We assume throughout the paper that the distribution of \tilde{e}_1 the production functions and the factor prices are such that the firm produces in both countries. The comparison between certainty and uncertainty is done through

the so-called *certainty equivalent* case, namely, as the random spot exchange rate \tilde{e}_1 is replaced by the expected exchange rate $E\tilde{e}_1 = \bar{e}_1$. To compare the firm's behavior under uncertain \tilde{e}_1 with the case where \bar{e}_1 is the exchange rate let us write the first order conditions for (1), which are necessary and sufficient conditions for optimum due to the concavity of U .

$$U'(W_0) - \delta(1+i)EU'(\tilde{W}_1) = 0 \quad (2)$$

$$-e_0r^*U'(W_0) + \delta E[\tilde{e}_1 \frac{\partial R}{\partial K^*} U'(\tilde{W}_1)] = 0 \quad (3)$$

$$\delta(\frac{\partial R}{\partial L^*} - w^*)E\tilde{e}_1 U'(\tilde{W}_1) = 0 \quad \text{or} \quad \frac{\partial R}{\partial L^*} = w^* \quad (4)$$

$$E(\tilde{e}_1 \frac{\partial R}{\partial L} - w)U'(\tilde{W}_1) = 0 \quad (5)$$

$$-rU'(W_0) + \delta E[\tilde{e}_1 \frac{\partial R}{\partial K} U'(\tilde{W}_1)] = 0. \quad (6)$$

From (6) and (3) we derive that

$$\frac{\partial R/\partial K^*}{\partial R/\partial K} = \frac{e_0r^*}{r}. \quad (7)$$

From (2), (5) and (6) we obtain,

$$\frac{\partial R/\partial L}{\partial R/\partial K} = \frac{w}{(1+i)r}. \quad (8)$$

However $Cov(\tilde{e}_1, U'(\tilde{W}_1)) < 0$ since U' is a decreasing function hence equation (5) implies that

$$\bar{e}_1 \frac{\partial R}{\partial L} - w > 0. \quad (9)$$

Also from (2) and (3) due to $Cov(\tilde{e}_1, U'(\tilde{W}_1)) < 0$, we obtain

$$\bar{e}_1 \frac{\partial R}{\partial K^*} - e_0r^*(1+i) > 0. \quad (10)$$

But this also implies that

$$\bar{e}_1 \frac{\partial R}{\partial K} - r(1+i) > 0. \quad (11)$$

Since $R(\cdot)$ as a function of K, L, K^*, L^* is a concave function, conditions (4), (9), (10) and (11) prove the following proposition:

Proposition 1. *In the certainty equivalent case, i.e., when $\bar{e}_1 = E\tilde{e}_1$ is fixed, the firm uses more labor and capital in both countries compared to the random \tilde{e}_1 case. Thus production declines in both countries as a result of uncertainty.*

If multinational firms are risk-averse, then exchange rate uncertainty reduces foreign *and* home production when there are no risk sharing markets or equivalent insurance devices. To reduce exchange rate risk, the multinational firm "hedges" through less home and foreign investments thus lowering its risk exposure.

3 Higher Exchange Rate Volatility

In this section we would like to study the effects of higher exchange rate volatility on the firms' investments in both countries. We shall add here some specific assumptions about the utility function to be made only throughout this subsection.

Assumption (A.1): $U'(x)$ is a strictly convex function and $xU'(x)$ is strictly concave.

Assumption (A.1) holds, for example, when U is a Cobb-Douglas or logarithmic utility or when it is a quadratic utility function.

Now we prove.

Proposition 2. *Assume that (A.1) holds. If \tilde{e}'_1 is a mean preserving spread (MPS) of \tilde{e}_1 , then the optimum production levels of the firm under the riskier exchange rate \tilde{e}'_1 are lower in both countries, and all levels of inputs decline.*

Proof. Let us consider the necessary and sufficient conditions for optimum, i.e., equations (2)–(6). Let us keep B, K, K^*, L, L^* as in the optimum levels for \tilde{e}_1 and replace the random variable in equations (2)–(6) by the random

\tilde{e}'_1 (which is a MPS of \tilde{e}_1). Since U' is strictly convex while $xU'(x)$ is strictly concave then

$$EU'(\tilde{W}'_1)|_{\tilde{e}'_1} > EU'(\tilde{W}_1)|_{\tilde{e}_1} \quad (12)$$

$$E\tilde{e}'_1U'(\tilde{W}'_1)|_{\tilde{e}'_1} < E\tilde{e}_1U'(\tilde{W}_1)|_{\tilde{e}_1}. \quad (13)$$

Using (12) and (13) in the first-order conditions equations (where the optimal inputs B, K, K^*, L, L^* remain unchanged) replacing \tilde{e}_1 by \tilde{e}'_1 results in equation (4) and

$$U'(W_0) - \delta(1+i)EU'(\tilde{W}'_1) < 0 \quad (14)$$

$$-e_{or}U'(W_0) + \delta E[\tilde{e}_1 \frac{\partial R}{\partial K^*} U'(\tilde{W}'_1)] < 0 \quad (15)$$

$$E(\tilde{e}'_1 \frac{\partial R}{\partial L} - w)U'(\tilde{W}'_1) < 0 \quad (16)$$

$$-rU'(W_0) + \tilde{E}[\tilde{e}'_1 \frac{\partial R}{\partial K} U'(\tilde{W}'_1)] < 0. \quad (17)$$

Since the maximand in (1) is a strictly concave function of K, K^*, L, L^* and since at the optimum under \tilde{e}'_1 we have equalities in (14)–(17), we derive that: $K' < K, K^{*'} < K^*, L' < L, L^{*'} < L^*$, where the prime denotes the optimum under the random exchange rate \tilde{e}'_1 . This proves the proposition. ||

As a reaction to a higher exchange rate volatility the risk-averse multinational firm reduces foreign *and* home production activities to deal with the increase in uncertainty. Hence the introduction of hedging devices will have real effects on this multinational firm's investments and production.

4 Currency Futures Markets

The financial hedging instrument we shall introduce now is the currency futures contract. Note that when currency put and call options are available it is equivalent to the futures market. Consider now the behavior of the

multinational firm when currency futures markets are available. The firm is a price taker in the currency futures market where the futures price is e_f for one unit of currency at the end of period 1, if commitment is made in date 0. We assume that no payment is made as the firm undertakes the futures contract at date 0, hence all transactions take place at the end of period 1.

Denote by Z the multinational firm's sale (or purchase) in the currency futures market. In maximization (1) we have in this case,

$$\tilde{W}_1 = [R(X + X^*) - w^*L^*]\tilde{e}_1 - wL - (1 + i)B + Z(e_f - \tilde{e}_1) \quad (18)$$

where the firm chooses Z optimally as well. This adds to the first order conditions (2)–(6) the following condition:

$$E(e_f - \tilde{e}_1)U'(\tilde{W}_1) = 0. \quad (19)$$

Using (2), (3) and (19) we derive that,

$$\frac{\partial R}{\partial K^*} = \frac{e_0 r^*(1 + i)}{e_f}. \quad (20)$$

Using (3), (6) and (19) we find that

$$\frac{\partial R}{\partial K} = \frac{r(1 + i)}{e_f} \quad (21)$$

But conditions (5) and (19) imply that

$$\frac{\partial R}{\partial L} = \frac{w}{e_f}. \quad (22)$$

Optimality conditions in this case require that the multinational firm sets the value of the (foreign) marginal revenue of each factor equal to its factor rental rate, where e_f plays the role of the exchange rate. Thus the multinational firm uses the forward market to reduce the uncertainty of the spot exchange rate.

By examining (20), (21), (22) and (4) we find that the firm chooses in this case its optimum levels of K, K^*, L, L^* regardless of its utility function U and the distribution function of \tilde{e}_1 .

Definition: *We say that the Separation property holds if the production of the firm in both countries is independent of its attitude towards risk and independent of the exchange rate distribution function.*

Now we prove that the Separation property holds in the presence of currency futures markets.

Proposition 3. *When currency futures markets exist the multinational firm's production levels are determined by equations (4), (20), (21) and (22), hence its demand for inputs in each country (and hence its output) is independent of its attitude towards risk and the distribution of the random exchange rate.*

The separation property implies that the optimum decisions regarding trade (home production), foreign production and hedging decisions can be economically devided. Risk related elements such as risk aversion and exchange rate expectations affect only the level of hedging but not the quantities of international investments and production.

Clearly, the optimal level of hedging can be computed from equation (19) once we determine the optimal levels of inputs using equations (20), (21), (22) and (4). We should also note that in the case of unbiased futures market, i.e., when $e_f = E\tilde{e}_1$, the firm will hedge the exchange rate risk altogether. Namely, when futures exchange rate price is unbiased \tilde{W}_1 is nonrandom at the optimum. This follows directly from equation (19) since,

$$(e_f - E\tilde{e}_1)EU'(\tilde{W}_1) - Cov(\tilde{e}_1, U'(\tilde{W}_1)) = 0.$$

Thus $Cov(\tilde{e}_1, U'(\tilde{W}_1)) = 0$. But U' is a strictly decreasing function hence \tilde{W}_1 must be independent of the realization of \tilde{e}_1 at the optimum. Thus the optimal hedge in this case is $Z = R(X + X^*) - w^*L^*$.

5 Stochastic Interest Rate

Let us examine the impact of uncertainty about the interest rate on the loan B . A potential change in interest rates is a risk faced by almost any investor or firm in many small countries. Risky interest rates are observed in many countries with highly volatile inflation rate. There are examples where banks index the rate of interest to the consumer price index. We will begin with a discussion of several important implications of interest rate risk. First we explore the effect of such unhedged risk on the separation result. Second we analyze the implications of uncertainty about interest rate for international investments and production levels of the multinational firm. Throughout this discussion we shall assume that currency futures market exist.

Let us rewrite the optimization problem (1) for the case where \tilde{i} is a random variable with a given probability distribution,

$$Max_{(B,K,K^*,L,L^*,Z)} [U(W_0) + \delta EU(\tilde{W}_1)] \quad \text{s.t.} \quad (23)$$

$$W_0 = I - e_0 r^* K^* - rK + B \geq 0$$

$$\tilde{W}_1 = [R(X + X^*) - w^* L^*] \tilde{e}_1 - wL - (1 + \tilde{i})B + Z(e_f - \tilde{e}_1),$$

where the expectation is with respect to the joint distribution of (\tilde{e}_1, \tilde{i}) . Necessary and sufficient conditions for optimum are equations (3), (4), (5), (6), (19) and

$$U'(W_0) - \delta E(1 + \tilde{i})U'(\tilde{W}_1) = 0. \quad (24)$$

1. Separation property. It is easy to derive from equations (3)–(6), (19) and (24) the following relationships:

$$\frac{\partial R}{\partial L^*} = w^* \quad \text{and} \quad \frac{\partial R}{\partial L} = \frac{w}{e_f} \quad (25)$$

$$\frac{\partial R}{\partial K^*} = \frac{e_0 r^* E(1 + \tilde{i}) U'(\tilde{W}_1)}{E \tilde{e}_1 U'(\tilde{W}_1)} \quad (26)$$

$$\frac{\partial R}{\partial K} = \frac{r E(1 + \tilde{i}) U'(\tilde{W}_1)}{E \tilde{e}_1 U'(\tilde{W}_1)}. \quad (27)$$

Since, even in the case where \tilde{i} and \tilde{e}_1 are independent, $Cov(\tilde{i}, U'(\tilde{W}_1)) \neq 0$ whenever $B \neq 0$ we cannot express $\frac{\partial R}{\partial K^*}$ and $\frac{\partial R}{\partial K}$ as a function of $e_0 r^*, r, w, w^*, e_f$ only. This implies that the Separation property does not hold if the firm borrows or lends at the optimum. Moreover, even if at the optimum $B = 0$ but $Cov(\tilde{i}, \tilde{e}_1) \neq 0$ the Separation property does not hold.

Proposition 4. *Assume that the interest rate \tilde{i} is a random variable and that a currency futures market exists. The Separation property does not hold in this case.*

The proof is a direct result of equations (25)–(27).

2. *Foreign direct investments.* Let us compare the optimum values of inputs, under stochastic interest rate \tilde{i} with the certainty equivalent case, i.e., for $\bar{i} = E\tilde{i}$. Denote by $\hat{B}, \hat{K}, \hat{K}^*, \hat{L}, \hat{L}^*$ and \hat{Z} the solution for maximization (23) and by B, K, K^*, L, L^*, Z the optimum levels for the certain interest rate \bar{i} case. Let us assume that the firm borrows money during its investments period, i.e., $\hat{B} > 0$ and $B > 0$.

To simplify the proof of our results let us make the following technical assumption regarding the total revenue function R and the production functions F and F^* .

Assumption (A.2): The following functions $R(X + X^*), X = F(K, L)$ and $X^* = F^*(K^*, L^*)$ satisfy:

$$-\frac{R''(X + X^*)}{R'(X + X^*)} \leq \min \left(\frac{F_{KL}(K, L)}{F_K F_L}, \frac{F_{K^*L^*}^*(K^*, L^*)}{F_{K^*}^* F_{L^*}^*} \right).$$

This assumption holds in many well-known cases if R satisfies certain conditions. For example if $F(K, L)$ and $F^*(K^*, L^*)$ are Cobb–Douglas pro-

duction functions then (A.2) holds if:

$$-\frac{yR''(y)}{R'(y)} \leq 2.$$

In the next Proposition we assume that (A.2) holds although it is a much stronger assumption than what we need to prove this proposition.

Proposition 5. *Assume that (A.2) holds and that a currency futures market exists. Introducing uncertainty in the interest rate \tilde{i} and assuming that $Cov(\tilde{i}, \tilde{e}_1) = 0$ or $Cov(\tilde{i}, \tilde{e}_1) < 0$ results in a lower capital investments and lower outputs in both countries. In particular,*

$$\hat{K}^* < K^*, \hat{K} < K, \hat{L}^* < L^* \quad \text{and} \quad \hat{L} < L.$$

Hence Proposition 5 demonstrates that this nondiversifiable interest rate risk affects the production level at home and abroad in the presence of currency futures markets.

Proof. By our assumptions $\hat{B} > 0$ hence for each fixed value of \tilde{e}_1 we have

$$Cov(\tilde{i}, U'(\tilde{W}|\tilde{e}_1 = e)) > 0.$$

Hence, since $R(X + X^*) - w^*L^* > 0$ and $Cov(\tilde{i}, \tilde{e}_1) \leq 0$, we obtain that

$$Cov(\tilde{i}, U'(\tilde{W}_1)) > 0. \tag{28}$$

From equation (26) and (19) we obtain that

$$\frac{\partial R}{\partial K^*} = \frac{e_0 r^* [(1 + \bar{i})EU'(\tilde{W}_1) + Cov(\tilde{i}, U'(\tilde{W}_1))]}{e_f EU'(\tilde{W}_1)}$$

Using (28) we derive,

$$\frac{\partial R}{\partial K^*} > \frac{e_0 r^* (1 + \bar{i})}{e_f}. \tag{29}$$

Similarly from equations (27), (19) and assumption (A.2) we derive that

$$\frac{\partial R}{\partial K} > \frac{r(1 + \bar{i})}{e_f}. \tag{30}$$

Now let us note that the optimal K^*, K, L^*, L (when the interest rate is fixed at $i = \bar{i}$) is a solution to equations (4), (20), (21) and (22), while $\hat{K}^*, \hat{K}, \hat{L}^*$ and \hat{L} solve equations (25) and inequalities (29) and (30). Since R is strictly concave, $F(K, L)$ and $F^*(K^*, L^*)$ are concave in (K, L) and (K^*, L^*) respectively we see that R as a function of K, L, K^*, L^* is strictly concave. Moreover, we have assumed that $F_{KL} > 0$ and $F_{K^*L^*}^* > 0$ hence by assumption (A.2) it can be verified that:

$$\frac{\partial^2 R}{\partial K \partial L} > 0 \quad \text{and} \quad \frac{\partial^2 R}{\partial K^* \partial L^*} > 0$$

Now comparing equation (4), (20), (21), (22) with equation (25) and the inequalities (29) and (30) implies that $\hat{K}^* < K^*, \hat{K} < K, \hat{L} < L$ and $\hat{L}^* < L^*$. This clearly proves that the output in each country declines as a result of the randomness of \tilde{i} .||

Corollary. *Under the assumptions of Proposition 5 if $Cov(\tilde{i}, \tilde{e}_1) > 0$ then under some conditions all the inequalities in Proposition 5 might be reversed. Namely, when the correlation between \tilde{i} and \tilde{e}_1 is sufficiently high we may observe higher investments and higher production in both countries as the variance of \tilde{i} increases (keeping the mean constant).*

Proof. This corollary can be proved as in the case of Proposition 5. However under certain conditions, and assuming $Cov(\tilde{i}, \tilde{e}_1) > 0$, we obtain that $Cov(\tilde{i}, U'(\tilde{W}_1)) < 0$. This results in reversing the inequalities in (29) and (30) which implies, together with (25) that $\hat{K}^* > K^*, \hat{K} > K, \hat{L} > L, \hat{L}^* > L^*$.||

6 International Borrowing

Assume now that international borrowing is possible and let us examine the existence of the Separation property under three different circumstances. Let us denote by B^* the loan made by this firm at the importing country and by i^* the interest rate there.

1. *Certain interest rates without currency futures market.* Consider first the solution for the optimal production and borrowing at the home and foreign capital markets when no futures market exists. Rewriting the wealth levels for the case where the domestic interest rate is certain and $Z \equiv 0$ we obtain,

$$\begin{aligned} W_0 &= I - e_0 r^* K^* - rK + B + e_0 B^* \\ \tilde{W}_1 &= [R(X + X^*) - w^* L^* - (1 + i^*)B^*] \tilde{e}_1 - wL - (1 + i)B. \end{aligned}$$

Maximizing with respect to B^*, B, K, K^*, L^*, L the necessary and sufficient conditions for problem (1) are

$$e_0 U'(W_0) - \delta E(1 + i^*) U'(\tilde{W}_1) \tilde{e}_1 = 0. \quad (31)$$

and the equations (3)–(6).

Proposition 6. *In the absence of currency futures market (and certain interest rates) the Separation property holds if international borrowing is available.*

In many small countries currency futures markets do not exist. Therefore, allowing exporting firms in these countries to borrow in the foreign markets constitute a substitute for the missing currency futures. Thus providing them with a hedging device which reduces the exchange rate risk and promote international trade.

Proof. From equation (31) we obtain

$$E \tilde{e}_1 U'(\tilde{W}_1) = \frac{e_0 U'(W_0)}{\delta(1 + i^*)}. \quad (32)$$

From equation (3) and (32) we get

$$\frac{\partial R}{\partial K^*} = r^*(1 + i^*). \quad (33)$$

Furthermore equations (6), (5) and (32) yields equation (4) and

$$\frac{\partial R}{\partial K} = \frac{r(1 + i^*)}{e_0} \quad (34)$$

$$\frac{\partial R}{\partial L} = \frac{w(1 + i^*)}{e_0(1 + i)}. \quad (35)$$

Conditions (33)–(35) and (4) prove Proposition 6.||

This observation, that without futures markets the possibility to borrow on international capital markets (with nonrandom interest rates) leads to the Separation property, is interesting. As one can see such a financial instrument plays the role of "currency futures market" due to the fixed interest rate in the international market and the revenue are in foreign currency.

2. *Random interest rate without currency futures markets.* Assume that international borrowing is available and that \tilde{i} is a random variable. We take i^* to be nonrandom. Therefore, incomes in the two periods are defined as follows

$$\begin{aligned} W_0 &= I - e_0 r^* K^* - rK + B + e_0 B^* \\ \tilde{W}_1 &= [R(X + X^*) - w^* L^* - (1 + i^*) B^*] \tilde{e}_1 - wL - (1 + \tilde{i})B. \end{aligned}$$

In this case we obtain from (31)

$$E\tilde{e}_1 U'(\tilde{W}_1) = \frac{e_0 U'(W_0)}{\delta(1 + i^*)}. \quad (36)$$

However equation (2) now reduces to (24).

With these W_0 and \tilde{W}_1 equations (3), (4), (5) and (24) hold as well. Therefore we obtain

$$\frac{\partial R}{\partial L} = \frac{w\delta(1 + i^*)EU'(\tilde{W}_1)}{e_0 U'(W_0)},$$

which cannot be expressed independently of $EU'(\tilde{W}_1)$ since futures markets are missing (hence equation (19) is not available). Thus the Separation property does not hold in this case.

3. *Random domestic interest rate with futures.* Consider the above analysis when currency futures market exists.

Proposition 7. *Assume that the domestic interest rate is random and that currency futures market exists. If international borrowing is available (at nonrandom rate) the Separation property holds.*

Proof. Let us rewrite \tilde{W}_1 for this case,

$$\tilde{W}_1 = [R(X + X^*) - w^*L^* - (1 + i^*)B^*]\tilde{e}_1 - wL - (1 + \tilde{i})B + Z(e_f - \tilde{e}_1). \quad (37)$$

In this case equation (19) is added hence

$$e_f EU'(\tilde{W}_1) = E\tilde{e}_1 U'(\tilde{W}_1) = \frac{e_0 U'(W_0)}{\delta(1 + i^*)}. \quad (38)$$

Therefore,

$$\frac{\partial R}{\partial L} = \frac{w}{e_f}. \quad (39)$$

From (24), (27) and (37), we obtain

$$\frac{\partial R}{\partial K} = \frac{r(1 + i^*)}{e_0}. \quad (40)$$

Similarly from (26), (27) and (37) we obtain

$$\frac{\partial R}{\partial K^*} = r^*(1 + i^*). \quad (41)$$

Thus the Separation property holds in the case where \tilde{i} is random, given that currency futures market exists and international borrowing is an available option.||

Proposition 7 follows from the fact the international borrowing at a certain rate of interest creates in fact a hedging device against the risky domestic interest rate due to the existence of currency futures market.

7 Concluding Remarks

The two-period framework of this paper has given rise to some interesting insights when exchange rate, as well as interest rate in some cases, are stochastic. We observed that international borrowing guarantees the Separation property if currency futures markets are absent (and the interest

rate is nonrandom). Thus in small countries where currency futures are not available allowing firms to borrow abroad may be a useful tool to reduce exchange rate risks. As we have noted currency futures market does not guarantee the Separation result if the interest rate is random. However, this is not the case if we allow the firm to borrow at the importing country (where it produces part of its output as well).

How rigorous are these results? We believe that these observations hold in a much more general framework. Particularly one can show that the same results hold for exporting competitive firms producing in the home country only. We do not study the case where interest rates in both countries may be random or the effect of such source of uncertainty on the currency hedging behavior of the firm. However, this paper extends many of the results in the literature dealing with firms facing price uncertainty and optimal hedging in currency or commodity futures markets.

References

- BENNINGA, S., R. ELDOR and ZILCHA, I. (1985): "Optimal International Hedging and Output Policies in Commodity and Currency Forward Markets", *Journal of International Money and Finance* 4, pp. 537-552.
- BROLL, U. and I. ZILCHA (1991): "Exchange Rate Uncertainty, Futures Markets and the Multinational Firm", *European Economic Review* (forthcoming).
- CALDERON-ROSSELL, J.R. (1985): "Towards the Theory of Foreign Direct Investment", *Oxford Economic Papers* 37, pp. 282-291.
- CUSHMAN, D.O. (1988): "Exchange Rate Uncertainty and Foreign Direct Investment in the United States", *Weltwirtschaftsarchiv* 124, pp. 322-336.
- DANTHINE, J.P. (1978): "Information, Futures Markets, and Stabilizing Speculation", *Journal of Economic Theory* 17, pp. 79-98.

ELDOR, R. and I. ZILCHA (1986): "Discriminating Monopoly, Forward Markets and International Trade", *International Economic Review* 28, pp. 459–468.

FRANKE, G. (1991): "Exchange Rate Volatility and International Trading Strategy", *Journal of International Money and Finance* 10, pp. 292–307.

HELPMAN, E. (1984): "A Simple Theory of International Trade with Multinational Corporations", *Journal of Political Economy* 92, pp. 451–471.

ITAGAKI, T. (1991): "A Two-step Decision Model of the Multinational Enterprise under Foreign Demand Uncertainty", *Journal of International Economics* 30, pp. 185–190.

KRUGMAN, P.R. (1989): *Exchange-Rate Instability*. Cambridge, London: MIT-Press.

SHAPIRO, A.C. [1991]: *Foundations of Multinational Financial Management*, Boston, London, Toronto (Allyn and Bacon).

Addresses of authors: Dr. Udo Broll, University of Konstanz, Department of Economics, D-7750 Konstanz, Federal Republic of Germany; Prof. Dr. Itzhak Zilcha, Tel Aviv University, Department of Economics, Ramat-Aviv, 69978 Tel Aviv, Israel .